Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.2-Cosine/208-4.2.4.1

Nasser M. Abbasi

May 18, 2024 Compiled on May 18, 2024 at 8:15am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**393**]. This is test number [208].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.75 (392)	0.25 (1)
Mathematica	98.98 (389)	1.02 (4)
Maple	60.56 (238)	39.44 (155)
Fricas	60.56 (238)	39.44 (155)
Maxima	30.28 (119)	69.72 (274)
Reduce	29.77 (117)	70.23 (276)
Mupad	19.08 (75)	80.92 (318)
Giac	18.07 (71)	81.93 (322)
Sympy	3.82 (15)	96.18 (378)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

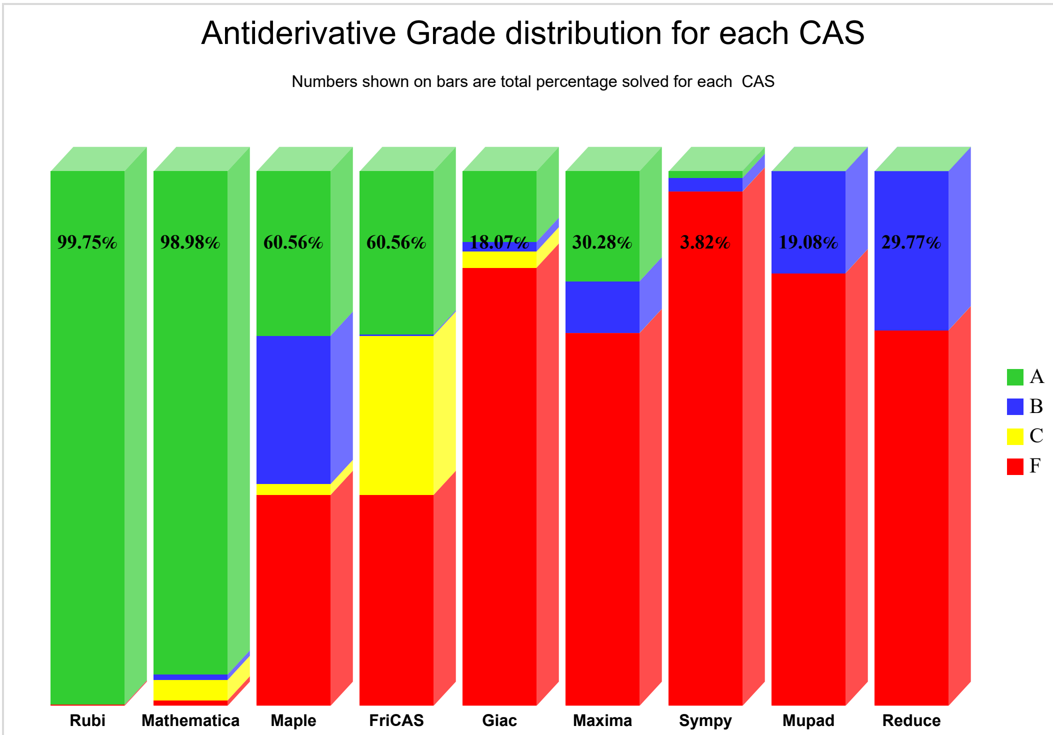
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

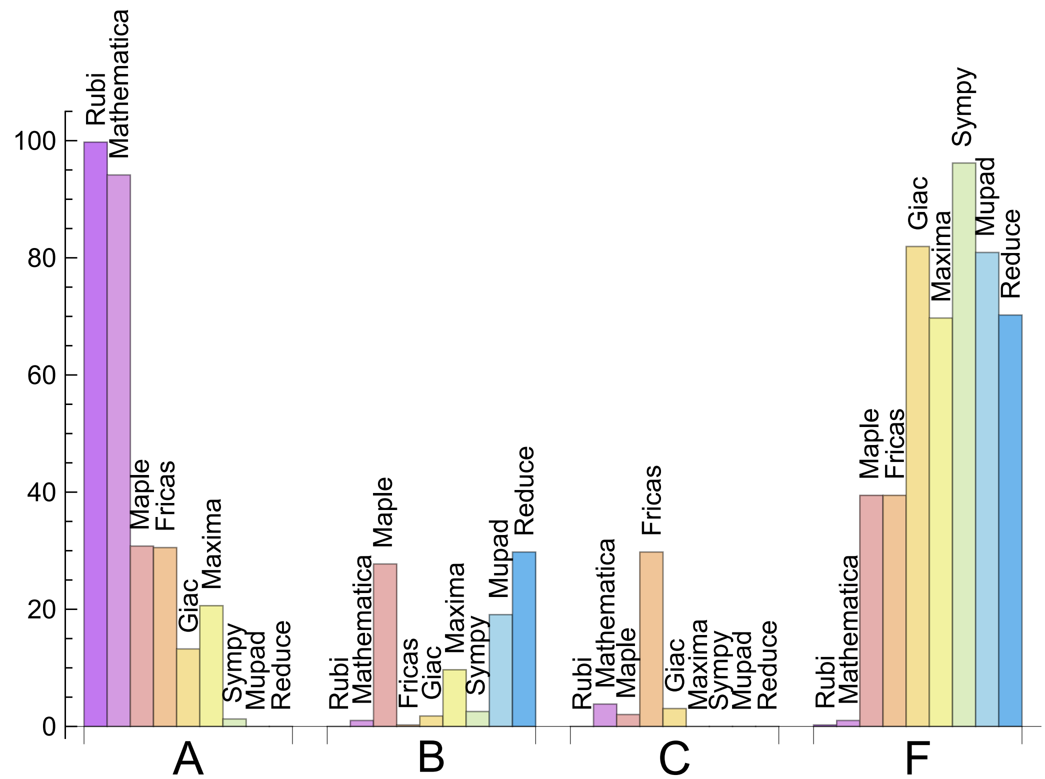
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.746	0.000	0.000	0.254
Mathematica	94.148	1.018	3.817	1.018
Maple	30.789	27.735	2.036	39.440
Fricas	30.534	0.254	29.771	39.440
Maxima	20.611	9.669	0.000	69.720
Giac	13.232	1.781	3.053	81.934
Sympy	1.272	2.545	0.000	96.183
Mupad	0.000	19.084	0.000	80.916
Reduce	0.000	29.771	0.000	70.229

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	155	100.00	0.00	0.00
Maple	155	100.00	0.00	0.00
Maxima	274	100.00	0.00	0.00
Reduce	276	100.00	0.00	0.00
Mupad	318	0.00	100.00	0.00
Giac	322	85.09	0.00	14.91
Sympy	378	21.69	78.31	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.11
Reduce	0.18
Maxima	0.28
Giac	0.34
Rubi	0.50
Mathematica	0.97
Maple	4.42
Sympy	10.58
Mupad	25.98

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	94.19	1.00	84.00	0.86
Reduce	112.75	1.04	66.00	0.65
Rubi	124.56	0.93	115.00	1.01
Giac	136.55	2.09	94.00	1.11
Fricas	172.12	1.52	170.00	1.35
Sympy	191.47	2.94	184.00	2.10
Mathematica	213.17	1.21	91.00	0.76
Maple	233.94	1.99	192.00	1.53
Maxima	473.18	3.74	107.00	1.18

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

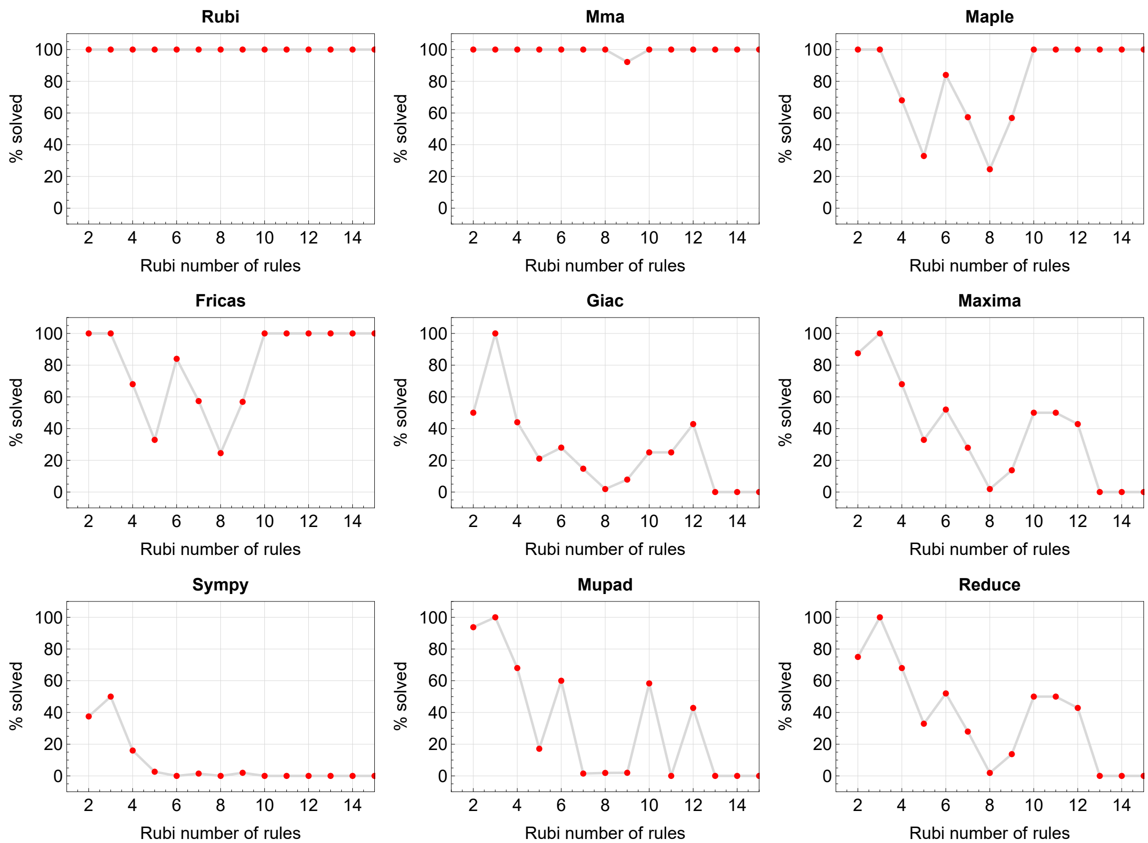


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

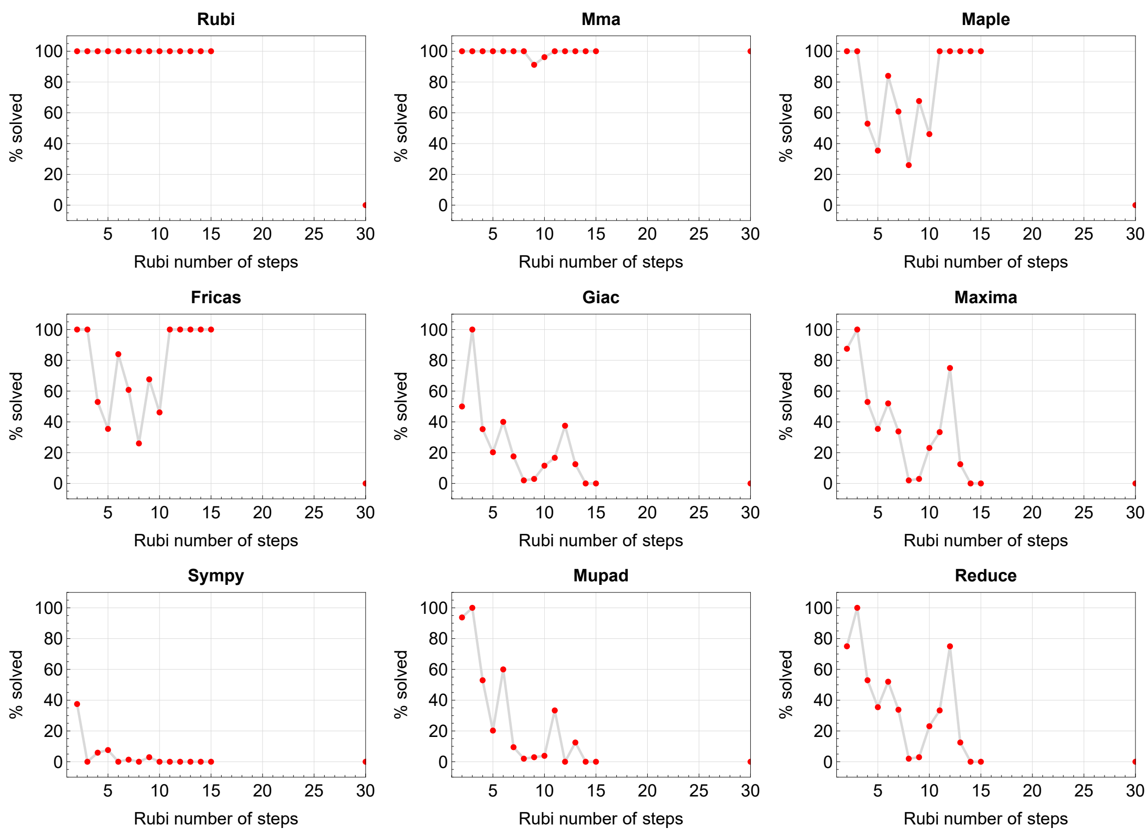


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

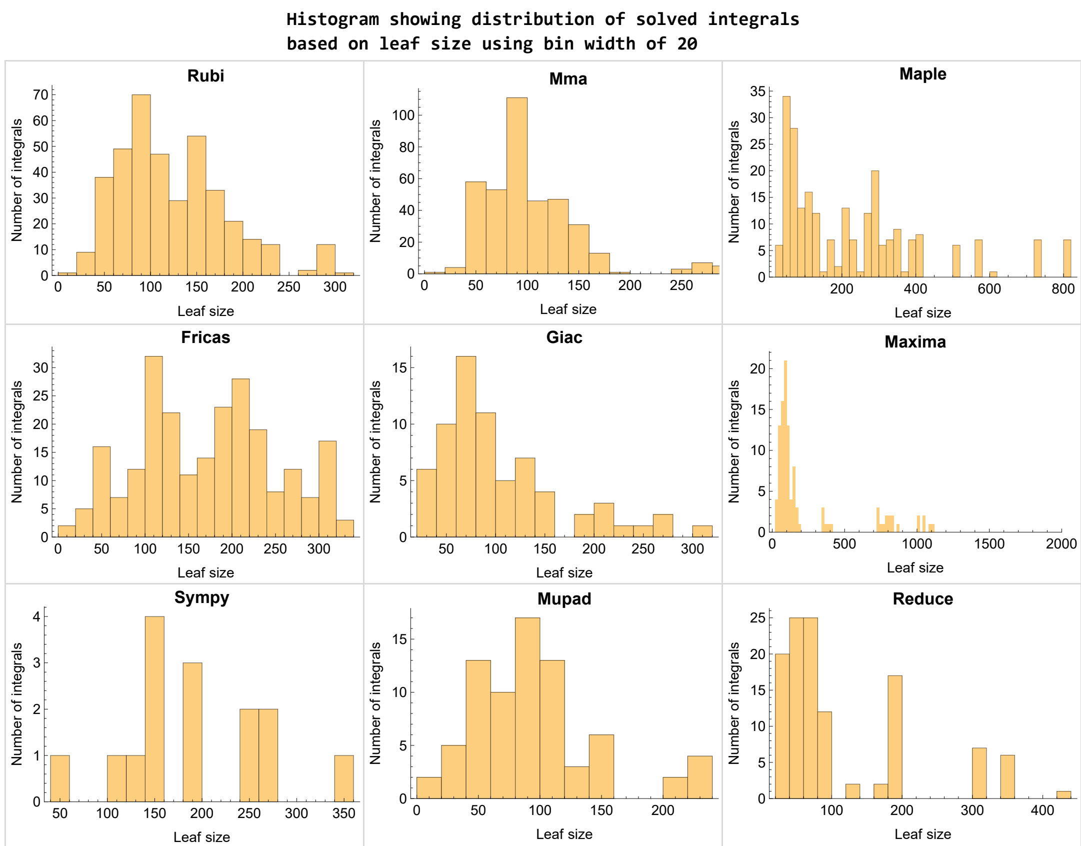


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

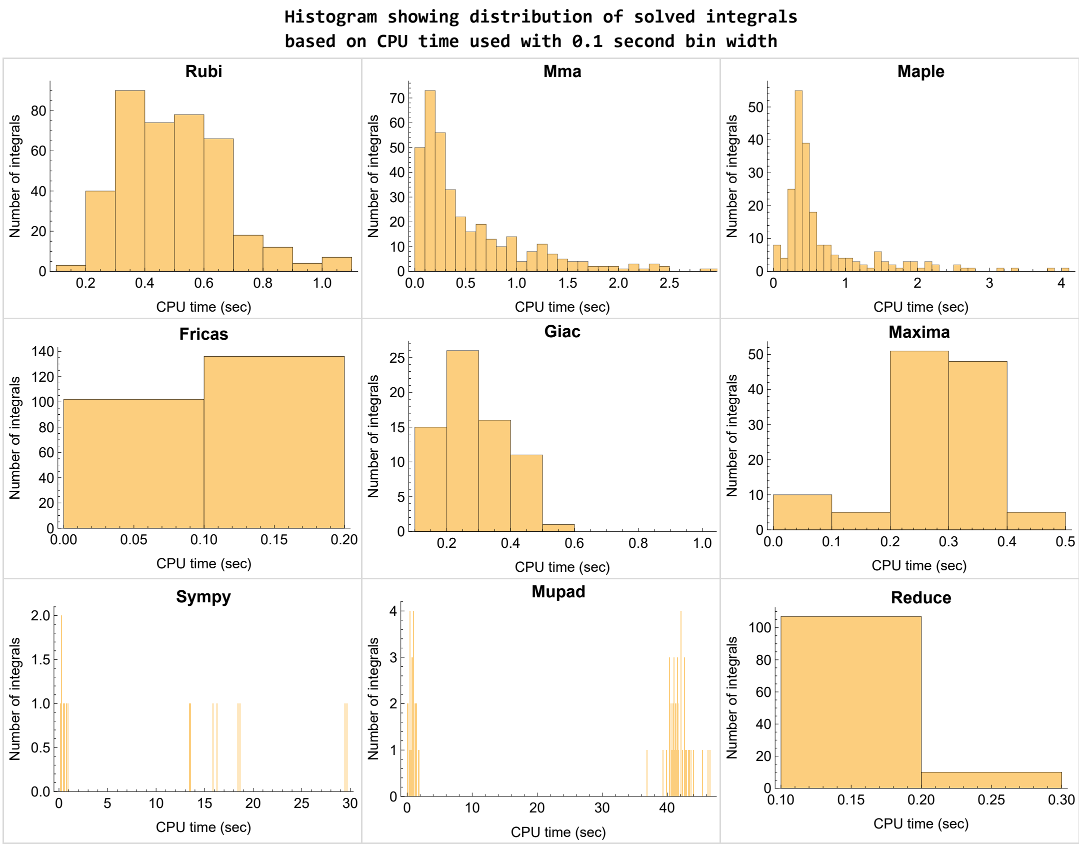


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

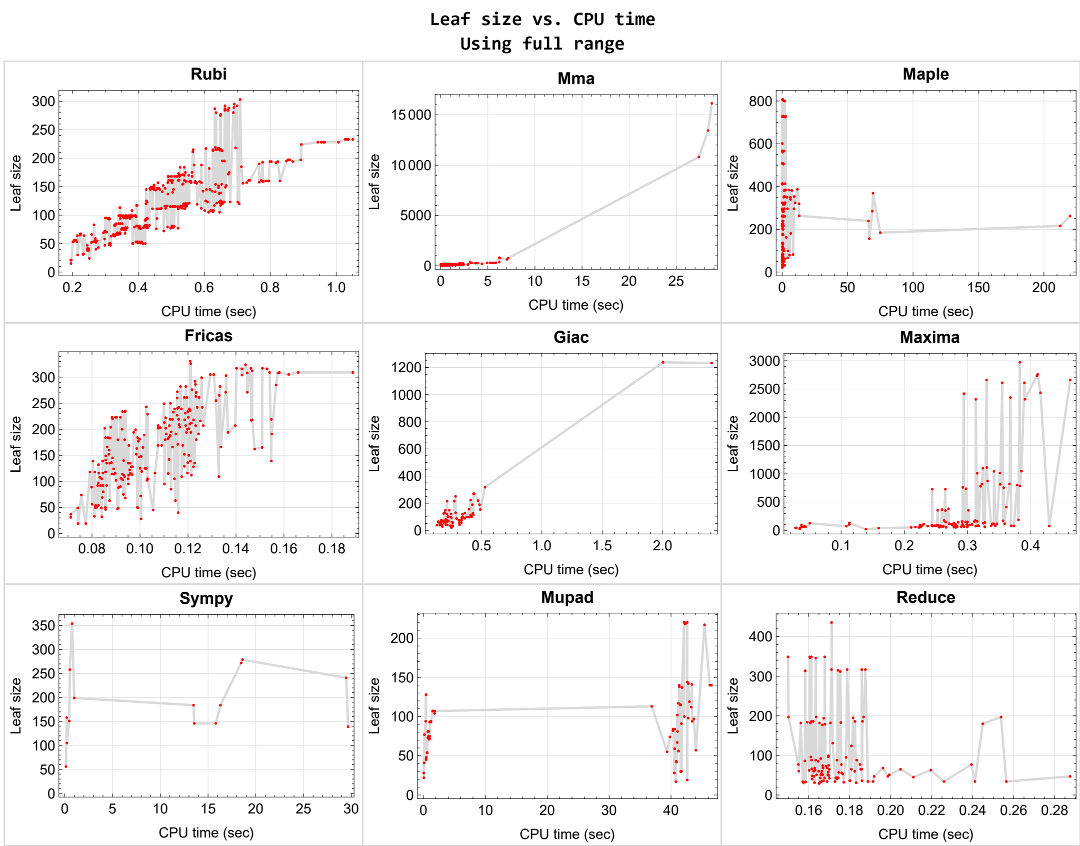


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {66, 67, 68, 76, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 232, 233, 234, 235, 236, 237, 267, 268, 276, 283, 383, 388, 389, 390, 391, 393}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

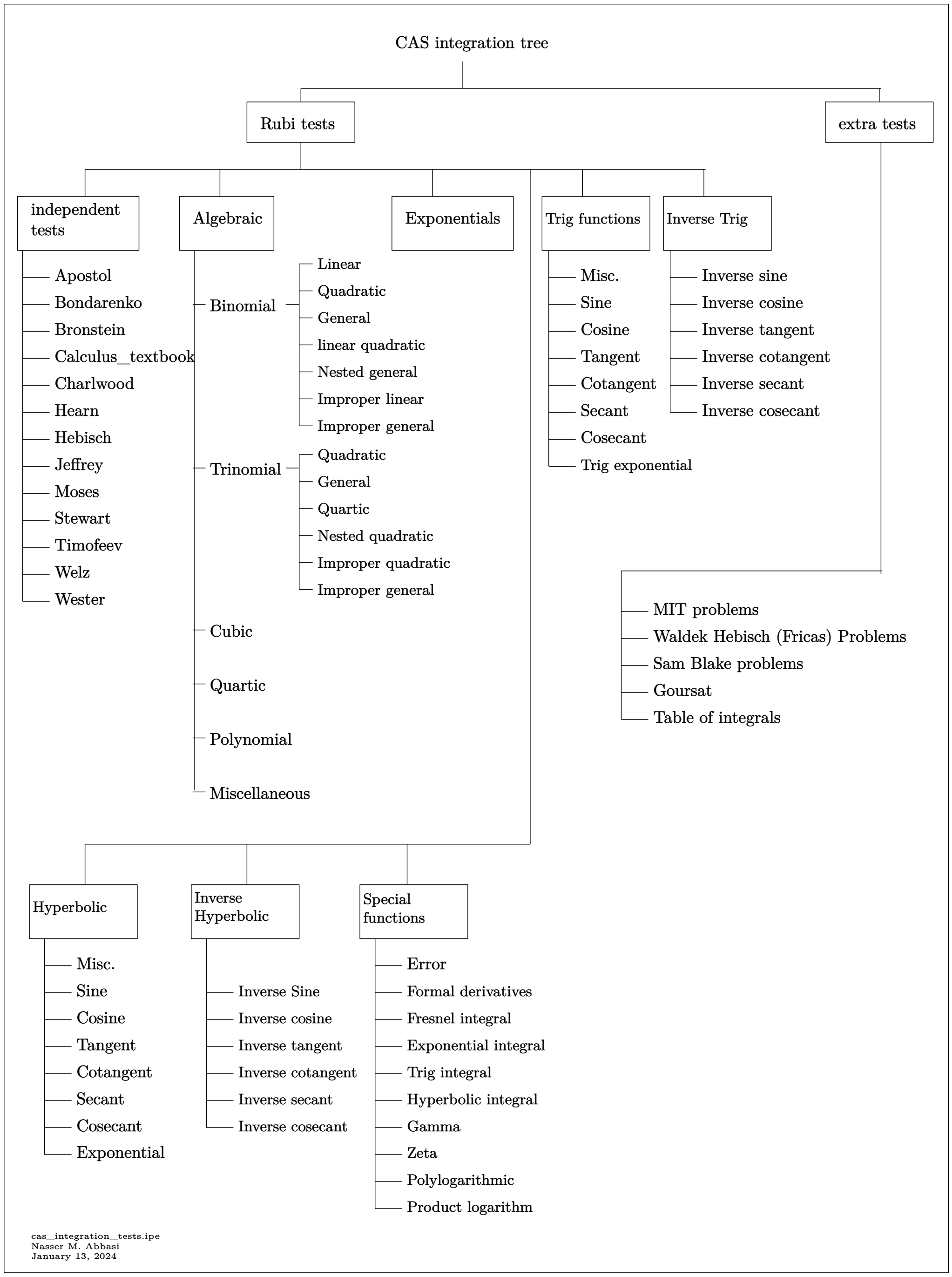
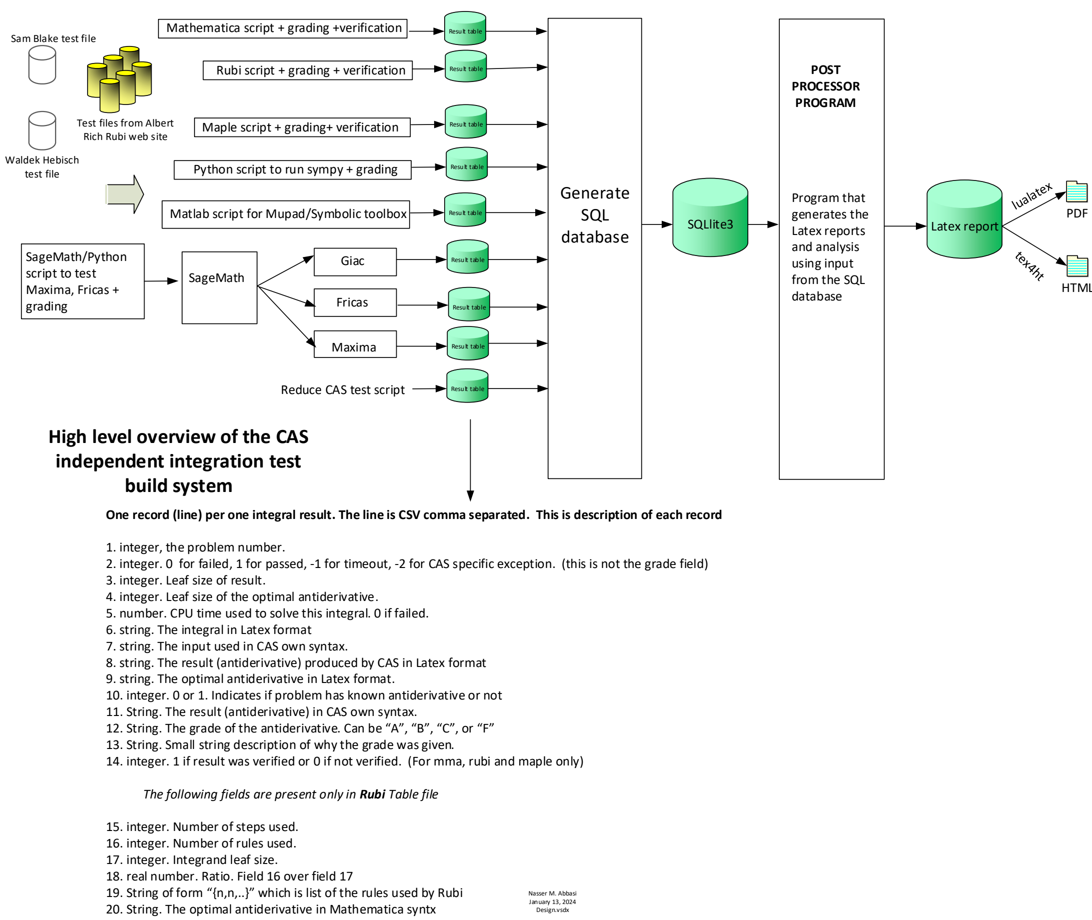


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

B grade { }

C grade { }

F normal fail { 368 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 201, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 388, 389, 390, 391 }

B grade { 200, 208, 233, 393 }

C grade { 35, 36, 66, 67, 68, 76, 198, 232, 267, 268, 276, 283, 383, 384, 386 }

F normal fail { 202, 385, 387, 392 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 35, 36, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 271, 279, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338 }

B grade { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287 }

C grade { 26, 27, 28, 29, 30, 31, 32, 33 }

F normal fail { 34, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 24, 25, 35, 36, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338 }

B grade { 12 }

C grade { 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287 }

F normal fail { 34, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 132, 133, 134, 135, 136, 288, 289, 290, 291, 292, 293, 297, 298, 299, 300, 301, 302, 306, 307, 308, 309, 310, 311, 315, 316, 317, 318, 319, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335 }

B grade { 35, 36, 95, 96, 97, 104, 105, 106, 113, 114, 115, 121, 122, 123, 129, 130, 131, 137, 138, 139, 294, 295, 296, 303, 304, 305, 312, 313, 314, 320, 321, 322, 328, 329, 330, 336, 337, 338 }

C grade { }

F normal fail { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 89, 90, 91, 92, 93, 95, 96, 98, 99, 100, 101, 102, 104, 105, 107, 108, 111, 113, 288, 289, 290, 291, 294, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 312, 313, 314 }

B grade { 35, 36, 97, 106, 109, 114, 115 }

C grade { 94, 103, 110, 112, 292, 293, 301, 302, 308, 309, 310, 311 }

F normal fail { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

F(-1) timedout fail { }

F(-2) exception fail { 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19, 25, 35, 36, 65, 89, 90, 91, 92, 94, 96, 98, 99, 100, 101, 103, 105, 107, 108, 109, 110, 112, 114, 116, 117, 118, 120, 122, 124, 125, 126, 128, 130, 132, 133, 134, 136, 138, 266, 288, 289, 290, 291, 297, 298, 299, 300, 306, 307, 308, 309, 315, 316, 317, 323, 324, 325, 331, 332, 333 }

C grade { }

F normal fail { }

F(-1) timedout fail { 16, 17, 18, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 93, 95, 97, 102, 104, 106, 111, 113, 115, 119, 121, 123, 127, 129, 131, 135, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 292, 293, 294, 295, 296, 301, 302, 303, 304, 305, 310, 311, 312, 313, 314, 318, 319, 320, 321, 322, 326, 327, 328, 329, 330, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

F(-2) exception fail { }

Sympy

A grade { 92, 118, 290, 291, 317 }

B grade { 1, 2, 3, 4, 9, 10, 11, 35, 36, 91 }

C grade { }

F normal fail { 5, 6, 7, 12, 13, 30, 31, 32, 33, 34, 40, 66, 67, 76, 77, 93, 119, 120, 128, 161, 162, 167, 168, 178, 179, 180, 181, 182, 185, 186, 193, 194, 198, 200, 201, 202, 204, 205, 206, 209, 210, 213, 214, 215, 216, 219, 220, 227, 228, 229, 232, 235, 236, 237, 238, 242, 267, 268, 276, 277, 292, 318, 319, 327, 354, 355, 365, 366, 367, 368, 369, 372, 373, 379, 380, 383, 385, 386, 387, 389, 390, 391 }

F(-1) timedout fail { 8, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 169, 170, 171, 172, 173, 174, 175, 176, 177, 183, 184, 187, 188, 189, 190, 191, 192, 195, 196, 197, 199, 203, 207, 208, 211, 212, 217, 218, 221, 222, 223, 224, 225, 226, 230, 231, 233, 234, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 269, 270, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 356, 357, 358, 359, 360, 361, 362, 363, 364, 370, 371, 374, 375, 376, 377, 378, 381, 382, 384, 388, 392, 393 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338 }

C grade { }

F normal fail { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	83	133	82	75	80	199	93	95	74
N.S.	1	0.90	1.45	0.89	0.82	0.87	2.16	1.01	1.03	0.80
time (sec)	N/A	0.266	0.062	8.384	0.042	0.085	0.990	0.250	0.178	39.822

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	66	101	66	60	63	151	76	73	59
N.S.	1	0.92	1.40	0.92	0.83	0.88	2.10	1.06	1.01	0.82
time (sec)	N/A	0.259	0.040	4.259	0.040	0.087	0.469	0.321	0.168	40.597

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	47	71	49	43	45	105	57	51	43
N.S.	1	0.94	1.42	0.98	0.86	0.90	2.10	1.14	1.02	0.86
time (sec)	N/A	0.245	0.028	1.477	0.029	0.105	0.217	0.186	0.164	40.830

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	50	31	34	28	56	34	29	28
N.S.	1	1.00	1.67	1.03	1.13	0.93	1.87	1.13	0.97	0.93
time (sec)	N/A	0.216	0.047	0.429	0.035	0.100	0.119	0.242	0.165	0.046

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	30	38	40	0	40	42	22
N.S.	1	1.00	1.46	1.25	1.58	1.67	0.00	1.67	1.75	0.92
time (sec)	N/A	0.251	0.048	0.237	0.159	0.116	0.000	0.218	0.170	0.059

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	55	58	72	0	60	177	41
N.S.	1	1.00	1.20	1.38	1.45	1.80	0.00	1.50	4.42	1.02
time (sec)	N/A	0.271	0.041	0.332	0.037	0.100	0.000	0.254	0.167	0.107

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	68	93	85	97	95	0	98	312	77
N.S.	1	0.97	1.33	1.21	1.39	1.36	0.00	1.40	4.46	1.10
time (sec)	N/A	0.360	0.028	0.507	0.037	0.110	0.000	0.245	0.175	0.151

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	94	137	108	126	114	0	121	436	102
N.S.	1	0.96	1.40	1.10	1.29	1.16	0.00	1.23	4.45	1.04
time (sec)	N/A	0.457	0.041	0.770	0.051	0.093	0.000	0.214	0.171	41.023

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	111	93	79	130	85	354	87	131	119
N.S.	1	0.95	0.79	0.68	1.11	0.73	3.03	0.74	1.12	1.02
time (sec)	N/A	0.443	0.335	5.707	0.113	0.113	0.765	0.293	0.172	42.974

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	85	68	61	103	68	258	68	97	91
N.S.	1	0.96	0.76	0.69	1.16	0.76	2.90	0.76	1.09	1.02
time (sec)	N/A	0.345	0.202	2.205	0.113	0.094	0.527	0.226	0.170	41.676

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	59	45	44	73	49	158	43	63	67
N.S.	1	0.97	0.74	0.72	1.20	0.80	2.59	0.70	1.03	1.10
time (sec)	N/A	0.270	0.186	0.717	0.108	0.074	0.201	0.204	0.163	41.006

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	21	20	31	0	20	31	17
N.S.	1	1.00	1.00	1.40	1.33	2.07	0.00	1.33	2.07	1.13
time (sec)	N/A	0.196	0.015	0.227	0.139	0.071	0.000	0.252	0.183	40.781

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	35	27	37	0	34	60	28
N.S.	1	1.00	0.84	0.81	0.63	0.86	0.00	0.79	1.40	0.65
time (sec)	N/A	0.279	0.116	0.267	0.034	0.071	0.000	0.262	0.162	40.544

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	58	61	58	43	56	0	57	92	42
N.S.	1	0.89	0.94	0.89	0.66	0.86	0.00	0.88	1.42	0.65
time (sec)	N/A	0.298	0.252	0.374	0.028	0.080	0.000	0.263	0.166	40.886

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	70	81	78	60	74	0	79	124	56
N.S.	1	0.80	0.93	0.90	0.69	0.85	0.00	0.91	1.43	0.64
time (sec)	N/A	0.304	0.363	0.435	0.044	0.076	0.000	0.203	0.181	41.210

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	111	88	324	0	128	0	0	47	0
N.S.	1	0.98	0.78	2.87	0.00	1.13	0.00	0.00	0.42	0.00
time (sec)	N/A	0.469	0.930	9.923	0.000	0.097	0.000	0.000	0.173	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	111	86	296	0	112	0	0	43	0
N.S.	1	0.98	0.76	2.62	0.00	0.99	0.00	0.00	0.38	0.00
time (sec)	N/A	0.463	0.772	4.099	0.000	0.096	0.000	0.000	0.183	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	70	261	0	102	0	0	35	0
N.S.	1	1.00	0.91	3.39	0.00	1.32	0.00	0.00	0.45	0.00
time (sec)	N/A	0.355	0.480	2.089	0.000	0.103	0.000	0.000	0.187	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	58	225	0	92	0	0	45	94
N.S.	1	1.00	0.77	3.00	0.00	1.23	0.00	0.00	0.60	1.25
time (sec)	N/A	0.349	0.399	0.714	0.000	0.086	0.000	0.000	0.171	0.350

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	118	0	0	38	0
N.S.	1	1.00	0.77	2.92	0.00	1.59	0.00	0.00	0.51	0.00
time (sec)	N/A	0.358	0.462	1.063	0.000	0.086	0.000	0.000	0.175	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	116	0	0	47	0
N.S.	1	1.00	0.74	3.77	0.00	1.49	0.00	0.00	0.60	0.00
time (sec)	N/A	0.366	0.551	0.951	0.000	0.090	0.000	0.000	0.188	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	112	81	566	0	139	0	0	47	0
N.S.	1	0.97	0.70	4.92	0.00	1.21	0.00	0.00	0.41	0.00
time (sec)	N/A	0.483	0.720	1.472	0.000	0.121	0.000	0.000	0.173	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	116	77	413	0	135	0	0	47	0
N.S.	1	1.01	0.67	3.59	0.00	1.17	0.00	0.00	0.41	0.00
time (sec)	N/A	0.488	0.836	2.125	0.000	0.095	0.000	0.000	0.171	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	99	0	19	0	0	32	0
N.S.	1	1.00	1.10	4.71	0.00	0.90	0.00	0.00	1.52	0.00
time (sec)	N/A	0.195	0.451	4.480	0.000	0.077	0.000	0.000	0.164	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	99	0	19	0	0	37	19
N.S.	1	1.00	1.00	4.71	0.00	0.90	0.00	0.00	1.76	0.90
time (sec)	N/A	0.197	0.225	1.405	0.000	0.074	0.000	0.000	0.180	42.636

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	112	78	184	0	142	0	0	55	0
N.S.	1	0.97	0.68	1.60	0.00	1.23	0.00	0.00	0.48	0.00
time (sec)	N/A	0.591	2.339	75.010	0.000	0.112	0.000	0.000	0.244	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	108	79	369	0	147	0	0	55	0
N.S.	1	0.94	0.69	3.21	0.00	1.28	0.00	0.00	0.48	0.00
time (sec)	N/A	0.601	1.905	69.420	0.000	0.100	0.000	0.000	0.241	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	76	58	156	0	112	0	0	55	0
N.S.	1	0.97	0.74	2.00	0.00	1.44	0.00	0.00	0.71	0.00
time (sec)	N/A	0.455	1.612	66.641	0.000	0.095	0.000	0.000	0.256	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	72	55	350	0	95	0	0	49	0
N.S.	1	0.97	0.74	4.73	0.00	1.28	0.00	0.00	0.66	0.00
time (sec)	N/A	0.477	1.557	3.832	0.000	0.095	0.000	0.000	0.226	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	79	58	162	0	97	0	0	35	0
N.S.	1	1.05	0.77	2.16	0.00	1.29	0.00	0.00	0.47	0.00
time (sec)	N/A	0.489	1.237	3.182	0.000	0.083	0.000	0.000	0.187	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	81	61	340	0	108	0	0	55	0
N.S.	1	1.05	0.79	4.42	0.00	1.40	0.00	0.00	0.71	0.00
time (sec)	N/A	0.502	0.944	6.404	0.000	0.083	0.000	0.000	0.208	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	117	79	181	0	119	0	0	55	0
N.S.	1	1.02	0.69	1.57	0.00	1.03	0.00	0.00	0.48	0.00
time (sec)	N/A	0.650	1.545	6.622	0.000	0.101	0.000	0.000	0.208	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	117	81	387	0	129	0	0	55	0
N.S.	1	1.02	0.70	3.37	0.00	1.12	0.00	0.00	0.48	0.00
time (sec)	N/A	0.631	1.692	11.717	0.000	0.119	0.000	0.000	0.201	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	114	0	0	0	0	0	38	0
N.S.	1	0.97	0.97	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.344	0.217	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	113	31	175	33	279	1238	78	30
N.S.	1	1.00	3.65	1.00	5.65	1.06	9.00	39.94	2.52	0.97
time (sec)	N/A	0.232	0.231	1.990	0.312	0.081	18.604	2.003	0.190	41.695

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	114	31	175	32	272	1233	76	30
N.S.	1	1.00	3.56	0.97	5.47	1.00	8.50	38.53	2.38	0.94
time (sec)	N/A	0.235	0.245	1.450	0.263	0.084	18.464	2.406	0.189	41.557

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	115	88	322	0	122	0	0	44	0
N.S.	1	1.03	0.79	2.88	0.00	1.09	0.00	0.00	0.39	0.00
time (sec)	N/A	0.544	0.688	1.875	0.000	0.096	0.000	0.000	0.177	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	115	89	294	0	106	0	0	42	0
N.S.	1	1.05	0.81	2.67	0.00	0.96	0.00	0.00	0.38	0.00
time (sec)	N/A	0.513	0.337	1.582	0.000	0.092	0.000	0.000	0.197	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	70	261	0	102	0	0	35	0
N.S.	1	1.00	0.91	3.39	0.00	1.32	0.00	0.00	0.45	0.00
time (sec)	N/A	0.351	0.102	0.000	0.000	0.090	0.000	0.000	0.182	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	77	59	237	0	89	0	0	48	0
N.S.	1	1.05	0.81	3.25	0.00	1.22	0.00	0.00	0.66	0.00
time (sec)	N/A	0.402	0.110	0.582	0.000	0.080	0.000	0.000	0.204	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	78	55	214	0	115	0	0	52	0
N.S.	1	1.13	0.80	3.10	0.00	1.67	0.00	0.00	0.75	0.00
time (sec)	N/A	0.415	0.952	0.415	0.000	0.112	0.000	0.000	0.208	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	82	56	292	0	113	0	0	52	0
N.S.	1	1.08	0.74	3.84	0.00	1.49	0.00	0.00	0.68	0.00
time (sec)	N/A	0.425	0.620	0.336	0.000	0.097	0.000	0.000	0.229	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	116	84	561	0	136	0	0	52	0
N.S.	1	1.05	0.76	5.10	0.00	1.24	0.00	0.00	0.47	0.00
time (sec)	N/A	0.538	0.890	0.348	0.000	0.088	0.000	0.000	0.271	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	120	83	411	0	132	0	0	52	0
N.S.	1	1.06	0.73	3.64	0.00	1.17	0.00	0.00	0.46	0.00
time (sec)	N/A	0.541	1.110	0.352	0.000	0.083	0.000	0.000	0.297	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	115	91	324	0	124	0	0	45	0
N.S.	1	1.05	0.83	2.95	0.00	1.13	0.00	0.00	0.41	0.00
time (sec)	N/A	0.497	0.374	2.190	0.000	0.089	0.000	0.000	0.286	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	111	86	296	0	112	0	0	43	0
N.S.	1	0.98	0.76	2.62	0.00	0.99	0.00	0.00	0.38	0.00
time (sec)	N/A	0.462	0.384	0.002	0.000	0.121	0.000	0.000	0.183	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	79	71	263	0	103	0	0	55	0
N.S.	1	1.05	0.95	3.51	0.00	1.37	0.00	0.00	0.73	0.00
time (sec)	N/A	0.423	0.094	0.867	0.000	0.093	0.000	0.000	0.213	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	79	61	239	0	88	0	0	59	0
N.S.	1	1.04	0.80	3.14	0.00	1.16	0.00	0.00	0.78	0.00
time (sec)	N/A	0.418	0.136	0.681	0.000	0.091	0.000	0.000	0.239	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	78	57	216	0	114	0	0	59	0
N.S.	1	1.08	0.79	3.00	0.00	1.58	0.00	0.00	0.82	0.00
time (sec)	N/A	0.427	0.765	0.340	0.000	0.088	0.000	0.000	0.235	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	82	58	294	0	112	0	0	59	0
N.S.	1	1.05	0.74	3.77	0.00	1.44	0.00	0.00	0.76	0.00
time (sec)	N/A	0.434	0.661	0.388	0.000	0.093	0.000	0.000	0.258	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	116	84	564	0	141	0	0	59	0
N.S.	1	1.03	0.74	4.99	0.00	1.25	0.00	0.00	0.52	0.00
time (sec)	N/A	0.533	0.892	0.402	0.000	0.086	0.000	0.000	0.250	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	120	83	413	0	136	0	0	59	0
N.S.	1	1.04	0.72	3.59	0.00	1.18	0.00	0.00	0.51	0.00
time (sec)	N/A	0.544	1.265	0.405	0.000	0.120	0.000	0.000	0.241	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	111	88	324	0	128	0	0	47	0
N.S.	1	0.98	0.78	2.87	0.00	1.13	0.00	0.00	0.42	0.00
time (sec)	N/A	0.456	0.389	0.000	0.000	0.122	0.000	0.000	0.177	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	113	87	296	0	116	0	0	59	0
N.S.	1	1.01	0.78	2.64	0.00	1.04	0.00	0.00	0.53	0.00
time (sec)	N/A	0.522	0.369	9.067	0.000	0.096	0.000	0.000	0.223	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	81	73	263	0	105	0	0	63	0
N.S.	1	1.04	0.94	3.37	0.00	1.35	0.00	0.00	0.81	0.00
time (sec)	N/A	0.422	0.101	13.043	0.000	0.103	0.000	0.000	0.236	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	79	65	239	0	90	0	0	63	0
N.S.	1	1.01	0.83	3.06	0.00	1.15	0.00	0.00	0.81	0.00
time (sec)	N/A	0.421	0.763	65.963	0.000	0.092	0.000	0.000	0.244	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	78	57	216	0	116	0	0	63	0
N.S.	1	1.05	0.77	2.92	0.00	1.57	0.00	0.00	0.85	0.00
time (sec)	N/A	0.429	0.572	212.383	0.000	0.120	0.000	0.000	0.233	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	82	58	294	0	114	0	0	63	0
N.S.	1	1.05	0.74	3.77	0.00	1.46	0.00	0.00	0.81	0.00
time (sec)	N/A	0.425	0.801	0.225	0.000	0.089	0.000	0.000	0.237	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	116	80	601	0	145	0	0	63	0
N.S.	1	1.01	0.70	5.23	0.00	1.26	0.00	0.00	0.55	0.00
time (sec)	N/A	0.540	1.294	0.170	0.000	0.089	0.000	0.000	0.246	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	120	83	413	0	140	0	0	63	0
N.S.	1	1.04	0.72	3.59	0.00	1.22	0.00	0.00	0.55	0.00
time (sec)	N/A	0.549	1.845	0.173	0.000	0.093	0.000	0.000	0.233	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	149	94	349	0	128	0	0	47	0
N.S.	1	1.01	0.64	2.37	0.00	0.87	0.00	0.00	0.32	0.00
time (sec)	N/A	0.664	1.289	6.289	0.000	0.096	0.000	0.000	0.184	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	83	321	0	125	0	0	47	0
N.S.	1	1.00	0.72	2.79	0.00	1.09	0.00	0.00	0.41	0.00
time (sec)	N/A	0.498	1.224	1.425	0.000	0.102	0.000	0.000	0.182	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	115	77	293	0	109	0	0	45	0
N.S.	1	1.03	0.69	2.62	0.00	0.97	0.00	0.00	0.40	0.00
time (sec)	N/A	0.505	0.859	0.967	0.000	0.094	0.000	0.000	0.178	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	81	73	260	0	105	0	0	38	0
N.S.	1	1.01	0.91	3.25	0.00	1.31	0.00	0.00	0.48	0.00
time (sec)	N/A	0.395	0.126	0.645	0.000	0.084	0.000	0.000	0.176	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	58	225	0	92	0	0	45	94
N.S.	1	1.00	0.77	3.00	0.00	1.23	0.00	0.00	0.60	1.25
time (sec)	N/A	0.355	0.125	0.000	0.000	0.084	0.000	0.000	0.176	43.433

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	76	198	213	0	118	0	0	57	0
N.S.	1	1.07	2.79	3.00	0.00	1.66	0.00	0.00	0.80	0.00
time (sec)	N/A	0.408	4.382	0.278	0.000	0.079	0.000	0.000	0.205	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	82	141	291	0	116	0	0	61	0
N.S.	1	1.12	1.93	3.99	0.00	1.59	0.00	0.00	0.84	0.00
time (sec)	N/A	0.421	2.422	0.306	0.000	0.085	0.000	0.000	0.205	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	116	631	563	0	139	0	0	61	0
N.S.	1	1.04	5.63	5.03	0.00	1.24	0.00	0.00	0.54	0.00
time (sec)	N/A	0.529	7.043	0.355	0.000	0.155	0.000	0.000	0.206	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	120	74	412	0	135	0	0	61	0
N.S.	1	1.09	0.67	3.75	0.00	1.23	0.00	0.00	0.55	0.00
time (sec)	N/A	0.547	0.951	0.341	0.000	0.123	0.000	0.000	0.208	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	152	97	729	0	160	0	0	61	0
N.S.	1	1.03	0.66	4.96	0.00	1.09	0.00	0.00	0.41	0.00
time (sec)	N/A	0.666	1.494	2.704	0.000	0.087	0.000	0.000	0.229	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	86	324	0	125	0	0	47	0
N.S.	1	1.00	0.75	2.82	0.00	1.09	0.00	0.00	0.41	0.00
time (sec)	N/A	0.510	1.329	2.179	0.000	0.100	0.000	0.000	0.260	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	80	296	0	109	0	0	45	0
N.S.	1	1.00	0.70	2.57	0.00	0.95	0.00	0.00	0.39	0.00
time (sec)	N/A	0.520	1.122	1.606	0.000	0.117	0.000	0.000	0.254	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	81	69	263	0	105	0	0	38	0
N.S.	1	1.01	0.86	3.29	0.00	1.31	0.00	0.00	0.48	0.00
time (sec)	N/A	0.403	0.932	0.635	0.000	0.114	0.000	0.000	0.269	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	79	61	239	0	92	0	0	45	0
N.S.	1	1.01	0.78	3.06	0.00	1.18	0.00	0.00	0.58	0.00
time (sec)	N/A	0.402	0.129	0.486	0.000	0.085	0.000	0.000	0.178	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	118	0	0	38	0
N.S.	1	1.00	0.77	2.92	0.00	1.59	0.00	0.00	0.51	0.00
time (sec)	N/A	0.362	0.128	0.000	0.000	0.081	0.000	0.000	0.177	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	80	140	294	0	116	0	0	51	0
N.S.	1	1.07	1.87	3.92	0.00	1.55	0.00	0.00	0.68	0.00
time (sec)	N/A	0.418	1.582	0.300	0.000	0.106	0.000	0.000	0.175	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	116	81	566	0	139	0	0	55	0
N.S.	1	1.03	0.72	5.01	0.00	1.23	0.00	0.00	0.49	0.00
time (sec)	N/A	0.547	0.316	0.376	0.000	0.084	0.000	0.000	0.211	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	120	77	413	0	135	0	0	55	0
N.S.	1	1.07	0.69	3.69	0.00	1.21	0.00	0.00	0.49	0.00
time (sec)	N/A	0.537	0.934	0.409	0.000	0.086	0.000	0.000	0.179	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	86	324	0	125	0	0	47	0
N.S.	1	1.00	0.75	2.82	0.00	1.09	0.00	0.00	0.41	0.00
time (sec)	N/A	0.507	1.508	2.256	0.000	0.103	0.000	0.000	0.209	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	80	296	0	109	0	0	45	0
N.S.	1	1.00	0.70	2.57	0.00	0.95	0.00	0.00	0.39	0.00
time (sec)	N/A	0.505	1.312	1.661	0.000	0.133	0.000	0.000	0.191	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	81	69	263	0	105	0	0	38	0
N.S.	1	1.01	0.86	3.29	0.00	1.31	0.00	0.00	0.48	0.00
time (sec)	N/A	0.396	1.062	0.799	0.000	0.095	0.000	0.000	0.180	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	79	61	239	0	92	0	0	45	0
N.S.	1	1.01	0.78	3.06	0.00	1.18	0.00	0.00	0.58	0.00
time (sec)	N/A	0.389	0.123	0.451	0.000	0.082	0.000	0.000	0.197	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	78	57	216	0	118	0	0	38	0
N.S.	1	1.05	0.77	2.92	0.00	1.59	0.00	0.00	0.51	0.00
time (sec)	N/A	0.405	0.135	0.214	0.000	0.086	0.000	0.000	0.179	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	116	0	0	47	0
N.S.	1	1.00	0.74	3.77	0.00	1.49	0.00	0.00	0.60	0.00
time (sec)	N/A	0.361	0.204	0.000	0.000	0.083	0.000	0.000	0.169	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	114	81	566	0	139	0	0	59	0
N.S.	1	1.02	0.72	5.05	0.00	1.24	0.00	0.00	0.53	0.00
time (sec)	N/A	0.523	0.258	0.344	0.000	0.080	0.000	0.000	0.187	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	120	77	413	0	135	0	0	63	0
N.S.	1	1.06	0.68	3.65	0.00	1.19	0.00	0.00	0.56	0.00
time (sec)	N/A	0.544	0.379	0.372	0.000	0.087	0.000	0.000	0.190	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	112	81	566	0	139	0	0	47	0
N.S.	1	0.97	0.70	4.92	0.00	1.21	0.00	0.00	0.41	0.00
time (sec)	N/A	0.470	0.064	0.000	0.000	0.085	0.000	0.000	0.187	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	116	77	413	0	135	0	0	47	0
N.S.	1	1.01	0.67	3.59	0.00	1.17	0.00	0.00	0.41	0.00
time (sec)	N/A	0.481	0.071	0.000	0.000	0.096	0.000	0.000	0.162	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	69	70	70	111	63	0	60	53	97
N.S.	1	0.59	0.60	0.60	0.96	0.54	0.00	0.52	0.46	0.84
time (sec)	N/A	0.294	0.250	0.801	0.231	0.115	0.000	0.160	0.177	43.690

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	82	67	82	75	200	0	47	65	112
N.S.	1	0.73	0.59	0.73	0.66	1.77	0.00	0.42	0.58	0.99
time (sec)	N/A	0.330	0.791	0.575	0.313	0.120	0.000	0.161	0.170	43.269

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	52	52	47	57	46	139	38	31	72
N.S.	1	0.70	0.70	0.64	0.77	0.62	1.88	0.51	0.42	0.97
time (sec)	N/A	0.271	0.104	0.391	0.279	0.086	29.668	0.131	0.157	0.998

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	53	52	54	52	162	146	28	31	45
N.S.	1	0.59	0.58	0.60	0.58	1.80	1.62	0.31	0.34	0.50
time (sec)	N/A	0.214	0.092	0.335	0.211	0.148	13.554	0.176	0.163	0.430

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	47	44	53	80	201	0	64	44	0
N.S.	1	0.69	0.65	0.78	1.18	2.96	0.00	0.94	0.65	0.00
time (sec)	N/A	0.315	0.061	0.375	0.344	0.120	0.000	0.198	0.174	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	38	45	45	80	185	0	70	33	81
N.S.	1	0.64	0.76	0.76	1.36	3.14	0.00	1.19	0.56	1.37
time (sec)	N/A	0.249	0.080	0.339	0.373	0.112	0.000	0.180	0.159	41.048

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	63	59	116	728	213	0	101	179	0
N.S.	1	0.81	0.76	1.49	9.33	2.73	0.00	1.29	2.29	0.00
time (sec)	N/A	0.328	0.124	0.494	0.244	0.110	0.000	0.223	0.168	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	66	51	54	350	47	0	130	62	217
N.S.	1	0.84	0.65	0.68	4.43	0.59	0.00	1.65	0.78	2.75
time (sec)	N/A	0.339	0.228	0.377	0.265	0.091	0.000	0.195	0.160	45.411

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	91	80	174	2318	255	0	215	314	0
N.S.	1	0.75	0.66	1.43	19.00	2.09	0.00	1.76	2.57	0.00
time (sec)	N/A	0.423	0.256	0.701	0.391	0.132	0.000	0.275	0.158	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	70	70	71	117	69	0	60	54	98
N.S.	1	0.59	0.59	0.60	0.98	0.58	0.00	0.50	0.45	0.82
time (sec)	N/A	0.296	0.276	0.424	0.271	0.084	0.000	0.144	0.166	42.784

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	83	67	83	82	209	0	47	66	113
N.S.	1	0.72	0.58	0.72	0.71	1.80	0.00	0.41	0.57	0.97
time (sec)	N/A	0.329	0.607	0.441	0.248	0.116	0.000	0.158	0.161	36.886

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	53	53	48	60	50	0	38	32	54
N.S.	1	0.70	0.70	0.63	0.79	0.66	0.00	0.50	0.42	0.71
time (sec)	N/A	0.264	0.058	0.292	0.300	0.099	0.000	0.155	0.166	0.593

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	54	52	55	55	165	0	40	32	46
N.S.	1	0.58	0.56	0.59	0.59	1.77	0.00	0.43	0.34	0.49
time (sec)	N/A	0.204	0.099	0.267	0.256	0.151	0.000	0.173	0.177	0.424

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	48	44	54	83	204	0	64	45	0
N.S.	1	0.69	0.63	0.77	1.19	2.91	0.00	0.91	0.64	0.00
time (sec)	N/A	0.312	0.070	0.298	0.255	0.117	0.000	0.217	0.211	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	39	45	46	80	188	0	70	34	82
N.S.	1	0.64	0.74	0.75	1.31	3.08	0.00	1.15	0.56	1.34
time (sec)	N/A	0.247	0.079	0.339	0.250	0.113	0.000	0.215	0.256	40.302

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	64	59	117	761	216	0	101	180	0
N.S.	1	0.80	0.74	1.46	9.51	2.70	0.00	1.26	2.25	0.00
time (sec)	N/A	0.325	0.119	0.412	0.292	0.125	0.000	0.221	0.245	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	67	52	55	355	50	0	130	63	218
N.S.	1	0.83	0.64	0.68	4.38	0.62	0.00	1.60	0.78	2.69
time (sec)	N/A	0.345	0.111	0.283	0.253	0.081	0.000	0.286	0.220	42.304

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	92	81	175	2434	260	0	215	315	0
N.S.	1	0.74	0.65	1.40	19.47	2.08	0.00	1.72	2.52	0.00
time (sec)	N/A	0.421	0.174	0.432	0.415	0.115	0.000	0.215	0.174	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	72	70	73	127	75	0	75	56	100
N.S.	1	0.58	0.56	0.58	1.02	0.60	0.00	0.60	0.45	0.80
time (sec)	N/A	0.299	0.298	0.441	0.278	0.093	0.000	0.167	0.176	42.149

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	85	67	85	92	219	0	64	68	72
N.S.	1	0.70	0.55	0.70	0.75	1.80	0.00	0.52	0.56	0.59
time (sec)	N/A	0.351	1.140	0.455	0.252	0.116	0.000	0.141	0.167	0.901

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	55	52	50	64	54	0	148	34	56
N.S.	1	0.69	0.65	0.62	0.80	0.68	0.00	1.85	0.42	0.70
time (sec)	N/A	0.276	0.158	0.299	0.282	0.083	0.000	0.201	0.157	41.218

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	56	52	57	59	171	0	124	34	48
N.S.	1	0.57	0.53	0.58	0.60	1.73	0.00	1.25	0.34	0.48
time (sec)	N/A	0.207	0.140	0.270	0.218	0.116	0.000	0.187	0.191	0.465

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	50	44	56	87	210	0	73	47	0
N.S.	1	0.68	0.59	0.76	1.18	2.84	0.00	0.99	0.64	0.00
time (sec)	N/A	0.315	0.096	0.301	0.244	0.121	0.000	0.218	0.192	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	41	45	48	80	194	0	79	36	84
N.S.	1	0.63	0.69	0.74	1.23	2.98	0.00	1.22	0.55	1.29
time (sec)	N/A	0.247	0.090	0.278	0.315	0.119	0.000	0.207	0.186	40.601

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	66	59	119	821	222	0	121	182	0
N.S.	1	0.79	0.70	1.42	9.77	2.64	0.00	1.44	2.17	0.00
time (sec)	N/A	0.335	0.138	0.355	0.366	0.111	0.000	0.248	0.156	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	69	51	57	367	54	0	148	65	220
N.S.	1	0.81	0.60	0.67	4.32	0.64	0.00	1.74	0.76	2.59
time (sec)	N/A	0.343	0.246	0.301	0.260	0.082	0.000	0.234	0.181	42.113

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	94	80	177	2662	270	0	251	317	0
N.S.	1	0.72	0.61	1.35	20.32	2.06	0.00	1.92	2.42	0.00
time (sec)	N/A	0.427	0.205	0.429	0.462	0.123	0.000	0.285	0.171	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	82	67	82	75	207	0	0	68	115
N.S.	1	0.73	0.59	0.73	0.66	1.83	0.00	0.00	0.60	1.02
time (sec)	N/A	0.327	0.846	0.635	0.429	0.119	0.000	0.000	0.163	41.579

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	52	52	47	57	49	0	0	34	75
N.S.	1	0.70	0.70	0.64	0.77	0.66	0.00	0.00	0.46	1.01
time (sec)	N/A	0.258	0.100	0.424	0.226	0.091	0.000	0.000	0.166	0.982

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	53	52	54	52	169	146	0	34	81
N.S.	1	0.59	0.58	0.60	0.58	1.88	1.62	0.00	0.38	0.90
time (sec)	N/A	0.202	0.080	0.317	0.281	0.108	15.815	0.000	0.169	0.838

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	47	44	53	80	207	0	0	47	0
N.S.	1	0.69	0.65	0.78	1.18	3.04	0.00	0.00	0.69	0.00
time (sec)	N/A	0.299	0.053	0.388	0.369	0.140	0.000	0.000	0.163	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	38	45	45	85	191	0	0	36	84
N.S.	1	0.64	0.76	0.76	1.44	3.24	0.00	0.00	0.61	1.42
time (sec)	N/A	0.245	0.060	0.342	0.228	0.155	0.000	0.000	0.167	40.400

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	63	59	116	728	219	0	0	182	0
N.S.	1	0.81	0.76	1.49	9.33	2.81	0.00	0.00	2.33	0.00
time (sec)	N/A	0.318	0.080	0.450	0.265	0.114	0.000	0.000	0.171	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	66	51	54	355	50	0	0	65	220
N.S.	1	0.84	0.65	0.68	4.49	0.63	0.00	0.00	0.82	2.78
time (sec)	N/A	0.332	0.142	0.361	0.301	0.086	0.000	0.000	0.186	42.625

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	91	80	174	2318	261	0	0	317	0
N.S.	1	0.75	0.66	1.43	19.00	2.14	0.00	0.00	2.60	0.00
time (sec)	N/A	0.420	0.145	0.658	0.313	0.120	0.000	0.000	0.179	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	85	67	85	75	207	0	0	68	115
N.S.	1	0.70	0.55	0.70	0.61	1.70	0.00	0.00	0.56	0.94
time (sec)	N/A	0.332	1.047	0.414	0.245	0.123	0.000	0.000	0.196	41.493

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	55	52	50	57	49	0	0	34	75
N.S.	1	0.69	0.65	0.62	0.71	0.61	0.00	0.00	0.42	0.94
time (sec)	N/A	0.271	0.122	0.358	0.327	0.084	0.000	0.000	0.226	0.837

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	56	52	57	52	169	0	0	34	81
N.S.	1	0.57	0.53	0.58	0.53	1.71	0.00	0.00	0.34	0.82
time (sec)	N/A	0.206	0.076	0.244	0.223	0.109	0.000	0.000	0.241	0.700

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	50	44	56	80	207	0	0	47	0
N.S.	1	0.68	0.59	0.76	1.08	2.80	0.00	0.00	0.64	0.00
time (sec)	N/A	0.312	0.062	0.275	0.258	0.108	0.000	0.000	0.287	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	41	45	48	93	191	0	0	36	84
N.S.	1	0.63	0.69	0.74	1.43	2.94	0.00	0.00	0.55	1.29
time (sec)	N/A	0.250	0.071	0.265	0.254	0.111	0.000	0.000	0.180	40.329

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	66	59	119	736	219	0	0	182	0
N.S.	1	0.79	0.70	1.42	8.76	2.61	0.00	0.00	2.17	0.00
time (sec)	N/A	0.333	0.087	0.326	0.298	0.155	0.000	0.000	0.165	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	69	51	57	380	50	0	0	65	220
N.S.	1	0.81	0.60	0.67	4.47	0.59	0.00	0.00	0.76	2.59
time (sec)	N/A	0.337	0.135	0.289	0.270	0.082	0.000	0.000	0.205	42.154

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	94	80	177	2350	261	0	0	317	0
N.S.	1	0.72	0.61	1.35	17.94	1.99	0.00	0.00	2.42	0.00
time (sec)	N/A	0.425	0.173	0.411	0.368	0.125	0.000	0.000	0.188	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	85	70	85	75	207	0	0	68	115
N.S.	1	0.70	0.57	0.70	0.61	1.70	0.00	0.00	0.56	0.94
time (sec)	N/A	0.340	1.204	0.420	0.237	0.126	0.000	0.000	0.169	41.509

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	55	55	50	57	49	0	0	34	75
N.S.	1	0.69	0.69	0.62	0.71	0.61	0.00	0.00	0.42	0.94
time (sec)	N/A	0.268	0.098	0.385	0.255	0.099	0.000	0.000	0.173	0.842

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	56	55	57	52	169	0	0	34	81
N.S.	1	0.57	0.56	0.58	0.53	1.71	0.00	0.00	0.34	0.82
time (sec)	N/A	0.211	0.060	0.267	0.266	0.118	0.000	0.000	0.189	0.712

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	50	47	56	80	207	0	0	47	0
N.S.	1	0.68	0.64	0.76	1.08	2.80	0.00	0.00	0.64	0.00
time (sec)	N/A	0.314	0.069	0.276	0.278	0.112	0.000	0.000	0.199	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	41	45	48	93	191	0	0	36	117
N.S.	1	0.63	0.69	0.74	1.43	2.94	0.00	0.00	0.55	1.80
time (sec)	N/A	0.245	0.077	0.266	0.225	0.115	0.000	0.000	0.180	41.187

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	66	59	119	754	219	0	0	182	0
N.S.	1	0.79	0.70	1.42	8.98	2.61	0.00	0.00	2.17	0.00
time (sec)	N/A	0.324	0.093	0.340	0.357	0.113	0.000	0.000	0.161	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	69	51	57	412	50	0	0	65	220
N.S.	1	0.81	0.60	0.67	4.85	0.59	0.00	0.00	0.76	2.59
time (sec)	N/A	0.345	0.146	0.302	0.361	0.081	0.000	0.000	0.185	42.192

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	94	80	177	2418	261	0	0	317	0
N.S.	1	0.72	0.61	1.35	18.46	1.99	0.00	0.00	2.42	0.00
time (sec)	N/A	0.428	0.176	0.417	0.294	0.118	0.000	0.000	0.186	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	96	0	0	0	0	0	29	0
N.S.	1	1.04	1.01	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.371	0.144	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	91	0	0	0	0	0	29	0
N.S.	1	1.04	0.96	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.363	0.199	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	29	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.304	0.130	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	95	88	0	0	0	0	0	43	0
N.S.	1	1.09	1.01	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.362	0.145	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	97	88	0	0	0	0	0	47	0
N.S.	1	1.07	0.97	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.377	0.186	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	99	96	0	0	0	0	0	47	0
N.S.	1	1.08	1.04	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.377	0.133	0.000	0.000	0.000	0.000	0.000	0.263	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	96	0	0	0	0	0	29	0
N.S.	1	1.04	1.01	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.386	0.134	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	91	0	0	0	0	0	29	0
N.S.	1	1.04	0.96	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.361	0.190	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	29	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.311	0.137	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	97	88	0	0	0	0	0	43	0
N.S.	1	1.09	0.99	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.378	0.140	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	97	88	0	0	0	0	0	47	0
N.S.	1	1.07	0.97	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.393	0.193	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	97	96	0	0	0	0	0	47	0
N.S.	1	1.08	1.07	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.380	0.142	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	96	0	0	0	0	0	29	0
N.S.	1	1.04	1.01	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.392	0.285	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	91	0	0	0	0	0	29	0
N.S.	1	1.04	0.96	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.350	0.195	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	29	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.311	0.012	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	97	89	0	0	0	0	0	43	0
N.S.	1	1.09	1.00	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.359	0.010	0.000	0.000	0.000	0.000	0.000	0.271	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	97	90	0	0	0	0	0	47	0
N.S.	1	1.09	1.01	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.374	0.023	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	97	90	0	0	0	0	0	47	0
N.S.	1	1.08	1.00	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.375	0.170	0.000	0.000	0.000	0.000	0.000	0.299	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	96	0	0	0	0	0	29	0
N.S.	1	1.04	1.01	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.365	0.122	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	91	0	0	0	0	0	29	0
N.S.	1	1.04	0.96	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.340	0.023	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	87	0	0	0	0	0	29	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.300	0.011	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	95	91	0	0	0	0	0	43	0
N.S.	1	1.06	1.01	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.356	0.539	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	97	89	0	0	0	0	0	47	0
N.S.	1	1.07	0.98	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.372	0.455	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	99	91	0	0	0	0	0	47	0
N.S.	1	1.08	0.99	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.374	0.516	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	96	0	0	0	0	0	29	0
N.S.	1	1.04	1.01	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.355	0.136	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	91	0	0	0	0	0	29	0
N.S.	1	1.04	0.96	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.344	0.030	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	87	0	0	0	0	0	29	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.310	0.012	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	95	91	0	0	0	0	0	43	0
N.S.	1	1.06	1.01	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.366	0.476	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	99	89	0	0	0	0	0	47	0
N.S.	1	1.06	0.96	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.381	0.460	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	99	91	0	0	0	0	0	47	0
N.S.	1	1.08	0.99	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.390	0.516	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	96	0	0	0	0	0	29	0
N.S.	1	1.04	1.01	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.347	0.206	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	90	0	0	0	0	0	29	0
N.S.	1	1.04	0.95	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.344	0.013	0.000	0.000	0.000	0.000	0.000	0.284	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	87	0	0	0	0	0	29	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.313	0.169	0.000	0.000	0.000	0.000	0.000	0.342	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	95	88	0	0	0	0	0	43	0
N.S.	1	1.06	0.98	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.365	0.019	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	99	90	0	0	0	0	0	47	0
N.S.	1	1.06	0.97	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.381	0.220	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	99	91	0	0	0	0	0	47	0
N.S.	1	1.08	0.99	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.378	0.236	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	150	142	0	0	0	0	0	49	0
N.S.	1	1.01	0.96	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.448	0.319	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	149	142	0	0	0	0	0	42	0
N.S.	1	1.02	0.97	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.446	0.254	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	149	142	0	0	0	0	0	42	0
N.S.	1	1.02	0.97	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.441	0.358	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	149	142	0	0	0	0	0	47	0
N.S.	1	1.02	0.97	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.438	0.320	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	147	142	0	0	0	0	0	47	0
N.S.	1	1.02	0.99	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.444	0.234	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	153	142	0	0	0	0	0	47	0
N.S.	1	1.03	0.95	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.447	0.309	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	132	0	0	0	0	0	45	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.453	0.274	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	122	0	0	0	0	0	47	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.397	0.218	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	120	0	0	0	0	0	45	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.395	0.198	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	114	0	0	0	0	0	38	0
N.S.	1	0.97	0.97	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.344	0.191	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	111	0	0	0	0	0	51	0
N.S.	1	1.08	1.11	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.381	0.255	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	120	117	0	0	0	0	0	55	0
N.S.	1	1.07	1.04	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.423	0.223	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	123	114	0	0	0	0	0	55	0
N.S.	1	0.98	0.91	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.427	0.185	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	125	122	0	0	0	0	0	55	0
N.S.	1	0.98	0.96	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.425	0.182	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	147	140	0	0	0	0	0	51	0
N.S.	1	1.04	0.99	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.463	0.254	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	147	140	0	0	0	0	0	49	0
N.S.	1	1.04	0.99	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.442	0.230	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	147	140	0	0	0	0	0	42	0
N.S.	1	1.04	0.99	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.435	0.200	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	145	140	0	0	0	0	0	49	0
N.S.	1	1.04	1.00	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.437	0.203	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	146	140	0	0	0	0	0	42	0
N.S.	1	1.07	1.03	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.446	0.216	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	147	140	0	0	0	0	0	51	0
N.S.	1	1.05	1.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.458	0.202	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	147	140	0	0	0	0	0	51	0
N.S.	1	1.04	0.99	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.463	0.213	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	147	140	0	0	0	0	0	51	0
N.S.	1	1.04	0.99	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.481	0.211	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	172	242	0	0	0	0	0	42	0
N.S.	1	1.01	1.42	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.673	1.977	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	145	175	0	0	0	0	0	42	0
N.S.	1	1.07	1.30	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.617	1.485	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	145	289	0	0	0	0	0	42	0
N.S.	1	1.07	2.14	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.607	4.935	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	142	144	0	0	0	0	0	42	0
N.S.	1	1.05	1.07	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.596	0.987	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	142	0	0	0	0	0	0	42	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.624	0.000	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	280	279	0	0	0	0	0	42	0
N.S.	1	1.02	1.02	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.688	3.312	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	280	276	0	0	0	0	0	42	0
N.S.	1	1.02	1.01	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.636	3.202	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	277	256	0	0	0	0	0	42	0
N.S.	1	1.01	0.93	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.650	2.099	0.000	0.000	0.000	0.000	0.000	0.264	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	272	275	256	0	0	0	0	0	42	0
N.S.	1	1.01	0.94	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.649	2.377	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	119	0	0	0	0	0	42	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.565	0.622	0.000	0.000	0.000	0.000	0.000	0.314	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	285	10805	0	0	0	0	0	42	0
N.S.	1	1.00	37.78	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.664	27.403	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	145	118	0	0	0	0	0	45	0
N.S.	1	1.03	0.84	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.424	0.290	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	170	140	0	0	0	0	0	49	0
N.S.	1	1.02	0.84	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.540	0.419	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	170	140	0	0	0	0	0	49	0
N.S.	1	1.02	0.84	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.553	0.423	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	171	140	0	0	0	0	0	51	0
N.S.	1	1.01	0.83	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.535	0.611	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	170	140	0	0	0	0	0	47	0
N.S.	1	1.02	0.84	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.528	0.458	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	170	140	0	0	0	0	0	47	0
N.S.	1	1.02	0.84	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.524	0.459	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	140	0	0	0	0	0	47	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.522	0.388	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	175	136	0	0	0	0	0	52	0
N.S.	1	1.05	0.81	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.543	0.374	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	145	120	0	0	0	0	0	47	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.506	0.492	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	145	120	0	0	0	0	0	47	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.465	0.302	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	145	118	0	0	0	0	0	45	0
N.S.	1	1.03	0.84	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.448	0.247	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	0	0	0	0	57	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.473	0.172	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	137	109	0	0	0	0	0	61	0
N.S.	1	1.04	0.83	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.490	0.198	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	140	109	0	0	0	0	0	61	0
N.S.	1	1.07	0.83	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.504	0.195	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	145	118	0	0	0	0	0	61	0
N.S.	1	1.04	0.85	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.507	0.178	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	51	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.563	0.303	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	51	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.517	0.421	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	49	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.500	0.346	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	40	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.510	0.254	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	42	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.490	0.245	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	168	133	0	0	0	0	0	51	0
N.S.	1	1.03	0.82	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.525	0.277	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	51	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.492	0.238	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	168	138	0	0	0	0	0	51	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.501	0.239	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	175	356	0	0	0	0	0	49	0
N.S.	1	1.01	2.06	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.676	6.212	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	296	295	13441	0	0	0	0	0	49	0
N.S.	1	1.00	45.41	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.691	28.382	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	287	290	0	0	0	0	0	49	0
N.S.	1	1.02	1.03	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.663	5.838	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	287	289	0	0	0	0	0	49	0
N.S.	1	1.02	1.03	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.632	5.715	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	284	263	0	0	0	0	0	49	0
N.S.	1	1.01	0.94	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.672	3.676	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	284	261	0	0	0	0	0	49	0
N.S.	1	1.01	0.93	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.662	3.680	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	184	144	0	0	0	0	0	57	0
N.S.	1	0.98	0.77	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.527	0.449	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	228	125	382	0	191	0	0	64	0
N.S.	1	1.09	0.60	1.83	0.00	0.91	0.00	0.00	0.31	0.00
time (sec)	N/A	1.007	2.922	7.458	0.000	0.124	0.000	0.000	0.199	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	194	111	351	0	177	0	0	62	0
N.S.	1	1.08	0.62	1.95	0.00	0.98	0.00	0.00	0.34	0.00
time (sec)	N/A	0.869	2.877	3.326	0.000	0.099	0.000	0.000	0.170	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	157	94	317	0	163	0	0	53	0
N.S.	1	1.08	0.65	2.19	0.00	1.12	0.00	0.00	0.37	0.00
time (sec)	N/A	0.729	2.412	1.899	0.000	0.099	0.000	0.000	0.170	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	123	83	283	0	149	0	0	72	0
N.S.	1	1.10	0.74	2.53	0.00	1.33	0.00	0.00	0.64	0.00
time (sec)	N/A	0.648	1.148	1.193	0.000	0.086	0.000	0.000	0.230	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	123	78	260	0	181	0	0	78	0
N.S.	1	1.13	0.72	2.39	0.00	1.66	0.00	0.00	0.72	0.00
time (sec)	N/A	0.701	1.191	1.071	0.000	0.100	0.000	0.000	0.231	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	160	90	505	0	199	0	0	78	0
N.S.	1	1.14	0.64	3.61	0.00	1.42	0.00	0.00	0.56	0.00
time (sec)	N/A	0.830	0.702	1.062	0.000	0.098	0.000	0.000	0.247	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	197	122	800	0	220	0	0	78	0
N.S.	1	1.09	0.67	4.42	0.00	1.22	0.00	0.00	0.43	0.00
time (sec)	N/A	0.893	0.909	1.915	0.000	0.089	0.000	0.000	0.237	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	233	143	725	0	231	0	0	78	0
N.S.	1	1.11	0.68	3.45	0.00	1.10	0.00	0.00	0.37	0.00
time (sec)	N/A	1.051	1.126	2.509	0.000	0.119	0.000	0.000	0.225	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	228	128	384	0	195	0	0	65	0
N.S.	1	1.09	0.61	1.83	0.00	0.93	0.00	0.00	0.31	0.00
time (sec)	N/A	0.965	1.881	2.582	0.000	0.113	0.000	0.000	0.167	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	190	108	353	0	183	0	0	63	0
N.S.	1	1.05	0.60	1.95	0.00	1.01	0.00	0.00	0.35	0.00
time (sec)	N/A	0.766	0.441	1.592	0.000	0.088	0.000	0.000	0.195	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	159	95	319	0	165	0	0	81	0
N.S.	1	1.09	0.65	2.18	0.00	1.13	0.00	0.00	0.55	0.00
time (sec)	N/A	0.779	0.208	1.200	0.000	0.101	0.000	0.000	0.229	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	125	85	285	0	148	0	0	87	0
N.S.	1	1.08	0.73	2.46	0.00	1.28	0.00	0.00	0.75	0.00
time (sec)	N/A	0.650	0.059	0.897	0.000	0.100	0.000	0.000	0.263	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	123	80	262	0	180	0	0	87	0
N.S.	1	1.08	0.70	2.30	0.00	1.58	0.00	0.00	0.76	0.00
time (sec)	N/A	0.661	0.685	0.489	0.000	0.091	0.000	0.000	0.238	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	160	92	506	0	200	0	0	87	0
N.S.	1	1.10	0.63	3.49	0.00	1.38	0.00	0.00	0.60	0.00
time (sec)	N/A	0.796	0.688	0.533	0.000	0.093	0.000	0.000	0.246	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	197	122	805	0	223	0	0	87	0
N.S.	1	1.06	0.66	4.33	0.00	1.20	0.00	0.00	0.47	0.00
time (sec)	N/A	0.860	0.790	0.564	0.000	0.092	0.000	0.000	0.243	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	233	134	727	0	235	0	0	87	0
N.S.	1	1.08	0.62	3.38	0.00	1.09	0.00	0.00	0.40	0.00
time (sec)	N/A	1.031	1.634	0.579	0.000	0.094	0.000	0.000	0.260	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	224	125	384	0	203	0	0	67	0
N.S.	1	1.06	0.59	1.81	0.00	0.96	0.00	0.00	0.32	0.00
time (sec)	N/A	0.894	0.464	5.319	0.000	0.108	0.000	0.000	0.174	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	192	109	353	0	189	0	0	85	0
N.S.	1	1.05	0.60	1.93	0.00	1.03	0.00	0.00	0.46	0.00
time (sec)	N/A	0.817	0.343	9.233	0.000	0.102	0.000	0.000	0.243	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	161	97	319	0	169	0	0	91	0
N.S.	1	1.07	0.64	2.11	0.00	1.12	0.00	0.00	0.60	0.00
time (sec)	N/A	0.773	0.216	12.933	0.000	0.095	0.000	0.000	0.252	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	125	79	285	0	150	0	0	91	0
N.S.	1	1.04	0.66	2.38	0.00	1.25	0.00	0.00	0.76	0.00
time (sec)	N/A	0.650	0.758	68.937	0.000	0.120	0.000	0.000	0.339	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	123	80	262	0	182	0	0	91	0
N.S.	1	1.06	0.69	2.26	0.00	1.57	0.00	0.00	0.78	0.00
time (sec)	N/A	0.656	0.606	219.993	0.000	0.112	0.000	0.000	0.379	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	160	92	508	0	204	0	0	91	0
N.S.	1	1.09	0.63	3.46	0.00	1.39	0.00	0.00	0.62	0.00
time (sec)	N/A	0.794	0.673	0.188	0.000	0.088	0.000	0.000	0.337	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	197	121	807	0	229	0	0	91	0
N.S.	1	1.05	0.64	4.29	0.00	1.22	0.00	0.00	0.48	0.00
time (sec)	N/A	0.855	0.947	0.199	0.000	0.103	0.000	0.000	0.271	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	233	134	727	0	243	0	0	91	0
N.S.	1	1.07	0.62	3.35	0.00	1.12	0.00	0.00	0.42	0.00
time (sec)	N/A	1.036	2.323	0.205	0.000	0.103	0.000	0.000	0.282	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	228	127	381	0	194	0	0	67	0
N.S.	1	1.07	0.59	1.78	0.00	0.91	0.00	0.00	0.31	0.00
time (sec)	N/A	0.944	1.478	1.492	0.000	0.137	0.000	0.000	0.179	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	194	108	350	0	180	0	0	65	0
N.S.	1	1.05	0.58	1.89	0.00	0.97	0.00	0.00	0.35	0.00
time (sec)	N/A	0.823	1.235	1.316	0.000	0.123	0.000	0.000	0.195	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	161	97	316	0	166	0	0	56	0
N.S.	1	1.07	0.65	2.11	0.00	1.11	0.00	0.00	0.37	0.00
time (sec)	N/A	0.735	0.228	0.902	0.000	0.114	0.000	0.000	0.200	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	121	82	282	0	152	0	0	56	128
N.S.	1	1.03	0.70	2.41	0.00	1.30	0.00	0.00	0.48	1.09
time (sec)	N/A	0.554	0.115	0.592	0.000	0.087	0.000	0.000	0.180	0.405

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	121	279	259	0	184	0	0	75	0
N.S.	1	1.10	2.54	2.35	0.00	1.67	0.00	0.00	0.68	0.00
time (sec)	N/A	0.638	5.519	0.898	0.000	0.087	0.000	0.000	0.192	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	160	757	508	0	202	0	0	81	0
N.S.	1	1.15	5.45	3.65	0.00	1.45	0.00	0.00	0.58	0.00
time (sec)	N/A	0.787	7.162	0.922	0.000	0.108	0.000	0.000	0.196	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	197	116	801	0	223	0	0	81	0
N.S.	1	1.09	0.64	4.45	0.00	1.24	0.00	0.00	0.45	0.00
time (sec)	N/A	0.854	1.056	1.215	0.000	0.090	0.000	0.000	0.214	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	233	133	726	0	234	0	0	81	0
N.S.	1	1.11	0.64	3.47	0.00	1.12	0.00	0.00	0.39	0.00
time (sec)	N/A	1.036	1.236	1.812	0.000	0.093	0.000	0.000	0.212	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	228	130	384	0	194	0	0	67	0
N.S.	1	1.05	0.60	1.77	0.00	0.89	0.00	0.00	0.31	0.00
time (sec)	N/A	0.955	2.156	2.644	0.000	0.121	0.000	0.000	0.188	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	194	108	353	0	180	0	0	65	0
N.S.	1	1.03	0.57	1.88	0.00	0.96	0.00	0.00	0.35	0.00
time (sec)	N/A	0.803	2.234	1.785	0.000	0.094	0.000	0.000	0.180	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	161	94	319	0	166	0	0	56	0
N.S.	1	1.05	0.61	2.08	0.00	1.08	0.00	0.00	0.37	0.00
time (sec)	N/A	0.736	1.719	1.013	0.000	0.086	0.000	0.000	0.192	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	125	85	285	0	152	0	0	56	0
N.S.	1	1.04	0.71	2.38	0.00	1.27	0.00	0.00	0.47	0.00
time (sec)	N/A	0.604	0.174	0.632	0.000	0.098	0.000	0.000	0.192	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	119	80	262	0	184	0	0	58	0
N.S.	1	1.03	0.69	2.26	0.00	1.59	0.00	0.00	0.50	0.00
time (sec)	N/A	0.578	0.401	0.259	0.000	0.086	0.000	0.000	0.170	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	158	761	508	0	202	0	0	77	0
N.S.	1	1.10	5.28	3.53	0.00	1.40	0.00	0.00	0.53	0.00
time (sec)	N/A	0.767	6.276	0.437	0.000	0.085	0.000	0.000	0.189	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	197	119	807	0	223	0	0	83	0
N.S.	1	1.08	0.65	4.41	0.00	1.22	0.00	0.00	0.45	0.00
time (sec)	N/A	0.859	0.879	0.517	0.000	0.090	0.000	0.000	0.190	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	233	136	729	0	234	0	0	83	0
N.S.	1	1.10	0.64	3.44	0.00	1.10	0.00	0.00	0.39	0.00
time (sec)	N/A	1.037	1.383	0.583	0.000	0.094	0.000	0.000	0.187	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	228	130	384	0	194	0	0	67	0
N.S.	1	1.05	0.60	1.77	0.00	0.89	0.00	0.00	0.31	0.00
time (sec)	N/A	0.961	2.134	1.998	0.000	0.097	0.000	0.000	0.191	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	194	111	353	0	180	0	0	65	0
N.S.	1	1.03	0.59	1.88	0.00	0.96	0.00	0.00	0.35	0.00
time (sec)	N/A	0.799	2.111	1.580	0.000	0.111	0.000	0.000	0.181	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	161	97	319	0	166	0	0	56	0
N.S.	1	1.05	0.63	2.08	0.00	1.08	0.00	0.00	0.37	0.00
time (sec)	N/A	0.738	1.753	1.128	0.000	0.134	0.000	0.000	0.180	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	125	85	285	0	152	0	0	56	0
N.S.	1	1.04	0.71	2.38	0.00	1.27	0.00	0.00	0.47	0.00
time (sec)	N/A	0.605	0.183	0.645	0.000	0.118	0.000	0.000	0.171	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	123	807	262	0	184	0	0	58	0
N.S.	1	1.06	6.96	2.26	0.00	1.59	0.00	0.00	0.50	0.00
time (sec)	N/A	0.610	6.199	0.264	0.000	0.099	0.000	0.000	0.170	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	156	92	508	0	202	0	0	67	0
N.S.	1	1.06	0.63	3.46	0.00	1.37	0.00	0.00	0.46	0.00
time (sec)	N/A	0.717	0.701	0.278	0.000	0.111	0.000	0.000	0.173	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	195	119	807	0	223	0	0	85	0
N.S.	1	1.05	0.64	4.36	0.00	1.21	0.00	0.00	0.46	0.00
time (sec)	N/A	0.849	0.445	0.471	0.000	0.088	0.000	0.000	0.172	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	233	136	729	0	234	0	0	91	0
N.S.	1	1.10	0.64	3.44	0.00	1.10	0.00	0.00	0.43	0.00
time (sec)	N/A	1.027	1.222	0.546	0.000	0.120	0.000	0.000	0.188	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	193	119	807	0	223	0	0	67	0
N.S.	1	1.03	0.63	4.29	0.00	1.19	0.00	0.00	0.36	0.00
time (sec)	N/A	0.777	0.109	0.440	0.000	0.120	0.000	0.000	0.162	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	138	109	109	159	292	0	94	96	141
N.S.	1	0.62	0.49	0.49	0.71	1.31	0.00	0.42	0.43	0.63
time (sec)	N/A	0.645	1.331	1.100	0.315	0.123	0.000	0.341	0.161	43.377

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	111	92	102	116	276	0	75	85	137
N.S.	1	0.60	0.50	0.55	0.63	1.50	0.00	0.41	0.46	0.74
time (sec)	N/A	0.540	1.066	0.549	0.341	0.122	0.000	0.313	0.165	41.790

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	94	75	77	80	236	241	57	57	104
N.S.	1	0.66	0.52	0.54	0.56	1.65	1.69	0.40	0.40	0.73
time (sec)	N/A	0.375	0.775	0.450	0.293	0.121	29.465	0.316	0.175	1.790

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	64	61	63	64	212	184	40	40	54
N.S.	1	0.52	0.50	0.51	0.52	1.72	1.50	0.33	0.33	0.44
time (sec)	N/A	0.235	0.119	0.406	0.260	0.121	13.483	0.318	0.162	0.613

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	50	93	61	104	304	0	98	48	0
N.S.	1	0.54	1.00	0.66	1.12	3.27	0.00	1.05	0.52	0.00
time (sec)	N/A	0.421	0.869	0.412	0.275	0.143	0.000	0.378	0.162	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	50	60	70	144	312	0	98	74	0
N.S.	1	0.54	0.65	0.75	1.55	3.35	0.00	1.05	0.80	0.00
time (sec)	N/A	0.414	0.088	0.421	0.285	0.147	0.000	0.398	0.167	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	77	83	130	780	233	0	127	194	0
N.S.	1	0.69	0.75	1.17	7.03	2.10	0.00	1.14	1.75	0.00
time (sec)	N/A	0.521	0.196	0.543	0.320	0.114	0.000	0.429	0.167	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	105	87	133	1009	265	0	189	183	0
N.S.	1	0.69	0.57	0.88	6.64	1.74	0.00	1.24	1.20	0.00
time (sec)	N/A	0.646	0.493	0.519	0.315	0.133	0.000	0.424	0.176	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	120	110	200	2611	299	0	270	346	0
N.S.	1	0.62	0.57	1.04	13.53	1.55	0.00	1.40	1.79	0.00
time (sec)	N/A	0.706	0.374	0.773	0.390	0.126	0.000	0.440	0.163	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	139	109	110	169	309	0	94	97	142
N.S.	1	0.61	0.48	0.48	0.74	1.35	0.00	0.41	0.42	0.62
time (sec)	N/A	0.650	0.973	0.684	0.356	0.158	0.000	0.337	0.170	42.804

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	112	92	103	126	285	0	75	86	138
N.S.	1	0.59	0.49	0.54	0.67	1.51	0.00	0.40	0.46	0.73
time (sec)	N/A	0.533	0.710	0.385	0.324	0.157	0.000	0.336	0.158	41.351

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	95	76	78	86	249	0	57	58	71
N.S.	1	0.65	0.52	0.53	0.59	1.69	0.00	0.39	0.39	0.48
time (sec)	N/A	0.361	0.171	0.350	0.300	0.110	0.000	0.334	0.170	0.906

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	65	61	64	67	217	0	89	41	55
N.S.	1	0.51	0.48	0.50	0.53	1.71	0.00	0.70	0.32	0.43
time (sec)	N/A	0.223	0.136	0.315	0.284	0.146	0.000	0.361	0.174	39.374

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	51	93	62	107	308	0	98	49	0
N.S.	1	0.53	0.97	0.65	1.11	3.21	0.00	1.02	0.51	0.00
time (sec)	N/A	0.399	0.685	0.330	0.299	0.145	0.000	0.385	0.170	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	51	60	71	147	316	0	98	75	0
N.S.	1	0.53	0.62	0.74	1.53	3.29	0.00	1.02	0.78	0.00
time (sec)	N/A	0.406	0.100	0.320	0.265	0.153	0.000	0.418	0.170	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	78	83	131	813	240	0	127	195	0
N.S.	1	0.68	0.73	1.15	7.13	2.11	0.00	1.11	1.71	0.00
time (sec)	N/A	0.505	0.146	0.427	0.352	0.116	0.000	0.362	0.182	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	106	88	134	1044	272	0	189	184	0
N.S.	1	0.68	0.56	0.86	6.69	1.74	0.00	1.21	1.18	0.00
time (sec)	N/A	0.627	0.180	0.352	0.342	0.116	0.000	0.484	0.159	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	121	111	201	2732	308	0	270	347	0
N.S.	1	0.61	0.56	1.02	13.80	1.56	0.00	1.36	1.75	0.00
time (sec)	N/A	0.649	0.321	0.461	0.410	0.157	0.000	0.434	0.161	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	141	109	112	185	331	0	118	99	144
N.S.	1	0.59	0.45	0.46	0.77	1.37	0.00	0.49	0.41	0.60
time (sec)	N/A	0.633	0.970	0.704	0.381	0.121	0.000	0.342	0.170	42.653

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	114	92	105	140	303	0	98	88	94
N.S.	1	0.57	0.46	0.53	0.70	1.52	0.00	0.49	0.44	0.47
time (sec)	N/A	0.525	1.378	0.383	0.328	0.136	0.000	0.346	0.174	1.237

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	97	75	80	94	263	0	223	60	73
N.S.	1	0.63	0.48	0.52	0.61	1.70	0.00	1.44	0.39	0.47
time (sec)	N/A	0.354	1.329	0.355	0.303	0.123	0.000	0.410	0.182	0.704

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	67	61	66	71	227	0	156	43	57
N.S.	1	0.50	0.45	0.49	0.53	1.68	0.00	1.16	0.32	0.42
time (sec)	N/A	0.226	0.174	0.322	0.262	0.115	0.000	0.397	0.168	44.021

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	53	93	64	111	316	0	113	51	0
N.S.	1	0.52	0.91	0.63	1.09	3.10	0.00	1.11	0.50	0.00
time (sec)	N/A	0.396	0.875	0.348	0.273	0.142	0.000	0.410	0.169	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	53	60	73	151	324	0	113	77	0
N.S.	1	0.52	0.59	0.72	1.48	3.18	0.00	1.11	0.75	0.00
time (sec)	N/A	0.398	0.141	0.332	0.300	0.144	0.000	0.433	0.185	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	80	83	133	873	250	0	153	197	0
N.S.	1	0.67	0.69	1.11	7.28	2.08	0.00	1.28	1.64	0.00
time (sec)	N/A	0.509	0.281	0.390	0.332	0.113	0.000	0.490	0.164	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	108	87	136	1112	286	0	219	186	0
N.S.	1	0.66	0.53	0.83	6.78	1.74	0.00	1.34	1.13	0.00
time (sec)	N/A	0.619	0.575	0.348	0.330	0.123	0.000	0.471	0.162	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	123	110	203	2972	326	0	318	349	0
N.S.	1	0.59	0.53	0.98	14.29	1.57	0.00	1.53	1.68	0.00
time (sec)	N/A	0.645	0.411	0.479	0.383	0.121	0.000	0.529	0.160	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	111	92	102	116	282	0	0	88	140
N.S.	1	0.60	0.50	0.55	0.63	1.53	0.00	0.00	0.48	0.76
time (sec)	N/A	0.521	1.215	0.845	0.359	0.133	0.000	0.000	0.163	46.539

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	94	75	77	80	242	0	0	60	107
N.S.	1	0.66	0.52	0.54	0.56	1.69	0.00	0.00	0.42	0.75
time (sec)	N/A	0.345	0.956	0.405	0.317	0.116	0.000	0.000	0.155	1.805

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	64	61	63	64	218	184	0	43	93
N.S.	1	0.52	0.50	0.51	0.52	1.77	1.50	0.00	0.35	0.76
time (sec)	N/A	0.215	0.119	0.371	0.290	0.147	16.297	0.000	0.173	1.206

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	50	93	61	104	309	0	0	51	0
N.S.	1	0.54	1.00	0.66	1.12	3.32	0.00	0.00	0.55	0.00
time (sec)	N/A	0.383	0.457	0.462	0.281	0.154	0.000	0.000	0.199	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	50	60	70	149	317	0	0	77	0
N.S.	1	0.54	0.65	0.75	1.60	3.41	0.00	0.00	0.83	0.00
time (sec)	N/A	0.384	0.082	0.412	0.297	0.140	0.000	0.000	0.239	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	77	83	131	785	239	0	0	197	0
N.S.	1	0.69	0.75	1.18	7.07	2.15	0.00	0.00	1.77	0.00
time (sec)	N/A	0.498	0.124	0.530	0.383	0.117	0.000	0.000	0.254	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	105	87	133	1014	271	0	0	186	0
N.S.	1	0.69	0.57	0.88	6.67	1.78	0.00	0.00	1.22	0.00
time (sec)	N/A	0.613	0.345	0.542	0.351	0.136	0.000	0.000	0.183	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	120	110	200	2611	305	0	0	349	0
N.S.	1	0.62	0.57	1.04	13.53	1.58	0.00	0.00	1.81	0.00
time (sec)	N/A	0.656	0.287	0.753	0.355	0.131	0.000	0.000	0.168	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	114	92	105	116	282	0	0	88	140
N.S.	1	0.57	0.46	0.53	0.58	1.42	0.00	0.00	0.44	0.70
time (sec)	N/A	0.529	1.290	0.539	0.346	0.117	0.000	0.000	0.183	46.275

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	97	75	80	80	242	0	0	60	107
N.S.	1	0.63	0.48	0.52	0.52	1.56	0.00	0.00	0.39	0.69
time (sec)	N/A	0.355	1.119	0.307	0.337	0.126	0.000	0.000	0.165	1.483

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	67	61	66	64	218	0	0	43	93
N.S.	1	0.50	0.45	0.49	0.47	1.61	0.00	0.00	0.32	0.69
time (sec)	N/A	0.224	0.127	0.293	0.327	0.119	0.000	0.000	0.166	1.011

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	53	48	64	104	309	0	0	51	0
N.S.	1	0.52	0.47	0.63	1.02	3.03	0.00	0.00	0.50	0.00
time (sec)	N/A	0.393	0.681	0.309	0.292	0.189	0.000	0.000	0.167	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	53	60	73	157	317	0	0	77	0
N.S.	1	0.52	0.59	0.72	1.54	3.11	0.00	0.00	0.75	0.00
time (sec)	N/A	0.399	0.103	0.346	0.310	0.146	0.000	0.000	0.176	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	80	83	134	802	239	0	0	197	0
N.S.	1	0.67	0.69	1.12	6.68	1.99	0.00	0.00	1.64	0.00
time (sec)	N/A	0.504	0.134	0.409	0.378	0.121	0.000	0.000	0.187	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	108	87	136	1048	271	0	0	186	0
N.S.	1	0.66	0.53	0.83	6.39	1.65	0.00	0.00	1.13	0.00
time (sec)	N/A	0.628	0.348	0.360	0.385	0.123	0.000	0.000	0.186	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	123	110	203	2660	305	0	0	349	0
N.S.	1	0.59	0.53	0.98	12.79	1.47	0.00	0.00	1.68	0.00
time (sec)	N/A	0.679	0.298	0.438	0.330	0.162	0.000	0.000	0.161	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	114	95	105	116	282	0	0	88	140
N.S.	1	0.57	0.48	0.53	0.58	1.42	0.00	0.00	0.44	0.70
time (sec)	N/A	0.529	1.329	0.573	0.339	0.119	0.000	0.000	0.161	41.333

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	97	78	80	80	242	0	0	60	107
N.S.	1	0.63	0.50	0.52	0.52	1.56	0.00	0.00	0.39	0.69
time (sec)	N/A	0.362	1.105	0.332	0.318	0.124	0.000	0.000	0.166	1.477

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	67	64	66	64	218	0	0	43	93
N.S.	1	0.50	0.47	0.49	0.47	1.61	0.00	0.00	0.32	0.69
time (sec)	N/A	0.225	0.158	0.303	0.295	0.122	0.000	0.000	0.174	1.016

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	53	53	64	104	309	0	0	51	0
N.S.	1	0.52	0.52	0.63	1.02	3.03	0.00	0.00	0.50	0.00
time (sec)	N/A	0.404	0.702	0.310	0.307	0.166	0.000	0.000	0.166	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	53	60	73	157	317	0	0	77	0
N.S.	1	0.52	0.59	0.72	1.54	3.11	0.00	0.00	0.75	0.00
time (sec)	N/A	0.410	0.100	0.312	0.291	0.151	0.000	0.000	0.155	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	80	83	134	820	239	0	0	197	0
N.S.	1	0.67	0.69	1.12	6.83	1.99	0.00	0.00	1.64	0.00
time (sec)	N/A	0.505	0.121	0.419	0.323	0.114	0.000	0.000	0.150	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	108	87	136	1098	271	0	0	186	0
N.S.	1	0.66	0.53	0.83	6.70	1.65	0.00	0.00	1.13	0.00
time (sec)	N/A	0.624	0.255	0.373	0.324	0.145	0.000	0.000	0.161	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	123	110	203	2760	305	0	0	349	0
N.S.	1	0.59	0.53	0.98	13.27	1.47	0.00	0.00	1.68	0.00
time (sec)	N/A	0.656	0.249	0.503	0.411	0.129	0.000	0.000	0.150	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	117	0	0	0	0	0	41	0
N.S.	1	1.05	0.76	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.524	0.357	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	157	121	0	0	0	0	0	41	0
N.S.	1	1.02	0.79	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.489	0.749	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	159	122	0	0	0	0	0	62	0
N.S.	1	1.07	0.82	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.556	0.611	0.000	0.000	0.000	0.000	0.000	0.279	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	156	116	0	0	0	0	0	68	0
N.S.	1	1.06	0.79	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.577	0.398	0.000	0.000	0.000	0.000	0.000	0.315	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	157	123	0	0	0	0	0	68	0
N.S.	1	1.08	0.85	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.591	0.249	0.000	0.000	0.000	0.000	0.000	0.307	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	159	123	0	0	0	0	0	68	0
N.S.	1	1.05	0.81	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.572	0.236	0.000	0.000	0.000	0.000	0.000	0.312	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	117	0	0	0	0	0	41	0
N.S.	1	1.05	0.76	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.539	0.407	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	157	114	0	0	0	0	0	41	0
N.S.	1	1.02	0.74	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.502	0.328	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	159	121	0	0	0	0	0	62	0
N.S.	1	1.07	0.82	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.560	0.695	0.000	0.000	0.000	0.000	0.000	0.341	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	157	122	0	0	0	0	0	68	0
N.S.	1	1.08	0.84	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.577	0.947	0.000	0.000	0.000	0.000	0.000	0.342	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	157	117	0	0	0	0	0	68	0
N.S.	1	1.08	0.81	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.571	0.266	0.000	0.000	0.000	0.000	0.000	0.339	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	159	124	0	0	0	0	0	68	0
N.S.	1	1.05	0.82	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.586	0.228	0.000	0.000	0.000	0.000	0.000	0.358	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	133	0	0	0	0	0	41	0
N.S.	1	1.05	0.86	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.521	1.403	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	124	0	0	0	0	0	41	0
N.S.	1	1.05	0.81	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.499	0.037	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	157	121	0	0	0	0	0	41	0
N.S.	1	1.02	0.79	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.460	0.218	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	154	116	0	0	0	0	0	62	0
N.S.	1	1.03	0.78	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.534	0.631	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	157	116	0	0	0	0	0	68	0
N.S.	1	1.08	0.80	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.560	0.687	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	159	118	0	0	0	0	0	68	0
N.S.	1	1.07	0.79	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.545	0.266	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	122	0	0	0	0	0	41	0
N.S.	1	1.05	0.79	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.504	0.355	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	122	0	0	0	0	0	41	0
N.S.	1	1.05	0.79	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.501	0.373	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	124	0	0	0	0	0	41	0
N.S.	1	1.05	0.81	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.492	0.023	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	115	0	0	0	0	0	41	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.476	0.236	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	155	115	0	0	0	0	0	62	0
N.S.	1	1.05	0.78	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.536	0.017	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	159	118	0	0	0	0	0	68	0
N.S.	1	1.07	0.79	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.555	0.295	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	232	218	178	0	0	0	0	0	72	0
N.S.	1	0.94	0.77	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.695	0.837	0.000	0.000	0.000	0.000	0.000	0.267	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	229	217	175	0	0	0	0	0	63	0
N.S.	1	0.95	0.76	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.645	0.603	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	229	217	175	0	0	0	0	0	63	0
N.S.	1	0.95	0.76	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.605	0.559	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	229	217	175	0	0	0	0	0	68	0
N.S.	1	0.95	0.76	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.642	0.573	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	215	173	0	0	0	0	0	68	0
N.S.	1	0.95	0.76	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.626	0.560	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	0	175	0	0	0	0	0	68	0
N.S.	1	0.00	0.74	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.000	0.669	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	219	157	0	0	0	0	0	66	0
N.S.	1	0.96	0.69	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.634	0.558	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	188	153	0	0	0	0	0	68	0
N.S.	1	1.01	0.82	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.591	0.431	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	188	151	0	0	0	0	0	66	0
N.S.	1	1.01	0.81	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.569	0.395	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	184	144	0	0	0	0	0	57	0
N.S.	1	0.98	0.77	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.540	0.385	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	175	138	0	0	0	0	0	76	0
N.S.	1	1.03	0.81	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.563	0.657	0.000	0.000	0.000	0.000	0.000	0.309	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	182	130	0	0	0	0	0	82	0
N.S.	1	1.05	0.75	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.615	0.956	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	195	150	0	0	0	0	0	82	0
N.S.	1	1.01	0.77	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.655	0.395	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	195	150	0	0	0	0	0	82	0
N.S.	1	0.99	0.77	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.661	0.395	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	215	173	0	0	0	0	0	72	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.651	0.513	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	215	173	0	0	0	0	0	63	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.635	0.527	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	213	171	0	0	0	0	0	63	0
N.S.	1	0.96	0.77	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.626	0.521	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	214	166	0	0	0	0	0	65	0
N.S.	1	0.99	0.76	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.648	0.510	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	213	173	0	0	0	0	0	74	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.654	0.507	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	215	173	0	0	0	0	0	74	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.639	0.506	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	183	185	376	0	0	0	0	0	65	0
N.S.	1	1.01	2.05	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.713	3.119	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	154	137	0	0	0	0	0	63	0
N.S.	1	1.07	0.95	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.653	1.658	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	154	0	0	0	0	0	0	63	0
N.S.	1	1.07	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.634	0.000	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	151	105	0	0	0	0	0	63	0
N.S.	1	1.05	0.73	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.643	1.404	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	148	0	0	0	0	0	0	63	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.643	0.000	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	292	296	0	0	0	0	0	65	0
N.S.	1	1.02	1.03	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.700	5.357	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	292	294	0	0	0	0	0	65	0
N.S.	1	1.02	1.02	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.663	5.258	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	289	268	0	0	0	0	0	65	0
N.S.	1	1.01	0.93	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.690	3.908	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	288	266	0	0	0	0	0	65	0
N.S.	1	1.01	0.93	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.675	3.883	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	0	0	0	0	0	0	88	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.567	0.000	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	303	16142	0	0	0	0	0	65	0
N.S.	1	1.00	53.10	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.709	28.779	0.000	0.000	0.000	0.000	0.000	0.182	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [9] had the largest ratio of [.4285709999999999980]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	0.90	21	0.190
2	A	5	4	0.92	21	0.190
3	A	5	4	0.94	21	0.190
4	A	4	3	1.00	19	0.158
5	A	4	4	1.00	19	0.211
6	A	4	4	1.00	21	0.190
7	A	6	6	0.97	21	0.286
8	A	8	8	0.96	21	0.381
9	A	9	9	0.95	21	0.429
10	A	7	7	0.96	21	0.333
11	A	5	5	0.97	21	0.238
12	A	3	3	1.00	21	0.143
13	A	6	5	1.00	21	0.238
14	A	6	5	0.89	21	0.238
15	A	6	5	0.80	21	0.238
16	A	8	8	0.98	25	0.320
17	A	8	8	0.98	25	0.320
18	A	6	6	1.00	25	0.240
19	A	6	6	1.00	25	0.240
20	A	6	6	1.00	25	0.240
21	A	6	6	1.00	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	8	8	0.97	25	0.320
23	A	8	8	1.01	25	0.320
24	A	2	2	1.00	23	0.087
25	A	2	2	1.00	23	0.087
26	A	10	10	0.97	25	0.400
27	A	10	10	0.94	25	0.400
28	A	8	8	0.97	25	0.320
29	A	8	8	0.97	25	0.320
30	A	8	8	1.05	25	0.320
31	A	8	8	1.05	25	0.320
32	A	10	10	1.02	25	0.400
33	A	10	10	1.02	25	0.400
34	A	4	4	0.97	23	0.174
35	A	2	2	1.00	33	0.061
36	A	2	2	1.00	32	0.062
37	A	9	9	1.03	33	0.273
38	A	9	9	1.05	31	0.290
39	A	6	6	1.00	25	0.240
40	A	7	7	1.05	31	0.226
41	A	7	7	1.13	33	0.212
42	A	7	7	1.08	33	0.212
43	A	9	9	1.05	33	0.273
44	A	9	9	1.06	33	0.273
45	A	9	9	1.05	31	0.290
46	A	8	8	0.98	25	0.320
47	A	7	7	1.05	31	0.226
48	A	7	7	1.04	33	0.212
49	A	7	7	1.08	33	0.212
50	A	7	7	1.05	33	0.212
51	A	9	9	1.03	33	0.273
52	A	9	9	1.04	33	0.273
53	A	8	8	0.98	25	0.320

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	9	9	1.01	31	0.290
55	A	7	7	1.04	33	0.212
56	A	7	7	1.01	33	0.212
57	A	7	7	1.05	33	0.212
58	A	7	7	1.05	33	0.212
59	A	9	9	1.01	33	0.273
60	A	9	9	1.04	33	0.273
61	A	11	11	1.01	33	0.333
62	A	9	9	1.00	33	0.273
63	A	9	9	1.03	33	0.273
64	A	7	7	1.01	31	0.226
65	A	6	6	1.00	25	0.240
66	A	7	7	1.07	31	0.226
67	A	7	7	1.12	33	0.212
68	A	9	9	1.04	33	0.273
69	A	9	9	1.09	33	0.273
70	A	11	11	1.03	33	0.333
71	A	9	9	1.00	33	0.273
72	A	9	9	1.00	33	0.273
73	A	7	7	1.01	33	0.212
74	A	7	7	1.01	31	0.226
75	A	6	6	1.00	25	0.240
76	A	7	7	1.07	31	0.226
77	A	9	9	1.03	33	0.273
78	A	9	9	1.07	33	0.273
79	A	9	9	1.00	33	0.273
80	A	9	9	1.00	33	0.273
81	A	7	7	1.01	33	0.212
82	A	7	7	1.01	33	0.212
83	A	7	7	1.05	31	0.226
84	A	6	6	1.00	25	0.240
85	A	9	9	1.02	31	0.290

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	9	9	1.06	33	0.273
87	A	8	8	0.97	25	0.320
88	A	8	8	1.01	25	0.320
89	A	6	5	0.59	35	0.143
90	A	6	6	0.73	35	0.171
91	A	5	4	0.70	35	0.114
92	A	2	2	0.59	35	0.057
93	A	5	5	0.69	35	0.143
94	A	4	4	0.64	35	0.114
95	A	5	5	0.81	35	0.143
96	A	7	6	0.84	35	0.171
97	A	7	7	0.75	35	0.200
98	A	6	5	0.59	35	0.143
99	A	6	6	0.72	35	0.171
100	A	5	4	0.70	35	0.114
101	A	2	2	0.58	35	0.057
102	A	5	5	0.69	35	0.143
103	A	4	4	0.64	35	0.114
104	A	5	5	0.80	35	0.143
105	A	7	6	0.83	35	0.171
106	A	7	7	0.74	35	0.200
107	A	6	5	0.58	35	0.143
108	A	6	6	0.70	35	0.171
109	A	5	4	0.69	35	0.114
110	A	2	2	0.57	35	0.057
111	A	5	5	0.68	35	0.143
112	A	4	4	0.63	35	0.114
113	A	5	5	0.79	35	0.143
114	A	7	6	0.81	35	0.171
115	A	7	7	0.72	35	0.200
116	A	6	6	0.73	35	0.171
117	A	5	4	0.70	35	0.114

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	0.59	35	0.057
119	A	5	5	0.69	35	0.143
120	A	4	4	0.64	35	0.114
121	A	5	5	0.81	35	0.143
122	A	7	6	0.84	35	0.171
123	A	7	7	0.75	35	0.200
124	A	6	6	0.70	35	0.171
125	A	5	4	0.69	35	0.114
126	A	2	2	0.57	35	0.057
127	A	5	5	0.68	35	0.143
128	A	4	4	0.63	35	0.114
129	A	5	5	0.79	35	0.143
130	A	7	6	0.81	35	0.171
131	A	7	7	0.72	35	0.200
132	A	6	6	0.70	35	0.171
133	A	5	4	0.69	35	0.114
134	A	2	2	0.57	35	0.057
135	A	5	5	0.68	35	0.143
136	A	4	4	0.63	35	0.114
137	A	5	5	0.79	35	0.143
138	A	7	6	0.81	35	0.171
139	A	7	7	0.72	35	0.200
140	A	5	5	1.04	33	0.152
141	A	5	5	1.04	31	0.161
142	A	4	4	1.00	25	0.160
143	A	5	5	1.09	31	0.161
144	A	5	5	1.07	33	0.152
145	A	5	5	1.08	33	0.152
146	A	5	5	1.04	33	0.152
147	A	5	5	1.04	31	0.161
148	A	4	4	1.00	25	0.160
149	A	5	5	1.09	31	0.161

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	5	5	1.07	33	0.152
151	A	5	5	1.08	33	0.152
152	A	5	5	1.04	33	0.152
153	A	5	5	1.04	31	0.161
154	A	4	4	1.00	25	0.160
155	A	5	5	1.09	31	0.161
156	A	5	5	1.09	33	0.152
157	A	5	5	1.08	33	0.152
158	A	5	5	1.04	33	0.152
159	A	5	5	1.04	31	0.161
160	A	4	4	1.00	25	0.160
161	A	5	5	1.06	31	0.161
162	A	5	5	1.07	33	0.152
163	A	5	5	1.08	33	0.152
164	A	5	5	1.04	33	0.152
165	A	5	5	1.04	31	0.161
166	A	4	4	1.00	25	0.160
167	A	5	5	1.06	31	0.161
168	A	5	5	1.06	33	0.152
169	A	5	5	1.08	33	0.152
170	A	5	5	1.04	33	0.152
171	A	5	5	1.04	31	0.161
172	A	4	4	1.00	25	0.160
173	A	5	5	1.06	31	0.161
174	A	5	5	1.06	33	0.152
175	A	5	5	1.08	33	0.152
176	A	5	5	1.01	33	0.152
177	A	5	5	1.02	33	0.152
178	A	5	5	1.02	33	0.152
179	A	5	5	1.02	33	0.152
180	A	5	5	1.02	33	0.152
181	A	5	5	1.03	33	0.152

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	5	5	1.00	33	0.152
183	A	5	5	1.00	31	0.161
184	A	5	5	1.00	29	0.172
185	A	4	4	0.97	23	0.174
186	A	5	5	1.08	29	0.172
187	A	5	5	1.07	31	0.161
188	A	5	5	0.98	31	0.161
189	A	5	5	0.98	31	0.161
190	A	5	5	1.04	33	0.152
191	A	5	5	1.04	33	0.152
192	A	5	5	1.04	33	0.152
193	A	5	5	1.04	33	0.152
194	A	5	5	1.07	33	0.152
195	A	5	5	1.05	33	0.152
196	A	5	5	1.04	33	0.152
197	A	5	5	1.04	33	0.152
198	A	8	8	1.01	25	0.320
199	A	9	9	1.07	27	0.333
200	A	9	9	1.07	27	0.333
201	A	9	9	1.05	27	0.333
202	A	9	9	1.03	27	0.333
203	A	10	9	1.02	27	0.333
204	A	10	9	1.02	27	0.333
205	A	10	9	1.01	27	0.333
206	A	10	9	1.01	27	0.333
207	A	10	9	1.00	26	0.346
208	A	9	8	1.00	25	0.320
209	A	6	6	1.03	30	0.200
210	A	7	7	1.02	40	0.175
211	A	7	7	1.02	40	0.175
212	A	7	7	1.01	40	0.175
213	A	7	7	1.02	40	0.175

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	7	7	1.02	40	0.175
215	A	7	7	1.00	40	0.175
216	A	7	7	1.05	40	0.175
217	A	7	7	1.03	38	0.184
218	A	7	7	1.03	36	0.194
219	A	6	6	1.03	30	0.200
220	A	7	7	1.00	36	0.194
221	A	7	7	1.04	38	0.184
222	A	7	7	1.07	38	0.184
223	A	7	7	1.04	38	0.184
224	A	7	7	1.03	40	0.175
225	A	7	7	1.03	40	0.175
226	A	7	7	1.03	40	0.175
227	A	7	7	1.03	40	0.175
228	A	7	7	1.03	40	0.175
229	A	7	7	1.03	40	0.175
230	A	7	7	1.03	40	0.175
231	A	7	7	1.03	40	0.175
232	A	8	8	1.01	32	0.250
233	A	9	8	1.00	32	0.250
234	A	10	9	1.02	34	0.265
235	A	10	9	1.02	34	0.265
236	A	10	9	1.01	34	0.265
237	A	10	9	1.01	34	0.265
238	A	6	6	0.98	31	0.194
239	A	15	15	1.09	41	0.366
240	A	13	13	1.08	39	0.333
241	A	12	12	1.08	33	0.364
242	A	11	11	1.10	39	0.282
243	A	11	11	1.13	41	0.268
244	A	13	13	1.14	41	0.317
245	A	13	13	1.09	41	0.317

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
246	A	15	15	1.11	41	0.366
247	A	15	15	1.09	39	0.385
248	A	12	12	1.05	33	0.364
249	A	13	13	1.09	39	0.333
250	A	11	11	1.08	41	0.268
251	A	11	11	1.08	41	0.268
252	A	13	13	1.10	41	0.317
253	A	13	13	1.06	41	0.317
254	A	15	15	1.08	41	0.366
255	A	14	14	1.06	33	0.424
256	A	13	13	1.05	39	0.333
257	A	13	13	1.07	41	0.317
258	A	11	11	1.04	41	0.268
259	A	11	11	1.06	41	0.268
260	A	13	13	1.09	41	0.317
261	A	13	13	1.05	41	0.317
262	A	15	15	1.07	41	0.366
263	A	15	15	1.07	41	0.366
264	A	13	13	1.05	41	0.317
265	A	13	13	1.07	39	0.333
266	A	10	10	1.03	33	0.303
267	A	11	11	1.10	39	0.282
268	A	13	13	1.15	41	0.317
269	A	13	13	1.09	41	0.317
270	A	15	15	1.11	41	0.366
271	A	15	15	1.05	41	0.366
272	A	13	13	1.03	41	0.317
273	A	13	13	1.05	41	0.317
274	A	11	11	1.04	39	0.282
275	A	10	10	1.03	33	0.303
276	A	13	13	1.10	39	0.333
277	A	13	13	1.08	41	0.317

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	15	15	1.10	41	0.366
279	A	15	15	1.05	41	0.366
280	A	13	13	1.03	41	0.317
281	A	13	13	1.05	41	0.317
282	A	11	11	1.04	41	0.268
283	A	11	11	1.06	39	0.282
284	A	12	12	1.06	33	0.364
285	A	13	13	1.05	39	0.333
286	A	15	15	1.10	41	0.366
287	A	12	12	1.03	33	0.364
288	A	13	12	0.62	43	0.279
289	A	11	10	0.60	43	0.233
290	A	5	5	0.66	43	0.116
291	A	2	2	0.52	43	0.047
292	A	7	7	0.54	43	0.163
293	A	7	7	0.54	43	0.163
294	A	10	9	0.69	43	0.209
295	A	12	11	0.69	43	0.256
296	A	12	11	0.62	43	0.256
297	A	13	12	0.61	43	0.279
298	A	11	10	0.59	43	0.233
299	A	5	5	0.65	43	0.116
300	A	2	2	0.51	43	0.047
301	A	7	7	0.53	43	0.163
302	A	7	7	0.53	43	0.163
303	A	10	9	0.68	43	0.209
304	A	12	11	0.68	43	0.256
305	A	12	11	0.61	43	0.256
306	A	13	12	0.59	43	0.279
307	A	11	10	0.57	43	0.233
308	A	5	5	0.63	43	0.116
309	A	2	2	0.50	43	0.047

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	7	7	0.52	43	0.163
311	A	7	7	0.52	43	0.163
312	A	10	9	0.67	43	0.209
313	A	12	11	0.66	43	0.256
314	A	12	11	0.59	43	0.256
315	A	11	10	0.60	43	0.233
316	A	5	5	0.66	43	0.116
317	A	2	2	0.52	43	0.047
318	A	7	7	0.54	43	0.163
319	A	7	7	0.54	43	0.163
320	A	10	9	0.69	43	0.209
321	A	12	11	0.69	43	0.256
322	A	12	11	0.62	43	0.256
323	A	11	10	0.57	43	0.233
324	A	5	5	0.63	43	0.116
325	A	2	2	0.50	43	0.047
326	A	7	7	0.52	43	0.163
327	A	7	7	0.52	43	0.163
328	A	10	9	0.67	43	0.209
329	A	12	11	0.66	43	0.256
330	A	12	11	0.59	43	0.256
331	A	11	10	0.57	43	0.233
332	A	5	5	0.63	43	0.116
333	A	2	2	0.50	43	0.047
334	A	7	7	0.52	43	0.163
335	A	7	7	0.52	43	0.163
336	A	10	9	0.67	43	0.209
337	A	12	11	0.66	43	0.256
338	A	12	11	0.59	43	0.256
339	A	8	8	1.05	39	0.205
340	A	7	7	1.02	33	0.212
341	A	8	8	1.07	39	0.205

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
342	A	8	8	1.06	41	0.195
343	A	8	8	1.08	41	0.195
344	A	8	8	1.05	41	0.195
345	A	8	8	1.05	39	0.205
346	A	7	7	1.02	33	0.212
347	A	8	8	1.07	39	0.205
348	A	8	8	1.08	41	0.195
349	A	8	8	1.08	41	0.195
350	A	8	8	1.05	41	0.195
351	A	8	8	1.05	41	0.195
352	A	8	8	1.05	39	0.205
353	A	7	7	1.02	33	0.212
354	A	8	8	1.03	39	0.205
355	A	8	8	1.08	41	0.195
356	A	8	8	1.07	41	0.195
357	A	8	8	1.05	41	0.195
358	A	8	8	1.05	41	0.195
359	A	8	8	1.05	39	0.205
360	A	7	7	1.00	33	0.212
361	A	8	8	1.05	39	0.205
362	A	8	8	1.07	41	0.195
363	A	8	8	0.94	41	0.195
364	A	8	8	0.95	41	0.195
365	A	8	8	0.95	41	0.195
366	A	8	8	0.95	41	0.195
367	A	8	8	0.95	41	0.195
368	F	0	0	N/A	0.000	N/A
369	A	7	7	0.96	41	0.171
370	A	7	7	1.01	39	0.179
371	A	7	7	1.01	37	0.189
372	A	6	6	0.98	31	0.194
373	A	7	7	1.03	37	0.189

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	8	8	1.05	39	0.205
375	A	8	8	1.01	39	0.205
376	A	8	8	0.99	39	0.205
377	A	8	8	0.96	41	0.195
378	A	8	8	0.96	41	0.195
379	A	8	8	0.96	41	0.195
380	A	8	8	0.99	41	0.195
381	A	8	8	0.96	41	0.195
382	A	8	8	0.96	41	0.195
383	A	8	8	1.01	33	0.242
384	A	9	9	1.07	35	0.257
385	A	9	9	1.07	35	0.257
386	A	9	9	1.05	35	0.257
387	A	9	9	1.03	35	0.257
388	A	10	9	1.02	35	0.257
389	A	10	9	1.02	35	0.257
390	A	10	9	1.01	35	0.257
391	A	10	9	1.01	35	0.257
392	A	10	9	1.00	35	0.257
393	A	9	8	1.00	33	0.242

CHAPTER 3

Listing of integrals

3.1 $\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(A+C)\sin(c+dx)}{d}-\frac{(3A+4C)\sin^3(c+dx)}{3d}+\frac{3(A+2C)\sin^5(c+dx)}{5d}-\frac{(A+4C)\sin^7(c+dx)}{7d}+\frac{C\sin^9(c+dx)}{9d}$$

output
```
(A+C)*sin(d*x+c)/d-1/3*(3*A+4*C)*sin(d*x+c)^3/d+3/5*(A+2*C)*sin(d*x+c)^5/d
-1/7*(A+4*C)*sin(d*x+c)^7/d+1/9*C*sin(d*x+c)^9/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.45

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{A\sin(c+dx)}{d}+\frac{C\sin(c+dx)}{d}-\frac{A\sin^3(c+dx)}{d}-\frac{4C\sin^3(c+dx)}{3d}+\frac{3A\sin^5(c+dx)}{5d}+\frac{6C\sin^5(c+dx)}{5d}-\frac{A\sin^7(c+dx)}{7d}-\frac{4C\sin^7(c+dx)}{7d}+\frac{C\sin^9(c+dx)}{9d}$$

input
```
Integrate[Cos[c + d*x]^7*(A + C*Cos[c + d*x]^2),x]
```

output
```
(A*Sin[c + d*x])/d + (C*Sin[c + d*x])/d - (A*Sin[c + d*x]^3)/d - (4*C*Sin[
c + d*x]^3)/(3*d) + (3*A*Sin[c + d*x]^5)/(5*d) + (6*C*Sin[c + d*x]^5)/(5*d
) - (A*Sin[c + d*x]^7)/(7*d) - (4*C*Sin[c + d*x]^7)/(7*d) + (C*Sin[c + d*x
]^9)/(9*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^7\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3492}$$

$$-\frac{\int \left(1-\sin^2(c+dx)\right)^3\left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{290}$$

$$-\frac{\int \left(C\sin^8(c+dx)-(A+4C)\sin^6(c+dx)+3(A+2C)\sin^4(c+dx)-(3A+4C)\sin^2(c+dx)+A\left(\frac{C}{A}+1\right)\right)d}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{7}(A+4C)\sin^7(c+dx)-\frac{3}{5}(A+2C)\sin^5(c+dx)+\frac{1}{3}(3A+4C)\sin^3(c+dx)-(A+C)\sin(c+dx)-\frac{1}{9}C\sin}{d}$$

input
```
Int[Cos[c + d*x]^7*(A + C*Cos[c + d*x]^2),x]
```

output
```
-((-((A + C)*Sin[c + d*x]) + ((3*A + 4*C)*Sin[c + d*x]^3)/3 - (3*(A + 2*C)
*Sin[c + d*x]^5)/5 + ((A + 4*C)*Sin[c + d*x]^7)/7 - (C*Sin[c + d*x]^9)/9)/
d)
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 8.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

method	result
parallelrisch	$\frac{8820(A+C)\sin(3dx+3c)+252(7A+9C)\sin(5dx+5c)+45(4A+9C)\sin(7dx+7c)+35C\sin(9dx+9c)+44100\left(A+\frac{9C}{10}\right)\mathrm{s}}{80640d}$
derivativedivides	$\frac{\frac{C\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8\cos(dx+c)^6}{7}+\frac{48\cos(dx+c)^4}{35}+\frac{64\cos(dx+c)^2}{35}\right)\sin(dx+c)}{9}+\frac{A\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+}{5}\right.}{7}}{d}$
default	$\frac{\frac{C\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8\cos(dx+c)^6}{7}+\frac{48\cos(dx+c)^4}{35}+\frac{64\cos(dx+c)^2}{35}\right)\sin(dx+c)}{9}+\frac{A\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+}{5}\right.}{7}}{d}$
parts	$\frac{A\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7d}+\frac{C\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8\cos(dx+c)^6}{7}+\frac{48\cos(dx+c)^4}{35}+\right.}{9d}$
risch	$\frac{35\sin(dx+c)A}{64d}+\frac{63C\sin(dx+c)}{128d}+\frac{C\sin(9dx+9c)}{2304d}+\frac{\sin(7dx+7c)A}{448d}+\frac{9\sin(7dx+7c)C}{1792d}+\frac{7\sin(5dx+5c)A}{320d}+$
norman	$\frac{\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{17}}{d}+\frac{8(3A+2C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{8(3A+2C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}}{3d}+\frac{8(17A+19C)\tan\left(\frac{dx}{2}+\right.}{5d}}{\left(1+\tan\right.}$
orering	$-\frac{117469\left(-7\cos(dx+c)^6\left(A+C\cos(dx+c)^2\right)d\sin(dx+c)-2\cos(dx+c)^8Cd\sin(dx+c)\right)}{99225d^2}-\frac{34562\left(-210\cos(dx+c)^4\right.}{}$

input
```
int(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/80640*(8820*(A+C)*sin(3*d*x+3*c)+252*(7*A+9*C)*sin(5*d*x+5*c)+45*(4*A+9*
C)*sin(7*d*x+7*c)+35*C*sin(9*d*x+9*c)+44100*(A+9/10*C)*sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\left(35\,C\cos\left(dx+c\right)^8+5\left(9\,A+8\,C\right)\cos\left(dx+c\right)^6+6\left(9\,A+8\,C\right)\cos\left(dx+c\right)^4+8\left(9\,A+8\,C\right)\cos\left(dx+\right.\right.}{315\,d}$$

input
```
integrate(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/315*(35*C*cos(d*x + c)^8 + 5*(9*A + 8*C)*cos(d*x + c)^6 + 6*(9*A + 8*C)*
cos(d*x + c)^4 + 8*(9*A + 8*C)*cos(d*x + c)^2 + 144*A + 128*C)*sin(d*x + c
)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(80) = 160$.

Time = 0.99 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.16

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{16A\sin^7(c+dx)}{35d} + \frac{8A\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2A\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{A\sin(c+dx)\cos^6(c+dx)}{d} + \frac{128C\sin^9(c+dx)}{315d} + \frac{64C}{} \\ x(A+C\cos^2(c))\cos^7(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**7*(A+C*cos(d*x+c)**2),x)
```

output
```
Piecewise((16*A*sin(c + d*x)**7/(35*d) + 8*A*sin(c + d*x)**5*cos(c + d*x)*
*2/(5*d) + 2*A*sin(c + d*x)**3*cos(c + d*x)**4/d + A*sin(c + d*x)*cos(c +
d*x)**6/d + 128*C*sin(c + d*x)**9/(315*d) + 64*C*sin(c + d*x)**7*cos(c + d
*x)**2/(35*d) + 16*C*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 8*C*sin(c + d
*x)**3*cos(c + d*x)**6/(3*d) + C*sin(c + d*x)*cos(c + d*x)**8/d, Ne(d, 0))
, (x*(A + C*cos(c)**2)*cos(c)**7, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{35\,C\sin(dx+c)^9 - 45\,(A+4\,C)\sin(dx+c)^7 + 189\,(A+2\,C)\sin(dx+c)^5 - 105\,(3\,A+4\,C)\sin(dx+}{315\,d}$$

input
```
integrate(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/315*(35*C*sin(d*x + c)^9 - 45*(A + 4*C)*sin(d*x + c)^7 + 189*(A + 2*C)*s
in(d*x + c)^5 - 105*(3*A + 4*C)*sin(d*x + c)^3 + 315*(A + C)*sin(d*x + c))
/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int \cos^7(c+dx) \left(A+C\cos^2(c+dx)\right) dx = \frac{C\sin(9dx+9c)}{2304d} + \frac{(4A+9C)\sin(7dx+7c)}{1792d} + \frac{(7A+9C)\sin(5dx+5c)}{320d} + \frac{7(A+C)\sin(3dx+3c)}{64d} + \frac{7(10A+9C)\sin(dx+c)}{128d}$$

input
```
integrate(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2304*C*sin(9*d*x + 9*c)/d + 1/1792*(4*A + 9*C)*sin(7*d*x + 7*c)/d + 1/32
0*(7*A + 9*C)*sin(5*d*x + 5*c)/d + 7/64*(A + C)*sin(3*d*x + 3*c)/d + 7/128
*(10*A + 9*C)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 39.82 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int \cos^7(c+dx) \left(A+C\cos^2(c+dx)\right) dx$$

$$= \frac{\frac{C\sin(c+dx)^9}{9} + \left(-\frac{A}{7}-\frac{4C}{7}\right)\sin(c+dx)^7 + \left(\frac{3A}{5}+\frac{6C}{5}\right)\sin(c+dx)^5 + \left(-A-\frac{4C}{3}\right)\sin(c+dx)^3 + (A+}{d}$$

input
```
int(cos(c + d*x)^7*(A + C*cos(c + d*x)^2),x)
```

output
```
((C*sin(c + d*x)^9)/9 - sin(c + d*x)^3*(A + (4*C)/3) + sin(c + d*x)*(A + C
) + sin(c + d*x)^5*((3*A)/5 + (6*C)/5) - sin(c + d*x)^7*(A/7 + (4*C)/7))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$=\frac{\sin\left(dx+c\right)\left(35\sin\left(dx+c\right)^{8}c-45\sin\left(dx+c\right)^{6}a-180\sin\left(dx+c\right)^{6}c+189\sin\left(dx+c\right)^{4}a+378\sin\left(d\right.\right.}{315d}$$

input
```
int(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x)
```

output
```
(sin(c + d*x)*(35*sin(c + d*x)**8*c - 45*sin(c + d*x)**6*a - 180*sin(c + d
*x)**6*c + 189*sin(c + d*x)**4*a + 378*sin(c + d*x)**4*c - 315*sin(c + d*x
)**2*a - 420*sin(c + d*x)**2*c + 315*a + 315*c))/(315*d)
```

3.2 $\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(A+C)\sin(c+dx)}{d}-\frac{(2A+3C)\sin^3(c+dx)}{3d}+\frac{(A+3C)\sin^5(c+dx)}{5d}-\frac{C\sin^7(c+dx)}{7d}$$

output
```
(A+C)*sin(d*x+c)/d-1/3*(2*A+3*C)*sin(d*x+c)^3/d+1/5*(A+3*C)*sin(d*x+c)^5/d
-1/7*C*sin(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{A\sin(c+dx)}{d}+\frac{C\sin(c+dx)}{d}-\frac{2A\sin^3(c+dx)}{3d}-\frac{C\sin^3(c+dx)}{d}+\frac{A\sin^5(c+dx)}{5d}+\frac{3C\sin^5(c+dx)}{5d}-\frac{C\sin^7(c+dx)}{7d}$$

input
```
Integrate[Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2),x]
```

output
```
(A*Sin[c + d*x])/d + (C*Sin[c + d*x])/d - (2*A*Sin[c + d*x]^3)/(3*d) - (C*
Sin[c + d*x]^3)/d + (A*Sin[c + d*x]^5)/(5*d) + (3*C*Sin[c + d*x]^5)/(5*d)
- (C*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^5\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3492}$$

$$-\frac{\int\left(1-\sin^2(c+dx)\right)^2\left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{290}$$

$$-\frac{\int\left(-C\sin^6(c+dx)+(A+3C)\sin^4(c+dx)-(2A+3C)\sin^2(c+dx)+A\left(\frac{C}{A}+1\right)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{5}(A+3C)\sin^5(c+dx)+\frac{1}{3}(2A+3C)\sin^3(c+dx)-(A+C)\sin(c+dx)+\frac{1}{7}C\sin^7(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2),x]
```

output
```
-((-((A + C)*Sin[c + d*x]) + ((2*A + 3*C)*Sin[c + d*x]^3)/3 - ((A + 3*C)*S
in[c + d*x]^5)/5 + (C*Sin[c + d*x]^7)/7)/d)
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{(700A+735C)\sin(3dx+3c)+(84A+147C)\sin(5dx+5c)+15C\sin(7dx+7c)+4200\left(A+\frac{7C}{8}\right)\sin(dx+c)}{6720d}$
derivativedivides	$\frac{\frac{C\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}+\frac{A\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}}{d}$
default	$\frac{\frac{C\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}+\frac{A\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}}{d}$
parts	$\frac{A\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5d}+\frac{C\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7d}$
risch	$\frac{5\sin(dx+c)A}{8d}+\frac{35C\sin(dx+c)}{64d}+\frac{\sin(7dx+7c)C}{448d}+\frac{\sin(5dx+5c)A}{80d}+\frac{7\sin(5dx+5c)C}{320d}+\frac{5\sin(3dx+3c)A}{48d}+7$
norman	$\frac{\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{d}+\frac{4(5A+3C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{4(5A+3C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{3d}+\frac{8(91A+53C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{35d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^7}$
orering	$-\frac{12916\left(-5\cos(dx+c)^4\left(A+C\cos(dx+c)^2\right)d\sin(dx+c)-2\cos(dx+c)^6Cd\sin(dx+c)\right)}{11025d^2}-\frac{94\left(-60\cos(dx+c)^2\left(A+C\right.\right.}{}$

input
```
int(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6720*((700*A+735*C)*sin(3*d*x+3*c)+(84*A+147*C)*sin(5*d*x+5*c)+15*C*sin(
7*d*x+7*c)+4200*(A+7/8*C)*sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\left(15\,C\cos\left(dx+c\right)^6+3\left(7\,A+6\,C\right)\cos\left(dx+c\right)^4+4\left(7\,A+6\,C\right)\cos\left(dx+c\right)^2+56\,A+48\,C\right)\sin\left(dx+\right.}{105\,d}$$

input
```
integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/105*(15*C*cos(d*x + c)^6 + 3*(7*A + 6*C)*cos(d*x + c)^4 + 4*(7*A + 6*C)*
cos(d*x + c)^2 + 56*A + 48*C)*sin(d*x + c)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(60) = 120$.

Time = 0.47 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.10

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{8A\sin^5(c+dx)}{15d} + \frac{4A\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{A\sin(c+dx)\cos^4(c+dx)}{d} + \frac{16C\sin^7(c+dx)}{35d} + \frac{8C\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2C\sin \\ x(A+C\cos^2(c))\cos^5(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(A+C*cos(d*x+c)**2),x)
```

output
```
Piecewise((8*A*sin(c + d*x)**5/(15*d) + 4*A*sin(c + d*x)**3*cos(c + d*x)**
2/(3*d) + A*sin(c + d*x)*cos(c + d*x)**4/d + 16*C*sin(c + d*x)**7/(35*d) +
 8*C*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*C*sin(c + d*x)**3*cos(c + d
*x)**4/d + C*sin(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(A + C*cos(c)**
2)*cos(c)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{15\,C\sin(dx+c)^7 - 21\,(A+3\,C)\sin(dx+c)^5 + 35\,(2\,A+3\,C)\sin(dx+c)^3 - 105\,(A+C)\sin(dx+}{105\,d}$$

input
```
integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/105*(15*C*sin(d*x + c)^7 - 21*(A + 3*C)*sin(d*x + c)^5 + 35*(2*A + 3*C)
*sin(d*x + c)^3 - 105*(A + C)*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{\left(4\,A+7\,C\right)\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{\left(20\,A+21\,C\right)\sin\left(3\,dx+3\,c\right)}{192\,d}+\frac{5\left(8\,A+7\,C\right)\sin\left(dx+c\right)}{64\,d}$$

input
```
integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
1/448*C*sin(7*d*x + 7*c)/d + 1/320*(4*A + 7*C)*sin(5*d*x + 5*c)/d + 1/192*
(20*A + 21*C)*sin(3*d*x + 3*c)/d + 5/64*(8*A + 7*C)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 40.60 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{\frac{C\sin(c+d\,x)^7}{7}+\left(-\frac{A}{5}-\frac{3\,C}{5}\right)\,\sin\left(c+d\,x\right)^5+\left(\frac{2\,A}{3}+C\right)\,\sin\left(c+d\,x\right)^3+\left(-A-C\right)\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^5*(A + C*cos(c + d*x)^2),x)
```

output
```
-(sin(c + d*x)^3*((2*A)/3 + C) + (C*sin(c + d*x)^7)/7 - sin(c + d*x)*(A +
C) - sin(c + d*x)^5*(A/5 + (3*C)/5))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sin\left(dx+c\right)\left(-15\sin\left(dx+c\right)^6c+21\sin\left(dx+c\right)^4a+63\sin\left(dx+c\right)^4c-70\sin\left(dx+c\right)^2a-105\sin\left(d\right.\right.}{105d}$$

input
```
int(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x)
```

output
```
(sin(c + d*x)*( - 15*sin(c + d*x)**6*c + 21*sin(c + d*x)**4*a + 63*sin(c +
 d*x)**4*c - 70*sin(c + d*x)**2*a - 105*sin(c + d*x)**2*c + 105*a + 105*c)
)/(105*d)
```

3.3 $\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(A+C)\sin(c+dx)}{d}-\frac{(A+2C)\sin^3(c+dx)}{3d}+\frac{C\sin^5(c+dx)}{5d}$$

output
```
(A+C)*sin(d*x+c)/d-1/3*(A+2*C)*sin(d*x+c)^3/d+1/5*C*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{A\sin(c+dx)}{d}+\frac{C\sin(c+dx)}{d}-\frac{A\sin^3(c+dx)}{3d}-\frac{2C\sin^3(c+dx)}{3d}+\frac{C\sin^5(c+dx)}{5d}$$

input
```
Integrate[Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2),x]
```

output
```
(A*Sin[c + d*x])/d + (C*Sin[c + d*x])/d - (A*Sin[c + d*x]^3)/(3*d) - (2*C*
Sin[c + d*x]^3)/(3*d) + (C*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3492}$$

$$-\frac{\int \left(1-\sin^2(c+dx)\right)\left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{290}$$

$$-\frac{\int \left(C\sin^4(c+dx)-(A+2C)\sin^2(c+dx)+A\left(\frac{C}{A}+1\right)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{3}(A+2C)\sin^3(c+dx)-(A+C)\sin(c+dx)-\frac{1}{5}C\sin^5(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2),x]
```

output
```
-((-((A + C)*Sin[c + d*x]) + ((A + 2*C)*Sin[c + d*x]^3)/3 - (C*Sin[c + d*x
]^5)/5)/d)
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{(20A+25C)\sin(3dx+3c)+3C\sin(5dx+5c)+180\left(A+\frac{5C}{6}\right)\sin(dx+c)}{240d}$
derivativedivides	$\frac{\frac{C\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+\frac{A\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{\frac{C\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+\frac{A\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
parts	$\frac{A\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3d}+\frac{C\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5d}$
risch	$\frac{3\sin(dx+c)A}{4d}+\frac{5C\sin(dx+c)}{8d}+\frac{\sin(5dx+5c)C}{80d}+\frac{\sin(3dx+3c)A}{12d}+\frac{5\sin(3dx+3c)C}{48d}$
norman	$\frac{\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}+\frac{8(2A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{8(2A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}+\frac{4(25A+29C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$
orering	$-\frac{259\left(-3\cos(dx+c)^2\left(A+C\cos(dx+c)^2\right)d\sin(dx+c)-2\cos(dx+c)^4Cd\sin(dx+c)\right)}{225d^2}-\frac{7\left(-6d^3\sin(dx+c)^3\left(A+C\right.\right.}{}$

input
```
int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/240*((20*A+25*C)*sin(3*d*x+3*c)+3*C*sin(5*d*x+5*c)+180*(A+5/6*C)*sin(d*x
+c))/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\left(3\,C\cos\left(dx+c\right)^4+\left(5\,A+4\,C\right)\cos\left(dx+c\right)^2+10\,A+8\,C\right)\sin\left(dx+c\right)}{15\,d}$$

input
```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/15*(3*C*cos(d*x + c)^4 + (5*A + 4*C)*cos(d*x + c)^2 + 10*A + 8*C)*sin(d*
x + c)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(42) = 84.

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.10

$$\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \begin{cases} \frac{2A\sin^3(c+dx)}{3d}+\frac{A\sin(c+dx)\cos^2(c+dx)}{d}+\frac{8C\sin^5(c+dx)}{15d}+\frac{4C\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{C\sin(c+dx)\cos^4(c+dx)}{d} & \text{for } d\neq \\ x(A+C\cos^2(c))\cos^3(c) & \text{otherwi} \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2),x)
```

output
```
Piecewise((2*A*sin(c + d*x)**3/(3*d) + A*sin(c + d*x)*cos(c + d*x)**2/d +
8*C*sin(c + d*x)**5/(15*d) + 4*C*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + C
*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**3
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^3(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = \frac{3\,C\sin\left(dx+c\right)^5 - 5\,(A+2\,C)\sin\left(dx+c\right)^3 + 15\,(A+C)\sin\left(dx+c\right)}{15\,d}$$

input
```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/15*(3*C*sin(d*x + c)^5 - 5*(A + 2*C)*sin(d*x + c)^3 + 15*(A + C)*sin(d*x
 + c))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \cos^3(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = \frac{3\,C\sin\left(dx+c\right)^5 - 5\,A\sin\left(dx+c\right)^3 - 10\,C\sin\left(dx+c\right)^3 + 15\,A\sin\left(dx+c\right) + 15\,C\sin\left(dx+c\right)}{15\,d}$$

input
```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
1/15*(3*C*sin(d*x + c)^5 - 5*A*sin(d*x + c)^3 - 10*C*sin(d*x + c)^3 + 15*A
*sin(d*x + c) + 15*C*sin(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 40.83 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\frac{C\sin(c+d\,x)^5}{5}+\left(-\frac{A}{3}-\frac{2\,C}{3}\right)\,\sin\left(c+d\,x\right)^3+\left(A+C\right)\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^3*(A + C*cos(c + d*x)^2),x)
```

output
```
((C*sin(c + d*x)^5)/5 + sin(c + d*x)*(A + C) - sin(c + d*x)^3*(A/3 + (2*C)
/3))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sin\left(dx+c\right)\left(3\sin\left(dx+c\right)^4c-5\sin\left(dx+c\right)^2a-10\sin\left(dx+c\right)^2c+15a+15c\right)}{15d}$$

input
```
int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x)
```

output
```
(sin(c + d*x)*(3*sin(c + d*x)**4*c - 5*sin(c + d*x)**2*a - 10*sin(c + d*x)
**2*c + 15*a + 15*c))/(15*d)
```

3.4 $\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(A+C)\sin(c+dx)}{d}-\frac{C\sin^3(c+dx)}{3d}$$

output
```
(A+C)*sin(d*x+c)/d-1/3*C*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{A\cos(dx)\sin(c)}{d}+\frac{A\cos(c)\sin(dx)}{d}+\frac{C\sin(c+dx)}{d}-\frac{C\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cos[c + d*x]*(A + C*Cos[c + d*x]^2),x]
```

output
```
(A*Cos[d*x]*Sin[c])/d + (A*Cos[c]*Sin[d*x])/d + (C*Sin[c + d*x])/d - (C*Si
n[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$
$$\Big\downarrow \text{3492}$$
$$-\frac{\int \left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\frac{1}{3}C\sin^3(c+dx)-(A+C)\sin(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]*(A + C*Cos[c + d*x]^2),x]
```

output
```
-((-((A + C)*Sin[c + d*x]) + (C*Sin[c + d*x]^3)/3)/d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result
parallelrisch	$\frac{C\sin(3dx+3c)+12\left(A+\frac{3C}{4}\right)\sin(dx+c)}{12d}$
derivativedivides	$\frac{\frac{C\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+A\sin(dx+c)}{d}$
default	$\frac{\frac{C\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+A\sin(dx+c)}{d}$
parts	$\frac{\sin(dx+c)A}{d}+\frac{C\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3d}$
risch	$\frac{\sin(dx+c)A}{d}+\frac{3C\sin(dx+c)}{4d}+\frac{\sin(3dx+3c)C}{12d}$
norman	$\frac{\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{4(3A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$
orering	$-\frac{10\left(-d\sin(dx+c)\left(A+C\cos(dx+c)^2\right)-2C\cos(dx+c)^2d\sin(dx+c)\right)}{9d^2}-\frac{d^3\sin(dx+c)\left(A+C\cos(dx+c)^2\right)+20C\cos}{9d}$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(C*sin(3*d*x+3*c)+12*(A+3/4*C)*sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int\cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{\left(C\cos\left(dx+c\right)^2+3\,A+2\,C\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sin(d*x + c)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \begin{cases} \frac{A\sin(c+dx)}{d} + \frac{2C\sin^3(c+dx)}{3d} + \frac{C\sin(c+dx)\cos^2(c+dx)}{d} & \text{for } d \neq 0 \\ x(A+C\cos^2(c))\cos(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2),x)
```

output
```
Piecewise((A*sin(c + d*x)/d + 2*C*sin(c + d*x)**3/(3*d) + C*sin(c + d*x)*c
os(c + d*x)**2/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)C - 3A\sin(dx+c)}{3d}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/3*((sin(d*x + c)^3 - 3*sin(d*x + c))*C - 3*A*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)C-3\,A\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/3*((sin(d*x + c)^3 - 3*sin(d*x + c))*C - 3*A*sin(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{\frac{C\sin(c+d\,x)^3}{3}-\sin\left(c+d\,x\right)\,\left(A+C\right)}{d}$$

input
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2),x)
```

output
```
-((C*sin(c + d*x)^3)/3 - sin(c + d*x)*(A + C))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sin\left(dx+c\right)\left(-\sin\left(dx+c\right)^2c+3a+3c\right)}{3d}$$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2),x)
```

output
```
(sin(c + d*x)*( - sin(c + d*x)**2*c + 3*a + 3*c))/(3*d)
```

3.5 $\int (A + C\cos^2(c + dx))\sec(c + dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int (A + C\cos^2(c + dx))\sec(c + dx)\,dx = \frac{A\text{arctanh}(\sin(c + dx))}{d} + \frac{C\sin(c + dx)}{d}$$

output
```
A*arctanh(sin(d*x+c))/d+C*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int (A + C\cos^2(c + dx))\sec(c + dx)\,dx = \frac{A\coth^{-1}(\sin(c + dx))}{d} + \frac{C\cos(dx)\sin(c)}{d} + \frac{C\cos(c)\sin(dx)}{d}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
(A*ArcCoth[Sin[c + d*x]])/d + (C*Cos[d*x]*Sin[c])/d + (C*Cos[c]*Sin[d*x])/
d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3493, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3493}$$

$$A\int \sec(c+dx)dx+\frac{C\sin(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{C\sin(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{A\text{arctanh}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}$$

input
```
Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
(A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))+C\sin(dx+c)}{d}$	30
default	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))+C\sin(dx+c)}{d}$	30
parts	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{C\sin(dx+c)}{d}$	32
parallelrisch	$\frac{A\left(-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)+C\sin(dx+c)}{d}$	43
risch	$-\frac{iC\,e^{i(dx+c)}}{2d}+\frac{iC\,e^{-i(dx+c)}}{2d}+\frac{A\ln(e^{i(dx+c)}+i)}{d}-\frac{A\ln(e^{i(dx+c)}-i)}{d}$	71
norman	$\frac{\frac{2C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	86

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(A*ln(sec(d*x+c)+tan(d*x+c))+C*sin(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \left(A + C\cos^2(c + dx)\right)\sec(c + dx)\,dx = \frac{A\log\left(\sin\left(dx + c\right) + 1\right) - A\log\left(-\sin\left(dx + c\right) + 1\right) + 2\,C\sin\left(dx + c\right)}{2\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(A*log(sin(d*x + c) + 1) - A*log(-sin(d*x + c) + 1) + 2*C*sin(d*x + c)
)/d
```

Sympy [F]

$$\int \left(A + C\cos^2(c + dx)\right)\sec(c + dx)\,dx = \int \left(A + C\cos^2\left(c + dx\right)\right)\sec\left(c + dx\right)\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \left(A + C\cos^2(c + dx)\right)\sec(c + dx)\,dx = \frac{A\log\left(\sin\left(dx + c\right) + 1\right) - A\log\left(\sin\left(dx + c\right) - 1\right) + 2\,C\sin\left(dx + c\right)}{2\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(A*log(sin(d*x + c) + 1) - A*log(sin(d*x + c) - 1) + 2*C*sin(d*x + c))
/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int (A + C\cos^2(c + dx)) \sec(c + dx) \, dx$$
$$= \frac{A\log(|\sin(dx + c) + 1|) - A\log(|\sin(dx + c) - 1|) + 2C\sin(dx + c)}{2d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")
```

output
```
1/2*(A*log(abs(sin(d*x + c) + 1)) - A*log(abs(sin(d*x + c) - 1)) + 2*C*sin
(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (A + C\cos^2(c + dx)) \sec(c + dx) \, dx = \frac{C \sin(c + d x) + A \operatorname{atanh}(\sin(c + d x))}{d}$$

input
```
int((A + C*cos(c + d*x)^2)/cos(c + d*x),x)
```

output
```
(C*sin(c + d*x) + A*atanh(sin(c + d*x)))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \left(A + C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a + \log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a + \sin\left(dx+c\right)c}{d}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a + sin(c + d*
x)*c)/d
```

3.6 $\int (A + C\cos^2(c + dx))\sec^3(c + dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 40

$$\int (A + C\cos^2(c + dx))\sec^3(c + dx)\,dx = \frac{(A + 2C)\text{arctanh}(\sin(c + dx))}{2d} + \frac{A\sec(c + dx)\tan(c + dx)}{2d}$$

output
```
1/2*(A+2*C)*arctanh(sin(d*x+c))/d+1/2*A*sec(d*x+c)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int (A + C\cos^2(c + dx))\sec^3(c + dx)\,dx = \frac{C\coth^{-1}(\sin(c + dx))}{d} + \frac{A\text{arctanh}(\sin(c + dx))}{2d} + \frac{A\sec(c + dx)\tan(c + dx)}{2d}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
(C*ArcCoth[Sin[c + d*x]])/d + (A*ArcTanh[Sin[c + d*x]])/(2*d) + (A*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3491, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{ 3491}$$

$$\frac{1}{2}(A+2C)\int \sec(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}(A+2C)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{(A+2C)\text{arctanh}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
((A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*Sec[c + d*x]*Tan[c + d*x])/(2
*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+C\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+C\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parts	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{C\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parallelrisch	$\frac{-(\cos(2dx+2c)+1)(A+2C)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(\cos(2dx+2c)+1)(A+2C)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2A\sin(dx+c)}{2d(\cos(2dx+2c)+1)}$
risch	$-\frac{iA\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{A\ln\left(e^{i(dx+c)}-i\right)}{2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)C}{d}+\frac{A\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}+i\right)C}{d}$
norman	$\frac{\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}+\frac{3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^2}-\frac{(A+2C)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{(A+2C)\ln(}{}$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+C*ln(sec(
d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \left(A + C\cos^2(c + dx)\right)\sec^3(c + dx)\,dx$$
$$= \frac{(A + 2\,C)\cos{(dx + c)}^2\log{(\sin{(dx + c)} + 1)} - (A + 2\,C)\cos{(dx + c)}^2\log{(-\sin{(dx + c)} + 1)} + 2\,A\sin}{4\,d\cos{(dx + c)}^2}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*((A + 2*C)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A + 2*C)*cos(d*x +
c)^2*log(-sin(d*x + c) + 1) + 2*A*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \left(A + C\cos^2(c + dx)\right)\sec^3(c + dx)\,dx = \int \left(A + C\cos^2{(c + dx)}\right)\sec^3{(c + dx)}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \left(A + C\cos^2(c + dx)\right)\sec^3(c + dx)\,dx$$
$$= \frac{(A + 2\,C)\log{(\sin{(dx + c)} + 1)} - (A + 2\,C)\log{(\sin{(dx + c)} - 1)} - \frac{2\,A\sin(dx + c)}{\sin(dx + c)^2 - 1}}{4\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*((A + 2*C)*log(sin(d*x + c) + 1) - (A + 2*C)*log(sin(d*x + c) - 1) - 2
*A*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{(A+2\,C)\log\left(|\sin\left(dx+c\right)+1|\right)-(A+2\,C)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\,A\sin(dx+c)}{\sin(dx+c)^2-1}}{4\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*((A + 2*C)*log(abs(sin(d*x + c) + 1)) - (A + 2*C)*log(abs(sin(d*x + c)
 - 1)) - 2*A*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{\mathrm{atanh}(\sin\left(c+d\,x\right))\,\left(\frac{A}{2}+C\right)}{d} - \frac{A\,\sin\left(c+d\,x\right)}{2\,d\,\left(\sin\left(c+d\,x\right)^2-1\right)}$$

input
```
int((A + C*cos(c + d*x)^2)/cos(c + d*x)^3,x)
```

output
```
(atanh(sin(c + d*x))*(A/2 + C))/d - (A*sin(c + d*x))/(2*d*(sin(c + d*x)^2
- 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.42

$$\int \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2 a-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2 c+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 2*log(tan((c + d*x)/2) -
 1)*sin(c + d*x)**2*c + log(tan((c + d*x)/2) - 1)*a + 2*log(tan((c + d*x)/
2) - 1)*c + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 2*log(tan((c + d
*x)/2) + 1)*sin(c + d*x)**2*c - log(tan((c + d*x)/2) + 1)*a - 2*log(tan((c
 + d*x)/2) + 1)*c - sin(c + d*x)*a)/(2*d*(sin(c + d*x)**2 - 1))
```

3.7 $\int \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\frac{(3A+4C)\text{arctanh}(\sin(c+dx))}{8d}+\frac{(3A+4C)\sec(c+dx)\tan(c+dx)}{8d}+\frac{A\sec^3(c+dx)\tan(c+dx)}{4d}$$

output

```
1/8*(3*A+4*C)*arctanh(sin(d*x+c))/d+1/8*(3*A+4*C)*sec(d*x+c)*tan(d*x+c)/d+
1/4*A*sec(d*x+c)^3*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int (A + C\cos^2(c + dx))\sec^5(c + dx)\,dx = \frac{3A\text{arctanh}(\sin(c + dx))}{8d} + \frac{C\text{arctanh}(\sin(c + dx))}{2d} + \frac{3A\sec(c + dx)\tan(c + dx)}{8d} + \frac{C\sec(c + dx)\tan(c + dx)}{2d} + \frac{A\sec^3(c + dx)\tan(c + dx)}{4d}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

output
```
(3*A*ArcTanh[Sin[c + d*x]])/(8*d) + (C*ArcTanh[Sin[c + d*x]])/(2*d) + (3*A
*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (C*Sec[c + d*x]*Tan[c + d*x])/(2*d) +
(A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3491, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c + dx)\left(A + C\cos^2(c + dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A + C\sin\left(c + dx + \frac{\pi}{2}\right)^2}{\sin\left(c + dx + \frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{1}{4}(3A+4C)\int \sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}(3A+4C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\big\downarrow$ 4255

$$\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}$$

$\big\downarrow$ 4257

$$\frac{1}{4}(3A+4C)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}$$

input
```
Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

output
```
(A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A + 4*C)*(ArcTanh[Sin[c + d*x]
]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(\ldots}{2}\right.}{d}$
default	$\frac{A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(\ldots}{2}\right.}{d}$
parts	$\frac{A\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\mathrm{t}\ldots}{2}\right.}{d}$
parallelrisch	$\frac{-6\left(A+\frac{4C}{3}\right)\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+6\left(A+\frac{4C}{3}\right)\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln\left(\mathrm{ta}\ldots\right.}{4d(\cos(4dx+4c)+4\cos(2dx+2c)+3)}$
risch	$-\frac{i\,e^{i(dx+c)}\left(3A\,e^{6i(dx+c)}+4C\,e^{6i(dx+c)}+11A\,e^{4i(dx+c)}+4C\,e^{4i(dx+c)}-11A\,e^{2i(dx+c)}-4C\,e^{2i(dx+c)}-3A-4C\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}-3\ldots$
norman	$\frac{\frac{(5A+4C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(5A+4C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{4d}+\frac{(7A-4C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2d}+\frac{(7A-4C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{2d}+\frac{(13A+4C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4d}\ldots}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^4}$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+t
an(d*x+c)))+C*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \left(A + C\cos^2(c + dx)\right)\sec^5(c + dx)\,dx$$
$$= \frac{(3\,A + 4\,C)\cos\left(dx + c\right)^4\log\left(\sin\left(dx + c\right) + 1\right) - (3\,A + 4\,C)\cos\left(dx + c\right)^4\log\left(-\sin\left(dx + c\right) + 1\right) + 2\,(\cdots}{16\,d\cos\left(dx + c\right)^4}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/16*((3*A + 4*C)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*A + 4*C)*cos(d
*x + c)^4*log(-sin(d*x + c) + 1) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*si
n(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \left(A + C\cos^2(c + dx)\right)\sec^5(c + dx)\,dx = \int \left(A + C\cos^2\left(c + dx\right)\right)\sec^5\left(c + dx\right)dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.39

$$\int \left(A + C\cos^2(c + dx)\right)\sec^5(c + dx)\,dx$$
$$= \frac{(3\,A + 4\,C)\log\left(\sin\left(dx + c\right) + 1\right) - (3\,A + 4\,C)\log\left(\sin\left(dx + c\right) - 1\right) - \frac{2\left((3\,A + 4\,C)\sin(dx + c)^3 - (5\,A + 4\,C)\sin(dx\cdots\right)}{\sin(dx + c)^4 - 2\,\sin(dx + c)^2 + 1}}{16\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/16*((3*A + 4*C)*log(sin(d*x + c) + 1) - (3*A + 4*C)*log(sin(d*x + c) - 1
) - 2*((3*A + 4*C)*sin(d*x + c)^3 - (5*A + 4*C)*sin(d*x + c))/(sin(d*x + c
)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int \left(A + C\cos^2(c + dx)\right)\sec^5(c + dx)\,dx$$
$$= \frac{(3\,A + 4\,C)\log\left(|\sin\left(dx + c\right) + 1|\right) - (3\,A + 4\,C)\log\left(|\sin\left(dx + c\right) - 1|\right) - \frac{2\left(3\,A\sin(dx+c)^3 + 4\,C\sin(dx+c)^3 - 5\,A\right.}{\left(\sin(dx+c)^2 - 1\right.}}{16\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="giac")
```

output
```
1/16*((3*A + 4*C)*log(abs(sin(d*x + c) + 1)) - (3*A + 4*C)*log(abs(sin(d*x
 + c) - 1)) - 2*(3*A*sin(d*x + c)^3 + 4*C*sin(d*x + c)^3 - 5*A*sin(d*x + c
) - 4*C*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \left(A + C\cos^2(c + dx)\right)\sec^5(c + dx)\,dx$$
$$= \frac{\sin\left(c + d\,x\right)\,\left(\frac{5\,A}{8} + \frac{C}{2}\right) - \sin\left(c + d\,x\right)^3\,\left(\frac{3\,A}{8} + \frac{C}{2}\right)}{d\,\left(\sin\left(c + d\,x\right)^4 - 2\sin\left(c + d\,x\right)^2 + 1\right)} + \frac{\mathrm{atanh}(\sin\left(c + d\,x\right))\,\left(\frac{3\,A}{8} + \frac{C}{2}\right)}{d}$$

input
```
int((A + C*cos(c + d*x)^2)/cos(c + d*x)^5,x)
```

output
```
(sin(c + d*x)*((5*A)/8 + C/2) - sin(c + d*x)^3*((3*A)/8 + C/2))/(d*(sin(c
+ d*x)^4 - 2*sin(c + d*x)^2 + 1)) + (atanh(sin(c + d*x))*((3*A)/8 + C/2))/
d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 312, normalized size of antiderivative = 4.46

$$\int \left(A + C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$= \frac{-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4 a - 4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4 c + 6\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

output
```
( - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 4*log(tan((c + d*x)/2)
 - 1)*sin(c + d*x)**4*c + 6*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a +
8*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 3*log(tan((c + d*x)/2) - 1
)*a - 4*log(tan((c + d*x)/2) - 1)*c + 3*log(tan((c + d*x)/2) + 1)*sin(c +
d*x)**4*a + 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c - 6*log(tan((c +
 d*x)/2) + 1)*sin(c + d*x)**2*a - 8*log(tan((c + d*x)/2) + 1)*sin(c + d*x)
**2*c + 3*log(tan((c + d*x)/2) + 1)*a + 4*log(tan((c + d*x)/2) + 1)*c - 3*
sin(c + d*x)**3*a - 4*sin(c + d*x)**3*c + 5*sin(c + d*x)*a + 4*sin(c + d*x
)*c)/(8*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.8 $\int (A + C\cos^2(c+dx))\sec^7(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\begin{aligned}\int (A+C\cos^2(c+dx))\sec^7(c+dx)\,dx = {} & \frac{(5A+6C)\text{arctanh}(\sin(c+dx))}{16d} \\ & + \frac{(5A+6C)\sec(c+dx)\tan(c+dx)}{16d} \\ & + \frac{(5A+6C)\sec^3(c+dx)\tan(c+dx)}{24d} \\ & + \frac{A\sec^5(c+dx)\tan(c+dx)}{6d}\end{aligned}$$

output
```
1/16*(5*A+6*C)*arctanh(sin(d*x+c))/d+1/16*(5*A+6*C)*sec(d*x+c)*tan(d*x+c)/
d+1/24*(5*A+6*C)*sec(d*x+c)^3*tan(d*x+c)/d+1/6*A*sec(d*x+c)^5*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.40

$$\int \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\frac{5A\text{arctanh}(\sin(c+dx))}{16d}+\frac{3C\text{arctanh}(\sin(c+dx))}{8d}+\frac{5A\sec(c+dx)\tan(c+dx)}{16d}+\frac{3C\sec(c+dx)\tan(c+dx)}{8d}+\frac{5A\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{C\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{A\sec^5(c+dx)\tan(c+dx)}{6d}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

output
```
(5*A*ArcTanh[Sin[c + d*x]])/(16*d) + (3*C*ArcTanh[Sin[c + d*x]])/(8*d) + (
5*A*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (3*C*Sec[c + d*x]*Tan[c + d*x])/(8
*d) + (5*A*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (C*Sec[c + d*x]^3*Tan[c +
 d*x])/(4*d) + (A*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3491, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\Big\downarrow \text{ 3491}$$

$$\frac{1}{6}(5A+6C)\int \sec^5(c+dx)dx+\frac{A\tan(c+dx)\sec^5(c+dx)}{6d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{6}(5A+6C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{A\tan(c+dx)\sec^5(c+dx)}{6d}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{1}{6}(5A+6C)\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{A\tan(c+dx)\sec^5(c+dx)}{6d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{6}(5A+6C)\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{A\tan(c+dx)\sec^5(c+dx)}{6d}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{1}{6}(5A+6C)\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{A\tan(c+dx)\sec^5(c+dx)}{6d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{6}(5A+6C)\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{A\tan(c+dx)\sec^5(c+dx)}{6d}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{1}{6}(5A+6C)\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{A\tan(c+dx)\sec^5(c+dx)}{6d}$$

input
```
Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

output
```
(A*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + ((5*A + 6*C)*((Sec[c + d*x]^3*Tan[
c + d*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c +
d*x])/(2*d)))/4))/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{A\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+C\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(}{8}\right.\right.}{d}$
default	$\frac{A\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+C\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(}{8}\right.\right.}{d}$
parts	$\frac{A\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}+\frac{C\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\,\mathrm{se}}{}\right.\right.}{}$
parallelrisch	$\frac{-225\left(\frac{\cos(6dx+6c)}{15}+\frac{2\cos(4dx+4c)}{5}+\cos(2dx+2c)+\frac{2}{3}\right)\left(A+\frac{6C}{5}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+225\left(\frac{\cos(6dx+6c)}{15}+\frac{2\cos(4dx+4c)}{5}\right.}{48d(\cos(6dx+6c)+6}$
risch	$-\frac{i e^{i(dx+c)}\left(15A\,e^{10i(dx+c)}+18C\,e^{10i(dx+c)}+85A\,e^{8i(dx+c)}+102C\,e^{8i(dx+c)}+198A\,e^{6i(dx+c)}+84C\,e^{6i(dx+c)}-198A\,e\right.}{24d\left(e^{2i(dx+c)}+1\right)^6}$
norman	$\frac{\frac{(11A+10C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{(11A+10C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}}{8d}+\frac{7(19A-6C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{24d}+\frac{7(19A-6C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{24d}+\frac{(71A+18C)\tan}{24d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
1/d*(A*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)+
5/16*ln(sec(d*x+c)+tan(d*x+c)))+C*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan
(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

$$\int \left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$$
$$=\frac{3\,(5\,A+6\,C)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,A+6\,C)\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+}{96\,d\cos\left(dx+c\right)^6}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/96*(3*(5*A + 6*C)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(5*A + 6*C)*c
os(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(3*(5*A + 6*C)*cos(d*x + c)^4 + 2
*(5*A + 6*C)*cos(d*x + c)^2 + 8*A)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)]

Timed out.

$$\int (A + C\cos^2(c + dx))\sec^7(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

$$\int (A + C\cos^2(c + dx))\sec^7(c + dx)\,dx$$
$$= \frac{3(5A + 6C)\log(\sin(dx + c) + 1) - 3(5A + 6C)\log(\sin(dx + c) - 1) - \frac{2\left(3(5A+6C)\sin(dx+c)^5 - 8(5A+6C\right.}{\sin(dx+c)^6 - 3\sin(dx}}{96\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/96*(3*(5*A + 6*C)*log(sin(d*x + c) + 1) - 3*(5*A + 6*C)*log(sin(d*x + c)
 - 1) - 2*(3*(5*A + 6*C)*sin(d*x + c)^5 - 8*(5*A + 6*C)*sin(d*x + c)^3 + 3
*(11*A + 10*C)*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*
x + c)^2 - 1))/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

$$\int \left(A + C\cos^2(c + dx)\right)\sec^7(c + dx)\,dx$$

$$= \frac{3\,(5\,A + 6\,C)\log\left(\left|\sin\left(dx + c\right) + 1\right|\right) - 3\,(5\,A + 6\,C)\log\left(\left|\sin\left(dx + c\right) - 1\right|\right) - \frac{2\left(15\,A\sin(dx+c)^5 + 18\,C\sin(dx+c\right.}{}}{96\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="giac")
```

output
```
1/96*(3*(5*A + 6*C)*log(abs(sin(d*x + c) + 1)) - 3*(5*A + 6*C)*log(abs(sin
(d*x + c) - 1)) - 2*(15*A*sin(d*x + c)^5 + 18*C*sin(d*x + c)^5 - 40*A*sin(
d*x + c)^3 - 48*C*sin(d*x + c)^3 + 33*A*sin(d*x + c) + 30*C*sin(d*x + c))/
(sin(d*x + c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 41.02 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \left(A + C\cos^2(c + dx)\right)\sec^7(c + dx)\,dx$$

$$= \frac{\operatorname{atanh}(\sin\left(c + d\,x\right))\,\left(\frac{5\,A}{16} + \frac{3\,C}{8}\right)}{d} - \frac{\left(\frac{5\,A}{16} + \frac{3\,C}{8}\right)\,\sin\left(c + d\,x\right)^5 + \left(-\frac{5\,A}{6} - C\right)\,\sin\left(c + d\,x\right)^3 + \left(\frac{11\,A}{16} + \frac{5\,C}{8}\right)\,\sin\left(c + d\,x\right)}{d\,\left(\sin\left(c + d\,x\right)^6 - 3\sin\left(c + d\,x\right)^4 + 3\sin\left(c + d\,x\right)^2 - 1\right)}$$

input
```
int((A + C*cos(c + d*x)^2)/cos(c + d*x)^7,x)
```

output
```
(atanh(sin(c + d*x))*((5*A)/16 + (3*C)/8))/d - (sin(c + d*x)*((11*A)/16 +
(5*C)/8) - sin(c + d*x)^3*((5*A)/6 + C) + sin(c + d*x)^5*((5*A)/16 + (3*C)
/8))/(d*(3*sin(c + d*x)^2 - 3*sin(c + d*x)^4 + sin(c + d*x)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 436, normalized size of antiderivative = 4.45

$$\int \left(A + C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$$
$$= \frac{-15\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^6 a - 18\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^6 c + 45\log\left(\tan\left(\frac{dx}{2}+\right.\right.}{}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

output
```
( - 15*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**6*a - 18*log(tan((c + d*x)/
2) - 1)*sin(c + d*x)**6*c + 45*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a
 + 54*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*c - 45*log(tan((c + d*x)/2
) - 1)*sin(c + d*x)**2*a - 54*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c
+ 15*log(tan((c + d*x)/2) - 1)*a + 18*log(tan((c + d*x)/2) - 1)*c + 15*log
(tan((c + d*x)/2) + 1)*sin(c + d*x)**6*a + 18*log(tan((c + d*x)/2) + 1)*si
n(c + d*x)**6*c - 45*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*a - 54*log(
tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c + 45*log(tan((c + d*x)/2) + 1)*sin
(c + d*x)**2*a + 54*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*c - 15*log(t
an((c + d*x)/2) + 1)*a - 18*log(tan((c + d*x)/2) + 1)*c - 15*sin(c + d*x)*
*5*a - 18*sin(c + d*x)**5*c + 40*sin(c + d*x)**3*a + 48*sin(c + d*x)**3*c
- 33*sin(c + d*x)*a - 30*sin(c + d*x)*c)/(48*d*(sin(c + d*x)**6 - 3*sin(c
+ d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.9 $\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 117

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{5}{128}(8A+7C)x+\frac{5(8A+7C)\cos(c+dx)\sin(c+dx)}{128d}+\frac{5(8A+7C)\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{(8A+7C)\cos^5(c+dx)\sin(c+dx)}{48d}+\frac{C\cos^7(c+dx)\sin(c+dx)}{8d}$$

output
```
5/128*(8*A+7*C)*x+5/128*(8*A+7*C)*cos(d*x+c)*sin(d*x+c)/d+5/192*(8*A+7*C)*
cos(d*x+c)^3*sin(d*x+c)/d+1/48*(8*A+7*C)*cos(d*x+c)^5*sin(d*x+c)/d+1/8*C*c
os(d*x+c)^7*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \cos^6(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = \frac{960Ac+840cC+960Adx+840Cdx+48(15A+14C)\sin(2(c+dx))+24(6A+7C)\sin(4(c+dx))+}{3072d}$$

input
```
Integrate[Cos[c + d*x]^6*(A + C*Cos[c + d*x]^2),x]
```

output
```
(960*A*c + 840*c*C + 960*A*d*x + 840*C*d*x + 48*(15*A + 14*C)*Sin[2*(c + d
*x)] + 24*(6*A + 7*C)*Sin[4*(c + d*x)] + 16*A*Sin[6*(c + d*x)] + 32*C*Sin[
6*(c + d*x)] + 3*C*Sin[8*(c + d*x)])/(3072*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3493, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^6 \left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right) dx$$

$$\Big\downarrow \text{ 3493}$$

$$\frac{1}{8}(8A+7C)\int \cos^6(c+dx)dx+\frac{C\sin(c+dx)\cos^7(c+dx)}{8d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{8}(8A+7C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^6 dx+\frac{C\sin(c+dx)\cos^7(c+dx)}{8d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{8}(8A+7C)\left(\frac{5}{6}\int \cos^4(c+dx)dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{C\sin(c+dx)\cos^7(c+dx)}{8d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}(8A+7C)\left(\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{C\sin(c+dx)\cos^7(c+dx)}{8d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{8}(8A+7C)\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{C\sin(c+dx)\cos^7(c+dx)}{8d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}(8A+7C)\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{C\sin(c+dx)\cos^7(c+dx)}{8d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{8}(8A+7C)\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{C\sin(c+dx)\cos^7(c+dx)}{8d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{8}(8A+7C)\left(\frac{\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5}{6}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)\right)+\frac{C\sin(c+dx)\cos^7(c+dx)}{8d}$$

input
```
Int[Cos[c + d*x]^6*(A + C*Cos[c + d*x]^2),x]
```

output
```
(C*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) + ((8*A + 7*C)*((Cos[c + d*x]^5*Sin[
c + d*x])/(6*d) + (5*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos
[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6))/8
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 5.71 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{(720A+672C)\sin(2dx+2c)+(144A+168C)\sin(4dx+4c)+(16A+32C)\sin(6dx+6c)+3C\sin(8dx+8c)+960x\left(A+\frac{7C}{8}\right)}{3072d}$
derivativedivides	$\frac{C\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)+A\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\right.}{6}\right.}{d}$
default	$\frac{C\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)+A\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\right.}{6}\right.}{d}$
parts	$\frac{A\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{C\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\right.}{8}\right.}{d}$
risch	$\frac{5xA}{16}+\frac{35Cx}{128}+\frac{C\sin(8dx+8c)}{1024d}+\frac{\sin(6dx+6c)A}{192d}+\frac{\sin(6dx+6c)C}{96d}+\frac{3\sin(4dx+4c)A}{64d}+\frac{7\sin(4dx+4c)C}{128d}+$
norman	$\frac{\left(\frac{5A}{16}+\frac{35C}{128}\right)x+\left(\frac{5A}{2}+\frac{35C}{16}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{5A}{2}+\frac{35C}{16}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}+\left(\frac{5A}{16}+\frac{35C}{128}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}+\left(\frac{35A}{2}+\frac{245C}{16}\right.}{}$
orering	Expression too large to display

input
```
int(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/3072*((720*A+672*C)*sin(2*d*x+2*c)+(144*A+168*C)*sin(4*d*x+4*c)+(16*A+32
*C)*sin(6*d*x+6*c)+3*C*sin(8*d*x+8*c)+960*x*(A+7/8*C)*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{15\,(8\,A+7\,C)dx+\left(48\,C\cos\left(dx+c\right)^7+8\,(8\,A+7\,C)\cos\left(dx+c\right)^5+10\,(8\,A+7\,C)\cos\left(dx+c\right)^3+15\right.}{384\,d}$$

input
```
integrate(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/384*(15*(8*A + 7*C)*d*x + (48*C*cos(d*x + c)^7 + 8*(8*A + 7*C)*cos(d*x +
 c)^5 + 10*(8*A + 7*C)*cos(d*x + c)^3 + 15*(8*A + 7*C)*cos(d*x + c))*sin(d
*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(109) = 218$.

Time = 0.76 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.03

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \begin{cases} \frac{5Ax\sin^6(c+dx)}{16} + \frac{15Ax\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15Ax\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5Ax\cos^6(c+dx)}{16} + \frac{5A\sin^5(c+dx)\cos(c+dx)}{16d} + \\ x(A+C\cos^2(c))\cos^6(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**6*(A+C*cos(d*x+c)**2),x)
```

output
```
Piecewise((5*A*x*sin(c + d*x)**6/16 + 15*A*x*sin(c + d*x)**4*cos(c + d*x)*
*2/16 + 15*A*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*A*x*cos(c + d*x)**6/
16 + 5*A*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*A*sin(c + d*x)**3*cos(c +
 d*x)**3/(6*d) + 11*A*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 35*C*x*sin(c +
 d*x)**8/128 + 35*C*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 105*C*x*sin(c +
 d*x)**4*cos(c + d*x)**4/64 + 35*C*x*sin(c + d*x)**2*cos(c + d*x)**6/32 +
35*C*x*cos(c + d*x)**8/128 + 35*C*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 3
85*C*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 511*C*sin(c + d*x)**3*cos(c
 + d*x)**5/(384*d) + 93*C*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(d, 0)),
 (x*(A + C*cos(c)**2)*cos(c)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{15\,(dx+c)(8\,A+7\,C) + \frac{15\,(8\,A+7\,C)\tan(dx+c)^7+55\,(8\,A+7\,C)\tan(dx+c)^5+73\,(8\,A+7\,C)\tan(dx+c)^3+3\,(88\,A+93\,C)\tan(dx+c)}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{384\,d}$$

input
```
integrate(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/384*(15*(d*x + c)*(8*A + 7*C) + (15*(8*A + 7*C)*tan(d*x + c)^7 + 55*(8*A
 + 7*C)*tan(d*x + c)^5 + 73*(8*A + 7*C)*tan(d*x + c)^3 + 3*(88*A + 93*C)*t
an(d*x + c))/(tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan
(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{5}{128}\,(8\,A+7\,C)x + \frac{C\sin\left(8\,dx+8\,c\right)}{1024\,d} + \frac{(A+2\,C)\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{(6\,A+7\,C)\sin\left(4\,dx+4\,c\right)}{128\,d} + \frac{(15\,A+14\,C)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

input
```
integrate(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
5/128*(8*A + 7*C)*x + 1/1024*C*sin(8*d*x + 8*c)/d + 1/192*(A + 2*C)*sin(6*
d*x + 6*c)/d + 1/128*(6*A + 7*C)*sin(4*d*x + 4*c)/d + 1/64*(15*A + 14*C)*s
in(2*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 42.97 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = x\left(\frac{5\,A}{16}+\frac{35\,C}{128}\right) + \frac{\left(\frac{5\,A}{16}+\frac{35\,C}{128}\right)\,\tan(c+d\,x)^7+\left(\frac{55\,A}{48}+\frac{385\,C}{384}\right)\,\tan(c+d\,x)^5+\left(\frac{73\,A}{48}+\frac{511\,C}{384}\right)\,\tan(c+d\,x)^3+\left(\frac{11\,A}{16}+\frac{93\,C}{128}\right.}{d\,\left({\tan\left(c+d\,x\right)}^8+4\,{\tan\left(c+d\,x\right)}^6+6\,{\tan\left(c+d\,x\right)}^4+4\,{\tan\left(c+d\,x\right)}^2+1\right)}$$

input
```
int(cos(c + d*x)^6*(A + C*cos(c + d*x)^2),x)
```

output
```
x*((5*A)/16 + (35*C)/128) + (tan(c + d*x)*((11*A)/16 + (93*C)/128) + tan(c
 + d*x)^7*((5*A)/16 + (35*C)/128) + tan(c + d*x)^5*((55*A)/48 + (385*C)/38
4) + tan(c + d*x)^3*((73*A)/48 + (511*C)/384))/(d*(4*tan(c + d*x)^2 + 6*ta
n(c + d*x)^4 + 4*tan(c + d*x)^6 + tan(c + d*x)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.12

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{-48\cos(dx+c)\sin(dx+c)^7c+64\cos(dx+c)\sin(dx+c)^5a+200\cos(dx+c)\sin(dx+c)^5c-208\,c}{}$$

input
```
int(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x)
```

output
```
( - 48*cos(c + d*x)*sin(c + d*x)**7*c + 64*cos(c + d*x)*sin(c + d*x)**5*a
+ 200*cos(c + d*x)*sin(c + d*x)**5*c - 208*cos(c + d*x)*sin(c + d*x)**3*a
- 326*cos(c + d*x)*sin(c + d*x)**3*c + 264*cos(c + d*x)*sin(c + d*x)*a + 2
79*cos(c + d*x)*sin(c + d*x)*c + 120*a*d*x + 105*c*d*x)/(384*d)
```

3.10 $\int \cos^4(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \cos^4(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = \frac{1}{16}(6A+5C)x + \frac{(6A+5C)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(6A+5C)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{C\cos^5(c+dx)\sin(c+dx)}{6d}$$

output
```
1/16*(6*A+5*C)*x+1/16*(6*A+5*C)*cos(d*x+c)*sin(d*x+c)/d+1/24*(6*A+5*C)*cos
(d*x+c)^3*sin(d*x+c)/d+1/6*C*cos(d*x+c)^5*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^4(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = \frac{72Ac+60cC+72Adx+60Cdx+(48A+45C)\sin(2(c+dx))+(6A+9C)\sin(4(c+dx))+C\sin(6(c}{192d}$$

input
```
Integrate[Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2),x]
```

output
```
(72*A*c + 60*c*C + 72*A*d*x + 60*C*d*x + (48*A + 45*C)*Sin[2*(c + d*x)] +
(6*A + 9*C)*Sin[4*(c + d*x)] + C*Sin[6*(c + d*x)])/(192*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3493, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^4\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{6}(6A+5C)\int \cos^4(c+dx)dx+\frac{C\sin(c+dx)\cos^5(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}(6A+5C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{C\sin(c+dx)\cos^5(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}(6A+5C)\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{C\sin(c+dx)\cos^5(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}(6A+5C)\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{C\sin(c+dx)\cos^5(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}(6A+5C)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{C\sin(c+dx)\cos^5(c+dx)}{6d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{6}(6A+5C)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)+\frac{C\sin(c+dx)\cos^5(c+dx)}{6d}$$

input
```
Int[Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2),x]
```

output
```
(C*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) + ((6*A + 5*C)*((Cos[c + d*x]^3*Sin[
c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{(48A+45C)\sin(2dx+2c)+(6A+9C)\sin(4dx+4c)+\sin(6dx+6c)C+72x\left(A+\frac{5C}{6}\right)d}{192d}$
risch	$\frac{3xA}{8}+\frac{5Cx}{16}+\frac{\sin(6dx+6c)C}{192d}+\frac{\sin(4dx+4c)A}{32d}+\frac{3\sin(4dx+4c)C}{64d}+\frac{\sin(2dx+2c)A}{4d}+\frac{15\sin(2dx+2c)C}{64d}$
derivativedivides	$\frac{C\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+A\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3}{8}\right.}{d}$
default	$\frac{C\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+A\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3}{8}\right.}{d}$
parts	$\frac{A\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{C\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}\right.}{d}$
norman	$\frac{\left(\frac{3A}{8}+\frac{5C}{16}\right)x+\left(\frac{3A}{8}+\frac{5C}{16}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+\left(\frac{9A}{4}+\frac{15C}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\left(\frac{9A}{4}+\frac{15C}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(\frac{15A}{2}+\frac{25C}{4}\right)}{}$
orering	$x\cos(dx+c)^4\left(A+C\cos(dx+c)^2\right)-\frac{49\left(-4\cos(dx+c)^3\left(A+C\cos(dx+c)^2\right)d\sin(dx+c)-2\cos(dx+\right.}{144d^2}$

input
```
int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/192*((48*A+45*C)*sin(2*d*x+2*c)+(6*A+9*C)*sin(4*d*x+4*c)+sin(6*d*x+6*c)*
C+72*x*(A+5/6*C)*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3\,(6\,A+5\,C)dx+\left(8\,C\cos(dx+c)^5+2\,(6\,A+5\,C)\cos(dx+c)^3+3\,(6\,A+5\,C)\cos(dx+c)\right)\sin(dx-}{48\,d}$$

input
```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/48*(3*(6*A + 5*C)*d*x + (8*C*cos(d*x + c)^5 + 2*(6*A + 5*C)*cos(d*x + c)
^3 + 3*(6*A + 5*C)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(82) = 164$.

Time = 0.53 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.90

$$\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\begin{cases}\frac{3Ax\sin^4(c+dx)}{8}+\frac{3Ax\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3Ax\cos^4(c+dx)}{8}+\frac{3A\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5A\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{5C\\ x(A+C\cos^2(c))\cos^4(c)\end{cases}$$

input
```
integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2),x)
```

output
```
Piecewise((3*A*x*sin(c + d*x)**4/8 + 3*A*x*sin(c + d*x)**2*cos(c + d*x)**2
/4 + 3*A*x*cos(c + d*x)**4/8 + 3*A*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*
A*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 5*C*x*sin(c + d*x)**6/16 + 15*C*x*s
in(c + d*x)**4*cos(c + d*x)**2/16 + 15*C*x*sin(c + d*x)**2*cos(c + d*x)**4
/16 + 5*C*x*cos(c + d*x)**6/16 + 5*C*sin(c + d*x)**5*cos(c + d*x)/(16*d) +
 5*C*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*C*sin(c + d*x)*cos(c + d*x
)**5/(16*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3\,(dx+c)(6\,A+5\,C)+\frac{3\,(6\,A+5\,C)\tan(dx+c)^5+8\,(6\,A+5\,C)\tan(dx+c)^3+3\,(10\,A+11\,C)\tan(dx+c)}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

input
```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/48*(3*(d*x + c)*(6*A + 5*C) + (3*(6*A + 5*C)*tan(d*x + c)^5 + 8*(6*A + 5
*C)*tan(d*x + c)^3 + 3*(10*A + 11*C)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan
(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^4(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = \frac{1}{16}\,(6\,A+5\,C)x + \frac{C\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{(2\,A+3\,C)\sin\left(4\,dx+4\,c\right)}{64\,d} + \frac{(16\,A+15\,C)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

input
```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
1/16*(6*A + 5*C)*x + 1/192*C*sin(6*d*x + 6*c)/d + 1/64*(2*A + 3*C)*sin(4*d
*x + 4*c)/d + 1/64*(16*A + 15*C)*sin(2*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 41.68 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \cos^4(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx = x\left(\frac{3\,A}{8}+\frac{5\,C}{16}\right) + \frac{\left(\frac{3\,A}{8}+\frac{5\,C}{16}\right)\,{\mathrm{tan}\left(c+d\,x\right)}^5+\left(A+\frac{5\,C}{6}\right)\,{\mathrm{tan}\left(c+d\,x\right)}^3+\left(\frac{5\,A}{8}+\frac{11\,C}{16}\right)\,\mathrm{tan}\left(c+d\,x\right)}{d\,\left({\mathrm{tan}\left(c+d\,x\right)}^6+3\,{\mathrm{tan}\left(c+d\,x\right)}^4+3\,{\mathrm{tan}\left(c+d\,x\right)}^2+1\right)}$$

input
```
int(cos(c + d*x)^4*(A + C*cos(c + d*x)^2),x)
```

output
```
x*((3*A)/8 + (5*C)/16) + (tan(c + d*x)*((5*A)/8 + (11*C)/16) + tan(c + d*x
)^3*(A + (5*C)/6) + tan(c + d*x)^5*((3*A)/8 + (5*C)/16))/(d*(3*tan(c + d*x
)^2 + 3*tan(c + d*x)^4 + tan(c + d*x)^6 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{8\cos(dx+c)\sin(dx+c)^5c-12\cos(dx+c)\sin(dx+c)^3a-26\cos(dx+c)\sin(dx+c)^3c+30\cos(d}{48d}$$

input
```
int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x)
```

output
```
(8*cos(c + d*x)*sin(c + d*x)**5*c - 12*cos(c + d*x)*sin(c + d*x)**3*a - 26
*cos(c + d*x)*sin(c + d*x)**3*c + 30*cos(c + d*x)*sin(c + d*x)*a + 33*cos(
c + d*x)*sin(c + d*x)*c + 18*a*d*x + 15*c*d*x)/(48*d)
```

3.11 $\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{1}{8}(4A+3C)x+\frac{(4A+3C)\cos(c+dx)\sin(c+dx)}{8d}+\frac{C\cos^3(c+dx)\sin(c+dx)}{4d}$$

output
```
1/8*(4*A+3*C)*x+1/8*(4*A+3*C)*cos(d*x+c)*sin(d*x+c)/d+1/4*C*cos(d*x+c)^3*s
in(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2),x]
```

output
```
(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + C*Sin[4*(c + d*x)]
)/(32*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3493, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{4}(4A+3C)\int \cos^2(c+dx)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}(4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}(4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}$$

input
```
Int[Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2),x]
```

output
```
(C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A + 3*C)*(x/2 + (Cos[c + d*x]*
Sin[c + d*x])/(2*d)))/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{(8A+8C)\sin(2dx+2c)+\sin(4dx+4c)C+16x\left(A+\frac{3C}{4}\right)d}{32d}$
risch	$\frac{xA}{2}+\frac{3Cx}{8}+\frac{\sin(4dx+4c)C}{32d}+\frac{\sin(2dx+2c)A}{4d}+\frac{\sin(2dx+2c)C}{4d}$
derivativedivides	$\frac{C\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{C\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{C\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
norman	$\frac{\left(\frac{A}{2}+\frac{3C}{8}\right)x+\left(2A+\frac{3C}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(2A+\frac{3C}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(3A+\frac{9C}{4}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(\frac{A}{2}+\frac{3C}{8}\right)x\tan\left(\frac{dx}{2}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}$
orering	$x\cos(dx+c)^2\left(A+C\cos(dx+c)^2\right)-\frac{5\left(-2\cos(dx+c)\left(A+C\cos(dx+c)^2\right)d\sin(dx+c)-2\cos(dx+c)\right.}{16d^2}$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/32*((8*A+8*C)*sin(2*d*x+2*c)+sin(4*d*x+4*c)*C+16*x*(A+3/4*C)*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(4\,A+3\,C)dx+\left(2\,C\cos(dx+c)^3+(4\,A+3\,C)\cos(dx+c)\right)\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/8*((4*A + 3*C)*d*x + (2*C*cos(d*x + c)^3 + (4*A + 3*C)*cos(d*x + c))*sin
(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(53) = 106$.

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.59

$$\int \cos^2(c+dx) \left(A + C\cos^2(c+dx)\right) \, dx$$
$$= \begin{cases} \frac{Ax\sin^2(c+dx)}{2} + \frac{Ax\cos^2(c+dx)}{2} + \frac{A\sin(c+dx)\cos(c+dx)}{2d} + \frac{3Cx\sin^4(c+dx)}{8} + \frac{3Cx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Cx\cos^4(c+dx)}{8} \\ x(A+C\cos^2(c))\cos^2(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2),x)
```

output
```
Piecewise((A*x*sin(c + d*x)**2/2 + A*x*cos(c + d*x)**2/2 + A*sin(c + d*x)*
cos(c + d*x)/(2*d) + 3*C*x*sin(c + d*x)**4/8 + 3*C*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + 3*C*x*cos(c + d*x)**4/8 + 3*C*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*C*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + C*cos(c
)**2)*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos^2(c+dx) \left(A + C\cos^2(c+dx)\right) \, dx$$
$$= \frac{(dx+c)(4A+3C) + \frac{(4A+3C)\tan(dx+c)^3 + (4A+5C)\tan(dx+c)}{\tan(dx+c)^4 + 2\tan(dx+c)^2 + 1}}{8d}$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/8*((d*x + c)*(4*A + 3*C) + ((4*A + 3*C)*tan(d*x + c)^3 + (4*A + 5*C)*tan
(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=\frac{1}{8}\,(4\,A+3\,C)x+\frac{C\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{(A+C)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(4*A + 3*C)*x + 1/32*C*sin(4*d*x + 4*c)/d + 1/4*(A + C)*sin(2*d*x + 2*
c)/d
```

Mupad [B] (verification not implemented)

Time = 41.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx=x\left(\frac{A}{2}+\frac{3\,C}{8}\right)+\frac{\left(\frac{A}{2}+\frac{3\,C}{8}\right)\,\tan(c+d\,x)^3+\left(\frac{A}{2}+\frac{5\,C}{8}\right)\,\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

input
```
int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2),x)
```

output
```
x*(A/2 + (3*C)/8) + (tan(c + d*x)*(A/2 + (5*C)/8) + tan(c + d*x)^3*(A/2 +
(3*C)/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{-2\cos(dx+c)\sin(dx+c)^3c+4\cos(dx+c)\sin(dx+c)a+5\cos(dx+c)\sin(dx+c)c+4adx+3cd}{8d}$$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)**3*c + 4*cos(c + d*x)*sin(c + d*x)*a + 5*c
os(c + d*x)*sin(c + d*x)*c + 4*a*d*x + 3*c*d*x)/(8*d)
```

3.12 $\int (A + C\cos^2(c + dx))\sec^2(c + dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 15

$$\int (A + C\cos^2(c + dx))\sec^2(c + dx)\,dx = Cx + \frac{A\tan(c + dx)}{d}$$

output

```
C*x+A*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (A + C\cos^2(c + dx))\sec^2(c + dx)\,dx = Cx + \frac{A\tan(c + dx)}{d}$$

input

```
Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output

```
C*x + (A*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3491}$$

$$C\int 1dx+\frac{A\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{A\tan(c+dx)}{d}+Cx$$

input
```
Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
C*x + (A*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{A\tan(dx+c)+C(dx+c)}{d}$
default	$\frac{A\tan(dx+c)+C(dx+c)}{d}$
parts	$\frac{A\tan(dx+c)}{d}+\frac{C(dx+c)}{d}$
risch	$Cx+\frac{2iA}{d\left(e^{2i(dx+c)}+1\right)}$
parallelrisch	$\frac{A\sin(dx+c)+dxC\cos(dx+c)}{d\cos(dx+c)}$
norman	$\frac{Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-Cx-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-Cx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(A*tan(d*x+c)+C*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{Cdx\cos\left(dx+c\right)+A\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
(C*d*x*cos(d*x + c) + A*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx=\int (A+C\cos^2{(c+dx)})\sec^2{(c+dx)}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx=\frac{(dx+c)C+A\tan{(dx+c)}}{d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
((d*x + c)*C + A*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx=\frac{(dx+c)C+A\tan{(dx+c)}}{d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
((d*x + c)*C + A*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 40.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{A\tan(c+d\,x)+C\,d\,x}{d}$$

input
```
int((A + C*cos(c + d*x)^2)/cos(c + d*x)^2,x)
```

output
```
(A*tan(c + d*x) + C*d*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{\cos\left(dx+c\right)cdx+\sin\left(dx+c\right)a}{\cos\left(dx+c\right)d}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
(cos(c + d*x)*c*d*x + sin(c + d*x)*a)/(cos(c + d*x)*d)
```

3.13 $\int (A + C\cos^2(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int (A + C\cos^2(c+dx))\sec^4(c+dx)\,dx = \frac{(2A+3C)\tan(c+dx)}{3d} + \frac{A\sec^2(c+dx)\tan(c+dx)}{3d}$$

output
```
1/3*(2*A+3*C)*tan(d*x+c)/d+1/3*A*sec(d*x+c)^2*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int (A + C\cos^2(c+dx))\sec^4(c+dx)\,dx = \frac{C\tan(c+dx)}{d} + \frac{A\left(\tan(c+dx) + \frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

output
```
(C*Tan[c + d*x])/d + (A*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3491, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx) \left(A+C\cos^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{ 3491}$$

$$\frac{1}{3}(2A+3C)\int \sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}(2A+3C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{3d}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}$$

input
```
Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

output
```
((2*A + 3*C)*Tan[c + d*x])/(3*d) + (A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+C\tan(dx+c)}{d}$	35
default	$\frac{-A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+C\tan(dx+c)}{d}$	35
parts	$-\frac{A\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{C\tan(dx+c)}{d}$	37
parallelrisch	$\frac{(2A+3C)\sin(3dx+3c)+6\left(A+\frac{C}{2}\right)\sin(dx+c)}{3d(\cos(3dx+3c)+3\cos(dx+c))}$	57
risch	$\frac{2i\left(3C\,\mathrm{e}^{4i(dx+c)}+6A\,\mathrm{e}^{2i(dx+c)}+6C\,\mathrm{e}^{2i(dx+c)}+2A+3C\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	63
norman	$\frac{-\frac{8A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{8A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}-\frac{4(A-3C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3d}-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^3}$	124

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+C*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{\left((2\,A+3\,C)\cos\left(dx+c\right)^2+A\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sin(d*x + c)/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(A+C\cos^2\left(c+dx\right)\right)\sec^4\left(c+dx\right)dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{A\tan\left(dx+c\right)^3+3\left(A+C\right)\tan\left(dx+c\right)}{3\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/3*(A*tan(d*x + c)^3 + 3*(A + C)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{A\tan\left(dx+c\right)^3+3\,A\tan\left(dx+c\right)+3\,C\tan\left(dx+c\right)}{3\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
1/3*(A*tan(d*x + c)^3 + 3*A*tan(d*x + c) + 3*C*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 40.54 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{A\tan(c+d\,x)^3}{3\,d}+\frac{\tan(c+d\,x)\,\left(A+C\right)}{d}$$

input
```
int((A + C*cos(c + d*x)^2)/cos(c + d*x)^4,x)
```

output
```
(A*tan(c + d*x)^3)/(3*d) + (tan(c + d*x)*(A + C))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx = \frac{\sin(dx+c)\left(2\sin(dx+c)^2 a+3\sin(dx+c)^2 c-3a-3c\right)}{3\cos(dx+c)\,d\left(\sin(dx+c)^2-1\right)}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
(sin(c + d*x)*(2*sin(c + d*x)**2*a + 3*sin(c + d*x)**2*c - 3*a - 3*c))/(3*
cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.14 $\int (A+C\cos^2(c+dx))\sec^6(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int (A+C\cos^2(c+dx))\sec^6(c+dx)\,dx = \frac{(4A+5C)\tan(c+dx)}{5d} + \frac{A\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{(4A+5C)\tan^3(c+dx)}{15d}$$

output
```
1/5*(4*A+5*C)*tan(d*x+c)/d+1/5*A*sec(d*x+c)^4*tan(d*x+c)/d+1/15*(4*A+5*C)*
tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (A+C\cos^2(c+dx))\sec^6(c+dx)\,dx = \frac{C\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d} + \frac{A\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

output
```
(C*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d + (A*(Tan[c + d*x] + (2*Tan[c + d*
x]^3)/3 + Tan[c + d*x]^5/5))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3491, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{ 3491}$$

$$\frac{1}{5}(4A+5C)\int \sec^4(c+dx)dx+\frac{A\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}(4A+5C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{A\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{A\tan(c+dx)\sec^4(c+dx)}{5d}-\frac{(4A+5C)\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{5d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{A\tan(c+dx)\sec^4(c+dx)}{5d}-\frac{(4A+5C)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{5d}$$

input
```
Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

output
```
(A*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) - ((4*A + 5*C)*(-Tan[c + d*x] - Tan[
c + d*x]^3/3))/(5*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-A\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-C\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{-A\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-C\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
parts	$-\frac{A\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}-\frac{C\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
parallelrisch	$\frac{(40A+50C)\sin(3dx+3c)+(8A+10C)\sin(5dx+5c)+80\left(A+\frac{C}{2}\right)\sin(dx+c)}{15d(\cos(5dx+5c)+5\cos(3dx+3c)+10\cos(dx+c))}$
risch	$\frac{4i\left(15C\,e^{6i(dx+c)}+40A\,e^{4i(dx+c)}+35C\,e^{4i(dx+c)}+20A\,e^{2i(dx+c)}+25C\,e^{2i(dx+c)}+4A+5C\right)}{15d\left(e^{2i(dx+c)}+1\right)^5}$
norman	$\frac{-\frac{4(A-C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{4(A-C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{3d}-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{d}-\frac{2(11A-5C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^5}$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-C*(-2/3-1/3*
sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \left(A + C\cos^2(c + dx)\right)\sec^6(c + dx)\,dx = \frac{\left(2\,(4\,A + 5\,C)\cos\left(dx + c\right)^4 + (4\,A + 5\,C)\cos\left(dx + c\right)^2 + 3\,A\right)\sin\left(dx + c\right)}{15\,d\cos\left(dx + c\right)^5}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/15*(2*(4*A + 5*C)*cos(d*x + c)^4 + (4*A + 5*C)*cos(d*x + c)^2 + 3*A)*sin
(d*x + c)/(d*cos(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{3\,A\tan\left(dx+c\right)^5+5\,(2\,A+C)\tan\left(dx+c\right)^3+15\,(A+C)\tan\left(dx+c\right)}{15\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/15*(3*A*tan(d*x + c)^5 + 5*(2*A + C)*tan(d*x + c)^3 + 15*(A + C)*tan(d*x
 + c))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{3\,A\tan\left(dx+c\right)^5+10\,A\tan\left(dx+c\right)^3+5\,C\tan\left(dx+c\right)^3+15\,A\tan\left(dx+c\right)+15\,C\tan\left(dx+c\right)}{15\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="giac")
```

output
```
1/15*(3*A*tan(d*x + c)^5 + 10*A*tan(d*x + c)^3 + 5*C*tan(d*x + c)^3 + 15*A
*tan(d*x + c) + 15*C*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 40.89 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{\frac{A\tan(c+d\,x)^5}{5}+\left(\frac{2\,A}{3}+\frac{C}{3}\right)\,\tan(c+d\,x)^3+(A+C)\,\tan(c+d\,x)}{d}$$

input
```
int((A + C*cos(c + d*x)^2)/cos(c + d*x)^6,x)
```

output
```
((A*tan(c + d*x)^5)/5 + tan(c + d*x)*(A + C) + tan(c + d*x)^3*((2*A)/3 + C
/3))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.42

$$\int \left(A + C\cos^2(c + dx)\right)\sec^6(c + dx)\,dx$$
$$= \frac{\sin(dx+c)\left(8\sin(dx+c)^4 a + 10\sin(dx+c)^4 c - 20\sin(dx+c)^2 a - 25\sin(dx+c)^2 c + 15a + 15c\right)}{15\cos(dx+c)\,d\left(\sin(dx+c)^4 - 2\sin(dx+c)^2 + 1\right)}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

output
```
(sin(c + d*x)*(8*sin(c + d*x)**4*a + 10*sin(c + d*x)**4*c - 20*sin(c + d*x
)**2*a - 25*sin(c + d*x)**2*c + 15*a + 15*c))/(15*cos(c + d*x)*d*(sin(c +
d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.15 $\int (A + C\cos^2(c + dx))\sec^8(c + dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int (A + C\cos^2(c + dx))\sec^8(c + dx)\,dx = \frac{(6A + 7C)\tan(c + dx)}{7d} + \frac{A\sec^6(c + dx)\tan(c + dx)}{7d} + \frac{2(6A + 7C)\tan^3(c + dx)}{21d} + \frac{(6A + 7C)\tan^5(c + dx)}{35d}$$

output
```
1/7*(6*A+7*C)*tan(d*x+c)/d+1/7*A*sec(d*x+c)^6*tan(d*x+c)/d+2/21*(6*A+7*C)*
tan(d*x+c)^3/d+1/35*(6*A+7*C)*tan(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int (A + C\cos^2(c + dx))\sec^8(c + dx)\,dx = \frac{C\left(\tan(c + dx) + \frac{2}{3}\tan^3(c + dx) + \frac{1}{5}\tan^5(c + dx)\right)}{d} + \frac{A\left(\tan(c + dx) + \tan^3(c + dx) + \frac{3}{5}\tan^5(c + dx) + \frac{1}{7}\tan^7(c + dx)\right)}{d}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^8,x]
```

output
```
(C*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d + (A*(Tan[c
 + d*x] + Tan[c + d*x]^3 + (3*Tan[c + d*x]^5)/5 + Tan[c + d*x]^7/7))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3491, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^8}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{1}{7}(6A+7C)\int \sec^6(c+dx)dx+\frac{A\tan(c+dx)\sec^6(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(6A+7C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^6dx+\frac{A\tan(c+dx)\sec^6(c+dx)}{7d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{A\tan(c+dx)\sec^6(c+dx)}{7d}-\frac{(6A+7C)\int\left(\tan^4(c+dx)+2\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{7d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{A\tan(c+dx)\sec^6(c+dx)}{7d}-\frac{(6A+7C)\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{7d}$$

input
```
Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^8,x]
```

output
```
(A*Sec[c + d*x]^6*Tan[c + d*x])/(7*d) - ((6*A + 7*C)*(-Tan[c + d*x] - (2*T
an[c + d*x]^3)/3 - Tan[c + d*x]^5/5))/(7*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-A\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)-C\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
default	$\frac{-A\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)-C\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
parts	$-\frac{A\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}-\frac{C\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
risch	$\frac{16i\left(70C\,e^{8i(dx+c)}+210A\,e^{6i(dx+c)}+175C\,e^{6i(dx+c)}+126A\,e^{4i(dx+c)}+147C\,e^{4i(dx+c)}+42A\,e^{2i(dx+c)}+49C\,e^{2i(dx+c)}\ldots\right.}{105d\left(e^{2i(dx+c)}+1\right)^7}$
parallelrisch	$\frac{(1008A+1176C)\sin(3dx+3c)+(336A+392C)\sin(5dx+5c)+(48A+56C)\sin(7dx+7c)+1680\left(A+\frac{C}{2}\right)\sin(dx+c)}{105d(\cos(7dx+7c)+7\cos(5dx+5c)+21\cos(3dx+3c)+35\cos(dx+c))}$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-A*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(
d*x+c)-C*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \left(A+C\cos^2(c+dx)\right)\sec^8(c+dx)\,dx$$
$$=\frac{\left(8\,(6\,A+7\,C)\cos\left(dx+c\right)^6+4\,(6\,A+7\,C)\cos\left(dx+c\right)^4+3\,(6\,A+7\,C)\cos\left(dx+c\right)^2+15\,A\right)\sin\left(dx+\ldots\right.}{105\,d\cos\left(dx+c\right)^7}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="fricas")
```

output
```
1/105*(8*(6*A + 7*C)*cos(d*x + c)^6 + 4*(6*A + 7*C)*cos(d*x + c)^4 + 3*(6*
A + 7*C)*cos(d*x + c)^2 + 15*A)*sin(d*x + c)/(d*cos(d*x + c)^7)
```

Sympy [F(-1)]

Timed out.

$$\int \left(A+C\cos^2(c+dx)\right)\sec^8(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \left(A+C\cos^2(c+dx)\right)\sec^8(c+dx)\,dx = \frac{15\,A\tan\left(dx+c\right)^7+21\,(3\,A+C)\tan\left(dx+c\right)^5+35\,(3\,A+2\,C)\tan\left(dx+c\right)^3+105\,(A+C)\tan\left(dx+\right.}{105\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="maxima")
```

output
```
1/105*(15*A*tan(d*x + c)^7 + 21*(3*A + C)*tan(d*x + c)^5 + 35*(3*A + 2*C)*
tan(d*x + c)^3 + 105*(A + C)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \left(A+C\cos^2(c+dx)\right)\sec^8(c+dx)\,dx = \frac{15\,A\tan\left(dx+c\right)^7+63\,A\tan\left(dx+c\right)^5+21\,C\tan\left(dx+c\right)^5+105\,A\tan\left(dx+c\right)^3+70\,C\tan\left(dx+c\right)^3}{105\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="giac")
```

output
```
1/105*(15*A*tan(d*x + c)^7 + 63*A*tan(d*x + c)^5 + 21*C*tan(d*x + c)^5 + 1
05*A*tan(d*x + c)^3 + 70*C*tan(d*x + c)^3 + 105*A*tan(d*x + c) + 105*C*tan
(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 41.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int (A + C\cos^2(c+dx))\sec^8(c+dx)\,dx$$
$$= \frac{\frac{A\tan(c+d\,x)^7}{7} + \left(\frac{3\,A}{5} + \frac{C}{5}\right)\,\tan(c+d\,x)^5 + \left(A + \frac{2\,C}{3}\right)\,\tan(c+d\,x)^3 + (A+C)\,\tan(c+d\,x)}{d}$$

input
```
int((A + C*cos(c + d*x)^2)/cos(c + d*x)^8,x)
```

output
```
((A*tan(c + d*x)^7)/7 + tan(c + d*x)^3*(A + (2*C)/3) + tan(c + d*x)*(A + C
) + tan(c + d*x)^5*((3*A)/5 + C/5))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.43

$$\int (A + C\cos^2(c+dx))\sec^8(c+dx)\,dx$$
$$= \frac{\sin(dx+c)\left(48\sin(dx+c)^6 a + 56\sin(dx+c)^6 c - 168\sin(dx+c)^4 a - 196\sin(dx+c)^4 c + 210\sin(c\right.}{105\cos(dx+c)\,d\left(\sin(dx+c)^6 - 3\sin(dx+c)^4 + 3\sin(dx+c\right.}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x)
```

output
```
(sin(c + d*x)*(48*sin(c + d*x)**6*a + 56*sin(c + d*x)**6*c - 168*sin(c + d
*x)**4*a - 196*sin(c + d*x)**4*c + 210*sin(c + d*x)**2*a + 245*sin(c + d*x
)**2*c - 105*a - 105*c))/(105*cos(c + d*x)*d*(sin(c + d*x)**6 - 3*sin(c +
d*x)**4 + 3*sin(c + d*x)**2 - 1))
```

3.16 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}$$

output
```
2/15*b^2*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))/d/cos(d*x+c)^(1/2)+2/45*b*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+
2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(b\cos(c+dx))^{5/2}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}(18A+19C+5C\ldots\right.}{180d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt
[Cos[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(18
0*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3493, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \; 3493$$

$$\frac{1}{9}(9A+7C)\int (b\cos(c+dx))^{5/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}(9A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$\Big\downarrow$ 3121

$$\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$\Big\downarrow$ 3119

$$\frac{1}{9}(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + ((9*A + 7*C)*((6*b^2*S
qrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) +
(2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(101) = 202$.

Time = 9.92 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.87

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b^3 \left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+320C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-72A-29\cdots\right.}{\cdots}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b^3 \left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\cdots\right.}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\cdots\right.}}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-160
*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos
(1/2*d*x+1/2*c)+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*
A+136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{2\left(-3i\sqrt{\frac{1}{2}}(9A+7C)b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
-2/45*(-3*I*sqrt(1/2)*(9*A + 7*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(1/2)*(9*A + 7
*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) - (5*C*b^2*cos(d*x + c)^3 + (9*A + 7*C)*b^2*cos(d*x +
c))*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\sqrt{b}\,b^2\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4dx\right)c+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c
 + d*x))*cos(c + d*x)**2,x)*a)
```

3.17 $\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}$$

output
```
2/21*b^2*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
/d/(b*cos(d*x+c))^(1/2)+2/21*b*(7*A+5*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
+2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(b\cos(c+dx))^{3/2}\left(4(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sqrt{\cos(c+dx)}(14A+13\right.}{42d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[
Cos[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*d*Co
s[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3493, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{7}(7A+5C)\int (b\cos(c+dx))^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\big\downarrow$ 3121

$$\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\big\downarrow$ 3042

$$\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\big\downarrow$ 3120

$$\frac{1}{7}(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

input

```
Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

output

```
(2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + ((7*A + 5*C)*((2*b^2*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) +
(2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(100) = 200$.

Time = 4.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.62

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b^{2}\left(48C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-72C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(28A+56C)\sin\left(\right.\right.}{21}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b^{2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\ldots}\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\, d}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(48*C
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*
C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(7A+5C)b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{\frac{1}{2}}(7A+5C)b^{\frac{3}{2}}\text{weie}\ldots\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
-2/21*(I*sqrt(1/2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - I*sqrt(1/2)*(7*A + 5*C)*b^(3/2)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (3*C*b*cos(d*x + c)^2 + (7*A +
5*C)*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\sqrt{b}\,b\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{3}dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x
))*cos(c + d*x)**3,x)*c)
```

3.18 $\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}$$

output
```
2/5*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d
/cos(d*x+c)^(1/2)+2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Co
s[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3493, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{5}(5A+3C)\int \sqrt{b\cos(c+dx)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

output
```
(2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[C
os[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(69) = 138.

Time = 2.09 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.39

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b\left(8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2}\cdots\right.}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}\cdots\right.\right.}}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\, d}-\frac{2C\sqrt{b\left(-1+\cdots\right.}}{\cdots}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(8*C*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/
2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.32

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{2\left(\sqrt{b\cos\left(dx+c\right)}C\cos\left(dx+c\right)\sin\left(dx+c\right)-\sqrt{\frac{1}{2}}(-5i\,A-3i\,C)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrass}\cdots\right.}{\cdots}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) - sqrt(1/2)*(-5*I*A
- 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - sqrt(1/2)*(5*I*A + 3*I*C)*sqrt(b)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x)
**2,x)*c)
```

3.19 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}$$

output
```
2/3*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d/(b*c
os(d*x+c))^(1/2)+2/3*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx))}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d
*x)])/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3493, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos
[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(66) = 132.

Time = 0.71 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.00

method	result
parts	$\frac{2A\sqrt{\cos(dx+c)}\,\mathrm{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\sqrt{2}\right)}{d\sqrt{b\cos(dx+c)}}-\frac{2C\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}\right.\right.}}$
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\mathrm{co}\right.\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\right.}$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*A/d/(b*cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))-2/3*C*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
(4*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*
d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1
/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A+i\,C)\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{\frac{1}{2}}(-3i\,A-i\,C)\sqrt{b}\right.}{3\,bd}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(1/2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(1/2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(b*cos(d*x + c))*C*sin(d*x
+ c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos(dx + c)^2 + A}{\sqrt{b\cos(dx + c)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos{(dx+c)}^2 + A}{\sqrt{b\cos{(dx+c)}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\,C\,\sin{(c+d\,x)}\,\sqrt{b\,\cos{(c+d\,x)}}}{3\,b\,d} + \frac{2\,A\,\sqrt{\cos{(c+d\,x)}}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{b\,\cos{(c+d\,x)}}} + \frac{2\,C\,\sqrt{\cos{(c+d\,x)}}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d\,\sqrt{b\,\cos{(c+d\,x)}}}$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/2),x)
```

output
```
(2*C*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*cos(c + d*x)^(1/2
)*ellipticF(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*C*cos(c + d
*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1/2))
```

Reduce [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$
$$= \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a + \left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)c\right)}{b}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
)*cos(c + d*x),x)*c))/b
```

3.20 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
-2*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/
cos(d*x+c)^(1/2)+2*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-2(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*A*Sin[c + d*x
])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3491, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int\sqrt{b\cos(c+dx)}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(70) = 140$.

Time = 1.06 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.92

method	result
default	$\frac{2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\right.}}$
parts	$-\frac{2A\left(-2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.}{b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/si
n(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(iA-iC)\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(1/2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(1/2)*(-I*A +
 I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*A*sin(d*x + c))/
(b^2*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}dx\right)c\right)}{b^2}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a + int(sqrt(cos(c + d
*x)),x)*c))/b**2
```

3.21 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

output
```
2/3*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^2/d/
(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left((A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\tan(c+dx)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*x
]))/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3491, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{(A+3C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+3C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b
*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(69) = 138$.

Time = 0.95 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.77

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,(A+3C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+A}{3b^2\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\mathrm{si}}}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(}{}}}{3b^2\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\mathrm{sir}}$

input

```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2)))/b^2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1+2*cos
(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2
)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(i\,A+3i\,C)\sqrt{b}\cos\left(dx+c\right)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{\frac{1}{2}}(-i\,A}{3\,b^3}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(1/2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-I*A - 3*I*C)*sqrt(b)*co
s(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - s
qrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos(c+d\,x)^2 + A}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)c + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a\right)}{b^3}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*c + int(sqrt(cos(c + d*x)
)/cos(c + d*x)**3,x)*a))/b**3
```

3.22 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

output
```
-2/5*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b^4/d/cos(d*x+c)^(1/2)+2/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+
5*C)*sin(d*x+c)/b^3/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \frac{2\left(-\left((3A+5C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\right)+(3A+5C)\sin(c+dx)+A}{5b^3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

output
```
(2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5
*C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b^3*d*Sqrt[b*Cos[c + d
*x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3491, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

output
```
(2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + ((3*A + 5*C)*((-2*Sqrt
[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (
2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(103) = 206$.

Time = 1.47 (sec) , antiderivative size = 566, normalized size of antiderivative = 4.92

method	result
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6-12\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\text{EllipticE}\left(c\right.\right.}{}$
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6-12A\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\right.}{}$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*A*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(
1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/
d-2*C/b^3*(-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4
-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*
c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A+5i\,C)\sqrt{b}\cos\left(dx+c\right)^3 \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\ \text{s}\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(sqrt(1/2)*(3*I*A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(1/2)
*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - ((3*A + 5*C)*cos(d*x +
c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^4*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \int \frac{C\cos(dx+c)^2 + A}{(b\cos(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+A}{\left(b\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx=\frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)c\right)}{b^4}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x)**4,x)*a + int(sqrt(cos(c + d
*x))/cos(c + d*x)**2,x)*c))/b**4
```

3.23 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^4d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}$$

output
```
2/21*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^4
/d/(b*cos(d*x+c))^(1/2)+2/7*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(7/2)+2/21*(5*
A+7*C)*sin(d*x+c)/b^3/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.67

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \frac{2\left((5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C+3A\sec^2(c-\right.}{21b^4d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2),x]
```

output
```
(2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C
+ 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b^4*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3491, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2),x]
```

output
```
(2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + ((5*A + 7*C)*((2*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) +
(2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(102) = 204$.

Time $= 2.12$ (sec) , antiderivative size $= 413$, normalized size of antiderivative $= 3.59$

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}}{56b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{4}}-\frac{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{42b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right.}\right.\right.}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+60\sqrt{\frac{1}{2}-\right.}{}$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4*(A*(-1/5
6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5
/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(5i\,A+7i\,C)\sqrt{b}\cos{(dx+c)}^4\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{\frac{1}{2}}(-5i\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
-2/21*(sqrt(1/2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-5*I*A - 7*I*C)*sqrt(
b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) - ((5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/
(b^5*d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\,dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{9}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\,dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{9}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\left(b\,\cos\left(c+d\,x\right)\right)^{9/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(9/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)c\right)}{b^5}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x)**5,x)*a + int(sqrt(cos(c + d
*x))/cos(c + d*x)**3,x)*c))/b**5
```

3.24 $\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx=-\frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}$$

output
```
-2*cos(d*x+c)^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx=-\frac{\sqrt{\cos(c+dx)}\sin(2(c+dx))}{d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(3 - 5*Cos[c + d*x]^2),x]
```

output
```
-((Sqrt[Cos[c + d*x]]*Sin[2*(c + d*x)])/d)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3042, 3490}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(3-5\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{ 3490}$$

$$-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(3 - 5*Cos[c + d*x]^2),x]
```

output
```
(-2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3490
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(19) = 38$.

Time = 4.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.71

method	result
default	$-\frac{4\sqrt{\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$
parts	$\frac{6\sqrt{\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d} + \frac{2\sqrt{\left(-1+2\cos\left(\right.\right.}}{}$

input
```
int(cos(d*x+c)^(1/2)*(3-5*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-4*((-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1+2*cos(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx = -\frac{2\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)}{d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(3-5*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
-2*cos(d*x + c)^(3/2)*sin(d*x + c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(3-5*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx = \int -\left(5\,\cos\left(dx+c\right)^2-3\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(3-5*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
-integrate((5*cos(d*x + c)^2 - 3)*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx = \int -\left(5\,\cos\left(dx+c\right)^2-3\right)\sqrt{\cos\left(dx+c\right)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(3-5*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate(-(5*cos(d*x + c)^2 - 3)*sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx=-\int \sqrt{\cos\left(c+d\,x\right)}\,\left(5\cos\left(c+d\,x\right)^2-3\right)\,dx$$

input
```
int(-cos(c + d*x)^(1/2)*(5*cos(c + d*x)^2 - 3),x)
```

output
```
-int(cos(c + d*x)^(1/2)*(5*cos(c + d*x)^2 - 3), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx=3\left(\int \sqrt{\cos\left(dx+c\right)}dx\right)-5\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)$$

input
```
int(cos(d*x+c)^(1/2)*(3-5*cos(d*x+c)^2),x)
```

output
```
3*int(sqrt(cos(c + d*x)),x) - 5*int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)
```

3.25 $\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{d}$$

output
```
-2*cos(d*x+c)^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{d}$$

input
```
Integrate[(1 - 3*Cos[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

output
```
(-2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3042, 3490}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-3\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3490}$$

$$-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

input
```
Int[(1 - 3*Cos[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

output
```
(-2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3490
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(19) = 38$.

Time = 1.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.71

method	result
default	$-\frac{4\sqrt{\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$
parts	$\frac{2\ \mathrm{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\sqrt{2}\right)}{d}+\frac{2\sqrt{\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\right.}$

input
```
int((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*((-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(-1+2*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=-\frac{2\sqrt{\cos(dx+c)}\sin(dx+c)}{d}$$

input
```
integrate((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(cos(d*x + c))*sin(d*x + c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((1-3*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int -\frac{3\,\cos\left(dx+c\right)^2-1}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((3*cos(d*x + c)^2 - 1)/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int -\frac{3\,\cos\left(dx+c\right)^2-1}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(3*cos(d*x + c)^2 - 1)/sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 42.64 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{d}$$

input
```
int(-(3*cos(c + d*x)^2 - 1)/cos(c + d*x)^(1/2),x)
```

output
```
-(2*cos(c + d*x)^(1/2)*sin(c + d*x))/d
```

Reduce [F]

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx - 3\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)$$

input
```
int((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cos(c + d*x))/cos(c + d*x),x) - 3*int(sqrt(cos(c + d*x))*cos(c +
d*x),x)
```

3.26 $\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{9/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{9/2}\,dx=\frac{2b^4(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^3(5A+7C)(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}+\frac{2Ab^2(b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}$$

output

```
2/21*b^4*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
*(b*sec(d*x+c))^(1/2)/d+2/21*b^3*(5*A+7*C)*(b*sec(d*x+c))^(3/2)*sin(d*x+c)
/d+2/7*A*b^2*(b*sec(d*x+c))^(5/2)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{9/2}\,dx=\frac{b^2(b\sec(c+dx))^{5/2}\left(2(5A+7C)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C)\sin(2(c\right.}{21d}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(9/2),x]
```

output
```
(b^2*(b*Sec[c + d*x])^(5/2)*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*EllipticF[(c
 + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))/(21*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3717, 3042, 4534, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(c+dx))^{9/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3717}$$

$$b^2\int (b\sec(c+dx))^{5/2}\left(A\sec^2(c+dx)+C\right)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)^2+C\right)dx$$

$$\Big\downarrow \text{4534}$$

$$b^2\left(\frac{1}{7}(5A+7C)\int (b\sec(c+dx))^{5/2}dx+\frac{2A\tan(c+dx)(b\sec(c+dx))^{5/2}}{7d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{1}{7}(5A+7C)\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2A\tan(c+dx)(b\sec(c+dx))^{5/2}}{7d}\right)$$

$$\Big\downarrow \text{4255}$$

$$b^2\left(\frac{1}{7}(5A+7C)\left(\frac{1}{3}b^2\int\sqrt{b\sec(c+dx)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2A\tan(c+dx)(b\sec(c+dx))}{7d}\right.$$

$\downarrow$ 3042

$$b^2\left(\frac{1}{7}(5A+7C)\left(\frac{1}{3}b^2\int\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\frac{2A\tan(c+dx)(b\sec(c}{7d}\right.$$

$\downarrow$ 4258

$$b^2\left(\frac{1}{7}(5A+7C)\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)+\right.$$

$\downarrow$ 3042

$$b^2\left(\frac{1}{7}(5A+7C)\left(\frac{1}{3}b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)\right.$$

$\downarrow$ 3120

$$b^2\left(\frac{1}{7}(5A+7C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}\right)\right.$$

input
```
Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(9/2),x]
```

output
```
b^2*(((5*A + 7*C)*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[b*Sec[c + d*x]])/(3*d) + (2*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
))/7 + (2*A*(b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 75.01 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.60

method	result
default	$\frac{b^4\left(\frac{2C\tan(dx+c)}{3}-\frac{2A\left(-5\tan(dx+c)-3\tan(dx+c)\sec(dx+c)^2\right)}{21}-\frac{10i(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,\text{EllipticF}(i(\csc(dx+c)-\cot}{21}\right.}{d}$
parts	$\frac{A\left(\frac{10\tan(dx+c)}{21}+\frac{2\tan(dx+c)\sec(dx+c)^2}{7}-\frac{10i(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,\text{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)}{21}\right)b^4\sqrt{b\,\text{se}}}{d}$

input
```
int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
b^4/d*(2/3*C*tan(d*x+c)-2/21*A*(-5*tan(d*x+c)-3*tan(d*x+c)*sec(d*x+c)^2)-1
0/21*I*(1+cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*A-2/3*I*(1+cos(d*x+c))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(d*x
+c)-cot(d*x+c)),I)*C)*(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.23

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{9/2}\,dx=\frac{-i\sqrt{2}(5A+7C)b^{\frac{9}{2}}\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
1/21*(-I*sqrt(2)*(5*A + 7*C)*b^(9/2)*cos(d*x + c)^3*weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(5*A + 7*C)*b^(9/2)*cos(d*
x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((5
*A + 7*C)*b^4*cos(d*x + c)^2 + 3*A*b^4)*sqrt(b/cos(d*x + c))*sin(d*x + c))
/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{9/2}\,dx=\text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{9/2}\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{9}{2}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{9/2}\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{9}{2}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{9/2}\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{9/2}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(9/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(9/2), x)
```

Reduce [F]

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{9/2}\,dx=\sqrt{b}\,b^4\left(\left(\int \sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)^4dx\right)c+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^4dx\right)a\right)$$

input
```
int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x)
```

output

```
sqrt(b)*b**4*(int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**4,x)*c
+ int(sqrt(sec(c + d*x))*sec(c + d*x)**4,x)*a)
```

3.27 $\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{7/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{7/2} dx = -\frac{2b^4(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^3(3A+5C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2Ab^2(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

output
```
-2/5*b^4*(3*A+5*C)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2
)/(b*sec(d*x+c))^(1/2)+2/5*b^3*(3*A+5*C)*(b*sec(d*x+c))^(1/2)*sin(d*x+c)/d
+2/5*A*b^2*(b*sec(d*x+c))^(3/2)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int (A+C\cos^2(c+dx)) (b\sec(c+dx))^{7/2} dx = -\frac{b^2(b\sec(c+dx))^{3/2}\left(2(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-(3A+5C)\sin(2(c+dx))-2A\tan(c-\right.}{5d}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(7/2),x]
```

output
```
-1/5*(b^2*(b*Sec[c + d*x])^(3/2)*(2*(3*A + 5*C)*Cos[c + d*x]^(3/2)*Ellipti
cE[(c + d*x)/2, 2] - (3*A + 5*C)*Sin[2*(c + d*x)] - 2*A*Tan[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3717, 3042, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(c+dx))^{7/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3717}$$

$$b^2\int (b\sec(c+dx))^{3/2}\left(A\sec^2(c+dx)+C\right)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)^2+C\right)dx$$

$$\Big\downarrow \text{4534}$$

$$b^2\left(\frac{1}{5}(3A+5C)\int (b\sec(c+dx))^{3/2}dx+\frac{2A\tan(c+dx)(b\sec(c+dx))^{3/2}}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{1}{5}(3A+5C)\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2A\tan(c+dx)(b\sec(c+dx))^{3/2}}{5d}\right)$$

$$\Big\downarrow \text{4255}$$

$$b^2\left(\frac{1}{5}(3A+5C)\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}dx\right)+\frac{2A\tan(c+dx)(b\sec(c+dx))^{3/2}}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{1}{5}(3A+5C)\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-b^2\int\frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2A\tan(c+dx)(b\sec(c+dx}{5d}\right.$$

$$\Big\downarrow \text{4258}$$

$$b^2\left(\frac{1}{5}(3A+5C)\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2A\tan(c+dx)(b\sec(c+dx}{5d}\right.$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{1}{5}(3A+5C)\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{b^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2A\tan(c+dx)(b\sec(c+d}{5d}\right.$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{1}{5}(3A+5C)\left(\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)+\frac{2A\tan(c+dx)(b\sec(c+}{5d}\right.$$

input
```
Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(7/2),x]
```

output
```
b^2*(((3*A + 5*C)*((-2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]*Sqrt[b*Sec[c + d*x]]) + (2*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d))/5 +
(2*A*(b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 69.42 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.21

method	result
default	$\frac{2b^3\left(3i\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)A\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticE}(i(\csc(dx+c)-\cot(dx+c)),i)+5i\left(-\cos(dx+c)^2-\right.\right.}{}$
parts	$\frac{2A\sqrt{b\sec(dx+c)}\,b^3\left(3\sin(dx+c)+\tan(dx+c)+\sec(dx+c)\tan(dx+c)+i\left(-3\cos(dx+c)^2-6\cos(dx+c)-3\right)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx}{1+\cos(d}}\right.}{}$

input
```
int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*b^3/d*(3*I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*A*(1/(1+cos(d*x+c)))^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+5*
I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+3*I*(cos(d*x+c)
^2+2*cos(d*x+c)+1)*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+5*I*(cos(d*x+c)^2+2*cos(d*x+c
)+1)*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elliptic
F(I*(csc(d*x+c)-cot(d*x+c)),I)+A*(3*sin(d*x+c)+tan(d*x+c)+sec(d*x+c)*tan(d
*x+c))+5*C*sin(d*x+c))*(b*sec(d*x+c))^(1/2)/(1+cos(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.28

$$\int (A + C\cos^2(c+dx)) (b\sec(c+dx))^{7/2}\,dx = \frac{-i\sqrt{2}(3A+5C)b^{\frac{7}{2}}\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/5*(-I*sqrt(2)*(3*A + 5*C)*b^(7/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(3*
A + 5*C)*b^(7/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*b^3*cos(d*x + c)^
2 + A*b^3)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{7/2}\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{7/2}\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{7/2}\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{7/2}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(7/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{7/2}\,dx=\sqrt{b}\,b^3\left(\left(\int \sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)^3dx\right)c+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^3dx\right)a\right)$$

input
```
int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x)
```

output
```
sqrt(b)*b**3*(int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**3,x)*c
+ int(sqrt(sec(c + d*x))*sec(c + d*x)**3,x)*a)
```

3.28 $\int (A + C\cos^2(c + dx)) (b\sec(c+dx))^{5/2}\, dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int (A + C\cos^2(c + dx)) (b\sec(c + dx))^{5/2}\, dx = \frac{2b^2(A + 3C)\sqrt{\cos(c + dx)}\,\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)\sqrt{b\sec(c + dx)}}{3d} + \frac{2Ab^2\sqrt{b\sec(c + dx)}\tan(c + dx)}{3d}$$

output
```
2/3*b^2*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b
*sec(d*x+c))^(1/2)/d+2/3*A*b^2*(b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int (A + C\cos^2(c + dx)) (b\sec(c + dx))^{5/2}\, dx = \frac{2b^2\sqrt{b\sec(c + dx)}\left((A + 3C)\sqrt{\cos(c + dx)}\,\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + A\tan(c + dx)\right)}{3d}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*b^2*Sqrt[b*Sec[c + d*x]]*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2] + A*Tan[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3717, 3042, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{ 3717}$$

$$b^2\int \sqrt{b\sec(c+dx)}\left(A\sec^2(c+dx)+C\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$b^2\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A\csc\left(c+dx+\frac{\pi}{2}\right)^2+C\right)dx$$

$$\Big\downarrow \text{ 4534}$$

$$b^2\left(\frac{1}{3}(A+3C)\int \sqrt{b\sec(c+dx)}dx+\frac{2A\tan(c+dx)\sqrt{b\sec(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^2\left(\frac{1}{3}(A+3C)\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2A\tan(c+dx)\sqrt{b\sec(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{ 4258}$$

$$b^2\left(\frac{1}{3}(A+3C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2A\tan(c+dx)\sqrt{b\sec(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{1}{3}(A+3C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2A\tan(c+dx)\sqrt{b\sec(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(\frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2A\tan(c+dx)\sqrt{b\sec(c+dx)}}{3d}\right)$$

input
```
Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(5/2),x]
```

output
```
b^2*((2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[
c + d*x]])/(3*d) + (2*A*Sqrt[b*Sec[c + d*x]]*Tan[c + d*x])/(3*d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 66.64 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.00

method	result
default	$\frac{b^2\left(\frac{2A\tan(dx+c)}{3}-\frac{2i(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,\text{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)A}{3}-2i(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+}}\right.}{d}$
parts	$\frac{A\left(-\frac{2i(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{1}{1+\cos(dx+c)}}\,\text{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)}{3}+\frac{2\tan(dx+c)}{3}\right)b^2\sqrt{b\sec(dx+c)}}{d}-\frac{2iC\,b^2(1+}{}$

input
```
int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
b^2/d*(2/3*A*tan(d*x+c)-2/3*I*(1+cos(d*x+c))*A*(1/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)-2*I
*(1+cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*C)*(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int \left(A+C\cos^2(c+dx)\right)\left(b\sec(c+dx)\right)^{5/2}dx = \frac{-i\sqrt{2}(A+3C)b^{\frac{5}{2}}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*A*b^2*sqrt(b
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \left(A + C\cos^2(c + dx)\right) (b\sec(c + dx))^{5/2}\, dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(A+C\cos^2(c+dx)\right) (b\sec(c+dx))^{5/2}\, dx = \int \left(C\cos\left(dx+c\right)^2 + A\right) \left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}\, dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{5/2}\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{5/2}\,dx = \int \left(C\cos\left(c+d\,x\right)^2+A\right)\left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(5/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{5/2}\,dx = \sqrt{b}\,b^2\left(\left(\int \sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2dx\right)c+\left(\int \sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)^2dx\right)a\right)$$

input
```
int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*b**2*(int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x)*c
+ int(sqrt(sec(c + d*x))*sec(c + d*x)**2,x)*a)
```

3.29 $\int (A+C\cos^2(c+dx))(b\sec(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int (A+C\cos^2(c+dx))(b\sec(c+dx))^{3/2}\,dx = -\frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2Ab^2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

output
```
-2*b^2*(A-C)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*s
ec(d*x+c))^(1/2)+2*A*b^2*tan(d*x+c)/d/(b*sec(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int (A+C\cos^2(c+dx))(b\sec(c+dx))^{3/2}\,dx = \frac{2b\sqrt{b\sec(c+dx)}\left(-\left((A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+A\sin(c+dx)\right)}{d}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*b*Sqrt[b*Sec[c + d*x]]*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2]) + A*Sin[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3717, 3042, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3717}$$

$$b^2\int \frac{A\sec^2(c+dx)+C}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)^2+C}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4534}$$

$$b^2\left(\frac{2A\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}-(A-C)\int \frac{1}{\sqrt{b\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{2A\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}-(A-C)\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$b^2\left(\frac{2A\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}-\frac{(A-C)\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{2A\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}-\frac{(A-C)\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{2A\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\right)$$

input
```
Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(3/2),x]
```

output
```
b^2*((-2*(A - C)*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*S
ec[c + d*x]]) + (2*A*Tan[c + d*x])/(d*Sqrt[b*Sec[c + d*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.83 (sec) , antiderivative size = 350, normalized size of antiderivative = 4.73

method	result
default	$\frac{2\left(i\left(-\cos(dx+c)^2-2\cos(dx+c)-1\right)A\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticE}(i(\csc(dx+c)-\cot(dx+c)),i)+i\left(\cos(dx+c)^2+2\cos(\right.\right.}{}$
parts	$\frac{2A\left(i\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)+i\left(-\cos(dx+c)^2-2\cos(\right.\right.}{d(1+\cos(dx+c))}$

input
```
int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+I*(cos(d*x
+c)^2+2*cos(d*x+c)+1)*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+I*(cos(d*x+c)^2+2*cos(d*x+
c)+1)*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellipti
cF(I*(csc(d*x+c)-cot(d*x+c)),I)+I*(-cos(d*x+c)^2-2*cos(d*x+c)-1)*C*(1/(1+c
os(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(d*x+c
)-cot(d*x+c)),I)+C*cos(d*x+c)*sin(d*x+c)+A*sin(d*x+c))*(b*sec(d*x+c))^(1/2
)*b/(1+cos(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int (A + C\cos^2(c + dx)) (b\sec(c + dx))^{3/2} \, dx = \frac{-i\sqrt{2}(A - C)b^{\frac{3}{2}}\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx -$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*(A - C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(A - C)*b^(3/2)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
2*A*b*sqrt(b/cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (A + C\cos^2(c + dx)) (b\sec(c + dx))^{3/2} \, dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{3/2}\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{3/2}\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{3/2}\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(3/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \left(A+C\cos^2(c+dx)\right)\left(b\sec(c+dx)\right)^{3/2}dx=\sqrt{b}\,b\left(\left(\int\sqrt{\sec\left(dx+c\right)}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)dx\right)c+\left(\int\sqrt{\sec\left(dx+c\right)}\,\sec\left(dx+c\right)dx\right)a\right)$$

input
```
int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x)
```

output
```
sqrt(b)*b*(int(sqrt(sec(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x)*c + int(
sqrt(sec(c + d*x))*sec(c + d*x),x)*a)
```

3.30 $\int (A+C\cos^2(c+dx))\sqrt{b\sec(c+dx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int (A+C\cos^2(c+dx))\sqrt{b\sec(c+dx)}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2b^2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}$$

output
```
2/3*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*sec
(d*x+c))^(1/2)/d+2/3*b^2*C*tan(d*x+c)/d/(b*sec(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int (A+C\cos^2(c+dx))\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\left(2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx))\right)}{3d}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)*Sqrt[b*Sec[c + d*x]],x]
```

output
```
(Sqrt[b*Sec[c + d*x]]*(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2] + C*Sin[2*(c + d*x)]))/(3*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3717, 3042, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sec(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \ 3717$$

$$b^2\int \frac{A\sec^2(c+dx)+C}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$b^2\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)^2+C}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \ 4533$$

$$b^2\left(\frac{(3A+C)\int\sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$b^2\left(\frac{(3A+C)\int\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \ 4258$$

$$b^2\left(\frac{(3A+C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}\right)$$

$\big\downarrow$ 3042

$$b^2\left(\frac{(3A+C)\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}\right)$$

$\big\downarrow$ 3120

$$b^2\left(\frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}\right)$$

input
```
Int[(A + C*Cos[c + d*x]^2)*Sqrt[b*Sec[c + d*x]],x]
```

output
```
b^2*((2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[
c + d*x]])/(3*b^2*d) + (2*C*Tan[c + d*x])/(3*d*(b*Sec[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.16

method	result
default	$-\frac{2\left(i(3\cos(dx+c)+3)A\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\text{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)+i(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{1+\cdots}\right.}{3d}$
parts	$-\frac{2iA(1+\cos(dx+c))\sqrt{b\sec(dx+c)}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\text{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)}{d}-\frac{2C\left(i(1+\cos(dx+c))\right.\cdots}{}$

input
```
int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*(I*(3*cos(d*x+c)+3)*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+I*(1+cos(d*x+c))*C*(1
/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(
d*x+c)-cot(d*x+c)),I)-C*cos(d*x+c)*sin(d*x+c))*(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \left(A+C\cos^2(c+dx)\right)\sqrt{b\sec(c+dx)}\,dx$$
$$=\frac{2\,C\sqrt{\frac{b}{\cos(dx+c)}}\cos{(dx+c)}\sin{(dx+c)}+\sqrt{2}(-3i\,A-i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+\cdots}{3\,d}$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*C*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(-3*I*A
- I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
 sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c)))/d
```

Sympy [F]

$$\int \left(A+C\cos^2(c+dx)\right)\sqrt{b\sec(c+dx)}\,dx=\int \sqrt{b\sec\left(c+dx\right)}\left(A+C\cos^2\left(c+dx\right)\right)\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(c + d*x))*(A + C*cos(c + d*x)**2), x)
```

Maxima [F]

$$\int \left(A+C\cos^2(c+dx)\right)\sqrt{b\sec(c+dx)}\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\sec\left(dx+c\right)}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \left(A+C\cos^2(c+dx)\right)\sqrt{b\sec(c+dx)}\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\sec\left(dx+c\right)}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+C\cos^2(c+dx))\sqrt{b\sec(c+dx)}\,dx = \int \left(C\cos\left(c+d\,x\right)^2+A\right)\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(1/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int (A+C\cos^2(c+dx))\sqrt{b\sec(c+dx)}\,dx = \sqrt{b}\left(\left(\int \sqrt{\sec\left(dx+c\right)}dx\right)a + \left(\int \sqrt{\sec\left(dx+c\right)}\cos\left(dx+c\right)^2dx\right)c\right)$$

input
```
int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*(int(sqrt(sec(c + d*x)),x)*a + int(sqrt(sec(c + d*x))*cos(c + d*x)
**2,x)*c)
```

3.31 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}$$

output
```
2/5*(5*A+3*C)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*
sec(d*x+c))^(1/2)+2/5*b^2*C*tan(d*x+c)/d/(b*sec(d*x+c))^(5/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\frac{\frac{4(5A+3C)E\left(\frac{1}{2}(c+dx)|2\right)}{\sqrt{\cos(c+dx)}}+2C\sin(2(c+dx))}{10d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]
```

output
```
((4*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*C*Sin[2*
(c + d*x)])/(10*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3717, 3042, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3717}$$

$$b^2\int \frac{A\sec^2(c+dx)+C}{(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)^2+C}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4533}$$

$$b^2\left(\frac{(5A+3C)\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(5A+3C)\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^2\left(\frac{(5A+3C)\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(5A+3C)\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}\right)$$

input
```
Int[(A + C*Cos[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]
```

output
```
b^2*((2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]
*Sqrt[b*Sec[c + d*x]]) + (2*C*Tan[c + d*x])/(5*d*(b*Sec[c + d*x])^(5/2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.40 (sec) , antiderivative size = 340, normalized size of antiderivative = 4.42

method	result
default	$\frac{2i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(2+\cos(dx+c)+\sec(dx+c))A\,\mathrm{EllipticE}(i(\csc(dx+c)-\cot(dx+c)),i)+\frac{6i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c}}}{}}{}$
parts	$\frac{2A\left(i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(-\cos(dx+c)-2-\sec(dx+c))+i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx}{1+\cos(d}}\right.}{d(1+\cos(dx+c))\sqrt{b\sec(dx+c)}}$

input
```
int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/(1+cos(d*x+c))/(b*sec(d*x+c))^(1/2)*(5*I*(1/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*A*EllipticE(I*(c
sc(d*x+c)-cot(d*x+c)),I)+3*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*C*EllipticE(I*(csc(d*x+c)-cot(d*x+
c)),I)-5*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2+c
os(d*x+c)+sec(d*x+c))*A*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)-3*I*(1/(1+c
os(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2+cos(d*x+c)+sec(d*x+
c))*C*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+5*A*sin(d*x+c)+sin(d*x+c)*(co
s(d*x+c)^2+cos(d*x+c)+3)*C)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\sec(c + dx)}}\, dx$$
$$= \frac{2\,C\sqrt{\frac{b}{\cos(dx+c)}}\cos\left(dx+c\right)^2\sin\left(dx+c\right) + \sqrt{2}(5i\,A + 3i\,C)\sqrt{b}\text{weierstrassZeta}(-4, 0, \text{weierstrassPInvers}}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/5*(2*C*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + sqrt(2)*(5*I*A
 + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d
)
```

Sympy [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\sec(c + dx)}}\, dx = \int \frac{A + C\cos^2\left(c + dx\right)}{\sqrt{b\sec\left(c + dx\right)}}\, dx$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*sec(d*x+c))**(1/2),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)/sqrt(b*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx = \int \frac{C\cos(dx + c)^2 + A}{\sqrt{b\sec(dx + c)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/sqrt(b*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx = \int \frac{C\cos(dx + c)^2 + A}{\sqrt{b\sec(dx + c)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/sqrt(b*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\sec(c + dx)}}\,dx = \int \frac{C\cos(c + d\,x)^2 + A}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(1/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx=\frac{\sqrt{b}\left(\left(\int\frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)}dx\right)a+\left(\int\frac{\sqrt{\sec(dx+c)}\cos(dx+c)^2}{\sec(dx+c)}dx\right)c\right)}{b}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(sec(c + d*x))/sec(c + d*x),x)*a + int((sqrt(sec(c + d*x
))*cos(c + d*x)**2)/sec(c + d*x),x)*c))/b
```

3.32 $\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{21b^2d}+\frac{2(7A+5C)\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}+\frac{2b^2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}$$

output
```
2/21*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))*(b*
sec(d*x+c))^(1/2)/b^2/d+2/21*(7*A+5*C)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(1/2)
+2/7*b^2*C*tan(d*x+c)/d/(b*sec(d*x+c))^(7/2)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx=\frac{\frac{4(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{\sqrt{\cos(c+dx)}}+2(14A+13C+3C\cos(2(c+dx)))\sin(c+dx)}{42bd\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(3/2),x]
```

output
```
((4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(14*A +
13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(42*b*d*Sqrt[b*Sec[c + d*x]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3717, 3042, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3717}$$

$$b^2\int \frac{A\sec^2(c+dx)+C}{(b\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$b^2\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)^2+C}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4533}$$

$$b^2\left(\frac{(7A+5C)\int \frac{1}{(b\sec(c+dx))^{3/2}}dx}{7b^2}+\frac{2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^2\left(\frac{(7A+5C)\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7b^2}+\frac{2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 4256}$$

$$b^2\left(\frac{(7A+5C)\left(\frac{\int \sqrt{b\sec(c+dx)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3042

$$b^2\left(\frac{(7A+5C)\left(\frac{\int \sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 4258

$$b^2\left(\frac{(7A+5C)\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3042

$$b^2\left(\frac{(7A+5C)\left(\frac{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3120

$$b^2\left(\frac{(7A+5C)\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}\right)}{7b^2}+\frac{2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}\right)$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(3/2),x]
```

output
```
b^2*(((7*A + 5*C)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*
Sec[c + d*x]])/(3*b^2*d) + (2*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])))
/(7*b^2) + (2*C*Tan[c + d*x])/(7*d*(b*Sec[c + d*x])^(7/2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.62 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.57

method	result
default	$\frac{-\frac{2iA\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(7+7\sec(dx+c))}{21}-\frac{2iC\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}(i(\csc}{21}}{d\sqrt{b\sec(dx+c)}\,b}$
parts	$\frac{A\left(-\frac{2i\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}(i(\csc(dx+c)-\cot(dx+c)),i)(1+\sec(dx+c))}{3}+\frac{2\sin(dx+c)}{3}\right)}{d\sqrt{b\sec(dx+c)}\,b}-\frac{2C\left(\sin(dx+c)\left(-3\cos(dx+\right.\right.}{}$

input
```
int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/21*I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(7+7*sec(d*x+c))-2/21*I*C*(1/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(csc(d*x+c)-
cot(d*x+c)),I)*(5+5*sec(d*x+c))+2/3*A*sin(d*x+c)-2/21*sin(d*x+c)*(-3*cos(d
*x+c)^2-5)*C)/(b*sec(d*x+c))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}(-7i\,A-5i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/21*(sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*cos(d*x + c)^3 + (7*A +
5*C)*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{A+C\cos^2{(c+dx)}}{(b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)/(b*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{C\cos{(dx+c)}^2+A}{(b\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{C\cos{(dx+c)}^2+A}{(b\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(3/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^2}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\cos(dx+c)^2}{\sec(dx+c)^2}dx\right)c\right)}{b^2}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(sec(c + d*x))/sec(c + d*x)**2,x)*a + int((sqrt(sec(c +
d*x))*cos(c + d*x)**2)/sec(c + d*x)**2,x)*c))/b**2
```

3.33 $\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}$$

output
```
2/15*(9*A+7*C)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d*x+c)^(1/2
)/(b*sec(d*x+c))^(1/2)+2/45*(9*A+7*C)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(3/2)+
2/9*b^2*C*tan(d*x+c)/d/(b*sec(d*x+c))^(9/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\frac{48(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 4(18A+19C+5C\cos(2(c+dx)))\sin(2(c+dx))}{360b^2d\sqrt{b\sec(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(5/2),x]
```

output
```
((48*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 4*(18*A +
 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(360*b^2*d*Sqrt[b*Sec[c +
d*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3717, 3042, 4533, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3717}$$

$$b^2\int \frac{A\sec^2(c+dx)+C}{(b\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\int \frac{A\csc\left(c+dx+\frac{\pi}{2}\right)^2+C}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4533}$$

$$b^2\left(\frac{(9A+7C)\int \frac{1}{(b\sec(c+dx))^{5/2}}dx}{9b^2}+\frac{2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(9A+7C)\int \frac{1}{\left(b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9b^2}+\frac{2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$b^2\left(\frac{(9A+7C)\left(\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(9A+7C)\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$b^2\left(\frac{(9A+7C)\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(9A+7C)\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{(9A+7C)\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}\right)}{9b^2}+\frac{2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}\right)$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(5/2),x]
```

output
```
b^2*(((9*A + 7*C)*((6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x
]]*Sqrt[b*Sec[c + d*x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2))
))/(9*b^2) + (2*C*Tan[c + d*x])/(9*d*(b*Sec[c + d*x])^(9/2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3717
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Simp[d^(n*p)   Int[(d*Csc[e + f*x])^(m - n*p
)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
  !IntegerQ[m] && IntegersQ[n, p]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 11.72 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.37

method	result
default	$-\frac{6i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(2+\cos(dx+c)+\sec(dx+c))A\,\text{EllipticE}(i(\csc(dx+c)-\cot(dx+c)),i)}{5}-\frac{14i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(2+c$
parts	$\frac{2A\left(\sin(dx+c)\left(\cos(dx+c)^2+\cos(dx+c)+3\right)-3i\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(2+\cos(dx+c)+\sec(dx+c))\,\text{EllipticF}(i(\csc(dx+c)-c}{5d(1+\cos(dx+c))\sqrt{b\sec(}}$

input
```
int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/d/b^2*(-27*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(2+cos(d*x+c)+sec(d*x+c))*A*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-21*I
*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2+cos(d*x+c)+
sec(d*x+c))*C*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+27*I*(1/(1+cos(d*x+c)
))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*A*Ell
ipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+21*I*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(2+cos(d*x+c)+sec(d*x+c))*C*EllipticF(I*(csc(d*x
+c)-cot(d*x+c)),I)+9*sin(d*x+c)*(-cos(d*x+c)^2-cos(d*x+c)-3)*A+sin(d*x+c)*
(-5*cos(d*x+c)^4-5*cos(d*x+c)^3-7*cos(d*x+c)^2-7*cos(d*x+c)-21)*C)/(1+cos(
d*x+c))/(b*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{3\sqrt{2}(-9i\,A-7i\,C)\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/45*(3*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(9*I*A + 7*I
*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) - 2*(5*C*cos(d*x + c)^4 + (9*A + 7*C)*cos(d*x + c)^2)*
sqrt(b/cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\sec(c + dx))^{5/2}}\, dx = \int \frac{A + C\cos^2{(c + dx)}}{(b\sec{(c + dx)})^{\frac{5}{2}}}\, dx$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)/(b*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\sec(c + dx))^{5/2}}\, dx = \int \frac{C\cos{(dx + c)}^2 + A}{(b\sec{(dx + c)})^{\frac{5}{2}}}\, dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{C\cos(dx+c)^2+A}{(b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \int \frac{C\cos(c+dx)^2+A}{\left(\frac{b}{\cos(c+dx)}\right)^{5/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(5/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\sec(dx+c)}}{\sec(dx+c)^3}dx\right)a+\left(\int \frac{\sqrt{\sec(dx+c)}\,\cos(dx+c)^2}{\sec(dx+c)^3}dx\right)c\right)}{b^3}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(sec(c + d*x))/sec(c + d*x)**3,x)*a + int((sqrt(sec(c +
d*x))*cos(c + d*x)**2)/sec(c + d*x)**3,x)*c))/b**3
```

3.34 $\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right) dx = \frac{C(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(2+m)} - \frac{(C(1+m)+A(2+m))(b\cos(c+dx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sin(c+d}{bd(1+m)(2+m)\sqrt{\sin^2(c+dx)}}$$

output
```
C*(b*cos(d*x+c))^(1+m)*sin(d*x+c)/b/d/(2+m)-(C*(1+m)+A*(2+m))*(b*cos(d*x+c
))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/b
/d/(1+m)/(2+m)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right) dx = -\frac{(b\cos(c+dx))^m\cot(c+dx)\left(A(3+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)+C(1+m)}{d(1+m)(3+m)}$$

input
```
Integrate[(b*Cos[c + d*x])^m*(A + C*Cos[c + d*x]^2),x]
```

output
```
-(((b*Cos[c + d*x])^m*Cot[c + d*x]*(A*(3 + m)*Hypergeometric2F1[1/2, (1 +
m)/2, (3 + m)/2, Cos[c + d*x]^2] + C*(1 + m)*Cos[c + d*x]^2*Hypergeometric
2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(
1 + m)*(3 + m)))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3493}$$

$$\left(A+\frac{C(m+1)}{m+2}\right)\int (b\cos(c+dx))^m dx+\frac{C\sin(c+dx)(b\cos(c+dx))^{m+1}}{bd(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\left(A+\frac{C(m+1)}{m+2}\right)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^m dx+\frac{C\sin(c+dx)(b\cos(c+dx))^{m+1}}{bd(m+2)}$$

$$\Big\downarrow \text{3122}$$

$$\frac{C\sin(c+dx)(b\cos(c+dx))^{m+1}}{bd(m+2)}-\frac{\left(A+\frac{C(m+1)}{m+2}\right)\sin(c+dx)(b\cos(c+dx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(c+dx)\right)}{bd(m+1)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^m*(A + C*Cos[c + d*x]^2),x]
```

output
```
(C*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(2 + m)) - ((A + (C*(1 + m)
)/(2 + m))*(b*Cos[c + d*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + m)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos (dx+c))^m \left(A+C\cos (dx+c)^2\right) dx$$

input
```
int((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x)
```

output
```
int((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^m\,dx$$

input
```
integrate((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^m, x)
```

Sympy [F]

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right)\,dx=\int \left(b\cos\left(c+dx\right)\right)^m\left(A+C\cos^2\left(c+dx\right)\right)\,dx$$

input
```
integrate((b*cos(d*x+c))**m*(A+C*cos(d*x+c)**2),x)
```

output
```
Integral((b*cos(c + d*x))**m*(A + C*cos(c + d*x)**2), x)
```

Maxima [F]

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^m\,dx$$

input
```
integrate((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^m, x)
```

Giac [F]

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^m\,dx$$

input
```
integrate((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^m\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^m,x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^m, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right)\,dx = b^m\left(\left(\int\cos\left(dx+c\right)^m dx\right)a+\left(\int\cos\left(dx+c\right)^m\cos\left(dx+c\right)^2 dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x)
```

output
```
b**m*(int(cos(c + d*x)**m,x)*a + int(cos(c + d*x)**m*cos(c + d*x)**2,x)*c)
```

3.35 $\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 31

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx=\frac{C(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(2+m)}$$

output
```
C*(b*cos(d*x+c))^(1+m)*sin(d*x+c)/b/d/(2+m)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.65

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{C(b\cos(c+dx))^m\cot(c+dx)\left((3+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)-(2+m)\cos^2}{d(2+m)(3+m)}$$

input
```
Integrate[(b*Cos[c + d*x])^m*(-((C*(1 + m))/(2 + m)) + C*Cos[c + d*x]^2),x
]
```

output
```
(C*(b*Cos[c + d*x])^m*Cot[c + d*x]*((3 + m)*Hypergeometric2F1[1/2, (1 + m)
/2, (3 + m)/2, Cos[c + d*x]^2] - (2 + m)*Cos[c + d*x]^2*Hypergeometric2F1[
1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(2 +
m)*(3 + m))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {3042, 3490}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^m \left(C\cos^2(c+dx) - \frac{C(m+1)}{m+2}\right) dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^m \left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2 - \frac{C(m+1)}{m+2}\right) dx$$

$$\Big\downarrow \; 3490$$

$$\frac{C\sin(c+dx)(b\cos(c+dx))^{m+1}}{bd(m+2)}$$

input
```
Int[(b*Cos[c + d*x])^m*(-((C*(1 + m))/(2 + m)) + C*Cos[c + d*x]^2),x]
```

output
```
(C*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(2 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3490
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{\sin(2dx+2c)(b\cos(dx+c))^m C}{2(2+m)d}$	31

input
```
int((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x,method=_RETURNVERBO
SE)
```

output
```
1/2*sin(2*d*x+2*c)/(2+m)/d*(b*cos(d*x+c))^m*C
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int (b\cos(c+dx))^m\left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(dx+c))^m C\cos(dx+c)\sin(dx+c)}{dm+2d}$$

input
```
integrate((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x, algorithm="f
ricas")
```

output
```
(b*cos(d*x + c))^m*C*cos(d*x + c)*sin(d*x + c)/(d*m + 2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(26) = 52$.

Time = 18.60 (sec) , antiderivative size = 279, normalized size of antiderivative = 9.00

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m} + C\cos^2(c+dx)\right) dx = \begin{cases} -\frac{2C\left(-\frac{b\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1} + \frac{b}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}\right)^m \tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{dm\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2dm\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dm+2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2d} + \frac{2C\left(-\frac{b\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1} + \frac{b}{\tan^2\left(\frac{c}{2}+\right.} \right.}{dm\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2dm\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dm+2} \\ x(b\cos(c))^m \left(-\frac{C(m+1)}{m+2} + C\cos^2(c)\right) \end{cases}$$

input
```
integrate((b*cos(d*x+c))**m*(-C*(1+m)/(2+m)+C*cos(d*x+c)**2),x)
```

output
```
Piecewise((-2*C*(-b*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + b/(tan
(c/2 + d*x/2)**2 + 1))**m*tan(c/2 + d*x/2)**3/(d*m*tan(c/2 + d*x/2)**4 + 2
*d*m*tan(c/2 + d*x/2)**2 + d*m + 2*d*tan(c/2 + d*x/2)**4 + 4*d*tan(c/2 + d
*x/2)**2 + 2*d) + 2*C*(-b*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) +
b/(tan(c/2 + d*x/2)**2 + 1))**m*tan(c/2 + d*x/2)/(d*m*tan(c/2 + d*x/2)**4
+ 2*d*m*tan(c/2 + d*x/2)**2 + d*m + 2*d*tan(c/2 + d*x/2)**4 + 4*d*tan(c/2
+ d*x/2)**2 + 2*d), Ne(d, 0)), (x*(b*cos(c))**m*(-C*(m + 1)/(m + 2) + C*co
s(c)**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(31) = 62$.

Time = 0.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 5.65

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m} + C\cos^2(c+dx)\right) dx = -\frac{\left(\cos(2\,dx+2\,c)^2 + \sin(2\,dx+2\,c)^2 + 2\,\cos(2\,dx+2\,c) + 1\right)^{\frac{1}{2}\,m} Cb^m \sin\left(-(dx+c)(m+2) + m\,\mathrm{arcta}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x, algorithm="m
axima")
```

output
```
-1/4*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/2*m)*C*b^m*sin(-(d*x + c)*(m + 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/2*m)*C*b^m*sin(-(d*x + c)*(m - 2) + m*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)))/(2^m*d*(m + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. $2(31) = 62$.

Time = 2.00 (sec) , antiderivative size = 1238, normalized size of antiderivative = 39.94

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x, algorithm="g
iac")
```

output
```
2*(C*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 + 1))
^m*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)
^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c))
 + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c
)) + 1/2*pi*m*floor(d*x/pi + c/pi + 1/2))^2*tan(1/2*d*x + 1/2*c)^3 - C*(ab
s(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 + 1))^m*tan(-
1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b
)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*p
i*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/2
*pi*m*floor(d*x/pi + c/pi + 1/2))^2*tan(1/2*d*x + 1/2*c) - C*(abs(tan(1/2*
d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 + 1))^m*tan(1/2*d*x + 1
/2*c)^3 + C*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^
2 + 1))^m*tan(1/2*d*x + 1/2*c))/(d*m*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1
/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 -
 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/2*pi*m*floor(d*x/pi + c/pi + 1/2)
)^2*tan(1/2*d*x + 1/2*c)^4 + 2*d*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c
)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sg
n(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*
sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/2*pi*m*floor(d*x/pi + c/pi + 1/2))...
```

Mupad [B] (verification not implemented)

Time = 41.69 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int (b\cos(c+dx))^m\left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx = \frac{C\,\sin\left(2\,c+2\,d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^m}{2\,d\,\left(m+2\right)}$$

input
```
int((b*cos(c + d*x))^m*(C*cos(c + d*x)^2 - (C*(m + 1))/(m + 2)),x)
```

output
```
(C*sin(2*c + 2*d*x)*(b*cos(c + d*x))^m)/(2*d*(m + 2))
```

Reduce [F]

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{b^m c\left(-\left(\int \cos\left(dx+c\right)^m dx\right) m-\left(\int \cos\left(dx+c\right)^m dx\right)+\left(\int \cos\left(dx+c\right)^m \cos\left(dx+c\right)^2 dx\right) m+2\left(\int \cos\right.}{m+2}$$

input
```
int((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x)
```

output
```
(b**m*c*( - int(cos(c + d*x)**m,x)*m - int(cos(c + d*x)**m,x) + int(cos(c
+ d*x)**m*cos(c + d*x)**2,x)*m + 2*int(cos(c + d*x)**m*cos(c + d*x)**2,x))
)/(m + 2)
```

3.36 $\int (b\cos(c+dx))^m \left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 32

$$\int (b\cos(c+dx))^m \left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right)\,dx=-\frac{A(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(1+m)}$$

output
```
-A*(b*cos(d*x+c))^(1+m)*sin(d*x+c)/b/d/(1+m)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.56

$$\int (b\cos(c+dx))^m \left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right)\,dx =$$
$$-\frac{A(b\cos(c+dx))^m\cot(c+dx)\left((3+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)-(2+m)\,\text{co}\right.}{d(1+m)(3+m)}$$

input
```
Integrate[(b*Cos[c + d*x])^m*(A - (A*(2 + m)*Cos[c + d*x]^2)/(1 + m)),x]
```

output
```
-((A*(b*Cos[c + d*x])^m*Cot[c + d*x]*((3 + m)*Hypergeometric2F1[1/2, (1 +
m)/2, (3 + m)/2, Cos[c + d*x]^2] - (2 + m)*Cos[c + d*x]^2*Hypergeometric2F
1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1
+ m)*(3 + m)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3042, 3490}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A-\frac{A(m+2)\cos^2(c+dx)}{m+1}\right)(b\cos(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(A-\frac{A(m+2)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{m+1}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3490}$$

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{m+1}}{bd(m+1)}$$

input
```
Int[(b*Cos[c + d*x])^m*(A - (A*(2 + m)*Cos[c + d*x]^2)/(1 + m)),x]
```

output
```
-((A*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(1 + m)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3490
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
parallelrisch	$-\frac{\sin(2dx+2c)(b\cos(dx+c))^m A}{2(1+m)d}$	31

input
```
int((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x,method=_RETURNVERBOS
E)
```

output
```
-1/2*sin(2*d*x+2*c)/(1+m)/d*(b*cos(d*x+c))^m*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (b\cos(c+dx))^m \left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right)\,dx = -\frac{(b\cos(dx+c))^m A\cos(dx+c)\sin(dx+c)}{dm+d}$$

input
```
integrate((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x, algorithm="fr
icas")
```

output
```
-(b*cos(d*x + c))^m*A*cos(d*x + c)*sin(d*x + c)/(d*m + d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(27) = 54$.

Time = 18.46 (sec) , antiderivative size = 272, normalized size of antiderivative = 8.50

$$\int (b\cos(c+dx))^m \left(A - \frac{A(2+m)\cos^2(c+dx)}{1+m}\right) dx$$
$$= \begin{cases} \frac{2A\left(-\frac{b\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}+\frac{b}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}\right)^m \tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{dm\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2dm\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dm+d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+d} - \frac{2A\left(-\frac{b\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}+\frac{b}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ \right.}{dm\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2dm\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dm+d\tan^4} \\ x(b\cos(c))^m\left(A-\frac{A(m+2)\cos^2(c)}{m+1}\right) \end{cases}$$

input
```
integrate((b*cos(d*x+c))**m*(A-A*(2+m)*cos(d*x+c)**2/(1+m)),x)
```

output
```
Piecewise((2*A*(-b*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + b/(tan(
c/2 + d*x/2)**2 + 1))**m*tan(c/2 + d*x/2)**3/(d*m*tan(c/2 + d*x/2)**4 + 2*
d*m*tan(c/2 + d*x/2)**2 + d*m + d*tan(c/2 + d*x/2)**4 + 2*d*tan(c/2 + d*x/
2)**2 + d) - 2*A*(-b*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + b/(ta
n(c/2 + d*x/2)**2 + 1))**m*tan(c/2 + d*x/2)/(d*m*tan(c/2 + d*x/2)**4 + 2*d
*m*tan(c/2 + d*x/2)**2 + d*m + d*tan(c/2 + d*x/2)**4 + 2*d*tan(c/2 + d*x/2
)**2 + d), Ne(d, 0)), (x*(b*cos(c))**m*(A - A*(m + 2)*cos(c)**2/(m + 1)),
True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(32) = 64$.

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 5.47

$$\int (b\cos(c+dx))^m \left(A - \frac{A(2+m)\cos^2(c+dx)}{1+m}\right) dx$$
$$= \frac{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{2}\,m} Ab^m \sin\left(-(dx+c)(m+2)+m\arctan\right.}{}$$

input
```
integrate((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x, algorithm="ma
xima")
```

output
```
1/4*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/2*m)*A*b^m*sin(-(d*x + c)*(m + 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/2*m)*A*b^m*sin(-(d*x + c)*(m - 2) + m*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)))/(2^m*d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1233 vs. $2(32) = 64$.

Time = 2.41 (sec) , antiderivative size = 1233, normalized size of antiderivative = 38.53

$$\int (b\cos(c+dx))^m \left(A - \frac{A(2+m)\cos^2(c+dx)}{1+m}\right) dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x, algorithm="gi
ac")
```

output
```
-2*(A*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 + 1)
)^m*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c
)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)
) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*
c)) + 1/2*pi*m*floor(d*x/pi + c/pi + 1/2))^2*tan(1/2*d*x + 1/2*c)^3 - A*(a
bs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 + 1))^m*tan(
-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*
b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*
pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/
2*pi*m*floor(d*x/pi + c/pi + 1/2))^2*tan(1/2*d*x + 1/2*c) - A*(abs(tan(1/2
*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 + 1))^m*tan(1/2*d*x +
1/2*c)^3 + A*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)
^2 + 1))^m*tan(1/2*d*x + 1/2*c))/(d*m*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x +
1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 -
1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2
- 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/2*pi*m*floor(d*x/pi + c/pi + 1/2
))^2*tan(1/2*d*x + 1/2*c)^4 + d*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)
^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn
(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*s
gn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/2*pi*m*floor(d*x/pi + c/pi + 1/2))^...
```

Mupad [B] (verification not implemented)

Time = 41.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int (b\cos(c+dx))^m \left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right)\,dx = -\frac{A\,\sin\left(2\,c+2\,d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^m}{2\,d\,\left(m+1\right)}$$

input
```
int((b*cos(c + d*x))^m*(A - (A*cos(c + d*x)^2*(m + 2))/(m + 1)),x)
```

output
```
-(A*sin(2*c + 2*d*x)*(b*cos(c + d*x))^m)/(2*d*(m + 1))
```

Reduce [F]

$$\int (b\cos(c+dx))^m \left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right)\,dx$$
$$=\frac{b^m a\left(\left(\int \cos\left(dx+c\right)^m dx\right)m+\int \cos\left(dx+c\right)^m dx-\left(\int \cos\left(dx+c\right)^m \cos\left(dx+c\right)^2 dx\right)m-2\left(\int \cos\left(dx\right.\right.\right.}{m+1}$$

input
```
int((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x)
```

output
```
(b**m*a*(int(cos(c + d*x)**m,x)*m + int(cos(c + d*x)**m,x) - int(cos(c + d
*x)**m*cos(c + d*x)**2,x)*m - 2*int(cos(c + d*x)**m*cos(c + d*x)**2,x)))/(
m + 1)
```

3.37 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 112

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}$$

output
```
2/15*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
d/cos(d*x+c)^(1/2)+2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/9*
C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^3/d
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b\cos(c+dx)}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}(18A+19C+5C\cos(2(c+dx)))\sin(2\right.}{180d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[C
os[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(180*
d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2030, 3042, 3493, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}\left(C\cos^2(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{9}(9A+7C)\int (b\cos(c+dx))^{5/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + ((9*A + 7*C)*((6*b^2*
Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) +
 (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9)/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(100) = 200$.

Time = 1.88 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.88

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,b\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+320C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-72A-296\right.}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,b\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\right.\right.}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\right.}}$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/45*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-160*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos(1
/2*d*x+1/2*c)+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+
136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/si
n(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{2\left(3\sqrt{\frac{1}{2}}(-9i\,A-7i\,C)\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+\dots}\right.}{\dots}$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
-2/45*(3*sqrt(1/2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(1/2)*(9*I*A +
 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))) - (5*C*cos(d*x + c)^3 + (9*A + 7*C)*cos(d*x + c))*
sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4dx\right)c + \left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)a\right)$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c + d*
x))*cos(c + d*x)**2,x)*a)
```

3.38 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 110

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}$$

output
```
2/21*b*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d
/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2/7
*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.81

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{3/2}\left(4(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sqrt{\cos(c+dx)}(14A+13C+3C\cos(2(c+d\right.}{42bd\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[
Cos[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*b*d*
Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2030, 3042, 3493, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{3/2}\left(C\cos^2(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{7}(7A+5C)\int (b\cos(c+dx))^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{7}(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b}$$

$$\downarrow \text{3042}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b}$$

$$\downarrow \text{3121}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b}$$

$$\downarrow \text{3042}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b}$$

$$\downarrow \text{3120}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b}$$

input
```
Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + ((7*A + 5*C)*((2*b^2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) +
 (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(97) = 194$.

Time = 1.58 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.67

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b\left(48C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-72C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(28A+56C)\sin\left(\right.\right.}{21}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVER
BOSE)
```

output
```
-2/21*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(48*C*s
in(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)
^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}(7iA+5iC)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{\frac{1}{2}}(-7iA-5iC)\cdots\right.}{}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm=
"fricas")
```

output
```
-2/21*(sqrt(1/2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (3*C*cos(d*x + c)^2 + 7*A
+ 5*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3dx\right)c\right)$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x))
*cos(c + d*x)**3,x)*c)
```

3.39 $\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}$$

output
```
2/5*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d
/cos(d*x+c)^(1/2)+2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Co
s[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3493, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{5}(5A+3C)\int \sqrt{b\cos(c+dx)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

output
```
(2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[C
os[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(69) = 138$.

Time = 0.00 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.39

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b\left(8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\cdots}\right.}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{\cdots}\right.\right.}}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\, d} - \frac{2C\sqrt{b\left(-1+\cdots\right.}}{\cdots}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(8*C*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/
2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.32

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{2\left(\sqrt{b\cos\left(dx+c\right)}C\cos\left(dx+c\right)\sin\left(dx+c\right)-\sqrt{\frac{1}{2}}(-5i\,A-3i\,C)\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrass}\cdots\right.}{\cdots}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) - sqrt(1/2)*(-5*I*A
- 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - sqrt(1/2)*(5*I*A + 3*I*C)*sqrt(b)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x)
**2,x)*c)
```

3.40 $\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 73

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec(c+dx)\,dx = \frac{2b(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*b*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d/(b
*cos(d*x+c))^(1/2)+2/3*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec(c+dx)\,dx = \frac{b\left(2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx))\right)}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
(b*(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c
+ d*x)]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3042, 2030, 3493, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{3493}$$

$$b\left(\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3121}$$

$$b\left(\frac{(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$\Big\downarrow$ 3042

$$b\left(\frac{(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$\Big\downarrow$ 3120

$$b\left(\frac{2(3A+C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
b*((2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(64) = 128$.

Time = 0.58 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.25

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\cdots\right.\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\cdots\right.}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,d}-\frac{2C\sqrt{b\left(-1\cdots\right.}}{\cdots}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x,method=_RETURNVER
BOSE)
```

output
```
-2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(4*C*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*s
in(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1
+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = -\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A+i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{\frac{1}{2}}(-3i\,A-i\,C)\sqrt{b}\right.}{3\,d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"fricas")
```

output
```
-2/3*(sqrt(1/2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(1/2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(b*cos(d*x + c))*C*sin(d*x
+ c))/d
```

Sympy [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*(A + C*cos(c + d*x)**2)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x), x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \sqrt{b}\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)c + \left(\int \sqrt{\cos(dx+c)}\,\sec(dx+c)\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x)*c + int(sq
rt(cos(c + d*x))*sec(c + d*x),x)*a)
```

3.41 $\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 69

$$\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
-2*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(
d*x+c)^(1/2)+2*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx = \frac{2b\left(-\left((A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
(2*b*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c +
d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 2030, 3491, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^2\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^2\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
b^2*((-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sq
rt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(65) = 130$.

Time = 0.42 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.10

method	result
default	$\frac{2b\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b}\,\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\mathrm{co}\right.}}$
parts	$-\frac{2Ab\left(-2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,method=_RETURNV
ERBOSE)
```

output
```
2*b*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/si
n(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.67

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(i\,A-i\,C)\sqrt{b}\cos\left(dx+c\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\text{si}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="fricas")
```

output
```
-2*(sqrt(1/2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(1/2)*(-I*A +
 I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*A*sin(d*x + c))/
(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx = \sqrt{b}\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)c + \left(\int \sqrt{\cos(dx+c)}\,\sec(dx+c)^2\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x)*c + int
(sqrt(cos(c + d*x))*sec(c + d*x)**2,x)*a)
```

3.42 $\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 76

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx = \frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

output
```
2/3*b*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d/(b
*cos(d*x+c))^(1/2)+2/3*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx = \frac{2b\left((A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
(2*b*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d
*x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 2030, 3491, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^3\left(\frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^3\left(\frac{(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^3\left(\frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
b^3*((2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*S
qrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(67) = 134$.

Time = 0.34 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.84

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,(A+3C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+A\cdots\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\cdots\right.}}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos\cdots}{}}\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\cdots}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2)))*b*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1+2*cos(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/
d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.49

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(iA+3iC)\sqrt{b}\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+\sqrt{\frac{1}{2}}(-\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="fricas")
```

output
```
-2/3*(sqrt(1/2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-I*A - 3*I*C)*sqrt(b)*co
s(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - s
qrt(b*cos(d*x + c))*A*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx$$
$$=\sqrt{b}\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\sec(dx+c)^3\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**3,x)*c + int
(sqrt(cos(c + d*x))*sec(c + d*x)**3,x)*a)
```

3.43 $\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 110

$$\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^4(c+dx)\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output

```
-2/5*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
d/cos(d*x+c)^(1/2)+2/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*b*(3*A+
5*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = -\frac{\sqrt{b\cos(c+dx)}\sec^2(c+dx)\left(2(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-(3A+5C)\sin(2(c+dx))-\right.}{5d}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

output
```
-1/5*(Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(2*(3*A + 5*C)*Cos[c + d*x]^(3/2
)*EllipticE[(c + d*x)/2, 2] - (3*A + 5*C)*Sin[2*(c + d*x)] - 2*A*Tan[c + d
*x]))/d
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^4\left(\frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^4\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^4\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^4\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

output
```
b^4*((2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + ((3*A + 5*C)*((-2
*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]
) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(98) = 196.

Time = 0.35 (sec) , antiderivative size = 561, normalized size of antiderivative = 5.10

method	result
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticE}\left(c\right.}{}$
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\right.}{}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,method=_RETURNV
ERBOSE)
```

output
```
-2/5*A*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x
+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d-2*
C*b*(-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))
^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.24

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A+5i\,C)\sqrt{b}\cos\left(dx+c\right)^3 \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorith
m="fricas")
```

output
```
-2/5*(sqrt(1/2)*(3*I*A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(1/2)
*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - ((3*A + 5*C)*cos(d*x +
c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^4,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)^4dx\right)c + \left(\int \sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)^4dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**4,x)*c + int
(sqrt(cos(c + d*x))*sec(c + d*x)**4,x)*a)
```

3.44 $\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 113

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \frac{2b(5A+7C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

output
```
2/21*b*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d
/(b*cos(d*x+c))^(1/2)+2/7*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*b^2
*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$= \frac{\sqrt{b\cos(c+dx)}\sec^3(c+dx)\left(2(5A+7C)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C)\sin(2(c+d\right.}{21d}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*Ell
ipticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))
/(21*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^5\left(\frac{(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3042

$$b^5\left(\frac{(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3116

$$b^5\left(\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3042

$$b^5\left(\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3121

$$b^5\left(\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3042

$$b^5\left(\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3120

$$b^5\left(\frac{(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

output
```
b^5*((2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + ((5*A + 7*C)*((2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(100) = 200$.

Time = 0.35 (sec) , antiderivative size = 411, normalized size of antiderivative = 3.64

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b\left(A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}}{56b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{4}}-\frac{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\right.\right.}}{42b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right.}\right.\right.}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+60\sqrt{\frac{1}{2}-\right.}{}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,method=_RETURNV
ERBOSE)
```

output
```
-2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(A*(-1/56*
cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/2
1*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))+C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4
-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(
1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^5(c+dx)\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(5i\,A+7i\,C)\sqrt{b}\cos{(dx+c)}^4\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{\frac{1}{2}}(\cdots\right.}{\cdots}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorith
m="fricas")
```

output
```
-2/21*(sqrt(1/2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-5*I*A - 7*I*C)*sqrt(
b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) - ((5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^5}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^5,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^5, x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\sqrt{b}\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^5\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\sec(dx+c)^5\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**5,x)*c + int
(sqrt(cos(c + d*x))*sec(c + d*x)**5,x)*a)
```

3.45 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 110

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{2b(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}$$

output

```
2/15*b*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)/d/cos(d*x+c)^(1/2)+2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/9*
C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(b\cos(c+dx))^{5/2}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}(18A+19C+5\right.}{180bd\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt
[Cos[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(18
0*b*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2030, 3042, 3493, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}\left(C\cos^2(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{9}(9A+7C)\int (b\cos(c+dx))^{5/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + ((9*A + 7*C)*((6*b^2*
Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) +
 (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9)/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(98) = 196$.

Time = 2.19 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.95

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b^2\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+320C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-72A-29\cdots\right.}{\cdots}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b^2\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\cdots\right.}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\cdots\right.}}$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVER
BOSE)
```

output
```
-2/45*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-160
*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos
(1/2*d*x+1/2*c)+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*
A+136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{2\left(-3i\sqrt{\frac{1}{2}}(9A+7C)b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))\right)}{}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm=
"fricas")
```

output
```
-2/45*(-3*I*sqrt(1/2)*(9*A + 7*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(1/2)*(9*A + 7
*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) - (5*C*b*cos(d*x + c)^3 + (9*A + 7*C)*b*cos(d*x + c))*
sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\sqrt{b}\,b\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4dx\right)c+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)a\right)$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c +
d*x))*cos(c + d*x)**2,x)*a)
```

3.46 $\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right) dx = \frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}$$

output
```
2/21*b^2*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
/d/(b*cos(d*x+c))^(1/2)+2/21*b*(7*A+5*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
+2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right) dx = \frac{(b\cos(c+dx))^{3/2} \left(4(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + 2\sqrt{\cos(c+dx)}(14A+13\right.}{42d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[
Cos[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*d*Co
s[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3493, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{7}(7A+5C)\int (b\cos(c+dx))^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + ((7*A + 5*C)*((2*b^2*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) +
(2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(100) = 200$.

Time = 0.00 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.62

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,b^2\left(48C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-72C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(28A+56C)\sin\left(\right.\right.}{21}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,b^2\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2s\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,d}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(48*C
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*
C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{2\left(i\sqrt{\frac{1}{2}}(7A+5C)b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{\frac{1}{2}}(7A+5C)b^{\frac{3}{2}}\text{weie}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
-2/21*(I*sqrt(1/2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - I*sqrt(1/2)*(7*A + 5*C)*b^(3/2)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (3*C*b*cos(d*x + c)^2 + (7*A +
5*C)*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\sqrt{b}\,b\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{3}dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x
))*cos(c + d*x)**3,x)*c)
```

3.47 $\int (b\cos(c+dx))^{3/2} (A+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 75

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2b(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/5*b*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
/d/cos(d*x+c)^(1/2)+2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{b\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[
Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3042, 2030, 3493, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A\right)dx$$

$$\Big\downarrow \text{3493}$$

$$b\left(\frac{1}{5}(5A+3C)\int \sqrt{b\cos(c+dx)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{1}{5}(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \text{3121}$$

$$b\left(\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
b*((2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqr
t[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(67) = 134$.

Time = 0.87 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.51

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b^{2}\left(8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\cdots}\right.}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{d\cdots}{}\right.\right.}}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b^{2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\, d}-\frac{2C\sqrt{b\left(-1+\cdots\right.}}{}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x,method=_RETURNVER
BOSE)
```

output
```
2/5*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(8*C*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x
+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(
1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.37

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2\left(\sqrt{b\cos\left(dx+c\right)}Cb\cos\left(dx+c\right)\sin\left(dx+c\right)+i\sqrt{\frac{1}{2}}(5\,A+3\,C)b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weie}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"fricas")
```

output
```
2/5*(sqrt(b*cos(d*x + c))*C*b*cos(d*x + c)*sin(d*x + c) + I*sqrt(1/2)*(5*A
 + 3*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) - I*sqrt(1/2)*(5*A + 3*C)*b^(3/2)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\sqrt{b}\,b\left(\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)a+\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^{3}\sec(dx+c)\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*a + int(sqr
t(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x),x)*c)
```

3.48 $\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 76

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*b^2*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d/
(b*cos(d*x+c))^(1/2)+2/3*b*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{b^2\left(2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx))\right)}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
(b^2*(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(
c + d*x)]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 2030, 3493, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{3493}$$

$$b^2\left(\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^2\left(\frac{(3A+C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$\downarrow$ 3042

$$b^2\left(\frac{(3A+C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$\downarrow$ 3120

$$b^2\left(\frac{2(3A+C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
b^2*((2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[
b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(67) = 134$.

Time = 0.68 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.14

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b^{2}\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \mathrm{EllipticF}\left(\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\ \sin\left(\right.}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b^{2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}-\frac{2C\sqrt{b\left(-\right.}}{}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(4*C*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2
*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(
-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$
$$-\frac{2\left(i\sqrt{\frac{1}{2}}(3\,A+C)b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-i\,\sqrt{\frac{1}{2}}(3\,A+C)b^{\frac{3}{2}}\text{weierst}\right.}{3\,d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="fricas")
```

output
```
-2/3*(I*sqrt(1/2)*(3*A + C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - I*sqrt(1/2)*(3*A + C)*b^(3/2)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(b*cos(d*x + c))*C*b*sin(d*x + c
))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2 + A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^2,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \sqrt{b}\,b\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)a + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^2\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*a + int(
sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**2,x)*c)
```

3.49 $\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 72

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = -\frac{2b(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
-2*b*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/co
s(d*x+c)^(1/2)+2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{2b^2\left(-\left((A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
(2*b^2*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c
+ d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3042, 2030, 3491, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^3\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^3\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^3\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
b^3*((-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sq
rt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(68) = 136$.

Time $= 0.34$ (sec) , antiderivative size $= 216$, normalized size of antiderivative $= 3.00$

method	result
default	$\frac{2b^2\sqrt{-2b\sin(\frac{dx}{2}+\frac{c}{2})^4+\sin(\frac{dx}{2}+\frac{c}{2})^2 b}\,(2A\cos(\frac{dx}{2}+\frac{c}{2})\sin(\frac{dx}{2}+\frac{c}{2})^2-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin(\frac{dx}{2}+\frac{c}{2})^2-1}\ \mathrm{EllipticE}(\cos(\frac{dx}{2}}{\sqrt{-b(2\sin(\frac{dx}{2}+\frac{c}{2})^4-\sin(\frac{dx}{2}+\frac{c}{2})^2)}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{b(-1+2\cos}}$
parts	$-\frac{2A\,b^2(-2\sqrt{-2b\sin(\frac{dx}{2}+\frac{c}{2})^4+\sin(\frac{dx}{2}+\frac{c}{2})^2 b}\,\cos(\frac{dx}{2}+\frac{c}{2})\sin(\frac{dx}{2}+\frac{c}{2})^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin(\frac{dx}{2}+\frac{c}{2})^2-1}\,\sqrt{-2b\sin(\frac{dx}{2}+}{\sqrt{-b(2\sin(\frac{dx}{2}+\frac{c}{2})^4-\sin(\frac{dx}{2}+\frac{c}{2})^2)}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{b(-1+2\cos(\frac{dx}{2}+\frac{c}{2})^2)}}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,method=_RETURNV
ERBOSE)
```

output
```
2*b^2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.58

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(A-C)b^{\frac{3}{2}}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(d\ldots\right.}{\ldots}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="fricas")
```

output
```
-2*(I*sqrt(1/2)*(A - C)*b^(3/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(1/2)*(A - C)
*b^(3/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*A*b*sin(d*x + c))/(d*
cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\sqrt{b}\,b\left(\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)a+\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^3\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*a + int(
sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**3,x)*c)
```

3.50 $\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 78

$$\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

output
```
2/3*b^2*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d/
(b*cos(d*x+c))^(1/2)+2/3*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{2b^2\left((A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + A\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

output
```
(2*b^2*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c +
 d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3042, 2030, 3491, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^4\left(\frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^4\left(\frac{(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^4\left(\frac{(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3120}$$

$$b^4\left(\frac{2(A+3C)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

output
```
b^4*((2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*S
qrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(69) = 138$.

Time = 0.39 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.77

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,(A+3C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+A}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\mathrm{si}\right.}}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\mathrm{si}}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,method=_RETURNV
ERBOSE)
```

output
```
-2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2)))*b^2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1+2*cos
(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2
)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(A+3\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^2\,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-i\sqrt{\frac{1}{2}}(A+3}{3\,d\,c}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorith
m="fricas")
```

output
```
-2/3*(I*sqrt(1/2)*(A + 3*C)*b^(3/2)*cos(d*x + c)^2*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) - I*sqrt(1/2)*(A + 3*C)*b^(3/2)*cos(d*x
 + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(b
*cos(d*x + c))*A*b*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^4,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \sqrt{b}\,b\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)a+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^4\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**4,x)*a + int(
sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**4,x)*c)
```

3.51 $\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right) \sec^5(c+dx)\, dx$

Optimal result

Integrand size = 33, antiderivative size = 113

$$\int (b\cos(c+dx))^{3/2} \left(A + C\cos^2(c+dx)\right) \sec^5(c+dx)\, dx = -\frac{2b(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output
```
-2/5*b*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)/d/cos(d*x+c)^(1/2)+2/5*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*b^2*(
3*A+5*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx =$$
$$-\frac{(b\cos(c+dx))^{3/2}\sec^3(c+dx)\left(2(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-(3A+5C)\sin(2(c+dx))-\right.}{5d}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

output
```
-1/5*((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*(2*(3*A + 5*C)*Cos[c + d*x]^(3
/2)*EllipticE[(c + d*x)/2, 2] - (3*A + 5*C)*Sin[2*(c + d*x)] - 2*A*Tan[c +
 d*x]))/d
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^5\left(\frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^5\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^5\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^5\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

output
```
b^5*((2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + ((3*A + 5*C)*((-2
*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]
) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(101) = 202$.

Time = 0.40 (sec) , antiderivative size = 564, normalized size of antiderivative = 4.99

method	result
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\operatorname{EllipticE}\left(\right.}{}$
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12A\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,method=_RETURNV
ERBOSE)
```

output
```
-2/5*A*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/
2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d
*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d-
2*C*b^2*(-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-s
in(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)
^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.25

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx =$$
$$-\frac{2\left(i\sqrt{\frac{1}{2}}(3A+5C)b^{\frac{3}{2}}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorith
m="fricas")
```

output
```
-2/5*(I*sqrt(1/2)*(3*A + 5*C)*b^(3/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(1/2)
*(3*A + 5*C)*b^(3/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - ((3*A + 5*C)*b*cos(d*x + c)^
2 + A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^5}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^5,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^5, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \sqrt{b}\,b\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^5\,dx\right)a + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^5\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**5,x)*a + int(
sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**5,x)*c)
```

3.52 $\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

output
```
2/21*b^2*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
/d/(b*cos(d*x+c))^(1/2)+2/7*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*b
^3*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{(b\cos(c+dx))^{3/2}\sec^4(c+dx)\left(2(5A+7C)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C)\right.}{21d}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

output
```
((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*E
llipticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]
))/(21*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^6\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 3491}$$

$$b^6\left(\frac{(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^6\left(\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^6\left(\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^6\left(\frac{(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

output
```
b^6*((2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + ((5*A + 7*C)*((2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(102) = 204$.

Time = 0.40 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.59

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b^{2}\left(A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}}{56b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{4}}-\frac{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\cdots\right.\right.}}{42b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\cdots\right.\right.}\cdots\right.\right.}{\cdots}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+60\sqrt{\frac{1}{2}-\cdots}\right.}{\cdots}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,method=_RETURNV
ERBOSE)
```

output
```
-2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(A*(-1/5
6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5
/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.18

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(5\,A+7\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^4\,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-i\sqrt{\frac{1}{2}}(5\,A\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorith
m="fricas")
```

output
```
-2/21*(I*sqrt(1/2)*(5*A + 7*C)*b^(3/2)*cos(d*x + c)^4*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) - I*sqrt(1/2)*(5*A + 7*C)*b^(3/2)*co
s(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (
(5*A + 7*C)*b*cos(d*x + c)^2 + 3*A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(
d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sec^6(c+dx)\, dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}} \sec\left(dx+c\right)^6\, dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sec^6(c+dx)\, dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}} \sec\left(dx+c\right)^6\, dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sec^6(c+dx)\, dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^6}\, dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^6,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^6, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\sqrt{b}\,b\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^6\,dx\right)a+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^6\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**6,x)*a + int(
sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**6,x)*c)
```

3.53 $\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) dx = \frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}$$

output
```
2/15*b^2*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))/d/cos(d*x+c)^(1/2)+2/45*b*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+
2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

$$\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) dx = \frac{(b\cos(c+dx))^{5/2}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2\sqrt{\cos(c+dx)}(18A+19C+5C\right.}{180d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt
[Cos[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(18
0*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3493, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{9}(9A+7C)\int (b\cos(c+dx))^{5/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}(9A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{9}(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + ((9*A + 7*C)*((6*b^2*S
qrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) +
(2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(101) = 202$.

Time = 0.00 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.87

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b^{3}\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+320C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-72A-29}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b^{3}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\right.}}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-160
*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos
(1/2*d*x+1/2*c)+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*
A+136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{2\left(-3i\sqrt{\frac{1}{2}}(9A+7C)b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
-2/45*(-3*I*sqrt(1/2)*(9*A + 7*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(1/2)*(9*A + 7
*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) - (5*C*b^2*cos(d*x + c)^3 + (9*A + 7*C)*b^2*cos(d*x +
c))*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\sqrt{b}\,b^2\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4dx\right)c+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c
 + d*x))*cos(c + d*x)**2,x)*a)
```

3.54 $\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) \sec(c+dx)\, dx$

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right) \sec(c+dx)\, dx = \frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

output
```
2/21*b^3*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
/d/(b*cos(d*x+c))^(1/2)+2/21*b^2*(7*A+5*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)
/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{b(b\cos(c+dx))^{3/2}\left(4(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sqrt{\cos(c+dx)}(14A+13C+3C\cos$$

$$42d\cos^{\frac{3}{2}}(c+dx)$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
(b*(b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqr
t[Cos[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*d*
Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {3042, 2030, 3493, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A\right)dx$$

$$\Big\downarrow \text{3493}$$

$$b\left(\frac{1}{7}(7A+5C)\int (b\cos(c+dx))^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}\right)$$

$\Big\downarrow$ 3042

$$b\left(\frac{1}{7}(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}\right)$$

$\Big\downarrow$ 3115

$$b\left(\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3042

$$b\left(\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+d}{7bd}$$

$\Big\downarrow$ 3121

$$b\left(\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+}{7bd}$$

$\Big\downarrow$ 3042

$$b\left(\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c}{7bd}$$

$\Big\downarrow$ 3120

$$b\left(\frac{1}{7}(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(}{7}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
b*((2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + ((7*A + 5*C)*((2*b^
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]])
 + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(99) = 198$.

Time = 9.07 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.64

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b^3 (48C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-72C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(28A+56C)\sin(}{21}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b^3 (4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2s}}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\, d}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x,method=_RETURNVER
BOSE)
```

output
```
-2/21*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(48*C
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*
C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(7A+5C)b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{\frac{1}{2}}(7A+5C)b^{\frac{5}{2}}\text{weie}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"fricas")
```

output
```
-2/21*(I*sqrt(1/2)*(7*A + 5*C)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - I*sqrt(1/2)*(7*A + 5*C)*b^(5/2)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (3*C*b^2*cos(d*x + c)^2 + (7*A
+ 5*C)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \frac{\left(C\cos(c+d\,x)^2+A\right)\,\left(b\,\cos(c+d\,x)\right)^{5/2}}{\cos(c+d\,x)}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\sqrt{b}\,b^2\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x),x)*c + i
nt(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x)*a)
```

3.55 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 78

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{2b^2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/5*b^2*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))/d/cos(d*x+c)^(1/2)+2/5*b*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{b^2\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqr
t[Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3042, 2030, 3493, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A\right)dx$$

$$\Big\downarrow \text{3493}$$

$$b^2\left(\frac{1}{5}(5A+3C)\int \sqrt{b\cos(c+dx)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{1}{5}(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^2\left(\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
b^2*((2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*S
qrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(70) = 140$.

Time = 13.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.37

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b^3 \left(8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\ldots}\right.}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{d\ldots}{}\right.\right.}}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\, d}-\frac{2C\sqrt{b\left(-1+\ldots\right.}}{}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,method=_RETURNV
ERBOSE)
```

output
```
2/5*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(8*C*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x
+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(
1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.35

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{2\left(\sqrt{b\cos(dx+c)}Cb^2\cos(dx+c)\sin(dx+c)+i\sqrt{\frac{1}{2}}(5A+3C)b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{wei}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="fricas")
```

output
```
2/5*(sqrt(b*cos(d*x + c))*C*b^2*cos(d*x + c)*sin(d*x + c) + I*sqrt(1/2)*(5
*A + 3*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(1/2)*(5*A + 3*C)*b^(5/2)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\sqrt{b}\,b^2\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^2\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**2,x)*c
+ int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x)*a)
```

3.56 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 78

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*b^3*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d/
(b*cos(d*x+c))^(1/2)+2/3*b^2*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{2(b\cos(c+dx))^{5/2}\left((3A+C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sqrt{\cos(c+dx)}\sin(c+dx)\right)}{3d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
(2*(b*Cos[c + d*x])^(5/2)*((3*A + C)*EllipticF[(c + d*x)/2, 2] + C*Sqrt[Co
s[c + d*x]]*Sin[c + d*x]))/(3*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3042, 2030, 3493, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{3493}$$

$$b^3\left(\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^3 \left(\frac{(3A+C)\sqrt{\cos(c+dx)} \int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd} \right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^3 \left(\frac{(3A+C)\sqrt{\cos(c+dx)} \int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd} \right)$$

$$\Big\downarrow \text{ 3120}$$

$$b^3 \left(\frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd} \right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
b^3*((2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[
b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(69) = 138$.

Time = 65.96 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.06

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b^{3}\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \mathrm{EllipticF}\left(\right.\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\ \sin\left(\right.}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b^{3}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\, d} - \frac{2C\sqrt{b\left(-\right.}}{}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(4*C*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2
*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(
-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.15

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(3A+C)b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - i\sqrt{\frac{1}{2}}(3A+C)b^{\frac{5}{2}}\text{weierst}\right.}{3d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="fricas")
```

output
```
-2/3*(I*sqrt(1/2)*(3*A + C)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - I*sqrt(1/2)*(3*A + C)*b^(5/2)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(b*cos(d*x + c))*C*b^2*sin(d*x +
 c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^3,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\sqrt{b}\,b^2\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^3\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**3,x)*c
+ int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**3,x)*a)
```

3.57 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 74

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = -\frac{2b^2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
-2*b^2*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/
cos(d*x+c)^(1/2)+2*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{2b^3\left(-\left((A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

output
```
(2*b^3*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c
+ d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 2030, 3491, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^4\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^4\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^4\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

output
```
b^4*((-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sq
rt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(70) = 140$.

Time = 212.38 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.92

method	result
default	$\frac{2b^3\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\right.}}$
parts	$-\frac{2A\,b^3\left(-2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{-2b\sin\left(\frac{dx}{2}+\right.}\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,method=_RETURNV
ERBOSE)
```

output
```
2*b^3*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.57

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(A-C)b^{\frac{5}{2}}\cos\left(dx+c\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(d\right.\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorith
m="fricas")
```

output
```
-2*(I*sqrt(1/2)*(A - C)*b^(5/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(1/2)*(A - C)
*b^(5/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*A*b^2*sin(d*x + c))/(
d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^4,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\sqrt{b}\,b^2\left(\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^4\,dx\right)c+\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^4\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**4,x)*c
+ int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**4,x)*a)
```

3.58 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 78

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

output
```
2/3*b^3*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d/
(b*cos(d*x+c))^(1/2)+2/3*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\frac{2b^3\left((A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

output
```
(2*b^3*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c +
 d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3042, 2030, 3491, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^5\left(\frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^5\left(\frac{(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^5\left(\frac{2(A+3C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

output
```
b^5*((2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*S
qrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(69) = 138$.

Time = 0.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.77

$$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

output
```
-2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2)))*b^3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1+2*cos
(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2
)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.46

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(A+3\,C)b^{\frac{5}{2}}\cos\left(dx+c\right)^2\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))-i\sqrt{\frac{1}{2}}(A+3\right.}{3\,d\,c}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorith
m="fricas")
```

output
```
-2/3*(I*sqrt(1/2)*(A + 3*C)*b^(5/2)*cos(d*x + c)^2*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) - I*sqrt(1/2)*(A + 3*C)*b^(5/2)*cos(d*x
 + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(b
*cos(d*x + c))*A*b^2*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)(b\cos\left(dx+c\right))^{\frac{5}{2}}\sec\left(dx+c\right)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^5}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^5,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^5, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\sqrt{b}\,b^2\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4\sec\left(dx+c\right)^5dx\right)c+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)^5dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**5,x)*c
+ int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**5,x)*a)
```

3.59 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = -\frac{2b^2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output
```
-2/5*b^2*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))/d/cos(d*x+c)^(1/2)+2/5*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*b^3
*(3*A+5*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = -\frac{2b^4\left((3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{1}{2}(3A+5C)\sin(2(c+dx))-A\tan(c+dx)\right)}{5d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

output
```
(-2*b^4*((3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - ((3*A
+ 5*C)*Sin[2*(c + d*x)])/2 - A*Tan[c + d*x]))/(5*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^6\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3491}$$

$$b^6\left(\frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$b^6\left(\frac{(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$\Big\downarrow$ 3116

$$b^6\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$b^6\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$\Big\downarrow$ 3121

$$b^6\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$b^6\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$\Big\downarrow$ 3119

$$b^6\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

output
```
b^6*((2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + ((3*A + 5*C)*((-2
*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]
) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(103) = 206$.

Time = 0.17 (sec) , antiderivative size = 601, normalized size of antiderivative = 5.23

$$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b^{2}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

output
```
-2/5*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/
2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d
*x+1/2*c)^2-1)*(24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*A*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+40*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)-20*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^2-40*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+20*C*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)-5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx =$$
$$-\frac{2\left(i\sqrt{\frac{1}{2}}(3A+5C)b^{\frac{5}{2}}\cos\left(dx+c\right)^3 \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorith
m="fricas")
```

output
```
-2/5*(I*sqrt(1/2)*(3*A + 5*C)*b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(1/2)
*(3*A + 5*C)*b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - ((3*A + 5*C)*b^2*cos(d*x + c
)^2 + A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^6\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^6\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^6}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^6,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^6, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\sqrt{b}\,b^2\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^6\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^6\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**6,x)*c
+ int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**6,x)*a)
```

3.60 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

output
```
2/21*b^3*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
/d/(b*cos(d*x+c))^(1/2)+2/7*A*b^6*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*b
^4*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{(b\cos(c+dx))^{5/2}\sec^5(c+dx)\left(2(5A+7C)\cos^{\frac{5}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C)\sin\ldots\right.}{21d}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

output
```
((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*E
llipticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]
))/(21*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\Big\downarrow \text{2030}$$

$$b^7\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^7\left(\frac{(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3042

$$b^7\left(\frac{(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3116

$$b^7\left(\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3042

$$b^7\left(\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3121

$$b^7\left(\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3042

$$b^7\left(\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$\Big\downarrow$ 3120

$$b^7\left(\frac{(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

output
```
b^7*((2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + ((5*A + 7*C)*((2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(102) = 204$.

Time = 0.17 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.59

$$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b^{3}\left(A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}}{56b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{4}}-\frac{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{}}{4}\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

output
```
-2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(A*(-1/5
6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5
/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.22

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(5A+7C)b^{\frac{5}{2}}\cos(dx+c)^{4}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{\frac{1}{2}}(5A\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorith
m="fricas")
```

output
```
-2/21*(I*sqrt(1/2)*(5*A + 7*C)*b^(5/2)*cos(d*x + c)^4*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) - I*sqrt(1/2)*(5*A + 7*C)*b^(5/2)*co
s(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (
(5*A + 7*C)*b^2*cos(d*x + c)^2 + 3*A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^7\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^7\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \int \frac{\left(C\cos(c+d\,x)^2+A\right)\,(b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^7}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^7,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^7, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \sqrt{b}\,b^2\left(\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^7\,dx\right)c + \left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^7\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**7,x)*c
+ int(sqrt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**7,x)*a)
```

3.61 $\int \frac{\cos^4(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 147

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{10(11A+9C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d\sqrt{b\cos(c+dx)}} + \frac{10(11A+9C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{231bd} + \frac{2(11A+9C)(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d} + \frac{2C(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d}$$

output
```
10/231*(11*A+9*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/
d/(b*cos(d*x+c))^(1/2)+10/231*(11*A+9*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b
/d+2/77*(11*A+9*C)*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^3/d+2/11*C*(b*cos(d*x
+c))^(9/2)*sin(d*x+c)/b^5/d
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{80(11A+9C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(572A+531C+12(11A+16C)\cos(2(c+dx))}{1848d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(80*(11*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (572*A + 5
31*C + 12*(11*A + 16*C)*Cos[2*(c + d*x)] + 21*C*Cos[4*(c + d*x)])*Sin[2*(c
 + d*x)])/(1848*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 3493, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (b\cos(c+dx))^{7/2}\left(C\cos^2(c+dx)+A\right)dx}{b^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^4}$$

$$\Big\downarrow \; 3493$$

$$\frac{\frac{1}{11}(11A+9C)\int (b\cos(c+dx))^{7/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{9/2}}{11bd}}{b^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{11}(11A+9C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{9/2}}{11bd}}{b^4}$$

$\Big\downarrow$ 3115

$$\frac{\frac{1}{11}(11A+9C)\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{9/2}}{11bd}}{b^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{11}(11A+9C)\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{9/2}}{11bd}}{b^4}$$

$\Big\downarrow$ 3115

$$\frac{\frac{1}{11}(11A+9C)\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2C\sin(c+dx)(b\cos}{11bd}}{b^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{11}(11A+9C)\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2C\sin(c+dx}{}}{b^4}$$

$\Big\downarrow$ 3121

$$\frac{\frac{1}{11}(11A+9C)\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2C\sin(c+}{}}{b^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{11}(11A+9C)\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2C\,\mathrm{si}}{}}{b^4}$$

$\Big\downarrow$ 3120

$$\frac{\frac{1}{11}(11A+9C)\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2C\,\text{s}}{}}{b^4}$$

input
```
Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
((2*C*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(11*b*d) + ((11*A + 9*C)*((2*b*
(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(3*d)))/7))/11)/b^4
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(130) = 260$.

Time = 6.29 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.37

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(1344C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}-3360C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(528A+379\right.}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5\sqrt{\frac{1}{2}-\right.}{21\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\right.}}$

input
```
int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/231*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1344*C*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-3360*C*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^10+(528*A+3792*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-79
2*A-2328*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(616*A+924*C)*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-176*A-186*C)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+55*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2
*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.87

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx =$$
$$-\frac{2\left(5\sqrt{\frac{1}{2}}(11iA+9iC)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{\frac{1}{2}}(-11iA-\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-2/231*(5*sqrt(1/2)*(11*I*A + 9*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c)) + 5*sqrt(1/2)*(-11*I*A - 9*I*C)*sqrt(b)*weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (21*C*cos(d*x + c)^
4 + 3*(11*A + 9*C)*cos(d*x + c)^2 + 55*A + 45*C)*sqrt(b*cos(d*x + c))*sin(
d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{{\cos\left(c+d\,x\right)}^4\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

output
```
int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{b}\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^5\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)a\right)}{b}$$

input
```
int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**5,x)*c + int(sqrt(cos(c + d
*x))*cos(c + d*x)**3,x)*a))/b
```

3.62 $\int \frac{\cos^3(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}$$

output
```
2/15*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b/d/cos(d*x+c)^(1/2)+2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+
2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^4/d
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{6(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(18A+19C+5C\cos(2(c+dx)))\sin(c+dx)}{45d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(6*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]
^2*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(45*d*Sqrt[b*Cos[c +
 d*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2030, 3042, 3493, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}\left(C\cos^2(c+dx)+A\right)dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^3}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{9}(9A+7C)\int (b\cos(c+dx))^{5/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^3}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^3}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^3}$$

input
```
Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
((2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + ((9*A + 7*C)*((6*b^2*
Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) +
 (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9)/b^3
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(103) = 206$.

Time = 1.42 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.79

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+320C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-72A-296C\right.}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\right.\right.}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\right.}}$

input
```
int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/45*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos(1/2
*d*x+1/2*c)+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+13
6*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(
1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx =$$

$$-\frac{2\left(3\sqrt{\frac{1}{2}}(-9i\,A-7i\,C)\sqrt{b}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-2/45*(3*sqrt(1/2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(1/2)*(9*I*A +
 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))) - (5*C*cos(d*x + c)^3 + (9*A + 7*C)*cos(d*x + c))*
sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

output
```
int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4dx\right)c+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)a\right)}{b}$$

input
```
int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c + d
*x))*cos(c + d*x)**2,x)*a))/b
```

3.63 $\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 112

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}$$

output
```
2/21*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d/(
b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d+2/7
*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^3/d
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{4(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(14A+13C+3C\cos(2(c+dx)))\sin(2(c+dx))}{42d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(4*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (14*A + 13*C
 + 3*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(42*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2030, 3042, 3493, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (b\cos(c+dx))^{3/2}\left(C\cos^2(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^2}$$

$$\Big\downarrow \; 3493$$

$$\frac{\frac{1}{7}(7A+5C)\int (b\cos(c+dx))^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{7}(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^2}$$

input

```
Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

output

```
((2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + ((7*A + 5*C)*((2*b^2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) +
 (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(99) = 198$.

Time = 0.97 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.62

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(48C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-72C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(28A+56C)\sin\left(\frac{d}{2}\right.\right.}{21\sqrt{}}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin}\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/21*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*C*sin
(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2
))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}(7i\,A+5i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{\frac{1}{2}}(-7i\,A-5i\,C)\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-2/21*(sqrt(1/2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (3*C*cos(d*x + c)^2 + 7*A
+ 5*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

output
```
int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3dx\right)c\right)}{b}$$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
)*cos(c + d*x)**3,x)*c))/b
```

3.64 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 80

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}$$

output
```
2/5*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b
/d/cos(d*x+c)^(1/2)+2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5bd\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Co
s[c + d*x]]*Sin[2*(c + d*x)]))/(5*b*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2030, 3042, 3493, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt{b\cos(c+dx)}\left(C\cos^2(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{5}(5A+3C)\int \sqrt{b\cos(c+dx)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
((2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[
Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d))/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(72) = 144$.

Time = 0.64 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.25

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\mathrm{s}}\right.}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}\right.\right.}}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}-\frac{2C\sqrt{b\left(-1+2\right.}}{}$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
2/5*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*C*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x
+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)
/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2\left(\sqrt{b\cos\left(dx+c\right)}C\cos\left(dx+c\right)\sin\left(dx+c\right)-\sqrt{\frac{1}{2}}(-5i\,A-3i\,C)\sqrt{b}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrass}\right.}{}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm=
"fricas")
```

output
```
2/5*(sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) - sqrt(1/2)*(-5*I*A
- 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - sqrt(1/2)*(5*I*A + 3*I*C)*sqrt(b)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

output
```
int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\frac{\sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)c\right)}{b}$$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x
)**2,x)*c))/b
```

3.65 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}$$

output
```
2/3*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d/(b*c
os(d*x+c))^(1/2)+2/3*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx))}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d
*x)])/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3493, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos
[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(66) = 132.

Time = 0.00 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.00

method	result
parts	$\frac{2A\sqrt{\cos(dx+c)}\,\mathrm{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\sqrt{2}\right)}{d\sqrt{b\cos(dx+c)}}-\frac{2C\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\mathrm{si}\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}\right.\right.}}$
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\mathrm{co}\right.\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\right.}$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*A/d/(b*cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))-2/3*C*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
(4*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*
d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1
/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2\left(\sqrt{\frac{1}{2}}(3iA+iC)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{\frac{1}{2}}(-3iA-iC)\sqrt{b}\right.}{3\,bd}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(1/2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(1/2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(b*cos(d*x + c))*C*sin(d*x
+ c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\sqrt{b\cos\left(dx + c\right)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos(dx+c)^2 + A}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 43.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\,C\,\sin(c+d\,x)\,\sqrt{b\,\cos(c+d\,x)}}{3\,b\,d} + \frac{2\,A\,\sqrt{\cos(c+d\,x)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{b\,\cos(c+d\,x)}} + \frac{2\,C\,\sqrt{\cos(c+d\,x)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d\,\sqrt{b\,\cos(c+d\,x)}}$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/2),x)
```

output
```
(2*C*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*cos(c + d*x)^(1/2
)*ellipticF(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*C*cos(c + d
*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1/2))
```

Reduce [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)c\right)}{b}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
)*cos(c + d*x),x)*c))/b
```

3.66 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 71

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
-2*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/co
s(d*x+c)^(1/2)+2*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.38 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.79

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{(A+C\cos^2(c+dx))\left(2(A-C)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sec(c)\sin(dx+\arctan(\tan(\right.}{}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

output
```
((A + C*Cos[c + d*x]^2)*(2*(A - C)*HypergeometricPFQ[{-1/2, -1/4}, {3/4},
Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c]]] + Csc[c]*(-3
*(A - C)*Cos[c - d*x - ArcTan[Tan[c]]]*Sec[c] - (A - C)*Cos[c + d*x + ArcT
an[Tan[c]]]*Sec[c] + 2*((2*A - C)*Cos[d*x] - C*Cos[2*c + d*x])*Sqrt[Sec[c]
^2])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C
+ C*Cos[2*(c + d*x)])*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3042, 2030, 3491, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3121}$$

$$b\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b*((-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt
[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(67) = 134$.

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.00

method	result
default	$\frac{2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \text{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\right.}}$
parts	$-\frac{2A\left(-2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}}$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(
1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(iA - iC)\sqrt{b}\cos{(dx + c)}\,\mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos{(dx + c)} + i\sin\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm=
"fricas")
```

output
```
-2*(sqrt(1/2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(1/2)*(-I*A +
 I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*A*sin(d*x + c))/
(b*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{(A + C\cos^2{(c + dx)})\sec{(c + dx)}}{\sqrt{b\cos{(c + dx)}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/sqrt(b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$
$$= \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)}dx\right)a + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)c\right)}{b}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x))/cos(c + d*x),x)*a + int(sq
rt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*c))/b
```

3.67 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 73

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

output
```
2/3*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d/(b*c
os(d*x+c))^(1/2)+2/3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.93

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{4b(A+C\cos^2(c+dx))\left((A+3C)\cos^2(c+dx)\sqrt{\cos^2(dx-\arctan(\cot(c)))}\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx\right.\right.}{3d(b\cos(c+dx))^{3/2}(2A+C+C\cos(2(c+}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(-4*b*(A + C*Cos[c + d*x]^2)*((A + 3*C)*Cos[c + d*x]^2*Sqrt[Cos[d*x - ArcT
an[Cot[c]]]^2]*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]] - A*Sqrt[Csc[c]^2]*Sin[c + d*x]))/
(3*d*(b*Cos[c + d*x])^(3/2)*(2*A + C + C*Cos[2*(c + d*x)])*Sqrt[Csc[c]^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3042, 2030, 3491, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^2\left(\frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3121$$

$$b^2\left(\frac{(A+3C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{(A+3C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3120$$

$$b^2\left(\frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b^2*((2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*S
qrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(64) = 128$.

Time = 0.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.99

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,(A+3C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+A\cdots\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\cdots\right.}}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(\cdots}{}}\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\cdots}$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2)))*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1+2*cos(1/2
*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(iA + 3iC)\sqrt{b}\cos(dx + c)^2\,\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + \sqrt{\frac{1}{2}}(-\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-2/3*(sqrt(1/2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-I*A - 3*I*C)*sqrt(b)*co
s(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - s
qrt(b*cos(d*x + c))*A*sin(d*x + c))/(b*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/sqrt(b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\sec\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\sec\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2 + A}{\cos\left(c+d\,x\right)^2\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$
$$= \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)}dx\right)a + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)c\right)}{b}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**2)/cos(c + d*x),x)*a + int
(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*c))/b
```

3.68 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 112

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output
```
-2/5*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b/d/cos(d*x+c)^(1/2)+2/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+
5*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.04 (sec) , antiderivative size = 631, normalized size of antiderivative = 5.63

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$

$$= b\left(\frac{\cos^4(c + dx)\,(C + A\sec^2(c + dx))\left(\frac{4(3A+5C)\csc(c)\sec(c)}{5d} + \frac{4A\sec(c)\sec^3(c+dx)\sin(dx)}{5d} + \frac{4\sec(c)\sec(c+dx)(3A\sin(d}{5d}\right.}{(b\cos(c + dx))^{3/2}(2A + C + C\cos(2c + 2dx))}\right.$$

$$+ \frac{6A\cos^{\frac{7}{2}}(c + dx)\csc(c)\,(C + A\sec^2(c + dx))\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+ar}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx-}}\right.}{5d(b\cos(c + dx))^{3/2}(2A + C +}$$

$$+ \frac{2C\cos^{\frac{7}{2}}(c + dx)\csc(c)\,(C + A\sec^2(c + dx))\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+ar}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx-}}\right.}{d(b\cos(c + dx))^{3/2}(2A + C + C}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b*((Cos[c + d*x]^4*(C + A*Sec[c + d*x]^2)*((4*(3*A + 5*C)*Csc[c]*Sec[c])/(
5*d) + (4*A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (4*Sec[c]*Sec[c + d*x]
*(3*A*Sin[d*x] + 5*C*Sin[d*x]))/(5*d) + (4*A*Sec[c + d*x]^2*Tan[c])/(5*d))
)/((b*Cos[c + d*x])^(3/2)*(2*A + C + C*Cos[2*c + 2*d*x])) + (6*A*Cos[c + d
*x]^(7/2)*Csc[c]*(C + A*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4},
{3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqr
t[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) -
 ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(b*Cos[c + d*x])^(
3/2)*(2*A + C + C*Cos[2*c + 2*d*x])) + (2*C*Cos[c + d*x]^(7/2)*Csc[c]*(C +
 A*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcT
an[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTa
n[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(b*Cos[c + d*x])^(3/2)*(2*A + C + C*Cos[2
*c + 2*d*x])))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3 \int \frac{C \sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2 + A}{\left(b \sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^3 \left(\frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3 \left(\frac{(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^3 \left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3 \left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^3 \left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3 \left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^3 \left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b^3*((2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + ((3*A + 5*C)*((-2
*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]
) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(100) = 200$.

Time = 0.36 (sec) , antiderivative size = 563, normalized size of antiderivative = 5.03

method	result
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticE}\left(\mathrm{c}\cdots\right.\right.}{\cdots}$
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\cdots}\right.}{\cdots}$

input

```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output

```
-2/5*A*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/
2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d
*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d-
2*C*(-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))
^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.24

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A + 5i\,C)\sqrt{b}\cos\left(dx + c\right)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos\left(dx + c\right) +\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-2/5*(sqrt(1/2)*(3*I*A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(1/2)
*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - ((3*A + 5*C)*cos(d*x +
c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^3\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$
$$= \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^3}{\cos(dx+c)}dx\right)a + \left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^3 dx\right)c\right)}{b}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**3)/cos(c + d*x),x)*a + int
(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*c))/b
```

3.69 $\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 110

$$\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

output
```
2/21*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/d/(
b*cos(d*x+c))^(1/2)+2/7*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*b*(5*
A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.67

$$\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\left((5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C+3A\sec^2(c+dx))\tan(c+dx)\right)}{21d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C
+ 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^4\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 3491}$$

$$b^4\left(\frac{(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^4\left(\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^4\left(\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^4\left(\frac{(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b^4*((2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + ((5*A + 7*C)*((2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(97) = 194.

Time = 0.34 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.75

method	result
default	$-\frac{\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{56b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^4}-\frac{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{42b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}\right.\right.}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+60\sqrt{\frac{1}{2}-\right.}{}$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-1/56*co
s(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(
1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))+2*C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4
-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(
1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23

$$\int \frac{(A + C\cos^2(c + dx))\sec^4(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(5i\,A + 7i\,C)\sqrt{b}\cos\left(dx + c\right)^4 \text{weierstrassPInverse}(-4, 0, \cos\left(dx + c\right) + i\,\sin\left(dx + c\right)) + \sqrt{\frac{1}{2}}(-\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-2/21*(sqrt(1/2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-5*I*A - 7*I*C)*sqrt(
b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) - ((5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/
(b*d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^4(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^4(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^4}{\sqrt{b\cos\left(dx + c\right)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^4(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^4}{\sqrt{b\cos\left(dx + c\right)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^4(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)^4\,\sqrt{b\,\cos\left(c + d\,x\right)}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^4}{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^4dx\right)c\right)}{b}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**4)/cos(c + d*x),x)*a + int
(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**4,x)*c))/b
```

3.70 $\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 147

$$\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2(7A+9C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{2b^2(7A+9C)\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{2(7A+9C)\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}$$

output
```
-2/15*(7*A+9*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
/b/d/cos(d*x+c)^(1/2)+2/9*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+2/45*b^2
*(7*A+9*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/15*(7*A+9*C)*sin(d*x+c)/d/(
b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

$$\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{-6(7A+9C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+6(7A+9C)\sin(c+dx)+2\sec(c+dx)\,(7A+9C+5A\,\mathrm{s}}{45d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(-6*(7*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*(7*A + 9*
C)*Sin[c + d*x] + 2*Sec[c + d*x]*(7*A + 9*C + 5*A*Sec[c + d*x]^2)*Tan[c +
d*x])/(45*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^5\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{11/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^5\left(\frac{(7A+9C)\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{9b^2}+\frac{2A\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\Big\downarrow$ 3042

$$b^5\left(\frac{(7A+9C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{9b^2}+\frac{2A\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\Big\downarrow$ 3116

$$b^5\left(\frac{(7A+9C)\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2A\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\Big\downarrow$ 3042

$$b^5\left(\frac{(7A+9C)\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2A\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\Big\downarrow$ 3116

$$b^5\left(\frac{(7A+9C)\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2A\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\Big\downarrow$ 3042

$$b^5\left(\frac{(7A+9C)\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2A\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^5\left(\frac{(7A+9C)\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2A\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{(7A+9C)\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2A\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^5\left(\frac{(7A+9C)\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2A\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b^5*((2*A*Sin[c + d*x])/(9*b*d*(b*Cos[c + d*x])^(9/2)) + ((7*A + 9*C)*((2*
Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])
/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2)))/(9*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(131) = 262$.

Time = 2.70 (sec) , antiderivative size = 729, normalized size of antiderivative = 4.96

method	result	size
default	Expression too large to display	729
parts	Expression too large to display	779

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-1/144*c
os(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/
15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*
sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))))+2/5*C/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c
)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-
24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*si
n(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2))/
sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.09

$$\int \frac{(A + C\cos^2(c + dx))\sec^5(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx =$$

$$-\frac{2\left(3\sqrt{\tfrac{1}{2}}(7i\,A + 9i\,C)\sqrt{b}\cos\left(dx + c\right)^5 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos\left(dx + c\right)\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-2/45*(3*sqrt(1/2)*(7*I*A + 9*I*C)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt
(1/2)*(-7*I*A - 9*I*C)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*(7*A + 9*C)*cos
(d*x + c)^4 + (7*A + 9*C)*cos(d*x + c)^2 + 5*A)*sqrt(b*cos(d*x + c))*sin(d
*x + c))/(b*d*cos(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^5(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^5(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^5}{\sqrt{b\cos\left(dx + c\right)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{(C\cos(dx+c)^2+A)\sec(dx+c)^5}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos(c+d\,x)^2+A}{\cos(c+d\,x)^5\,\sqrt{b\,\cos(c+d\,x)}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^5}{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^5\,dx\right)c\right)}{b}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**5)/cos(c + d*x),x)*a + int
(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**5,x)*c))/b
```

3.71 $\int \frac{\cos^4(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d}$$

output
```
2/15*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b^2/d/cos(d*x+c)^(1/2)+2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/
d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^5/d
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{6(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(18A}{45bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x
]
```

output
```
(6*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]
^2*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(45*b*d*Sqrt[b*Cos[c
 + d*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2030, 3042, 3493, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx) \left(A+C \cos^2(c+dx)\right)}{(b \cos(c+dx))^{3/2}} \, dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b \cos(c+dx))^{5/2} \left(C \cos^2(c+dx)+A\right) dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b \sin \left(c+dx+\frac{\pi}{2}\right)\right)^{5/2} \left(C \sin \left(c+dx+\frac{\pi}{2}\right)^2+A\right) dx}{b^4}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{9}(9A+7C) \int (b \cos(c+dx))^{5/2} dx+\frac{2C \sin(c+dx)(b \cos(c+dx))^{7/2}}{9bd}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C) \int \left(b \sin \left(c+dx+\frac{\pi}{2}\right)\right)^{5/2} dx+\frac{2C \sin(c+dx)(b \cos(c+dx))^{7/2}}{9bd}}{b^4}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{9}(9A+7C) \left(\frac{3}{5} b^2 \int \sqrt{b \cos(c+dx)} dx+\frac{2b \sin(c+dx)(b \cos(c+dx))^{3/2}}{5d}\right)+\frac{2C \sin(c+dx)(b \cos(c+dx))^{7/2}}{9bd}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^4}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^4}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^4}$$

input
```
Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + ((9*A + 7*C)*((6*b^2*
Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) +
 (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9)/b^4
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(103) = 206$.

Time = 2.18 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.82

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+320C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-72A-296C}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos(}{5b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b(-1+2c}}$

input
```
int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/45*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-160*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos(1
/2*d*x+1/2*c)+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+
136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/si
n(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(3\sqrt{\frac{1}{2}}(-9i\,A-7i\,C)\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
-2/45*(3*sqrt(1/2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(1/2)*(9*I*A +
 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))) - (5*C*cos(d*x + c)^3 + (9*A + 7*C)*cos(d*x + c))*
sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^4}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^4}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^4\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4dx\right)c+\left(\int \sqrt{\cos\left(dx+c\right)}\right.\right.}{b^2}$$

input
```
int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c + d
*x))*cos(c + d*x)**2,x)*a))/b**2
```

3.72 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}$$

output
```
2/21*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b/d
/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^2/d
+2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^4/d
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{4(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(14A+13}{42bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x
]
```

output
```
(4*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (14*A + 13*C
 + 3*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(42*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2030, 3042, 3493, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{3/2}\left(C\cos^2(c+dx)+A\right)dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^3}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{7}(7A+5C)\int (b\cos(c+dx))^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{7}(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^3}$$

$$\Big\downarrow \; 3121$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^3}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^3}$$

input

```
Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

output

```
((2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + ((7*A + 5*C)*((2*b^2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) +
 (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)/b^3
```

Defintions of rubi rules used

rule 2030

```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(102) = 204$.

Time = 1.61 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.57

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(48C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-72C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(28A+56C)\sin\left(\frac{d}{2}\right.\right.}{21b}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\right.}{3b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}d}$

input
```
int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/21*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(48*C*s
in(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)
^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2\left(\sqrt{\frac{1}{2}}(7i\,A+5i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{\frac{1}{2}}(-7i\,A-5i\,C)\sqrt{b}\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
-2/21*(sqrt(1/2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (3*C*cos(d*x + c)^2 + 7*A
+ 5*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^3\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\,\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\right.\right.}{b^2}$$

input
```
int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
)*cos(c + d*x)**3,x)*c))/b**2
```

3.73 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 80

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}$$

output
```
2/5*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b
^2/d/cos(d*x+c)^(1/2)+2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\cos(c+dx)\sin(}{5bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x
]
```

output
```
(2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + C*Cos[c + d*
x]*Sin[2*(c + d*x)])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2030, 3042, 3493, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt{b\cos(c+dx)}\left(C\cos^2(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{5}(5A+3C)\int \sqrt{b\cos(c+dx)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[
Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d))/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(72) = 144$.

Time = 0.64 (sec), antiderivative size = 263, normalized size of antiderivative = 3.29

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2s}\right.}{5b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}\right.\right.}}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}-\frac{2C\sqrt{b\left(-1+2\right.}}{}$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/5*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(8*C*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/
2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec), antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(\sqrt{b\cos(dx+c)}C\cos(dx+c)\sin(dx+c)-\sqrt{\frac{1}{2}}(-5iA-3\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
2/5*(sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) - sqrt(1/2)*(-5*I*A
- 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - sqrt(1/2)*(5*I*A + 3*I*C)*sqrt(b)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2
*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int\sqrt{\cos\left(dx+c\right)}dx\right)a+\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2\right.\right.}{b^2}$$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x
)**2,x)*c))/b**2
```

3.74 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 78

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}$$

output
```
2/3*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b/d/(b
*cos(d*x+c))^(1/2)+2/3*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+}{3bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d
*x)])/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2030, 3042, 3493, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \frac{C\cos^2(c+dx)+A}{\sqrt{b\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{ 3493}$$

$$\frac{\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\frac{(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Co
s[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d))/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(69) = 138$.

Time = 0.49 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.06

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\mathrm{co}\right.\right.}{3b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\right.}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}-\frac{2C\sqrt{b\left(-1\right.}}{}$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(4*C*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*s
in(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1
+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A+i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{\frac{1}{2}}(-3i\,A-i\,C)\sqrt{b}\,\mathrm{we}\right.}{3\,b^2d}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm=
"fricas")
```

output
```
-2/3*(sqrt(1/2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(1/2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(b*cos(d*x + c))*C*sin(d*x
+ c))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)c\right)}{b^2}$$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
)*cos(c + d*x),x)*c))/b**2
```

3.75 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
-2*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/
cos(d*x+c)^(1/2)+2*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-2(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*A*Sin[c + d*x
])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3491, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int\sqrt{b\cos(c+dx)}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(70) = 140$.

Time = 0.00 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.92

method	result
default	$\frac{2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\,\mathrm{co}\right.}}$
parts	$-\frac{2A\left(-2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.}{b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/si
n(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(i\,A-i\,C)\sqrt{b}\cos\left(dx+c\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(\right.\right.\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(1/2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(1/2)*(-I*A +
 I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*A*sin(d*x + c))/
(b^2*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(dx+c\right)^2 + A}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2 + A}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a + \left(\int \sqrt{\cos\left(dx+c\right)}dx\right)c\right)}{b^2}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a + int(sqrt(cos(c + d
*x)),x)*c))/b**2
```

3.76 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 75

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

output
```
2/3*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b/d/(b
*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.87

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{4(A+C\cos^2(c+dx))\left((A+3C)\cos^2(c+dx)\sqrt{\cos^2(dx-\arctan(\cot(c)))}\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\right.\right.}{3d(b\cos(c+dx))^{3/2}(2A+C+C\cos(2(c+dx)}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(-4*(A + C*Cos[c + d*x]^2)*((A + 3*C)*Cos[c + d*x]^2*Sqrt[Cos[d*x - ArcTan
[Cot[c]]]^2]*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]] - A*Sqrt[Csc[c]^2]*Sin[c + d*x]))/(3
*d*(b*Cos[c + d*x])^(3/2)*(2*A + C + C*Cos[2*(c + d*x)])*Sqrt[Csc[c]^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3042, 2030, 3491, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3491}$$

$$b\left(\frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3121}$$

$$b\left(\frac{(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3120}$$

$$b\left(\frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

output
```
b*((2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqr
t[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(66) = 132$.

Time = 0.30 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.92

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,(A+3C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+A\right.}{3b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\mathrm{si}\right.}}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{3b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\mathrm{sin}}$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2)))/b*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1+2*cos(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/
d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.55

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(i\,A + 3i\,C)\sqrt{b}\cos\left(dx + c\right)^2 \text{weierstrassPInverse}(-4, 0, \cos\left(dx + c\right) + i\,\sin\left(dx + c\right)) + \sqrt{\frac{1}{2}}(-i\,A\right.}{3\,b^2}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm=
"fricas")
```

output
```
-2/3*(sqrt(1/2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-I*A - 3*I*C)*sqrt(b)*co
s(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - s
qrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^2*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{(A + C\cos^2\left(c + dx\right))\sec\left(c + dx\right)}{(b\cos\left(c + dx\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)^2}dx\right)a + \left(\int \sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right.\right.\right.}{b^2}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x))/cos(c + d*x)**2,x)*a + int
(sqrt(cos(c + d*x))*sec(c + d*x),x)*c))/b**2
```

3.77 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 113

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

output
```
-2/5*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b^2/d/cos(d*x+c)^(1/2)+2/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+
5*C)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(-\left((3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+(3A+5C)\,\mathrm{s}\right.}{5bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x
]
```

output
```
(2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5
*C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b*d*Sqrt[b*Cos[c + d*x
]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^2\left(\frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3116$$

$$b^2\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3121$$

$$b^2\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3119$$

$$b^2\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
b^2*((2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + ((3*A + 5*C)*((-2
*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]
) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(101) = 202$.

Time = 0.38 (sec) , antiderivative size = 566, normalized size of antiderivative = 5.01

method	result
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticE}\left(c\right.\right.}{}$
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\right.}{}$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/5*A*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(
1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/
d-2*C/b*(-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-s
in(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)
^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.23

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A + 5i\,C)\sqrt{b}\cos{(dx + c)}^3\,\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos{(dx + c)} + i\,\text{s}\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
-2/5*(sqrt(1/2)*(3*I*A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(1/2)
*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - ((3*A + 5*C)*cos(d*x +
c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{(A + C\cos^2{(c + dx)})\sec^2{(c + dx)}}{(b\cos{(c + dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c + d*x))**(3/2),
x)
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2 + A}{\cos\left(c+d\,x\right)^2\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)^2}dx\right)a + \left(\int \sqrt{\cos\left(dx + c\right)}\,\sec\left(dx + \right.\right.\right.}{b^2}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**2)/cos(c + d*x)**2,x)*a +
int(sqrt(cos(c + d*x))*sec(c + d*x)**2,x)*c))/b**2
```

3.78 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 112

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

output
```
2/21*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b/d
/(b*cos(d*x+c))^(1/2)+2/7*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*(5*
A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left((5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C\right.}{21bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(3/2),x
]
```

output
```
(2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C
+ 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^3\left(\frac{(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^3\left(\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^3\left(\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^3\left(\frac{(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
b^3*((2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + ((5*A + 7*C)*((2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(99) = 198$.

Time = 0.41 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.69

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}}{56b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{4}}-\frac{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{42b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right.}\right.\right.}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+60\right.}{}$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(A*(-1/56*
cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/2
1*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))+C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4
-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(
1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.21

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(5iA + 7iC)\sqrt{b}\cos(dx+c)^4 \,\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + \sqrt{\frac{1}{2}}(-5i\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
-2/21*(sqrt(1/2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-5*I*A - 7*I*C)*sqrt(
b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) - ((5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/
(b^2*d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2 + A}{\cos\left(c+d\,x\right)^3\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^3}{\cos(dx+c)^2}dx\right)a + \left(\int \sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+\right.\right.\right.}{b^2}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**3)/cos(c + d*x)**2,x)*a +
int(sqrt(cos(c + d*x))*sec(c + d*x)**3,x)*c))/b**2
```

3.79 $\int \frac{\cos^5(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int \frac{\cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}$$

output
```
2/15*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b^3/d/cos(d*x+c)^(1/2)+2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/
d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^6/d
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{\cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{6(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(18A}{45b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x
]
```

output
```
(6*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]
^2*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(45*b^2*d*Sqrt[b*Cos
[c + d*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2030, 3042, 3493, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}\left(C\cos^2(c+dx)+A\right)dx}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^5}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{9}(9A+7C)\int (b\cos(c+dx))^{5/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^5}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^5}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^5}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{1}{9}(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^5}$$

input

```
Int[(Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

output

```
((2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + ((9*A + 7*C)*((6*b^2*
Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) +
 (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9)/b^5
```

Defintions of rubi rules used

rule 2030

```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(103) = 206$.

Time = 2.26 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.82

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+320C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-72A-296C\cdots}{\cdots}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\cdots\right.\right.}{5b^{2}\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cdots\right.}}$

input
```
int(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/45*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-160
*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos
(1/2*d*x+1/2*c)+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*
A+136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{\cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(3\sqrt{\frac{1}{2}}(-9i\,A-7i\,C)\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-2/45*(3*sqrt(1/2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(1/2)*(9*I*A +
 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))) - (5*C*cos(d*x + c)^3 + (9*A + 7*C)*cos(d*x + c))*
sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^5(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^5}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^5(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^5}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^5\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^5*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^5*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{b}\left(\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4dx\right)c+\left(\int\sqrt{\cos\left(dx+c\right)}\right.\right.}{b^3}$$

input
```
int(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c + d
*x))*cos(c + d*x)**2,x)*a))/b**3
```

3.80 $\int \frac{\cos^4(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}$$

output
```
2/21*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^2
/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^3
/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^5/d
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{4(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(14A+13}{42b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x
]
```

output
```
(4*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (14*A + 13*C
 + 3*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(42*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2030, 3042, 3493, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{3/2}\left(C\cos^2(c+dx)+A\right)dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^4}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{7}(7A+5C)\int (b\cos(c+dx))^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{7}(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^4}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^4}$$

$$\Big\downarrow \; 3121$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^4}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{1}{7}(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^4}$$

input

```
Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

output

```
((2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + ((7*A + 5*C)*((2*b^2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) +
 (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)/b^4
```

Defintions of rubi rules used

rule 2030

```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(102) = 204$.

Time = 1.66 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.57

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(48C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-72C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(28A+56C)\sin\left(\frac{dx}{2}\right.\right.}{21b^{2}}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\right.}{3b^{2}\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)d}}$

input
```
int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/21*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(48*C
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*
C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2\left(\sqrt{\frac{1}{2}}(7i\,A+5i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{\frac{1}{2}}(-7i\,A-5i\,C)\sqrt{b}\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-2/21*(sqrt(1/2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (3*C*cos(d*x + c)^2 + 7*A
+ 5*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^4}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^4}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{{\cos\left(c+d\,x\right)}^4\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a+\left(\int \sqrt{\cos(dx+c)}\right.\right.}{b^3}$$

input
```
int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
)*cos(c + d*x)**3,x)*c))/b**3
```

3.81 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 80

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}$$

output
```
2/5*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b
^3/d/cos(d*x+c)^(1/2)+2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(5A+3C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+C\cos(c+dx)\sin(}{5b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x
]
```

output
```
(2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + C*Cos[c + d*
x]*Sin[2*(c + d*x)])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2030, 3042, 3493, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \ 2030$$

$$\frac{\int \sqrt{b\cos(c+dx)}\left(C\cos^2(c+dx)+A\right)dx}{b^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^3}$$

$$\Big\downarrow \ 3493$$

$$\frac{\frac{1}{5}(5A+3C)\int \sqrt{b\cos(c+dx)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{5}(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \ 3121$$

$$\frac{\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

input
```
Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[
Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d))/b^3
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(72) = 144$.

Time = 0.80 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.29

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-8C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+5A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\,\mathrm{s}}}{5b^{2}\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{d}{2}\right.\right.}}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^{2}\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}-\frac{2C\sqrt{b\left(-1+2\right.}}{}$

input
```
int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/5*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(8*C*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x
+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(
1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \frac{\cos^{3}(c+dx)\left(A+C\cos^{2}(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(\sqrt{b\cos\left(dx+c\right)}C\cos\left(dx+c\right)\sin\left(dx+c\right)-\sqrt{\frac{1}{2}}(-5i\,A-3\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
2/5*(sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) - sqrt(1/2)*(-5*I*A
- 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - sqrt(1/2)*(5*I*A + 3*I*C)*sqrt(b)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3
*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^3\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{b}\left(\left(\int\sqrt{\cos\left(dx+c\right)}dx\right)a+\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2\right.\right.}{b^3}$$

input
```
int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x
)**2,x)*c))/b**3
```

3.82 $\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 78

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d}$$

output
```
2/3*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^2/d/
(b*cos(d*x+c))^(1/2)+2/3*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^3/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx))}{3b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x
]
```

output
```
(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d
*x)])/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2030, 3042, 3493, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx) \left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{C\cos^2(c+dx)+A}{\sqrt{b\cos(c+dx)}}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Co
s[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d))/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(69) = 138$.

Time = 0.45 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.06

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\text{EllipticF}\left(\text{co}\right.\right.}{3b^{2}\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\right.}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^{2}\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}-\frac{2C\sqrt{b\left(-1-\right.}}{}$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(4*C*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2
*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(
-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A+i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{\frac{1}{2}}(-3i\,A-i\,C)\sqrt{b}\text{we}\right.}{3\,b^{3}d}$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-2/3*(sqrt(1/2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(1/2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(b*cos(d*x + c))*C*sin(d*x
+ c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)c}{b^3}$$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
)*cos(c + d*x),x)*c))/b**3
```

3.83 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 74

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

output
```
-2*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d/
cos(d*x+c)^(1/2)+2*A*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-2(A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*A*Sin[c + d*x
])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2030, 3042, 3491, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \frac{C\cos^2(c+dx)+A}{(b\cos(c+dx))^{3/2}}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}$$

$$\Big\downarrow \text{ 3491}$$

$$\frac{\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int \sqrt{b\cos(c+dx)}dx}{b^2}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{b}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{(A-C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[C
os[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]))/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(70) = 140$.

Time = 0.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.92

method	result
default	$\frac{2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\,\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{b^2\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\right.}}$
parts	$-\frac{2A\left(-2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.}{b^2\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
2/b^2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}(i\,A-i\,C)\sqrt{b}\cos\left(dx+c\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm=
"fricas")
```

output
```
-2*(sqrt(1/2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(1/2)*(-I*A +
 I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*A*sin(d*x + c))/
(b^3*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}dx\right)c\right)}{b^3}$$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*a + int(sqrt(cos(c + d
*x)),x)*c))/b**3
```

3.84 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

output
```
2/3*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^2/d/
(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left((A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\tan(c+dx)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*x
]))/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3491, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{(A+3C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+3C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b
*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(69) = 138$.

Time = 0.00 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.77

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,(A+3C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+A}{3b^2\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\mathrm{si}\right.}}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{3b^2\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\mathrm{si}}$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2)))/b^2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1+2*cos
(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2
)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}(i\,A+3i\,C)\sqrt{b}\cos\left(dx+c\right)^2\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{\frac{1}{2}}(-i\,A\right.}{3\,b^3}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(1/2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-I*A - 3*I*C)*sqrt(b)*co
s(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - s
qrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \int \frac{C\cos(c + d\,x)^2 + A}{(b\,\cos(c + d\,x))^{5/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)c + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a\right)}{b^3}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*c + int(sqrt(cos(c + d*x)
)/cos(c + d*x)**3,x)*a))/b**3
```

3.85 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

output

```
-2/5*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b^3/d/cos(d*x+c)^(1/2)+2/5*A*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+5*
C)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(-\left((3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+(3A+5C)\,\mathrm{si}\right.}{5b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5
*C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b^2*d*Sqrt[b*Cos[c + d
*x]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \ 2030$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \ 3491$$

$$b\left(\frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

output
```
b*((2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + ((3*A + 5*C)*((-2*S
qrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]])
+ (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(100) = 200$.

Time = 0.34 (sec) , antiderivative size = 566, normalized size of antiderivative = 5.05

method	result
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6-12\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \mathrm{EllipticE}(\mathrm{c}}{}$
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{}}{}$

input

```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVER
BOSE)
```

output

```
-2/5*A*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(
1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/
d-2*C/b^2*(-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4
-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*
c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.24

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A + 5i\,C)\sqrt{b}\cos{(dx+c)}^3\,\mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos{(dx+c)} + i\,\mathrm{s}\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm=
"fricas")
```

output
```
-2/5*(sqrt(1/2)*(3*I*A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(1/2)
*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - ((3*A + 5*C)*cos(d*x +
c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)}dx\right)c+\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)^3}dx\right)a\right)}{b^3}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x))/cos(c + d*x),x)*c + int((s
qrt(cos(c + d*x))*sec(c + d*x))/cos(c + d*x)**3,x)*a))/b**3
```

3.86 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 113

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}$$

output
```
2/21*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^2
/d/(b*cos(d*x+c))^(1/2)+2/7*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*(5*
A+7*C)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left((5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C\right.}{21b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x
]
```

output
```
(2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C
+ 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 2030, 3491, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^2\left(\frac{(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^2\left(\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^2\left(\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(\frac{(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
b^2*((2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + ((5*A + 7*C)*((2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(100) = 200$.

Time = 0.37 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.65

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}}{56b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{4}}-\frac{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{42b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right.}\right.\right.}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\;\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+60\sqrt{\frac{1}{2}-\right.}{}$

input

```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output

```
-2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(A*(-1/5
6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5
/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.19

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = -\frac{2\left(\sqrt{\frac{1}{2}}(5i\,A + 7i\,C)\sqrt{b}\cos\left(dx + c\right)^4 \text{weierstrassPInverse}(-4, 0, \cos\left(dx + c\right) + i\,\sin\left(dx + c\right)) + \sqrt{\frac{1}{2}}(-5i\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-2/21*(sqrt(1/2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-5*I*A - 7*I*C)*sqrt(
b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) - ((5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/
(b^3*d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2 + A}{\cos\left(c+d\,x\right)^2\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)}dx\right)c + \left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)^3}dx\right)a\right.}{b^3}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**2)/cos(c + d*x),x)*c + int
((sqrt(cos(c + d*x))*sec(c + d*x)**2)/cos(c + d*x)**3,x)*a))/b**3
```

3.87 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

output

```
-2/5*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b^4/d/cos(d*x+c)^(1/2)+2/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+
5*C)*sin(d*x+c)/b^3/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \frac{2\left(-\left((3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right) + (3A+5C)\sin(c+dx) + A}{5b^3d\sqrt{b\cos(c+dx)}}$$

input

```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

output
```
(2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5
*C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b^3*d*Sqrt[b*Cos[c + d
*x]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3491, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

output
```
(2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + ((3*A + 5*C)*((-2*Sqrt
[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (
2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(103) = 206$.

Time = 0.00 (sec) , antiderivative size = 566, normalized size of antiderivative = 4.92

method	result
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\operatorname{EllipticE}\left(c\right.}{}$
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-12A\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\right.}{}$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*A*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(
1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/
d-2*C/b^3*(-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4
-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*
c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A + 5i\,C)\sqrt{b}\cos{(dx + c)}^3\,\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos{(dx + c)} + i\,\text{s}\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(sqrt(1/2)*(3*I*A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(1/2)
*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - ((3*A + 5*C)*cos(d*x +
c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^4*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\, dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{7}{2}}}\, dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{7/2}}\, dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{7}{2}}}\, dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+A}{\left(b\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx=\frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)c\right)}{b^4}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x)**4,x)*a + int(sqrt(cos(c + d
*x))/cos(c + d*x)**2,x)*c))/b**4
```

3.88 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^4d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}$$

output
```
2/21*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))/b^4
/d/(b*cos(d*x+c))^(1/2)+2/7*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(7/2)+2/21*(5*
A+7*C)*sin(d*x+c)/b^3/d/(b*cos(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.67

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \frac{2\left((5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5A+7C+3A\sec^2(c-\right.}{21b^4d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2),x]
```

output
```
(2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C
+ 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b^4*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3491, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2),x]
```

output
```
(2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + ((5*A + 7*C)*((2*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) +
(2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(102) = 204$.

Time = 0.00 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.59

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(A\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}{56b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^4}-\frac{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{42b\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\right)\right)}{}$
parts	$-\frac{2A\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+60\sqrt{\frac{1}{2}}\right)}{}$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4*(A*(-1/5
6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5
/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(5i\,A+7i\,C)\sqrt{b}\cos{(dx+c)}^4\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{\frac{1}{2}}(-5i\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
-2/21*(sqrt(1/2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-5*I*A - 7*I*C)*sqrt(
b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) - ((5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/
(b^5*d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\, dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{9}{2}}}\, dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{9/2}}\, dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{9}{2}}}\, dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{9/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(9/2),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx=\frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^5}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)c\right)}{b^5}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x)**5,x)*a + int(sqrt(cos(c + d
*x))/cos(c + d*x)**3,x)*c))/b**5
```

3.89 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 116

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(A+2C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{b\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
(A+C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-1/3*(A+2*C)*(b*co
s(d*x+c))^(1/2)*sin(d*x+c)^3/d/cos(d*x+c)^(1/2)+1/5*C*(b*cos(d*x+c))^(1/2)
*sin(d*x+c)^5/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b\cos(c+dx)}(100A+89C+4(5A+7C)\cos(2(c+dx))+3C\cos(4(c+dx)))\sin(c+dx)}{120d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x
]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(100*A + 89*C + 4*(5*A + 7*C)*Cos[2*(c + d*x)] + 3*C
*Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2031, 3042, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3492}$$

$$-\frac{\sqrt{b\cos(c+dx)}\int \left(1-\sin^2(c+dx)\right)\left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{290}$$

$$-\frac{\sqrt{b\cos(c+dx)}\int \left(C\sin^4(c+dx)-(A+2C)\sin^2(c+dx)+A\left(\frac{C}{A}+1\right)\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}(A+2C)\sin^3(c+dx)-(A+C)\sin(c+dx)-\frac{1}{5}C\sin^5(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

output
```
-((Sqrt[b*Cos[c + d*x]]*(-((A + C)*Sin[c + d*x]) + ((A + 2*C)*Sin[c + d*x]
^3)/3 - (C*Sin[c + d*x]^5)/5))/(d*Sqrt[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

method	result
default	$\frac{\sin(dx+c)\left(3C\cos(dx+c)^{4}+5A\cos(dx+c)^{2}+4C\cos(dx+c)^{2}+10A+8C\right)\sqrt{b\cos(dx+c)}}{15d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\left(2+\cos(dx+c)^{2}\right)\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}+\frac{C\sin(dx+c)\left(3\cos(dx+c)^{4}+4\cos(dx+c)^{2}+8\right)\sqrt{b\cos(dx+c)}}{15d\sqrt{\cos(dx+c)}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{6i(dx+c)}C}{80\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{2i(dx+c)}(6A+5C)}{8\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,(6.}{8\left(\mathrm{e}^{2i(dx+c)}+1\right)d}$

input
```
int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x,method=_RET
URNVERBOSE)
```

output
```
1/15/d*sin(d*x+c)*(3*C*cos(d*x+c)^4+5*A*cos(d*x+c)^2+4*C*cos(d*x+c)^2+10*A
+8*C)/cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.54

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\left(3\,C\cos\left(dx+c\right)^{4}+\left(5\,A+4\,C\right)\cos\left(dx+c\right)^{2}+10\,A+8\,C\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\,d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="fricas")
```

output
```
1/15*(3*C*cos(d*x + c)^4 + (5*A + 4*C)*cos(d*x + c)^2 + 10*A + 8*C)*sqrt(b
*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{C\sqrt{b}\left(3\sin(5\,dx+5\,c)+25\sin\left(\frac{3}{5}\arctan(\sin(5\,dx+5\,c),\cos(5\,dx+5\,c))\right)+150\sin\left(\frac{1}{5}\arctan(\sin(5\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="maxima")
```

output
```
1/240*(C*sqrt(b)*(3*sin(5*d*x + 5*c) + 25*sin(3/5*arctan2(sin(5*d*x + 5*c)
, cos(5*d*x + 5*c))) + 150*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5
*c)))) + 20*A*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{1}{240}\sqrt{b}\left(\frac{3\,C\sin\left(5\,dx+5\,c\right)}{d}+\frac{5\,(4\,A+5\,C)\sin\left(3\,dx+3\,c\right)}{d}+\frac{30\,(6\,A+5\,C)\sin\left(dx+c\right)}{d}\right)$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="giac")
```

output
```
1/240*sqrt(b)*(3*C*sin(5*d*x + 5*c)/d + 5*(4*A + 5*C)*sin(3*d*x + 3*c)/d +
 30*(6*A + 5*C)*sin(d*x + c)/d)
```

Mupad [B] (verification not implemented)

Time = 43.69 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(200\,A\,\sin\left(2\,c+2\,d\,x\right)+20\,A\,\sin\left(4\,c+4\,d\,x\right)+175\,C\,\sin\left(2\,c+2\,d\,x\right)\right.}{240\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(200*A*sin(2*c + 2*d*x) + 20*A*
sin(4*c + 4*d*x) + 175*C*sin(2*c + 2*d*x) + 28*C*sin(4*c + 4*d*x) + 3*C*si
n(6*c + 6*d*x)))/(240*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b}\,\sin(dx+c)\left(3\sin(dx+c)^4c-5\sin(dx+c)^2a-10\sin(dx+c)^2c+15a+15c\right)}{15d}$$

input
```
int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*(3*sin(c + d*x)**4*c - 5*sin(c + d*x)**2*a - 10*sin(
c + d*x)**2*c + 15*a + 15*c))/(15*d)
```

3.90 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 113

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$

output
```
1/8*(4*A+3*C)*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/8*(4*A+3*C)*cos(d*
x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d+1/4*C*cos(d*x+c)^(5/2)*(b*cos
(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b\cos(c+dx)}(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))}{32d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x
]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)
] + C*Sin[4*(c + d*x)]))/(32*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2031, 3042, 3493, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\int \cos^2(c+dx)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A + 3*C
)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

method	result
default	$\frac{\left(4A(dx+c)+3C(dx+c)+4A\cos(dx+c)\sin(dx+c)+\sin(dx+c)\cos(dx+c)\left(2\cos(dx+c)^2+3\right)C\right)\sqrt{b\cos(dx+c)}}{8d\sqrt{\cos(dx+c)}}$
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}+\frac{C\left(2\cos(dx+c)^3\sin(dx+c)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)\sqrt{b\cos(dx+c)}}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,e^{i(dx+c)}(8A+6C)x}{8\,e^{2i(dx+c)}+8}-\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,e^{5i(dx+c)}C}{32\left(e^{2i(dx+c)}+1\right)d}+\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,e^{-i(d}}{4\left(e^{2i(dx+c)}+1\right)d}$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x,method=_RET
URNVERBOSE)
```

output
```
1/8/d*(4*A*(d*x+c)+3*C*(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*cos(d*
x+c)*(2*cos(d*x+c)^2+3)*C)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.77

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$=\left[\frac{2\left(2\,C\cos\left(dx+c\right)^2+4\,A+3\,C\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(4\,A+3\,C\right)\sqrt{-b}\log\left(2\,b\right.}{16\,d}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="fricas")
```

output
```
[1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c) + (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqr
t(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/8*((
2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{8\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))A\sqrt{b}+\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right)\right.\right.\right.}{32\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="maxima")
```

output
```
1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*sqrt(b) + (12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*s
qrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.42

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{1}{32}\left(4\,(4\,A+3\,C)x+\frac{C\sin\left(4\,dx+4\,c\right)}{d}+\frac{8\,(A+C)\sin\left(2\,dx+2\,c\right)}{d}\right)\sqrt{b}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="giac")
```

output
```
1/32*(4*(4*A + 3*C)*x + C*sin(4*d*x + 4*c)/d + 8*(A + C)*sin(2*d*x + 2*c)/
d)*sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 43.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(c+d\,x\right)+8\,C\,\sin\left(c+d\,x\right)+8\,A\,\sin\left(3\,c+3\,d\,x\right)+9\,C\,\sin\left(3\right.}{32\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right.}$$

input
```
int(cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c +
 d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) +
 32*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*d*(cos(2*c + 2*d*x) +
 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b}\,(-2\cos(dx+c)\sin(dx+c)^3c+4\cos(dx+c)\sin(dx+c)a+5\cos(dx+c)\sin(dx+c)c+4adx+$$
$$8d$$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
(sqrt(b)*( - 2*cos(c + d*x)*sin(c + d*x)**3*c + 4*cos(c + d*x)*sin(c + d*x
)*a + 5*cos(c + d*x)*sin(c + d*x)*c + 4*a*d*x + 3*c*d*x))/(8*d)
```

3.91 $\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 74

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{C\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

output
```
(A+C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-1/3*C*(b*cos(d*x+
c))^(1/2)*sin(d*x+c)^3/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b\cos(c+dx)}(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x
]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*
Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2031, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \textbf{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \cos(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \textbf{3492}$$

$$-\frac{\sqrt{b\cos(c+dx)}\int \left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}C\sin^3(c+dx)-(A+C)\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

output
```
-((Sqrt[b*Cos[c + d*x]]*(-((A + C)*Sin[c + d*x]) + (C*Sin[c + d*x]^3)/3))/
(d*Sqrt[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result
default	$\frac{\sin(dx+c)\left(C\cos(dx+c)^{2}+3A+2C\right)\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}+\frac{C\sin(dx+c)\left(2+\cos(dx+c)^{2}\right)\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{4i(dx+c)}C}{12\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{2i(dx+c)}(4A+3C)}{4\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,(4A}{4\left(\mathrm{e}^{2i(dx+c)}+1\right)d}$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x,method=_RET
URNVERBOSE)
```

output
```
1/3/d*sin(d*x+c)*(C*cos(d*x+c)^2+3*A+2*C)/cos(d*x+c)^(1/2)*(b*cos(d*x+c))^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.62

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\left(C\cos\left(dx+c\right)^2+3\,A+2\,C\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="fricas")
```

output
```
1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sq
rt(cos(d*x + c)))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(68) = 136.

Time = 29.67 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.88

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \begin{cases} x\sqrt{b\cos\left(c\right)}\left(A+C\cos^2\left(c\right)\right)\sqrt{\cos\left(c\right)} & \text{for}\ d=0 \\ 0 & \text{for}\ c=-dx+\frac{\pi}{2}\vee c= \\ \frac{A\sqrt{b\cos\left(c+dx\right)}\sin\left(c+dx\right)}{d\sqrt{\cos\left(c+dx\right)}}+\frac{2C\sqrt{b\cos\left(c+dx\right)}\sin^3\left(c+dx\right)}{3d\sqrt{\cos\left(c+dx\right)}}+\frac{C\sqrt{b\cos\left(c+dx\right)}\sin\left(c+dx\right)\cos^{\frac{3}{2}}\left(c+dx\right)}{d} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Piecewise((x*sqrt(b*cos(c))*(A + C*cos(c)**2)*sqrt(cos(c)), Eq(d, 0)), (0,
 Eq(c, -d*x + pi/2) | Eq(c, -d*x + 3*pi/2)), (A*sqrt(b*cos(c + d*x))*sin(c
 + d*x)/(d*sqrt(cos(c + d*x))) + 2*C*sqrt(b*cos(c + d*x))*sin(c + d*x)**3/
(3*d*sqrt(cos(c + d*x))) + C*sqrt(b*cos(c + d*x))*sin(c + d*x)*cos(c + d*x
)**(3/2)/d, True))
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\sqrt{b}\left(\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)+12\,A\sqrt{b}\sin\left(dx+c\right)}{12\,d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="maxima")
```

output
```
1/12*(C*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))) + 12*A*sqrt(b)*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.51

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{1}{12}\,\sqrt{b}\left(\frac{C\sin\left(3\,dx+3\,c\right)}{d}+\frac{3\,\left(4\,A+3\,C\right)\sin\left(dx+c\right)}{d}\right)$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="giac")
```

output
```
1/12*sqrt(b)*(C*sin(3*d*x + 3*c)/d + 3*(4*A + 3*C)*sin(d*x + c)/d)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(2\,c+2\,d\,x\right)+10\,C\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(4\,c+4\,d\,x\right)\right)}{12\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(12*A*sin(2*c + 2*d*x) + 10*C*s
in(2*c + 2*d*x) + C*sin(4*c + 4*d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.42

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b}\,\sin\left(dx+c\right)\left(-\sin\left(dx+c\right)^2c+3a+3c\right)}{3d}$$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*( - sin(c + d*x)**2*c + 3*a + 3*c))/(3*d)
```

3.92 $\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 90

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
A*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*C*x*(b*cos(d*x+c))^(1/2)/cos
(d*x+c)^(1/2)+1/2*C*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(c+dx)}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]]
,x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*S
qrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.59, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(Ax+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(A*x + (C*x)/2 + (C*Cos[c + d*x]*Sin[c + d*x])/(2*d)
))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\left(C\cos\left(dx+c\right)\sin\left(dx+c\right)+2A\left(dx+c\right)+C\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}}{2d\sqrt{\cos\left(dx+c\right)}}$	54
risch	$\frac{\sqrt{b\cos\left(dx+c\right)}\,\left(4A+2C\right)x}{4\sqrt{\cos\left(dx+c\right)}}+\frac{\sqrt{b\cos\left(dx+c\right)}\,C\sin\left(2dx+2c\right)}{4\sqrt{\cos\left(dx+c\right)}\,d}$	63
parts	$\frac{C\left(\cos\left(dx+c\right)\sin\left(dx+c\right)+dx+c\right)\sqrt{b\cos\left(dx+c\right)}}{2d\sqrt{\cos\left(dx+c\right)}}+\frac{A\left(dx+c\right)\sqrt{b\cos\left(dx+c\right)}}{d\sqrt{\cos\left(dx+c\right)}}$	72

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/2/d*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))*(b*cos(d*x+c))^(1/2)
/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \left[\frac{2\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+(2A+C)\sqrt{-b}\log\left(2b\cos\left(dx+c\right)^2-2\sqrt{b\cos\left(dx+\right.}\right.}{4d}\right.$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
[1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)
*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(co
s(d*x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x
 + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*
x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [A] (verification not implemented)

Time = 13.55 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \begin{cases} \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}\sin^2(c+dx)}{2\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}{2} + \frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)\sqrt{\cos(c+dx)}}{2d} & \text{for} \\ \frac{x\sqrt{b\cos(c)}(A+C\cos^2(c))}{\sqrt{\cos(c)}} & \text{oth} \end{cases}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

output
```
Piecewise((A*x*sqrt(b*cos(c + d*x))/sqrt(cos(c + d*x)) + C*x*sqrt(b*cos(c
+ d*x))*sin(c + d*x)**2/(2*sqrt(cos(c + d*x))) + C*x*sqrt(b*cos(c + d*x))*
cos(c + d*x)**(3/2)/2 + C*sqrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d
*x))/(2*d), Ne(d, 0)), (x*sqrt(b*cos(c))*(A + C*cos(c)**2)/sqrt(cos(c)), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C\sqrt{b}+8\,A\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{4\,d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C*sqrt(b) + 8*A*sqrt(b)*arctan(sin(d
*x + c)/(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{1}{4}\left(2\,(2\,A+C)x+\frac{C\sin(2\,dx+2\,c)}{d}\right)\sqrt{b}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
1/4*(2*(2*A + C)*x + C*sin(2*d*x + 2*c)/d)*sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(C*sin(2*c + 2*d*x) + 4*A*d*x + 2*C*d*x))/(4*d*cos
(c + d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b}\left(\cos\left(dx+c\right)\sin\left(dx+c\right)c+2adx+cdx\right)}{2d}$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

output
```
(sqrt(b)*(cos(c + d*x)*sin(c + d*x)*c + 2*a*d*x + c*d*x))/(2*d)
```

3.93 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 68

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{A\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
A*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+C*(b*cos(d*x
+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}(A\text{arctanh}(\sin(c+dx))+C\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2)
,x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Sqrt[
Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2031, 3042, 3493, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \sec(c+dx)dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d))/
Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

method	result
default	$-\frac{(2A\ \text{arctanh}(-\csc(dx+c)+\cot(dx+c))-C\sin(dx+c))\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}$
parts	$\frac{C\sqrt{b\cos(dx+c)}\ \sin(dx+c)}{d\sqrt{\cos(dx+c)}}-\frac{2A\sqrt{b\cos(dx+c)}\ \text{arctanh}(-\csc(dx+c)+\cot(dx+c))}{d\sqrt{\cos(dx+c)}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\,C\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}+\frac{i\sqrt{b\cos(dx+c)}\,C\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}+\frac{\sqrt{b\cos(dx+c)}\,A\ln(\mathrm{e}^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}-\frac{\sqrt{b\cos(dx+c)}\,A\ln(\mathrm{e}^{i(dx+c)}}{\sqrt{\cos(dx+c)}\,d}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/d*(2*A*arctanh(-csc(d*x+c)+cot(d*x+c))-C*sin(d*x+c))*(b*cos(d*x+c))^(1/
2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\left[\frac{A\sqrt{b}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}C\sqrt{}}{2\,d\cos(dx+c)}\right.$$
$$\left.-\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)-\sqrt{b\cos(dx+c)}C\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)}\right]$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
[1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c
))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*(A + C*cos(c + d*x)**2)/cos(c + d*x)**(3/2),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{A\sqrt{b}\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin\right.}{2d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
1/2*(A*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
- log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 2*C*sqrt(b)
*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(A\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-A\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}\right)\sqrt{b}}{d}$$

input

```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algo
rithm="giac")
```

output

```
(A*log(tan(1/2*d*x + 1/2*c) + 1) - A*log(tan(1/2*d*x + 1/2*c) - 1) + 2*C*t
an(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input

```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),x)
```

output

```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a+\sin\left(dx+c\right)c\right)}{d}$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a + s
in(c + d*x)*c))/d
```

3.94 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 59

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
C*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*(b*cos(d*x+c))^(1/2)*sin(d*x+c
)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}(Cdx\cos(c+dx)+A\sin(c+dx))}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2)
,x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*
x]^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2031, 3042, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \ 2031$$

$$\frac{\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3491$$

$$\frac{\sqrt{b\cos(c+dx)}\left(C\int 1dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 24$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+Cx\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(C*x + (A*Tan[c + d*x])/d))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\left(C\left(dx+c\right)\cos\left(dx+c\right)+A\sin\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}}{d\cos\left(dx+c\right)^{\frac{3}{2}}}$	45
parts	$\frac{A\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^{\frac{3}{2}}}+\frac{C\left(dx+c\right)\sqrt{b\cos\left(dx+c\right)}}{d\sqrt{\cos\left(dx+c\right)}}$	59
risch	$\frac{Cx\sqrt{b\cos\left(dx+c\right)}}{\sqrt{\cos\left(dx+c\right)}}+\frac{2i\sqrt{b\cos\left(dx+c\right)}\,A}{\sqrt{\cos\left(dx+c\right)}\,d\left(\mathrm{e}^{2i\left(dx+c\right)}+1\right)}$	61

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/d*(C*(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3
/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.14

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \left[\frac{C\sqrt{-b}\cos(dx+c)^2\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2}{2\,d\cos(dx+c)^2}\right.$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
[1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c
))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), (C*sqrt(b)*arcta
n(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
+ c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(C\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\frac{A\sqrt{b}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1}\right)}{d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
2*(C*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + A*sqrt(b)*sin(2*d*x
 + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
)/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{\left(-i\,C\log\left(i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+i\,C\log\left(-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}\right)\sqrt{b}}{d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
-(-I*C*log(I*tan(1/2*d*x + 1/2*c) - 1) + I*C*log(-I*tan(1/2*d*x + 1/2*c) -
 1) + 2*A*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))*sqrt(b)/d
```

Mupad [B] (verification not implemented)

Time = 41.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A\,\sin\left(2\,c+2\,d\,x\right)+C\,d\,x+C\,d\,x\,\cos\left(2\,c+2\,d\,x\right)+A\,1\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d*x)
 + C*d*x + C*d*x*cos(2*c + 2*d*x)))/(d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x
) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,(\cos\left(dx+c\right)cdx+\sin\left(dx+c\right)a)}{\cos\left(dx+c\right)d}$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

output
```
(sqrt(b)*(cos(c + d*x)*c*d*x + sin(c + d*x)*a))/(cos(c + d*x)*d)
```

3.95 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 78

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
1/2*(A+2*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/
2*A*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}((A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+A\sin(c+dx))}{2d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2)
,x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*
Sin[c + d*x]))/(2*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2031, 3042, 3491, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{2}(A+2C)\int \sec(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{2}(A+2C)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{(A+2C)\text{arctanh}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(((A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*Sec[c
+ d*x]*Tan[c + d*x])/(2*d)))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

method	result
default	$-\frac{\left(A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2-A\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2+4C\ \mathrm{arctanh}(-\csc(dx+c)+\cot(\ldots\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$-\frac{2C\sqrt{b\cos(dx+c)}\ \mathrm{arctanh}(-\csc(dx+c)+\cot(dx+c))}{d\sqrt{\cos(dx+c)}}+\frac{A\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(\ldots\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\,A\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(e^{i(dx+c)}+i\right)}{2\sqrt{\cos(dx+c)}\,d}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2/d*(A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^2-A*ln(-cot(d*x+c)+csc(d
*x+c)+1)*cos(d*x+c)^2+4*C*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)^2-A*s
in(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.73

$$\int\frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\left[\frac{(A+2\,C)\sqrt{b}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,\sqrt{b\cos\left(dx\ldots\right.}}{4\,d\cos\left(dx+c\right)^3}\right.$$
$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(d\ldots\right.}{2\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
[1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*c
os(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/c
os(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))
/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*
sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(
d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(66) = 132.

Time = 0.24 (sec) , antiderivative size = 728, normalized size of antiderivative = 9.33

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
1/4*(2*C*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1
) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) - (4*(sin(4
*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
+ 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*c
os(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x +
4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*co
s(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)
^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos...
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left((A+2C)\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-(A+2C)\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)+\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+A\tan\left(\frac{1}{2}\right.\right.}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2\tan\left(\frac{1}{2}dx+\right.}\right.}{2d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
1/2*((A + 2*C)*log(tan(1/2*d*x + 1/2*c) + 1) - (A + 2*C)*log(tan(1/2*d*x +
 1/2*c) - 1) + 2*(A*tan(1/2*d*x + 1/2*c)^3 + A*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^4 - 2*tan(1/2*d*x + 1/2*c)^2 + 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.29

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2 a-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2 c+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 2*log(tan((c +
d*x)/2) - 1)*sin(c + d*x)**2*c + log(tan((c + d*x)/2) - 1)*a + 2*log(tan((
c + d*x)/2) - 1)*c + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 2*log(t
an((c + d*x)/2) + 1)*sin(c + d*x)**2*c - log(tan((c + d*x)/2) + 1)*a - 2*l
og(tan((c + d*x)/2) + 1)*c - sin(c + d*x)*a))/(2*d*(sin(c + d*x)**2 - 1))
```

3.96 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 79

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
1/3*A*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+1/3*(2*A+3*C)*(b*
cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2)
,x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*Co
s[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2031, 3042, 3491, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}(2A+3C)\int \sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}(2A+3C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(((2*A + 3*C)*Tan[c + d*x])/(3*d) + (A*Sec[c + d*x]^
2*Tan[c + d*x])/(3*d)))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\sin(dx+c)\left(2A\cos(dx+c)^{2}+3C\cos(dx+c)^{2}+A\right)\sqrt{b\cos(dx+c)}}{3d\cos(dx+c)^{\frac{7}{2}}}$	54
parts	$\frac{C\sin(dx+c)\sqrt{b\cos(dx+c)}}{d\cos(dx+c)^{\frac{3}{2}}}+\frac{A\sin(dx+c)\left(2\cos(dx+c)^{2}+1\right)\sqrt{b\cos(dx+c)}}{3d\cos(dx+c)^{\frac{7}{2}}}$	73
risch	$\frac{2i\sqrt{b\cos(dx+c)}\,\left(3C\,\mathrm{e}^{4i(dx+c)}+6A\,\mathrm{e}^{2i(dx+c)}+6C\,\mathrm{e}^{2i(dx+c)}+2A+3C\right)}{3\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}$	81

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
1/3/d*sin(d*x+c)*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*(b*cos(d*x+c))^(1/2
)/cos(d*x+c)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\frac{\left((2\,A+3\,C)\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*
cos(d*x + c)^(7/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(67) = 134$.

Time = 0.27 (sec) , antiderivative size = 350, normalized size of antiderivative = 4.43

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(\frac{2((3\cos(2dx+2c)+1)\sin(6dx+6c)+3(3\cos(2dx+2c)+1)\,\mathrm{s}\cdots}{2(3\cos(4dx+4c)+3\cos(2dx+2c)+1)\cos(6dx+6c)+\cos(6dx+6c)^2+6(3\cos(2dx+2c)+1)\cos(4dx+4c)+9\cos(4dx+4c)^2+9\cos(2dx\cdots}\cdots\right.}{\cdots}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
2/3*(2*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
+ 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x +
2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin
(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 +
9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x
+ 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 3*C*sqrt(b)*sin(2*d*x + 2*c)/(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left(3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,A\right.}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
-2/3*(3*A*tan(1/2*d*x + 1/2*c)^5 + 3*C*tan(1/2*d*x + 1/2*c)^5 - 2*A*tan(1/
2*d*x + 1/2*c)^3 - 6*C*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x + 1/2*c) +
 3*C*tan(1/2*d*x + 1/2*c))*sqrt(b)/((tan(1/2*d*x + 1/2*c)^6 - 3*tan(1/2*d*
x + 1/2*c)^4 + 3*tan(1/2*d*x + 1/2*c)^2 - 1)*d)
```

Mupad [B] (verification not implemented)

Time = 45.41 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)+2\,A\,\sin\left(6\,c+6\,d\,x\right)+15\,C\,\sin\left(2\,c+2\right.\right.}{}$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(9/2),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4*
c + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*
c + 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(
4*c + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c
 + 4*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2
*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{b}\,\sin\left(dx+c\right)\left(2\sin\left(dx+c\right)^2a+3\sin\left(dx+c\right)^2c-3a-3c\right)}{3\cos\left(dx+c\right)d\left(\sin\left(dx+c\right)^2-1\right)}$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*(2*sin(c + d*x)**2*a + 3*sin(c + d*x)**2*c - 3*a - 3
*c))/(3*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.97 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 122

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
1/8*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+
1/4*A*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)+1/8*(3*A+4*C)*(b*
cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{b\cos(c+dx)}((3A+4C)\operatorname{arctanh}(\sin(c+dx))\cos^4(c+dx)+(2A+(3A+4C)\cos^2(c+dx))\sin(c+dx))}{8d\cos^{\frac{9}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2
),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 +
(2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2031, 3042, 3491, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{2031}$$
$$\frac{\sqrt{b\cos(c+dx)}\int (C\cos^2(c+dx)+A)\sec^5(c+dx)dx}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\int\sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \textbf{4255}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \textbf{4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A + 4*C
)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/S
qrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\left(3A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^{4}+4C\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^{4}-3A\ln(-\cot(dx+c)+\csc(dx+\cdots\right.}{}$
parts	$\frac{C\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^{2}-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^{2}+\sin(dx+c)\right)\sqrt{b\cos(dx+c)}}{2d\cos(dx+c)^{\frac{5}{2}}}-\frac{A\left(3\ln\cdots\right.}{}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\,\left(3A\,\mathrm{e}^{7i(dx+c)}+4C\,\mathrm{e}^{7i(dx+c)}+11A\,\mathrm{e}^{5i(dx+c)}+4C\,\mathrm{e}^{5i(dx+c)}-11A\,\mathrm{e}^{3i(dx+c)}-4C\,\mathrm{e}^{3i(dx+c)}-3A\,\mathrm{e}^{i(dx+c)}-4C\,\mathrm{e}^{i(dx\cdots}\right.}{4\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
-1/8/d*(3*A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^4+4*C*ln(-cot(d*x+c)+c
sc(d*x+c)-1)*cos(d*x+c)^4-3*A*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4-4*
C*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+(-3*cos(d*x+c)^2-2)*sin(d*x+c)
*A-4*C*cos(d*x+c)^2*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$= \left[\frac{(3A+4C)\sqrt{b}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2((3A+\cdots}{16d\cos(dx+c)^5}\right.$$
$$-\frac{(3A+4C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-((3A+4C)\cos(dx+c)^2+2A)\sqrt{b\cdots}}{8d\cos(dx+c)^5}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
[1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(
b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*((3*A + 4*C)
*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*
x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2318 vs. $2(104) = 208$.

Time = 0.39 (sec) , antiderivative size = 2318, normalized size of antiderivative = 19.00

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
-1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4
*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d
*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(
8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c
))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x +
8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d
*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c)
+ 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x +
8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 1
6*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*
d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(104) = 208.

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$= \frac{\left((3\,A+4\,C)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-(3\,A+4\,C)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+4\,C\right.}{}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
1/8*((3*A + 4*C)*log(tan(1/2*d*x + 1/2*c) + 1) - (3*A + 4*C)*log(tan(1/2*d
*x + 1/2*c) - 1) + 2*(5*A*tan(1/2*d*x + 1/2*c)^7 + 4*C*tan(1/2*d*x + 1/2*c
)^7 + 3*A*tan(1/2*d*x + 1/2*c)^5 - 4*C*tan(1/2*d*x + 1/2*c)^5 + 3*A*tan(1/
2*d*x + 1/2*c)^3 - 4*C*tan(1/2*d*x + 1/2*c)^3 + 5*A*tan(1/2*d*x + 1/2*c) +
 4*C*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^8 - 4*tan(1/2*d*x + 1/2*c
)^6 + 6*tan(1/2*d*x + 1/2*c)^4 - 4*tan(1/2*d*x + 1/2*c)^2 + 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^{11/2}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(11/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(11/2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{b}\left(-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4 a-4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4 c+6\log\left(\tan\left(\frac{dx}{2}+\right.\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

output
```
(sqrt(b)*( - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 4*log(tan((c
+ d*x)/2) - 1)*sin(c + d*x)**4*c + 6*log(tan((c + d*x)/2) - 1)*sin(c + d*x
)**2*a + 8*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 3*log(tan((c + d*
x)/2) - 1)*a - 4*log(tan((c + d*x)/2) - 1)*c + 3*log(tan((c + d*x)/2) + 1)
*sin(c + d*x)**4*a + 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c - 6*log
(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - 8*log(tan((c + d*x)/2) + 1)*sin
(c + d*x)**2*c + 3*log(tan((c + d*x)/2) + 1)*a + 4*log(tan((c + d*x)/2) +
1)*c - 3*sin(c + d*x)**3*a - 4*sin(c + d*x)**3*c + 5*sin(c + d*x)*a + 4*si
n(c + d*x)*c))/(8*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.98 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right) d$

Optimal result

Integrand size = 35, antiderivative size = 119

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right) dx = \frac{b(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{b(A+2C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{b\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output

```
b*(A+C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-1/3*b*(A+2*C)*(
b*cos(d*x+c))^(1/2)*sin(d*x+c)^3/d/cos(d*x+c)^(1/2)+1/5*b*C*(b*cos(d*x+c))
^(1/2)*sin(d*x+c)^5/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right) dx = \frac{(b\cos(c+dx))^{3/2}(100A+89C+4(5A+7C)\cos(2(c+dx))+3C\cos(4(c+dx)))\,\mathrm{s}}{120d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)
,x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(100*A + 89*C + 4*(5*A + 7*C)*Cos[2*(c + d*x)] + 3
*C*Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2031, 3042, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3492}$$

$$-\frac{b\sqrt{b\cos(c+dx)}\int \left(1-\sin^2(c+dx)\right)\left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{290}$$

$$-\frac{b\sqrt{b\cos(c+dx)}\int \left(C\sin^4(c+dx)-(A+2C)\sin^2(c+dx)+A\left(\frac{C}{A}+1\right)\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}(A+2C)\sin^3(c+dx)-(A+C)\sin(c+dx)-\frac{1}{5}C\sin^5(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

output
```
-((b*Sqrt[b*Cos[c + d*x]]*(-((A + C)*Sin[c + d*x]) + ((A + 2*C)*Sin[c + d*
x]^3)/3 - (C*Sin[c + d*x]^5)/5))/(d*Sqrt[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.60

method	result
default	$\frac{b\sin(dx+c)\left(3C\cos(dx+c)^4+5A\cos(dx+c)^2+4C\cos(dx+c)^2+10A+8C\right)\sqrt{b\cos(dx+c)}}{15d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\left(2+\cos(dx+c)^2\right)b\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}+\frac{C\sin(dx+c)\left(3\cos(dx+c)^4+4\cos(dx+c)^2+8\right)b\sqrt{b\cos(dx+c)}}{15d\sqrt{\cos(dx+c)}}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\,e^{6i(dx+c)}C}{80(e^{2i(dx+c)}+1)d}-\frac{ib\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\,e^{2i(dx+c)}(6A+5C)}{8(e^{2i(dx+c)}+1)d}+\frac{ib\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}}{8(e^{2i(dx+c)}+1)d}$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RET
URNVERBOSE)
```

output
```
1/15*b/d*sin(d*x+c)*(3*C*cos(d*x+c)^4+5*A*cos(d*x+c)^2+4*C*cos(d*x+c)^2+10
*A+8*C)/cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.58

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{\left(3\,Cb\cos\left(dx+c\right)^4+\left(5\,A+4\,C\right)b\cos\left(dx+c\right)^2+2\,(5\,A+4\,C)b\right)\sqrt{b\cos\left(dx+c\right)}\,\mathrm{si}}{15\,d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="fricas")
```

output
```
1/15*(3*C*b*cos(d*x + c)^4 + (5*A + 4*C)*b*cos(d*x + c)^2 + 2*(5*A + 4*C)*
b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{20\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)A\sqrt{b}+(}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="maxima")
```

output
```
1/240*(20*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))))*A*sqrt(b) + (3*b*sin(5*d*x + 5*c) + 25*b*sin(3/5*arctan2(s
in(5*d*x + 5*c), cos(5*d*x + 5*c))) + 150*b*sin(1/5*arctan2(sin(5*d*x + 5*
c), cos(5*d*x + 5*c))))*C*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{1}{240}\,b^{\frac{3}{2}}\left(\frac{3\,C\sin\left(5\,dx+5\,c\right)}{d}+\frac{5\,(4\,A+5\,C)\sin\left(3\,dx+3\,c\right)}{d}+\frac{30\,(6\,A+5\,C)\sin\left(dx\right.}{d}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="giac")
```

output
```
1/240*b^(3/2)*(3*C*sin(5*d*x + 5*c)/d + 5*(4*A + 5*C)*sin(3*d*x + 3*c)/d +
 30*(6*A + 5*C)*sin(d*x + c)/d)
```

Mupad [B] (verification not implemented)

Time = 42.78 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(200\,A\,\sin\left(2\,c+2\,d\,x\right)+20\,A\,\sin\left(4\,c+4\,d\,x\right)+1\right.}{240\,d\,\left(\cos\left(2\,c+2\,d\,x\right.\right.}$$

input
```
int(cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
```

output
```
(b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(200*A*sin(2*c + 2*d*x) + 20*
A*sin(4*c + 4*d*x) + 175*C*sin(2*c + 2*d*x) + 28*C*sin(4*c + 4*d*x) + 3*C*
sin(6*c + 6*d*x)))/(240*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.45

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b}\,\sin(dx+c)\,b\left(3\sin(dx+c)^4 c-5\sin(dx+c)^2 a-10\sin(dx+c)^2 c+15a+15\right.}{15d}$$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b*(3*sin(c + d*x)**4*c - 5*sin(c + d*x)**2*a - 10*si
n(c + d*x)**2*c + 15*a + 15*c))/(15*d)
```

3.99 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 35, antiderivative size = 116

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{b(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}+\frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$

output
```
1/8*b*(4*A+3*C)*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/8*b*(4*A+3*C)*co
s(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d+1/4*b*C*cos(d*x+c)^(5/2)*
(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(b\cos(c+dx))^{3/2}(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx$$
$$\overline{32d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)
,x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*
x)] + C*Sin[4*(c + d*x)]))/(32*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2031, 3042, 3493, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3493}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\int\cos^2(c+dx)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A + 3
*C)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])    Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

method	result
default	$\frac{b\left(4A(dx+c)+3C(dx+c)+4A\cos(dx+c)\sin(dx+c)+\sin(dx+c)\cos(dx+c)\left(2\cos(dx+c)^{2}+3\right)C\right)\sqrt{b\cos(dx+c)}}{8d\sqrt{\cos(dx+c)}}$
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)b\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}+\frac{C\left(2\cos(dx+c)^{3}\sin(dx+c)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)b\sqrt{b\cos(dx+c)}}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{b\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{i(dx+c)}(8A+6C)x}{8\,\mathrm{e}^{2i(dx+c)}+8}-\frac{ib\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{5i(dx+c)}C}{32\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{ib\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{-}}{4\left(\mathrm{e}^{2i(dx+c)}+1\right)d}$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RET
URNVERBOSE)
```

output
```
1/8*b/d*(4*A*(d*x+c)+3*C*(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*cos(
d*x+c)*(2*cos(d*x+c)^2+3)*C)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.80

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \left[\frac{(4A+3C)\sqrt{-b}b\log\left(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="fricas")
```

output
```
[1/16*((4*A + 3*C)*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b*cos(d*x + c)^
2 + (4*A + 3*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d
, 1/8*((4*A + 3*C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(
b)*cos(d*x + c)^(3/2))) + (2*C*b*cos(d*x + c)^2 + (4*A + 3*C)*b)*sqrt(b*co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{8\left(2\left(dx+c\right)b+b\sin\left(2\,dx+2\,c\right)\right)A\sqrt{b}+\left(12\left(dx+c\right)b+b\sin\left(4\,dx+4\,c\right)+8\,b\sin\left(\right.\right.}{32\,d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="maxima")
```

output
```
1/32*(8*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*A*sqrt(b) + (12*(d*x + c)*b +
 b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))))*C*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.41

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{1}{32}\left(4\left(4\,A+3\,C\right)x+\frac{C\sin\left(4\,dx+4\,c\right)}{d}+\frac{8\left(A+C\right)\sin\left(2\,dx+2\,c\right)}{d}\right)b^{\frac{3}{2}}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="giac")
```

output
```
1/32*(4*(4*A + 3*C)*x + C*sin(4*d*x + 4*c)/d + 8*(A + C)*sin(2*d*x + 2*c)/
d)*b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 36.89 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{b\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,(8A\sin(c+dx)+8C\sin(c+dx)+8A\sin(3c$$

$$32d$$

input
```
int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
```

output
```
(b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c
 + d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x)
 + 32*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*d*(cos(2*c + 2*d*x)
 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.57

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b}\,b\left(-2\cos(dx+c)\sin(dx+c)^3c+4\cos(dx+c)\sin(dx+c)a+5\cos(dx+c)\mathrm{si}\right.}{8d}$$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
(sqrt(b)*b*( - 2*cos(c + d*x)*sin(c + d*x)**3*c + 4*cos(c + d*x)*sin(c + d
*x)*a + 5*cos(c + d*x)*sin(c + d*x)*c + 4*a*d*x + 3*c*d*x))/(8*d)
```

3.100 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 76

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{b(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{bC\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

output
```
b*(A+C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-1/3*b*C*(b*cos(
d*x+c))^(1/2)*sin(d*x+c)^3/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{b\sqrt{b\cos(c+dx)}(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x
]],x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*
d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2031, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3492}$$

$$-\frac{b\sqrt{b\cos(c+dx)}\int \left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}C\sin^3(c+dx)-(A+C)\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

output
```
-((b*Sqrt[b*Cos[c + d*x]]*(-((A + C)*Sin[c + d*x]) + (C*Sin[c + d*x]^3)/3)
)/(d*Sqrt[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{b\sin(dx+c)\left(C\cos(dx+c)^{2}+3A+2C\right)\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}$	48
risch	$\frac{b\sqrt{b\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{b\cos(dx+c)}\,C\sin(3dx+3c)}{12\sqrt{\cos(dx+c)}\,d}$	73
parts	$\frac{A\sin(dx+c)b\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}+\frac{C\sin(dx+c)\left(2+\cos(dx+c)^{2}\right)b\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}$	73

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/3*b/d*sin(d*x+c)*(C*cos(d*x+c)^2+3*A+2*C)/cos(d*x+c)^(1/2)*(b*cos(d*x+c)
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.66

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{\left(Cb\cos\left(dx+c\right)^2+(3\,A+2\,C)b\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx-\right.}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
1/3*(C*b*cos(d*x + c)^2 + (3*A + 2*C)*b)*sqrt(b*cos(d*x + c))*sin(d*x + c)
/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{12\,Ab^{\frac{3}{2}}\sin\left(dx+c\right)+\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\,\text{arctan}\right.\right.}{12\,d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/12*(12*A*b^(3/2)*sin(d*x + c) + (b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*C*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.50

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{1}{12}\,b^{\frac{3}{2}}\left(\frac{C\sin\left(3\,dx+3\,c\right)}{d}+\frac{3\left(4\,A+3\,C\right)\sin\left(dx+c\right)}{d}\right)$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
1/12*b^(3/2)*(C*sin(3*d*x + 3*c)/d + 3*(4*A + 3*C)*sin(d*x + c)/d)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(c+d\,x\right)+9\,C\,\sin\left(c+d\,x\right)+\right.}{12\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x)
```

output
```
(b*(b*cos(c + d*x))^(1/2)*(12*A*sin(c + d*x) + 9*C*sin(c + d*x) + C*sin(3*
c + 3*d*x)))/(12*d*cos(c + d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.42

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b}\,\sin(dx+c)\,b\left(-\sin(dx+c)^2 c+3a+3c\right)}{3d}$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b*( - sin(c + d*x)**2*c + 3*a + 3*c))/(3*d)
```

3.101 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 93

$$\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
A*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b*C*x*(b*cos(d*x+c))^(1/2)
/cos(d*x+c)^(1/2)+1/2*b*C*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)
/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.56

$$\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/
2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d
*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.58, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(Ax+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(A*x + (C*x)/2 + (C*Cos[c + d*x]*Sin[c + d*x])/(2*
d)))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{b(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}$	55
risch	$\frac{b\sqrt{b\cos(dx+c)}\,(4A+2C)x}{4\sqrt{\cos(dx+c)}}+\frac{b\sqrt{b\cos(dx+c)}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	65
parts	$\frac{A(dx+c)b\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}+\frac{C(\cos(dx+c)\sin(dx+c)+dx+c)b\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}$	74

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/2*b/d*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))*(b*cos(d*x+c))^(1/
2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.77

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{2\sqrt{b\cos\left(dx+c\right)}Cb\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(2A+\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
[1/4*(2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A +
C)*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqr
t(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*C*b*sqrt(c
os(d*x + c))*sin(d*x + c) + (2*A + C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*
sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{8\,Ab^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+(2\,(dx+c)b+b\sin\left(2\,dx+2\right.}{4\,d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
1/4*(8*A*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (2*(d*x + c)*b
+ b*sin(2*d*x + 2*c))*C*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left((dx+c)(2\,A+C)+\frac{C\tan(dx+c)}{\tan(dx+c)^2+1}\right)b^{\frac{3}{2}}}{2\,d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
1/2*((d*x + c)*(2*A + C) + C*tan(d*x + c)/(tan(d*x + c)^2 + 1))*b^(3/2)/d
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.49

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),x)
```

output
```
(b*(b*cos(c + d*x))^(1/2)*(C*sin(2*c + 2*d*x) + 4*A*d*x + 2*C*d*x))/(4*d*c
os(c + d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.34

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b(\cos\left(dx+c\right)\sin\left(dx+c\right)c+2adx+cdx)}{2d}$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

output
```
(sqrt(b)*b*(cos(c + d*x)*sin(c + d*x)*c + 2*a*d*x + c*d*x))/(2*d)
```

3.102 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 70

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{A\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
A*b*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+b*C*(b*cos
(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}(A\text{arctanh}(\sin(c+dx))+C\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/
2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Cos
[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2031, 3042, 3493, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3493}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \sec(c+dx)dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d)
)/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

method	result
default	$-\frac{b\left(2A\,\operatorname{arctanh}\left(-\csc\left(dx+c\right)+\cot\left(dx+c\right)\right)-C\sin\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}}{d\sqrt{\cos\left(dx+c\right)}}$
parts	$-\frac{2A\,\operatorname{arctanh}\left(-\csc\left(dx+c\right)+\cot\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\,b}{d\sqrt{\cos\left(dx+c\right)}}+\frac{bC\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}$
risch	$-\frac{ib\sqrt{b\cos\left(dx+c\right)}\,C\,\mathrm{e}^{i\left(dx+c\right)}}{2\sqrt{\cos\left(dx+c\right)}\,d}+\frac{ib\sqrt{b\cos\left(dx+c\right)}\,C\,\mathrm{e}^{-i\left(dx+c\right)}}{2\sqrt{\cos\left(dx+c\right)}\,d}-\frac{b\sqrt{b\cos\left(dx+c\right)}\,A\ln\left(\mathrm{e}^{i\left(dx+c\right)}-i\right)}{\sqrt{\cos\left(dx+c\right)}\,d}+\frac{b\sqrt{b\cos\left(dx+c\right)}\,A\ln\left(\mathrm{e}^{i\left(dx\right.}\right.}{\sqrt{\cos\left(dx+c\right)}\,d}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-b/d*(2*A*arctanh(-csc(d*x+c)+cot(d*x+c))-C*sin(d*x+c))*(b*cos(d*x+c))^(1/
2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.91

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[\frac{Ab^{\frac{3}{2}}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\ldots}{\cos(dx+c)^3}\ldots\right.}{2\ldots}\right.$$

$$\left. -\frac{A\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)-\sqrt{b\cos(dx+c)}Cb\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)}\right]$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
[1/2*(A*b^(3/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c
))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)), -(A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*b*sqrt(cos
(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,Cb^{\frac{3}{2}}\sin\left(dx+c\right)+\left(b\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
1/2*(2*C*b^(3/2)*sin(d*x + c) + (b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2
*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c
) + 1))*A*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(A\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-A\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\right.\right.}{d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
(A*log(tan(1/2*d*x + 1/2*c) + 1) - A*log(tan(1/2*d*x + 1/2*c) - 1) + 2*C*t
an(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))*b^(3/2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{\sqrt{b}\,b\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a}{d}$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

output
```
(sqrt(b)*b*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a +
 sin(c + d*x)*c))/d
```

3.103 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 61

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
b*C*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*(b*cos(d*x+c))^(1/2)*sin(d
*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}(Cdx\cos(c+dx)+A\sin(c+dx))}{d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/
2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c +
d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2031, 3042, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3491}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(C\int 1dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+Cx\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(C*x + (A*Tan[c + d*x])/d))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{b(C(dx+c)\cos(dx+c)+A\sin(dx+c))\sqrt{b\cos(dx+c)}}{d\cos(dx+c)^{\frac{3}{2}}}$	46
parts	$\frac{Ab\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}}+\frac{C(dx+c)b\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}$	61
risch	$\frac{bCx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}}+\frac{2ib\sqrt{b\cos(dx+c)}\,A}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)}$	63

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
b/d*(C*(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3
/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.08

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left[\frac{C\sqrt{-b}b\cos\left(dx+c\right)^2\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(d\right.}\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
[1/2*(C*sqrt(-b)*b*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x +
 c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), (C*b^(3/2)*a
rctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(
d*x + c)^2 + sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(Cb^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\frac{Ab^{\frac{3}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\mathrm{co}}\right.}{d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
2*(C*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + A*b^(3/2)*sin(2*d*x
 + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
)/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{\left(-i\,C\log\left(i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+i\,C\log\left(-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}\right)b^{\frac{3}{2}}}{d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
-(-I*C*log(I*tan(1/2*d*x + 1/2*c) - 1) + I*C*log(-I*tan(1/2*d*x + 1/2*c) -
 1) + 2*A*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))*b^(3/2)/d
```

Mupad [B] (verification not implemented)

Time = 40.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\cos(c+d\,x)}\,(A\sin(2\,c+2\,d\,x)+C\,d\,x+C\,d\,x\,\cos}{d\,\sqrt{\cos(c+d\,x)}\,(\cos(2\,c+}$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(7/2),x)
```

output
```
(b*(b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d*
x) + C*d*x + C*d*x*cos(2*c + 2*d*x)))/(d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d
*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b(\cos(dx+c)\,cdx+\sin(dx+c)\,a)}{\cos(dx+c)\,d}$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

output
```
(sqrt(b)*b*(cos(c + d*x)*c*d*x + sin(c + d*x)*a))/(cos(c + d*x)*d)
```

3.104 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 80

$$\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{b(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
1/2*b*(A+2*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+
1/2*A*b*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

$$\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{3/2}\left((A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+\right.}{2d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/
2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 +
A*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2031, 3042, 3491, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{2}(A+2C)\int \sec(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{2}(A+2C)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{(A+2C)\text{arctanh}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(((A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*Sec[
c + d*x]*Tan[c + d*x])/(2*d)))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46

method	result
default	$-\frac{b\left(A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2-A\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2+4C\ \mathrm{arctanh}(-\csc(dx+c)+\cot}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+\sin(dx+c)\right)\sqrt{b\cos(dx+c)}\,b}{2d\cos(dx+c)^{\frac{5}{2}}}-\frac{2C\ a}{}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\,A\left(\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{b\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2\sqrt{\cos(dx+c)}\,d}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2*b/d*(A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^2-A*ln(-cot(d*x+c)+csc
(d*x+c)+1)*cos(d*x+c)^2+4*C*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)^2-A
*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.70

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left[\frac{(A+2\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{\cdots}}{\mathrm{co}\cdots}\right.}{}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{3}-\sqrt{b\cos\left(dx+c\right)}Ab\sqrt{\cos\left(dx+c\right)}\sin\left(dx\cdots\right.}{2\,d\cos\left(dx+c\right)^{3}}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
[1/4*((A + 2*C)*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*c
os(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/c
os(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b*arctan(sqrt(b*cos(d*x +
c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*
cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(68) = 136$.

Time = 0.29 (sec) , antiderivative size = 761, normalized size of antiderivative = 9.51

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
1/4*(2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*C*sqrt(b) - (4*(b
*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 +
4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*
d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b*
cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d
*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2
*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)...
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left((A+2\,C)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-(A+2\,C)\log\left(\mathrm{ta}\right.\right.}{2\,}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
1/2*((A + 2*C)*log(tan(1/2*d*x + 1/2*c) + 1) - (A + 2*C)*log(tan(1/2*d*x +
 1/2*c) - 1) + 2*(A*tan(1/2*d*x + 1/2*c)^3 + A*tan(1/2*d*x + 1/2*c))/(tan(
1/2*d*x + 1/2*c)^4 - 2*tan(1/2*d*x + 1/2*c)^2 + 1))*b^(3/2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.25

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2 a-2\log\left(\tan\left(\right.\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

output
```
(sqrt(b)*b*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 2*log(tan((c
+ d*x)/2) - 1)*sin(c + d*x)**2*c + log(tan((c + d*x)/2) - 1)*a + 2*log(tan
((c + d*x)/2) - 1)*c + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 2*log
(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*c - log(tan((c + d*x)/2) + 1)*a - 2
*log(tan((c + d*x)/2) + 1)*c - sin(c + d*x)*a))/(2*d*(sin(c + d*x)**2 - 1)
)
```

3.105 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 81

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
1/3*A*b*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+1/3*b*(2*A+3*C)
*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11
/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*
Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2031, 3042, 3491, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}(2A+3C)\int \sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}(2A+3C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(((2*A + 3*C)*Tan[c + d*x])/(3*d) + (A*Sec[c + d*x
]^2*Tan[c + d*x])/(3*d)))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{b\sin(dx+c)\left(2A\cos(dx+c)^2+3C\cos(dx+c)^2+A\right)\sqrt{b\cos(dx+c)}}{3d\cos(dx+c)^{\frac{7}{2}}}$	55
parts	$\frac{A\sin(dx+c)\left(2\cos(dx+c)^2+1\right)\sqrt{b\cos(dx+c)}\,b}{3d\cos(dx+c)^{\frac{7}{2}}}+\frac{C\sin(dx+c)\sqrt{b\cos(dx+c)}\,b}{d\cos(dx+c)^{\frac{3}{2}}}$	75
risch	$\frac{2ib\sqrt{b\cos(dx+c)}\,\left(3C\,e^{4i(dx+c)}+6A\,e^{2i(dx+c)}+6C\,e^{2i(dx+c)}+2A+3C\right)}{3\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^3}$	82

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
1/3*b/d*sin(d*x+c)*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*(b*cos(d*x+c))^(1
/2)/cos(d*x+c)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{\left((2\,A+3\,C)b\cos\left(dx+c\right)^2+Ab\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+\right.}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
1/3*((2*A + 3*C)*b*cos(d*x + c)^2 + A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c)
/(d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(69) = 138$.

Time = 0.25 (sec) , antiderivative size = 355, normalized size of antiderivative = 4.38

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2\left(\frac{3\,Cb^{\frac{3}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1} - \frac{}{2\,(3\,\cos(4\,dx+4\,c)+}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
2/3*(3*C*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1) - 2*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*
b*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x
 + 6*c) - 3*(3*b*cos(2*d*x + 2*c) + b)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*c
os(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6
*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2
 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*
x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.60

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = -\frac{2\left(3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,C\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,A\tan\right.}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\right.}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
-2/3*(3*A*tan(1/2*d*x + 1/2*c)^5 + 3*C*tan(1/2*d*x + 1/2*c)^5 - 2*A*tan(1/
2*d*x + 1/2*c)^3 - 6*C*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x + 1/2*c) +
 3*C*tan(1/2*d*x + 1/2*c))*b^(3/2)/((tan(1/2*d*x + 1/2*c)^6 - 3*tan(1/2*d*
x + 1/2*c)^4 + 3*tan(1/2*d*x + 1/2*c)^2 - 1)*d)
```

Mupad [B] (verification not implemented)

Time = 42.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.69

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+\right.\right.}{}$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(11/2),x)
```

output
```
(b*(b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(
4*c + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(
4*c + 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*si
n(4*c + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4
*c + 4*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c +
 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,\sin\left(dx+c\right)b\left(2\sin\left(dx+c\right)^2 a+3\sin\left(dx+c\right)^2 c-3a\right.}{3\cos\left(dx+c\right)d\left(\sin\left(dx+c\right)^2-1\right)}$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b*(2*sin(c + d*x)**2*a + 3*sin(c + d*x)**2*c - 3*a -
 3*c))/(3*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.106 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 125

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
1/8*b*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2
)+1/4*A*b*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)+1/8*b*(3*A+4*
C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.65

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\cos(c+dx)}\left((3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+\right.}{8d\cos^{\frac{9}{2}}(c+}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13
/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4
+ (2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2
))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2031, 3042, 3491, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^5(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\int \sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A + 4
*C)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))
/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

method	result
default	$-\frac{b(3A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+4C\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-3A\ln(-\cot(dx+c)+\csc(dx-}{}$
parts	$\frac{A(-3\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+3\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^4+3\cos(dx+c)^2\sin(dx+c)+2\sin(}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\,(3A\,e^{7i(dx+c)}+4C\,e^{7i(dx+c)}+11A\,e^{5i(dx+c)}+4C\,e^{5i(dx+c)}-11A\,e^{3i(dx+c)}-4C\,e^{3i(dx+c)}-3A\,e^{i(dx+c)}-4C\,e^{i(d}{4\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^4}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x,method=_RE
TURNVERBOSE)
```

output
```
-1/8*b/d*(3*A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^4+4*C*ln(-cot(d*x+c)
+csc(d*x+c)-1)*cos(d*x+c)^4-3*A*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4-
4*C*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+(-3*cos(d*x+c)^2-2)*sin(d*x+
c)*A-4*C*cos(d*x+c)^2*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.08

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \left[\frac{(3\,A+4\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c}}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{(3\,A+4\,C)\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left((3\,A+4\,C)b\cos\left(dx+c\right)^2+2\,Ab\right)\sqrt{b}\cdots}{8\,d\cos\left(dx+c\right)^5}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, alg
orithm="fricas")
```

output
```
[1/16*((3*A + 4*C)*b^(3/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(
b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*((3*A + 4*C)*b*cos(d*x + c)^2 + 2*A*b)*sqrt(b*cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*((3*A +
4*C)*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(
cos(d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*b*cos(d*x + c)^2 + 2*A*b)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2434 vs. $2(107) = 214$.

Time = 0.42 (sec) , antiderivative size = 2434, normalized size of antiderivative = 19.47

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, alg
orithm="maxima")
```

output
```
-1/16*((12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*
c) + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*
c) + 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*
c) + 4*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*
c) + 4*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x
 + 4*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*
x + 6*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x +
4*c) + 4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c
) + 4*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c)
+ b)*cos(4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3
*b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin
(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) + 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 3...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(107) = 214$.

Time = 0.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.72

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left((3\,A+4\,C)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-(3\,A+4\,C)\log\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, alg
orithm="giac")
```

output
```
1/8*((3*A + 4*C)*log(tan(1/2*d*x + 1/2*c) + 1) - (3*A + 4*C)*log(tan(1/2*d
*x + 1/2*c) - 1) + 2*(5*A*tan(1/2*d*x + 1/2*c)^7 + 4*C*tan(1/2*d*x + 1/2*c
)^7 + 3*A*tan(1/2*d*x + 1/2*c)^5 - 4*C*tan(1/2*d*x + 1/2*c)^5 + 3*A*tan(1/
2*d*x + 1/2*c)^3 - 4*C*tan(1/2*d*x + 1/2*c)^3 + 5*A*tan(1/2*d*x + 1/2*c) +
 4*C*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^8 - 4*tan(1/2*d*x + 1/2*c
)^6 + 6*tan(1/2*d*x + 1/2*c)^4 - 4*tan(1/2*d*x + 1/2*c)^2 + 1))*b^(3/2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{13/2}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(13/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(13/2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.52

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b\left(-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4 a-4\log\left(\tan\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x)
```

output
```
(sqrt(b)*b*( - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 4*log(tan((
c + d*x)/2) - 1)*sin(c + d*x)**4*c + 6*log(tan((c + d*x)/2) - 1)*sin(c + d
*x)**2*a + 8*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 3*log(tan((c +
d*x)/2) - 1)*a - 4*log(tan((c + d*x)/2) - 1)*c + 3*log(tan((c + d*x)/2) +
1)*sin(c + d*x)**4*a + 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c - 6*l
og(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - 8*log(tan((c + d*x)/2) + 1)*s
in(c + d*x)**2*c + 3*log(tan((c + d*x)/2) + 1)*a + 4*log(tan((c + d*x)/2)
+ 1)*c - 3*sin(c + d*x)**3*a - 4*sin(c + d*x)**3*c + 5*sin(c + d*x)*a + 4*
sin(c + d*x)*c))/(8*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.107 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx\right.$

Optimal result

Integrand size = 35, antiderivative size = 125

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{b^2(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b^2(A+2C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{b\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
b^2*(A+C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-1/3*b^2*(A+2*
C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)^3/d/cos(d*x+c)^(1/2)+1/5*b^2*C*(b*cos(d
*x+c))^(1/2)*sin(d*x+c)^5/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{(b\cos(c+dx))^{5/2}(100A+89C+4(5A+7C)\cos(2(c+dx))+3C\cos(4(c+dx)))\,\mathrm{s}}{120d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)
,x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(100*A + 89*C + 4*(5*A + 7*C)*Cos[2*(c + d*x)] + 3
*C*Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2031, 3042, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3492}$$

$$-\frac{b^2\sqrt{b\cos(c+dx)}\int \left(1-\sin^2(c+dx)\right)\left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{290}$$

$$-\frac{b^2\sqrt{b\cos(c+dx)}\int \left(C\sin^4(c+dx)-(A+2C)\sin^2(c+dx)+A\left(\frac{C}{A}+1\right)\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}(A+2C)\sin^3(c+dx)-(A+C)\sin(c+dx)-\frac{1}{5}C\sin^5(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

output
```
-((b^2*Sqrt[b*Cos[c + d*x]]*(-((A + C)*Sin[c + d*x]) + ((A + 2*C)*Sin[c +
d*x]^3)/3 - (C*Sin[c + d*x]^5)/5))/(d*Sqrt[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

method	result
default	$\frac{b^{2} \sin(dx+c)\left(3C\cos(dx+c)^{4}+5A\cos(dx+c)^{2}+4C\cos(dx+c)^{2}+10A+8C\right)\sqrt{b\cos(dx+c)}}{15d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\left(2+\cos(dx+c)^{2}\right)b^{2}\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}+\frac{C\sin(dx+c)\left(3\cos(dx+c)^{4}+4\cos(dx+c)^{2}+8\right)b^{2}\sqrt{b\cos(dx+c)}}{15d\sqrt{\cos(dx+c)}}$
risch	$-\frac{ib^{2}\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{6i(dx+c)}C}{80\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{ib^{2}\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{2i(dx+c)}(6A+5C)}{8\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{ib^{2}\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx}}{8\left(\mathrm{e}^{2i(dx+c)}+1\right.}$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x,method=_RET
URNVERBOSE)
```

output
```
1/15*b^2/d*sin(d*x+c)*(3*C*cos(d*x+c)^4+5*A*cos(d*x+c)^2+4*C*cos(d*x+c)^2+
10*A+8*C)/cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^{2}(c+dx)\right)\,dx=\frac{\left(3\,Cb^{2}\cos\left(dx+c\right)^{4}+\left(5\,A+4\,C\right)b^{2}\cos\left(dx+c\right)^{2}+2\left(5\,A+4\,C\right)b^{2}\right)\sqrt{b\cos\left(dx+c\right.}}{15\,d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="fricas")
```

output
```
1/15*(3*C*b^2*cos(d*x + c)^4 + (5*A + 4*C)*b^2*cos(d*x + c)^2 + 2*(5*A + 4
*C)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{20\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)A\sqrt{b}+$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="maxima")
```

output
```
1/240*(20*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))))*A*sqrt(b) + (3*b^2*sin(5*d*x + 5*c) + 25*b^2*sin(3/5*a
rctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 150*b^2*sin(1/5*arctan2(sin(
5*d*x + 5*c), cos(5*d*x + 5*c))))*C*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{1}{240}\left(\frac{3\,Cb^2\sin\left(5\,dx+5\,c\right)}{d}+\frac{5\left(4\,Ab^2+5\,Cb^2\right)\sin\left(3\,dx+3\,c\right)}{d}+\frac{30\left(6\,Ab^2+5\,Cb^2\right)}{d}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algo
rithm="giac")
```

output
```
1/240*(3*C*b^2*sin(5*d*x + 5*c)/d + 5*(4*A*b^2 + 5*C*b^2)*sin(3*d*x + 3*c)
/d + 30*(6*A*b^2 + 5*C*b^2)*sin(d*x + c)/d)*sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 42.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{b^2\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\left(200\,A\,\sin\left(2\,c+2\,d\,x\right)+20\,A\,\sin\left(4\,c+4\,d\,x\right)+\right.}{240\,d\,\left(\cos\left(2\,c+2\,d\,x\right.\right.}$$

input
```
int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2),x)
```

output
```
(b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(200*A*sin(2*c + 2*d*x) + 2
0*A*sin(4*c + 4*d*x) + 175*C*sin(2*c + 2*d*x) + 28*C*sin(4*c + 4*d*x) + 3*
C*sin(6*c + 6*d*x)))/(240*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.45

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A + C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b}\,\sin(dx+c)\,b^2\left(3\sin(dx+c)^4c - 5\sin(dx+c)^2a - 10\sin(dx+c)^2c + 15a + 1\right.}{15d}$$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b**2*(3*sin(c + d*x)**4*c - 5*sin(c + d*x)**2*a - 10
*sin(c + d*x)**2*c + 15*a + 15*c))/(15*d)
```

3.108 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 122

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{b^2(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}+\frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$

output

```
1/8*b^2*(4*A+3*C)*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/8*b^2*(4*A+3*C
)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d+1/4*b^2*C*cos(d*x+c)^
(5/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.55

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{(b\cos(c+dx))^{5/2}(4(4A+3C)(c+dx)+8(A+C)\sin(2(c}{32d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x
]],x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*
x)] + C*Sin[4*(c + d*x)]))/(32*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2031, 3042, 3493, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3493}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\int \cos^2(c+dx)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 24$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A +
 3*C)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

method	result
default	$\frac{b^{2}\left(4A(dx+c)+3C(dx+c)+4A\cos(dx+c)\sin(dx+c)+\sin(dx+c)\cos(dx+c)\left(2\cos(dx+c)^{2}+3\right)C\right)\sqrt{b\cos(dx+c)}}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{b^{2}\sqrt{b\cos(dx+c)}\,(8A+6C)x}{16\sqrt{\cos(dx+c)}}+\frac{b^{2}\sqrt{b\cos(dx+c)}\,C\sin(4dx+4c)}{32\sqrt{\cos(dx+c)}\,d}+\frac{b^{2}\sqrt{b\cos(dx+c)}\,(A+C)\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)b^{2}\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}+\frac{C\left(2\cos(dx+c)^{3}\sin(dx+c)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)b^{2}\sqrt{b\cos(dx+c}}{8d\sqrt{\cos(dx+c)}}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/8*b^2/d*(4*A*(d*x+c)+3*C*(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*co
s(d*x+c)*(2*cos(d*x+c)^2+3)*C)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.80

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\left[\frac{(4\,A+3\,C)\sqrt{-b}b^{2}\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx-\right.}\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
[1/16*((4*A + 3*C)*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b^2*cos(d*x +
 c)^2 + (4*A + 3*C)*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c))/d, 1/8*((4*A + 3*C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2))) + (2*C*b^2*cos(d*x + c)^2 + (4*A + 3*C)*b^2)
*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{8\left(2\left(dx+c\right)b^2+b^2\sin\left(2\,dx+2\,c\right)\right)A\sqrt{b}+\left(12\left(dx+c\right)b^2\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
1/32*(8*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*A*sqrt(b) + (12*(d*x + c)
*b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))))*C*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{1}{32}\left(\frac{Cb^2\sin\left(4\,dx+4\,c\right)}{d}+4\left(4\,Ab^2+3\,Cb^2\right)x+\frac{8\left(Ab^2+\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
1/32*(C*b^2*sin(4*d*x + 4*c)/d + 4*(4*A*b^2 + 3*C*b^2)*x + 8*(A*b^2 + C*b^
2)*sin(2*d*x + 2*c)/d)*sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(2\,c+2\,d\,x\right)+8\,C\,\sin\left(2\,c+2\right.\right.}{32\,d\,\sqrt{\cos\left(c+\right.}}$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x)
```

output
```
(b^2*(b*cos(c + d*x))^(1/2)*(8*A*sin(2*c + 2*d*x) + 8*C*sin(2*c + 2*d*x) +
 C*sin(4*c + 4*d*x) + 16*A*d*x + 12*C*d*x))/(32*d*cos(c + d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.56

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b}\,b^2\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)^3c+4\cos\left(dx+c\right)\sin\left(c\right.\right.}{8d}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

output
```
(sqrt(b)*b**2*( - 2*cos(c + d*x)*sin(c + d*x)**3*c + 4*cos(c + d*x)*sin(c
+ d*x)*a + 5*cos(c + d*x)*sin(c + d*x)*c + 4*a*d*x + 3*c*d*x))/(8*d)
```

3.109 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 80

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{b^2C\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

output
```
b^2*(A+C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-1/3*b^2*C*(b*
cos(d*x+c))^(1/2)*sin(d*x+c)^3/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/
2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*
d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2031, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3492}$$

$$-\frac{b^2\sqrt{b\cos(c+dx)}\int \left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}C\sin^3(c+dx)-(A+C)\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

output
```
-((b^2*Sqrt[b*Cos[c + d*x]]*(-((A + C)*Sin[c + d*x]) + (C*Sin[c + d*x]^3)/
3))/(d*Sqrt[Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{b^{2} \sin(dx+c)\left(C\cos(dx+c)^{2}+3A+2C\right)\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}$	50
risch	$\frac{b^{2}\sqrt{b\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4\sqrt{\cos(dx+c)}\,d}+\frac{b^{2}\sqrt{b\cos(dx+c)}\,C\sin(3dx+3c)}{12\sqrt{\cos(dx+c)}\,d}$	77
parts	$\frac{C\sin(dx+c)\left(2+\cos(dx+c)^{2}\right)b^{2}\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}+\frac{A\sin(dx+c)b^{2}\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}$	77

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/3*b^2/d*sin(d*x+c)*(C*cos(d*x+c)^2+3*A+2*C)/cos(d*x+c)^(1/2)*(b*cos(d*x+
c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(Cb^2\cos\left(dx+c\right)^2+(3\,A+2\,C)b^2\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx\right.}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
1/3*(C*b^2*cos(d*x + c)^2 + (3*A + 2*C)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x
+ c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{12\,Ab^{\frac{5}{2}}\sin\left(dx+c\right)+\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\,\text{arcta}\right.\right.}{12\,d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
1/12*(12*A*b^(5/2)*sin(d*x + c) + (b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*C*sqrt(b))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(70) = 140.

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.85

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left(3\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Cb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,A\right.}{3\left(\tan\left(\frac{1}{2}\,dx+\right.\right.}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
2/3*(3*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 3*C*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*A
*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*A*b^2*tan
(1/2*d*x + 1/2*c) + 3*C*b^2*tan(1/2*d*x + 1/2*c))*sqrt(b)/((tan(1/2*d*x +
1/2*c)^6 + 3*tan(1/2*d*x + 1/2*c)^4 + 3*tan(1/2*d*x + 1/2*c)^2 + 1)*d)
```

Mupad [B] (verification not implemented)

Time = 41.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(c+d\,x\right)+9\,C\,\sin\left(c+d\,x\right)-\right.}{12\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),x)
```

output
```
(b^2*(b*cos(c + d*x))^(1/2)*(12*A*sin(c + d*x) + 9*C*sin(c + d*x) + C*sin(
3*c + 3*d*x)))/(12*d*cos(c + d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.42

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{\sqrt{b}\,\sin\left(dx+c\right)b^2\left(-\sin\left(dx+c\right)^2c+3a+3c\right)}{3d}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b**2*( - sin(c + d*x)**2*c + 3*a + 3*c))/(3*d)
```

3.110 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 99

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
A*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b^2*C*x*(b*cos(d*x+c))^(
1/2)/cos(d*x+c)^(1/2)+1/2*b^2*C*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*sin(
d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/
2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d
*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(Ax+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(A*x + (C*x)/2 + (C*Cos[c + d*x]*Sin[c + d*x])/(
2*d)))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{b^{2}\left(C\cos\left(dx+c\right)\sin\left(dx+c\right)+2A\left(dx+c\right)+C\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}}{2d\sqrt{\cos\left(dx+c\right)}}$	57
risch	$\frac{b^{2}\sqrt{b\cos\left(dx+c\right)}\,\left(4A+2C\right)x}{4\sqrt{\cos\left(dx+c\right)}}+\frac{b^{2}\sqrt{b\cos\left(dx+c\right)}\,C\sin\left(2dx+2c\right)}{4\sqrt{\cos\left(dx+c\right)}\,d}$	69
parts	$\frac{A\left(dx+c\right)b^{2}\sqrt{b\cos\left(dx+c\right)}}{d\sqrt{\cos\left(dx+c\right)}}+\frac{C\left(\cos\left(dx+c\right)\sin\left(dx+c\right)+dx+c\right)b^{2}\sqrt{b\cos\left(dx+c\right)}}{2d\sqrt{\cos\left(dx+c\right)}}$	78

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/2*b^2/d*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))*(b*cos(d*x+c))^(
1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.73

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[\frac{2\sqrt{b\cos\left(dx+c\right)}Cb^{2}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+(2\,A+\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
[1/4*(2*sqrt(b*cos(d*x + c))*C*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A
+ C)*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)
*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*C*b^2*
sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*b^(5/2)*arctan(sqrt(b*cos(d*x
+ c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.60

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{8\,Ab^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + (2\,(dx+c)b^2 + b^2\sin(2\,dx +}{4\,d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
1/4*(8*A*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (2*(d*x + c)*b^
2 + b^2*sin(2*d*x + 2*c))*C*sqrt(b))/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.25

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left((2i\,Ab^2+i\,Cb^2)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-(2i\,Ab^2+i\,C\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
1/2*((2*I*A*b^2 + I*C*b^2)*log(tan(1/2*d*x + 1/2*c) + I) - (2*I*A*b^2 + I*
C*b^2)*log(tan(1/2*d*x + 1/2*c) - I) - 2*(C*b^2*tan(1/2*d*x + 1/2*c)^3 - C
*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 + 2*tan(1/2*d*x + 1/2*c
)^2 + 1))*sqrt(b)/d
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2),x)
```

output
```
(b^2*(b*cos(c + d*x))^(1/2)*(C*sin(2*c + 2*d*x) + 4*A*d*x + 2*C*d*x))/(4*d
*cos(c + d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.34

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b^2(\cos\left(dx+c\right)\sin\left(dx+c\right)c+2adx+cdx)}{2d}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

output
```
(sqrt(b)*b**2*(cos(c + d*x)*sin(c + d*x)*c + 2*a*d*x + c*d*x))/(2*d)
```

3.111 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 74

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{Ab^2\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
A*b^2*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+b^2*C*(b
*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{5/2}(A\operatorname{arctanh}(\sin(c+dx))+C\sin(c+dx))}{d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/
2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Cos
[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2031, 3042, 3493, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3493}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \sec(c+dx)dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/
d))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

method	result
default	$-\frac{b^{2}\left(2A\,\operatorname{arctanh}\left(-\csc\left(dx+c\right)+\cot\left(dx+c\right)\right)-C\sin\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}}{d\sqrt{\cos\left(dx+c\right)}}$
parts	$-\frac{2A\sqrt{b\cos\left(dx+c\right)}\,\operatorname{arctanh}\left(-\csc\left(dx+c\right)+\cot\left(dx+c\right)\right)b^{2}}{d\sqrt{\cos\left(dx+c\right)}}+\frac{b^{2}C\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}$
risch	$-\frac{i b^{2}\sqrt{b\cos\left(dx+c\right)}\,C\,{\mathrm e}^{i\left(dx+c\right)}}{2\sqrt{\cos\left(dx+c\right)}\,d}+\frac{i b^{2}\sqrt{b\cos\left(dx+c\right)}\,C\,{\mathrm e}^{-i\left(dx+c\right)}}{2\sqrt{\cos\left(dx+c\right)}\,d}+\frac{b^{2}\sqrt{b\cos\left(dx+c\right)}\,A\ln\left({\mathrm e}^{i\left(dx+c\right)}+i\right)}{\sqrt{\cos\left(dx+c\right)}\,d}-\frac{b^{2}\sqrt{b\cos\left(dx+c\right)}\,A\ln(}{\sqrt{\cos\left(dx+c\right)}}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-b^2/d*(2*A*arctanh(-csc(d*x+c)+cot(d*x+c))-C*sin(d*x+c))*(b*cos(d*x+c))^(
1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.84

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\left[\frac{Ab^{\frac{5}{2}}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\cdots}{\cos(dx+c)^3}\cdots\right.}{\cdots}\right.$$

$$\left.-\frac{A\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)-\sqrt{b\cos\left(dx+c\right)}Cb^2\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}\right]$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
[1/2*(A*b^(5/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c
))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*C*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)), -(A*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x
 + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*b^2*sq
rt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\,Cb^{\frac{5}{2}}\sin\left(dx+c\right)+\left(b^2\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
1/2*(2*C*b^(5/2)*sin(d*x + c) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 +
 2*sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x
 + c) + 1))*A*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(Ab^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-Ab^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.}{d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
(A*b^2*log(tan(1/2*d*x + 1/2*c) + 1) - A*b^2*log(tan(1/2*d*x + 1/2*c) - 1)
 + 2*C*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{\sqrt{b}\,b^2\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a\right.}{d}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

output
```
(sqrt(b)*b**2*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*
a + sin(c + d*x)*c))/d
```

3.112 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 65

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
b^2*C*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b^2*(b*cos(d*x+c))^(1/2)*s
in(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(Cdx\cos(c+dx)+A\sin(c+dx))}{d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/
2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c +
d*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2031, 3042, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \ 2031$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3491$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(C\int 1dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 24$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+Cx\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(C*x + (A*Tan[c + d*x])/d))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{b^{2}\left(C(dx+c)\cos(dx+c)+A\sin(dx+c)\right)\sqrt{b\cos(dx+c)}}{d\cos(dx+c)^{\frac{3}{2}}}$	48
parts	$\frac{A\,b^{2}\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}}+\frac{C(dx+c)b^{2}\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}$	65
risch	$\frac{b^{2}Cx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}}+\frac{2ib^{2}\sqrt{b\cos(dx+c)}\,A}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}$	67

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
b^2/d*(C*(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^
(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.98

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left[\frac{C\sqrt{-b}b^2\cos(dx+c)^2\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
[1/2*(C*sqrt(-b)*b^2*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(
d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x
 + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), (C*b^(5/
2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*
cos(d*x + c)^2 + sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\left(Cb^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\frac{Ab^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\mathrm{co}}\right.}{d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
2*(C*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + A*b^(5/2)*sin(2*d*x
 + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
)/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{\left(-i\,Cb^2\log\left(i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+i\,Cb^2\log\left(-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}\right)\sqrt{b}}{d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
-(-I*C*b^2*log(I*tan(1/2*d*x + 1/2*c) - 1) + I*C*b^2*log(-I*tan(1/2*d*x +
1/2*c) - 1) + 2*A*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))*s
qrt(b)/d
```

Mupad [B] (verification not implemented)

Time = 40.60 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A\,\sin\left(2\,c+2\,d\,x\right)+C\,d\,x+C\,d\,x\,\cos\right.}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c\right.\right.}$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(9/2),x)
```

output
```
(b^2*(b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*
d*x) + C*d*x + C*d*x*cos(2*c + 2*d*x)))/(d*cos(c + d*x)^(1/2)*(cos(2*c + 2
*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b^2(\cos\left(dx+c\right)cdx+\sin\left(dx+c\right)a)}{\cos\left(dx+c\right)d}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

output
```
(sqrt(b)*b**2*(cos(c + d*x)*c*d*x + sin(c + d*x)*a))/(cos(c + d*x)*d)
```

3.113 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 84

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{b^2(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
1/2*b^2*(A+2*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2
)+1/2*A*b^2*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{5/2}\left((A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+\right.}{2d\cos^{\frac{9}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11
/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 +
A*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(9/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2031, 3042, 3491, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{2}(A+2C)\int \sec(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{2}(A+2C)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{(A+2C)\text{arctanh}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(((A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*Se
c[c + d*x]*Tan[c + d*x])/(2*d)))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.42

method	result
default	$-\frac{b^2\left(A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2-A\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2+4C\ \operatorname{arctanh}(-\csc(dx+c)+\mathrm{co}\cdots\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+\sin(dx+c)\right)\sqrt{b\cos(dx+c)}\,b^2}{2d\cos(dx+c)^{\frac{5}{2}}}-\frac{2C\cdots}{}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\,A\left(\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{b^2\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}\cdots\right.}{2\sqrt{\cos(dx+c)}\,d}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
-1/2*b^2/d*(A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^2-A*ln(-cot(d*x+c)+c
sc(d*x+c)+1)*cos(d*x+c)^2+4*C*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)^2
-A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.64

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \left[\frac{(A+2\,C)b^{\frac{5}{2}}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{\cdots}}{\cos\cdots}\right.}{\cdots}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-\sqrt{b\cos\left(dx+c\right)}Ab^2\sqrt{\cos\left(dx+c\right)}\sin\left(\cdots\right.}{2\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
[1/4*((A + 2*C)*b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*c
os(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/c
os(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x +
 c))/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*
x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqr
t(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 821 vs. 2(72) = 144.

Time = 0.37 (sec) , antiderivative size = 821, normalized size of antiderivative = 9.77

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
1/4*(2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^
2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*C*sqrt(b) - (
4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*d
*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d
*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c
)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*
x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2*c
os(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2
*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)
/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*...
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.44

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left((Ab^2+2\,Cb^2)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-(Ab^2+2\,Cb^2)\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
1/2*((A*b^2 + 2*C*b^2)*log(tan(1/2*d*x + 1/2*c) + 1) - (A*b^2 + 2*C*b^2)*l
og(tan(1/2*d*x + 1/2*c) - 1) + 2*(A*b^2*tan(1/2*d*x + 1/2*c)^3 + A*b^2*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 2*tan(1/2*d*x + 1/2*c)^2 + 1)
)*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{11/2}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.17

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx=\frac{\sqrt{b}\,b^2\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2a-2\log\left(\tan\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)
```

output
```
(sqrt(b)*b**2*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 2*log(tan(
(c + d*x)/2) - 1)*sin(c + d*x)**2*c + log(tan((c + d*x)/2) - 1)*a + 2*log(
tan((c + d*x)/2) - 1)*c + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 2*
log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*c - log(tan((c + d*x)/2) + 1)*a
- 2*log(tan((c + d*x)/2) + 1)*c - sin(c + d*x)*a))/(2*d*(sin(c + d*x)**2 -
 1))
```

3.114 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 85

$$\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{b^2(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
1/3*A*b^2*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+1/3*b^2*(2*A+
3*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\sin(c+dx)\,(3(A+C)+A\tan^2(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13
/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*
Cos[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2031, 3042, 3491, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}(2A+3C)\int \sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}(2A+3C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(((2*A + 3*C)*Tan[c + d*x])/(3*d) + (A*Sec[c + d
*x]^2*Tan[c + d*x])/(3*d)))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{b^{2} \sin \left(d x+c\right) \left(2 A \cos \left(d x+c\right)^{2}+3 C \cos \left(d x+c\right)^{2}+A\right) \sqrt{b \cos \left(d x+c\right)}}{3 d \cos \left(d x+c\right)^{\frac{7}{2}}}$	57
parts	$\frac{A \sin \left(d x+c\right) \left(2 \cos \left(d x+c\right)^{2}+1\right) \sqrt{b \cos \left(d x+c\right)}\, b^{2}}{3 d \cos \left(d x+c\right)^{\frac{7}{2}}}+\frac{C \sin \left(d x+c\right) \sqrt{b \cos \left(d x+c\right)}\, b^{2}}{d \cos \left(d x+c\right)^{\frac{3}{2}}}$	79
risch	$\frac{2 i b^{2} \sqrt{b \cos \left(d x+c\right)}\, \left(3 C \,{\mathrm e}^{4 i \left(d x+c\right)}+6 A \,{\mathrm e}^{2 i \left(d x+c\right)}+6 C \,{\mathrm e}^{2 i \left(d x+c\right)}+2 A+3 C\right)}{3 \sqrt{\cos \left(d x+c\right)}\, d \left({\mathrm e}^{2 i \left(d x+c\right)}+1\right)^{3}}$	84

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x,method=_RE
TURNVERBOSE)
```

output
```
1/3*b^2/d*sin(d*x+c)*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*(b*cos(d*x+c))^
(1/2)/cos(d*x+c)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left((2\,A+3\,C)b^{2}\cos\left(dx+c\right)^{2}+Ab^{2}\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx\right.}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, alg
orithm="fricas")
```

output
```
1/3*((2*A + 3*C)*b^2*cos(d*x + c)^2 + A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x
+ c)/(d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(73) = 146.

Time = 0.26 (sec) , antiderivative size = 367, normalized size of antiderivative = 4.32

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2\left(\frac{3\,Cb^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1} - \frac{\cdots}{2\,(3\,\cos(4\,dx+4\,c)+\cdots}\right.}{\cdots}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, alg
orithm="maxima")
```

output
```
2/3*(3*C*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1) - 2*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) +
9*b^2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*s
in(6*d*x + 6*c) - 3*(3*b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*A*sqr
t(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + c
os(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*
d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*
c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c)
+ 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(73) = 146$.

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.74

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = -\frac{2\left(3\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Cb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,Cb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 -\right.}{3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, alg
orithm="giac")
```

output
```
-2/3*(3*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 3*C*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*
A*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*A*b^2*ta
n(1/2*d*x + 1/2*c) + 3*C*b^2*tan(1/2*d*x + 1/2*c))*sqrt(b)/((tan(1/2*d*x +
 1/2*c)^6 - 3*tan(1/2*d*x + 1/2*c)^4 + 3*tan(1/2*d*x + 1/2*c)^2 - 1)*d)
```

Mupad [B] (verification not implemented)

Time = 42.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.59

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+\right.\right.}{}$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(13/2),x)
```

output
```
(b^2*(b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*co
s(4*c + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*co
s(4*c + 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*
sin(4*c + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin
(4*c + 4*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c
 + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,\sin\left(dx+c\right)b^2\left(2\sin\left(dx+c\right)^2a+3\sin\left(dx+c\right)^2c-3c\right.}{3\cos\left(dx+c\right)d\left(\sin\left(dx+c\right)^2-1\right)}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*b**2*(2*sin(c + d*x)**2*a + 3*sin(c + d*x)**2*c - 3*
a - 3*c))/(3*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.115 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 131

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx=\frac{b^2(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{b^2(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
1/8*b^2*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1
/2)+1/4*A*b^2*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)+1/8*b^2*(
3*A+4*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{5/2}\left((3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c-\right.}{8d\cos^{\frac{13}{2}}(c-}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(15
/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4
+ (2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(13/
2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2031, 3042, 3491, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int\left(C\cos^2(c+dx)+A\right)\sec^5(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int\frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\int\sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(15/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A +
 4*C)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4
))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.35

method	result
default	$-\frac{b^2\left(3A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+4C\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-3A\ln(-\cot(dx+c)+\csc(d$
parts	$\frac{A\left(-3\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+3\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^4+3\cos(dx+c)^2\sin(dx+c)+2\sin(\right.}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\,(3A\,e^{7i(dx+c)}+4C\,e^{7i(dx+c)}+11A\,e^{5i(dx+c)}+4C\,e^{5i(dx+c)}-11A\,e^{3i(dx+c)}-4C\,e^{3i(dx+c)}-3A\,e^{i(dx+c)}-4C\,e^{i(}}{4\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^4}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x,method=_RE
TURNVERBOSE)
```

output
```
-1/8*b^2/d*(3*A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^4+4*C*ln(-cot(d*x+
c)+csc(d*x+c)-1)*cos(d*x+c)^4-3*A*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^
4-4*C*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+(-3*cos(d*x+c)^2-2)*sin(d*
x+c)*A-4*C*cos(d*x+c)^2*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.06

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \left[\frac{(3\,A+4\,C)b^{\frac{5}{2}}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}}{\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{(3\,A+4\,C)\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left((3\,A+4\,C)b^2\cos\left(dx+c\right)^2+2\,Ab^2\right)\cdots}{8\,d\cos\left(dx+c\right)^5}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, alg
orithm="fricas")
```

output
```
[1/16*((3*A + 4*C)*b^(5/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(
b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*((3*A + 4*C)*b^2*cos(d*x + c)^2 + 2*A*b^2)*sqrt(b*co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*((3*
A + 4*C)*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*b^2*cos(d*x + c)^2 + 2
*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(15/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2662 vs. $2(113) = 226$.

Time = 0.46 (sec) , antiderivative size = 2662, normalized size of antiderivative = 20.32

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, alg
orithm="maxima")
```

output
```
-1/16*((12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d*
x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*
sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c)
+ 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x
 + 6*c) + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2
*cos(6*d*x + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^
2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x
+ 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2
*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*c
os(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^
2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(
4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c)
 + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*
(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(113) = 226$.

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.92

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{\left((3\,Ab^2+4\,Cb^2)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-(3\,Ab^2+4\,C\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, alg
orithm="giac")
```

output
```
1/8*((3*A*b^2 + 4*C*b^2)*log(tan(1/2*d*x + 1/2*c) + 1) - (3*A*b^2 + 4*C*b^
2)*log(tan(1/2*d*x + 1/2*c) - 1) + 2*(5*A*b^2*tan(1/2*d*x + 1/2*c)^7 + 4*C
*b^2*tan(1/2*d*x + 1/2*c)^7 + 3*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*C*b^2*tan
(1/2*d*x + 1/2*c)^5 + 3*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*C*b^2*tan(1/2*d*x
 + 1/2*c)^3 + 5*A*b^2*tan(1/2*d*x + 1/2*c) + 4*C*b^2*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^8 - 4*tan(1/2*d*x + 1/2*c)^6 + 6*tan(1/2*d*x + 1/2*
c)^4 - 4*tan(1/2*d*x + 1/2*c)^2 + 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{15/2}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(15/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(15/2), x
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.42

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b^2\left(-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4 a-4\log\left(\tan\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x)
```

output
```
(sqrt(b)*b**2*( - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 4*log(ta
n((c + d*x)/2) - 1)*sin(c + d*x)**4*c + 6*log(tan((c + d*x)/2) - 1)*sin(c
+ d*x)**2*a + 8*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 3*log(tan((c
 + d*x)/2) - 1)*a - 4*log(tan((c + d*x)/2) - 1)*c + 3*log(tan((c + d*x)/2)
 + 1)*sin(c + d*x)**4*a + 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c -
6*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - 8*log(tan((c + d*x)/2) + 1
)*sin(c + d*x)**2*c + 3*log(tan((c + d*x)/2) + 1)*a + 4*log(tan((c + d*x)/
2) + 1)*c - 3*sin(c + d*x)**3*a - 4*sin(c + d*x)**3*c + 5*sin(c + d*x)*a +
 4*sin(c + d*x)*c))/(8*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.116 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 113

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}$$

output
```
1/8*(4*A+3*C)*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*cos(d*
x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/4*C*cos(d*x+c)^(7/2)*sin(d*
x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))}{32d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]]
,x]
```

output
```
(Sqrt[Cos[c + d*x]]*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)]
+ C*Sin[4*(c + d*x)]))/(32*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2031, 3042, 3493, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\int\cos^2(c+dx)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*((C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A + 3*C)*
(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])    Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

method	result	si
default	$\frac{\left(4A(dx+c)+3C(dx+c)+4A\cos(dx+c)\sin(dx+c)+\sin(dx+c)\cos(dx+c)\left(2\cos(dx+c)^{2}+3\right)C\right)\sqrt{\cos(dx+c)}}{8d\sqrt{b\cos(dx+c)}}$	82
risch	$\frac{\sqrt{\cos(dx+c)}\,(8A+6C)x}{16\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,C\sin(4dx+4c)}{32\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,(A+C)\sin(2dx+2c)}{4\sqrt{b\cos(dx+c)}\,d}$	98
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}}+\frac{C\left(2\cos(dx+c)^{3}\sin(dx+c)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)\sqrt{\cos(dx+c)}}{8d\sqrt{b\cos(dx+c)}}$	10

input
```
int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/8/d*(4*A*(d*x+c)+3*C*(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*cos(d*
x+c)*(2*cos(d*x+c)^2+3)*C)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.83

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left[\frac{2\left(2\,C\cos\left(dx+c\right)^2+4\,A+3\,C\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-(4\,A+3\,C)\sqrt{-b}\log\left(2\,b\right.}{16\,bd}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
[1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c) - (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqr
t(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b*d), 1/
8*((2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x +
 c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{8\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{\sqrt{b}}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right))C}{\sqrt{b}}}{32\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/sqrt(b) + (12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/s
qrt(b))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 41.58 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(8\,A\,\sin(c+d\,x)+8\,C\,\sin(c+d\,x)+8\,A\,\sin(3\,c+3\,d\,x)+9\,C\,\sin(3}{32\,b\,d\,\left(\cos(2\,c+2\,d\,x)+1\right.}$$

input
```
int((cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c +
 d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) +
 32*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*b*d*(cos(2*c + 2*d*x)
 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{b}\left(-2\cos(dx+c)\sin(dx+c)^3c+4\cos(dx+c)\sin(dx+c)a+5\cos(dx+c)\sin(dx+c)c+4adx+\right.}{8bd}$$

input
```
int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*( - 2*cos(c + d*x)*sin(c + d*x)**3*c + 4*cos(c + d*x)*sin(c + d*x
)*a + 5*cos(c + d*x)*sin(c + d*x)*c + 4*a*d*x + 3*c*d*x))/(8*b*d)
```

3.117 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 74

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{C\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

output
```
(A+C)*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-1/3*C*cos(d*x+c)^
(1/2)*sin(d*x+c)^3/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]]
,x]
```

output
```
(Sqrt[Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Sq
rt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2031, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) \left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx) \left(C\cos^2(c+dx)+A\right)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3492}$$

$$-\frac{\sqrt{\cos(c+dx)}\int \left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}C\sin^3(c+dx)-(A+C)\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
-((Sqrt[Cos[c + d*x]]*(-((A + C)*Sin[c + d*x]) + (C*Sin[c + d*x]^3)/3))/(d
*Sqrt[b*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\sin(dx+c)\left(C\cos(dx+c)^2+3A+2C\right)\sqrt{\cos(dx+c)}}{3d\sqrt{b\cos(dx+c)}}$	47
risch	$\frac{\sqrt{\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,C\sin(3dx+3c)}{12\sqrt{b\cos(dx+c)}\,d}$	71
parts	$\frac{A\sin(dx+c)\sqrt{\cos(dx+c)}}{d\sqrt{b\cos(dx+c)}}+\frac{C\sin(dx+c)\left(2+\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}}{3d\sqrt{b\cos(dx+c)}}$	71

input
```
int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/3/d*sin(d*x+c)*(C*cos(d*x+c)^2+3*A+2*C)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.66

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\left(C\cos\left(dx+c\right)^2+3\,A+2\,C\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,bd\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*
sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{\sqrt{b}}+\frac{12\,A\sin(dx+c)}{\sqrt{b}}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
1/12*(C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))))/sqrt(b) + 12*A*sin(d*x + c)/sqrt(b))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(2\,c+2\,d\,x\right)+10\,C\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(4\,c+4\,d\,x\right)\right)}{12\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int((cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(12*A*sin(2*c + 2*d*x) + 10*C*s
in(2*c + 2*d*x) + C*sin(4*c + 4*d*x)))/(12*b*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.46

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b}\,\sin\left(dx+c\right)\left(-\sin\left(dx+c\right)^2c+3a+3c\right)}{3bd}$$

input
```
int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*( - sin(c + d*x)**2*c + 3*a + 3*c))/(3*b*d)
```

3.118 $\int \frac{\sqrt{\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 90

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

output
```
A*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/(b*cos(
d*x+c))^(1/2)+1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]]
,x]
```

output
```
(Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*Sqr
t[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.59, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \ 2031$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{\cos(c+dx)}\left(Ax+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*x + (C*x)/2 + (C*Cos[c + d*x]*Sin[c + d*x])/(2*d)))
/Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}}$	54
risch	$\frac{\sqrt{\cos(dx+c)}\,(4A+2C)x}{4\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,C\sin(2dx+2c)}{4\sqrt{b\cos(dx+c)}\,d}$	63
parts	$\frac{A(dx+c)\sqrt{\cos(dx+c)}}{d\sqrt{b\cos(dx+c)}}+\frac{C(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}}$	72

input
```
int(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/2/d*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))*cos(d*x+c)^(1/2)/(b*
cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[\frac{2\sqrt{b\cos(dx+c)}C\sqrt{\cos(dx+c)}\sin(dx+c)-(2A+C)\sqrt{-b}\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\right.}{4bd}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
[1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) - (2*A + C)
*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(co
s(d*x + c))*sin(d*x + c) - b))/(b*d), 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos
(d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*si
n(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b*d)]
```

Sympy [A] (verification not implemented)

Time = 15.82 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \begin{cases} \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{Cx\sin^2(c+dx)\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{Cx\cos^{\frac{5}{2}}(c+dx)}{2\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}} & \text{for } d \neq 0 \\ \frac{x(A+C\cos^2(c))\sqrt{\cos(c)}}{\sqrt{b\cos(c)}} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Piecewise((A*x*sqrt(cos(c + d*x))/sqrt(b*cos(c + d*x)) + C*x*sin(c + d*x)*
*2*sqrt(cos(c + d*x))/(2*sqrt(b*cos(c + d*x))) + C*x*cos(c + d*x)**(5/2)/(
2*sqrt(b*cos(c + d*x))) + C*sin(c + d*x)*cos(c + d*x)**(3/2)/(2*d*sqrt(b*c
os(c + d*x))), Ne(d, 0)), (x*(A + C*cos(c)**2)*sqrt(cos(c))/sqrt(b*cos(c))
, True))
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{\sqrt{b}} + \frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}}{4\,d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/sqrt(b) + 8*A*arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/sqrt(b))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(c+d\,x\right)+C\,\sin\left(3\,c+3\,d\,x\right)+8\,A\,d\,x\,\cos\left(c+d\,x\right)+4\,C\,d\,x\,\mathrm{co}\right.}{4\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int((cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + C*sin(3*c + 3
*d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))/(4*b*d*(cos(2*c + 2*
d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b}\,(\cos(dx+c)\sin(dx+c)\,c+2adx+cdx)}{2bd}$$

input
```
int(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(cos(c + d*x)*sin(c + d*x)*c + 2*a*d*x + c*d*x))/(2*b*d)
```

3.119 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 68

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{A\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+C*cos(d*x+c)
^(1/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(A\text{arctanh}(\sin(c+dx))+C\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
,x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Sqrt[b*
Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 3042, 3493, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3493}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec(c+dx)dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d))/Sq
rt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\left(2 A \,\operatorname{arctanh}\left(-\csc \left(d x +c \right)+\cot \left(d x +c \right)\right)-C \sin \left(d x +c \right)\right) \sqrt{\cos \left(d x +c \right)}}{d \sqrt{b \cos \left(d x +c \right)}}$	53
parts	$\frac{C \sqrt{\cos \left(d x +c \right)}\, \sin \left(d x +c \right)}{d \sqrt{b \cos \left(d x +c \right)}}-\frac{2 A \,\operatorname{arctanh}\left(-\csc \left(d x +c \right)+\cot \left(d x +c \right)\right) \sqrt{\cos \left(d x +c \right)}}{d \sqrt{b \cos \left(d x +c \right)}}$	71
risch	$\frac{\sqrt{\cos \left(d x +c \right)}\, A \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{\sqrt{b \cos \left(d x +c \right)}\, d}-\frac{\sqrt{\cos \left(d x +c \right)}\, A \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{\sqrt{b \cos \left(d x +c \right)}\, d}+\frac{C \sin \left(2 d x +2 c \right)}{2 d \sqrt{\cos \left(d x +c \right)}\, \sqrt{b \cos \left(d x +c \right)}}$	108

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/d*(2*A*arctanh(-csc(d*x+c)+cot(d*x+c))-C*sin(d*x+c))*cos(d*x+c)^(1/2)/(
b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.04

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[\frac{A\sqrt{b}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}C\sqrt{}}{2\,bd\cos(dx+c)}\right.$$
$$\left.-\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)-\sqrt{b\cos(dx+c)}C\sqrt{\cos(dx+c)}\sin(dx+c)}{bd\cos(dx+c)}\right]$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
[1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c
))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(
d*x + c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x
 + c))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{A + C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)/(sqrt(b*cos(c + d*x))*sqrt(cos(c + d*x)))
, x)
```

Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)}{\sqrt{b}} + \frac{2C\sin(dx+c)}{\sqrt{b}}}{2d}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 2*C*sin(d*x
 + c)/sqrt(b))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.69

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a+\sin\left(dx+c\right)c\right)}{bd}$$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a + s
in(c + d*x)*c))/(b*d)
```

3.120 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 59

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
C*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/
(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\frac{Cdx\cos(c+dx)+A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
,x]
```

output
```
(C*d*x*Cos[c + d*x] + A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c +
 d*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2032, 3042, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \ 2032$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^2(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \ 3491$$

$$\frac{\sqrt{\cos(c+dx)}\left(C\int 1dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \ 24$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+Cx\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(C*x + (A*Tan[c + d*x])/d))/Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{C(dx+c)\cos(dx+c)+A\sin(dx+c)}{d\sqrt{\cos(dx+c)}\,\sqrt{b\cos(dx+c)}}$	45
risch	$\frac{Cx\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)}}+\frac{i\mathrm{e}^{-i(dx+c)}A}{\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,d}$	57
parts	$\frac{A\sin(dx+c)}{d\sqrt{\cos(dx+c)}\,\sqrt{b\cos(dx+c)}}+\frac{C(dx+c)\sqrt{\cos(dx+c)}}{d\sqrt{b\cos(dx+c)}}$	59

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/d*(C*(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.24

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$=\left[-\frac{C\sqrt{-b}\cos(dx+c)^2\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-}{2\,bd\cos(dx+c)^2}\right.$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
[-1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x +
c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^2), (C*sqrt(b)*ar
ctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d
*x + c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*c
os(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)/(sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2)
), x)
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.44

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx$$
$$= \frac{2\left(\frac{C\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}} + \frac{A\sqrt{b}\sin(2\,dx+2\,c)}{b\cos(2\,dx+2\,c)^2+b\sin(2\,dx+2\,c)^2+2\,b\cos(2\,dx+2\,c)+b}\right)}{d}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
2*(C*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + A*sqrt(b)*sin(2*d*x
 + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*x + 2*c
) + b))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 40.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.42

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A\,\sin\left(2\,c+2\,d\,x\right)+C\,d\,x+C\,d\,x\,\cos\left(2\,c+2\,d\,x\right)+A\,1\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{b\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d*x)
 + C*d*x + C*d*x*cos(2*c + 2*d*x)))/(b*d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d
*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b}\,(\cos\left(dx+c\right)cdx+\sin\left(dx+c\right)a)}{\cos\left(dx+c\right)bd}$$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(cos(c + d*x)*c*d*x + sin(c + d*x)*a))/(cos(c + d*x)*b*d)
```

3.121 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 78

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/2*(A+2*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+1/
2*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
,x]
```

output
```
((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin[c + d*x])/(2*d*Cos
[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 3042, 3491, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^3(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3491}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}(A+2C)\int \sec(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}(A+2C)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{(A+2C)\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(((A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*Sec[c +
d*x]*Tan[c + d*x])/(2*d)))/Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

method	result
default	$-\frac{A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2-A\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2+4C\,\operatorname{arctanh}(-\csc(dx+c)+\cot(d}{2d\cos(dx+c)^{\frac{3}{2}}\sqrt{b\cos(dx+c)}}$
parts	$\frac{A\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+\sin(dx+c)\right)}{2d\cos(dx+c)^{\frac{3}{2}}\sqrt{b\cos(dx+c)}}-\frac{2C\,\operatorname{arctanh}(-\csc(d}{d\sqrt{}}$
risch	$-\frac{iA(e^{2i(dx+c)}-1)}{2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)d}-\frac{\sqrt{\cos(dx+c)}\,(A+2C)\ln(e^{i(dx+c)}-i)}{2\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,(A+2C)\ln(e^{i(dx+c)}+i)}{2\sqrt{b\cos(dx+c)}\,d}$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2/d*(A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^2-A*ln(-cot(d*x+c)+csc(d
*x+c)+1)*cos(d*x+c)^2+4*C*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)^2-A*s
in(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.81

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left[\frac{(A+2\,C)\sqrt{b}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,\sqrt{b\cos\left(d\right.}}{4\,bd\cos\left(dx+c\right)^3}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(d\right.}{2\,bd\cos\left(dx+c\right)^3}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
[1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*c
os(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/c
os(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))
/(b*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c)
)*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*co
s(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(66) = 132$.

Time = 0.26 (sec) , antiderivative size = 728, normalized size of antiderivative = 9.33

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
1/4*(2*C*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(
cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) - (4*(sin(4
*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
+ 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*c
os(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x +
4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*co
s(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*A/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.33

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b}\,\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2a-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2c+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 2*log(tan((c +
d*x)/2) - 1)*sin(c + d*x)**2*c + log(tan((c + d*x)/2) - 1)*a + 2*log(tan((
c + d*x)/2) - 1)*c + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 2*log(t
an((c + d*x)/2) + 1)*sin(c + d*x)**2*c - log(tan((c + d*x)/2) + 1)*a - 2*l
og(tan((c + d*x)/2) + 1)*c - sin(c + d*x)*a))/(2*b*d*(sin(c + d*x)**2 - 1)
)
```

3.122 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 79

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
1/3*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*sin
(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
,x]
```

output
```
(Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*Sqrt[Cos[c + d*x]]*Sqrt
[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2032, 3042, 3491, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^4(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}(2A+3C)\int \sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}(2A+3C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(((2*A + 3*C)*Tan[c + d*x])/(3*d) + (A*Sec[c + d*x]^2*
Tan[c + d*x])/(3*d)))/Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\sin(dx+c)\left(2A\cos(dx+c)^{2}+3C\cos(dx+c)^{2}+A\right)}{3d\cos(dx+c)^{\frac{5}{2}}\sqrt{b\cos(dx+c)}}$	54
parts	$\frac{A\sin(dx+c)\left(2\cos(dx+c)^{2}+1\right)}{3d\cos(dx+c)^{\frac{5}{2}}\sqrt{b\cos(dx+c)}}+\frac{C\sin(dx+c)}{d\sqrt{\cos(dx+c)}\sqrt{b\cos(dx+c)}}$	73
risch	$\frac{i\left(3C\,e^{3i(dx+c)}+(9C+8A)\cos(dx+c)+i(4A+3C)\sin(dx+c)\right)}{3\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\left(e^{2i(dx+c)}+1\right)^{2}d}$	81

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/3/d*sin(d*x+c)*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)/cos(d*x+c)^(5/2)/(b
*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\left((2\,A+3\,C)\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,bd\cos\left(dx+c\right)^{\frac{7}{2}}}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*
d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(67) = 134.

Time = 0.30 (sec) , antiderivative size = 355, normalized size of antiderivative = 4.49

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{2\left(\frac{3C\sqrt{b}\sin(2dx+2c)}{b\cos(2dx+2c)^2+b\sin(2dx+2c)^2+2b\cos(2dx+2c)+b}+\frac{}{\left(2(3\cos(4dx+4c)+3\cos(2dx+2c)+1)\cos(6dx+6c)+\cos(6dx+6c)^2+6(3c\right.}\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
2/3*(3*C*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*
c)^2 + 2*b*cos(2*d*x + 2*c) + b) + 2*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x +
 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*s
in(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((2*(3*cos(4*d*x
+ 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6
*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(
2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
+ sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 42.63 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.78

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(c+dx)}\,(18\,A\sin(2\,c+2\,d\,x)+12\,A\sin(4\,c+4\,d\,x)+2\,A\sin(6\,c+6\,d\,x)+15\,C\sin(2\,c+2}{}$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4*
c + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*
c + 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(
4*c + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c
 + 4*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*b*d*cos(c + d*x)^(1/2)*(15*cos(2*c +
 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \frac{\sqrt{b}\,\sin(dx + c)\left(2\sin(dx + c)^2 a + 3\sin(dx + c)^2 c - 3a - 3c\right)}{3\cos(dx + c)\,bd\left(\sin(dx + c)^2 - 1\right)}$$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*(2*sin(c + d*x)**2*a + 3*sin(c + d*x)**2*c - 3*a - 3
*c))/(3*cos(c + d*x)*b*d*(sin(c + d*x)**2 - 1))
```

3.123 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 122

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/8*(3*A+4*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+
1/4*A*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*sin
(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+(2A+(3A+4C)\cos^2(c+dx))\sin(c+dx)}{8d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]])
,x]
```

output
```
((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos
[c + d*x]^2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2032, 3042, 3491, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$\Big\downarrow \text{2032}$$
$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^5(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx}{\sqrt{b\cos(c+dx)}}$$
$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\int \sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A + 4*C)*
(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/Sqr
t[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.43

method	result
default	$\frac{-3A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-4C\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+3A\ln(-\cot(dx+c)+\csc(dx+c)}{8d\cos(dx+c)^{\frac{7}{2}}\sqrt{}}$
parts	$-\frac{A\left(3\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-3\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^4-3\cos(dx+c)^2\sin(dx+c)-2\sin(\right.}{8d\cos(dx+c)^{\frac{7}{2}}\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i(3A\,e^{6i(dx+c)}+4C\,e^{6i(dx+c)}+11A\,e^{4i(dx+c)}+4C\,e^{4i(dx+c)}-11A\,e^{2i(dx+c)}-4C\,e^{2i(dx+c)}-3A-4C)}{8\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^3d}+\frac{\sqrt{\cos(dx+c)}\,(3A+4C)}{8\sqrt{b\cos(dx-}}$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/8/d*(-3*A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^4-4*C*ln(-cot(d*x+c)+c
sc(d*x+c)-1)*cos(d*x+c)^4+3*A*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+4*
C*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+(3*cos(d*x+c)^2+2)*sin(d*x+c)*
A+4*C*cos(d*x+c)^2*sin(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.14

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{9}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx$$
$$= \left[\frac{(3\,A + 4\,C)\sqrt{b}\cos{(dx + c)}^5\log\left(-\frac{b\cos(dx+c)^3 - 2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c) - 2\,b\cos(dx+c)}{\cos(dx+c)^3}\right) + 2\left((3\,A + \right.}{16\,bd\cos{(dx + c)}^5}\right.$$
$$-\frac{(3\,A + 4\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos{(dx + c)}^5 - \left((3\,A + 4\,C)\cos{(dx + c)}^2 + 2\,A\right)\sqrt{b}}{8\,bd\cos{(dx + c)}^5}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
[1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(
b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^5), -1/8*((3*A + 4*
C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(
d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{9}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2318 vs. $2(104) = 208$.

Time = 0.31 (sec) , antiderivative size = 2318, normalized size of antiderivative = 19.00

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{9}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
-1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4
*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d
*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(
8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c
))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x +
8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d
*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c)
+ 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x +
8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 1
6*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*
d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos(c+dx)^2+A}{\cos(c+dx)^{9/2}\sqrt{b\cos(c+dx)}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(9/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(9/2)*(b*cos(c + d*x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.60

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b}\left(-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4 a-4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4 c+6\log\left(\tan\left(\frac{dx}{2}+\right.\right.\right.}{}$$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*( - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 4*log(tan((c
+ d*x)/2) - 1)*sin(c + d*x)**4*c + 6*log(tan((c + d*x)/2) - 1)*sin(c + d*x
)**2*a + 8*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 3*log(tan((c + d*
x)/2) - 1)*a - 4*log(tan((c + d*x)/2) - 1)*c + 3*log(tan((c + d*x)/2) + 1)
*sin(c + d*x)**4*a + 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c - 6*log
(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - 8*log(tan((c + d*x)/2) + 1)*sin
(c + d*x)**2*c + 3*log(tan((c + d*x)/2) + 1)*a + 4*log(tan((c + d*x)/2) +
1)*c - 3*sin(c + d*x)**3*a - 4*sin(c + d*x)**3*c + 5*sin(c + d*x)*a + 4*si
n(c + d*x)*c))/(8*b*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.124 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 122

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}}$$

output
```
1/8*(4*A+3*C)*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*cos(
d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/4*C*cos(d*x+c)^(7/2)*si
n(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.55

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))}{32d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/
2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)]
+ C*Sin[4*(c + d*x)]))/(32*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2031, 3042, 3493, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3493}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\int \cos^2(c+dx)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A + 3*C)*
(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

method	result	si
default	$\frac{\left(4A(dx+c)+3C(dx+c)+4A\cos(dx+c)\sin(dx+c)+\sin(dx+c)\cos(dx+c)\left(2\cos(dx+c)^{2}+3\right)C\right)\sqrt{\cos(dx+c)}}{8bd\sqrt{b\cos(dx+c)}}$	85
risch	$\frac{\sqrt{\cos(dx+c)}\,(8A+6C)x}{16b\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,C\sin(4dx+4c)}{32b\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,(A+C)\sin(2dx+2c)}{4b\sqrt{b\cos(dx+c)}\,d}$	10
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}\,b}+\frac{C\left(2\cos(dx+c)^{3}\sin(dx+c)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)\sqrt{\cos(dx+c)}}{8d\sqrt{b\cos(dx+c)}\,b}$	11

input
```
int(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/8/b/d*(4*A*(d*x+c)+3*C*(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*cos(
d*x+c)*(2*cos(d*x+c)^2+3)*C)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.70

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\left[\frac{2\left(2\,C\cos\left(dx+c\right)^{2}+4\,A+3\,C\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+\right.}}{}\right.$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
[1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c) - (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqr
t(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d),
1/8*((2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x
 + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^2*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{8\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{3}{2}}} + \frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx\right.}{b^{\frac{3}{2}}}}{32\,d}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(3/2) + (12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/b
^(3/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 41.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(8\,A\,\sin(c+d\,x)+8\,C\,\sin(c+$$

input
```
int((cos(c + d*x)^(7/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c +
 d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) +
 32*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*b^2*d*(cos(2*c + 2*d*
x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.56

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\,\left(-2\cos(dx+c)\sin(dx+c)^3c+4\cos(dx+c)\sin(dx+c)\right.}{8b^2d}$$

input
```
int(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*( - 2*cos(c + d*x)*sin(c + d*x)**3*c + 4*cos(c + d*x)*sin(c + d*x
)*a + 5*cos(c + d*x)*sin(c + d*x)*c + 4*a*d*x + 3*c*d*x))/(8*b**2*d)
```

3.125 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 80

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{C\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

output
```
(A+C)*cos(d*x+c)^(1/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)-1/3*C*cos(d*x+c
)^(1/2)*sin(d*x+c)^3/b/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/
2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*(b
*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2031, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3492}$$

$$-\frac{\sqrt{\cos(c+dx)}\int \left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}C\sin^3(c+dx)-(A+C)\sin(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
-((Sqrt[Cos[c + d*x]]*(-((A + C)*Sin[c + d*x]) + (C*Sin[c + d*x]^3)/3))/(b
*d*Sqrt[b*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{\sin(dx+c)\left(C\cos(dx+c)^2+3A+2C\right)\sqrt{\cos(dx+c)}}{3bd\sqrt{b\cos(dx+c)}}$	50
risch	$\frac{\sqrt{\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4b\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,C\sin(3dx+3c)}{12b\sqrt{b\cos(dx+c)}\,d}$	77
parts	$\frac{A\sin(dx+c)\sqrt{\cos(dx+c)}}{d\sqrt{b\cos(dx+c)}\,b}+\frac{C\sin(dx+c)\left(2+\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}}{3d\sqrt{b\cos(dx+c)}\,b}$	77

input
```
int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/3/b/d*sin(d*x+c)*(C*cos(d*x+c)^2+3*A+2*C)*cos(d*x+c)^(1/2)/(b*cos(d*x+c)
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.61

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\left(C\cos\left(dx+c\right)^2+3\,A+2\,C\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,b^2d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2*
d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{b^{\frac{3}{2}}}+\frac{12\,A\sin(dx+c)}{b^{\frac{3}{2}}}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
1/12*(C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))))/b^(3/2) + 12*A*sin(d*x + c)/b^(3/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(2\,c+2\,d\,x\right)+10\,C\,\mathrm{s}\right.}{12\,b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right.}$$

input
```
int((cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(12*A*sin(2*c + 2*d*x) + 10*C*s
in(2*c + 2*d*x) + C*sin(4*c + 4*d*x)))/(12*b^2*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.42

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sin(dx+c)\left(-\sin(dx+c)^2 c+3a+3c\right)}{3b^2d}$$

input

```
int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output

```
(sqrt(b)*sin(c + d*x)*( - sin(c + d*x)**2*c + 3*a + 3*c))/(3*b**2*d)
```

3.126 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 99

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

output
```
A*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/b/(b*
cos(d*x+c))^(1/2)+1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/
2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/
2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*(b*
Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \ 2031$$
$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)dx}{b\sqrt{b\cos(c+dx)}}$$
$$\Big\downarrow \ 2009$$
$$\frac{\sqrt{\cos(c+dx)}\left(Ax+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*x + (C*x)/2 + (C*Cos[c + d*x]*Sin[c + d*x])/(2*d)))
/(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))\sqrt{\cos(dx+c)}}{2bd\sqrt{b\cos(dx+c)}}$	57
risch	$\frac{\sqrt{\cos(dx+c)}\,(4A+2C)x}{4b\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,C\sin(2dx+2c)}{4b\sqrt{b\cos(dx+c)}\,d}$	69
parts	$\frac{A(dx+c)\sqrt{\cos(dx+c)}}{db\sqrt{b\cos(dx+c)}}+\frac{C(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}\,b}$	78

input
```
int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/2/b/d*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))*cos(d*x+c)^(1/2)/(
b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.71

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{2\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-(2A+C)\sqrt{\cdots}}{\cdots}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
[1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) - (2*A + C)
*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(co
s(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(c
os(d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*
sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^2*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{b^{\frac{3}{2}}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}}{4\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(3/2) + 8*A*arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/b^(3/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(c+d\,x\right)+C\,\sin\left(3\,c+\right.\right.}{4\,b^2\,d\,\left(\cos\left(2\,c+2\right.\right.}$$

input
```
int((cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + C*sin(3*c + 3
*d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))/(4*b^2*d*(cos(2*c +
2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.34

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\cos\left(dx+c\right)\sin\left(dx+c\right)c+2adx+cdx\right)}{2b^2d}$$

input
```
int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(cos(c + d*x)*sin(c + d*x)*c + 2*a*d*x + c*d*x))/(2*b**2*d)
```

3.127 $\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 74

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{A\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+C*cos(d*x+
c)^(1/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(A\text{arctanh}(\sin(c+dx))+C\sin(c+dx))}{d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/
2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*(b*Cos[
c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2031, 3042, 3493, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec(c+dx)dx+\frac{C\sin(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{C\sin(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d))/(b
*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\left(2 A \,\operatorname{arctanh}\left(-\csc \left(d x +c \right)+\cot \left(d x +c \right)\right)-C \sin \left(d x +c \right)\right) \sqrt{\cos \left(d x +c \right)}}{b d \sqrt{b \cos \left(d x +c \right)}}$
parts	$-\frac{2 A \,\operatorname{arctanh}\left(-\csc \left(d x +c \right)+\cot \left(d x +c \right)\right) \sqrt{\cos \left(d x +c \right)}}{d b \sqrt{b \cos \left(d x +c \right)}}+\frac{C \sqrt{\cos \left(d x +c \right)}\, \sin \left(d x +c \right)}{b d \sqrt{b \cos \left(d x +c \right)}}$
risch	$-\frac{i \sqrt{\cos \left(d x +c \right)}\, C \,{\mathrm e}^{i \left(d x +c \right)}}{2 b \sqrt{b \cos \left(d x +c \right)}\, d}+\frac{i \sqrt{\cos \left(d x +c \right)}\, C \,{\mathrm e}^{-i \left(d x +c \right)}}{2 b \sqrt{b \cos \left(d x +c \right)}\, d}+\frac{\sqrt{\cos \left(d x +c \right)}\, A \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{b \sqrt{b \cos \left(d x +c \right)}\, d}-\frac{\sqrt{\cos \left(d x +c \right)}\, A \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{b \sqrt{b \cos \left(d x +c \right)}\, d}$

input
```
int(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/b/d*(2*A*arctanh(-csc(d*x+c)+cot(d*x+c))-C*sin(d*x+c))*cos(d*x+c)^(1/2)
/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.80

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{A\sqrt{b}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin}{\cos(dx+c)^3}\right.}{2b^2d}\right.$$
$$\left.-\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)-\sqrt{b\cos(dx+c)}C\sqrt{\cos(dx+c)}\sin(dx+c)}{b^2d\cos(dx+c)}\right]$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
[1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c
))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*co
s(d*x + c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d
*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\right.\right.}{b^{\frac{3}{2}}}}{2\,d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2) + 2*C*sin(d*x
 + c)/b^(3/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a+\sin}{b^2d}$$

input
```
int(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a + s
in(c + d*x)*c))/(b**2*d)
```

3.128 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 65

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx=\frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
C*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/b/d/cos(d*x+c)^(1
/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{\cos(c+dx)}(Cdx\cos(c+dx)+A\sin(c+dx))}{d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2
)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d
*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2032, 3042, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^2(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{\cos(c+dx)}\left(C\int 1dx+\frac{A\tan(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+Cx\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(C*x + (A*Tan[c + d*x])/d))/(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{C(dx+c)\cos(dx+c)+A\sin(dx+c)}{bd\sqrt{\cos(dx+c)}\,\sqrt{b\cos(dx+c)}}$	48
risch	$\frac{Cx\sqrt{\cos(dx+c)}}{b\sqrt{b\cos(dx+c)}}+\frac{i\mathrm{e}^{-i(dx+c)}A}{b\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,d}$	63
parts	$\frac{A\sin(dx+c)}{bd\sqrt{\cos(dx+c)}\,\sqrt{b\cos(dx+c)}}+\frac{C(dx+c)\sqrt{\cos(dx+c)}}{db\sqrt{b\cos(dx+c)}}$	65

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/b/d*(C*(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.94

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{C\sqrt{-b}\cos{(dx+c)}^2\log\left(2\,b\cos{(dx+c)}^2+2\,\sqrt{b\cos{(dx+c)}}\right.}{}\right.$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
[-1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x +
c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^2), (C*sqrt(b)*
arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos
(d*x + c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2
*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \int \frac{A+C\cos^2{(c+dx)}}{(b\cos{(c+dx)})^{\frac{3}{2}}\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)/((b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x
))), x)
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(\frac{A\sqrt{b}\sin(2\,dx+2\,c)}{b^2\cos(2\,dx+2\,c)^2+b^2\sin(2\,dx+2\,c)^2+2\,b^2\cos(2\,dx+2\,c)+b^2}+\frac{C\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}\right)}{d}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
2*(A*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*
c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + C*arctan(sin(d*x + c)/(cos(d*x + c)
 + 1))/b^(3/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 40.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A\,\sin\left(2\,c+2\,d\,x\right)+C\,d\,x+C\,d\,x\,\cos\left(2\,c+2\,d\right.}{b^2\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\right.}$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d*x)
 + C*d*x + C*d*x*cos(2*c + 2*d*x)))/(b^2*d*cos(c + d*x)^(1/2)*(cos(2*c + 2
*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\,(\cos(dx+c)\,cdx+\sin(dx+c)\,a)}{\cos(dx+c)\,b^2d}$$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(cos(c + d*x)*c*d*x + sin(c + d*x)*a))/(cos(c + d*x)*b**2*d)
```

3.129 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 84

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/2*(A+2*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+
1/2*A*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+A\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2
)),x]
```

output
```
((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin[c + d*x])/(2*d*Sqr
t[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 3042, 3491, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^3(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}(A+2C)\int \sec(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}(A+2C)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{(A+2C)\text{arctanh}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(((A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*Sec[c +
d*x]*Tan[c + d*x])/(2*d)))/(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.42

method	result
default	$-\frac{A \ln(-\cot(dx+c)+\csc(dx+c)-1) \cos(dx+c)^2 - A \ln(-\cot(dx+c)+\csc(dx+c)+1) \cos(dx+c)^2 + 4C \operatorname{arctanh}(-\csc(dx+c)+\cot(d \cdots}{2bd \cos(dx+c)^{\frac{3}{2}} \sqrt{b \cos(dx+c)}}$
parts	$\frac{A\left(\ln(-\cot(dx+c)+\csc(dx+c)+1) \cos(dx+c)^2 - \ln(-\cot(dx+c)+\csc(dx+c)-1) \cos(dx+c)^2 + \sin(dx+c)\right)}{2d \cos(dx+c)^{\frac{3}{2}} b \sqrt{b \cos(dx+c)}} - \frac{2C \operatorname{arctanh}(-\csc(d \cdots}{db \cdots}$
risch	$-\frac{iA\left(e^{2i(dx+c)}-1\right)}{2b\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\left(e^{2i(dx+c)}+1\right)d} - \frac{\sqrt{\cos(dx+c)}\,(A+2C) \ln\left(e^{i(dx+c)}-i\right)}{2b\sqrt{b\cos(dx+c)}\,d} + \frac{\sqrt{\cos(dx+c)}\,(A+2C) \ln\left(e^{i(dx+c)}+ \cdots\right)}{2b\sqrt{b\cos(dx+c)}\,d}$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2/b/d*(A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^2-A*ln(-cot(d*x+c)+csc
(d*x+c)+1)*cos(d*x+c)^2+4*C*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)^2-A
*sin(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.61

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{(A+2\,C)\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\right.}{\cos(dx+c)^3}}{4\,b^2d\,c}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-\sqrt{b\cos(dx+c)}A\sqrt{\cos(dx+c)}\sin(dx+}{2\,b^2d\cos(dx+c)^3}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
[1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*c
os(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/c
os(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))
/(b^2*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x +
c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*
cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. 2(72) = 144.

Time = 0.30 (sec) , antiderivative size = 736, normalized size of antiderivative = 8.76

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
-1/4*((4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c
) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(
4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b*cos(4*d*x + 4*c)^2 + 4
*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d
*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b)) - 2*C*(log(cos(d*x + c)^2 +
 sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos(c+d\,x)^2+A}{\cos(c+d\,x)^{3/2}\,(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.17

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 a-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.\right.}{}$$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 2*log(tan((c +
d*x)/2) - 1)*sin(c + d*x)**2*c + log(tan((c + d*x)/2) - 1)*a + 2*log(tan((
c + d*x)/2) - 1)*c + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 2*log(t
an((c + d*x)/2) + 1)*sin(c + d*x)**2*c - log(tan((c + d*x)/2) + 1)*a - 2*l
og(tan((c + d*x)/2) + 1)*c - sin(c + d*x)*a))/(2*b**2*d*(sin(c + d*x)**2 -
 1))
```

3.130 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 85

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
1/3*A*sin(d*x+c)/b/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*s
in(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2
)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*(b*C
os[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2032, 3042, 3491, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^4(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}(2A+3C)\int \sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}(2A+3C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(((2*A + 3*C)*Tan[c + d*x])/(3*d) + (A*Sec[c + d*x]^2*
Tan[c + d*x])/(3*d)))/(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\sin(dx+c)\left(2A\cos(dx+c)^2+3C\cos(dx+c)^2+A\right)}{3bd\cos(dx+c)^{\frac{5}{2}}\sqrt{b\cos(dx+c)}}$	57
parts	$\frac{A\sin(dx+c)\left(2\cos(dx+c)^2+1\right)}{3d\cos(dx+c)^{\frac{5}{2}}b\sqrt{b\cos(dx+c)}}+\frac{C\sin(dx+c)}{d\sqrt{\cos(dx+c)}\,b\sqrt{b\cos(dx+c)}}$	79
risch	$\frac{i\left(3C\,\mathrm{e}^{3i(dx+c)}+(9C+8A)\cos(dx+c)+i(4A+3C)\sin(dx+c)\right)}{3b\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)^2d}$	84

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/3/b/d*sin(d*x+c)*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)/cos(d*x+c)^(5/2)/
(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.59

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\frac{\left((2\,A+3\,C)\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,b^2d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^
2*d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(73) = 146$.

Time = 0.27 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.47

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(\frac{3\,C\sqrt{b}\sin(2\,dx+2\,c)}{b^2\cos(2\,dx+2\,c)^2+b^2\sin(2\,dx+2\,c)^2+2\,b^2\cos(2\,dx+2\,c)+b^2} + \frac{}{\left(b\cos(6\,dx+6\,c)^2+9\,b\,c\right.}\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
2/3*(3*C*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x
+ 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + 2*((3*cos(2*d*x + 2*c) + 1)*sin
(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x
+ 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b*cos(6
*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*cos(2*d*x + 2*c)^2 + b*sin(6*
d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x + 4*c) + 3*b*cos(2*d*x +
2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c)
 + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) + b*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + b)*sqrt(b)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 42.15 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.59

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)+2\,\right.}{}$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4*
c + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*
c + 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(
4*c + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c
 + 4*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*b^2*d*cos(c + d*x)^(1/2)*(15*cos(2*c
 + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sin(dx+c)\left(2\sin(dx+c)^2 a+3\sin(dx+c)^2 c-3a-3c\right)}{3\cos(dx+c)\,b^2 d\left(\sin(dx+c)^2-1\right)}$$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*(2*sin(c + d*x)**2*a + 3*sin(c + d*x)**2*c - 3*a - 3
*c))/(3*cos(c + d*x)*b**2*d*(sin(c + d*x)**2 - 1))
```

3.131 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 131

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/8*(3*A+4*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2
)+1/4*A*sin(d*x+c)/b/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)
*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+(2A+(3A+4C)\,\text{c}}{8d\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2
)),x]
```

output
```
((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos
[c + d*x]^2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2032, 3042, 3491, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^5(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\int \sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A + 4*C)*
(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/(b*
Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.35

method	result
default	$\frac{-3A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-4C\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+3A\ln(-\cot(dx+c)+\csc(dx+c)}{8bd\cos(dx+c)^{\frac{7}{2}}\sqrt{}}$
parts	$\frac{A\left(-3\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+3\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^4+3\cos(dx+c)^2\sin(dx+c)+2\sin(\right.}{8d\cos(dx+c)^{\frac{7}{2}}b\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i(3A\,e^{6i(dx+c)}+4C\,e^{6i(dx+c)}+11A\,e^{4i(dx+c)}+4C\,e^{4i(dx+c)}-11A\,e^{2i(dx+c)}-4C\,e^{2i(dx+c)}-3A-4C)}{8b\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\left(e^{2i(dx+c)}+1\right)^3d}+\frac{\sqrt{\cos(dx+c)}\,(3A+4C)}{8b\sqrt{b\cos(dx}}$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/8/b/d*(-3*A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^4-4*C*ln(-cot(d*x+c)
+csc(d*x+c)-1)*cos(d*x+c)^4+3*A*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+
4*C*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+(3*cos(d*x+c)^2+2)*sin(d*x+c
)*A+4*C*cos(d*x+c)^2*sin(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.99

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{(3\,A+4\,C)\sqrt{b}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(d}}{\cos(dx+c)^3}\right.}{}\right.$$

$$-\frac{(3\,A+4\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left((3\,A+4\,C)\cos\left(dx+c\right)^2+2\,A\right)\sqrt{b\cos}}{8\,b^2d\cos\left(dx+c\right)^5}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
[1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(
b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^5), -1/8*((3*A +
4*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(co
s(d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2350 vs. 2(113) = 226.

Time = 0.37 (sec) , antiderivative size = 2350, normalized size of antiderivative = 17.94

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
-1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4
*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d
*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(
8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c
))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x +
8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d
*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c)
+ 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x +
8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 1
6*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*
d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\int \frac{C\cos(c+d\,x)^2+A}{\cos(c+d\,x)^{7/2}\,(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.42

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{b}\left(-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^4a-4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*( - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 4*log(tan((c
+ d*x)/2) - 1)*sin(c + d*x)**4*c + 6*log(tan((c + d*x)/2) - 1)*sin(c + d*x
)**2*a + 8*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 3*log(tan((c + d*
x)/2) - 1)*a - 4*log(tan((c + d*x)/2) - 1)*c + 3*log(tan((c + d*x)/2) + 1)
*sin(c + d*x)**4*a + 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c - 6*log
(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - 8*log(tan((c + d*x)/2) + 1)*sin
(c + d*x)**2*c + 3*log(tan((c + d*x)/2) + 1)*a + 4*log(tan((c + d*x)/2) +
1)*c - 3*sin(c + d*x)**3*a - 4*sin(c + d*x)**3*c + 5*sin(c + d*x)*a + 4*si
n(c + d*x)*c))/(8*b**2*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.132 $\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 122

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}$$

output
```
1/8*(4*A+3*C)*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*co
s(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/4*C*cos(d*x+c)^(7/2
)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.57

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))}{32b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(9/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/
2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)]
+ C*Sin[4*(c + d*x)]))/(32*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2031, 3042, 3493, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3493}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\int \cos^2(c+dx)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(9/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A + 3*C)*
(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/(b^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

method	result	si
default	$\frac{\left(4A(dx+c)+3C(dx+c)+4A\cos(dx+c)\sin(dx+c)+\sin(dx+c)\cos(dx+c)\left(2\cos(dx+c)^{2}+3\right)C\right)\sqrt{\cos(dx+c)}}{8b^{2}d\sqrt{b\cos(dx+c)}}$	85
risch	$\frac{\sqrt{\cos(dx+c)}\,(8A+6C)x}{16b^{2}\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,C\sin(4dx+4c)}{32b^{2}\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,(A+C)\sin(2dx+2c)}{4b^{2}\sqrt{b\cos(dx+c)}\,d}$	10
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}\,b^{2}}+\frac{C\left(2\cos(dx+c)^{3}\sin(dx+c)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)\sqrt{\cos(dx+c)}}{8d\sqrt{b\cos(dx+c)}\,b^{2}}$	11

input
```
int(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/8/b^2/d*(4*A*(d*x+c)+3*C*(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)+sin(d*x+c)*co
s(d*x+c)*(2*cos(d*x+c)^2+3)*C)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.70

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{2\left(2\,C\cos\left(dx+c\right)^{2}+4\,A+3\,C\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+\right.}}{}\right.$$

input
```
integrate(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
[1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c) - (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqr
t(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^3*d),
1/8*((2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x
 + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{8\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{5}{2}}} + \frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx}{b^{\frac{5}{2}}}}{32\,d}$$

input
```
integrate(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(5/2) + (12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/b
^(5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 41.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(c+d\,x\right)+8\,C\,\sin\left(c+\right.\right.}{}$$

input
```
int((cos(c + d*x)^(9/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c +
 d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) +
 32*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*b^3*d*(cos(2*c + 2*d*
x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.56

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)^3c+4\cos\left(dx+c\right)\sin\left(dx+c\right)\right.}{8b^3d}$$

input
```
int(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*( - 2*cos(c + d*x)*sin(c + d*x)**3*c + 4*cos(c + d*x)*sin(c + d*x
)*a + 5*cos(c + d*x)*sin(c + d*x)*c + 4*a*d*x + 3*c*d*x))/(8*b**3*d)
```

3.133 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 80

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{(A+C)\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{C\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

output
```
(A+C)*cos(d*x+c)^(1/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)-1/3*C*cos(d*x
+c)^(1/2)*sin(d*x+c)^3/b^2/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/
2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*b^2*
d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2031, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3492}$$

$$-\frac{\sqrt{\cos(c+dx)}\int \left(-C\sin^2(c+dx)+A+C\right)d(-\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}C\sin^3(c+dx)-(A+C)\sin(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
-((Sqrt[Cos[c + d*x]]*(-((A + C)*Sin[c + d*x]) + (C*Sin[c + d*x]^3)/3))/(b
^2*d*Sqrt[b*Cos[c + d*x]]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{\sin(dx+c)\left(C\cos(dx+c)^2+3A+2C\right)\sqrt{\cos(dx+c)}}{3b^2d\sqrt{b\cos(dx+c)}}$	50
risch	$\frac{\sqrt{\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4b^2\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,C\sin(3dx+3c)}{12b^2\sqrt{b\cos(dx+c)}\,d}$	77
parts	$\frac{A\sin(dx+c)\sqrt{\cos(dx+c)}}{d\sqrt{b\cos(dx+c)}\,b^2}+\frac{C\sin(dx+c)\left(2+\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}}{3d\sqrt{b\cos(dx+c)}\,b^2}$	77

input
```
int(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/3/b^2/d*sin(d*x+c)*(C*cos(d*x+c)^2+3*A+2*C)*cos(d*x+c)^(1/2)/(b*cos(d*x+
c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.61

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\left(C\cos\left(dx+c\right)^2+3\,A+2\,C\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,b^3d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3*
d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{b^{\frac{5}{2}}}+\frac{12\,A\sin(dx+c)}{b^{\frac{5}{2}}}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
1/12*(C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))))/b^(5/2) + 12*A*sin(d*x + c)/b^(5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(2\,c+2\,d\,x\right)+10\,C\,\mathrm{s}\right.}{12\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right.}$$

input
```
int((cos(c + d*x)^(7/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(12*A*sin(2*c + 2*d*x) + 10*C*s
in(2*c + 2*d*x) + C*sin(4*c + 4*d*x)))/(12*b^3*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.42

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{b}\,\sin\left(dx+c\right)\left(-\sin\left(dx+c\right)^2c+3a+3c\right)}{3b^3d}$$

input
```
int(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*( - sin(c + d*x)**2*c + 3*a + 3*c))/(3*b**3*d)
```

3.134 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 99

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

output
```
A*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/b^2
/(b*cos(d*x+c))^(1/2)+1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c
))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/
2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*b^2*d
*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(Ax+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*x + (C*x)/2 + (C*Cos[c + d*x]*Sin[c + d*x])/(2*d)))
/(b^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))\sqrt{\cos(dx+c)}}{2b^2d\sqrt{b\cos(dx+c)}}$	57
risch	$\frac{\sqrt{\cos(dx+c)}\,(4A+2C)x}{4b^2\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,C\sin(2dx+2c)}{4b^2\sqrt{b\cos(dx+c)}\,d}$	69
parts	$\frac{A(dx+c)\sqrt{\cos(dx+c)}}{d\,b^2\sqrt{b\cos(dx+c)}}+\frac{C(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}\,b^2}$	78

input
```
int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/2/b^2/d*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))*cos(d*x+c)^(1/2)
/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.71

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\left[\frac{2\sqrt{b\cos(dx+c)}C\sqrt{\cos(dx+c)}\sin(dx+c)-(2A+C)\sqrt{\cdots}}{\cdots}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
[1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) - (2*A + C)
*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(co
s(d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(c
os(d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*
sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{b^{\frac{5}{2}}} + \frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}}{4\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(5/2) + 8*A*arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/b^(5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(c+d\,x\right)+C\,\sin\left(3\,c+\right.\right.}{4\,b^3\,d\,\left(\cos\left(2\,c+2\right.\right.}$$

input
```
int((cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + C*sin(3*c + 3
*d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))/(4*b^3*d*(cos(2*c +
2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.34

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,(\cos(dx+c)\sin(dx+c)\,c+2adx+cdx)}{2b^3d}$$

input
```
int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(cos(c + d*x)*sin(c + d*x)*c + 2*a*d*x + c*d*x))/(2*b**3*d)
```

3.135 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 74

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{A\,\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

output
```
A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+C*cos(d*
x+c)^(1/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(A\,\text{arctanh}(\sin(c+dx))+C\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/
2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(b^2*d*Sqr
t[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2031, 3042, 3493, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec(c+dx)dx+\frac{C\sin(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{C\sin(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d))/(b
^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

method	result
default	$-\frac{(2A\,\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))-C\sin(dx+c))\sqrt{\cos(dx+c)}}{b^2 d\sqrt{b\cos(dx+c)}}$
parts	$-\frac{2A\,\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))\sqrt{\cos(dx+c)}}{d\,b^2\sqrt{b\cos(dx+c)}}+\frac{C\sqrt{\cos(dx+c)}\,\sin(dx+c)}{b^2 d\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i\sqrt{\cos(dx+c)}\,C\,\mathrm{e}^{i(dx+c)}}{2b^2\sqrt{b\cos(dx+c)}\,d}+\frac{i\sqrt{\cos(dx+c)}\,C\,\mathrm{e}^{-i(dx+c)}}{2b^2\sqrt{b\cos(dx+c)}\,d}-\frac{\sqrt{\cos(dx+c)}\,A\ln(\mathrm{e}^{i(dx+c)}-i)}{b^2\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,A\ln(\mathrm{e}^{i(dx+c)}+i)}{b^2\sqrt{b\cos(dx+c)}\,d}$

input
```
int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/b^2/d*(2*A*arctanh(-csc(d*x+c)+cot(d*x+c))-C*sin(d*x+c))*cos(d*x+c)^(1/
2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.80

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{A\sqrt{b}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(}{\cos(dx+c)^3}\right.}{2\,b^3d\,c} \right.$$
$$\left. -\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)-\sqrt{b\cos(dx+c)}C\sqrt{\cos(dx+c)}\sin(dx+c)}{b^3d\cos(dx+c)}\right]$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
[1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c
))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*co
s(d*x + c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d
*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,s\right.}{b^{\frac{5}{2}}}}{2\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2) + 2*C*sin(d*x
 + c)/b^(5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a+\sin\right.}{b^3d}$$

input
```
int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a + s
in(c + d*x)*c))/(b**3*d)
```

3.136 $\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 65

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
C*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/b^2/d/cos(d*x+c
)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(Cdx\cos(c+dx)+A\sin(c+dx))}{d(b\cos(c+dx))^{5/2}}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/
2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d
*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2031, 3042, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^2(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{\cos(c+dx)}\left(C\int 1dx+\frac{A\tan(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+Cx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(C*x + (A*Tan[c + d*x])/d))/(b^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{C(dx+c)\cos(dx+c)+A\sin(dx+c)}{b^{2}d\sqrt{\cos(dx+c)}\,\sqrt{b\cos(dx+c)}}$	48
parts	$\frac{A\sin(dx+c)}{b^{2}d\sqrt{\cos(dx+c)}\,\sqrt{b\cos(dx+c)}}+\frac{C(dx+c)\sqrt{\cos(dx+c)}}{d\,b^{2}\sqrt{b\cos(dx+c)}}$	65
risch	$\frac{Cx\sqrt{\cos(dx+c)}}{b^{2}\sqrt{b\cos(dx+c)}}+\frac{2i\sqrt{\cos(dx+c)}\,A}{b^{2}\sqrt{b\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}$	67

input
```
int(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/b^2/d*(C*(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c)
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.94

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \left[-\frac{C\sqrt{-b}\cos{(dx+c)}^2\log\left(2\,b\cos{(dx+c)}^2+2\,\sqrt{b\cos{(dx+}}\right.}{}\right.$$

input

```
integrate(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output

```
[-1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x +
c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^2), (C*sqrt(b)*
arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos
(d*x + c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3
*d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)**(1/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output

```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(\frac{A\sqrt{b}\sin(2\,dx+2\,c)}{b^3\cos(2\,dx+2\,c)^2+b^3\sin(2\,dx+2\,c)^2+2\,b^3\cos(2\,dx+2\,c)+b^3} + \frac{C\arctan\left(\frac{\sin(d}{\cos(dx}\right.}{b^{\frac{5}{2}}}\right.}{d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
2*(A*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*
c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + C*arctan(sin(d*x + c)/(cos(d*x + c)
 + 1))/b^(5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 41.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\sqrt{b\,\cos(c+dx)}\,(A\,\sin(c+dx)+A\,\sin(3\,c}{b^3\,d}$$

input
```
int((cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

output
```
(2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(A*cos(c + d*x)*3i + A*sin(c
+ d*x) + A*cos(3*c + 3*d*x)*1i + A*sin(3*c + 3*d*x) + C*d*x*cos(3*c + 3*d*
x) + 3*C*d*x*cos(c + d*x)))/(b^3*d*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x)
+ 3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,(\cos(dx+c)\,cdx+\sin(dx+c)\,a)}{\cos(dx+c)\,b^3d}$$

input
```
int(cos(d*x+c)^(1/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(cos(c + d*x)*c*d*x + sin(c + d*x)*a))/(cos(c + d*x)*b**3*d)
```

3.137 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 84

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx=\frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/2*(A+2*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2
)+1/2*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{\cos(c+dx)}((A+2C)\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+A\sin}{2d(b\cos(c+dx))^{5/2}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2
)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Si
n[c + d*x]))/(2*d*(b*Cos[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 3042, 3491, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^3(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}(A+2C)\int \sec(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}(A+2C)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{(A+2C)\text{arctanh}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(((A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*Sec[c +
d*x]*Tan[c + d*x])/(2*d)))/(b^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.42

method	result
default	$-\frac{A\ln\left(-\cot(dx+c)+\csc(dx+c)-1\right)\cos(dx+c)^{2}-A\ln\left(-\cot(dx+c)+\csc(dx+c)+1\right)\cos(dx+c)^{2}+4C\,\operatorname{arctanh}(-\csc(dx+c)+\cot(d\ldots)}{2b^{2}d\cos(dx+c)^{\frac{3}{2}}\sqrt{b\cos(dx+c)}}$
parts	$\frac{A\left(\ln\left(-\cot(dx+c)+\csc(dx+c)+1\right)\cos(dx+c)^{2}-\ln\left(-\cot(dx+c)+\csc(dx+c)-1\right)\cos(dx+c)^{2}+\sin(dx+c)\right)}{2d\cos(dx+c)^{\frac{3}{2}}b^{2}\sqrt{b\cos(dx+c)}}-\frac{2C\,\operatorname{arctanh}(-\csc(d\ldots)}{d\,b^{2}\ldots}$
risch	$-\frac{iA\left(e^{2i(dx+c)}-1\right)}{2b^{2}\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)d}-\frac{\sqrt{\cos(dx+c)}\,(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2b^{2}\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,(A+2C)\ln\left(e^{i(dx+c)}+\ldots\right)}{2b^{2}\sqrt{b\cos(dx+c)}\,d}$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2/b^2/d*(A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^2-A*ln(-cot(d*x+c)+c
sc(d*x+c)+1)*cos(d*x+c)^2+4*C*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)^2
-A*sin(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.61

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{(A+2\,C)\sqrt{b}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}\right.}{4\,b^3d}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+\right.}{2\,b^3d\cos\left(dx+c\right)^3}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
[1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*c
os(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/c
os(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))
/(b^3*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x +
c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*
cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. 2(72) = 144.

Time = 0.36 (sec) , antiderivative size = 754, normalized size of antiderivative = 8.98

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b\cos(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
-1/4*((4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c
) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(
4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b^2*cos(4*d*x + 4*c)^2 +
 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c
)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b
^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b)) - 2*C*(lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x +...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\sqrt{\cos\left(c+d\,x\right)}\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.17

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2 a-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.\right.}{}$$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a - 2*log(tan((c +
d*x)/2) - 1)*sin(c + d*x)**2*c + log(tan((c + d*x)/2) - 1)*a + 2*log(tan((
c + d*x)/2) - 1)*c + log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a + 2*log(t
an((c + d*x)/2) + 1)*sin(c + d*x)**2*c - log(tan((c + d*x)/2) + 1)*a - 2*l
og(tan((c + d*x)/2) + 1)*c - sin(c + d*x)*a))/(2*b**3*d*(sin(c + d*x)**2 -
 1))
```

3.138 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 85

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
1/3*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)
*sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d(b\cos(c+dx))^{5/2}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2
)),x]
```

output
```
(Cos[c + d*x]^(3/2)*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*(b*C
os[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2032, 3042, 3491, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^4(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}(2A+3C)\int \sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}(2A+3C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(((2*A + 3*C)*Tan[c + d*x])/(3*d) + (A*Sec[c + d*x]^2*
Tan[c + d*x])/(3*d)))/(b^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\sin(dx+c)\left(2A\cos(dx+c)^{2}+3C\cos(dx+c)^{2}+A\right)}{3b^{2}d\cos(dx+c)^{\frac{5}{2}}\sqrt{b\cos(dx+c)}}$	57
parts	$\frac{A\sin(dx+c)\left(2\cos(dx+c)^{2}+1\right)}{3d\cos(dx+c)^{\frac{5}{2}}b^{2}\sqrt{b\cos(dx+c)}}+\frac{C\sin(dx+c)}{d\sqrt{\cos(dx+c)}\,b^{2}\sqrt{b\cos(dx+c)}}$	79
risch	$\frac{i\left(3C\,e^{3i(dx+c)}+(9C+8A)\cos(dx+c)+i(4A+3C)\sin(dx+c)\right)}{3b^{2}\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)^{2}d}$	84

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/3/b^2/d*sin(d*x+c)*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)/cos(d*x+c)^(5/2
)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.59

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx=\frac{\left((2\,A+3\,C)\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,b^{3}d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^
3*d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(73) = 146$.

Time = 0.36 (sec) , antiderivative size = 412, normalized size of antiderivative = 4.85

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(\frac{3\,C\sqrt{b}\sin(2\,dx+2\,c)}{b^3\cos(2\,dx+2\,c)^2+b^3\sin(2\,dx+2\,c)^2+2\,b^3\cos(2\,dx+2\,c)+b^3}+\frac{}{\left(b^2\cos(6\,dx+6\,c)^2+9\,b\right.}\right.}{}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
2/3*(3*C*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x
+ 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + 2*((3*cos(2*d*x + 2*c) + 1)*sin
(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x
+ 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b^2*cos
(6*d*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^
2*sin(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2 + 6*b^2*cos(2*d*x + 2*c) + b^2
 + 2*(3*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6
*c) + 6*(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 6*(b^2*sin(4*d*x
 + 4*c) + b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 42.19 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.59

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)+2\,\right.}{}$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2)),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4*
c + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*
c + 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(
4*c + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c
 + 4*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*b^3*d*cos(c + d*x)^(1/2)*(15*cos(2*c
 + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,\sin(dx+c)\left(2\sin(dx+c)^2a+3\sin(dx+c)^2c-3a-3c\right)}{3\cos(dx+c)\,b^3d\left(\sin(dx+c)^2-1\right)}$$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*sin(c + d*x)*(2*sin(c + d*x)**2*a + 3*sin(c + d*x)**2*c - 3*a - 3
*c))/(3*cos(c + d*x)*b**3*d*(sin(c + d*x)**2 - 1))
```

3.139 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 131

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/8*(3*A+4*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1
/2)+1/4*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+
4*C)*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx) + (2A+(3A+4C)\,\text{c}\cdots}{8d\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2
)),x]
```

output
```
((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos
[c + d*x]^2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2032, 3042, 3491, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+A\right)\sec^5(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3491}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\int \sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}(3A+4C)\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A + 4*C)*
(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/(b^
2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.35

method	result
default	$\frac{-3A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-4C\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+3A\ln(-\cot(dx+c)+\csc(dx+c)}{8b^2d\cos(dx+c)^{\frac{7}{2}}\sqrt{}}$
parts	$-\frac{A\left(3\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-3\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^4-3\cos(dx+c)^2\sin(dx+c)-2\sin(\right.}{8d\cos(dx+c)^{\frac{7}{2}}b^2\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i(3A\,e^{6i(dx+c)}+4C\,e^{6i(dx+c)}+11A\,e^{4i(dx+c)}+4C\,e^{4i(dx+c)}-11A\,e^{2i(dx+c)}-4C\,e^{2i(dx+c)}-3A-4C)}{8b^2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^3d}+\frac{\sqrt{\cos(dx+c)}\,(3A+4C)}{8b^2\sqrt{b\cos(da}}$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/8/b^2/d*(-3*A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^4-4*C*ln(-cot(d*x+
c)+csc(d*x+c)-1)*cos(d*x+c)^4+3*A*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^
4+4*C*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+(3*cos(d*x+c)^2+2)*sin(d*x
+c)*A+4*C*cos(d*x+c)^2*sin(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.99

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{(3\,A+4\,C)\sqrt{b}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(d}}{\cos(dx+c)^3}\right.}{}\right.$$

$$-\frac{(3\,A+4\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left((3\,A+4\,C)\cos\left(dx+c\right)^2+2\,A\right)\sqrt{b\cos}}{8\,b^3d\cos\left(dx+c\right)^5}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
[1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(
b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^5), -1/8*((3*A +
4*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(co
s(d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2418 vs. $2(113) = 226$.

Time = 0.29 (sec) , antiderivative size = 2418, normalized size of antiderivative = 18.46

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
-1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4
*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d
*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(
8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c
))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x +
8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d
*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c)
+ 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x +
8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 1
6*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*
d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^{5/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(5/2)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.42

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{b}\left(-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^4a-4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

input
```
int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*( - 3*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 4*log(tan((c
+ d*x)/2) - 1)*sin(c + d*x)**4*c + 6*log(tan((c + d*x)/2) - 1)*sin(c + d*x
)**2*a + 8*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 3*log(tan((c + d*
x)/2) - 1)*a - 4*log(tan((c + d*x)/2) - 1)*c + 3*log(tan((c + d*x)/2) + 1)
*sin(c + d*x)**4*a + 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c - 6*log
(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - 8*log(tan((c + d*x)/2) + 1)*sin
(c + d*x)**2*c + 3*log(tan((c + d*x)/2) + 1)*a + 4*log(tan((c + d*x)/2) +
1)*c - 3*sin(c + d*x)**3*a - 4*sin(c + d*x)**3*c + 5*sin(c + d*x)*a + 4*si
n(c + d*x)*c))/(8*b**3*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.140 $\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^3d}-\frac{3(13A+10C)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{130b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
3/13*C*(b*cos(d*x+c))^(10/3)*sin(d*x+c)/b^3/d-3/130*(13*A+10*C)*(b*cos(d*x
+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin
(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=-\frac{3\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(8A\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)+5C\cos^4(c}{80d}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(8*A*Cos[c + d*x]^2*Hypergeometric
2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^4*Hypergeometric2F1[
1/2, 8/3, 11/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(80*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{7/3}\left(C\cos^2(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{13}(13A+10C)\int (b\cos(c+dx))^{7/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{13}(13A+10C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13bd}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13bd}-\frac{3(13A+10C)\sin(c+dx)(b\cos(c+dx))^{10/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{130bd\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b*d) - (3*(13*A + 10*C)*(b
*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin
[c + d*x])/(130*b*d*Sqrt[Sin[c + d*x]^2]))/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos (dx+c)^2 (b\cos (dx+c))^{\frac{1}{3}} \left(A+C\cos (dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

input
```
int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=b^{\frac{1}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{13}{3}}dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{7}{3}}dx\right)a\right)$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*(int(cos(c + d*x)**(1/3)*cos(c + d*x)**4,x)*c + int(cos(c + d*x)*
*(1/3)*cos(c + d*x)**2,x)*a)
```

3.141 $\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^2d}-\frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{70b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3/10*C*(b*cos(d*x+c))^(7/3)*sin(d*x+c)/b^2/d-3/70*(10*A+7*C)*(b*cos(d*x+c)
)^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*
x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=$$
$$-\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\left(13A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)+7C\cos^2(c+dx)\,\text{Hy}\right.}{91bd}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(13*A*Hypergeometric2F1[1/2, 7/6,
13/6, Cos[c + d*x]^2] + 7*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 13/6, 19
/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(91*b*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{4/3}\left(C\cos^2(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{10}(10A+7C)\int (b\cos(c+dx))^{4/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{10}(10A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}-\frac{3(10A+7C)\sin(c+dx)(b\cos(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{70bd\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b*d) - (3*(10*A + 7*C)*(b*C
os[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(70*b*d*Sqrt[Sin[c + d*x]^2]))/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos(dx+c)\,(b\cos(dx+c))^{\frac{1}{3}}\left(A+C\cos(dx+c)^2\right)dx$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm=
"fricas")
```

output
```
integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

input
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=b^{\frac{1}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{4}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{10}{3}}dx\right)c\right)$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*(int(cos(c + d*x)**(1/3)*cos(c + d*x),x)*a + int(cos(c + d*x)**(1
/3)*cos(c + d*x)**3,x)*c)
```

3.142 $\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx = \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{28bd\sqrt{\sin^2(c+dx)}}$$

output
```
3/7*C*(b*cos(d*x+c))^(4/3)*sin(d*x+c)/b/d-3/28*(7*A+4*C)*(b*cos(d*x+c))^(4
/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\,dx = -\frac{3\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(5A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,\text{Hyper}\right.}{20d}$$

input
```
Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/2, 2/3, 5
/3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3, 8/3,
Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(20*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{7}(7A+4C)\int \sqrt[3]{b\cos(c+dx)}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A+4C)\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}-\frac{3(7A+4C)\sin(c+dx)(b\cos(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{28bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) - (3*(7*A + 4*C)*(b*Cos[
c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d
*x])/(28*b*d*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{1}{3}} \left(A+C\cos(dx+c)^2\right) dx$$

input
```
int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=b^{\frac{1}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{1}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{7}{3}}dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*(int(cos(c + d*x)**(1/3),x)*a + int(cos(c + d*x)**(1/3)*cos(c + d
*x)**2,x)*c)
```

3.143 $\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 87

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{3C\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d} - \frac{3(4A+C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*C*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d-3/4*(4*A+C)*(b*cos(d*x+c))^(1/3)*h
ypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = -\frac{3b\cot(c+dx)\left(7A\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\right.\right.}{7d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
(-3*b*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] +
 C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2])*Sqrt[
Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3493}$$

$$b\left(\frac{1}{4}(4A+C)\int \frac{1}{(b\cos(c+dx))^{2/3}}dx+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{1}{4}(4A+C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}\right)$$

$$\Big\downarrow \text{3122}$$

$$b\left(\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd} - \frac{3(4A+C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+\right.}{4bd\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
b*((3*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) - (3*(4*A + C)*(b*Cos
[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos (dx+c))^{\frac{1}{3}} \left(A+C\cos (dx+c)^{2}\right) \sec (dx+c)\, dx$$

input
```
int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos (dx+c)^{2}+A\right)(b\cos (dx+c))^{\frac{1}{3}} \sec (dx+c)\, dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x), x)
```

Reduce [F]

$$\int \sqrt[3]{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec(c+dx)\,dx$$
$$=b^{\frac{1}{3}}\left(\left(\int \cos(dx+c)^{\frac{7}{3}}\sec(dx+c)\,dx\right)c+\left(\int \cos(dx+c)^{\frac{1}{3}}\sec(dx+c)\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
b**(1/3)*(int(cos(c + d*x)**(1/3)*cos(c + d*x)**2*sec(c + d*x),x)*c + int(
cos(c + d*x)**(1/3)*sec(c + d*x),x)*a)
```

3.144 $\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 91

$$\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx = \frac{3Ab\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}}$$

output

```
3/2*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)+3/8*(A-2*C)*(b*cos(d*x+c))^(4/3)
*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$
$$-\frac{3b\csc(c+dx)\left(-2A\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2I}\right.}{4d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
(-3*b*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]
 + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt
[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \; 2030$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/3}}dx$$

$$\Big\downarrow \; 3491$$

$$b^2\left(\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}-\frac{(A-2C)\int\sqrt[3]{b\cos(c+dx)}dx}{2b^2}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}-\frac{(A-2C)\int\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b^2}\right)$$

$$\Big\downarrow \; 3122$$

$$b^2\left(\frac{3(A-2C)\sin(c+dx)(b\cos(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}+\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
b^2*((3*A*Sin[c + d*x])/(2*b*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*C
os[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c
+ d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{1}{3}} \left(A+C\cos(dx+c)^{2}\right)\sec(dx+c)^{2} dx$$

input
```
int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos(dx+c)^{2}+A\right)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^{2}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\cos\left(c+d\,x\right)\right)^{1/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x)^2,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=b^{\frac{1}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{7}{3}}\sec\left(dx+c\right)^2dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{1}{3}}\sec\left(dx+c\right)^2dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
b**(1/3)*(int(cos(c + d*x)**(1/3)*cos(c + d*x)**2*sec(c + d*x)**2,x)*c + i
nt(cos(c + d*x)**(1/3)*sec(c + d*x)**2,x)*a)
```

3.145 $\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 92

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{3Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} - \frac{3(2A+5C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d\sqrt{\sin^2(c+dx)}}$$

output

```
3/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)-3/5*(2*A+5*C)*(b*cos(d*x+c))^(
1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$
$$-\frac{3\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,\text{Hyp}\right.}{5d}$$

input
```
Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
(-3*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(-(A*Hypergeometric2F1[-5/6, 1/2,
1/6, Cos[c + d*x]^2]) + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/6, 1/2, 7/6
, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(5*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{8/3}}dx$$

$$\Big\downarrow \text{ 3491}$$

$$b^3\left(\frac{(2A+5C)\int \frac{1}{(b\cos(c+dx))^{2/3}}dx}{5b^2}+\frac{3A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/3}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{(2A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{5b^2}+\frac{3A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/3}}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^3\left(\frac{3A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/3}}-\frac{3(2A+5C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
b^3*((3*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/3)) - (3*(2*A + 5*C)*(b
*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[
c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{1}{3}} \left(A+C\cos(dx+c)^2\right) \sec(dx+c)^3 \, dx$$

input
```
int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x)^3,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=b^{\frac{1}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{7}{3}}\sec\left(dx+c\right)^3dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{1}{3}}\sec\left(dx+c\right)^3dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
b**(1/3)*(int(cos(c + d*x)**(1/3)*cos(c + d*x)**2*sec(c + d*x)**3,x)*c + i
nt(cos(c + d*x)**(1/3)*sec(c + d*x)**3,x)*a)
```

3.146 $\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{11/3}\sin(c+dx)}{14b^3d}-\frac{3(14A+11C)(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{154b^3d\sqrt{\sin^2(c+dx)}}$$

output

```
3/14*C*(b*cos(d*x+c))^(11/3)*sin(d*x+c)/b^3/d-3/154*(14*A+11*C)*(b*cos(d*x
+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(s
in(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=-\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\left(17A\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)+11C\cos\right.}{187d}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*(17*A*Cos[c + d*x]^2*Hypergeometri
c2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2] + 11*C*Cos[c + d*x]^4*Hypergeometric
2F1[1/2, 17/6, 23/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(187*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{8/3}\left(C\cos^2(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{8/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{14}(14A+11C)\int (b\cos(c+dx))^{8/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{11/3}}{14bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{14}(14A+11C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{8/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{11/3}}{14bd}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{3C\sin(c+dx)(b\cos(c+dx))^{11/3}}{14bd}-\frac{3(14A+11C)\sin(c+dx)(b\cos(c+dx))^{11/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)}{154bd\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(11/3)*Sin[c + d*x])/(14*b*d) - (3*(14*A + 11*C)*(b
*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*S
in[c + d*x])/(154*b*d*Sqrt[Sin[c + d*x]^2]))/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos(dx+c)^2 (b\cos(dx+c))^{\frac{2}{3}} \left(A+C\cos(dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,dx$$

input
```
int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = b^{\frac{2}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{14}{3}}dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{8}{3}}dx\right)a\right)$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**(2/3)*cos(c + d*x)**4,x)*c + int(cos(c + d*x)*
*(2/3)*cos(c + d*x)**2,x)*a)
```

3.147 $\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d}-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^2/d-3/88*(11*A+8*C)*(b*cos(d*x+c)
)^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x
+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=$$

$$-\frac{3(b\cos(c+dx))^{5/3}\cot(c+dx)\left(7A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)+4C\cos^2(c+dx)\,\text{Hyperg}\right.}{56bd}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*(b*Cos[c + d*x])^(5/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 4/3, 7
/3, Cos[c + d*x]^2] + 4*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/3, 10/3,
 Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(56*b*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/3}\left(C\cos^2(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{11}(11A+8C)\int (b\cos(c+dx))^{5/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{11}(11A+8C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}-\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{88bd\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b*d) - (3*(11*A + 8*C)*(b*C
os[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c
+ d*x])/(88*b*d*Sqrt[Sin[c + d*x]^2]))/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos (dx+c) (b\cos (dx+c))^{\frac{2}{3}} \left(A+C\cos (dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm=
"fricas")
```

output
```
integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,dx$$

input
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=b^{\frac{2}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{11}{3}}dx\right)c\right)$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**(2/3)*cos(c + d*x),x)*a + int(cos(c + d*x)**(2
/3)*cos(c + d*x)**3,x)*c)
```

3.148 $\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx = \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40bd\sqrt{\sin^2(c+dx)}}$$

output
```
3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(5
/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx = -\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\left(11A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,\text{Hype}\right.}{55d}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*(11*A*Hypergeometric2F1[1/2, 5/6,
11/6, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17
/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(55*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{8}(8A+5C)\int (b\cos(c+dx))^{2/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}(8A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{40bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*(8*A + 5*C)*(b*Cos[
c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(40*b*d*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{2}{3}} \left(A + C\cos(dx+c)^2\right) dx$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=b^{\frac{2}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{8}{3}}dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**(2/3),x)*a + int(cos(c + d*x)**(2/3)*cos(c + d
*x)**2,x)*c)
```

3.149 $\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 89

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10d\sqrt{\sin^2(c+dx)}}$$

output
```
3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2/3
)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = -\frac{3b\cot(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{8d\sqrt[3]{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
(-3*b*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] +
 C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2])*Sqrt[S
in[c + d*x]^2])/(8*d*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\sqrt[3]{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{3493}$$

$$b\left(\frac{1}{5}(5A+2C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{1}{5}(5A+2C)\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}\right)$$

$$\Big\downarrow \text{3122}$$

$$b\left(\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos\right.}{10bd\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
b*((3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*(5*A + 2*C)*(b*C
os[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c
+ d*x])/(10*b*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{2}{3}} \left(A+C\cos(dx+c)^2\right)\sec(dx+c)\,dx$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b^{\frac{2}{3}}\left(\left(\left(\int \cos(dx+c)^{\frac{8}{3}}\sec(dx+c)\,dx\right)c+\left(\int \cos(dx+c)^{\frac{2}{3}}\sec(dx+c)\,dx\right)a\right)\right)$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**(2/3)*cos(c + d*x)**2*sec(c + d*x),x)*c + int(
cos(c + d*x)**(2/3)*sec(c + d*x),x)*a)
```

3.150 $\int (b\cos(c+dx))^{2/3} (A+C\cos^2(c+dx)) \sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 91

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \sec^2(c+dx)\,dx = \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

output
```
3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)+3/5*(2*A-C)*(b*cos(d*x+c))^(5/3)*h
ypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \sec^2(c+dx)\,dx = -\frac{3b\csc(c+dx)\left(-5A\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\right.}{5d\sqrt[3]{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
(-3*b*Csc[c + d*x]*(-5*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]
 + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2])*Sqr
t[Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = $\{3042, 2030, 3491, 3042, 3122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^2\left(\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}-\frac{(2A-C)\int(b\cos(c+dx))^{2/3}dx}{b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}-\frac{(2A-C)\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b^2}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
b^2*((3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos
[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
 d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{2}{3}}\left(A+C\cos(dx+c)^2\right)\sec(dx+c)^2\,dx$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\cos\left(c+d\,x\right)\right)^{2/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x)^2,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^{\frac{2}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{8}{3}}\sec\left(dx+c\right)^2 dx\right)c + \left(\int \cos\left(dx+c\right)^{\frac{2}{3}}\sec\left(dx+c\right)^2 dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**(2/3)*cos(c + d*x)**2*sec(c + d*x)**2,x)*c + i
nt(cos(c + d*x)**(2/3)*sec(c + d*x)**2,x)*a)
```

3.151 $\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 90

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}-\frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)-3/8*(A+4*C)*(b*cos(d*x+c))^(2/
3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=$$
$$-\frac{3(b\cos(c+dx))^{2/3}\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,\text{Hyp}\right.}{4d}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2,
1/3, Cos[c + d*x]^2]) + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/3, 1/2, 4/3
, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/3}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^3\left(\frac{(A+4C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx}{4b^2}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{(A+4C)\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b^2}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$\Big\downarrow$ 3122

$$b^3\left(\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}-\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
b^3*((3*A*Sin[c + d*x])/(4*b*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*C
os[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c
+ d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int (b\cos (dx+c))^{\frac{2}{3}} \left(A+C\cos (dx+c)^2\right) \sec (dx+c)^3\,dx$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx = \int \left(C\cos (dx+c)^2+A\right)(b\cos (dx+c))^{\frac{2}{3}} \sec (dx+c)^3\;dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x)^3,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^{\frac{2}{3}}\left(\left(\int \cos(dx+c)^{\frac{8}{3}}\sec(dx+c)^3\,dx\right)c + \left(\int \cos(dx+c)^{\frac{2}{3}}\sec(dx+c)^3\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**(2/3)*cos(c + d*x)**2*sec(c + d*x)**3,x)*c + i
nt(cos(c + d*x)**(2/3)*sec(c + d*x)**3,x)*a)
```

3.152 $\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{13/3}\sin(c+dx)}{16b^3d}-\frac{3(16A+13C)(b\cos(c+dx))^{13/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sin(c+dx)}{208b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
3/16*C*(b*cos(d*x+c))^(13/3)*sin(d*x+c)/b^3/d-3/208*(16*A+13*C)*(b*cos(d*x
+c))^(13/3)*hypergeom([1/2, 13/6],[19/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(s
in(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=-\frac{3\cos^2(c+dx)(b\cos(c+dx))^{4/3}\cot(c+dx)\left(19A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)+13C\cos\right.}{247d}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(19*A*Hypergeometri
c2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2] + 13*C*Cos[c + d*x]^2*Hypergeometric
2F1[1/2, 19/6, 25/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(247*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{10/3}\left(C\cos^2(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{10/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{16}(16A+13C)\int (b\cos(c+dx))^{10/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{13/3}}{16bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{16}(16A+13C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{10/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{13/3}}{16bd}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{3C\sin(c+dx)(b\cos(c+dx))^{13/3}}{16bd}-\frac{3(16A+13C)\sin(c+dx)(b\cos(c+dx))^{13/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)}{208bd\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(13/3)*Sin[c + d*x])/(16*b*d) - (3*(16*A + 13*C)*(b
*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*S
in[c + d*x])/(208*b*d*Sqrt[Sin[c + d*x]^2]))/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos(dx+c)^2 (b\cos(dx+c))^{\frac{4}{3}} \left(A+C\cos(dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*b*cos(d*x + c)^5 + A*b*cos(d*x + c)^3)*(b*cos(d*x + c))^(1/3),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)^2\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,dx$$

input
```
int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,(A+C\cos^2(c+dx))\ dx = b^{\frac{4}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{16}{3}}dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{10}{3}}dx\right)a\right)$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**(1/3)*cos(c + d*x)**5,x)*c + int(cos(c + d*x
)**(1/3)*cos(c + d*x)**3,x)*a)
```

3.153 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d}-\frac{3(13A+10C)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{130b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3/13*C*(b*cos(d*x+c))^(10/3)*sin(d*x+c)/b^2/d-3/130*(13*A+10*C)*(b*cos(d*x
+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin
(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=$$

$$-\frac{3(b\cos(c+dx))^{7/3}\cot(c+dx)\left(8A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,\text{Hyperg}\right.}{80bd}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*(b*Cos[c + d*x])^(7/3)*Cot[c + d*x]*(8*A*Hypergeometric2F1[1/2, 5/3, 8
/3, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 8/3, 11/3,
 Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(80*b*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{7/3}\left(C\cos^2(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{13}(13A+10C)\int (b\cos(c+dx))^{7/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{13}(13A+10C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13bd}}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13bd}-\frac{3(13A+10C)\sin(c+dx)(b\cos(c+dx))^{10/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{130bd\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b*d) - (3*(13*A + 10*C)*(b
*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin
[c + d*x])/(130*b*d*Sqrt[Sin[c + d*x]^2]))/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos (dx+c) (b\cos (dx+c))^{\frac{4}{3}} \left(A+C\cos (dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm=
"fricas")
```

output
```
integral((C*b*cos(d*x + c)^4 + A*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,dx$$

input
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = b^{\frac{4}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{13}{3}}dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{7}{3}}dx\right)a\right)$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**(1/3)*cos(c + d*x)**4,x)*c + int(cos(c + d*x
)**(1/3)*cos(c + d*x)**2,x)*a)
```

3.154 $\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right) dx = \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd} - \frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{70bd\sqrt{\sin^2(c+dx)}}$$

output
```
3/10*C*(b*cos(d*x+c))^(7/3)*sin(d*x+c)/b/d-3/70*(10*A+7*C)*(b*cos(d*x+c))^
(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)
^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right) dx = -\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\left(13A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)+7C\cos^2(c+dx)\,\text{Hype}\right.}{91d}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(13*A*Hypergeometric2F1[1/2, 7/6,
13/6, Cos[c + d*x]^2] + 7*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 13/6, 19
/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(91*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$
$$\Big\downarrow \text{3493}$$
$$\frac{1}{10}(10A+7C)\int (b\cos(c+dx))^{4/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{10}(10A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}$$
$$\Big\downarrow \text{3122}$$
$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}-\frac{3(10A+7C)\sin(c+dx)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{70bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b*d) - (3*(10*A + 7*C)*(b*Co
s[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(70*b*d*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{4}{3}} \left(A+C\cos(dx+c)^2\right) dx$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right) dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}} dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x
)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right) dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right) dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}} dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=b^{\frac{4}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{4}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{10}{3}}dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**(1/3)*cos(c + d*x),x)*a + int(cos(c + d*x)**
(1/3)*cos(c + d*x)**3,x)*c)
```

3.155 $\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 89

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d}-\frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{28d\sqrt{\sin^2(c+dx)}}$$

output

```
3/7*C*(b*cos(d*x+c))^(4/3)*sin(d*x+c)/d-3/28*(7*A+4*C)*(b*cos(d*x+c))^(4/3
)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=$$

$$-\frac{3b\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(5A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,\text{Hyperge}\right.}{20d}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
(-3*b*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/2, 2/3,
 5/3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3, 8/3
, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(20*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b\int \sqrt[3]{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A\right)dx$$

$$\Big\downarrow \text{ 3493}$$

$$b\left(\frac{1}{7}(7A+4C)\int \sqrt[3]{b\cos(c+dx)}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{1}{7}(7A+4C)\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}\right)$$

$$\Big\downarrow \text{ 3122}$$

$$b\left(\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}-\frac{3(7A+4C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos\right.}{28bd\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
b*((3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) - (3*(7*A + 4*C)*(b*C
os[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c
+ d*x])/(28*b*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{4}{3}}\left(A+C\cos(dx+c)^2\right)\sec(dx+c)\,dx$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"fricas")
```

output
```
integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*se
c(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3} \left(A + C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3} \left(A + C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A + C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2 + A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b^{\frac{4}{3}}\left(\left(\int \cos(dx+c)^{\frac{4}{3}}\sec(dx+c)\,dx\right)a+\left(\int \cos(dx+c)^{\frac{10}{3}}\sec(dx+c)\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**(1/3)*cos(c + d*x)*sec(c + d*x),x)*a + int(c
os(c + d*x)**(1/3)*cos(c + d*x)**3*sec(c + d*x),x)*c)
```

3.156 $\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 89

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{3bC\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d} - \frac{3b(4A+C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*b*C*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d-3/4*b*(4*A+C)*(b*cos(d*x+c))^(1/
3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$
$$-\frac{3b^2\cot(c+dx)\left(7A\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{7d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
(-3*b^2*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]
 + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2])*Sqr
t[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 2030, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3493}$$

$$b^2\left(\frac{1}{4}(4A+C)\int \frac{1}{(b\cos(c+dx))^{2/3}}dx+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{1}{4}(4A+C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}-\frac{3(4A+C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+\right.}{4bd\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
b^2*((3*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) - (3*(4*A + C)*(b*C
os[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos (dx+c))^{\frac{4}{3}} \left(A+C\cos (dx+c)^2\right) \sec (dx+c)^2 \, dx$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right) \sec^2(c+dx)\, dx = \int \left(C\cos (dx+c)^2+A\right)(b\cos (dx+c))^{\frac{4}{3}} \sec (dx+c)^2 \, dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="fricas")
```

output
```
integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*se
c(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right) \sec^2(c+dx)\, dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x)^2,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^{\frac{4}{3}}\left(\left(\int \cos(dx+c)^{\frac{4}{3}}\sec(dx+c)^2\,dx\right)a+\left(\int \cos(dx+c)^{\frac{10}{3}}\sec(dx+c)^2\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**(1/3)*cos(c + d*x)*sec(c + d*x)**2,x)*a + in
t(cos(c + d*x)**(1/3)*cos(c + d*x)**3*sec(c + d*x)**2,x)*c)
```

3.157 $\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 90

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}$$

output

```
3/2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)+3/8*(A-2*C)*(b*cos(d*x+c))^(4/
3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$

$$-\frac{3b^2\csc(c+dx)\left(-2A\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\right.}{4d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
(-3*b^2*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^
2] + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sq
rt[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^3\left(\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}-\frac{(A-2C)\int \sqrt[3]{b\cos(c+dx)}dx}{2b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}-\frac{(A-2C)\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b^2}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^3\left(\frac{3(A-2C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}+\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
b^3*((3*A*Sin[c + d*x])/(2*b*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*C
os[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c
+ d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int (b\cos (dx+c))^{\frac{4}{3}} \left(A+C\cos (dx+c)^2\right) \sec (dx+c)^3\, dx$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\, dx = \int \left(C\cos (dx+c)^2+A\right)(b\cos (dx+c))^{\frac{4}{3}} \sec (dx+c)^3\, dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="fricas")
```

output
```
integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*se
c(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\, dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x)^3,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^{\frac{4}{3}}\left(\left(\int \cos(dx+c)^{\frac{4}{3}}\sec(dx+c)^3\,dx\right)a + \left(\int \cos(dx+c)^{\frac{10}{3}}\sec(dx+c)^3\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**(1/3)*cos(c + d*x)*sec(c + d*x)**3,x)*a + in
t(cos(c + d*x)**(1/3)*cos(c + d*x)**3*sec(c + d*x)**3,x)*c)
```

3.158 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d} - \frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^3/d-3/88*(11*A+8*C)*(b*cos(d*x+c)
)^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x
+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\cot(c+dx)\left(7A\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)+4C\cos^4(c+dx)\,\text{Hyperge}\right.}{56d\sqrt[3]{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x
]
```

output
```
(-3*Cot[c + d*x]*(7*A*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[
c + d*x]^2] + 4*C*Cos[c + d*x]^4*Hypergeometric2F1[1/2, 7/3, 10/3, Cos[c +
 d*x]^2])*Sqrt[Sin[c + d*x]^2])/(56*d*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/3}\left(C\cos^2(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{11}(11A+8C)\int (b\cos(c+dx))^{5/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{11}(11A+8C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd} - \frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{88bd\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b*d) - (3*(11*A + 8*C)*(b*C
os[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c
+ d*x])/(88*b*d*Sqrt[Sin[c + d*x]^2]))/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos(dx+c)^2 \left(A+C\cos(dx+c)^2\right)}{\left(b\cos(dx+c)\right)^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\cos(dx+c)^2}{\left(b\cos(dx+c)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(2/3)/b, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3),x)
```

output
```
int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)\,(A+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= \frac{\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{11}{3}}dx\right)c}{b^{\frac{1}{3}}}$$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(cos(c + d*x)**4/cos(c + d*x)**(1/3),x)*c + int(cos(c + d*x)**2/cos(c
+ d*x)**(1/3),x)*a)/b**(1/3)
```

3.159 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d} - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b^2/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^
(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+
c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\left(11A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,\text{Hy}\right.}{55bd}$$

input
```
Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*(11*A*Hypergeometric2F1[1/2, 5/6,
11/6, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17
/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(55*b*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{2/3}\left(C\cos^2(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{8}(8A+5C)\int (b\cos(c+dx))^{2/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{8}(8A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{40bd\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*(8*A + 5*C)*(b*Cos
[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
 d*x])/(40*b*d*Sqrt[Sin[c + d*x]^2]))/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos (dx+c) \left(A+C\cos (dx+c)^2\right)}{(b\cos (dx+c))^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm=
"fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/b, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3),x)
```

output
```
int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\frac{\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{8}{3}}dx\right)c}{b^{\frac{1}{3}}}$$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(cos(c + d*x)/cos(c + d*x)**(1/3),x)*a + int(cos(c + d*x)**3/cos(c + d
*x)**(1/3),x)*c)/b**(1/3)
```

3.160 $\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10bd\sqrt{\sin^2(c+dx)}}$$

output

```
3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2
/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\cot(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{8d\sqrt[3]{b\cos(c+dx)}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + C
*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(8*d*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{5}(5A+2C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(5A+2C)\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{10bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*(5*A + 2*C)*(b*Cos[
c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d
*x])/(10*b*d*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [F]

$$\int \frac{A + C\cos\left(dx + c\right)^{2}}{\left(b\cos\left(dx + c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(dx+c\right)^2 + A}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/(b*cos(d*x + c)), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(dx+c\right)^2 + A}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{C\cos(dx+c)^2+A}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{C\cos(c+d\,x)^2+A}{(b\,\cos(c+d\,x))^{1/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\frac{\left(\int \cos(dx+c)^{\frac{5}{3}}\,dx\right)c+\left(\int \frac{1}{\cos(dx+c)^{\frac{1}{3}}}dx\right)a}{b^{\frac{1}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(cos(c + d*x)**2/cos(c + d*x)**(1/3),x)*c + int(1/cos(c + d*x)**(1/3),
x)*a)/b**(1/3)
```

3.161 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 90

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)+3/5*(2*A-C)*(b*cos(d*x+c))^(5/3)*hyp
ergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\left(-5A\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)+C\cos(c+dx)\cot(c+dx)\,\text{Hyperge}\right.}{5d\sqrt[3]{b\cos(c+dx)}}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*(-5*A*Csc[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] +
 C*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x
]^2])*Sqrt[Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{3491}$$

$$b\left(\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}-\frac{(2A-C)\int(b\cos(c+dx))^{2/3}dx}{b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}-\frac{(2A-C)\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b^2}\right)$$

$$\Big\downarrow \text{3122}$$

$$b\left(\frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(1/3),x]
```

output
```
b*((3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c
 + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d
*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{\left(A+C\cos\left(dx+c\right)^{2}\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm=
"fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b*cos
(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(A+C\cos^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)}{\sqrt[3]{b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2 + A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(1/3)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx$$
$$= \frac{\left(\int \frac{\sec(dx+c)}{\cos(dx+c)^{\frac{1}{3}}}dx\right)a + \left(\int \cos\left(dx + c\right)^{\frac{5}{3}}\sec\left(dx + c\right)dx\right)c}{b^{\frac{1}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(sec(c + d*x)/cos(c + d*x)**(1/3),x)*a + int((cos(c + d*x)**2*sec(c +
d*x))/cos(c + d*x)**(1/3),x)*c)/b**(1/3)
```

3.162 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 91

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)-3/8*(A+4*C)*(b*cos(d*x+c))^(2/3)
*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3b\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,\text{Hypergeometric2F}\right.}{4d(b\cos(c+dx))^{4/3}}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x
]
```

output
```
(-3*b*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2])
 + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2])*Sq
rt[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/3}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^2\left(\frac{(A+4C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx}{4b^2}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(A+4C)\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b^2}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}-\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

output
```
b^2*((3*A*Sin[c + d*x])/(4*b*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*C
os[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c
+ d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{\left(A+C\cos\left(dx+c\right)^{2}\right)\sec\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^{2}+A\right)\sec\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b*c
os(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{(A+C\cos^2{(c+dx)})\sec^2{(c+dx)}}{\sqrt[3]{b\cos{(c+dx)}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c + d*x))**(1/3),
x)
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^2\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{\left(\int \frac{\sec(dx+c)^2}{\cos(dx+c)^{\frac{1}{3}}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}\sec\left(dx+c\right)^2dx\right)c}{b^{\frac{1}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(sec(c + d*x)**2/cos(c + d*x)**(1/3),x)*a + int((cos(c + d*x)**2*sec(c
 + d*x)**2)/cos(c + d*x)**(1/3),x)*c)/b**(1/3)
```

3.163 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 92

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3/7*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/7*(4*A+7*C)*hypergeom([-1/6,
 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3\left(7C\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)+A\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2\right.\right.}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(1/3),x
]
```

output
```
(3*(7*C*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x] + A
*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sec[c + d*x]*Tan[c + d
*x]))/(7*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{10/3}}dx$$

$$\Big\downarrow \text{ 3491}$$

$$b^3\left(\frac{(4A+7C)\int \frac{1}{(b\cos(c+dx))^{4/3}}dx}{7b^2}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^3\left(\frac{(4A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{7b^2}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \text{ 3122}$$

$$b^3\left(\frac{3(4A+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(1/3),x]
```

output
```
b^3*((3*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/3)) + (3*(4*A + 7*C)*Hy
pergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^3*d*(b*
Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{(A + C\cos(dx + c)^2) \sec(dx + c)^3}{(b\cos(dx + c))^{\frac{1}{3}}} dx$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{(A + C\cos^2(c + dx)) \sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}} dx = \int \frac{(C\cos(dx + c)^2 + A) \sec(dx + c)^3}{(b\cos(dx + c))^{\frac{1}{3}}} dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b*c
os(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\, dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\, dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^3}{\left(b\cos\left(dx + c\right)\right)^{\frac{1}{3}}}\, dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\, dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^3}{\left(b\cos\left(dx + c\right)\right)^{\frac{1}{3}}}\, dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx = \int \frac{C\cos(c + d\,x)^2 + A}{\cos(c + d\,x)^3\,(b\,\cos(c + d\,x))^{1/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/3)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx$$
$$= \frac{\left(\int \frac{\sec(dx+c)^3}{\cos(dx+c)^{\frac{1}{3}}}dx\right)a + \left(\int \cos(dx + c)^{\frac{5}{3}}\sec(dx + c)^3\,dx\right)c}{b^{\frac{1}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(sec(c + d*x)**3/cos(c + d*x)**(1/3),x)*a + int((cos(c + d*x)**2*sec(c
 + d*x)**3)/cos(c + d*x)**(1/3),x)*c)/b**(1/3)
```

3.164 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^3d} - \frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{70b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
3/10*C*(b*cos(d*x+c))^(7/3)*sin(d*x+c)/b^3/d-3/70*(10*A+7*C)*(b*cos(d*x+c)
)^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*
x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\cot(c+dx)\left(13A\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)+7C\cos^4(c+dx)\,\text{Hypergeo}\right.}{91d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x
]
```

output
```
(-3*Cot[c + d*x]*(13*A*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Co
s[c + d*x]^2] + 7*C*Cos[c + d*x]^4*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[
c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(91*d*(b*Cos[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{4/3}\left(C\cos^2(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{10}(10A+7C)\int (b\cos(c+dx))^{4/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{10}(10A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}-\frac{3(10A+7C)\sin(c+dx)(b\cos(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{70bd\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b*d) - (3*(10*A + 7*C)*(b*C
os[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(70*b*d*Sqrt[Sin[c + d*x]^2]))/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos (dx+c)^2 \left(A+C\cos (dx+c)^2\right)}{(b\cos (dx+c))^{\frac{2}{3}}}dx$$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

output
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3)/b, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3),x)
```

output
```
int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\frac{\left(\int \cos\left(dx+c\right)^{\frac{4}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{10}{3}}dx\right)c}{b^{\frac{2}{3}}}$$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

output
```
(int(cos(c + d*x)**4/cos(c + d*x)**(2/3),x)*c + int(cos(c + d*x)**2/cos(c
+ d*x)**(2/3),x)*a)/b**(2/3)
```

3.165 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7b^2d} - \frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{28b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3/7*C*(b*cos(d*x+c))^(4/3)*sin(d*x+c)/b^2/d-3/28*(7*A+4*C)*(b*cos(d*x+c))^
(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c
)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(5A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,\text{Hyperge}\right.}{20bd}$$

input
```
Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/2, 2/3, 5
/3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3, 8/3,
Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(20*b*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt[3]{b\cos(c+dx)}\left(C\cos^2(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{7}(7A+4C)\int \sqrt[3]{b\cos(c+dx)}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{7}(7A+4C)\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}}{b}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}-\frac{3(7A+4C)\sin(c+dx)(b\cos(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{28bd\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) - (3*(7*A + 4*C)*(b*Cos
[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(28*b*d*Sqrt[Sin[c + d*x]^2]))/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos (dx+c) \left(A+C \cos (dx+c)^2\right)}{(b \cos (dx+c))^{\frac{2}{3}}} dx$$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

output
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm=
"fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)/b, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

input
```
int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3),x)
```

output
```
int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\left(\int \cos\left(dx+c\right)^{\frac{1}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{7}{3}}dx\right)c}{b^{\frac{2}{3}}}$$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

output
```
(int(cos(c + d*x)/cos(c + d*x)**(2/3),x)*a + int(cos(c + d*x)**3/cos(c + d
*x)**(2/3),x)*c)/b**(2/3)
```

3.166 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4bd} - \frac{3(4A+C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*C*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/b/d-3/4*(4*A+C)*(b*cos(d*x+c))^(1/3)
*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\cot(c+dx)\left(7A\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6}\right.\right.}{7d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + C
*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2])*Sqrt[Si
n[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3493}$$

$$\frac{1}{4}(4A+C)\int \frac{1}{(b\cos(c+dx))^{2/3}}dx+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(4A+C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}-\frac{3(4A+C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) - (3*(4*A + C)*(b*Cos[c
+ d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x
])/(4*b*d*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{A + C\cos(dx + c)^2}{(b\cos(dx + c))^{\frac{2}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)/(b*cos(d*x + c)), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+A}{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(2/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\frac{\left(\int \cos\left(dx+c\right)^{\frac{4}{3}}dx\right)c+\left(\int \frac{1}{\cos\left(dx+c\right)^{\frac{2}{3}}}dx\right)a}{b^{\frac{2}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

output
```
(int(cos(c + d*x)**2/cos(c + d*x)**(2/3),x)*c + int(1/cos(c + d*x)**(2/3),
x)*a)/b**(2/3)
```

3.167 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 90

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3A\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3/2*A*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)+3/8*(A-2*C)*(b*cos(d*x+c))^(4/3)*h
ypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\left(-2A\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)+C\cos(c+dx)\cot(c+dx)\,\text{Hypergeom}\right.}{4d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*(-2*A*Csc[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] +
 C*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]
^2])*Sqrt[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{3491}$$

$$b\left(\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}-\frac{(A-2C)\int \sqrt[3]{b\cos(c+dx)}dx}{2b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}-\frac{(A-2C)\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b^2}\right)$$

$$\Big\downarrow \text{3122}$$

$$b\left(\frac{3(A-2C)\sin(c+dx)(b\cos(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}+\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

output
```
b*((3*A*Sin[c + d*x])/(2*b*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos
[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{\left(A + C\cos\left(dx + c\right)^{2}\right)\sec\left(dx + c\right)}{\left(b\cos\left(dx + c\right)\right)^{\frac{2}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm=
"fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)/(b*cos
(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(A + C\cos^2\left(c+dx\right))\sec\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)), x)
```

Reduce [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\left(\int \frac{\sec(dx+c)}{\cos(dx+c)^{\frac{2}{3}}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{4}{3}}\sec\left(dx+c\right)dx\right)c}{b^{\frac{2}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

output
```
(int(sec(c + d*x)/cos(c + d*x)**(2/3),x)*a + int((cos(c + d*x)**2*sec(c +
d*x))/cos(c + d*x)**(2/3),x)*c)/b**(2/3)
```

3.168 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 93

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\frac{3Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}}-\frac{3(2A+5C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

output
```
3/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)-3/5*(2*A+5*C)*(b*cos(d*x+c))^(1/
3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=-\frac{3b\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,\text{Hypergeometric2F1}\right.}{5d(b\cos(c+dx))^{5/3}}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x
]
```

output
```
(-3*b*Csc[c + d*x]*(-(A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2])
 + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sq
rt[Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(5/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{8/3}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^2\left(\frac{(2A+5C)\int \frac{1}{(b\cos(c+dx))^{2/3}}dx}{5b^2}+\frac{3A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/3}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(2A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{5b^2}+\frac{3A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/3}}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{3A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/3}}-\frac{3(2A+5C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

output
```
b^2*((3*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/3)) - (3*(2*A + 5*C)*(b
*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[
c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{(A + C\cos(dx+c)^2)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(C\cos(dx+c)^2 + A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2/(b*c
os(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c + d*x))**(2/3),
x)
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^2\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)), x)
```

Reduce [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\left(\int \frac{\sec(dx+c)^2}{\cos(dx+c)^{\frac{2}{3}}}dx\right)a + \left(\int \cos(dx+c)^{\frac{4}{3}}\sec(dx+c)^2\,dx\right)c}{b^{\frac{2}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

output
```
(int(sec(c + d*x)**2/cos(c + d*x)**(2/3),x)*a + int((cos(c + d*x)**2*sec(c
 + d*x)**2)/cos(c + d*x)**(2/3),x)*c)/b**(2/3)
```

3.169 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 92

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3Ab^2\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}} + \frac{3(5A+8C)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{16d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/8*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(8/3)+3/16*(5*A+8*C)*hypergeom([-1/3
, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3\big(4C\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx) -}{8d(b\cos(c -}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x
]
```

output
```
(3*(4*C*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x] + A
*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sec[c + d*x]*Tan[c + d
*x]))/(8*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{11/3}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^3\left(\frac{(5A+8C)\int \frac{1}{(b\cos(c+dx))^{5/3}}dx}{8b^2}+\frac{3A\sin(c+dx)}{8bd(b\cos(c+dx))^{8/3}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{(5A+8C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx}{8b^2}+\frac{3A\sin(c+dx)}{8bd(b\cos(c+dx))^{8/3}}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^3\left(\frac{3(5A+8C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{16b^3d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}+\frac{3A\sin(c+dx)}{8bd(b\cos(c+dx))^{8/3}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x]
```

output
```
b^3*((3*A*Sin[c + d*x])/(8*b*d*(b*Cos[c + d*x])^(8/3)) + (3*(5*A + 8*C)*Hy
pergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(16*b^3*d*(b
*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{(A + C\cos(dx + c)^2)\sec(dx + c)^3}{(b\cos(dx + c))^{\frac{2}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^3}{\left(b\cos\left(dx + c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3/(b*c
os(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^3}{\left(b\cos\left(dx + c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^3\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)), x)
```

Reduce [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\frac{\left(\int \frac{\sec(dx+c)^3}{\cos(dx+c)^{\frac{2}{3}}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{4}{3}}\sec\left(dx+c\right)^3dx\right)c}{b^{\frac{2}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
```

output
```
(int(sec(c + d*x)**3/cos(c + d*x)**(2/3),x)*a + int((cos(c + d*x)**2*sec(c
 + d*x)**3)/cos(c + d*x)**(2/3),x)*c)/b**(2/3)
```

3.170 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d}-\frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b^3/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^
(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+
c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=-\frac{3\cos^2(c+dx)\cot(c+dx)\left(11A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,\text{Hypergeo}\right.}{55d(b\cos(c+dx))^{4/3}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x
]
```

output
```
(-3*Cos[c + d*x]^2*Cot[c + d*x]*(11*A*Hypergeometric2F1[1/2, 5/6, 11/6, Co
s[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[
c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(55*d*(b*Cos[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{2/3}\left(C\cos^2(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\frac{1}{8}(8A+5C)\int (b\cos(c+dx))^{2/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{8}(8A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{40bd\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*(8*A + 5*C)*(b*Cos
[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
 d*x])/(40*b*d*Sqrt[Sin[c + d*x]^2]))/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos\left(dx+c\right)^{2}\left(A+C\cos\left(dx+c\right)^{2}\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/b^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3),x)
```

output
```
int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{8}{3}}dx\right)c}{b^{\frac{4}{3}}}$$

input
```
int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(cos(c + d*x)/cos(c + d*x)**(1/3),x)*a + int(cos(c + d*x)**3/cos(c + d
*x)**(1/3),x)*c)/(b**(1/3)*b)
```

3.171 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/b^2/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^
(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c
)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\cot(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3}\right.\right.}{8bd\sqrt[3]{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + C
*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(8*b*d*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \frac{C\cos^2(c+dx)+A}{\sqrt[3]{b\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{ 3493}$$

$$\frac{\frac{1}{5}(5A+2C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{5}(5A+2C)\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}}{b}$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{10bd\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*(5*A + 2*C)*(b*Cos
[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(10*b*d*Sqrt[Sin[c + d*x]^2]))/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos \left(dx+c\right) \left(A+C \cos \left(dx+c\right)^{2}\right)}{\left(b \cos \left(dx+c\right)\right)^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm=
"fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}\,dx$$

input
```
int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3),x)
```

output
```
int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}dx\right)c+\left(\int \frac{1}{\cos\left(dx+c\right)^{\frac{1}{3}}}dx\right)a}{b^{\frac{4}{3}}}$$

input
```
int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(cos(c + d*x)**2/cos(c + d*x)**(1/3),x)*c + int(1/cos(c + d*x)**(1/3),
x)*a)/(b**(1/3)*b)
```

3.172 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)+3/5*(2*A-C)*(b*cos(d*x+c))^(5/3)*h
ypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\cot(c+dx)\left(-5A\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)+C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\right.\right.}{5d(b\cos(c+dx))^{4/3}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*Cot[c + d*x]*(-5*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] +
 C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2])*Sqrt[
Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{3491}$$

$$\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}-\frac{(2A-C)\int(b\cos(c+dx))^{2/3}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}-\frac{(2A-C)\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c +
d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x]
)/(5*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{A + C\cos(dx + c)^2}{(b\cos(dx + c))^{\frac{4}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)^2
), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(b\cos\left(dx + c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{C\cos(dx + c)^2 + A}{(b\cos(dx + c))^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{C\cos(c + d\,x)^2 + A}{(b\,\cos(c + d\,x))^{4/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(4/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{A + C\cos^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \frac{\left(\int \cos(dx + c)^{\frac{2}{3}}\,dx\right)c + \left(\int \frac{1}{\cos(dx+c)^{\frac{4}{3}}}dx\right)a}{b^{\frac{4}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(cos(c + d*x)/cos(c + d*x)**(1/3),x)*c + int(1/(cos(c + d*x)**(1/3)*co
s(c + d*x)),x)*a)/(b**(1/3)*b)
```

3.173 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 90

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*A*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)-3/8*(A+4*C)*(b*cos(d*x+c))^(2/3)*h
ypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\right.\right.}{4d(b\cos(c+dx))^{4/3}}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]) +
 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2])*Sqrt
[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/3}}dx$$

$$\Big\downarrow \text{ 3491}$$

$$b\left(\frac{(A+4C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx}{4b^2}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{(A+4C)\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b^2}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \text{ 3122}$$

$$b\left(\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}-\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

output
```
b*((3*A*Sin[c + d*x])/(4*b*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos
[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c +
d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{\left(A + C\cos\left(dx + c\right)^{2}\right)\sec\left(dx + c\right)}{\left(b\cos\left(dx + c\right)\right)^{\frac{4}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^{2} + A\right)\sec\left(dx + c\right)}{\left(b\cos\left(dx + c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm=
"fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b^2*c
os(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)}{\left(b\cos\left(dx + c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)}{\left(b\cos\left(dx + c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm=
"giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)\,\left(b\,\cos\left(c + d\,x\right)\right)^{4/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)), x)
```

Reduce [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \frac{\left(\int \frac{\sec(dx+c)}{\cos(dx+c)^{\frac{4}{3}}}dx\right)a + \left(\int \cos(dx+c)^{\frac{2}{3}}\sec(dx+c)\,dx\right)c}{b^{\frac{4}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(sec(c + d*x)/(cos(c + d*x)**(1/3)*cos(c + d*x)),x)*a + int((cos(c + d
*x)*sec(c + d*x))/cos(c + d*x)**(1/3),x)*c)/(b**(1/3)*b)
```

3.174 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 93

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3/7*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/7*(4*A+7*C)*hypergeom([-1/6, 1
/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3b^2\cot(c+dx)\left(A\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx\right.\right.}{7}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x
]
```

output
```
(3*b^2*Cot[c + d*x]*(A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]
+ 7*C*Cos[c + d*x]^2*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2])*Sq
rt[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(10/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{10/3}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^2\left(\frac{(4A+7C)\int \frac{1}{(b\cos(c+dx))^{4/3}}dx}{7b^2}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{(4A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{7b^2}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{3(4A+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

output
```
b^2*((3*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/3)) + (3*(4*A + 7*C)*Hy
pergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^3*d*(b*
Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{(A+C\cos(dx+c)^2)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b^2
*cos(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + A\right)\sec\left(dx + c\right)^2}{\left(b\cos\left(dx + c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)^2\left(b\,\cos\left(c + d\,x\right)\right)^{4/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)), x)
```

Reduce [F]

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \frac{\left(\int \frac{\sec(dx+c)^2}{\cos(dx+c)^{\frac{4}{3}}}dx\right)a + \left(\int \cos\left(dx + c\right)^{\frac{2}{3}}\sec\left(dx + c\right)^2 dx\right)c}{b^{\frac{4}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(sec(c + d*x)**2/(cos(c + d*x)**(1/3)*cos(c + d*x)),x)*a + int((cos(c
+ d*x)*sec(c + d*x)**2)/cos(c + d*x)**(1/3),x)*c)/(b**(1/3)*b)
```

3.175 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 92

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3Ab^2\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}} + \frac{3(7A+10C)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{40d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/10*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(10/3)+3/40*(7*A+10*C)*hypergeom([-
2/3, 1/2],[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c
)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3b^2\csc(c+dx)\left(2A\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},\cos^2(c+d\right.\right.}{2}$$

input
```
Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x
]
```

output
```
(3*b^2*Csc[c + d*x]*(2*A*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2
] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2])*
Sqrt[Sin[c + d*x]^2])/(20*d*(b*Cos[c + d*x])^(10/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3042, 2030, 3491, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{13/3}}dx$$

$$\Big\downarrow \text{3491}$$

$$b^3\left(\frac{(7A+10C)\int \frac{1}{(b\cos(c+dx))^{7/3}}dx}{10b^2}+\frac{3A\sin(c+dx)}{10bd(b\cos(c+dx))^{10/3}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{(7A+10C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}}dx}{10b^2}+\frac{3A\sin(c+dx)}{10bd(b\cos(c+dx))^{10/3}}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^3\left(\frac{3(7A+10C)\sin(c+dx)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{40b^3d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}+\frac{3A\sin(c+dx)}{10bd(b\cos(c+dx))^{10/3}}\right)$$

input
```
Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]
```

output
```
b^3*((3*A*Sin[c + d*x])/(10*b*d*(b*Cos[c + d*x])^(10/3)) + (3*(7*A + 10*C)
*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b^3*d
*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{(A + C\cos(dx+c)^2)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{4}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(C\cos(dx+c)^2 + A)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b^2
*cos(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{(C\cos(dx+c)^2 + A)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^3\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)), x)
```

Reduce [F]

$$\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{\left(\int \frac{\sec(dx+c)^3}{\cos(dx+c)^{\frac{4}{3}}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}\sec\left(dx+c\right)^3dx\right)c}{b^{\frac{4}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(sec(c + d*x)**3/(cos(c + d*x)**(1/3)*cos(c + d*x)),x)*a + int((cos(c
+ d*x)*sec(c + d*x)**3)/cos(c + d*x)**(1/3),x)*c)/(b**(1/3)*b)
```

3.176 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 33, antiderivative size = 148

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(10+3m)}-\frac{3b(C(7+3m)+A(10+3m))\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+\right.}{d(7+3m)(10+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
3*b*C*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d/(10+3*m)-3*b*(C*(
7+3*m)+A*(10+3*m))*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7
/6+1/2*m],[13/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(10+3*m)/(sin(d*
x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.96

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{4/3}\csc(c+dx)\left(A(13+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(A*(13 + 3*m)
*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2] + C*(7
+ 3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (13 + 3*m)/6, (19 + 3*m)/6, C
os[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 3*m)*(13 + 3*m))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{4/3}\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\int \cos^{m+\frac{4}{3}}(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3493}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(\frac{(A(3m+10)+C(3m+7))\int\cos^{m+\frac{4}{3}}(c+dx)dx}{3m+10}+\frac{3C\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)}{d(3m+10)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(\frac{(A(3m+10)+C(3m+7))\int\sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}dx}{3m+10}+\frac{3C\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)}{d(3m+10)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$\Big\downarrow$ 3122

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)}{d(3m+10)}-\frac{3(A(3m+10)+C(3m+7))\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+7),\right.}{d(3m+7)(3m+10)\sqrt{\sin^2(c+dx)}}\right.}{\sqrt[3]{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(b*(b*Cos[c + d*x])^(1/3)*((3*C*Cos[c + d*x]^(7/3 + m)*Sin[c + d*x])/(d*(1
0 + 3*m)) - (3*(C*(7 + 3*m) + A*(10 + 3*m))*Cos[c + d*x]^(7/3 + m)*Hyperge
ometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
d*(7 + 3*m)*(10 + 3*m)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^(1/3)
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos \left(dx+c\right)^{m} \left(b \cos \left(dx+c\right)\right)^{\frac{4}{3}} \left(A+C \cos \left(dx+c\right)^{2}\right)dx$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C \cos \left(dx+c\right)^{2}+A\right) \left(b \cos \left(dx+c\right)\right)^{\frac{4}{3}} \cos \left(dx+c\right)^{m}\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*co
s(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,dx$$

input
```
int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=b^{\frac{4}{3}}\left(\left(\int \cos\left(dx+c\right)^{m+\frac{1}{3}}\cos\left(dx+c\right)dx\right)a+\left(\int \cos\left(dx+c\right)^{m+\frac{1}{3}}\cos\left(dx+c\right)^{3}dx\right)c\right)$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**((3*m + 1)/3)*cos(c + d*x),x)*a + int(cos(c
+ d*x)**((3*m + 1)/3)*cos(c + d*x)**3,x)*c)
```

3.177 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 33, antiderivative size = 146

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+dx)}{d(8+3m)}-\frac{3(C(5+3m)+A(8+3m))\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+\right.}{d(5+3m)(8+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
3*C*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/d/(8+3*m)-3*(C*(5+3*m
)+A*(8+3*m))*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 5/6+1/2
*m],[11/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(5+3*m)/(8+3*m)/(sin(d*x+c)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\csc(c+dx)\left(A(11+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(A*(11 + 3*m)
*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + C*(5
+ 3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (11 + 3*m)/6, (17 + 3*m)/6, C
os[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(5 + 3*m)*(11 + 3*m))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{2/3}\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 2034$$

$$\frac{(b\cos(c+dx))^{2/3}\int \cos^{m+\frac{2}{3}}(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{(b\cos(c+dx))^{2/3}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 3493$$

$$\frac{(b\cos(c+dx))^{2/3}\left(\frac{(A(3m+8)+C(3m+5))\int\cos^{m+\frac{2}{3}}(c+dx)dx}{3m+8}+\frac{3C\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)}{d(3m+8)}\right)}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(b\cos(c+dx))^{2/3}\left(\frac{(A(3m+8)+C(3m+5))\int\sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}dx}{3m+8}+\frac{3C\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)}{d(3m+8)}\right)}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{(b\cos(c+dx))^{2/3}\left(\frac{3C\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)}{d(3m+8)}-\frac{3(A(3m+8)+C(3m+5))\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+5)\right.}{d(3m+5)(3m+8)\sqrt{\sin^2(c+dx)}}\right.}{\cos^{\frac{2}{3}}(c+dx)}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(2/3)*((3*C*Cos[c + d*x]^(5/3 + m)*Sin[c + d*x])/(d*(8 +
 3*m)) - (3*(C*(5 + 3*m) + A*(8 + 3*m))*Cos[c + d*x]^(5/3 + m)*Hypergeomet
ric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5
 + 3*m)*(8 + 3*m)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^(2/3)
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [F]

$$\int \cos \left(dx+c\right)^{m} \left(b\cos \left(dx+c\right)\right)^{\frac{2}{3}} \left(A+C\cos \left(dx+c\right)^{2}\right)dx$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos \left(dx+c\right)^{2}+A\right)\left(b\cos \left(dx+c\right)\right)^{\frac{2}{3}}\cos \left(dx+c\right)^{m}\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,dx$$

input
```
int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=b^{\frac{2}{3}}\left(\left(\int \cos\left(dx+c\right)^{m+\frac{2}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{m+\frac{2}{3}}\cos\left(dx+c\right)^2dx\right)c\right)$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**((3*m + 2)/3),x)*a + int(cos(c + d*x)**((3*m +
 2)/3)*cos(c + d*x)**2,x)*c)
```

3.178 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx))$

Optimal result

Integrand size = 33, antiderivative size = 146

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(7+3m)}$$
$$-\frac{3(C(4+3m)+A(7+3m))\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+\right.}{d(4+3m)(7+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
3*C*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d/(7+3*m)-3*(C*(4+3*m
)+A*(7+3*m))*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 2/3+1/2
*m],[5/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(4+3*m)/(7+3*m)/(sin(d*x+c)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\left(C(4+3m)\cos^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3}+\frac{m}{2},\right.\right.}{d(4+}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(C*(4 + 3*m)*
Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2
] + A*(10 + 3*m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*
x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(4 + 3*m)*(10 + 3*m))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\cos(c+dx)}\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 2034}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{m+\frac{1}{3}}(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3493}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(\frac{(A(3m+7)+C(3m+4))\int \cos^{m+\frac{1}{3}}(c+dx)dx}{3m+7}+\frac{3C\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)}{d(3m+7)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(\frac{(A(3m+7)+C(3m+4))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}dx}{3m+7}+\frac{3C\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)}{d(3m+7)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \; 3122$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)}{d(3m+7)}-\frac{3(A(3m+7)+C(3m+4))\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+4),\frac{1}{6}(\right.}{d(3m+4)(3m+7)\sqrt{\sin^2(c+dx)}}\right.}{\sqrt[3]{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(1/3)*((3*C*Cos[c + d*x]^(4/3 + m)*Sin[c + d*x])/(d*(7 +
 3*m)) - (3*(C*(4 + 3*m) + A*(7 + 3*m))*Cos[c + d*x]^(4/3 + m)*Hypergeomet
ric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4
 + 3*m)*(7 + 3*m)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^(1/3)
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [F]

$$\int \cos (dx+c)^m (b\cos (dx+c))^{\frac{1}{3}} \left(A+C\cos (dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos (dx+c)^2+A\right)(b\cos (dx+c))^{\frac{1}{3}}\cos (dx+c)^m \, dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Sympy [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\cos^m(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Integral((b*cos(c + d*x))**(1/3)*(A + C*cos(c + d*x)**2)*cos(c + d*x)**m,
x)
```

Maxima [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

input
```
int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=b^{\frac{1}{3}}\left(\left(\int \cos\left(dx+c\right)^{m+\frac{1}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{m+\frac{1}{3}}\cos\left(dx+c\right)^2dx\right)c\right)$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*(int(cos(c + d*x)**((3*m + 1)/3),x)*a + int(cos(c + d*x)**((3*m +
 1)/3)*cos(c + d*x)**2,x)*c)
```

3.179 $\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 146

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} - \frac{3(C(2+3m)+A(5+3m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+d}{d(2+3m)(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3*C*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)-3*(C*(2+3*m
)+A*(5+3*m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d
*x+c)^2)*sin(d*x+c)/d/(2+3*m)/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(A(8+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right.}{d(2+3m)(8+3n}$$

input
```
Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x
]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(8 + 3*m)*Hypergeometric2F1[1/2,
(2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + C*(2 + 3*m)*Cos[c + d*x]^2*Hyp
ergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(2 + 3*m)*(8 + 3*m)*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx) \left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \cos^{m-\frac{1}{3}}(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{(A(3m+5)+C(3m+2))\int \cos^{m-\frac{1}{3}}(c+dx)dx}{3m+5}+\frac{3C\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)}{d(3m+5)}\right)}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{(A(3m+5)+C(3m+2))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}dx}{3m+5}+\frac{3C\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)}{d(3m+5)}\right)}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)}{d(3m+5)}-\frac{3(A(3m+5)+C(3m+2))\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+2),\frac{1}{6}(3m\right.}{d(3m+2)(3m+5)\sqrt{\sin^2(c+dx)}}\right.}{\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(Cos[c + d*x]^(1/3)*((3*C*Cos[c + d*x]^(2/3 + m)*Sin[c + d*x])/(d*(5 + 3*m
)) - (3*(C*(2 + 3*m) + A*(5 + 3*m))*Cos[c + d*x]^(2/3 + m)*Hypergeometric2
F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*
m)*(5 + 3*m)*Sqrt[Sin[c + d*x]^2])))/(b*Cos[c + d*x])^(1/3)
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos(dx+c)^m \left(A + C\cos(dx+c)^2\right)}{\left(b\cos(dx+c)\right)^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(A + C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2 + A\right)\cos(dx+c)^m}{\left(b\cos(dx+c)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b*c
os(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(A + C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(A + C\cos^2(c+dx)\right)\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(1/3),
x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^m\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3),x)
```

output
```
int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\frac{\left(\int \frac{\cos(dx+c)^m}{\cos(dx+c)^{\frac{1}{3}}}dx\right)a+\left(\int \cos\left(dx+c\right)^m\cos\left(dx+c\right)^{\frac{5}{3}}dx\right)c}{b^{\frac{1}{3}}}$$

input
```
int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(cos(c + d*x)**m/cos(c + d*x)**(1/3),x)*a + int((cos(c + d*x)**m*cos(c
 + d*x)**2)/cos(c + d*x)**(1/3),x)*c)/b**(1/3)
```

3.180 $\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 144

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}} - \frac{3(C+3Cm+A(4+3m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1+3m),\frac{1}{6}(7+3m),\cos^2(c+dx)\right)}{d(1+3m)(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3*C*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(4+3*m)/(b*cos(d*x+c))^(2/3)-3*(C+3*C*m+
A*(4+3*m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/6+1/2*m],[7/6+1/2*m],cos(d*x
+c)^2)*sin(d*x+c)/d/(1+3*m)/(4+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx =$$

$$-\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(A(7+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1+3m),\frac{1}{6}(7+3m),\cos^2(c+dx)\right)+\right.}{d(1+3m)(7+3m}$$

input
```
Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x
]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(7 + 3*m)*Hypergeometric2F1[1/2,
(1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2] + C*(1 + 3*m)*Cos[c + d*x]^2*Hyp
ergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(d*(1 + 3*m)*(7 + 3*m)*(b*Cos[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{m-\frac{2}{3}}(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{2}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(\frac{(A(3m+4)+3Cm+C)\int \cos^{m-\frac{2}{3}}(c+dx)dx}{3m+4}+\frac{3C\sin(c+dx)\cos^{m+\frac{1}{3}}(c+dx)}{d(3m+4)}\right)}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(\frac{(A(3m+4)+3Cm+C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{2}{3}}dx}{3m+4}+\frac{3C\sin(c+dx)\cos^{m+\frac{1}{3}}(c+dx)}{d(3m+4)}\right)}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(\frac{3C\sin(c+dx)\cos^{m+\frac{1}{3}}(c+dx)}{d(3m+4)}-\frac{3(A(3m+4)+3Cm+C)\sin(c+dx)\cos^{m+\frac{1}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+1),\frac{1}{6}(3m\right.}{d(3m+1)(3m+4)\sqrt{\sin^2(c+dx)}}\right.}{(b\cos(c+dx))^{2/3}}$$

input

```
Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

output

```
(Cos[c + d*x]^(2/3)*((3*C*Cos[c + d*x]^(1/3 + m)*Sin[c + d*x])/(d*(4 + 3*m
)) - (3*(C + 3*C*m + A*(4 + 3*m))*Cos[c + d*x]^(1/3 + m)*Hypergeometric2F1
[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)
*(4 + 3*m)*Sqrt[Sin[c + d*x]^2])))/(b*Cos[c + d*x])^(2/3)
```

Defintions of rubi rules used

rule 2034

```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] &&
 !IntegerQ[m + n]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3493

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos(dx+c)^m \left(A+C\cos(dx+c)^2\right)}{\left(b\cos(dx+c)\right)^{\frac{2}{3}}}dx$$

input
```
int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

output
```
int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\cos(dx+c)^m}{\left(b\cos(dx+c)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/(b*c
os(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(A+C\cos^2(c+dx)\right)\cos^m(c+dx)}{\left(b\cos(c+dx)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(2/3),
x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\cos\left(c+d\,x\right)^m\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

input
```
int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3),x)
```

output
```
int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{\cos^m(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\left(\int \frac{\cos(dx+c)^m}{\cos(dx+c)^{\frac{2}{3}}}dx\right)a + \left(\int \cos(dx+c)^m \cos(dx+c)^{\frac{4}{3}}\,dx\right)c}{b^{\frac{2}{3}}}$$

input
```
int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

output
```
(int(cos(c + d*x)**m/cos(c + d*x)**(2/3),x)*a + int((cos(c + d*x)**m*cos(c
 + d*x)**2)/cos(c + d*x)**(2/3),x)*c)/b**(2/3)
```

3.181 $\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 149

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}}-\frac{3(C(1-3m)-A(2+3m))\cos^m(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m),\cos^2(c+dx)\right)}{bd(1-3m)(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3*C*cos(d*x+c)^m*sin(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)-3*(C*(1-3*m)-
A*(2+3*m))*cos(d*x+c)^m*hypergeom([1/2, -1/6+1/2*m],[5/6+1/2*m],cos(d*x+c)
^2)*sin(d*x+c)/b/d/(1-3*m)/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=$$

$$-\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(A(5+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m),\cos^2(c+dx)\right)\right.}{d(-1+3m)(5+}$$

input
```
Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x
]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(5 + 3*m)*Hypergeometric2F1[1/2,
(-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2] + C*(-1 + 3*m)*Cos[c + d*x]^2*H
ypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(d*(-1 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{ 2034}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \cos^{m-\frac{4}{3}}(c+dx)\left(C\cos^2(c+dx)+A\right)dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{4}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3493}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{(C(1-3m)-A(3m+2))\int \cos^{m-\frac{4}{3}}(c+dx)dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{(C(1-3m)-A(3m+2))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{4}{3}}dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{3(C(1-3m)-A(3m+2))\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m-1),\frac{1}{6}(3\right.}{d(1-3m)(3m+2)\sqrt{\sin^2(c+dx)}}\right.}{b\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(Cos[c + d*x]^(1/3)*((3*C*Cos[c + d*x]^(-1/3 + m)*Sin[c + d*x])/(d*(2 + 3*
m)) - (3*(C*(1 - 3*m) - A*(2 + 3*m))*Cos[c + d*x]^(-1/3 + m)*Hypergeometri
c2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 -
 3*m)*(2 + 3*m)*Sqrt[Sin[c + d*x]^2])))/(b*(b*Cos[c + d*x])^(1/3))
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos(dx+c)^m \left(A+C\cos(dx+c)^2\right)}{(b\cos(dx+c))^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2
*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(A+C\cos^2(c+dx)\right)\cos^m(c+dx)}{(b\cos(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(4/3),
x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)^m\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}\,dx$$

input
```
int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3),x)
```

output
```
int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\left(\int \frac{\cos(dx+c)^m}{\cos(dx+c)^{\frac{4}{3}}}dx\right)a+\left(\int \cos\left(dx+c\right)^m\cos\left(dx+c\right)^{\frac{2}{3}}dx\right)c}{b^{\frac{4}{3}}}$$

input
```
int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(cos(c + d*x)**m/(cos(c + d*x)**(1/3)*cos(c + d*x)),x)*a + int((cos(c
+ d*x)**m*cos(c + d*x))/cos(c + d*x)**(1/3),x)*c)/(b**(1/3)*b)
```

3.182 $\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+C\cos^2(c+dx\right.$

Optimal result

Integrand size = 33, antiderivative size = 144

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+dx)}{ad(2+m+n)} - \frac{(C(1+m+n)+A(2+m+n))(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m\right.}{ad(1+m+n)(2+m+n)\sqrt{\sin^2(c+dx)}}$$

output
```
C*(a*cos(d*x+c))^(1+m)*(b*cos(d*x+c))^n*sin(d*x+c)/a/d/(2+m+n)-(C*(1+m+n)+
A*(2+m+n))*(a*cos(d*x+c))^(1+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2+1/2*m
+1/2*n],[3/2+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a/d/(1+m+n)/(2+m+n)/(si
n(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx = -\frac{(a\cos(c+dx))^m(b\cos(c+dx))^n\cot(c+dx)\left(A(3+m+n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(\right.\right.}{}$$

input
```
Integrate[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
-(((a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(3 + m + n)*Hyper
geometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2] + C*(1 + m
 + n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 + m + n)/2, (5 + m + n)/2,
Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + m + n)*(3 + m + n)))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx))^m\left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{2034}$$

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\int (a\cos(c+dx))^{m+n}\left(C\cos^2(c+dx)+A\right)dx$$

$$\Big\downarrow \text{3042}$$

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{m+n}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx$$

$$\Big\downarrow \text{3493}$$

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\left(\left(A+\frac{C(m+n+1)}{m+n+2}\right)\int(a\cos(c+dx))^{m+n}dx+\frac{C\sin(c+dx)(a\cos(c+dx))^{m+n+1}}{ad(m+n+2)}\right)$$

$\Big\downarrow$ 3042

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\left(\left(A+\frac{C(m+n+1)}{m+n+2}\right)\int\left(a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{m+n}dx+\frac{C\sin(c+dx)(a\cos(c+dx))^{m+n+1}}{ad(m+n+2)}\right)$$

$\Big\downarrow$ 3122

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\left(\frac{C\sin(c+dx)(a\cos(c+dx))^{m+n+1}}{ad(m+n+2)}-\frac{\left(A+\frac{C(m+n+1)}{m+n+2}\right)\sin(c+dx)(a\cos(c+dx))^{m+n+1}\text{Hypergeometr}}{ad(m+n+1)\sqrt{\sin^2}}\right.$$

input
```
Int[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^n*((C*(a*Cos[c + d*x])^(1 + m + n)*Sin[c + d*x])/(a*d*(2
 + m + n)) - ((A + (C*(1 + m + n))/(2 + m + n))*(a*Cos[c + d*x])^(1 + m +
n)*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2]*Si
n[c + d*x])/(a*d*(1 + m + n)*Sqrt[Sin[c + d*x]^2])))/(a*Cos[c + d*x])^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] &&
 !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a\cos(dx+c))^m (b\cos(dx+c))^n \left(A+C\cos(dx+c)^2\right) dx$$

input
```
int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a\cos(c+dx))^m (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos(dx+c)^2+A\right)(a\cos(dx+c))^m (b\cos(dx+c))^n \, dx$$

input
```
integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

Sympy [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int (a\cos{(c+dx)})^m\,(b\cos{(c+dx)})^n\,\left(A+C\cos^2{(c+dx)}\right)\,dx$$

input
```
integrate((a*cos(d*x+c))**m*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

output
```
Integral((a*cos(c + d*x))**m*(b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2),
x)
```

Maxima [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos{(dx+c)}^2+A\right)(a\cos{(dx+c)})^m\,(b\cos{(dx+c)})^n\,dx$$

input
```
integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

Giac [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos{(dx+c)}^2+A\right)(a\cos{(dx+c)})^m\,(b\cos{(dx+c)})^n\,dx$$

input
```
integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(a\,\cos\left(c+d\,x\right)\right)^m\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(a*cos(c + d*x))^m*(b*cos(c + d*x))^n,x)
```

output
```
int((A + C*cos(c + d*x)^2)*(a*cos(c + d*x))^m*(b*cos(c + d*x))^n, x)
```

Reduce [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx = b^n a^m\left(\left(\int \cos\left(dx+c\right)^{m+n}dx\right)a+\left(\int \cos\left(dx+c\right)^{m+n}\cos\left(dx+c\right)^2dx\right)c\right)$$

input
```
int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
b**n*a**m*(int(cos(c + d*x)**(m + n),x)*a + int(cos(c + d*x)**(m + n)*cos(
c + d*x)**2,x)*c)
```

3.183 $\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) d$

Optimal result

Integrand size = 31, antiderivative size = 117

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)} - \frac{(C(3+n)+A(4+n))(b\cos(c+dx))^{3+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)(4+n)\sqrt{\sin^2(c+dx)}}$$

output
```
C*(b*cos(d*x+c))^(3+n)*sin(d*x+c)/b^3/d/(4+n)-(C*(3+n)+A*(4+n))*(b*cos(d*x
+c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)
/b^3/d/(3+n)/(4+n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = -\frac{\cos^2(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(A(5+n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)+\right.}{d(3+n)(5+n)}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
-((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(5 + n)*Hypergeometri
c2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + C*(3 + n)*Cos[c + d*x]^2
*Hypergeometric2F1[1/2, (5 + n)/2, (7 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c
+ d*x]^2])/(d*(3 + n)*(5 + n)))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{n+2}\left(C\cos^2(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3493}$$

$$\frac{\left(A+\frac{C(n+3)}{n+4}\right)\int (b\cos(c+dx))^{n+2}dx+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}}{bd(n+4)}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(A+\frac{C(n+3)}{n+4}\right)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}dx+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}}{bd(n+4)}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}}{bd(n+4)}-\frac{\left(A+\frac{C(n+3)}{n+4}\right)\sin(c+dx)(b\cos(c+dx))^{n+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{n+5}{2},\cos^2(c+dx)\right)}{bd(n+3)\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
((C*(b*Cos[c + d*x])^(3 + n)*Sin[c + d*x])/(b*d*(4 + n)) - ((A + (C*(3 + n
))/(4 + n))*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5
+ n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(3 + n)*Sqrt[Sin[c + d*x]^2]))/
b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos (dx+c)^2 (b\cos (dx+c))^n \left(A+C\cos (dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \int (C\cos(dx+c)^2+A)(b\cos(dx+c))^n\cos(dx+c)^2 dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="f
ricas")
```

output
```
integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \int (C\cos(dx+c)^2+A)(b\cos(dx+c))^n\cos(dx+c)^2 dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="m
axima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \int (C\cos(dx+c)^2+A)(b\cos(dx+c))^n\cos(dx+c)^2 dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="g
iac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \int \cos(c+d\,x)^2 \left(C\cos(c+d\,x)^2+A\right) (b\,\cos(c+d\,x))^n dx$$

input
```
int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

Reduce [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= b^n\left(\left(\int \cos\left(dx+c\right)^n \cos\left(dx+c\right)^4 dx\right)c+\left(\int \cos\left(dx+c\right)^n \cos\left(dx+c\right)^2 dx\right)a\right)$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)**4,x)*c + int(cos(c + d*x)**n*cos(c
 + d*x)**2,x)*a)
```

3.184 $\int \cos(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)dx$

Optimal result

Integrand size = 29, antiderivative size = 117

$$\int \cos(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)dx=\frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)}-\frac{(C(2+n)+A(3+n))(b\cos(c+dx))^{2+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)(3+n)\sqrt{\sin^2(c+dx)}}$$

output
```
C*(b*cos(d*x+c))^(2+n)*sin(d*x+c)/b^2/d/(3+n)-(C*(2+n)+A*(3+n))*(b*cos(d*x
+c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2
/d/(2+n)/(3+n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \cos(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)dx=$$
$$-\frac{\cos(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(A(4+n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)+C\right.}{d(2+n)(4+n)}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
-((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(4 + n)*Hypergeometric2
F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + C*(2 + n)*Cos[c + d*x]^2*H
ypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(2 + n)*(4 + n)))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2030, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (b\cos(c+dx))^{n+1}\left(C\cos^2(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx}{b}$$

$$\Big\downarrow \; 3493$$

$$\frac{\left(A+\frac{C(n+2)}{n+3}\right)\int (b\cos(c+dx))^{n+1}dx+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}}{bd(n+3)}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\left(A+\frac{C(n+2)}{n+3}\right)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}dx+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}}{bd(n+3)}}{b}$$

$$\Big\downarrow \; 3122$$

$$\frac{\frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}}{bd(n+3)}-\frac{\left(A+\frac{C(n+2)}{n+3}\right)\sin(c+dx)(b\cos(c+dx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\cos^2(c+dx)\right)}{bd(n+2)\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
((C*(b*Cos[c + d*x])^(2 + n)*Sin[c + d*x])/(b*d*(3 + n)) - ((A + (C*(2 + n
))/(3 + n))*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4
+ n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 + n)*Sqrt[Sin[c + d*x]^2]))/
b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos(dx+c)\,(b\cos(dx+c))^n\left(A+C\cos(dx+c)^2\right)dx$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx = \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right) \, dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fri
cas")
```

output
```
integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="max
ima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="gia
c")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,dx$$

input
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

Reduce [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= b^n \left(\left(\int \cos{(dx+c)}^n \cos{(dx+c)}\, dx \right) a + \left(\int \cos{(dx+c)}^n \cos{(dx+c)}^3\, dx \right) c \right)$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x),x)*a + int(cos(c + d*x)**n*cos(c +
d*x)**3,x)*c)
```

3.185 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)} - \frac{(C(1+n)+A(2+n))(b\cos(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)(2+n)\sqrt{\sin^2(c+dx)}}$$

output
```
C*(b*cos(d*x+c))^(1+n)*sin(d*x+c)/b/d/(2+n)-(C*(1+n)+A*(2+n))*(b*cos(d*x+c
))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b
/d/(1+n)/(2+n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = -\frac{(b\cos(c+dx))^n\cot(c+dx)\left(A(3+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)+C(1+n)\cos\right.}{d(1+n)(3+n)}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
-(((b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(3 + n)*Hypergeometric2F1[1/2, (1 +
n)/2, (3 + n)/2, Cos[c + d*x]^2] + C*(1 + n)*Cos[c + d*x]^2*Hypergeometric
2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(
1 + n)*(3 + n)))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3493}$$

$$\left(A+\frac{C(n+1)}{n+2}\right)\int (b\cos(c+dx))^n dx+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}}{bd(n+2)}$$

$$\Big\downarrow \text{3042}$$

$$\left(A+\frac{C(n+1)}{n+2}\right)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n dx+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}}{bd(n+2)}$$

$$\Big\downarrow \text{3122}$$

$$\frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}}{bd(n+2)}-\frac{\left(A+\frac{C(n+1)}{n+2}\right)\sin(c+dx)(b\cos(c+dx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
(C*(b*Cos[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(2 + n)) - ((A + (C*(1 + n)
)/(2 + n))*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 +
 n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^n \left(A + C\cos(dx+c)^2\right) dx$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right)\right)^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \int \left(b\cos\left(c+dx\right)\right)^n \left(A+C\cos^2\left(c+dx\right)\right)\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2), x)
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right)\right)^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \int (C\cos{(dx+c)}^2+A)(b\cos{(dx+c)})^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos{(c+d\,x)}^2+A\right)\,(b\,\cos{(c+d\,x)})^n\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
```

output
```
int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = b^n\left(\left(\int \cos{(dx+c)}^n\,dx\right)a+\left(\int \cos{(dx+c)}^n\cos{(dx+c)}^2\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**n,x)*a + int(cos(c + d*x)**n*cos(c + d*x)**2,x)*c)
```

3.186 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 100

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)} - \frac{(A+An+Cn)(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn(1+n)\sqrt{\sin^2(c+dx)}}$$

output
```
C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1+n)-(A*n+C*n+A)*(b*cos(d*x+c))^n*hyperge
om([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/n/(1+n)/(sin(d*x+c)^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = -\frac{b(b\cos(c+dx))^{-1+n}\cot(c+dx)\left(A(2+n)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)+Cn\cos^2(c\right.}{dn(2+n)}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
-((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(A*(2 + n)*Hypergeometric2F1[1
/2, n/2, (2 + n)/2, Cos[c + d*x]^2] + C*n*Cos[c + d*x]^2*Hypergeometric2F1
[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*n*(2
 + n)))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3042, 2030, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-1}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A\right)dx$$

$$\Big\downarrow \text{ 3493}$$

$$b\left(\frac{(An+A+Cn)\int(b\cos(c+dx))^{n-1}dx}{n+1}+\frac{C\sin(c+dx)(b\cos(c+dx))^n}{bd(n+1)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\left(\frac{(An+A+Cn)\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-1}dx}{n+1}+\frac{C\sin(c+dx)(b\cos(c+dx))^n}{bd(n+1)}\right)$$

$$\Big\downarrow \text{ 3122}$$

$$b\left(\frac{C\sin(c+dx)(b\cos(c+dx))^n}{bd(n+1)} - \frac{(An+A+Cn)\sin(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},c\right.}{bdn(n+1)\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
b*((C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(b*d*(1 + n)) - ((A + A*n + C*n)*(b
*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Si
n[c + d*x])/(b*d*n*(1 + n)*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^n \left(A+C\cos(dx+c)^2\right)\sec(dx+c)\,dx$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fri
cas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

Sympy [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2)*sec(c + d*x), x)
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int (C\cos(dx+c)^2+A)(b\cos(dx+c))^n\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="max
ima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int (C\cos(dx+c)^2+A)(b\cos(dx+c))^n\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="gia
c")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \frac{\left(C\cos(c+d\,x)^2+A\right)\,\left(b\,\cos(c+d\,x)\right)^n}{\cos(c+d\,x)}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b^n\left(\left(\int \cos(dx+c)^n\cos(dx+c)^2\sec(dx+c)\,dx\right)c + \left(\int \cos(dx+c)^n\sec(dx+c)\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)**2*sec(c + d*x),x)*c + int(cos(c +
d*x)**n*sec(c + d*x),x)*a)
```

3.187 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn} - \frac{b(C(1-n)-An)(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+d}{d(1-n)n\sqrt{\sin^2(c+dx)}}$$

output

```
b*C*(b*cos(d*x+c))^(-1+n)*sin(d*x+c)/d/n-b*(C*(1-n)-A*n)*(b*cos(d*x+c))^(-
1+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1
-n)/n/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$
$$-\frac{b(b\cos(c+dx))^{-1+n}\csc(c+dx)\left(A(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)+C\right.}{d(-1+n)(1+n}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
-((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*(A*(1 + n)*Hypergeometric2F1[1
/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2] + C*(-1 + n)*Cos[c + d*x]^2*Hyp
ergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*
x]^2])/(d*(-1 + n)*(1 + n)))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-2}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A\right)dx$$

$$\Big\downarrow \text{3493}$$

$$b^2\left(\frac{C\sin(c+dx)(b\cos(c+dx))^{n-1}}{bdn}-\frac{(C(1-n)-An)\int(b\cos(c+dx))^{n-2}dx}{n}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{C\sin(c+dx)(b\cos(c+dx))^{n-1}}{bdn}-\frac{(C(1-n)-An)\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-2}dx}{n}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{C\sin(c+dx)(b\cos(c+dx))^{n-1}}{bdn}-\frac{(C(1-n)-An)\sin(c+dx)(b\cos(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{}\right.}{bd(1-n)n\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

output
```
b^2*((C*(b*Cos[c + d*x])^(-1 + n)*Sin[c + d*x])/(b*d*n) - ((C*(1 - n) - A*
n)*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2,
 Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 - n)*n*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos (dx+c))^{n} \left(A+C\cos (dx+c)^{2}\right) \sec (dx+c)^{2} dx$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos (dx+c)^{2}+A\right)(b\cos (dx+c))^{n}\sec (dx+c)^{2}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="f
ricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="m
axima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="g
iac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^2,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^2, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=b^n\left(\left(\int \cos\left(dx+c\right)^n\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2dx\right)c+\left(\int \cos\left(dx+c\right)^n\sec\left(dx+c\right)^2dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)**2*sec(c + d*x)**2,x)*c + int(cos(c
 + d*x)**n*sec(c + d*x)**2,x)*a)
```

3.188 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 125

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)} + \frac{b^2(A(1-n)+C(2-n))(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}}$$

output
```
-b^2*C*(b*cos(d*x+c))^(-2+n)*sin(d*x+c)/d/(1-n)+b^2*(A*(1-n)+C*(2-n))*(b*c
os(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*sin(d*x+
c)/d/(1-n)/(2-n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.91

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$
$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\left(An\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)+C(-2+n)\cos\right.}{d(-2+n)n}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
-(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*n*Hypergeometric2F1[1/2, (-2 + n)/2,
 n/2, Cos[c + d*x]^2] + C*(-2 + n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, n
/2, (2 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(d*(-
2 + n)*n))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 2030, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-3}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A\right)dx$$

$$\Big\downarrow \text{3493}$$

$$b^3\left(\left(A+\frac{C(2-n)}{1-n}\right)\int (b\cos(c+dx))^{n-3}dx-\frac{C\sin(c+dx)(b\cos(c+dx))^{n-2}}{bd(1-n)}\right)$$

$\Big\downarrow$ 3042

$$b^3\left(\left(A+\frac{C(2-n)}{1-n}\right)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-3}dx-\frac{C\sin(c+dx)(b\cos(c+dx))^{n-2}}{bd(1-n)}\right)$$

$\Big\downarrow$ 3122

$$b^3\left(\frac{\left(A+\frac{C(2-n)}{1-n}\right)\sin(c+dx)(b\cos(c+dx))^{n-2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-2}{2},\frac{n}{2},\cos^2(c+dx)\right)}{bd(2-n)\sqrt{\sin^2(c+dx)}}-\frac{C\sin(c+dx)(}{bd(}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

output
```
b^3*(-((C*(b*Cos[c + d*x])^(-2 + n)*Sin[c + d*x])/(b*d*(1 - n))) + ((A + (
C*(2 - n))/(1 - n))*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 +
 n)/2, n/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 - n)*Sqrt[Sin[c + d*x]^2
]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos (dx+c))^{n} \left(A+C\cos (dx+c)^{2}\right) \sec (dx+c)^{3}\, dx$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \sec^3(c+dx)\, dx$$
$$= \int \left(C\cos (dx+c)^{2}+A\right)(b\cos (dx+c))^{n} \sec (dx+c)^{3}\, dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="f
ricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="m
axima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="g
iac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n\,(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx$$
$$=\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^3,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^3, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n\,(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx$$
$$=b^n\left(\left(\int \cos\left(dx+c\right)^n\cos\left(dx+c\right)^2\sec\left(dx+c\right)^3dx\right)c+\left(\int \cos\left(dx+c\right)^n\sec\left(dx+c\right)^3dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)**2*sec(c + d*x)**3,x)*c + int(cos(c
 + d*x)**n*sec(c + d*x)**3,x)*a)
```

3.189 $\int (b\cos(c+dx))^n \left(A + C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 127

$$\int (b\cos(c+dx))^n \left(A + C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)}$$
$$+ \frac{b^3(A(2-n)+C(3-n))(b\cos(c+dx))^{-3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+\right.}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}}$$

output
```
-b^3*C*(b*cos(d*x+c))^(-3+n)*sin(d*x+c)/d/(2-n)+b^3*(A*(2-n)+C*(3-n))*(b*c
os(d*x+c))^(-3+n)*hypergeom([1/2, -3/2+1/2*n],[-1/2+1/2*n],cos(d*x+c)^2)*s
in(d*x+c)/d/(2-n)/(3-n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx =$$
$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\left(A(-1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+dx)\right)\right.}{d(-3}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

output
```
-(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + n)*Hypergeometric2F1[1/2, (-3
+ n)/2, (-1 + n)/2, Cos[c + d*x]^2] + C*(-3 + n)*Cos[c + d*x]^2*Hypergeome
tric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^3*Sqrt[S
in[c + d*x]^2])/(d*(-3 + n)*(-1 + n)))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 2030, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-4}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+A\right)dx$$

$$\Big\downarrow \text{3493}$$

$$b^4\left(\left(A+\frac{C(3-n)}{2-n}\right)\int (b\cos(c+dx))^{n-4}dx-\frac{C\sin(c+dx)(b\cos(c+dx))^{n-3}}{bd(2-n)}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\left(A+\frac{C(3-n)}{2-n}\right)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-4}dx-\frac{C\sin(c+dx)(b\cos(c+dx))^{n-3}}{bd(2-n)}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^4\left(\frac{\left(A+\frac{C(3-n)}{2-n}\right)\sin(c+dx)(b\cos(c+dx))^{n-3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-3}{2},\frac{n-1}{2},\cos^2(c+dx)\right)}{bd(3-n)\sqrt{\sin^2(c+dx)}}-\frac{C\sin(c+dx}{b}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

output
```
b^4*(-((C*(b*Cos[c + d*x])^(-3 + n)*Sin[c + d*x])/(b*d*(2 - n))) + ((A + (
C*(3 - n))/(2 - n))*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 +
 n)/2, (-1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(3 - n)*Sqrt[Sin[c +
 d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos (dx+c))^n \left(A+C\cos (dx+c)^2\right) \sec (dx+c)^4 \,dx$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \sec^4(c+dx)\,dx = \int \left(C\cos (dx+c)^2+A\right)(b\cos (dx+c))^n \sec (dx+c)^4 \,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="f
ricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c))^n \sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="m
axima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c))^n \sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="g
iac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^4,x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^4, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = b^n\left(\left(\int \cos\left(dx+c\right)^n\cos\left(dx+c\right)^2\sec\left(dx+c\right)^4dx\right)c + \left(\int \cos\left(dx+c\right)^n\sec\left(dx+c\right)^4dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)**2*sec(c + d*x)**4,x)*c + int(cos(c
 + d*x)**n*sec(c + d*x)**4,x)*a)
```

3.190 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) d$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \frac{2C\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(9+2n)}$$
$$- \frac{2(C(7+2n)+A(9+2n))\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n\right.}{d(7+2n)(9+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
2*C*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(9+2*n)-2*(C*(7+2*n)+A*
(9+2*n))*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/
4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(7+2*n)/(9+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx =$$
$$- \frac{2\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(A(11+2n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n)\right.\right.}{d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(11 + 2*n)*Hyper
geometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] + C*(7 + 2*n)
*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (11 + 2*n)/4, (15 + 2*n)/4, Cos[c +
 d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 2*n)*(11 + 2*n))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx) \left(A+C\cos^2(c+dx)\right) (b\cos(c+dx))^n \, dx$$

$$\Big\downarrow \text{ 2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \cos^{n+\frac{5}{2}}(c+dx) \left(C\cos^2(c+dx)+A\right) dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{5}{2}} \left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right) dx$$

$$\Big\downarrow \text{ 3493}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \left(\frac{(A(2n+9)+C(2n+7)) \int \cos^{n+\frac{5}{2}}(c+dx)dx}{2n+9}+\frac{2C\sin(c+dx)\cos^{n+\frac{7}{2}}(c+dx)}{d(2n+9)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \left(\frac{(A(2n+9)+C(2n+7)) \int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{5}{2}} dx}{2n+9}+\frac{2C\sin(c+dx)\cos^{n+\frac{7}{2}}(c+dx)}{d(2n+9)}\right)$$

$$\Big\downarrow \text{ 3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2C\sin(c+dx)\cos^{n+\frac{7}{2}}(c+dx)}{d(2n+9)}-\frac{2(A(2n+9)+C(2n+7))\sin(c+dx)\cos^{n+\frac{7}{2}}(c+dx)\,\text{Hypergeometric}}{d(2n+7)(2n+9)\sqrt{\sin^2(c+}}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*C*Cos[c + d*x]^(7/2 + n)*Sin[c + d*x])/(d*(9 + 2*n
)) - (2*(C*(7 + 2*n) + A*(9 + 2*n))*Cos[c + d*x]^(7/2 + n)*Hypergeometric2
F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2
*n)*(9 + 2*n)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos (dx+c)^{\frac{5}{2}} (b\cos (dx+c))^n \left(A+C\cos (dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \int \left(C\cos (dx+c)^2+A\right)(b\cos (dx+c))^n \cos (dx+c)^{\frac{5}{2}} dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n*sqrt(cos
(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \int (C\cos(dx+c)^2+A)(b\cos(dx+c))^n\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \int (C\cos(dx+c)^2+A)(b\cos(dx+c))^n\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \int \cos(c+d\,x)^{5/2}\,\left(C\cos(c+d\,x)^2+A\right)\,\left(b\,\cos(c+d\,x)\right)^n\,dx$$

input
```
int(cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=b^n\left(\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}\cos\left(dx+c\right)^4dx\right)c+\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}\cos\left(dx+c\right)^2dx\right)a\right)$$

input
```
int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)*cos(c + d*x)**4,x)*c + int(cos(c + d
*x)**((2*n + 1)/2)*cos(c + d*x)**2,x)*a)
```

3.191 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) d$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)}$$
$$-\frac{2(C(5+2n)+A(7+2n))\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n)\right.}{d(5+2n)(7+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
2*C*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(7+2*n)-2*(C*(5+2*n)+A*
(7+2*n))*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4
+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(7+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx =$$
$$-\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(A(9+2n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\text{c}\right.\right.}{d(}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(9 + 2*n)*Hyperg
eometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + C*(5 + 2*n)*C
os[c + d*x]^2*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*
x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(5 + 2*n)*(9 + 2*n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx) \left(A+C\cos^2(c+dx)\right) (b\cos(c+dx))^n \, dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \cos^{n+\frac{3}{2}}(c+dx) \left(C\cos^2(c+dx)+A\right) dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}} \left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right) dx$$

$$\Big\downarrow \text{3493}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \left(\frac{(A(2n+7)+C(2n+5))\int \cos^{n+\frac{3}{2}}(c+dx)dx}{2n+7}+\frac{2C\sin(c+dx)\cos^{n+\frac{5}{2}}(c+dx)}{d(2n+7)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \left(\frac{(A(2n+7)+C(2n+5))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}dx}{2n+7}+\frac{2C\sin(c+dx)\cos^{n+\frac{5}{2}}(c+dx)}{d(2n+7)}\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2C\sin(c+dx)\cos^{n+\frac{5}{2}}(c+dx)}{d(2n+7)}-\frac{2(A(2n+7)+C(2n+5))\sin(c+dx)\cos^{n+\frac{5}{2}}(c+dx)\,\text{Hypergeometric}}{d(2n+5)(2n+7)\sqrt{\sin^2(c+}}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*C*Cos[c + d*x]^(5/2 + n)*Sin[c + d*x])/(d*(7 + 2*n
)) - (2*(C*(5 + 2*n) + A*(7 + 2*n))*Cos[c + d*x]^(5/2 + n)*Hypergeometric2
F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*
n)*(7 + 2*n)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos(dx+c)^{\frac{3}{2}}(b\cos(dx+c))^n\left(A+C\cos(dx+c)^2\right)dx$$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d
*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \int (C\cos(dx+c)^2+A)(b\cos(dx+c))^n \cos(dx+c)^{\frac{3}{2}} dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \int (C\cos(dx+c)^2+A)(b\cos(dx+c))^n \cos(dx+c)^{\frac{3}{2}} dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= \int \cos(c+d\,x)^{3/2} \left(C\cos(c+d\,x)^2+A\right) \left(b\,\cos(c+d\,x)\right)^n dx$$

input
```
int(cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx$$
$$=b^n\left(\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}\cos\left(dx+c\right)dx\right)a+\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}\cos\left(dx+c\right)^3dx\right)c\right)$$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)*cos(c + d*x),x)*a + int(cos(c + d*x)
**((2*n + 1)/2)*cos(c + d*x)**3,x)*c)
```

3.192 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)} - \frac{2(C(3+2n)+A(5+2n))\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\right.}{d(3+2n)(5+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
2*C*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(5+2*n)-2*(C*(3+2*n)+A*
(5+2*n))*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4
+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(5+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = -\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(A(7+2n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),c\right.\right.}{d(}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(7 + 2*n)*Hyperg
eometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + C*(3 + 2*n)*C
os[c + d*x]^2*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*
x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(3 + 2*n)*(7 + 2*n))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n+\frac{1}{2}}(c+dx)\left(C\cos^2(c+dx)+A\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(2n+5)+C(2n+3))\int \cos^{n+\frac{1}{2}}(c+dx)dx}{2n+5}+\frac{2C\sin(c+dx)\cos^{n+\frac{3}{2}}(c+dx)}{d(2n+5)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(2n+5)+C(2n+3))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}dx}{2n+5}+\frac{2C\sin(c+dx)\cos^{n+\frac{3}{2}}(c+dx)}{d(2n+5)}\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2C\sin(c+dx)\cos^{n+\frac{3}{2}}(c+dx)}{d(2n+5)}-\frac{2(A(2n+5)+C(2n+3))\sin(c+dx)\cos^{n+\frac{3}{2}}(c+dx)\,\text{Hypergeometric}}{d(2n+3)(2n+5)\sqrt{\sin^2(c+}}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*C*Cos[c + d*x]^(3/2 + n)*Sin[c + d*x])/(d*(5 + 2*n
)) - (2*(C*(3 + 2*n) + A*(5 + 2*n))*Cos[c + d*x]^(3/2 + n)*Hypergeometric2
F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*
n)*(5 + 2*n)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \sqrt{\cos(dx+c)}\,(b\cos(dx+c))^n\left(A+C\cos(dx+c)^2\right)dx$$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \int \left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx$$
$$= \int (C\cos{(dx+c)}^2 + A)(b\cos{(dx+c)})^n \sqrt{\cos{(dx+c)}}\, dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx$$
$$= \int (C\cos{(dx+c)}^2 + A)(b\cos{(dx+c)})^n \sqrt{\cos{(dx+c)}}\, dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) \, dx$$
$$= \int \sqrt{\cos{(c+d\,x)}}\,\left(C\cos{(c+d\,x)}^2 + A\right)\,\left(b\,\cos{(c+d\,x)}\right)^n\, dx$$

input
```
int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$$
$$= b^n\left(\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}} dx\right) a+\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}} \cos\left(dx+c\right)^2 dx\right) c\right)$$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2),x)*a + int(cos(c + d*x)**((2*n + 1)/
2)*cos(c + d*x)**2,x)*c)
```

3.193 $\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 140

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$
$$=\frac{2C\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)}$$
$$-\frac{2(C+2Cn+A(3+2n))\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\right.}{d(1+2n)(3+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
2*C*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(3+2*n)-2*(C+2*C*n+A*(3
+2*n))*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1
/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1+2*n)/(3+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx =$$
$$-\frac{2\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\csc(c+dx)\left(A(5+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\right.\right.}{d(}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x
]
```

output
```
(-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(5 + 2*n)*Hyperg
eometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + C*(1 + 2*n)*C
os[c + d*x]^2*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x
]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + 2*n)*(5 + 2*n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{1}{2}}(c+dx)\left(C\cos^2(c+dx)+A\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(2n+3)+2Cn+C)\int\cos^{n-\frac{1}{2}}(c+dx)dx}{2n+3}+\frac{2C\sin(c+dx)\cos^{n+\frac{1}{2}}(c+dx)}{d(2n+3)}\right)$$

$\big\downarrow$ 3042

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(2n+3)+2Cn+C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}dx}{2n+3}+\frac{2C\sin(c+dx)\cos^{n+\frac{1}{2}}(c+dx)}{d(2n+3)}\right)$$

$\big\downarrow$ 3122

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2C\sin(c+dx)\cos^{n+\frac{1}{2}}(c+dx)}{d(2n+3)}-\frac{2(A(2n+3)+2Cn+C)\sin(c+dx)\cos^{n+\frac{1}{2}}(c+dx)\text{Hypergeometric2}}{d(2n+1)(2n+3)\sqrt{\sin^2(c+d}}\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

output
```
((b*Cos[c + d*x])^n*((2*C*Cos[c + d*x]^(1/2 + n)*Sin[c + d*x])/(d*(3 + 2*n
)) - (2*(C + 2*C*n + A*(3 + 2*n))*Cos[c + d*x]^(1/2 + n)*Hypergeometric2F1
[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)
*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [F]

$$\int \frac{(b\cos (dx+c))^n \left(A+C\cos (dx+c)^2\right)}{\sqrt{\cos (dx+c)}}dx$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

output
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos (dx+c)^2+A\right)(b\cos (dx+c))^n}{\sqrt{\cos (dx+c)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos{(c+dx)})^n\,(A+C\cos^2{(c+dx)})}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2)/sqrt(cos(c + d*x)), x
)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(C\cos{(dx+c)}^2+A)(b\cos{(dx+c)})^n}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(C\cos{(dx+c)}^2+A)(b\cos{(dx+c)})^n}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(1/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = b^n\left(\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)}dx\right)a+\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}\cos\left(dx+c\right)dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)/cos(c + d*x),x)*a + int(cos(c + d*x)
**((2*n + 1)/2)*cos(c + d*x),x)*c)
```

3.194 $\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 136

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}} + \frac{2(A-C(1-2n)+2An)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx\right.}{d\left(1-4n^2\right)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1+2*n)/cos(d*x+c)^(1/2)+2*(A-C*(1-2*n)+
2*A*n)*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)
^2)*sin(d*x+c)/d/(-4*n^2+1)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(3+2n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx\right.\right.}{d(-1+2n)(3\ldots}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x
]
```

output
```
(-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(3 + 2*n)*Hypergeometric2F1[1/2, (-
1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + C*(-1 + 2*n)*Cos[c + d*x]^2*Hyp
ergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c
+ d*x]^2])/(d*(-1 + 2*n)*(3 + 2*n)*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{3}{2}}(c+dx)\left(C\cos^2(c+dx)+A\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx$$

$$\Big\downarrow \text{ 3493}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(2An+A-C(1-2n))\int \cos^{n-\frac{3}{2}}(c+dx)dx}{2n+1}+\frac{2C\sin(c+dx)\cos^{n-\frac{1}{2}}(c+dx)}{d(2n+1)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(2An+A-C(1-2n))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}dx}{2n+1}+\frac{2C\sin(c+dx)\cos^{n-\frac{1}{2}}(c+dx)}{d(2n+1)}\right)$$

$\Big\downarrow$ 3122

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2(2An+A-C(1-2n))\sin(c+dx)\cos^{n-\frac{1}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\cos^2(\right.}{d(1-2n)(2n+1)\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*C*Cos[c + d*x]^(-1/2 + n)*Sin[c + d*x])/(d*(1 + 2*
n)) + (2*(A - C*(1 - 2*n) + 2*A*n)*Cos[c + d*x]^(-1/2 + n)*Hypergeometric2
F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2
*n)*(1 + 2*n)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{(b\cos(dx+c))^n \left(A + C\cos(dx+c)^2\right)}{\cos(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

output
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n (A + C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2 + A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n (A + C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(c+dx))^n \left(A + C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2)/cos(c + d*x)**(3/2),
x)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(3/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = b^n\left(\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}dx\right)c+\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^2}dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2),x)*c + int(cos(c + d*x)**((2*n + 1)/
2)/cos(c + d*x)**2,x)*a)
```

3.195 $\int \frac{(b\cos(c+dx))^n(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 140

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)} + \frac{2(A+C(3-2n)-2An)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx\right)}{d(1-2n)(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(3/2)+2*(A+C*(3-2*n)
-2*A*n)*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c
)^2)*sin(d*x+c)/d/(1-2*n)/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(1+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx\right)}{d(-3+2n)(1}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x
]
```

output
```
(-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(1 + 2*n)*Hypergeometric2F1[1/2, (-
3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] + C*(-3 + 2*n)*Cos[c + d*x]^2*Hyp
ergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c
+ d*x]^2])/(d*(-3 + 2*n)*(1 + 2*n)*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+C\cos^2(c+dx))\,(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{5}{2}}(c+dx)\left(C\cos^2(c+dx)+A\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(1-2n)+C(3-2n))\int \cos^{n-\frac{5}{2}}(c+dx)dx}{1-2n}-\frac{2C\sin(c+dx)\cos^{n-\frac{3}{2}}(c+dx)}{d(1-2n)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(1-2n)+C(3-2n))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}dx}{1-2n}-\frac{2C\sin(c+dx)\cos^{n-\frac{3}{2}}(c+dx)}{d(1-2n)}\right)$$

$\big\downarrow$ 3122

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2(A(1-2n)+C(3-2n))\sin(c+dx)\cos^{n-\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-3),\frac{1}{4}(2n+1),\cos^2\right.}{d(1-2n)(3-2n)\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

output
```
((b*Cos[c + d*x])^n*((-2*C*Cos[c + d*x]^(-3/2 + n)*Sin[c + d*x])/(d*(1 - 2
*n)) + (2*(A*(1 - 2*n) + C*(3 - 2*n))*Cos[c + d*x]^(-3/2 + n)*Hypergeometr
ic2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1
- 2*n)*(3 - 2*n)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{(b\cos(dx+c))^n (A+C\cos(dx+c)^2)}{\cos(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

output
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n (A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(C\cos(dx+c)^2+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n (A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(5/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = b^n\left(\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)}dx\right)c+\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^3}dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)/cos(c + d*x),x)*c + int(cos(c + d*x)
**((2*n + 1)/2)/cos(c + d*x)**3,x)*a)
```

3.196 $\int \frac{(b\cos(c+dx))^n(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} + \frac{2(A(3-2n)+C(5-2n))(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+\right.}{d(3-2n)(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(5/2)+2*(A*(3-2*n)+C
*(5-2*n))*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*
x+c)^2)*sin(d*x+c)/d/(3-2*n)/(5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx =$$
$$-\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(-1+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+\right.\right.}{d(-5+2n)(}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x
]
```

output
```
(-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + 2*n)*Hypergeometric2F1[1/2, (
-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + C*(-5 + 2*n)*Cos[c + d*x]^2*H
ypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(d*(-5 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A + C\cos^2(c+dx))(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{7}{2}}(c+dx)\left(C\cos^2(c+dx)+A\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{7}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(3-2n)+C(5-2n))\int \cos^{n-\frac{7}{2}}(c+dx)dx}{3-2n}-\frac{2C\sin(c+dx)\cos^{n-\frac{5}{2}}(c+dx)}{d(3-2n)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(3-2n)+C(5-2n))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{7}{2}}dx}{3-2n}-\frac{2C\sin(c+dx)\cos^{n-\frac{5}{2}}(c+dx)}{d(3-2n)}\right)$$

$\big\downarrow$ 3122

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2(A(3-2n)+C(5-2n))\sin(c+dx)\cos^{n-\frac{5}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-5),\frac{1}{4}(2n-1),\cos^2\right.}{d(3-2n)(5-2n)\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

output
```
((b*Cos[c + d*x])^n*((-2*C*Cos[c + d*x]^(-5/2 + n)*Sin[c + d*x])/(d*(3 - 2
*n)) + (2*(A*(3 - 2*n) + C*(5 - 2*n))*Cos[c + d*x]^(-5/2 + n)*Hypergeometr
ic2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3
 - 2*n)*(5 - 2*n)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{(b\cos(dx+c))^n \left(A+C\cos(dx+c)^2\right)}{\cos(dx+c)^{\frac{7}{2}}}dx$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

output
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n \left(A + C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n \left(A + C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \int \frac{\left(C\cos\left(dx+c\right)^2 + A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(7/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = b^n\left(\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^4}dx\right)a+\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^2}dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)/cos(c + d*x)**4,x)*a + int(cos(c + d
*x)**((2*n + 1)/2)/cos(c + d*x)**2,x)*c)
```

3.197 $\int \frac{(b\cos(c+dx))^n(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=-\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(5-2n)\cos^{\frac{7}{2}}(c+dx)}+\frac{2(A(5-2n)+C(7-2n))(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-7+2n),\frac{1}{4}(-3+2n),\cos^2(c+\right.}{d(5-2n)(7-2n)\cos^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(5-2*n)/cos(d*x+c)^(7/2)+2*(A*(5-2*n)+C
*(7-2*n))*(b*cos(d*x+c))^n*hypergeom([1/2, -7/4+1/2*n],[-3/4+1/2*n],cos(d*
x+c)^2)*sin(d*x+c)/d/(5-2*n)/(7-2*n)/cos(d*x+c)^(7/2)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=$$

$$-\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(-3+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-7+2n),\frac{1}{4}(-3+2n),\cos^2(c+\right.\right.}{d(-7+2n)(}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x
]
```

output
```
(-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-3 + 2*n)*Hypergeometric2F1[1/2, (
-7 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2] + C*(-7 + 2*n)*Cos[c + d*x]^2*H
ypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(d*(-7 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2034, 3042, 3493, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(A+C\cos^2(c+dx)\right)(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{9}{2}}(c+dx)\left(C\cos^2(c+dx)+A\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{9}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+A\right)dx$$

$$\Big\downarrow \text{3493}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(5-2n)+C(7-2n))\int \cos^{n-\frac{9}{2}}(c+dx)dx}{5-2n}-\frac{2C\sin(c+dx)\cos^{n-\frac{7}{2}}(c+dx)}{d(5-2n)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(5-2n)+C(7-2n))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{9}{2}}dx}{5-2n}-\frac{2C\sin(c+dx)\cos^{n-\frac{7}{2}}(c+dx)}{d(5-2n)}\right)$$

$\Big\downarrow$ 3122

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2(A(5-2n)+C(7-2n))\sin(c+dx)\cos^{n-\frac{7}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-7),\frac{1}{4}(2n-3),\cos^2\right.}{d(5-2n)(7-2n)\sqrt{\sin^2(c+dx)}}\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

output
```
((b*Cos[c + d*x])^n*((-2*C*Cos[c + d*x]^(-7/2 + n)*Sin[c + d*x])/(d*(5 - 2
*n)) + (2*(A*(5 - 2*n) + C*(7 - 2*n))*Cos[c + d*x]^(-7/2 + n)*Hypergeometr
ic2F1[1/2, (-7 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5
 - 2*n)*(7 - 2*n)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3493
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Simp[(A*(m + 2) + C*(m + 1))/(m + 2)   Int[(b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{(b\cos(dx+c))^n \left(A+C\cos(dx+c)^2\right)}{\cos(dx+c)^{\frac{9}{2}}}dx$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

output
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(9/2),x)
```

output
```
int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = b^n \left(\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^5}dx\right) a + \left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^3}dx\right) c\right)$$

input
```
int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)/cos(c + d*x)**5,x)*a + int(cos(c + d
*x)**((2*n + 1)/2)/cos(c + d*x)**3,x)*c)
```

3.198 $\int (a+a\cos(e+fx))^m (A+C\cos^2(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 170

$$\int (a+a\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right)\,dx$$
$$= -\frac{C(a+a\cos(e+fx))^m\sin(e+fx)}{f\,(2+3m+m^2)} + \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}$$
$$+ \frac{2^{\frac{1}{2}+m}(C(1+m+m^2)+A(2+3m+m^2))\,(1+\cos(e+fx))^{-\frac{1}{2}-m}(a+a\cos(e+fx))^m\,\text{Hypergeomet}}{f(1+m)(2+m)}$$

output
```
-C*(a+a*cos(f*x+e))^m*sin(f*x+e)/f/(m^2+3*m+2)+C*(a+a*cos(f*x+e))^(1+m)*si
n(f*x+e)/a/f/(2+m)+2^(1/2+m)*(C*(m^2+m+1)+A*(m^2+3*m+2))*(1+cos(f*x+e))^(-
1/2-m)*(a+a*cos(f*x+e))^m*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*cos(f*x+e))
*sin(f*x+e)/f/(1+m)/(2+m)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.42

$$\int (a + a\cos(e + fx))^m \left(A + C\cos^2(e + fx)\right) dx$$
$$= \frac{i4^{-1-m}e^{-i(2+m)(e+fx)}\left(1 + e^{i(e+fx)}\right)\left(e^{-\frac{1}{2}i(e+fx)}\left(1 + e^{i(e+fx)}\right)\right)^{2m}\cos^{-2m}\left(\frac{1}{2}(e + fx)\right)(a(1 + \cos(e + fx)))}{}$$

input
```
Integrate[(a + a*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2),x]
```

output
```
(I*4^(-1 - m)*(1 + E^(I*(e + f*x)))*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f
*x)))^(2*m)*(a*(1 + Cos[e + f*x]))^m*(C*E^(I*m*(e + f*x))*(-2 + m)*m*Hyper
geometric2F1[1, -1 + m, -1 - m, -E^(I*(e + f*x))] + E^(I*(2 + m)*(e + f*x)
)*(2 + m)*(2*(2*A + C)*(-2 + m)*Hypergeometric2F1[1, 1 + m, 1 - m, -E^(I*(
e + f*x))] + C*E^((2*I)*(e + f*x))*m*Hypergeometric2F1[1, 3 + m, 3 - m, -E
^(I*(e + f*x))])))/(E^(I*(2 + m)*(e + f*x))*f*(-2 + m)*m*(2 + m)*Cos[(e +
f*x)/2]^(2*m))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3503, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(e + fx) + a)^m \left(A + C\cos^2(e + fx)\right) dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(e + fx + \frac{\pi}{2}\right) + a\right)^m \left(A + C\sin\left(e + fx + \frac{\pi}{2}\right)^2\right) dx$$

$$\Big\downarrow \text{3503}$$

$$\frac{\int (\cos(e+fx)a+a)^m (a(C(m+1)+A(m+2))-aC\cos(e+fx))dx}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$\Big\downarrow$ 3042

$$\frac{\int \left(\sin\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m \left(a(C(m+1)+A(m+2))-aC\sin\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$\Big\downarrow$ 3230

$$\frac{\frac{a(A(m^2+3m+2)+C(m^2+m+1))\int (\cos(e+fx)a+a)^m dx}{m+1}-\frac{aC\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a(A(m^2+3m+2)+C(m^2+m+1))\int \left(\sin\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx}{m+1}-\frac{aC\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$\Big\downarrow$ 3131

$$\frac{\frac{a(A(m^2+3m+2)+C(m^2+m+1))(\cos(e+fx)+1)^{-m}(a\cos(e+fx)+a)^m\int (\cos(e+fx)+1)^m dx}{m+1}-\frac{aC\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a(A(m^2+3m+2)+C(m^2+m+1))(\cos(e+fx)+1)^{-m}(a\cos(e+fx)+a)^m\int \left(\sin\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx}{m+1}-\frac{aC\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$\Big\downarrow$ 3130

$$\frac{\frac{a2^{m+\frac{1}{2}}\left(A\left(m^2+3m+2\right)+C\left(m^2+m+1\right)\right)\sin(e+fx)(\cos(e+fx)+1)^{-m-\frac{1}{2}}(a\cos(e+fx)+a)^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx))\right)}{f(m+1)}}{a(m+2)}$$

$$\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

input
```
Int[(a + a*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2),x]
```

output
```
(C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (-((a*C*(a +
 a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(1 + m))) + (2^(1/2 + m)*a*(C*(1 + m +
 m^2) + A*(2 + 3*m + m^2))*(1 + Cos[e + f*x])^(-1/2 - m)*(a + a*Cos[e + f*
x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Cos[e + f*x])/2]*Sin[e + f
*x])/(f*(1 + m)))/(a*(2 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3503
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a+a\cos(fx+e))^m \left(A+C\cos(fx+e)^2\right) dx$$

input
```
int((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)
```

output
```
int((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+a\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx = \int \left(C\cos(fx+e)^2+A\right)(a\cos(fx+e)+a)^m dx$$

input
```
integrate((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="fricas")
```

output
```
integral((C*cos(f*x + e)^2 + A)*(a*cos(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a+a\cos(e+fx))^m\left(A+C\cos^2(e+fx)\right)\,dx = \int (a(\cos(e+fx)+1))^m\left(A+C\cos^2(e+fx)\right)\,dx$$

input
```
integrate((a+a*cos(f*x+e))**m*(A+C*cos(f*x+e)**2),x)
```

output
```
Integral((a*(cos(e + f*x) + 1))**m*(A + C*cos(e + f*x)**2), x)
```

Maxima [F]

$$\int (a+a\cos(e+fx))^m\left(A+C\cos^2(e+fx)\right)\,dx = \int \left(C\cos(fx+e)^2+A\right)(a\cos(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(f*x + e)^2 + A)*(a*cos(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a+a\cos(e+fx))^m\left(A+C\cos^2(e+fx)\right)\,dx = \int \left(C\cos(fx+e)^2+A\right)(a\cos(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(f*x + e)^2 + A)*(a*cos(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(e+f\,x\right)^2+A\right)\,\left(a+a\,\cos\left(e+f\,x\right)\right)^m\,dx$$

input
```
int((A + C*cos(e + f*x)^2)*(a + a*cos(e + f*x))^m,x)
```

output
```
int((A + C*cos(e + f*x)^2)*(a + a*cos(e + f*x))^m, x)
```

Reduce [F]

$$\int (a+a\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right)\,dx$$
$$=\left(\int (\cos(fx+e)\,a+a)^m\,dx\right)a+\left(\int (\cos(fx+e)\,a+a)^m\cos(fx+e)^2\,dx\right)c$$

input
```
int((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)
```

output
```
int((cos(e + f*x)*a + a)**m,x)*a + int((cos(e + f*x)*a + a)**m*cos(e + f*x
)**2,x)*c
```

3.199 $\int (a+a\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 135

$$\int (a+a\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx = -\frac{9C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d} + \frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad} + \frac{(40A+19C)(a+a\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{10\;2^{5/6}d(1+\cos(c+dx))^{7/6}}$$

output
```
-9/40*C*(a+a*cos(d*x+c))^(2/3)*sin(d*x+c)/d+3/8*C*(a+a*cos(d*x+c))^(5/3)*s
in(d*x+c)/a/d+1/20*(40*A+19*C)*(a+a*cos(d*x+c))^(2/3)*hypergeom([-1/6, 1/2
],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(7/6)
```

Mathematica [A] (warning: unable to verify)

Time = 1.48 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.30

$$\int (a+a\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx = \frac{(a(1+\cos(c+dx)))^{2/3}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(6\;2^{5/6}(40A+28C+14C\cos(c+dx)+5\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((a*(1 + Cos[c + d*x]))^(2/3)*Sec[(c + d*x)/2]^2*(6*2^(5/6)*(40*A + 28*C +
 14*C*Cos[c + d*x] + 5*C*Cos[2*(c + d*x)])*(1 - Cos[d*x - 2*ArcTan[Cot[c/2
]]])^(1/6)*Sin[c + d*x] - 4*(40*A + 19*C)*Hypergeometric2F1[1/2, 5/6, 3/2,
 Cos[(d*x)/2 - ArcTan[Cot[c/2]]]^2]*Sin[d*x - 2*ArcTan[Cot[c/2]]]))/(320*2
^(5/6)*d*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3503, 27, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^{2/3}\left(A+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3503}$$

$$\frac{3\int \frac{1}{3}(\cos(c+dx)a+a)^{2/3}(a(8A+5C)-3aC\cos(c+dx))dx}{8a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (\cos(c+dx)a+a)^{2/3}(a(8A+5C)-3aC\cos(c+dx))dx}{8a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(\sin \left(c+dx+\frac{\pi}{2}\right) a+a\right)^{2/3} \left(a(8A+5C)-3aC \sin \left(c+dx+\frac{\pi}{2}\right)\right) dx}{8a}+\frac{3C \sin(c+dx)(a \cos(c+dx)+a)^{5/3}}{8ad}$$

$\Big\downarrow$ 3230

$$\frac{\frac{1}{5}a(40A+19C)\int (\cos(c+dx)a+a)^{2/3}dx-\frac{9aC \sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3C \sin(c+dx)(a \cos(c+dx)+a)^{5/3}}{8ad}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{5}a(40A+19C)\int \left(\sin \left(c+dx+\frac{\pi}{2}\right) a+a\right)^{2/3}dx-\frac{9aC \sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3C \sin(c+dx)(a \cos(c+dx)+a)^{5/3}}{8ad}$$

$\Big\downarrow$ 3131

$$\frac{\frac{a(40A+19C)(a\cos(c+dx)+a)^{2/3}\int (\cos(c+dx)+1)^{2/3}dx}{5(\cos(c+dx)+1)^{2/3}}-\frac{9aC \sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3C \sin(c+dx)(a \cos(c+dx)+a)^{5/3}}{8ad}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a(40A+19C)(a\cos(c+dx)+a)^{2/3}\int \left(\sin\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{5(\cos(c+dx)+1)^{2/3}}-\frac{9aC \sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3C \sin(c+dx)(a \cos(c+dx)+a)^{5/3}}{8ad}$$

$\Big\downarrow$ 3130

$$\frac{\frac{2\sqrt[6]{2}a(40A+19C)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{5d(\cos(c+dx)+1)^{7/6}}-\frac{9aC \sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3C \sin(c+dx)(a \cos(c+dx)+a)^{5/3}}{8ad}$$

input
```
Int[(a + a*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(a + a*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*a*d) + ((-9*a*C*(a + a*Co
s[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) + (2*2^(1/6)*a*(40*A + 19*C)*(a + a*
Cos[c + d*x])^(2/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2
]*Sin[c + d*x])/(5*d*(1 + Cos[c + d*x])^(7/6)))/(8*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3503
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [F]

$$\int (a+a\cos(dx+c))^{\frac{2}{3}}\left(A+C\cos(dx+c)^{2}\right)dx$$

input
```
int((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a+a\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int\left(C\cos(dx+c)^2+A\right)(a\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int (a+a\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{2/3}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(a + a*cos(c + d*x))^(2/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(a + a*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=a^{\frac{2}{3}}\left(\left(\int \left(\cos\left(dx+c\right)+1\right)^{\frac{2}{3}}dx\right)a+\left(\int \left(\cos\left(dx+c\right)+1\right)^{\frac{2}{3}}\cos\left(dx+c\right)^2dx\right)c\right)$$

input
```
int((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
a**(2/3)*(int((cos(c + d*x) + 1)**(2/3),x)*a + int((cos(c + d*x) + 1)**(2/
3)*cos(c + d*x)**2,x)*c)
```

3.200 $\int \sqrt[3]{a+a\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 135

$$\int \sqrt[3]{a+a\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$$
$$= -\frac{9C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{28d} + \frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad}$$
$$+ \frac{(28A+13C)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{14\sqrt[6]{2}d(1+\cos(c+dx))^{5/6}}$$

output
```
-9/28*C*(a+a*cos(d*x+c))^(1/3)*sin(d*x+c)/d+3/7*C*(a+a*cos(d*x+c))^(4/3)*s
in(d*x+c)/a/d+1/28*(28*A+13*C)*(a+a*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2]
,[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/d/(1+cos(d*x+c))^(5/6)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 289 vs. 2(135) = 270.

Time = 4.93 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.14

$$\int \sqrt[3]{a+a\cos(c+dx)}(A+C\cos^2(c+dx))\,dx$$
$$= \frac{\sqrt[3]{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(-2(28A+13C)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\cos^2\left(\frac{dx}{2}+\arctan\left(\tan\left(\frac{c}{2}\right)\right)\right)\right)\sec\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
((a*(1 + Cos[c + d*x]))^(1/3)*Sec[(c + d*x)/2]*(-2*(28*A + 13*C)*Hypergeom
etricPFQ[{-1/2, -1/6}, {5/6}, Cos[(d*x)/2 + ArcTan[Tan[c/2]]]^2]*Sec[c/2]*
Sin[(d*x)/2 + ArcTan[Tan[c/2]]] + ((5*(28*A + 13*C)*Cos[(c - d*x - 2*ArcTa
n[Tan[c/2]])/2]*Csc[c/2]*Sec[c/2] + (28*A + 13*C)*Cos[(c + d*x + 2*ArcTan[
Tan[c/2]])/2]*Csc[c/2]*Sec[c/2] + 6*Cos[(c + d*x)/2]*Sqrt[Sec[c/2]^2]*(-((
28*A + 13*C)*Cot[c/2]) + C*(Sin[c + d*x] + 2*Sin[2*(c + d*x)])))*Sqrt[Sin[
(d*x)/2 + ArcTan[Tan[c/2]]]^2])/2))/(28*d*Sqrt[Sec[c/2]^2]*Sqrt[Sin[(d*x)/
2 + ArcTan[Tan[c/2]]]^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3503, 27, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a\cos(c+dx)+a}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3503}$$

$$\frac{3\int \frac{1}{3}\sqrt[3]{\cos(c+dx)a+a}(a(7A+4C)-3aC\cos(c+dx))dx}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt[3]{\cos(c+dx)a+a}(a(7A+4C)-3aC\cos(c+dx))dx}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(7A+4C)-3aC\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{1}{4}a(28A+13C)\int \sqrt[3]{\cos(c+dx)a+a}dx-\frac{9aC\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}a(28A+13C)\int \sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{9aC\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{3131}$$

$$\frac{\frac{a(28A+13C)\sqrt[3]{a\cos(c+dx)+a}\int \sqrt[3]{\cos(c+dx)+1}dx}{4\sqrt[3]{\cos(c+dx)+1}}-\frac{9aC\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(28A+13C)\sqrt[3]{a\cos(c+dx)+a}\int \sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx}{4\sqrt[3]{\cos(c+dx)+1}}-\frac{9aC\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{3130}$$

$$\frac{\frac{a(28A+13C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{2\sqrt[6]{2}d(\cos(c+dx)+1)^{5/6}}-\frac{9aC\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

input
```
Int[(a + a*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(a + a*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*a*d) + ((-9*a*C*(a + a*Co
s[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) + (a*(28*A + 13*C)*(a + a*Cos[c + d*
x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d
*x])/(2*2^(1/6)*d*(1 + Cos[c + d*x])^(5/6)))/(7*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3503
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a+a\cos (dx+c))^{\frac{1}{3}} \left(A+C\cos (dx+c)^{2}\right) dx$$

input
```
int((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos (dx+c)^{2}+A\right)(a\cos (dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt[3]{a\left(\cos\left(c+dx\right)+1\right)}\left(A+C\cos^2\left(c+dx\right)\right)\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(1/3)*(A + C*cos(c + d*x)**2), x)
```

Maxima [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(a + a*cos(c + d*x))^(1/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(a + a*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = a^{\frac{1}{3}}\left(\left(\int \left(\cos\left(dx+c\right)+1\right)^{\frac{1}{3}}dx\right)a+\left(\int \left(\cos\left(dx+c\right)+1\right)^{\frac{1}{3}}\cos\left(dx+c\right)^2dx\right)c\right)$$

input
```
int((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
a**(1/3)*(int((cos(c + d*x) + 1)**(1/3),x)*a + int((cos(c + d*x) + 1)**(1/
3)*cos(c + d*x)**2,x)*c)
```

3.201 $\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 135

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = -\frac{9C\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}}+\frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad} + \frac{(10A+7C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{5\ 2^{5/6}d\sqrt[6]{1+\cos(c+dx)}\sqrt[3]{a+a\cos(c+dx)}}$$

output
```
-9/10*C*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/3)+3/5*C*(a+a*cos(d*x+c))^(2/3)*s
in(d*x+c)/a/d+1/10*(10*A+7*C)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*cos(d*x+c
))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(1/6)/(a+a*cos(d*x+c))^(1/3)
```

Mathematica [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx =$$
$$-\frac{3\,2^{5/6}C\sqrt[6]{1-\cos\left(dx-2\arctan\left(\cot\left(\frac{c}{2}\right)\right)\right)}(\sin(c+dx)-\sin(2(c+dx)))+2(10A+7C)\,\text{Hyperge}}{20d\sqrt[3]{a(1+\cos(c+dx))}\sqrt[6]{\sin^2\left(\frac{dx}{2}-\text{a}\right.}}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3),x]
```

output
```
-1/20*(3*2^(5/6)*C*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6)*(Sin[c + d*x]
 - Sin[2*(c + d*x)]) + 2*(10*A + 7*C)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos
[(d*x)/2 - ArcTan[Cot[c/2]]]^2]*Sin[d*x - 2*ArcTan[Cot[c/2]]])/(d*(a*(1 +
Cos[c + d*x]))^(1/3)*(Sin[(d*x)/2 - ArcTan[Cot[c/2]]]^2)^(1/6))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3503, 27, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt[3]{a\cos(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A + C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt[3]{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3503}$$

$$\frac{3\int \frac{a(5A+2C)-3aC\cos(c+dx)}{3\sqrt[3]{\cos(c+dx)a+a}}dx}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a(5A+2C)-3aC\cos(c+dx)}{\sqrt[3]{\cos(c+dx)a+a}}dx}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a(5A+2C)-3aC\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \; 3230$$

$$\frac{\frac{1}{2}a(10A+7C)\int \frac{1}{\sqrt[3]{\cos(c+dx)a+a}}dx-\frac{9aC\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{2}a(10A+7C)\int \frac{1}{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{9aC\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \; 3131$$

$$\frac{\frac{a(10A+7C)\sqrt[3]{\cos(c+dx)+1}\int \frac{1}{\sqrt[3]{\cos(c+dx)+1}}dx}{2\sqrt[3]{a\cos(c+dx)+a}}-\frac{9aC\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{a(10A+7C)\sqrt[3]{\cos(c+dx)+1}\int \frac{1}{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{2\sqrt[3]{a\cos(c+dx)+a}}-\frac{9aC\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \text{3130}$$

$$\frac{\frac{a(10A+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{2^{5/6}d\sqrt[6]{\cos(c+dx)+1}\sqrt[3]{a\cos(c+dx)+a}}-\frac{9aC\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3),x]
```

output
```
(3*C*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*a*d) + ((-9*a*C*Sin[c + d
*x])/(2*d*(a + a*Cos[c + d*x])^(1/3)) + (a*(10*A + 7*C)*Hypergeometric2F1[
1/2, 5/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2^(5/6)*d*(1 + Cos[c +
 d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3)))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3503
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{A+C\cos\left(dx+c\right)^{2}}{\left(a+a\cos\left(dx+c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{C\cos\left(dx+c\right)^{2}+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{A+C\cos^2{(c+dx)}}{\sqrt[3]{a\,(\cos{(c+dx)}+1)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/3),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)/(a*(cos(c + d*x) + 1))**(1/3), x)
```

Maxima [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{C\cos{(dx+c)}^2+A}{(a\cos{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{C\cos{(dx+c)}^2+A}{(a\cos{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt[3]{a + a\cos(c + dx)}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\left(a + a\,\cos\left(c + d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(1/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{A + C\cos^2(c + dx)}{\sqrt[3]{a + a\cos(c + dx)}}\,dx = \frac{\left(\int \frac{\cos(dx+c)^2}{(\cos(dx+c)+1)^{\frac{1}{3}}}dx\right)c + \left(\int \frac{1}{(\cos(dx+c)+1)^{\frac{1}{3}}}dx\right)a}{a^{\frac{1}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x)
```

output
```
(int(cos(c + d*x)**2/(cos(c + d*x) + 1)**(1/3),x)*c + int(1/(cos(c + d*x)
+ 1)**(1/3),x)*a)/a**(1/3)
```

3.202 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 138

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \frac{3(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} + \frac{3C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4ad} - \frac{(4A+7C)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{2\sqrt[6]{2}ad(1+\cos(c+dx))^{5/6}}$$

output
```
3*(A+C)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(2/3)+3/4*C*(a+a*cos(d*x+c))^(1/3)*s
in(d*x+c)/a/d-1/4*(4*A+7*C)*(a+a*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[3
/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/a/d/(1+cos(d*x+c))^(5/6)
```

Mathematica [F]

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3),x]
```

output
```
Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3), x]
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3503, 27, 3042, 3229, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(a\cos(c+dx)+a)^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}}dx$$

$$\Big\downarrow \text{3503}$$

$$\frac{3\int \frac{a(4A+C)-3aC\cos(c+dx)}{3(\cos(c+dx)a+a)^{2/3}}dx}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(4A+C)-3aC\cos(c+dx)}{(\cos(c+dx)a+a)^{2/3}}dx}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(4A+C)-3aC\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}}dx}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$$\Big\downarrow \text{3229}$$

$$\frac{\frac{12a(A+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-(4A+7C)\int \sqrt[3]{\cos(c+dx)a+a}dx}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{12a(A+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-(4A+7C)\int \sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$\Big\downarrow$ 3131

$$\frac{\frac{12a(A+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{(4A+7C)\sqrt[3]{a\cos(c+dx)+a}\int \sqrt[3]{\cos(c+dx)+1}dx}{\sqrt[3]{\cos(c+dx)+1}}}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$\Big\downarrow$ 3042

$$\frac{\frac{12a(A+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{(4A+7C)\sqrt[3]{a\cos(c+dx)+a}\int \sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx}{\sqrt[3]{\cos(c+dx)+1}}}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$\Big\downarrow$ 3130

$$\frac{\frac{12a(A+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{2^{5/6}(4A+7C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{5/6}}}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

input

```
Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3),x]
```

output

```
(3*C*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*a*d) + ((12*a*(A + C)*Sin
[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) - (2^(5/6)*(4*A + 7*C)*(a + a*Co
s[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*S
in[c + d*x])/(d*(1 + Cos[c + d*x])^(5/6)))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3503
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{A + C\cos(dx+c)^2}{(a + a\cos(dx+c))^{\frac{2}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{A + C\cos^2(c+dx)}{(a + a\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos(dx+c)^2 + A}{(a\cos(dx+c) + a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F]

$$\int \frac{A + C\cos^2(c+dx)}{(a + a\cos(c+dx))^{2/3}}\,dx = \int \frac{A + C\cos^2(c+dx)}{(a(\cos(c+dx)+1))^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)/(a*(cos(c + d*x) + 1))**(2/3), x)
```

Maxima [F]

$$\int \frac{A + C\cos^2(c + dx)}{(a + a\cos(c + dx))^{2/3}}\, dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(a\cos\left(dx + c\right) + a\right)^{\frac{2}{3}}}\, dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int \frac{A + C\cos^2(c + dx)}{(a + a\cos(c + dx))^{2/3}}\, dx = \int \frac{C\cos\left(dx + c\right)^2 + A}{\left(a\cos\left(dx + c\right) + a\right)^{\frac{2}{3}}}\, dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + C\cos^2(c + dx)}{(a + a\cos(c + dx))^{2/3}}\, dx = \int \frac{C\cos\left(c + d\,x\right)^2 + A}{\left(a + a\,\cos\left(c + d\,x\right)\right)^{2/3}}\, dx$$

input
```
int((A + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(2/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{A + C\cos^2(c + dx)}{(a + a\cos(c + dx))^{2/3}}\,dx = \frac{\left(\int \frac{\cos(dx+c)^2}{(\cos(dx+c)+1)^{\frac{2}{3}}}dx\right)c + \left(\int \frac{1}{(\cos(dx+c)+1)^{\frac{2}{3}}}dx\right)a}{a^{\frac{2}{3}}}$$

input
```
int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x)
```

output
```
(int(cos(c + d*x)**2/(cos(c + d*x) + 1)**(2/3),x)*c + int(1/(cos(c + d*x)
+ 1)**(2/3),x)*a)/a**(2/3)
```

3.203 $\int (a+b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 274

$$\int (a+b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) \, dx = \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}$$
$$-\frac{3aC \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{5/3}}$$
$$+\frac{(3a^2C+b^2(8A+5C)) \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

output
```
3/8*C*(a+b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d-3/8*a*C*AppellF1(1/2,-5/3,1/2,
3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(5/3)*sin(
d*x+c)*2^(1/2)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b))^(5/3)+1
/8*(3*a^2*C+b^2*(8*A+5*C))*AppellF1(1/2,-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b
),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2)/b^2/d/(1+c
os(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b))^(2/3)
```

Mathematica [A] (warning: unable to verify)

Time = 3.31 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.02

$$\int (a+b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(60a(a^2-b^2)\,C\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(}{}}\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(60*a*(a^2 - b^2)*C*AppellF1[2
/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a +
 b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x])
)/(a - b))] + 4*(40*A*b^2 - 6*a^2*C + 25*b^2*C)*AppellF1[5/3, 1/2, 1/2, 8/
3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(
-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b
*Cos[c + d*x]) - 20*b^2*C*(2*a + 5*b*Cos[c + d*x])*Sin[c + d*x]^2))/(800*b
^3*d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3503, 27, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3503}$$

$$\frac{3\int \frac{1}{3}(a+b\cos(c+dx))^{2/3}(b(8A+5C)-3aC\cos(c+dx))dx}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\cos(c+dx))^{2/3}(b(8A+5C)-3aC\cos(c+dx))dx}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(b(8A+5C)-3aC\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{3235}$$

$$\frac{\frac{(3a^2C+b^2(8A+5C))\int (a+b\cos(c+dx))^{2/3}dx}{b}-\frac{3aC\int (a+b\cos(c+dx))^{5/3}dx}{b}}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(3a^2C+b^2(8A+5C))\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}-\frac{3aC\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{b}}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{3144}$$

$$\frac{\frac{3aC\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{5/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{(3a^2C+b^2(8A+5C))\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{2/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{156}$$

$$\frac{\frac{3aC(a+b)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}\right)^{5/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} - \frac{(3a^2C+b^2(8A+5C))\sin(c+dx)(a+b\cos(c+dx))^{2/3}\int \frac{(}{\sqrt{1}}}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\left(\frac{a}{}\right.}}{8b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$\Big\downarrow$ 155

$$\frac{\frac{\sqrt{2}(3a^2C+b^2(8A+5C))\sin(c+dx)(a+b\cos(c+dx))^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} - \frac{3\sqrt{2}aC(a+b)\sin(c+dx)(a+b}{}}{8b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

input
```
Int[(a + b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((-3*Sqrt[2]*a*(a
+ b)*C*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
 d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Co
s[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(3*a^2*C + b^
2*(8*A + 5*C))*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 -
 Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqr
t[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3503
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Maple [F]

$$\int (a+b\cos (dx+c))^{\frac{2}{3}} \left(A+C\cos (dx+c)^{2}\right) dx$$

input
```
int((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{2/3} \left(A + C\cos^2(c+dx)\right)\,dx = \int \left(C\cos (dx+c)^{2}+A\right)(b\cos (dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{2/3}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(2/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \left(\int (\cos(dx+c)\,b+a)^{\frac{2}{3}}\,dx\right)a + \left(\int (\cos(dx+c)\,b+a)^{\frac{2}{3}}\cos(dx+c)^2\,dx\right)c$$

input
```
int((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int((cos(c + d*x)*b + a)**(2/3),x)*a + int((cos(c + d*x)*b + a)**(2/3)*cos
(c + d*x)**2,x)*c
```

3.204 $\int \sqrt[3]{a+b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 274

$$\int \sqrt[3]{a+b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx = \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \frac{3\sqrt{2}aC\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{4/3}} + \frac{\sqrt{2}(3a^2C+b^2(7A+4C))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx}}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

output
```
3/7*C*(a+b*cos(d*x+c))^(4/3)*sin(d*x+c)/b/d-3/7*2^(1/2)*a*C*AppellF1(1/2,-
4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(4
/3)*sin(d*x+c)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b))^(4/3)+1
/7*2^(1/2)*(3*a^2*C+b^2*(7*A+4*C))*AppellF1(1/2,-1/3,1/2,3/2,b*(1-cos(d*x+
c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)/b^2/d/(1+c
os(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b))^(1/3)
```

Mathematica [A] (warning: unable to verify)

Time = 3.20 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(12a(a^2-b^2)C\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\cdots}\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(12*a*(a^2 - b^2)*C*AppellF1[1
/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a +
 b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x])
)/(a - b))] + (28*A*b^2 - 3*a^2*C + 16*b^2*C)*AppellF1[4/3, 1/2, 1/2, 7/3,
 (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1
 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*C
os[c + d*x]) - 4*b^2*C*(a + 4*b*Cos[c + d*x])*Sin[c + d*x]^2))/(112*b^3*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3503, 27, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3503}$$

$$\frac{3\int \frac{1}{3}\sqrt[3]{a+b\cos(c+dx)}(b(7A+4C)-3aC\cos(c+dx))dx}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt[3]{a+b\cos(c+dx)}(b(7A+4C)-3aC\cos(c+dx))dx}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(7A+4C)-3aC\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{3235}$$

$$\frac{\frac{(3a^2C+b^2(7A+4C))\int \sqrt[3]{a+b\cos(c+dx)}dx}{b}-\frac{3aC\int (a+b\cos(c+dx))^{4/3}dx}{b}}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(3a^2C+b^2(7A+4C))\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{3aC\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx}{b}}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{3144}$$

$$\frac{\frac{3aC\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{4/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{(3a^2C+b^2(7A+4C))\sin(c+dx)\int \frac{\sqrt[3]{a+b\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{156}$$

$$\frac{\frac{3aC(a+b)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\int\frac{\left(\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}\right)^{4/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(3a^2C+b^2(7A+4C))\sin(c+dx)\sqrt[3]{a+b\cos(c+}}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx}}}{7b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$\downarrow$ 155

$$\frac{\frac{\sqrt{2}(3a^2C+b^2(7A+4C))\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{3\sqrt{2}aC(a+b)\sin(c+dx}{}}{7b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

input
```
Int[(a + b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) + ((-3*Sqrt[2]*a*(a
+ b)*C*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
 d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Co
s[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(3*a^2*C + b^
2*(7*A + 4*C))*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 -
 Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqr
t[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)))/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3503
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a+b\cos(dx+c))^{\frac{1}{3}} \left(A+C\cos(dx+c)^{2}\right) dx$$

input
```
int((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^{2}+A\right)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(A+C\cos^2{(c+dx)}\right)\sqrt[3]{a+b\cos{(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)*(a + b*cos(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos{(dx+c)}^2+A\right)(b\cos{(dx+c)}+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos{(dx+c)}^2+A\right)(b\cos{(dx+c)}+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(1/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$=\left(\int \left(\cos\left(dx+c\right)b+a\right)^{\frac{1}{3}}dx\right)a+\left(\int \left(\cos\left(dx+c\right)b+a\right)^{\frac{1}{3}}\cos\left(dx+c\right)^2dx\right)c$$

input
```
int((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

output
```
int((cos(c + d*x)*b + a)**(1/3),x)*a + int((cos(c + d*x)*b + a)**(1/3)*cos
(c + d*x)**2,x)*c
```

3.205 $\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 274

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{3\sqrt{2}aC\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} + \frac{\sqrt{2}(3a^2C+b^2(5A+2C))\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}{5b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

output
```
3/5*C*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d-3/5*2^(1/2)*a*C*AppellF1(1/2,-
2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2
/3)*sin(d*x+c)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b))^(2/3)+1
/5*2^(1/2)*(3*a^2*C+b^2*(5*A+2*C))*AppellF1(1/2,1/3,1/2,3/2,b*(1-cos(d*x+c
))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*sin(d*x+c)/b^2
/d/(1+cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/3)
```

Mathematica [A] (warning: unable to verify)

Time = 2.10 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.93

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx =$$

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(5(5Ab^2+3a^2C+2b^2C)\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(}{a+b}\right.\right.}{}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(5*A*b^2 + 3*a^2*C + 2*b^2*
C)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c
 + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Co
s[c + d*x]))/(-a + b)] - 6*a*C*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c +
 d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]
))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) -
10*b^2*C*Sin[c + d*x]^2))/(50*b^3*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3503, 27, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 3503$$

$$\frac{3\int \frac{b(5A+2C)-3aC\cos(c+dx)}{3\sqrt[3]{a+b\cos(c+dx)}}dx}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$\big\downarrow$ 27

$$\frac{\int \frac{b(5A+2C)-3aC\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}dx}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{b(5A+2C)-3aC\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$\big\downarrow$ 3235

$$\frac{\frac{(3a^2C+b^2(5A+2C))\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}dx}{b}-\frac{3aC\int (a+b\cos(c+dx))^{2/3}dx}{b}}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$\big\downarrow$ 3042

$$\frac{\frac{(3a^2C+b^2(5A+2C))\int \frac{1}{\sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{3aC\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$\big\downarrow$ 3144

$$\frac{\frac{3aC\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{2/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{(3a^2C+b^2(5A+2C))\sin(c+dx)\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+}}}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$\big\downarrow$ 156

$$\frac{\frac{3aC\sin(c+dx)(a+b\cos(c+dx))^{2/3}\int\frac{\left(\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}-\frac{(3a^2C+b^2(5A+2C))\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\int -}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+}}}{5b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$\Big\downarrow$ 155

$$\frac{\frac{\sqrt{2}(3a^2C+b^2(5A+2C))\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}-\frac{3\sqrt{2}aC\sin(c+dx)(a+b\,c}{}}{5b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

output
```
(3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + ((-3*Sqrt[2]*a*C*A
ppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/
(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Cos[c + d*
x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(3*a^2*C + b^2*(5*A +
 2*C))*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(b*d*Sq
rt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3)))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3503
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{A + C\cos\left(dx + c\right)^{2}}{\left(a + b\cos\left(dx + c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{A + C\cos^{2}(c + dx)}{\sqrt[3]{a + b\cos(c + dx)}}\,dx = \int \frac{C\cos\left(dx + c\right)^{2} + A}{\left(b\cos\left(dx + c\right) + a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \frac{A + C\cos^{2}(c + dx)}{\sqrt[3]{a + b\cos(c + dx)}}\,dx = \int \frac{A + C\cos^{2}\left(c + dx\right)}{\sqrt[3]{a + b\cos\left(c + dx\right)}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/3),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)/(a + b*cos(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{C\cos{(dx+c)}^2+A}{(b\cos{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{C\cos{(dx+c)}^2+A}{(b\cos{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{C\cos{(c+d\,x)}^2+A}{{(a+b\,\cos{(c+d\,x)})}^{1/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{A + C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \left(\int \frac{\cos(dx+c)^2}{(\cos(dx+c)\,b+a)^{\frac{1}{3}}}dx\right)c + \left(\int \frac{1}{(\cos(dx+c)\,b+a)^{\frac{1}{3}}}dx\right)a$$

input
```
int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

output
```
int(cos(c + d*x)**2/(cos(c + d*x)*b + a)**(1/3),x)*c + int(1/(cos(c + d*x)
*b + a)**(1/3),x)*a
```

3.206 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 272

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}$$
$$-\frac{3aC\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{(3a^2C+b^2(4A+C))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\sin(c+dx}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

output
```
3/4*C*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)/b/d-3/4*a*C*AppellF1(1/2,-1/3,1/2,
3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(
d*x+c)*2^(1/2)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b))^(1/3)+1
/4*(3*a^2*C+b^2*(4*A+C))*AppellF1(1/2,2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1
/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(d*x+c)*2^(1/2)/b^2/d
/(1+cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(2/3)
```

Mathematica [A] (warning: unable to verify)

Time = 2.38 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.94

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx =$$

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(4Ab^2+(3a^2+b^2)\,C)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\right.}{}$$

input
```
Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(4*A*b^2 + (3*a^2 + b^2)*C)
*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
 d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[
c + d*x]))/(-a + b)] + C*(-3*a*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c +
 d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]
))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) -
4*b^2*Sin[c + d*x]^2)))/(16*b^3*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3503, 27, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 3503}$$

$$\frac{3\int \frac{b(4A+C)-3aC\cos(c+dx)}{3(a+b\cos(c+dx))^{2/3}}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{b(4A+C)-3aC\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{b(4A+C)-3aC\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$\Big\downarrow$ 3235

$$\frac{\frac{(3a^2C+b^2(4A+C))\int \frac{1}{(a+b\cos(c+dx))^{2/3}}dx}{b}-\frac{3aC\int \sqrt[3]{a+b\cos(c+dx)}dx}{b}}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(3a^2C+b^2(4A+C))\int \frac{1}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}-\frac{3aC\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$\Big\downarrow$ 3144

$$\frac{\frac{3aC\sin(c+dx)\int \frac{\sqrt[3]{a+b\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{(3a^2C+b^2(4A+C))\sin(c+dx)\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}d\cos}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}}{4b}$$

$$\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$\Big\downarrow$ 156

$$\frac{\frac{3aC\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\int \frac{\sqrt[3]{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(3a^2C+b^2(4A+C))\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\int}{bd\sqrt{1-\cos(c+dx)}\sqrt{co}}}{4b}$$

$$\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$$\Big\downarrow \; 155$$

$$\frac{\frac{\sqrt{2}(3a^2C+b^2(4A+C))\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}} - \frac{3\sqrt{2}aC\sin(c+dx)\sqrt[3]{a+b\cos}}{bd\sqrt{}}}{4b}$$

$$\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

input
```
Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) + ((-3*Sqrt[2]*a*C*A
ppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/
(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Cos[c + d*
x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(3*a^2*C + b^2*(4*A +
 C))*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*
x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(b*d*Sqrt
[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3)))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3503
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{A + C\cos(dx+c)^2}{(a+b\cos(dx+c))^{\frac{2}{3}}}dx$$

input
```
int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

output
```
int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F]

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{A+C\cos^2\left(c+dx\right)}{\left(a+b\cos\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + C*cos(c + d*x)**2)/(a + b*cos(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos\left(dx+c\right)^2+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos{(dx+c)}^2+A}{(b\cos{(dx+c)}+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos{(c+d\,x)}^2+A}{(a+b\,\cos{(c+d\,x)})^{2/3}}\,dx$$

input
```
int((A + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3),x)
```

output
```
int((A + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \left(\int \frac{\cos{(dx+c)}^2}{(\cos{(dx+c)}\,b+a)^{\frac{2}{3}}}dx\right)c + \left(\int \frac{1}{(\cos{(dx+c)}\,b+a)^{\frac{2}{3}}}dx\right)a$$

input
```
int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

output
```
int(cos(c + d*x)**2/(cos(c + d*x)*b + a)**(2/3),x)*c + int(1/(cos(c + d*x)
*b + a)**(2/3),x)*a
```

3.207 $\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 211

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right)\,dx = -\frac{4\sqrt{2}A\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right.}{f\sqrt{1+\cos(e+fx)}} + \frac{4\sqrt{2}A\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right.}{f\sqrt{1+\cos(e+fx)}}$$

output
```
-4*2^(1/2)*A*AppellF1(1/2,-m,-3/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f
*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)/f/(1+cos(f*x+e))^(1/2)/(((a+b*cos(f*x
+e))/(a+b))^m)+4*2^(1/2)*A*AppellF1(1/2,-m,-1/2,3/2,b*(1-cos(f*x+e))/(a+b)
,1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)/f/(1+cos(f*x+e))^(1/2)/
(((a+b*cos(f*x+e))/(a+b))^m)
```

Mathematica [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.56

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right)\,dx$$
$$= \frac{4A\,\text{AppellF1}\left(\frac{3}{2},-\frac{1}{2},-m,\frac{5}{2},\sin^2\left(\frac{1}{2}(e+fx)\right),\frac{2b\sin^2\left(\frac{1}{2}(e+fx)\right)}{a+b}\right)\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)}(a+b\cos(e+fx))^m\left(\right.}{3f}$$

input
```
Integrate[(a + b*Cos[e + f*x])^m*(A - A*Cos[e + f*x]^2),x]
```

output
```
(4*A*AppellF1[3/2, -1/2, -m, 5/2, Sin[(e + f*x)/2]^2, (2*b*Sin[(e + f*x)/2
]^2)/(a + b)]*Sqrt[Cos[(e + f*x)/2]^2]*(a + b*Cos[e + f*x])^m*Sin[e + f*x]
*Tan[(e + f*x)/2]^2)/(3*f*((a + b*Cos[e + f*x])/(a + b))^m)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3497, 3042, 3234, 156, 155, 3263, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A-A\cos^2(e+fx)\right)(a+b\cos(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(A-A\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \; 3497$$

$$2A\int (\cos(e+fx)+1)(a+b\cos(e+fx))^m dx - A\int (\cos(e+fx)+1)^2(a+b\cos(e+fx))^m dx$$

$$\Big\downarrow \; 3042$$

$$2A\int\left(\sin\left(e+fx+\frac{\pi}{2}\right)+1\right)\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx-A\int\left(\sin\left(e+fx+\frac{\pi}{2}\right)+1\right)^2\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$\big\downarrow$ 3234

$$-A\int\left(\sin\left(e+fx+\frac{\pi}{2}\right)+1\right)^2\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx-\frac{2A\sin(e+fx)\int\frac{\sqrt{\cos(e+fx)+1}(a+b\cos(e+fx))^m}{\sqrt{1-\cos(e+fx)}}d\cos(e+fx)}{f\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}$$

$\big\downarrow$ 156

$$-A\int\left(\sin\left(e+fx+\frac{\pi}{2}\right)+1\right)^2\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx-\frac{2A\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\int\frac{\sqrt{\cos(e+fx)+1}\left(\frac{a}{a+b}+\frac{b\cos(e+fx)}{a+b}\right)^m}{\sqrt{1-\cos(e+fx)}}d\cos(e+fx)}{f\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}$$

$\big\downarrow$ 155

$$\frac{4\sqrt{2}A\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e-}{a+b}\right.}{f\sqrt{\cos(e+fx)+1}}$$
$$A\int\left(\sin\left(e+fx+\frac{\pi}{2}\right)+1\right)^2\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$\big\downarrow$ 3263

$$\frac{A\sin(e+fx)\int\frac{(\cos(e+fx)+1)^{3/2}(a+b\cos(e+fx))^m}{\sqrt{1-\cos(e+fx)}}d\cos(e+fx)}{f\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}+\frac{4\sqrt{2}A\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e-}{a+b}\right.}{f\sqrt{\cos(e+fx)+1}}$$

$\big\downarrow$ 156

$$\frac{A\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\int\frac{(\cos(e+fx)+1)^{3/2}\left(\frac{a}{a+b}+\frac{b\cos(e+fx)}{a+b}\right)^m}{\sqrt{1-\cos(e+fx)}}d\cos(e+fx)}{f\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}+\frac{4\sqrt{2}A\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e-}{a+b}\right.}{f\sqrt{\cos(e+fx)+1}}$$

$\big\downarrow$ 155

$$\frac{4\sqrt{2}A\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e-}{a+b}\right.}{f\sqrt{\cos(e+fx)+1}}$$
$$\frac{4\sqrt{2}A\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e-}{a+b}\right.}{f\sqrt{\cos(e+fx)+1}}$$

input
```
Int[(a + b*Cos[e + f*x])^m*(A - A*Cos[e + f*x]^2),x]
```

output
```
(-4*Sqrt[2]*A*AppellF1[1/2, -3/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - C
os[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Co
s[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (4*Sqrt[2]*A*AppellF1[1/2,
 -1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a +
 b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e +
 f*x])/(a + b))^m)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3234
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]]))   Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 -
(d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2,
0]
```

rule 3263
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]]))   Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d
*x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& IntegerQ[m]
```

rule 3497
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A - C)   Int[(a + b*Sin[e + f*x])^m*(1
+ Sin[e + f*x]), x], x] + Simp[C   Int[(a + b*Sin[e + f*x])^m*(1 + Sin[e +
f*x])^2, x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A + C, 0] &&  !In
tegerQ[2*m]
```

Maple [F]

$$\int (a + b\cos(fx + e))^m \left(A - A\cos(fx + e)^2\right) dx$$

input
```
int((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x)
```

output
```
int((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right) dx = \int -\left(A\cos\left(fx+e\right)^2-A\right)\left(b\cos\left(fx+e\right)+a\right)^m dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x, algorithm="fricas")
```

output
```
integral(-(A*cos(f*x + e)^2 - A)*(b*cos(f*x + e) + a)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right) dx = \text{Timed out}$$

input
```
integrate((a+b*cos(f*x+e))**m*(A-A*cos(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right) dx = \int -\left(A\cos\left(fx+e\right)^2-A\right)\left(b\cos\left(fx+e\right)+a\right)^m dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x, algorithm="maxima")
```

output
```
-integrate((A*cos(f*x + e)^2 - A)*(b*cos(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right) dx$$
$$= \int -\left(A\cos\left(fx+e\right)^2-A\right)\left(b\cos\left(fx+e\right)+a\right)^m dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x, algorithm="giac")
```

output
```
integrate(-(A*cos(f*x + e)^2 - A)*(b*cos(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right) dx$$
$$= \int \left(A-A\cos\left(e+f\,x\right)^2\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)^m dx$$

input
```
int((A - A*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m,x)
```

output
```
int((A - A*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m, x)
```

Reduce [F]

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right) dx$$
$$= a\left(\int \left(\cos\left(fx+e\right)b+a\right)^m dx-\left(\int \left(\cos\left(fx+e\right)b+a\right)^m \cos\left(fx+e\right)^2 dx\right)\right)$$

input
```
int((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x)
```

output
```
a*(int((cos(e + f*x)*b + a)**m,x) - int((cos(e + f*x)*b + a)**m*cos(e + f*
x)**2,x))
```

3.208 $\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 286

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right)\,dx = \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$
$$-\frac{\sqrt{2}aC\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^{1+m}\left(\frac{a+b\cos(e-}{a+b}\right.}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$
$$+\frac{\sqrt{2}(a^2C+b^2(C(1+m)+A(2+m)))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$

output
```
C*(a+b*cos(f*x+e))^(1+m)*sin(f*x+e)/b/f/(2+m)-2^(1/2)*a*C*AppellF1(1/2,-1-
m,1/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^(1+m
)*((a+b*cos(f*x+e))/(a+b))^(-1-m)*sin(f*x+e)/b^2/f/(2+m)/(1+cos(f*x+e))^(1
/2)+2^(1/2)*(a^2*C+b^2*(C*(1+m)+A*(2+m)))*AppellF1(1/2,-m,1/2,3/2,b*(1-cos
(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)/b^2/f/(2+
m)/(1+cos(f*x+e))^(1/2)/(((a+b*cos(f*x+e))/(a+b))^m)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 10805 vs. $2(286) = 572$.

Time = 27.40 (sec) , antiderivative size = 10805, normalized size of antiderivative = 37.78

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3503, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+C\cos^2(e+fx)\right)(a+b\cos(e+fx))^m dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(A+C\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \; 3503$$

$$\frac{\int (a+b\cos(e+fx))^m(b(C(m+1)+A(m+2))-aC\cos(e+fx))dx}{b(m+2)}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m \left(b(C(m+1)+A(m+2))-aC\sin\left(e+fx+\frac{\pi}{2}\right)\right)dx}{b(m+2)}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{ 3235}$$

$$\frac{\frac{\left(a^2C+b^2(A(m+2)+C(m+1))\right)\int(a+b\cos(e+fx))^m dx}{b}-\frac{aC\int(a+b\cos(e+fx))^{m+1}dx}{b}}{b(m+2)}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\left(a^2C+b^2(A(m+2)+C(m+1))\right)\int\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx}{b}-\frac{aC\int\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}dx}{b}}{b(m+2)}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{ 3144}$$

$$\frac{\frac{aC\sin(e+fx)\int\frac{(a+b\cos(e+fx))^{m+1}}{\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}d\cos(e+fx)}{bf\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}-\frac{\sin(e+fx)\left(a^2C+b^2(A(m+2)+C(m+1))\right)\int\frac{(a+b\cos(e+fx))^m}{\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}d\cos(e+fa}{bf\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}}{b(m+2)}$$

$$\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{ 156}$$

$$\frac{\frac{aC(a+b)\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\int\frac{\left(\frac{a}{a+b}+\frac{b\cos(e+fx)}{a+b}\right)^{m+1}}{\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}d\cos(e+fx)}{bf\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}-\frac{\sin(e+fx)\left(a^2C+b^2(A(m+2)+C(m+\right.}{}}{b(m+2)}$$

$$\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{ 155}$$

$$\frac{\frac{\sqrt{2}\sin(e+fx)\left(a^2C+b^2(A(m+2)+C(m+1))\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{bf\sqrt{\cos(e+fx)+1}}-}{b(m+2)}$$

$$\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

input
```
Int[(a + b*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2),x]
```

output
```
(C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) + (-((Sqrt[2]*
a*(a + b)*C*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 -
Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b*f*Sqrt[1 +
 Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m)) + (Sqrt[2]*(a^2*C + b^2*
(C*(1 + m) + A*(2 + m)))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Cos[e + f*x])/2,
 (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b*f
*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m))/(b*(2 + m))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3503
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a+b\cos(fx+e))^m \left(A+C\cos(fx+e)^2\right) dx$$

input
```
int((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)
```

output
```
int((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx = \int \left(C\cos(fx+e)^2+A\right)(b\cos(fx+e)+a)^m dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="fricas")
```

output
```
integral((C*cos(f*x + e)^2 + A)*(b*cos(f*x + e) + a)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) \, dx = \text{Timed out}$$

input
```
integrate((a+b*cos(f*x+e))**m*(A+C*cos(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) \, dx = \int \left(C\cos{(fx+e)}^2+A\right)(b\cos{(fx+e)}+a)^m \, dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="maxima")
```

output
```
integrate((C*cos(f*x + e)^2 + A)*(b*cos(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) \, dx = \int \left(C\cos{(fx+e)}^2+A\right)(b\cos{(fx+e)}+a)^m \, dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="giac")
```

output
```
integrate((C*cos(f*x + e)^2 + A)*(b*cos(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) \, dx = \int \left(C\cos\left(e+f\,x\right)^2+A\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)^m \,dx$$

input
```
int((A + C*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m,x)
```

output
```
int((A + C*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m, x)
```

Reduce [F]

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) \, dx = \left(\int \left(\cos\left(fx+e\right)b+a\right)^m dx\right)a+\left(\int \left(\cos\left(fx+e\right)b+a\right)^m \cos\left(fx+e\right)^2 dx\right)c$$

input
```
int((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)
```

output
```
int((cos(e + f*x)*b + a)**m,x)*a + int((cos(e + f*x)*b + a)**m*cos(e + f*x
)**2,x)*c
```

3.209 $\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)$

Optimal result

Integrand size = 30, antiderivative size = 141

$$\begin{aligned}\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx\\ =-\frac{B(a\cos(e+fx))^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{a^2f(2+m)\sqrt{\sin^2(e+fx)}}\\ -\frac{C(a\cos(e+fx))^{3+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{a^3f(3+m)\sqrt{\sin^2(e+fx)}}\end{aligned}$$

output
```
-B*(a*cos(f*x+e))^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],cos(f*x+e)^2)*s
in(f*x+e)/a^2/f/(2+m)/(sin(f*x+e)^2)^(1/2)-C*(a*cos(f*x+e))^(3+m)*hypergeo
m([1/2, 3/2+1/2*m],[5/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/a^3/f/(3+m)/(sin(f
*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx =$$
$$-\frac{\cos(e+fx)(a\cos(e+fx))^m\cot(e+fx)\left(B(3+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(e+fx)\right)\right.}{f(2+m)(3+}$$

input
```
Integrate[(a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

output
```
-((Cos[e + f*x]*(a*Cos[e + f*x])^m*Cot[e + f*x]*(B*(3 + m)*Hypergeometric2
F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2] + C*(2 + m)*Cos[e + f*x]*Hyp
ergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*
x]^2])/(f*(2 + m)*(3 + m)))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(B\sin\left(e+fx+\frac{\pi}{2}\right)+C\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3489}$$

$$\frac{\int (a\cos(e+fx))^{m+1}(B+C\cos(e+fx))dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}\left(B+C\sin\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a}$$

$$\Big\downarrow \; 3227$$

$$\frac{B\int (a\cos(e+fx))^{m+1}dx+\frac{C\int (a\cos(e+fx))^{m+2}dx}{a}}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{B\int \left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}dx+\frac{C\int \left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{m+2}dx}{a}}{a}$$

$$\Big\downarrow \; 3122$$

$$\frac{-\frac{C\sin(e+fx)(a\cos(e+fx))^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},\cos^2(e+fx)\right)}{a^2f(m+3)\sqrt{\sin^2(e+fx)}}-\frac{B\sin(e+fx)(a\cos(e+fx))^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{}\right.}{af(m+2)\sqrt{\sin^2(e+fx)}}}{a}$$

input
```
Int[(a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

output
```
(-((B*(a*Cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2
, Cos[e + f*x]^2]*Sin[e + f*x])/(a*f*(2 + m)*Sqrt[Sin[e + f*x]^2])) - (C*(
a*Cos[e + f*x])^(3 + m)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[e
 + f*x]^2]*Sin[e + f*x])/(a^2*f*(3 + m)*Sqrt[Sin[e + f*x]^2]))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int (a\cos(fx+e))^m \left(B\cos(fx+e)+C\cos(fx+e)^2\right)dx$$

input
```
int((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
int((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos(fx+e)^2+B\cos(fx+e)\right)(a\cos(fx+e))^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fri
cas")
```

output
```
integral((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e))^m, x)
```

Sympy [F]

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \int (a\cos{(e+fx)})^m (B+C\cos{(e+fx)})\cos{(e+fx)}\,dx$$

input
```
integrate((a*cos(f*x+e))**m*(B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

output
```
Integral((a*cos(e + f*x))**m*(B + C*cos(e + f*x))*cos(e + f*x), x)
```

Maxima [F]

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \int \left(C\cos{(fx+e)}^2 + B\cos{(fx+e)}\right)(a\cos{(fx+e)})^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="max
ima")
```

output
```
integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \int \left(C\cos{(fx+e)}^2 + B\cos{(fx+e)}\right)(a\cos{(fx+e)})^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="gia
c")
```

output
```
integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \int \left(a\,\cos\left(e+f\,x\right)\right)^m\,\left(C\cos\left(e+f\,x\right)^2+B\,\cos\left(e+f\,x\right)\right)\,dx$$

input
```
int((a*cos(e + f*x))^m*(B*cos(e + f*x) + C*cos(e + f*x)^2),x)
```

output
```
int((a*cos(e + f*x))^m*(B*cos(e + f*x) + C*cos(e + f*x)^2), x)
```

Reduce [F]

$$\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = a^m\left(\left(\int \cos\left(fx+e\right)^m\cos\left(fx+e\right)dx\right)b+\left(\int \cos\left(fx+e\right)^m\cos\left(fx+e\right)^2dx\right)c\right)$$

input
```
int((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
a**m*(int(cos(e + f*x)**m*cos(e + f*x),x)*b + int(cos(e + f*x)**m*cos(e +
f*x)**2,x)*c)
```

3.210 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(B\cos(c+dx)+C\cos$

Optimal result

Integrand size = 40, antiderivative size = 167

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3B\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)\sin(}{d(7+3m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3C\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(10+3m),\frac{1}{6}(16+3m),\cos^2(c+dx)\right)\sin}{d(10+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
-3*B*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6+1/2*m],[13/
6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d
*x+c)^(3+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 5/3+1/2*m],[8/3+1/2*m],co
s(d*x+c)^2)*sin(d*x+c)/d/(10+3*m)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\left(C(7+3m)\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3}+\frac{m}{2},\frac{8}{3}\right.\right.}{d(7}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c
+ d*x]^2),x]
```

output
```
(-3*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(C*(7 + 3*m)*
Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2]
+ B*(10 + 3*m)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d
*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 3*m)*(10 + 3*m))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\cos(c+dx)}\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 2034$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{m+\frac{1}{3}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3489}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{m+\frac{4}{3}}(c+dx)(B+C\cos(c+dx))dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(B\int \cos^{m+\frac{4}{3}}(c+dx)dx+C\int \cos^{m+\frac{7}{3}}(c+dx)dx\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}dx+C\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{7}{3}}dx\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(-\frac{3B\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+7),\frac{1}{6}(3m+13),\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}-\frac{3C\sin(c+dx)\cos^{m+\frac{10}{3}}(c+}{}\right.}{\sqrt[3]{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]
^2),x]
```

output
```
((b*Cos[c + d*x])^(1/3)*((-3*B*Cos[c + d*x]^(7/3 + m)*Hypergeometric2F1[1/
2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*S
qrt[Sin[c + d*x]^2]) - (3*C*Cos[c + d*x]^(10/3 + m)*Hypergeometric2F1[1/2,
 (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(10 + 3*m)*S
qrt[Sin[c + d*x]^2])))/Cos[c + d*x]^(1/3)
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \cos (dx+c)^m (b\cos (dx+c))^{\frac{1}{3}} \left(B\cos (dx+c)+C\cos (dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*
x + c)^m, x)
```

Sympy [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt[3]{b\cos\left(c+dx\right)}\left(B+C\cos\left(c+dx\right)\right)\cos\left(c+dx\right)\cos^m\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

output
```
Integral((b*cos(c + d*x))**(1/3)*(B + C*cos(c + d*x))*cos(c + d*x)*cos(c +
 d*x)**m, x)
```

Maxima [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d
*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d
*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)^m\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(B*cos(c + d*x) + C*cos(c + d*x)
^2),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(B*cos(c + d*x) + C*cos(c + d*x)
^2), x)
```

Reduce [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=b^{\frac{1}{3}}\left(\left(\int \cos\left(dx+c\right)^{m+\frac{1}{3}}\cos\left(dx+c\right)dx\right)b+\left(\int \cos\left(dx+c\right)^{m+\frac{1}{3}}\cos\left(dx+c\right)^2dx\right)c\right)$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*(int(cos(c + d*x)**((3*m + 1)/3)*cos(c + d*x),x)*b + int(cos(c +
d*x)**((3*m + 1)/3)*cos(c + d*x)**2,x)*c)
```

3.211 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\,\text{c}\right.$

Optimal result

Integrand size = 40, antiderivative size = 167

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3B\cos^{2+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(8+3m),\frac{1}{6}(14+3m),\cos^2(c+dx)\right)\sin(c}{d(8+3m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3C\cos^{3+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(11+3m),\frac{1}{6}(17+3m),\cos^2(c+dx)\right)\sin(}{d(11+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
-3*B*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 4/3+1/2*m],[7/3
+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(8+3*m)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d*
x+c)^(3+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 11/6+1/2*m],[17/6+1/2*m],c
os(d*x+c)^2)*sin(d*x+c)/d/(11+3*m)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\cos^{2+m}(c+dx)(b\cos(c+dx))^{2/3}\csc(c+dx)\left(B(11+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(8+3m),\frac{7}{3}+\frac{m}{2},\text{c}\right.\right.}{d(8}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c
+ d*x]^2),x]
```

output
```
(-3*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(B*(11 + 3*m)
*Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2] + C*(8 + 3
*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (11 + 3*m)/6, (17 + 3*m)/6, Cos[c
+ d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(8 + 3*m)*(11 + 3*m))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{2/3}\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{(b\cos(c+dx))^{2/3}\int \cos^{m+\frac{2}{3}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(b\cos(c+dx))^{2/3}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 3489$$

$$\frac{(b\cos(c+dx))^{2/3}\int \cos^{m+\frac{5}{3}}(c+dx)(B+C\cos(c+dx))dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{(b\cos(c+dx))^{2/3}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{5}{3}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 3227$$

$$\frac{(b\cos(c+dx))^{2/3}\left(B\int \cos^{m+\frac{5}{3}}(c+dx)dx+C\int \cos^{m+\frac{8}{3}}(c+dx)dx\right)}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{(b\cos(c+dx))^{2/3}\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{5}{3}}dx+C\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{8}{3}}dx\right)}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 3122$$

$$\frac{(b\cos(c+dx))^{2/3}\left(-\frac{3B\sin(c+dx)\cos^{m+\frac{8}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+8),\frac{1}{6}(3m+14),\cos^2(c+dx)\right)}{d(3m+8)\sqrt{\sin^2(c+dx)}}-\frac{3C\sin(c+dx)\cos^{m+\frac{11}{3}}(\cdots}{\cdots}\right.}{\cos^{\frac{2}{3}}(c+dx)}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]
^2),x]
```

output
```
((b*Cos[c + d*x])^(2/3)*((-3*B*Cos[c + d*x]^(8/3 + m)*Hypergeometric2F1[1/
2, (8 + 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(8 + 3*m)*S
qrt[Sin[c + d*x]^2]) - (3*C*Cos[c + d*x]^(11/3 + m)*Hypergeometric2F1[1/2,
 (11 + 3*m)/6, (17 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(11 + 3*m)*S
qrt[Sin[c + d*x]^2])))/Cos[c + d*x]^(2/3)
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \cos(dx+c)^m (b\cos(dx+c))^{\frac{2}{3}} \left(B\cos(dx+c)+C\cos(dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(2/3)*cos(d*
x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(2/3)*cos(d
*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(2/3)*cos(d
*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)^m\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(B*cos(c + d*x) + C*cos(c + d*x)
^2),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(B*cos(c + d*x) + C*cos(c + d*x)
^2), x)
```

Reduce [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = b^{\frac{2}{3}}\left(\left(\int \cos(dx+c)^{m+\frac{2}{3}}\cos(dx+c)\,dx\right)b+\left(\int \cos(dx+c)^{m+\frac{2}{3}}\cos(dx+c)^2\,dx\right)c\right)$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**((3*m + 2)/3)*cos(c + d*x),x)*b + int(cos(c +
d*x)**((3*m + 2)/3)*cos(c + d*x)**2,x)*c)
```

3.212 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\text{ c}\right.$

Optimal result

Integrand size = 40, antiderivative size = 169

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3bB\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(10+3m),\frac{1}{6}(16+3m),\cos^2(c+dx)\right)\sin(c}{d(10+3m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3bC\cos^{4+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(13+3m),\frac{1}{6}(19+3m),\cos^2(c+dx)\right)\sin(c}{d(13+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
-3*b*B*cos(d*x+c)^(3+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 5/3+1/2*m],[8
/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(10+3*m)/(sin(d*x+c)^2)^(1/2)-3*b*C*c
os(d*x+c)^(4+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 13/6+1/2*m],[19/6+1/2
*m],cos(d*x+c)^2)*sin(d*x+c)/d/(13+3*m)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.83

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3\cos^{2+m}(c+dx)(b\cos(c+dx))^{4/3}\csc(c+dx)\left(B(13+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3}+\frac{m}{2},\frac{8}{3}+\frac{m}{2},\cos^2(\right.\right.}{d(10}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(B*Cos[c + d*x] + C*Cos[c
+ d*x]^2),x]
```

output
```
(-3*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(B*(13 + 3*m)
*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] + C*(10 + 3*
m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (13 + 3*m)/6, (19 + 3*m)/6, Cos[c +
 d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(10 + 3*m)*(13 + 3*m))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{4/3}\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2034}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\int \cos^{m+\frac{4}{3}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \; 3489$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\int \cos^{m+\frac{7}{3}}(c+dx)(B+C\cos(c+dx))dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{7}{3}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \; 3227$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(B\int \cos^{m+\frac{7}{3}}(c+dx)dx+C\int \cos^{m+\frac{10}{3}}(c+dx)dx\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{7}{3}}dx+C\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{10}{3}}dx\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \; 3122$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(-\frac{3B\sin(c+dx)\cos^{m+\frac{10}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+10),\frac{1}{6}(3m+16),\cos^2(c+dx)\right)}{d(3m+10)\sqrt{\sin^2(c+dx)}}-\frac{3C\sin(c+dx)\cos^{m+\frac{13}{3}}}{\cdots}\right.}{\sqrt[3]{\cos(c+dx)}}$$

input

```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]
^2),x]
```

output

```
(b*(b*Cos[c + d*x])^(1/3)*((-3*B*Cos[c + d*x]^(10/3 + m)*Hypergeometric2F1
[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(10 + 3
*m)*Sqrt[Sin[c + d*x]^2]) - (3*C*Cos[c + d*x]^(13/3 + m)*Hypergeometric2F1
[1/2, (13 + 3*m)/6, (19 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(13 + 3
*m)*Sqrt[Sin[c + d*x]^2])))/Cos[c + d*x]^(1/3)
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \cos (dx+c)^m (b\cos (dx+c))^{\frac{4}{3}} \left(B\cos (dx+c)+C\cos (dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="fricas")
```

output
```
integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3)*
cos(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(4/3)*cos(d
*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(4/3)*cos(d
*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int {\cos\left(c+d\,x\right)}^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(B*cos(c + d*x) + C*cos(c + d*x)
^2),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(B*cos(c + d*x) + C*cos(c + d*x)
^2), x)
```

Reduce [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = b^{\frac{4}{3}}\left(\left(\int \cos(dx+c)^{m+\frac{1}{3}}\cos(dx+c)^3\,dx\right)c + \left(\int \cos(dx+c)^{m+\frac{1}{3}}\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**((3*m + 1)/3)*cos(c + d*x)**3,x)*c + int(cos
(c + d*x)**((3*m + 1)/3)*cos(c + d*x)**2,x)*b)
```

3.213 $\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 167

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3C\cos^{3+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(8+3m),\frac{1}{6}(14+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(8+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
-3*B*cos(d*x+c)^(2+m)*hypergeom([1/2, 5/6+1/2*m],[11/6+1/2*m],cos(d*x+c)^2
)*sin(d*x+c)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d
*x+c)^(3+m)*hypergeom([1/2, 4/3+1/2*m],[7/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c
)/d/(8+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\cos^{2+m}(c+dx)\csc(c+dx)\left(B(8+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx\right.\right.}{d(5+3m)(8+3r}$$

input
```
Integrate[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
d*x])^(1/3),x]
```

output
```
(-3*Cos[c + d*x]^(2 + m)*Csc[c + d*x]*(B*(8 + 3*m)*Hypergeometric2F1[1/2,
(5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + C*(5 + 3*m)*Cos[c + d*x]*Hype
rgeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(5 + 3*m)*(8 + 3*m)*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \cos^{m-\frac{1}{3}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt[3]{b\cos(c+dx)}}$$

$\downarrow$ 3489

$$\frac{\sqrt[3]{\cos(c+dx)}\int \cos^{m+\frac{2}{3}}(c+dx)(B+C\cos(c+dx))dx}{\sqrt[3]{b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt[3]{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{\sqrt[3]{b\cos(c+dx)}}$$

$\downarrow$ 3227

$$\frac{\sqrt[3]{\cos(c+dx)}\left(B\int \cos^{m+\frac{2}{3}}(c+dx)dx+C\int \cos^{m+\frac{5}{3}}(c+dx)dx\right)}{\sqrt[3]{b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt[3]{\cos(c+dx)}\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}dx+C\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{5}{3}}dx\right)}{\sqrt[3]{b\cos(c+dx)}}$$

$\downarrow$ 3122

$$\frac{\sqrt[3]{\cos(c+dx)}\left(-\frac{3B\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+5),\frac{1}{6}(3m+11),\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}}-\frac{3C\sin(c+dx)\cos^{m+\frac{8}{3}}(c+dx)}{}\right.}{\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^
(1/3),x]
```

output
```
(Cos[c + d*x]^(1/3)*((-3*B*Cos[c + d*x]^(5/3 + m)*Hypergeometric2F1[1/2, (
5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*Sqrt[
Sin[c + d*x]^2]) - (3*C*Cos[c + d*x]^(8/3 + m)*Hypergeometric2F1[1/2, (8 +
 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(8 + 3*m)*Sqrt[Sin
[c + d*x]^2])))/(b*Cos[c + d*x])^(1/3)
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \frac{\cos\left(dx+c\right)^{m}\left(B\cos\left(dx+c\right)+C\cos\left(dx+c\right)^{2}\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/b, x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(B+C\cos\left(c+dx\right)\right)\cos\left(c+dx\right)\cos^m\left(c+dx\right)}{\sqrt[3]{b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**m*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/
3),x)
```

output
```
Integral((B + C*cos(c + d*x))*cos(c + d*x)*cos(c + d*x)**m/(b*cos(c + d*x)
)**(1/3), x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x +
c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x +
c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

input
```
int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^
(1/3),x)
```

output
```
int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^
(1/3), x)
```

Reduce [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{\left(\int \cos\left(dx+c\right)^m \cos\left(dx+c\right)^{\frac{2}{3}}dx\right)b+\left(\int \cos\left(dx+c\right)^m \cos\left(dx+c\right)^{\frac{5}{3}}dx\right)c}{b^{\frac{1}{3}}}$$

input
```
int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int((cos(c + d*x)**m*cos(c + d*x))/cos(c + d*x)**(1/3),x)*b + int((cos(c
+ d*x)**m*cos(c + d*x)**2)/cos(c + d*x)**(1/3),x)*c)/b**(1/3)
```

3.214 $\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 167

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3C\cos^{3+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(7+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3*B*cos(d*x+c)^(2+m)*hypergeom([1/2, 2/3+1/2*m],[5/3+1/2*m],cos(d*x+c)^2)
*sin(d*x+c)/d/(4+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d*
x+c)^(3+m)*hypergeom([1/2, 7/6+1/2*m],[13/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c
)/d/(7+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx =$$
$$-\frac{3\cos^{2+m}(c+dx)\csc(c+dx)\left(B(7+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{5}{3}+\frac{m}{2},\cos^2(c+dx)\right)+C(}{d(4+3m)(7+3m)(b}$$

input
```
Integrate[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
d*x])^(2/3),x]
```

output
```
(-3*Cos[c + d*x]^(2 + m)*Csc[c + d*x]*(B*(7 + 3*m)*Hypergeometric2F1[1/2,
(4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2] + C*(4 + 3*m)*Cos[c + d*x]*Hyperge
ometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(4 + 3*m)*(7 + 3*m)*(b*Cos[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$$
$$\Big\downarrow \ 2034$$
$$\frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{m-\frac{2}{3}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx}{(b\cos(c+dx))^{2/3}}$$
$$\Big\downarrow \ 3042$$
$$\frac{\cos^{\frac{2}{3}}(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{2}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3489}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{m+\frac{1}{3}}(c+dx)(B+C\cos(c+dx))dx}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(B\int \cos^{m+\frac{1}{3}}(c+dx)dx+C\int \cos^{m+\frac{4}{3}}(c+dx)dx\right)}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}dx+C\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}dx\right)}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(-\frac{3B\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+4),\frac{1}{6}(3m+10),\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}}-\frac{3C\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)}{}\right.}{(b\cos(c+dx))^{2/3}}$$

input
```
Int[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^
(2/3),x]
```

output
```
(Cos[c + d*x]^(2/3)*((-3*B*Cos[c + d*x]^(4/3 + m)*Hypergeometric2F1[1/2, (
4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*Sqrt[
Sin[c + d*x]^2]) - (3*C*Cos[c + d*x]^(7/3 + m)*Hypergeometric2F1[1/2, (7 +
 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[Sin
[c + d*x]^2])))/(b*Cos[c + d*x])^(2/3)
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \frac{\cos(dx+c)^m \left(B\cos(dx+c)+C\cos(dx+c)^2\right)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

output
```
int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}$$

input
```
integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/b, x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\left(B+C\cos\left(c+dx\right)\right)\cos\left(c+dx\right)\cos^m\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{2}{3}}}$$

input
```
integrate(cos(d*x+c)**m*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/
3),x)
```

output
```
Integral((B + C*cos(c + d*x))*cos(c + d*x)*cos(c + d*x)**m/(b*cos(c + d*x)
)**(2/3), x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}$$

input
```
integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x +
c))^(2/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}$$

input
```
integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x +
c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}$$

input
```
int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^
(2/3),x)
```

output
```
int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^
(2/3), x)
```

Reduce [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\left(\int \cos\left(dx+c\right)^m\cos\left(dx+c\right)^{\frac{1}{3}}dx\right)b+\left(\int \cos\left(dx\right.\right.}{b^{\frac{2}{3}}}$$

input
```
int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

output
```
(int((cos(c + d*x)**m*cos(c + d*x))/cos(c + d*x)**(2/3),x)*b + int((cos(c
+ d*x)**m*cos(c + d*x)**2)/cos(c + d*x)**(2/3),x)*c)/b**(2/3)
```

3.215 $\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 173

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3B\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3C\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+dx)}{bd(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
-3*B*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x+c)^2)
*sin(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*C*cos(
d*x+c)^(2+m)*hypergeom([1/2, 5/6+1/2*m],[11/6+1/2*m],cos(d*x+c)^2)*sin(d*x
+c)/b/d/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx =$$
$$-\frac{3\cos^{2+m}(c+dx)\csc(c+dx)\left(B(5+3m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)+\right.}{d(2+3m)(5+3m)}$$

input
```
Integrate[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
d*x])^(4/3),x]
```

output
```
(-3*Cos[c + d*x]^(2 + m)*Csc[c + d*x]*(B*(5 + 3*m)*Hypergeometric2F1[1/2,
(2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + C*(2 + 3*m)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c +
 d*x]^2])/(d*(2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \cos^{m-\frac{4}{3}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{4}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3489}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \cos^{m-\frac{1}{3}}(c+dx)(B+C\cos(c+dx))dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(B\int \cos^{m-\frac{1}{3}}(c+dx)dx+C\int \cos^{m+\frac{2}{3}}(c+dx)dx\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}dx+C\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}dx\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(-\frac{3B\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+2),\frac{1}{6}(3m+8),\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}}-\frac{3C\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)}{}\right.}{b\sqrt[3]{b\cos(c+dx)}}$$

input

```
Int[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^
(4/3),x]
```

output

```
(Cos[c + d*x]^(1/3)*((-3*B*Cos[c + d*x]^(2/3 + m)*Hypergeometric2F1[1/2, (
2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*Sqrt[S
in[c + d*x]^2]) - (3*C*Cos[c + d*x]^(5/3 + m)*Hypergeometric2F1[1/2, (5 +
3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*Sqrt[Sin[
c + d*x]^2])))/(b*(b*Cos[c + d*x])^(1/3))
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] &&
 !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \frac{\cos (dx+c)^m \left(B\cos (dx+c)+C\cos (dx+c)^2\right)}{(b\cos (dx+c))^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*c
os(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(B+C\cos\left(c+dx\right)\right)\cos\left(c+dx\right)\cos^m\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)**m*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/
3),x)
```

output
```
Integral((B + C*cos(c + d*x))*cos(c + d*x)*cos(c + d*x)**m/(b*cos(c + d*x)
)**(4/3), x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x +
c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x +
c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\cos\left(c+d\,x\right)^m\,\left(C\,\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}$$

input
```
int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^
(4/3),x)
```

output
```
int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^
(4/3), x)
```

Reduce [F]

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\left(\int \frac{\cos(dx+c)^m}{\cos(dx+c)^{\frac{1}{3}}}dx\right)b+\left(\int \cos\left(dx+c\right)^m \cos\left(dx+c\right.\right.}{b^{\frac{4}{3}}}$$

input
```
int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(cos(c + d*x)**m/cos(c + d*x)**(1/3),x)*b + int((cos(c + d*x)**m*cos(c
 + d*x))/cos(c + d*x)**(1/3),x)*c)/(b**(1/3)*b)
```

3.216 $\int (a\cos(c+dx))^m(b\cos(c+dx))^n\,(B\cos(c+dx)+C$

Optimal result

Integrand size = 40, antiderivative size = 167

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{B(a\cos(c+dx))^{2+m}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2+m+n),\frac{1}{2}(4+m+n),\cos^2(c+d\ldots\right.}{a^2d(2+m+n)\sqrt{\sin^2(c+dx)}}$$

$$-\frac{C(a\cos(c+dx))^{3+m}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(3+m+n),\frac{1}{2}(5+m+n),\cos^2(c+d\ldots\right.}{a^3d(3+m+n)\sqrt{\sin^2(c+dx)}}$$

output
```
-B*(a*cos(d*x+c))^(2+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1+1/2*m+1/2*n],[2
+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a^2/d/(2+m+n)/(sin(d*x+c)^2)^(1/2)-
C*(a*cos(d*x+c))^(3+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/2+1/2*m+1/2*n],[
5/2+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a^3/d/(3+m+n)/(sin(d*x+c)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.81

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx =$$
$$-\frac{\cos(c+dx)(a\cos(c+dx))^m(b\cos(c+dx))^n\cot(c+dx)\left(B(3+m+n)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2+\right.\right.}{}$$

input
```
Integrate[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c
+ d*x]^2),x]
```

output
```
-((Cos[c + d*x]*(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(3 +
 m + n)*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^
2] + C*(2 + m + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + m + n)/2, (5 +
 m + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(2 + m + n)*(3 + m +
n)))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$

$$\Big\downarrow \text{2034}$$

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\int (a\cos(c+dx))^{m+n}\left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{m+n}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3489}$$

$$\frac{(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\int(a\cos(c+dx))^{m+n+1}(B+C\cos(c+dx))dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\int\left(a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{m+n+1}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a}$$

$$\Big\downarrow \text{3227}$$

$$\frac{(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\left(B\int(a\cos(c+dx))^{m+n+1}dx+\frac{C\int(a\cos(c+dx))^{m+n+2}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\left(B\int\left(a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{m+n+1}dx+\frac{C\int\left(a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{m+n+2}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3122}$$

$$\frac{(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\left(-\frac{C\sin(c+dx)(a\cos(c+dx))^{m+n+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m+n+3),\frac{1}{2}(m+n+5),\cos^2(c+dx)\right)}{a^2d(m+n+3)\sqrt{\sin^2(c+dx)}}\right.}{a}$$

input
```
Int[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]
^2),x]
```

output
```
((b*Cos[c + d*x])^n*(-((B*(a*Cos[c + d*x])^(2 + m + n)*Hypergeometric2F1[1
/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a*d*(2 +
m + n)*Sqrt[Sin[c + d*x]^2])) - (C*(a*Cos[c + d*x])^(3 + m + n)*Hypergeome
tric2F1[1/2, (3 + m + n)/2, (5 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(
a^2*d*(3 + m + n)*Sqrt[Sin[c + d*x]^2])))/(a*(a*Cos[c + d*x])^n)
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int (a\cos(dx+c))^m (b\cos(dx+c))^n \left(B\cos(dx+c)+C\cos(dx+c)^2\right) dx$$

input
```
int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(a\cos\left(dx+c\right)\right)^m\left(b\cos\left(dx+c\right)\right)^n \, dx$$

input
```
integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c))^m*(b*cos(d*x
 + c))^n, x)
```

Sympy [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$
$$= \int \left(a\cos\left(c+dx\right)\right)^m\left(b\cos\left(c+dx\right)\right)^n\left(B+C\cos\left(c+dx\right)\right)\cos\left(c+dx\right)dx$$

input
```
integrate((a*cos(d*x+c))**m*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

output
```
Integral((a*cos(c + d*x))**m*(b*cos(c + d*x))**n*(B + C*cos(c + d*x))*cos(
c + d*x), x)
```

Maxima [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(a\cos\left(dx+c\right)\right)^m\left(b\cos\left(dx+c\right)\right)^n\,dx$$

input
```
integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c))^m*(b*cos(d*
x + c))^n, x)
```

Giac [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(a\cos\left(dx+c\right)\right)^m\left(b\cos\left(dx+c\right)\right)^n\,dx$$

input
```
integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c))^m*(b*cos(d*
x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cos(c+dx))^m (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (a\,\cos\left(c+d\,x\right))^m\,(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int((a*cos(c + d*x))^m*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)
^2),x)
```

output
```
int((a*cos(c + d*x))^m*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)
^2), x)
```

Reduce [F]

$$\int (a\cos(c+dx))^m (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=b^n a^m \left(\left(\int \cos\left(dx+c\right)^{m+n}\cos\left(dx+c\right)dx\right)b + \left(\int \cos\left(dx+c\right)^{m+n}\cos\left(dx+c\right)^2 dx\right)c\right)$$

input
```
int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*a**m*(int(cos(c + d*x)**(m + n)*cos(c + d*x),x)*b + int(cos(c + d*x)*
*(m + n)*cos(c + d*x)**2,x)*c)
```

3.217 $\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 38, antiderivative size = 141

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = -\frac{B(b\cos(c+dx))^{4+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{2},\frac{6+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^4d(4+n)\sqrt{\sin^2(c+dx)}} - \frac{C(b\cos(c+dx))^{5+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+n}{2},\frac{7+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^5d(5+n)\sqrt{\sin^2(c+dx)}}$$

output
```
-B*(b*cos(d*x+c))^(4+n)*hypergeom([1/2, 2+1/2*n],[3+1/2*n],cos(d*x+c)^2)*s
in(d*x+c)/b^4/d/(4+n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(5+n)*hypergeo
m([1/2, 5/2+1/2*n],[7/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^5/d/(5+n)/(sin(d
*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx =$$
$$-\frac{\cos^3(c+dx)(b\cos(c+dx))^n \cot(c+dx) \left(B(5+n) \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{2},\frac{6+n}{2},\cos^2(c+dx)\right)+\right.}{d(4+n)(5+n)}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*
x]^2),x]
```

output
```
-((Cos[c + d*x]^3*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(5 + n)*Hypergeometri
c2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2] + C*(4 + n)*Cos[c + d*x]*H
ypergeometric2F1[1/2, (5 + n)/2, (7 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(4 + n)*(5 + n)))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2030, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{n+2} \left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2} \left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2}$$

$$\Big\downarrow \text{3489}$$

$$\frac{\int (b\cos(c+dx))^{n+3}(B+C\cos(c+dx))dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+3}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^3}$$

$$\Big\downarrow \text{3227}$$

$$\frac{B\int (b\cos(c+dx))^{n+3}dx+\frac{C\int (b\cos(c+dx))^{n+4}dx}{b}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+3}dx+\frac{C\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+4}dx}{b}}{b^3}$$

$$\Big\downarrow \text{3122}$$

$$\frac{-\frac{C\sin(c+dx)(b\cos(c+dx))^{n+5}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+5}{2},\frac{n+7}{2},\cos^2(c+dx)\right)}{b^2d(n+5)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+4}{2},\frac{n}{}\right.}{bd(n+4)\sqrt{\sin^2(c+dx)}}}{b^3}$$

input

```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),
x]
```

output

```
(-((B*(b*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2
, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(4 + n)*Sqrt[Sin[c + d*x]^2])) - (C*(
b*Cos[c + d*x])^(5 + n)*Hypergeometric2F1[1/2, (5 + n)/2, (7 + n)/2, Cos[c
 + d*x]^2]*Sin[c + d*x])/(b^2*d*(5 + n)*Sqrt[Sin[c + d*x]^2]))/b^3
```

Defintions of rubi rules used

rule 2030

```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \cos(dx+c)^2 (b\cos(dx+c))^n \left(B\cos(dx+c)+C\cos(dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n \cos(dx+c)^2 \, dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, a
lgorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3)*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2),x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\cos(dx+c)^2 \, dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, a
lgorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
 c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\cos(dx+c)^2 \, dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, a
lgorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
 c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)\right)\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),
x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),
 x)
```

Reduce [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=b^n\left(\left(\int \cos(dx+c)^n\cos(dx+c)^4\,dx\right)c+\left(\int \cos(dx+c)^n\cos(dx+c)^3\,dx\right)b\right)$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)**4,x)*c + int(cos(c + d*x)**n*cos(c
 + d*x)**3,x)*b)
```

3.218 $\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2\right.$

Optimal result

Integrand size = 36, antiderivative size = 141

$$\begin{aligned}\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx \\ = -\frac{B(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}} \\ -\frac{C(b\cos(c+dx))^{4+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{2},\frac{6+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^4d(4+n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
-B*(b*cos(d*x+c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(4+n)*hype
rgeom([1/2, 2+1/2*n],[3+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(4+n)/(sin(d
*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx =$$
$$-\frac{\cos^2(c+dx)(b\cos(c+dx))^n \cot(c+dx) \left(B(4+n) \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)+\right.}{d(3+n)(4+n)}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]
^2),x]
```

output
```
-((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(4 + n)*Hypergeometri
c2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + C*(3 + n)*Cos[c + d*x]*H
ypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(3 + n)*(4 + n)))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2030, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{n+1} \left(C\cos^2(c+dx)+B\cos(c+dx)\right) dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1} \left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right) dx}{b}$$

$$\Big\downarrow \text{3489}$$

$$\frac{\int (b\cos(c+dx))^{n+2}(B+C\cos(c+dx))dx}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b^2}$$

$$\Big\downarrow \; 3227$$

$$\frac{B\int (b\cos(c+dx))^{n+2}dx+\frac{C\int (b\cos(c+dx))^{n+3}dx}{b}}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}dx+\frac{C\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+3}dx}{b}}{b^2}$$

$$\Big\downarrow \; 3122$$

$$\frac{-\frac{C\sin(c+dx)(b\cos(c+dx))^{n+4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+4}{2},\frac{n+6}{2},\cos^2(c+dx)\right)}{b^2d(n+4)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\ldots\right)}{bd(n+3)\sqrt{\sin^2(c+dx)}}}{b^2}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(-((B*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2
, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(3 + n)*Sqrt[Sin[c + d*x]^2])) - (C*(
b*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c
 + d*x]^2]*Sin[c + d*x])/(b^2*d*(4 + n)*Sqrt[Sin[c + d*x]^2]))/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \cos(dx+c)(b\cos(dx+c))^n\left(B\cos(dx+c)+C\cos(dx+c)^2\right)dx$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, alg
orithm="fricas")
```

output
```
integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$
$$= \int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n \cos\left(dx+c\right) \, dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, alg
orithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
 c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$
$$= \int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n \cos\left(dx+c\right) \, dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, alg
orithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
 c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)\right)\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2), x
)
```

Reduce [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=b^n\left(\left(\int \cos(dx+c)^n\cos(dx+c)^3\,dx\right)c+\left(\int \cos(dx+c)^n\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)**3,x)*c + int(cos(c + d*x)**n*cos(c
 + d*x)**2,x)*b)
```

3.219 $\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 141

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = -\frac{B(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}} - \frac{C(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}$$

output
```
-B*(b*cos(d*x+c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*s
in(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(3+n)*hypergeo
m([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(3+n)/(sin(d
*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx =$$
$$-\frac{\cos(c+dx)(b\cos(c+dx))^n \cot(c+dx) \left(B(3+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)+C\right.}{d(2+n)(3+n)}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
-((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(3 + n)*Hypergeometric2
F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + C*(2 + n)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*
x]^2])/(d*(2 + n)*(3 + n)))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n \left(B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right) dx$$

$$\Big\downarrow \text{3489}$$

$$\frac{\int (b\cos(c+dx))^{n+1}(B+C\cos(c+dx))dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{B\int (b\cos(c+dx))^{n+1}dx+\frac{C\int (b\cos(c+dx))^{n+2}dx}{b}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}dx+\frac{C\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}dx}{b}}{b}$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{-\frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{n+5}{2},\cos^2(c+dx)\right)}{b^2d(n+3)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},n\right.}{bd(n+2)\sqrt{\sin^2(c+dx)}}}{b}$$

input
```
Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(-((B*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2
, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 + n)*Sqrt[Sin[c + d*x]^2])) - (C*(
b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c
 + d*x]^2]*Sin[c + d*x])/(b^2*d*(3 + n)*Sqrt[Sin[c + d*x]^2]))/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int (b\cos (dx+c))^n \left(B\cos (dx+c)+C\cos (dx+c)^2\right) dx$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos (dx+c)^2+B\cos (dx+c)\right)(b\cos (dx+c))^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fri
cas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int (b\cos{(c+dx)})^n\,(B+C\cos{(c+dx)})\cos{(c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))*cos(c + d*x), x)
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos{(dx+c)}^2+B\cos{(dx+c)}\right)(b\cos{(dx+c)})^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="max
ima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos{(dx+c)}^2+B\cos{(dx+c)}\right)(b\cos{(dx+c)})^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="gia
c")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (b\,\cos{(c+d\,x)})^n\,\left(C\cos{(c+d\,x)}^2+B\,\cos{(c+d\,x)}\right)\,dx$$

input
```
int((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

output
```
int((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=b^n\left(\left(\int \cos{(dx+c)}^n\cos{(dx+c)}\,dx\right)b+\left(\int \cos{(dx+c)}^n\cos{(dx+c)}^2\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x),x)*b + int(cos(c + d*x)**n*cos(c +
d*x)**2,x)*c)
```

3.220 $\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal result

Integrand size = 36, antiderivative size = 141

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = -\frac{B(b\cos(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}} - \frac{C(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}$$

output
```
-B*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(2+n)*hyperg
eom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(2+n)/(sin(d*x
+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx =$$
$$-\frac{(b\cos(c+dx))^n\cot(c+dx)\left(B(2+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)+C(1+n)\cos\right.}{d(1+n)(2+n)}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d
*x],x]
```

output
```
-(((b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(2 + n)*Hypergeometric2F1[1/2, (1 +
n)/2, (3 + n)/2, Cos[c + d*x]^2] + C*(1 + n)*Cos[c + d*x]*Hypergeometric2F
1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1
+ n)*(2 + n)))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3042, 2030, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n\left(B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-1}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\Big\downarrow \text{3489}$$

$$\int (b\cos(c+dx))^n(B+C\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$B\int (b\cos(c+dx))^n dx+\frac{C\int (b\cos(c+dx))^{n+1}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n dx+\frac{C\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

output
```
-((B*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
 Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])) - (C*(b
*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c
+ d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int (b\cos(dx+c))^n \left(B\cos(dx+c)+C\cos(dx+c)^2\right)\sec(dx+c)\,dx$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, alg
orithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c), x)
```

Sympy [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int (b\cos{(c+dx)})^n\,(B+C\cos{(c+dx)})\cos{(c+dx)}\sec{(c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

output
```
Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))*cos(c + d*x)*sec(c + d*x
), x)
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos{(dx+c)}^2+B\cos{(dx+c)}\right)(b\cos{(dx+c)})^n\sec{(dx+c)}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, alg
orithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
 c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, alg
orithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
 c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x),
x)
```

output
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x),
 x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= b^n\left(\left(\int \cos(dx+c)^n\cos(dx+c)\sec(dx+c)\,dx\right)b + \left(\int \cos(dx+c)^n\cos(dx+c)^2\sec(dx+c)\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)*sec(c + d*x),x)*b + int(cos(c + d*x
)**n*cos(c + d*x)**2*sec(c + d*x),x)*c)
```

3.221 $\int (b\cos(c+dx))^n \left(B\cos(c+dx) + C\cos^2(c+dx)\right)$ se $dx)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 132

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= -\frac{B(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}$$
$$- \frac{C(b\cos(c+dx))^{1+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

output
```
-B*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x
+c)/d/n/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/
2*n],[3/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$
$$-\frac{b(b\cos(c+dx))^{-1+n}\cot(c+dx)\left(B(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)+Cn\cos(c\right.}{dn(1+n)}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d
*x]^2,x]
```

output
```
-((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(B*(1 + n)*Hypergeometric2F1[1
/2, n/2, (2 + n)/2, Cos[c + d*x]^2] + C*n*Cos[c + d*x]*Hypergeometric2F1[1
/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*n*(1 +
 n)))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 2030, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n \left(B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-2}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\Big\downarrow \text{3489}$$

$$b\int (b\cos(c+dx))^{n-1}(B+C\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$b\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-1}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$b\left(B\int (b\cos(c+dx))^{n-1}dx+\frac{C\int (b\cos(c+dx))^{n}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-1}dx+\frac{C\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b\left(-\frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(c+dx)\right)}{b^2d(n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,
x]
```

output
```
b*(-((B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c +
d*x]^2]*Sin[c + d*x])/(b*d*n*Sqrt[Sin[c + d*x]^2])) - (C*(b*Cos[c + d*x])^
(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c
 + d*x])/(b^2*d*(1 + n)*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int (b\cos (dx+c))^{n} \left(B\cos (dx+c)+C\cos (dx+c)^{2}\right) \sec (dx+c)^{2}dx$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos (dx+c)^{2}+B\cos (dx+c)\right)(b\cos (dx+c))^{n}\sec (dx+c)^{2}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, a
lgorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,x
)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c))^n \sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, a
lgorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
 c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, a
lgorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
 c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
2,x)
```

output
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
2, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= b^n\left(\left(\int \cos(dx+c)^n \cos(dx+c)\sec(dx+c)^2\,dx\right)b + \left(\int \cos(dx+c)^n \cos(dx+c)^2\sec(dx+c)^2\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)*sec(c + d*x)**2,x)*b + int(cos(c +
d*x)**n*cos(c + d*x)**2*sec(c + d*x)**2,x)*c)
```

3.222 $\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 131

$$\begin{aligned}&\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx\\&=\frac{bB(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}\\&\quad-\frac{C(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
b*B*(b*cos(d*x+c))^(-1+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+
c)^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^n*hypergeom
([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/n/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$
$$-\frac{b(b\cos(c+dx))^{-1+n}\csc(c+dx)\left(Bn\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)+C(-1 \cdots\right.}{d(-1+n)n}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d
*x]^3,x]
```

output
```
-((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*(B*n*Hypergeometric2F1[1/2, (-
1 + n)/2, (1 + n)/2, Cos[c + d*x]^2] + C*(-1 + n)*Cos[c + d*x]*Hypergeomet
ric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1
+ n)*n))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 2030, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n\left(B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-3}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\Big\downarrow \; 3489$$

$$b^2 \int (b\cos(c+dx))^{n-2}(B+C\cos(c+dx))dx$$

$$\Big\downarrow \; 3042$$

$$b^2 \int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-2}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 3227$$

$$b^2\left(B\int (b\cos(c+dx))^{n-2}dx+\frac{C\int (b\cos(c+dx))^{n-1}dx}{b}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-2}dx+\frac{C\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-1}dx}{b}\right)$$

$$\Big\downarrow \; 3122$$

$$b^2\left(\frac{B\sin(c+dx)(b\cos(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\cos^2(c+dx)\right)}{bd(1-n)\sqrt{\sin^2(c+dx)}}-\frac{C\sin(c+dx)(b\cos(c+d\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,
x]
```

output
```
b^2*((B*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 +
n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 - n)*Sqrt[Sin[c + d*x]^2]) - (
C*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2
]*Sin[c + d*x])/(b^2*d*n*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int (b\cos(dx+c))^n \left(B\cos(dx+c)+C\cos(dx+c)^2\right)\sec(dx+c)^3\,dx$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, a
lgorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x
)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n \sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, a
lgorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
 c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, a
lgorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
 c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
3,x)
```

output
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
3, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= b^n\left(\left(\int \cos(dx+c)^n \cos(dx+c)\sec(dx+c)^3\,dx\right)b + \left(\int \cos(dx+c)^n \cos(dx+c)^2\sec(dx+c)^3\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)*sec(c + d*x)**3,x)*b + int(cos(c +
d*x)**n*cos(c + d*x)**2*sec(c + d*x)**3,x)*c)
```

3.223 $\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 139

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{b^2B(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}} + \frac{bC(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

output
```
b^2*B*(b*cos(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(2-n)/(sin(d*x+c)^2)^(1/2)+b*C*(b*cos(d*x+c))^(-1+n)*hyperg
eom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-n)/(sin(d*
x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.85

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx =$$
$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\left(B(-1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)+C(-2}{d(-2+n)}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d
*x]^4,x]
```

output
```
-(((b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(-1 + n)*Hypergeometric2F1[1/2, (-2
+ n)/2, n/2, Cos[c + d*x]^2] + C*(-2 + n)*Cos[c + d*x]*Hypergeometric2F1[1
/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*
x]^2])/(d*(-2 + n)*(-1 + n)))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 2030, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n\left(B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-4}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)dx$$

$$\Big\downarrow \text{3489}$$

$$b^3\int (b\cos(c+dx))^{n-3}(B+C\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$b^3\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-3}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$b^3\left(B\int (b\cos(c+dx))^{n-3}dx+\frac{C\int (b\cos(c+dx))^{n-2}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-3}dx+\frac{C\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-2}dx}{b}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^3\left(\frac{C\sin(c+dx)(b\cos(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\cos^2(c+dx)\right)}{b^2d(1-n)\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+dx)(b\cos(c+d}{}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,
x]
```

output
```
b^3*((B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 - n)*Sqrt[Sin[c + d*x]^2]) + (C*(b*C
os[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c
+ d*x]^2]*Sin[c + d*x])/(b^2*d*(1 - n)*Sqrt[Sin[c + d*x]^2]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int (b\cos(dx+c))^n \left(B\cos(dx+c)+C\cos(dx+c)^2\right)\sec(dx+c)^4\,dx$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, a
lgorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,x
)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2 + B\cos(dx+c)\right)(b\cos(dx+c))^n \sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, a
lgorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
 c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, a
lgorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
 c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
4,x)
```

output
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
4, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= b^n\left(\left(\int \cos(dx+c)^n \cos(dx+c)\sec(dx+c)^4\,dx\right)b + \left(\int \cos(dx+c)^n \cos(dx+c)^2\sec(dx+c)^4\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)*sec(c + d*x)**4,x)*b + int(cos(c +
d*x)**n*cos(c + d*x)**2*sec(c + d*x)**4,x)*c)
```

3.224 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos\right.$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{2B\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(9+2n),\frac{1}{4}(13+2n),\cos^2(c+dx)\right)\sin(c+}{d(9+2n)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{2C\cos^{\frac{11}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(11+2n),\frac{1}{4}(15+2n),\cos^2(c+dx)\right)\sin(c}{d(11+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*B*cos(d*x+c)^(9/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 9/4+1/2*n],[13/4+1/
2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(9+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c
)^(11/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 11/4+1/2*n],[15/4+1/2*n],cos(d*x
+c)^2)*sin(d*x+c)/d/(11+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{2\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(B(11+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(9+2n),\frac{1}{4}(13+2n\right.\right.}{d}$$

input

```
Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c
+ d*x]^2),x]
```

output

```
(-2*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(11 + 2*n)*Hyper
geometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2] + C*(9 + 2*n)
*Cos[c + d*x]*Hypergeometric2F1[1/2, (11 + 2*n)/4, (15 + 2*n)/4, Cos[c + d
*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(9 + 2*n)*(11 + 2*n))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n+\frac{5}{2}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{5}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3489}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int\cos^{n+\frac{7}{2}}(c+dx)(B+C\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{7}{2}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int\cos^{n+\frac{7}{2}}(c+dx)dx+C\int\cos^{n+\frac{9}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{7}{2}}dx+C\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{9}{2}}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(-\frac{2B\sin(c+dx)\cos^{n+\frac{9}{2}}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+9),\frac{1}{4}(2n+13),\cos^2(c+dx)\right)}{d(2n+9)\sqrt{\sin^2(c+dx)}}-\frac{2C\sin(\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]
^2),x]
```

output
```
((b*Cos[c + d*x])^n*((-2*B*Cos[c + d*x]^(9/2 + n)*Hypergeometric2F1[1/2, (
9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(9 + 2*n)*Sqrt[
Sin[c + d*x]^2]) - (2*C*Cos[c + d*x]^(11/2 + n)*Hypergeometric2F1[1/2, (11
 + 2*n)/4, (15 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(11 + 2*n)*Sqrt[
Sin[c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \cos\left(dx+c\right)^{\frac{5}{2}} \left(b\cos\left(dx+c\right)\right)^{n} \left(B\cos\left(dx+c\right)+C\cos\left(dx+c\right)^{2}\right)dx$$

input
```
int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3)*(b*cos(d*x + c))^n*sqrt(cos
(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int (C\cos(dx+c)^2 + B\cos(dx+c))(b\cos(dx+c))^n\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
 c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (C\cos{(dx+c)}^2+B\cos{(dx+c)})(b\cos{(dx+c)})^n\cos{(dx+c)}^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
 c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos{(c+d\,x)}^{5/2}\,(b\,\cos{(c+d\,x)})^n\,\left(C\cos{(c+d\,x)}^2+B\,\cos{(c+d\,x)}\right)\,dx$$

input
```
int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)
^2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)
^2), x)
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=b^n\left(\left(\int \cos(dx+c)^{n+\frac{1}{2}}\cos(dx+c)^4\,dx\right)c+\left(\int \cos(dx+c)^{n+\frac{1}{2}}\cos(dx+c)^3\,dx\right)b\right)$$

input
```
int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)*cos(c + d*x)**4,x)*c + int(cos(c + d
*x)**((2*n + 1)/2)*cos(c + d*x)**3,x)*b)
```

3.225 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos\right.$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{2B\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\sin(c+}{d(7+2n)\sqrt{\sin^2(c+dx)}} - \frac{2C\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(9+2n),\frac{1}{4}(13+2n),\cos^2(c+dx)\right)\sin(c+}{d(9+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*B*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/
2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c
)^(9/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 9/4+1/2*n],[13/4+1/2*n],cos(d*x+c
)^2)*sin(d*x+c)/d/(9+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx =$$
$$-\frac{2\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(B(9+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\right.\right.}{d\ldots}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c
+ d*x]^2),x]
```

output
```
(-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(9 + 2*n)*Hyperg
eometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] + C*(7 + 2*n)*
Cos[c + d*x]*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x
]^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 2*n)*(9 + 2*n))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$

$$\Big\downarrow \; 2034$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \cos^{n+\frac{3}{2}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)\right) dx$$

$$\Big\downarrow \; 3042$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right) dx$$

$$\Big\downarrow \; 3489$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \cos^{n+\frac{5}{2}}(c+dx)(B+C\cos(c+dx))dx$$

$\big\downarrow$ 3042

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{5}{2}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$\big\downarrow$ 3227

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \left(B\int \cos^{n+\frac{5}{2}}(c+dx)dx+C\int \cos^{n+\frac{7}{2}}(c+dx)dx\right)$$

$\big\downarrow$ 3042

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{5}{2}}dx+C\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{7}{2}}dx\right)$$

$\big\downarrow$ 3122

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \left(-\frac{2B\sin(c+dx)\cos^{n+\frac{7}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+7),\frac{1}{4}(2n+11),\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}-\frac{2C\sin(\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]
^2),x]
```

output
```
((b*Cos[c + d*x])^n*((-2*B*Cos[c + d*x]^(7/2 + n)*Hypergeometric2F1[1/2, (
7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[
Sin[c + d*x]^2]) - (2*C*Cos[c + d*x]^(9/2 + n)*Hypergeometric2F1[1/2, (9 +
 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(9 + 2*n)*Sqrt[Sin
[c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] &&
 !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \cos (dx+c)^{\frac{3}{2}} (b\cos (dx+c))^{n} \left(B\cos (dx+c)+C\cos (dx+c)^{2}\right) dx$$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$=\int \left(C\cos (dx+c)^{2}+B\cos (dx+c)\right)(b\cos (dx+c))^{n}\cos (dx+c)^{\frac{3}{2}} dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*(b*cos(d*x + c))^n*sqrt(cos
(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int (C\cos(dx+c)^2+B\cos(dx+c))(b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
 c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
 c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)^{3/2}\left(b\,\cos\left(c+d\,x\right)\right)^n\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)
^2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)
^2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=b^n\left(\left(\int \cos(dx+c)^{n+\frac{1}{2}}\cos(dx+c)^3\,dx\right)c+\left(\int \cos(dx+c)^{n+\frac{1}{2}}\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)*cos(c + d*x)**3,x)*c + int(cos(c + d
*x)**((2*n + 1)/2)*cos(c + d*x)**2,x)*b)
```

3.226 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = -\frac{2B\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+2n)\sqrt{\sin^2(c+dx)}} - \frac{2C\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*B*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2
*n],cos(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c)
^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2*n],cos(d*x+c)
^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx =$$
$$-\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \csc(c+dx) \left(B(7+2n) \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),c\right.\right.}{d(}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c
+ d*x]^2),x]
```

output
```
(-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(7 + 2*n)*Hyperg
eometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + C*(5 + 2*n)*C
os[c + d*x]*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]
^2])*Sqrt[Sin[c + d*x]^2])/(d*(5 + 2*n)*(7 + 2*n))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \cos^{n+\frac{1}{2}}(c+dx) \left(C\cos^2(c+dx)+B\cos(c+dx)\right) dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}} \left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right) dx$$

$$\Big\downarrow \text{3489}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int\cos^{n+\frac{3}{2}}(c+dx)(B+C\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int\cos^{n+\frac{3}{2}}(c+dx)dx+C\int\cos^{n+\frac{5}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}dx+C\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{5}{2}}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(-\frac{2B\sin(c+dx)\cos^{n+\frac{5}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+5),\frac{1}{4}(2n+9),\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}-\frac{2C\sin(c$$

input
```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]
^2),x]
```

output
```
((b*Cos[c + d*x])^n*((-2*B*Cos[c + d*x]^(5/2 + n)*Hypergeometric2F1[1/2, (
5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[S
in[c + d*x]^2]) - (2*C*Cos[c + d*x]^(7/2 + n)*Hypergeometric2F1[1/2, (7 +
2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[
c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \sqrt{\cos\left(dx+c\right)}\,\left(b\cos\left(dx+c\right)\right)^{n}\left(B\cos\left(dx+c\right)+C\cos\left(dx+c\right)^{2}\right)dx$$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{n}\left(B\cos(c+dx)+C\cos^{2}(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{n}\sqrt{\cos\left(dx+c\right)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d
*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**
2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (C\cos(dx+c)^2+B\cos(dx+c))(b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(
d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\sqrt{\cos\left(dx+c\right)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(
d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \sqrt{\cos\left(c+d\,x\right)}\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)
^2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)
^2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=b^n\left(\left(\int \cos(dx+c)^{n+\frac{1}{2}}\cos(dx+c)\,dx\right)b+\left(\int \cos(dx+c)^{n+\frac{1}{2}}\cos(dx+c)^2\,dx\right)c\right)$$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)*cos(c + d*x),x)*b + int(cos(c + d*x)
**((2*n + 1)/2)*cos(c + d*x)**2,x)*c)
```

3.227 $\int \frac{(b\cos(c+dx))^n(B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2B\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin(c+d}{d(3+2n)\sqrt{\sin^2(c+dx)}} - \frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin(c+d}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*B*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2
*n],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c)
^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx =$$
$$-\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \csc(c+dx) \left(B(5+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),c\right.\right.}{d(3}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Co
s[c + d*x]],x]
```

output
```
(-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(5 + 2*n)*Hyperg
eometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + C*(3 + 2*n)*C
os[c + d*x]*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^
2])*Sqrt[Sin[c + d*x]^2])/(d*(3 + 2*n)*(5 + 2*n))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \cos^{n-\frac{1}{2}}(c+dx) \left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}} \left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 3489$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int\cos^{n+\frac{1}{2}}(c+dx)(B+C\cos(c+dx))dx$$

$$\Big\downarrow \; 3042$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 3227$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int\cos^{n+\frac{1}{2}}(c+dx)dx+C\int\cos^{n+\frac{3}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \; 3042$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}dx+C\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}dx\right)$$

$$\Big\downarrow \; 3122$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(-\frac{2B\sin(c+dx)\cos^{n+\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}-\frac{2C\sin(c$$

input
```
Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c +
d*x]],x]
```

output
```
((b*Cos[c + d*x])^n*((-2*B*Cos[c + d*x]^(3/2 + n)*Hypergeometric2F1[1/2, (
3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[S
in[c + d*x]^2]) - (2*C*Cos[c + d*x]^(5/2 + n)*Hypergeometric2F1[1/2, (5 +
2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c
 + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \frac{(b\cos(dx+c))^n \left(B\cos(dx+c)+C\cos(dx+c)^2\right)}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

output
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \left(b\cos\left(c+dx\right)\right)^n\left(B+C\cos\left(c+dx\right)\right)\sqrt{\cos\left(c+dx\right)}\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/
2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))*sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/sqrt(cos(
d*x + c)), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/sqrt(cos(
d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
(1/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
(1/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= b^n\left(\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}dx\right)b+\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}\cos\left(dx+c\right)dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2),x)*b + int(cos(c + d*x)**((2*n + 1)/
2)*cos(c + d*x),x)*c)
```

3.228 $\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2B\sqrt{\cos(c+dx)}(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos^2(c+dx)\right)\sin(c+}{d(1+2n)\sqrt{\sin^2(c+dx)}} - \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin(c+d}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*B*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2
*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c)
^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{2\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\csc(c+dx)\left(B(3+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\right.\right.}{d(}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
 d*x]^(3/2),x]
```

output
```
(-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(3 + 2*n)*Hyperg
eometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + C*(1 + 2*n)*C
os[c + d*x]*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^
2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + 2*n)*(3 + 2*n))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 2034$$
$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{3}{2}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx$$
$$\Big\downarrow \; 3042$$
$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3489}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int\cos^{n-\frac{1}{2}}(c+dx)(B+C\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int\cos^{n-\frac{1}{2}}(c+dx)dx+C\int\cos^{n+\frac{1}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}dx+C\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(-\frac{2B\sin(c+dx)\cos^{n+\frac{1}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+1),\frac{1}{4}(2n+5),\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}-\frac{2C\sin(c$$

input
```
Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(3/2),x]
```

output
```
((b*Cos[c + d*x])^n*((-2*B*Cos[c + d*x]^(1/2 + n)*Hypergeometric2F1[1/2, (
1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[S
in[c + d*x]^2]) - (2*C*Cos[c + d*x]^(3/2 + n)*Hypergeometric2F1[1/2, (3 +
2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c
 + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \frac{(b\cos(dx+c))^n \left(B\cos(dx+c)+C\cos(dx+c)^2\right)}{\cos(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

output
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(b\cos\left(c+dx\right)\right)^n\left(B+C\cos\left(c+dx\right)\right)}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/
2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
 c)^(3/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
 c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
(3/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
(3/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= b^n\left(\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}dx\right)c+\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)}dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2),x)*c + int(cos(c + d*x)**((2*n + 1)/
2)/cos(c + d*x),x)*b)
```

3.229 $\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2B(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{2C\sqrt{\cos(c+dx)}(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos^2(c+dx)\right)\sin(c+}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c)
^(1/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx =$$
$$-\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(B(1+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx\right)\right.}{d\left(-1+4n^2\right.}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
 d*x]^(5/2),x]
```

output
```
(-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(1 + 2*n)*Hypergeometric2F1[1/2, (-
1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + C*(-1 + 2*n)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(-1 + 4*n^2)*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{5}{2}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3489}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{3}{2}}(c+dx)(B+C\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int \cos^{n-\frac{3}{2}}(c+dx)dx+C\int \cos^{n-\frac{1}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}dx+C\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2B\sin(c+dx)\cos^{n-\frac{1}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}}-\frac{2C\sin(c+\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(5/2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*B*Cos[c + d*x]^(-1/2 + n)*Hypergeometric2F1[1/2, (
-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[
Sin[c + d*x]^2]) - (2*C*Cos[c + d*x]^(1/2 + n)*Hypergeometric2F1[1/2, (1 +
 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[
c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \frac{(b\cos(dx+c))^n \left(B\cos(dx+c)+C\cos(dx+c)^2\right)}{\cos(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

output
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(b\cos\left(c+dx\right)\right)^n\left(B+C\cos\left(c+dx\right)\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/
2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
 c)^(5/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(C\cos(dx+c)^2+B\cos(dx+c))(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
 c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)^{5/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
(5/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
(5/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= b^n\left(\left(\int \frac{\cos(dx+c)^{n+\frac{1}{2}}}{\cos(dx+c)}dx\right)c+\left(\int \frac{\cos(dx+c)^{n+\frac{1}{2}}}{\cos(dx+c)^2}dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)/cos(c + d*x),x)*c + int(cos(c + d*x)
**((2*n + 1)/2)/cos(c + d*x)**2,x)*b)
```

3.230 $\int \frac{(b\cos(c+dx))^n(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\begin{aligned}\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2B(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}\\&\quad+\frac{2C(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)+2*C*(b*cos(d*x
+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/
(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(B(-1+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+d\right.\right.}{d(-3+2n)(-}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
 d*x]^(7/2),x]
```

output
```
(-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(-1 + 2*n)*Hypergeometric2F1[1/2, (
-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] + C*(-3 + 2*n)*Cos[c + d*x]*Hype
rgeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c
+ d*x]^2])/(d*(-3 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{7}{2}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{7}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3489}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{5}{2}}(c+dx)(B+C\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int \cos^{n-\frac{5}{2}}(c+dx)dx+C\int \cos^{n-\frac{3}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}dx+C\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2B\sin(c+dx)\cos^{n-\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-3),\frac{1}{4}(2n+1),\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}}+\frac{2C\sin(c+\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(7/2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*B*Cos[c + d*x]^(-3/2 + n)*Hypergeometric2F1[1/2, (
-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Sqrt[
Sin[c + d*x]^2]) + (2*C*Cos[c + d*x]^(-1/2 + n)*Hypergeometric2F1[1/2, (-1
 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Si
n[c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \frac{(b\cos(dx+c))^n (B\cos(dx+c)+C\cos(dx+c)^2)}{\cos(dx+c)^{\frac{7}{2}}}dx$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

output
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/
2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
 c)^(7/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
 c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
(7/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
(7/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= b^n\left(\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^3}dx\right)b+\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^2}dx\right)c\right)$$

input

```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

output

```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)/cos(c + d*x)**3,x)*b + int(cos(c + d
*x)**((2*n + 1)/2)/cos(c + d*x)**2,x)*c)
```

3.231 $\int \frac{(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 163

$$\begin{aligned}\int \frac{(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2B(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}\\ &\quad+\frac{2C(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)+2*C*(b*cos(d*
x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d
/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(B(-3+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+\right.\right.}{d(-5+2n)(}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
 d*x]^(9/2),x]
```

output
```
(-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(-3 + 2*n)*Hypergeometric2F1[1/2, (
-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + C*(-5 + 2*n)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(d*(-5 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2034, 3042, 3489, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{9}{2}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{9}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3489}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int\cos^{n-\frac{7}{2}}(c+dx)(B+C\cos(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{7}{2}}\left(B+C\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int\cos^{n-\frac{7}{2}}(c+dx)dx+C\int\cos^{n-\frac{5}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(B\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{7}{2}}dx+C\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2B\sin(c+dx)\cos^{n-\frac{5}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-5),\frac{1}{4}(2n-1),\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}}+\frac{2C\sin(c+\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^
(9/2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*B*Cos[c + d*x]^(-5/2 + n)*Hypergeometric2F1[1/2, (
-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Sqrt
[Sin[c + d*x]^2]) + (2*C*Cos[c + d*x]^(-3/2 + n)*Hypergeometric2F1[1/2, (-
3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Sqrt[S
in[c + d*x]^2])))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3489
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b   Int[(b*Sin[e + f*
x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Maple [F]

$$\int \frac{(b\cos(dx+c))^n \left(B\cos(dx+c)+C\cos(dx+c)^2\right)}{\cos(dx+c)^{\frac{9}{2}}}dx$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

output
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/
2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
 c)^(9/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
 c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^{9/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
(9/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^
(9/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= b^n\left(\left(\int \frac{\cos(dx+c)^{n+\frac{1}{2}}}{\cos(dx+c)^4}dx\right)b+\left(\int \frac{\cos(dx+c)^{n+\frac{1}{2}}}{\cos(dx+c)^3}dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)/cos(c + d*x)**4,x)*b + int(cos(c + d
*x)**((2*n + 1)/2)/cos(c + d*x)**3,x)*c)
```

3.232 $\int (a+a\cos(e+fx))^m (B\cos(e+fx)+C\cos^2(e+f$

Optimal result

Integrand size = 32, antiderivative size = 173

$$\int (a+a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e+fx)}{f(1+m)(2+m)}$$
$$+\frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}$$
$$+\frac{2^{\frac{1}{2}+m}(Bm(2+m)+C(1+m+m^2))\,(1+\cos(e+fx))^{-\frac{1}{2}-m}(a+a\cos(e+fx))^m\,\text{Hypergeometric2F}}{f(1+m)(2+m)}$$

output
```
-(C-B*(2+m))*(a+a*cos(f*x+e))^m*sin(f*x+e)/f/(1+m)/(2+m)+C*(a+a*cos(f*x+e)
)^(1+m)*sin(f*x+e)/a/f/(2+m)+2^(1/2+m)*(B*m*(2+m)+C*(m^2+m+1))*(1+cos(f*x+
e))^(-1/2-m)*(a+a*cos(f*x+e))^m*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*cos(f
*x+e))*sin(f*x+e)/f/(1+m)/(2+m)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.21 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.06

$$\int (a+a\cos(e+fx))^m\left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$= \frac{i4^{-1-m}e^{-2i(e+fx)}\left(1+e^{i(e+fx)}\right)^{-2m}\left(e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\right)^{2m}\cos^{-2m}\left(\frac{1}{2}(e+fx)\right)(a(1+\cos(e+fx)))}{}$$

input
```
Integrate[(a + a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

output
```
(I*4^(-1 - m)*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f*x)))^(2*m)*(a*(1 + Co
s[e + f*x]))^m*(C*m*(2 - m - 2*m^2 + m^3)*Hypergeometric2F1[-2 - m, -2*m,
-1 - m, -E^(I*(e + f*x))] + E^(I*(e + f*x))*(2 + m)*(2*B*m*(2 - 3*m + m^2)
*Hypergeometric2F1[-1 - m, -2*m, -m, -E^(I*(e + f*x))] + E^(I*(e + f*x))*(
1 + m)*(2*B*E^(I*(e + f*x))*(-2 + m)*m*Hypergeometric2F1[1 - m, -2*m, 2 -
m, -E^(I*(e + f*x))] + C*(-1 + m)*(E^((2*I)*(e + f*x))*m*Hypergeometric2F1
[2 - m, -2*m, 3 - m, -E^(I*(e + f*x))] + 2*(-2 + m)*Hypergeometric2F1[-2*m
, -m, 1 - m, -E^(I*(e + f*x))])))))/(E^((2*I)*(e + f*x))*(1 + E^(I*(e + f*
x)))^(2*m)*f*(-2 + m)*(-1 + m)*m*(1 + m)*(2 + m)*Cos[(e + f*x)/2]^(2*m))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3502, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(e+fx)+a)^m\left(B\cos(e+fx)+C\cos^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\left(B\sin\left(e+fx+\frac{\pi}{2}\right)+C\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \; 3502$$

$$\frac{\int (\cos(e+fx)a+a)^m (aC(m+1)-a(C-B(m+2))\cos(e+fx))dx}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(\sin\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m \left(aC(m+1)-a(C-B(m+2))\sin\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \; 3230$$

$$\frac{\frac{a\left(Bm(m+2)+C\left(m^2+m+1\right)\right)\int (\cos(e+fx)a+a)^m dx}{m+1}-\frac{a(C-B(m+2))\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{a\left(Bm(m+2)+C\left(m^2+m+1\right)\right)\int \left(\sin\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx}{m+1}-\frac{a(C-B(m+2))\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \; 3131$$

$$\frac{\frac{a\left(Bm(m+2)+C\left(m^2+m+1\right)\right)(\cos(e+fx)+1)^{-m}(a\cos(e+fx)+a)^m\int (\cos(e+fx)+1)^m dx}{m+1}-\frac{a(C-B(m+2))\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{a\left(Bm(m+2)+C\left(m^2+m+1\right)\right)(\cos(e+fx)+1)^{-m}(a\cos(e+fx)+a)^m\int \left(\sin\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx}{m+1}-\frac{a(C-B(m+2))\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)}}{a(m+2)}$$
$$\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \; 3130$$

$$\frac{\frac{a2^{m+\frac{1}{2}}(Bm(m+2)+C(m^2+m+1))\sin(e+fx)(\cos(e+fx)+1)^{-m-\frac{1}{2}}(a\cos(e+fx)+a)^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx))\right)}{f(m+1)} -}{a(m+2)}$$

$$\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

input
```
Int[(a + a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

output
```
(C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (-((a*(C - B
*(2 + m))*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(1 + m))) + (2^(1/2 + m)
*a*(B*m*(2 + m) + C*(1 + m + m^2))*(1 + Cos[e + f*x])^(-1/2 - m)*(a + a*Co
s[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Cos[e + f*x])/2]*S
in[e + f*x])/(f*(1 + m)))/(a*(2 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a + a\cos(fx + e))^m \left(B\cos(fx + e) + C\cos(fx + e)^2\right) dx$$

input
```
int((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
int((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a + a\cos(e + fx))^m \left(B\cos(e + fx) + C\cos^2(e + fx)\right) dx$$
$$= \int \left(C\cos(fx + e)^2 + B\cos(fx + e)\right)(a\cos(fx + e) + a)^m dx$$

input
```
integrate((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="f
ricas")
```

output
```
integral((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a+a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int (a(\cos(e+fx)+1))^m (B+C\cos(e+fx))\cos(e+fx)\,dx$$

input
```
integrate((a+a*cos(f*x+e))**m*(B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

output
```
Integral((a*(cos(e + f*x) + 1))**m*(B + C*cos(e + f*x))*cos(e + f*x), x)
```

Maxima [F]

$$\int (a+a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int (C\cos(fx+e)^2+B\cos(fx+e))(a\cos(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="m
axima")
```

output
```
integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a+a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int (C\cos(fx+e)^2+B\cos(fx+e))(a\cos(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="g
iac")
```

output
```
integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\cos(e + fx))^m \left(B\cos(e + fx) + C\cos^2(e + fx)\right) dx$$
$$= \int \left(C\cos\left(e + f\,x\right)^2 + B\,\cos\left(e + f\,x\right)\right)\,\left(a + a\,\cos\left(e + f\,x\right)\right)^m dx$$

input
```
int((B*cos(e + f*x) + C*cos(e + f*x)^2)*(a + a*cos(e + f*x))^m,x)
```

output
```
int((B*cos(e + f*x) + C*cos(e + f*x)^2)*(a + a*cos(e + f*x))^m, x)
```

Reduce [F]

$$\int (a + a\cos(e + fx))^m \left(B\cos(e + fx) + C\cos^2(e + fx)\right) dx$$
$$= \left(\int \left(\cos\left(fx + e\right)a + a\right)^m \cos\left(fx + e\right)dx\right)b$$
$$+ \left(\int \left(\cos\left(fx + e\right)a + a\right)^m \cos\left(fx + e\right)^2 dx\right)c$$

input
```
int((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
int((cos(e + f*x)*a + a)**m*cos(e + f*x),x)*b + int((cos(e + f*x)*a + a)**
m*cos(e + f*x)**2,x)*c
```

3.233 $\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 296

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$
$$- \frac{\sqrt{2}(aC-bB(2+m))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx\ldots}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$
$$+ \frac{\sqrt{2}(a^2C+b^2C(1+m)-abB(2+m))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+\ldots}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$

output
```
C*(a+b*cos(f*x+e))^(1+m)*sin(f*x+e)/b/f/(2+m)-2^(1/2)*(a*C-b*B*(2+m))*Appe
llF1(1/2,-1-m,1/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(
f*x+e))^(1+m)*((a+b*cos(f*x+e))/(a+b))^(-1-m)*sin(f*x+e)/b^2/f/(2+m)/(1+co
s(f*x+e))^(1/2)+2^(1/2)*(a^2*C+b^2*C*(1+m)-a*b*B*(2+m))*AppellF1(1/2,-m,1/
2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*
x+e)/b^2/f/(2+m)/(1+cos(f*x+e))^(1/2)/(((a+b*cos(f*x+e))/(a+b))^m)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 13441 vs. $2(296) = 592$.

Time = 28.38 (sec) , antiderivative size = 13441, normalized size of antiderivative = 45.41

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3502, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(B\cos(e+fx)+C\cos^2(e+fx)\right)(a+b\cos(e+fx))^m dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(B\sin\left(e+fx+\frac{\pi}{2}\right)+C\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int (a+b\cos(e+fx))^m(bC(m+1)-(aC-bB(m+2))\cos(e+fx))dx}{b(m+2)}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m \left(bC(m+1)+(bB(m+2)-aC)\sin\left(e+fx+\frac{\pi}{2}\right)\right)dx}{b(m+2)}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{ 3235}$$

$$\frac{\frac{(a^2C-abB(m+2)+b^2C(m+1))\int(a+b\cos(e+fx))^m dx}{b}-\frac{(aC-bB(m+2))\int(a+b\cos(e+fx))^{m+1}dx}{b}}{b(m+2)}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(a^2C-abB(m+2)+b^2C(m+1))\int\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx}{b}-\frac{(aC-bB(m+2))\int\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}dx}{b}}{b(m+2)}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{ 3144}$$

$$\frac{\frac{\sin(e+fx)(aC-bB(m+2))\int\frac{(a+b\cos(e+fx))^{m+1}}{\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}d\cos(e+fx)}{bf\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}-\frac{\sin(e+fx)(a^2C-abB(m+2)+b^2C(m+1))\int\frac{(a+b\cos(e+fx))^m}{\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)}}}{bf\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}}{b(m+2)}$$

$$\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{ 156}$$

$$\frac{\frac{(a+b)\sin(e+fx)(aC-bB(m+2))(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\int\frac{\left(\frac{a}{a+b}+\frac{b\cos(e+fx)}{a+b}\right)^{m+1}}{\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}d\cos(e+fx)}{bf\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}-\frac{\sin(e+fx)(a^2C-abB(m}{}}{b(m+2)}$$

$$\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{ 155}$$

$$\frac{\frac{\sqrt{2}\sin(e+fx)(a^2C-abB(m+2)+b^2C(m+1))(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{bf\sqrt{\cos(e+fx)+1}}}{b(m+}$$

$$\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

input
```
Int[(a + b*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

output
```
(C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) + (-((Sqrt[2]*
(a + b)*(a*C - b*B*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f
*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x
])/(b*f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m)) + (Sqrt[
2]*(a^2*C + b^2*C*(1 + m) - a*b*B*(2 + m))*AppellF1[1/2, 1/2, -m, 3/2, (1
- Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*
Sin[e + f*x])/(b*f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m
))/(b*(2 + m))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a+b\cos (fx+e))^m \left(B\cos (fx+e)+C\cos (fx+e)^2\right) dx$$

input
```
int((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
int((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \int \left(C\cos (fx+e)^2+B\cos (fx+e)\right)(b\cos (fx+e)+a)^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="f
ricas")
```

output
```
integral((C*cos(f*x + e)^2 + B*cos(f*x + e))*(b*cos(f*x + e) + a)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\cos(e + fx))^m \left(B\cos(e + fx) + C\cos^2(e + fx)\right) dx = \text{Timed out}$$

input
```
integrate((a+b*cos(f*x+e))**m*(B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + b\cos(e + fx))^m \left(B\cos(e + fx) + C\cos^2(e + fx)\right) dx$$
$$= \int (C\cos(fx + e)^2 + B\cos(fx + e))(b\cos(fx + e) + a)^m dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="m
axima")
```

output
```
integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(b*cos(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \int \left(C\cos(fx+e)^2+B\cos(fx+e)\right)(b\cos(fx+e)+a)^m dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="g
iac")
```

output
```
integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(b*cos(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \int \left(C\cos(e+fx)^2+B\cos(e+fx)\right) (a+b\cos(e+fx))^m dx$$

input
```
int((B*cos(e + f*x) + C*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m,x)
```

output
```
int((B*cos(e + f*x) + C*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m, x)
```

Reduce [F]

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \left(\int (\cos(fx+e)b+a)^m \cos(fx+e) dx\right) b$$
$$+ \left(\int (\cos(fx+e)b+a)^m \cos(fx+e)^2 dx\right) c$$

input
```
int((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
int((cos(e + f*x)*b + a)**m*cos(e + f*x),x)*b + int((cos(e + f*x)*b + a)**
m*cos(e + f*x)**2,x)*c
```

3.234 $\int (a+b\cos(c+dx))^{2/3} \left(B\cos(c+dx) + C\cos^2(c+d\right.$

Optimal result

Integrand size = 34, antiderivative size = 281

$$\int (a+b\cos(c+dx))^{2/3} \left(B\cos(c+dx) + C\cos^2(c+dx)\right) \, dx = \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{(8bB-3aC)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{5/3}} - \frac{(8abB-3a^2C-5b^2C)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\,\text{si}}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

output
```
3/8*C*(a+b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d+1/8*(8*B*b-3*C*a)*AppellF1(1/2
,-5/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^
(5/3)*sin(d*x+c)*2^(1/2)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b
))^(5/3)-1/8*(8*B*a*b-3*C*a^2-5*C*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-cos(
d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2
)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b))^(2/3)
```

Mathematica [A] (warning: unable to verify)

Time = 5.84 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.03

$$\int (a+b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(5(-a^2+b^2)(8bB-3aC)\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right.\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x
]
```

output
```
(-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(-a^2 + b^2)*(8*b*B - 3*a*C
)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c
+ d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + C
os[c + d*x]))/(a - b))] + (16*a*b*B - 6*a^2*C + 25*b^2*C)*AppellF1[5/3, 1/
2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*S
qrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a -
 b))]*(a + b*Cos[c + d*x]) - 5*b^2*(8*b*B + 2*a*C + 5*b*C*Cos[c + d*x])*Si
n[c + d*x]^2))/(200*b^3*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {3042, 3502, 27, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{3\int \frac{1}{3}(a+b\cos(c+dx))^{2/3}(5bC+(8bB-3aC)\cos(c+dx))dx}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; \textcolor{blue}{27}$$

$$\frac{\int (a+b\cos(c+dx))^{2/3}(5bC+(8bB-3aC)\cos(c+dx))dx}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; \textcolor{blue}{3042}$$

$$\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(5bC+(8bB-3aC)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; \textcolor{blue}{3235}$$

$$\frac{\frac{(8bB-3aC)\int (a+b\cos(c+dx))^{5/3}dx}{b}-\frac{\left(-3a^2C+8abB-5b^2C\right)\int (a+b\cos(c+dx))^{2/3}dx}{b}}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; \textcolor{blue}{3042}$$

$$\frac{\frac{(8bB-3aC)\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{b}-\frac{\left(-3a^2C+8abB-5b^2C\right)\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; \textcolor{blue}{3144}$$

$$\frac{\frac{\left(-3a^2C+8abB-5b^2C\right)\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{2/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{(8bB-3aC)\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{5/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; \textcolor{blue}{156}$$

$$\frac{\frac{(-3a^2C+8abB-5b^2C)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\int\frac{\left(\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}-\frac{(a+b)(8bB-3aC)\sin(c+dx)(a+b\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c}}}{8b}$$
$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$\Big\downarrow$ 155

$$\frac{\frac{\sqrt{2}(a+b)(8bB-3aC)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}-\frac{\sqrt{2}(-3a^2C+8abB-5b^2C)\sin(c}{}}{8b}$$
$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

input
```
Int[(a + b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((Sqrt[2]*(a + b)*
(8*b*B - 3*a*C)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1
- Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*(8*a
*b*B - 3*a^2*C - 5*b^2*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])
/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x
])/(b*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)))/(8*b
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a+b\cos(dx+c))^{\frac{2}{3}}\left(B\cos(dx+c)+C\cos(dx+c)^2\right)dx$$

input
```
int((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(2/3), x
)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3} \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{2/3} \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(2/3),
x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{2/3} \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(2/3),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3} \left(B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx = \int \left(C\cos\left(c+d\,x\right)^2 + B\,\cos\left(c+d\,x\right)\right) \left(a+b\,\cos\left(c+d\,x\right)\right)^{2/3} dx$$

input
```
int((B*cos(c + d*x) + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(2/3),x)
```

output
```
int((B*cos(c + d*x) + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{2/3} \left(B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx = \left(\int \left(\cos\left(dx+c\right)b+a\right)^{\frac{2}{3}}\cos\left(dx+c\right)dx\right)b + \left(\int \left(\cos\left(dx+c\right)b+a\right)^{\frac{2}{3}}\cos\left(dx+c\right)^{2}dx\right)c$$

input
```
int((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((cos(c + d*x)*b + a)**(2/3)*cos(c + d*x),x)*b + int((cos(c + d*x)*b +
a)**(2/3)*cos(c + d*x)**2,x)*c
```

3.235 $\int \sqrt[3]{a+b\cos(c+dx)}(B\cos(c+dx)+C\cos^2(c+dx)$

Optimal result

Integrand size = 34, antiderivative size = 281

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd}$$
$$+ \frac{\sqrt{2}(7bB-3aC)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{4/3}}$$
$$- \frac{\sqrt{2}(7abB-3a^2C-4b^2C)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

output

```
3/7*C*(a+b*cos(d*x+c))^(4/3)*sin(d*x+c)/b/d+1/7*2^(1/2)*(7*B*b-3*C*a)*Appe
llF1(1/2,-4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(
d*x+c))^(4/3)*sin(d*x+c)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b
))^(4/3)-1/7*2^(1/2)*(7*B*a*b-3*C*a^2-4*C*b^2)*AppellF1(1/2,-1/3,1/2,3/2,b
*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c
)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b))^(1/3)
```

Mathematica [A] (warning: unable to verify)

Time = 5.71 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.03

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(-a^2+b^2)(7bB-3aC)\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c}{a+b}\right.\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x
]
```

output
```
(-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*(7*b*B - 3*a*C
)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c
+ d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + C
os[c + d*x]))/(a - b))] + (7*a*b*B - 3*a^2*C + 16*b^2*C)*AppellF1[4/3, 1/2
, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sq
rt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a -
b))]*(a + b*Cos[c + d*x]) - 4*b^2*(7*b*B + a*C + 4*b*C*Cos[c + d*x])*Sin[c
 + d*x]^2))/(112*b^3*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 3502, 27, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{3\int \frac{1}{3}\sqrt[3]{a+b\cos(c+dx)}(4bC+(7bB-3aC)\cos(c+dx))dx}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt[3]{a+b\cos(c+dx)}(4bC+(7bB-3aC)\cos(c+dx))dx}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(4bC+(7bB-3aC)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{3235}$$

$$\frac{\frac{(7bB-3aC)\int (a+b\cos(c+dx))^{4/3}dx}{b}-\frac{(-3a^2C+7abB-4b^2C)\int \sqrt[3]{a+b\cos(c+dx)}dx}{b}}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(7bB-3aC)\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx}{b}-\frac{(-3a^2C+7abB-4b^2C)\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{3144}$$

$$\frac{\frac{(-3a^2C+7abB-4b^2C)\sin(c+dx)\int \frac{\sqrt[3]{a+b\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{(7bB-3aC)\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{4/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}}{7b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \text{156}$$

$$\frac{\frac{(-3a^2C+7abB-4b^2C)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\int\frac{\sqrt[3]{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a+b)(7bB-3aC)\sin(c+dx)\sqrt[3]{a}}{bd\sqrt{1-\cos(c+}}}{7b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$\Big\downarrow$ 155

$$\frac{\frac{\sqrt{2}(a+b)(7bB-3aC)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\sqrt{2}(-3a^2C+7abB-4b^2C)}{}}{7b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

input
```
Int[(a + b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) + ((Sqrt[2]*(a + b)*
(7*b*B - 3*a*C)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1
- Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) - (Sqrt[2]*(7*a
*b*B - 3*a^2*C - 4*b^2*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])
/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x
])/(b*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)))/(7*b
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a+b\cos(dx+c))^{\frac{1}{3}} \left(B\cos(dx+c)+C\cos(dx+c)^2\right) dx$$

input
```
int((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(1/3), x
)
```

Sympy [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (B+C\cos(c+dx))\sqrt[3]{a+b\cos(c+dx)}\cos(c+dx)\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Integral((B + C*cos(c + d*x))*(a + b*cos(c + d*x))**(1/3)*cos(c + d*x), x)
```

Maxima [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(1/3),
x)
```

Giac [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)\right)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(1/3),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

input
```
int((B*cos(c + d*x) + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(1/3),x)
```

output
```
int((B*cos(c + d*x) + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\left(\int (\cos(dx+c)\,b+a)^{\frac{1}{3}}\cos(dx+c)\,dx\right)b$$
$$+\left(\int (\cos(dx+c)\,b+a)^{\frac{1}{3}}\cos(dx+c)^2\,dx\right)c$$

input
```
int((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((cos(c + d*x)*b + a)**(1/3)*cos(c + d*x),x)*b + int((cos(c + d*x)*b +
a)**(1/3)*cos(c + d*x)**2,x)*c
```

3.236 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 281

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{\sqrt{2}(5bB-3aC)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} - \frac{\sqrt{2}(5abB-3a^2C-2b^2C)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\sin(c+dx)}{5b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

output
```
3/5*C*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d+1/5*2^(1/2)*(5*B*b-3*C*a)*Appe
llF1(1/2,-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(
d*x+c))^(2/3)*sin(d*x+c)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b
))^(2/3)-1/5*2^(1/2)*(5*B*a*b-3*C*a^2-2*C*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*
(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*si
n(d*x+c)/b^2/d/(1+cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/3)
```

Mathematica [A] (warning: unable to verify)

Time = 3.68 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.94

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx =$$

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(5(-5abB+3a^2C+2b^2C)\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\right.}{}$$

input
```
Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x
]
```

output
```
(-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(-5*a*b*B + 3*a^2*C + 2*b^2
*C)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[
c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 +
 Cos[c + d*x]))/(a - b))] + 2*(5*b*B - 3*a*C)*AppellF1[5/3, 1/2, 1/2, 8/3,
 (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1
 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*C
os[c + d*x]) - 10*b^2*C*Sin[c + d*x]^2))/(50*b^3*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 3502, 27, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 3502$$

$$\frac{3\int \frac{2bC+(5bB-3aC)\cos(c+dx)}{3\sqrt[3]{a+b\cos(c+dx)}}dx}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2bC+(5bB-3aC)\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}dx}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{2bC+(5bB-3aC)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 3235$$

$$\frac{\frac{(5bB-3aC)\int (a+b\cos(c+dx))^{2/3}dx}{b}-\frac{\left(-3a^2C+5abB-2b^2C\right)\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}dx}{b}}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(5bB-3aC)\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}-\frac{\left(-3a^2C+5abB-2b^2C\right)\int \frac{1}{\sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 3144$$

$$\frac{\frac{\left(-3a^2C+5abB-2b^2C\right)\sin(c+dx)\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{(5bB-3aC)\sin(c+dx)\int \frac{(a+b\cos(}{\sqrt{1-\cos(c+dx}}}{bd\sqrt{1-\cos(c+dx)}\sqrt{co}}}{5b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 156$$

$$\frac{\frac{\left(-3a^2C+5abB-2b^2C\right)\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\int\frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}-\frac{(5bB-}{}}{5b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$\Big\downarrow$ 155

$$\frac{\frac{\sqrt{2}(5bB-3aC)\sin(c+dx)(a+b\cos(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}-\frac{\sqrt{2}\left(-3a^2C+5abB-2b^2C\right)\sin(c+dx)}{b}}{5b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

input
```
Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

output
```
(3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + ((Sqrt[2]*(5*b*B -
 3*a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c
+ d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 + C
os[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*(5*a*b*B - 3
*a^2*C - 2*b^2*C)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1
 - Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*
x])/(b*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3)))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{B\cos(dx+c)+C\cos(dx+c)^2}{(a+b\cos(dx+c))^{\frac{1}{3}}}dx$$

input
```
int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

output
```
int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{C\cos(dx+c)^2+B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(1/3), x
)
```

Sympy [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{(B+C\cos{(c+dx)})\cos{(c+dx)}}{\sqrt[3]{a+b\cos{(c+dx)}}}\,dx$$

input
```
integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/3),x)
```

output
```
Integral((B + C*cos(c + d*x))*cos(c + d*x)/(a + b*cos(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{C\cos{(dx+c)}^2+B\cos{(dx+c)}}{(b\cos{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(1/3),
x)
```

Giac [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \int \frac{C\cos{(dx+c)}^2+B\cos{(dx+c)}}{(b\cos{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(1/3),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3),x)
```

output
```
int((B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\left(\int \frac{\cos\left(dx+c\right)}{\left(\cos\left(dx+c\right)b+a\right)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{\cos\left(dx+c\right)^2}{\left(\cos\left(dx+c\right)b+a\right)^{\frac{1}{3}}}dx\right)c$$

input
```
int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

output
```
int(cos(c + d*x)/(cos(c + d*x)*b + a)**(1/3),x)*b + int(cos(c + d*x)**2/(c
os(c + d*x)*b + a)**(1/3),x)*c
```

3.237 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 281

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}$$
$$+\frac{(4bB-3aC)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$-\frac{(4abB-3a^2C-b^2C)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\sin(c+d}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

output
```
3/4*C*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)/b/d+1/4*(4*B*b-3*C*a)*AppellF1(1/2
,-1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^
(1/3)*sin(d*x+c)*2^(1/2)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b
))^(1/3)-1/4*(4*B*a*b-3*C*a^2-C*b^2)*AppellF1(1/2,2/3,1/2,3/2,b*(1-cos(d*x
+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(d*x+c)*2
^(1/2)/b^2/d/(1+cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(2/3)
```

Mathematica [A] (warning: unable to verify)

Time = 3.68 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.93

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx =$$

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(-4abB+3a^2C+b^2C)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right.\right.}{}$$

input
```
Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x
]
```

output
```
(-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-4*a*b*B + 3*a^2*C + b^2*C
)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c
+ d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + C
os[c + d*x]))/(a - b))] + (4*b*B - 3*a*C)*AppellF1[4/3, 1/2, 1/2, 7/3, (a
+ b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + C
os[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c
 + d*x]) - 4*b^2*C*Sin[c + d*x]^2))/(16*b^3*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 3502, 27, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{3\int \frac{bC+(4bB-3aC)\cos(c+dx)}{3(a+b\cos(c+dx))^{2/3}}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$\downarrow$ 27

$$\frac{\int \frac{bC+(4bB-3aC)\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$\downarrow$ 3042

$$\frac{\int \frac{bC+(4bB-3aC)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$\downarrow$ 3235

$$\frac{\frac{(4bB-3aC)\int \sqrt[3]{a+b\cos(c+dx)}dx}{b}-\frac{(-3a^2C+4abB-b^2C)\int \frac{1}{(a+b\cos(c+dx))^{2/3}}dx}{b}}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$\downarrow$ 3042

$$\frac{\frac{(4bB-3aC)\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{(-3a^2C+4abB-b^2C)\int \frac{1}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$\downarrow$ 3144

$$\frac{\frac{(-3a^2C+4abB-b^2C)\sin(c+dx)\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{(4bB-3aC)\sin(c+dx)\int \frac{\sqrt[3]{a+b\cos(c+}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+}}}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)-}}}{4b}$$

$$\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$\downarrow$ 156

$$\frac{\frac{(-3a^2C+4abB-b^2C)\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}\right)^{2/3}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}-\frac{(4bB-3aC)\sin(c+dx}{bd\sqrt{}}}{4b}$$

$$\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$$\Big\downarrow \text{155}$$

$$\frac{\frac{\sqrt{2}(4bB-3aC)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{\sqrt{2}(-3a^2C+4abB-b^2C)\sin(c+}{}}{4b}$$

$$\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

input
```
Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) + ((Sqrt[2]*(4*b*B -
 3*a*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c
+ d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 + C
os[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) - (Sqrt[2]*(4*a*b*B - 3
*a^2*C - b^2*C)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 -
 Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x]
)/(b*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3)))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{B\cos(dx+c)+C\cos(dx+c)^2}{(a+b\cos(dx+c))^{\frac{2}{3}}}dx$$

input
```
int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

output
```
int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos{(dx+c)}^2+B\cos{(dx+c)}}{(b\cos{(dx+c)}+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(2/3), x
)
```

Sympy [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{(B+C\cos{(c+dx)})\cos{(c+dx)}}{(a+b\cos{(c+dx)})^{\frac{2}{3}}}\,dx$$

input
```
integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((B + C*cos(c + d*x))*cos(c + d*x)/(a + b*cos(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos{(dx+c)}^2+B\cos{(dx+c)}}{(b\cos{(dx+c)}+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(2/3),
x)
```

Giac [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos(dx+c)^2+B\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(2/3),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos(c+d\,x)^2+B\,\cos(c+d\,x)}{(a+b\,\cos(c+d\,x))^{2/3}}\,dx$$

input
```
int((B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3),x)
```

output
```
int((B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \left(\int \frac{\cos(dx+c)}{(\cos(dx+c)\,b+a)^{\frac{2}{3}}}dx\right)b + \left(\int \frac{\cos(dx+c)^2}{(\cos(dx+c)\,b+a)^{\frac{2}{3}}}dx\right)c$$

input
```
int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

output
```
int(cos(c + d*x)/(cos(c + d*x)*b + a)**(2/3),x)*b + int(cos(c + d*x)**2/(c
os(c + d*x)*b + a)**(2/3),x)*c
```

3.238 $\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+\right.$

Optimal result

Integrand size = 31, antiderivative size = 187

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\frac{C(a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}$$
$$-\frac{(C(1+m)+A(2+m))(a\cos(e+fx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e+f}{af(1+m)(2+m)\sqrt{\sin^2(e+fx)}}$$
$$-\frac{B(a\cos(e+fx))^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{a^2f(2+m)\sqrt{\sin^2(e+fx)}}$$

output
```
C*(a*cos(f*x+e))^(1+m)*sin(f*x+e)/a/f/(2+m)-(C*(1+m)+A*(2+m))*(a*cos(f*x+e
))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/a
/f/(1+m)/(2+m)/(sin(f*x+e)^2)^(1/2)-B*(a*cos(f*x+e))^(2+m)*hypergeom([1/2,
 1+1/2*m],[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/a^2/f/(2+m)/(sin(f*x+e)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.77

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \frac{(a\cos(e+fx))^m \cot(e+fx) \left(-\left((C(1+m)+A(2+m)) \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+f\right.\right.\right.}{}$$

input
```
Integrate[(a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

output
```
((a*Cos[e + f*x])^m*Cot[e + f*x]*(-((C*(1 + m) + A*(2 + m))*Hypergeometric
2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2]) + (1
+ m)*(C*Sin[e + f*x]^2 - B*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + m)/2,
(4 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])))/(f*(1 + m)*(2 + m))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m \left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)+C\sin\left(e+fx+\frac{\pi}{2}\right)^2\right) dx$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{\int (a\cos(e+fx))^m(a(C(m+1)+A(m+2))+aB(m+2)\cos(e+fx))dx}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx))^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(a \sin \left(e+fx+\frac{\pi}{2}\right)\right)^m \left(a(C(m+1)+A(m+2))+aB(m+2)\sin \left(e+fx+\frac{\pi}{2}\right)\right)dx}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx))^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{a(A(m+2)+C(m+1))\int (a\cos(e+fx))^m dx+B(m+2)\int (a\cos(e+fx))^{m+1}dx}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx))^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(A(m+2)+C(m+1))\int \left(a\sin \left(e+fx+\frac{\pi}{2}\right)\right)^m dx+B(m+2)\int \left(a\sin \left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}dx}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx))^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{3122}$$

$$\frac{-\frac{(A(m+2)+C(m+1))\sin(e+fx)(a\cos(e+fx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{f(m+1)\sqrt{\sin^2(e+fx)}}-\frac{B\sin(e+fx)(a\cos(e+fx))^{m+2}\text{Hyperg}\ldots}{af\sqrt{\sin\ldots}}}{a(m+2)}$$
$$\frac{C\sin(e+fx)(a\cos(e+fx))^{m+1}}{af(m+2)}$$

input
```
Int[(a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

output
```
(C*(a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (-(((C*(1 + m) +
 A*(2 + m))*(a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3
+ m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 + m)*Sqrt[Sin[e + f*x]^2])) -
(B*(a*Cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, C
os[e + f*x]^2]*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]))/(a*(2 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a\cos(fx+e))^m \left(A+B\cos(fx+e)+C\cos(fx+e)^2\right) dx$$

input
```
int((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
int((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)+A\right)(a\cos\left(fx+e\right))^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="f
ricas")
```

output
```
integral((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e))^m, x)
```

Sympy [F]

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int (a\cos\left(e+fx\right))^m \left(A+B\cos\left(e+fx\right)+C\cos^2\left(e+fx\right)\right)\,dx$$

input
```
integrate((a*cos(f*x+e))**m*(A+B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

output
```
Integral((a*cos(e + f*x))**m*(A + B*cos(e + f*x) + C*cos(e + f*x)**2), x)
```

Maxima [F]

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)+A\right)(a\cos\left(fx+e\right))^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="m
axima")
```

output
```
integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \int \left(C\cos\left(fx+e\right)^2+B\cos\left(fx+e\right)+A\right)\left(a\cos\left(fx+e\right)\right)^m dx$$

input
```
integrate((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="g
iac")
```

output
```
integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \int \left(a\,\cos\left(e+f\,x\right)\right)^m\,\left(C\cos\left(e+f\,x\right)^2+B\,\cos\left(e+f\,x\right)+A\right) dx$$

input
```
int((a*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2),x)
```

output
```
int((a*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2), x)
```

Reduce [F]

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx = a^m\left(\left(\int \cos(fx+e)^m dx\right)a+\left(\int \cos(fx+e)^m \cos(fx+e) dx\right)b + \left(\int \cos(fx+e)^m \cos(fx+e)^2 dx\right)c\right)$$

input
```
int((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
a**m*(int(cos(e + f*x)**m,x)*a + int(cos(e + f*x)**m*cos(e + f*x),x)*b + i
nt(cos(e + f*x)**m*cos(e + f*x)**2,x)*c)
```

3.239 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+$

Optimal result

Integrand size = 41, antiderivative size = 209

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{10bB\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd}+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}$$

output

```
2/15*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
d/cos(d*x+c)^(1/2)+10/21*b*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+10/21*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/
d+2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*B*(b*cos(d*x+c))^
(5/2)*sin(d*x+c)/b^2/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^3/d
```

Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.60

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b\cos(c+dx)}\left(84(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+300B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(7(36A\right.}{630d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[
c + d*x]^2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(84*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 300*B*El
lipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A + 43*C)*Cos[c + d*x]
 + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[c + d*x]))
/(630*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.366$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$\Big\downarrow \text{2030}$$
$$\frac{\int (b\cos(c+dx))^{5/2}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^2}$$

$\downarrow$ 3502

$$\frac{\frac{2\int \frac{1}{2}(b\cos(c+dx))^{5/2}(b(9A+7C)+9bB\cos(c+dx))dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^2}$$

$\downarrow$ 27

$$\frac{\frac{\int (b\cos(c+dx))^{5/2}(b(9A+7C)+9bB\cos(c+dx))dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(b(9A+7C)+9bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^2}$$

$\downarrow$ 3227

$$\frac{\frac{b(9A+7C)\int (b\cos(c+dx))^{5/2}dx+9B\int (b\cos(c+dx))^{7/2}dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^2}$$

$\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+9B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^2}$$

$\downarrow$ 3115

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{9b}+2C\sin(c+d}{b^2}$$

$\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{9b}+2C}{b^2}$$

$\downarrow$ 3115

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+2b\sin(c+dx)\right.}{9b}}{b^2}$$

$\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\cdots}{9b}}{b^2}$$

$\downarrow$ 3121

$$\frac{\frac{b(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)\cdots}{9b}}{b^2}$$

$\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos\cdots}}{3d}\cdots\right.\right.}{9b}}{b^2}$$

$\downarrow$ 3119

$$\frac{\frac{9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+b(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+\cdots}}{5d\sqrt{\cos(c+dx)}}\cdots\right.}{9b}}{b^2}$$

$\downarrow$ 3120

$$\frac{\frac{b(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+d\cdots}}{3d}\cdots\right.\right.}{9b}}{b^2}$$

input

```
Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*
x]^2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + (b*(9*A + 7*C)*((6*b^
2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)) + 9*B*((2*b*(b*Cos[c +
 d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x
]]*Sin[c + d*x])/(3*d)))/7))/(9*b))/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(184) = 368$.

Time = 7.46 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.83

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,b\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(720B+2240C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(\right.}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,b\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\right.\right.}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\right.}}$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,me
thod=_RETURNVERBOSE)
```

output
```
-2/315*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-1120
*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2
*c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1
/2*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-126*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*
d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.91

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{2\left(75i\sqrt{\frac{1}{2}}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-75i\sqrt{\frac{1}{2}}B\sqrt{b}\text{weierstrassPIn}\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="fricas")
```

output
```
-2/315*(75*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) - 75*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 21*sqrt(1/2)*(-9*I*A - 7*I*C)*sqrt(b)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 21*sqrt(1/2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*C*cos(d*x + c)^3 +
45*B*cos(d*x + c)^2 + 7*(9*A + 7*C)*cos(d*x + c) + 75*B)*sqrt(b*cos(d*x +
c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)
**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos
(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos
(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2),x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\sqrt{b}\left(\left(\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4dx\right)c+\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3dx\right)b\right.\right.$$
$$\left.+\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2dx\right)a\right)$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c + d*
x))*cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a)
```

3.240 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C$

Optimal result

Integrand size = 39, antiderivative size = 180

$$\begin{aligned}&\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx\\&=\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}\\&\quad+\frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}\\&\quad+\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}\\&\quad+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}\end{aligned}$$

output

```
6/5*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x
+c)^(1/2)+2/21*b*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*(b*cos(d*x+c))^(1/2)*sin(d*
x+c)/d+2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*C*(b*cos(d*x+c))^(5/2
)*sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 2.88 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.62

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{(b\cos(c+dx))^{3/2}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(70A\right.}{105bd\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c
+ d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*
EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c +
 d*x] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*b*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (b\cos(c+dx))^{3/2}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b}$$

$$\Big\downarrow \; 3502$$

$$\frac{\frac{2\int \frac{1}{2}(b\cos(c+dx))^{3/2}(b(7A+5C)+7bB\cos(c+dx))dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int (b\cos(c+dx))^{3/2}(b(7A+5C)+7bB\cos(c+dx))dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b(7A+5C)+7bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b}$$

$$\Big\downarrow \ 3227$$

$$\frac{\frac{b(7A+5C)\int (b\cos(c+dx))^{3/2}dx+7B\int (b\cos(c+dx))^{5/2}dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{b(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+7B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b}$$

$$\Big\downarrow \ 3115$$

$$\frac{\frac{b(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+\frac{2C\sin(c+dx)(b\cos}{7bd}}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{b(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+2C\sin(c+}{b}$$

$$\Big\downarrow \ 3121$$

$$\frac{\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+2}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}}{b}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+}{b}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{b(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}}{b}$$

input
```
Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]
^2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + (b*(7*A + 5*C)*((2*b^
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]])
 + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + 7*B*((6*b^2*Sqrt[b*Cos
[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(b*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/(7*b))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(159) = 318.

Time = 3.33 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.95

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b\left(240C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(-168B-360C\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(140}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin}\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/105*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(240*C
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-360*C)*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x
+1/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

$$\int\cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{2\left(5\sqrt{\frac{1}{2}}(7i\,A+5i\,C)\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\sqrt{\frac{1}{2}}(-7i\,A-5i\right.}{}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="fricas")
```

output
```
-2/105*(5*sqrt(1/2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*sqrt(1/2)*(-7*I*A - 5*I*C)*sqrt(b)*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(1/2)*B*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + 63*I*sqrt(1/2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*C*cos(d*x + c)^2 + 21
*B*cos(d*x + c) + 35*A + 25*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos
(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos
(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*
x)^2),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*
x)^2), x)
```

Reduce [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\sqrt{b}\left(\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a+\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)c+\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x))
*cos(c + d*x)**3,x)*c + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.241 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+$

Optimal result

Integrand size = 33, antiderivative size = 145

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}$$

output
```
2/5*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d
/cos(d*x+c)^(1/2)+2/3*b*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/3*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d+2/5
*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{2\sqrt{b\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5B+3C\right.}{15d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Ell
ipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c
 + d*x]))/(15*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \; 3502$$

$$\frac{2\int \frac{1}{2}\sqrt{b\cos(c+dx)}(b(5A+3C)+5bB\cos(c+dx))dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sqrt{b\cos(c+dx)}(b(5A+3C)+5bB\cos(c+dx))dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(5A+3C)+5bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b(5A+3C)\int \sqrt{b\cos(c+dx)}dx+5B\int (b\cos(c+dx))^{3/2}dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{b(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}+5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(5A+3C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}+5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3119}$$

$$\frac{5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+5B\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + ((2*b*(5*A + 3*C)*Sqrt
[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + 5*B*(
(2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d
*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(128) = 256$.

Time = 1.90 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.19

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b\left(24C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-20B-24C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(10B+6C\right.}{}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\, d}-\frac{2B\sqrt{b\left(-1+\right.}}{}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,method=_RETURNV
ERBOSE)
```

output
```
2/15*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(24*C*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{\frac{1}{2}}B\sqrt{b}\,\mathrm{weierstrassPInve}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - 5*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) + 3*sqrt(1/2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
3*sqrt(1/2)*(5*I*A + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*C*cos(d*x + c) + 5*B)*sqr
t(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \sqrt{b\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

output
```
int((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\cos\left(dx+c\right)dx\right)b + \left(\int \sqrt{\cos\left(dx+c\right)}\cos\left(dx+c\right)^2dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x)
,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*c)
```

3.242 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal result

Integrand size = 39, antiderivative size = 112

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}}$$
$$+\frac{2b(3A+C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}$$
$$+\frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c
)^(1/2)+2/3*b*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/
2))/d/(b*cos(d*x+c))^(1/2)+2/3*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\frac{b\left(6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+d\right.}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec
[c + d*x],x]
```

output
```
(b*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)]))/(3*d*Sqrt[b*C
os[c + d*x]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.282$, Rules used = {3042, 2030, 3502, 27, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{2030}$$
$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{3502}$$

$$b\left(\frac{2\int \frac{b(3A+C)+3bB\cos(c+dx)}{2\sqrt{b\cos(c+dx)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{27}$$

$$b\left(\frac{\int \frac{b(3A+C)+3bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{\int \frac{b(3A+C)+3bB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3227}$$

$$b\left(\frac{b(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+3B\int \sqrt{b\cos(c+dx)}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{b(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3B\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3121}$$

$$b\left(\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{3B\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{3B\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{6BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3120}$$

$$b\left(\frac{\frac{2b(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{6BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d
*x],x]
```

output
```
b*(((6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d
*x]]) + (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sq
rt[b*Cos[c + d*x]]))/(3*b) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*
d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(101) = 202.

Time = 1.19 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.53

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \text{EllipticF}\left(c\right.}{3\sqrt{-b\left(2s\right.}}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\, d}+\frac{2B\sqrt{b\left(-1\right.}}{}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(4*C*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.33

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = -\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A+i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{\frac{1}{2}}(-3i\,A-i\,C)\sqrt{b}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="fricas")
```

output
```
-2/3*(sqrt(1/2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(1/2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(1/2)*B*sqrt(b)*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
 + 3*I*sqrt(1/2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
d
```

Sympy [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \sqrt{b\cos\left(c+dx\right)}\left(A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec
(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec
(d*x + c), x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec
(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x),x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x), x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\sqrt{b}\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\sec(dx+c)\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*b + int(sqrt(
cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x),x)*c + int(sqrt(cos(c + d*x))*s
ec(c + d*x),x)*a)
```

3.243 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 109

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output

```
-2*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(
d*x+c)^(1/2)+2*b*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
/d/(b*cos(d*x+c))^(1/2)+2*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\frac{2b\left(-\left((A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\sin(c+d}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec
[c + d*x]^2,x]
```

output
```
(2*b*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c +
d*x]])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.268, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^2\left(\frac{2\int \frac{b^2B-b^2(A-C)\cos(c+dx)}{2\sqrt{b\cos(c+dx)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$b^2\left(\frac{\int \frac{b^2B-b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\int \frac{b^2B-b^2(A-C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(\frac{b^2B\int \frac{1}{\sqrt{b\cos(c+dx)}}dx-b(A-C)\int \sqrt{b\cos(c+dx)}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{b^2B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b(A-C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^2\left(\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{b(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{b(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$\Big\downarrow$ 3120

$$b^2\left(\frac{\frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d
*x]^2,x]
```

output
```
b^2*(((-2*b*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqr
t[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/
(d*Sqrt[b*Cos[c + d*x]]))/b^3 + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x
]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(102) = 204$.

Time = 1.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.39

method	result
default	$\frac{2b\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\cdots\right.\right.\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}\cdots\right.\right.}}$
parts	$-\frac{2Ab\left(-2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\cdots\right.}\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,me
thod=_RETURNVERBOSE)
```

output
```
2*b*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin
(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2
))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.66

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$
$$-\frac{2\left(i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\left(dx+c\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\left(dx\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="fricas")
```

output
```
-2*(I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(1/2)*(I*A - I*C)*sqrt(b
)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + sqrt(1/2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) - sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec
(d*x + c)^2, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec
(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^2,x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^2, x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^2dx\right)b+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2dx\right)c+\left(\int \sqrt{\cos\left(dx+c\right)}\,\sec\left(dx+c\right)^2dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*b + int(sq
rt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x)*c + int(sqrt(cos(c + d
*x))*sec(c + d*x)**2,x)*a)
```

3.244 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 140

$$\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx = -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
-2*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+
c)^(1/2)+2/3*b*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1
/2))/d/(b*cos(d*x+c))^(1/2)+2/3*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*
b*B*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.64

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\frac{2b\Big(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(A+3B\cos}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec
[c + d*x]^3,x]
```

output
```
(2*b*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*
x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.317$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^3\left(\frac{2\int\frac{3Bb^2+(A+3C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$b^3\left(\frac{\int\frac{3Bb^2+(A+3C)\cos(c+dx)b^2}{(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{\int\frac{3Bb^2+(A+3C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^3\left(\frac{b(A+3C)\int\frac{1}{\sqrt{b\cos(c+dx)}}dx+3b^2B\int\frac{1}{(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{b(A+3C)\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\int\frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^3\left(\frac{b(A+3C)\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{b(A+3C)\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^3\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$b^3\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+d}{3bd(b\cos(c+d}\right)$$

$\Big\downarrow$ 3119

$$b^3\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^3}\right)$$

$\Big\downarrow$ 3120

$$b^3\left(\frac{\frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx}{3bd(b\cos(c+dx)}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d
*x]^3,x]
```

output
```
b^3*((2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + ((2*b*(A + 3*C)*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + 3*
b^2*B*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos
[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(3*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(127) = 254$.

Time = 1.06 (sec) , antiderivative size = 505, normalized size of antiderivative = 3.61

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-12\cdots\right.}{\cdots}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(\cdots}{}}\cdots\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\cdots}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,me
thod=_RETURNVERBOSE)
```

output
```
2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/2*
c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.42

$$\int \sqrt{b\cos(c+dx)}\big(A+B\cos(c+dx)+C\cos^2(c+dx)\big)\sec^3(c+dx)\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}(i\,A+3i\,C)\sqrt{b}\cos{(dx+c)}^2\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{\frac{1}{2}}(-\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="fricas")
```

output
```
-2/3*(sqrt(1/2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-I*A - 3*I*C)*sqrt(b)*co
s(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3
*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(1/2)*B*sqrt(b)*co
s(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) - (3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx$$
$$= \int (C\cos(dx+c)^2 + B\cos(dx+c) + A)\sqrt{b\cos(dx+c)}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec
(d*x + c)^3, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)\,dx$$
$$= \int (C\cos(dx+c)^2 + B\cos(dx+c) + A)\sqrt{b\cos(dx+c)}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec
(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^3,x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^3, x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\sqrt{b}\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\sec(dx+c)^3\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*b + int(sq
rt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**3,x)*c + int(sqrt(cos(c + d
*x))*sec(c + d*x)**3,x)*a)
```

3.245 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 181

$$\begin{aligned}
&\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx \\
&= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} \\
&\quad+\frac{2bB\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} \\
&\quad+\frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

output

```
-2/5*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
d/cos(d*x+c)^(1/2)+2/3*b*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2
)+2/3*b^2*B*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/5*b*(3*A+5*C)*sin(d*x+c)/d
/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx =$$
$$-\frac{\sqrt{b\cos(c+dx)}\sec^2(c+dx)\left(6(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-10B\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\right.}{15d}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec
[c + d*x]^4,x]
```

output
```
-1/15*(Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(6*(3*A + 5*C)*Cos[c + d*x]^(3/
2)*EllipticE[(c + d*x)/2, 2] - 10*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)
/2, 2] - 10*B*Sin[c + d*x] - 9*A*Sin[2*(c + d*x)] - 15*C*Sin[2*(c + d*x)]
- 6*A*Tan[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.317$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \; 3500$$

$$b^4\left(\frac{2\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 27$$

$$b^4\left(\frac{\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^4\left(\frac{\int \frac{5Bb^2+(3A+5C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3227$$

$$b^4\left(\frac{b(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+5b^2B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^4\left(\frac{b(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+5b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3116$$

$$b^4\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A\sin(c+}{5bd(b\cos(c+}\right.$$

$$\Big\downarrow \; 3042$$

$$b^4\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A}{5bd(b}\right.$$

$$\Big\downarrow \; 3121$$

$$b^4\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right.}{5b^3}\right.$$

$\downarrow$ 3042

$$b^4\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c}{3bd(b\cos(c}\right.}{5b^3}\right.$$

$\downarrow$ 3119

$$b^4\left(\frac{5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c-}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^3}\right.$$

$\downarrow$ 3120

$$b^4\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+d}{3bd(b\cos(c+d}\right.}{5b^3}\right.$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d
*x]^4,x]
```

output
```
b^4*((2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (5*b^2*B*((2*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) +
 (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + b*(3*A + 5*C)*((-2*Sqr
t[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) +
(2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(160) = 320$.

Time $= 1.92$ (sec) , antiderivative size $= 800$, normalized size of antiderivative $= 4.42$

method	result	size
parts	Expression too large to display	800
default	Expression too large to display	804

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,me
thod=_RETURNVERBOSE)
```

output
```
-2/5*A*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x
+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d-2/
3*B*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*b*(b*(-1+2*cos(1/2*d*x+1
/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(-1+2*cos(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(b*(-1+
2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d-2*C*b*(-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*b*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.22

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\left(dx+c\right)^3 \text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^3*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(1/2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(1/2)*(-3*I*A - 5*I*C)*s
qrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))) - (3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*
x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec
(d*x + c)^4, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec
(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^4,x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^4, x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\sqrt{b}\left(\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^4\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\sec(dx+c)^4\,dx\right)a\right)\right.$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**4,x)*b + int(sq
rt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**4,x)*c + int(sqrt(cos(c + d
*x))*sec(c + d*x)**4,x)*a)
```

3.246 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^5(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 210

$$\begin{aligned}&\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx\\&=-\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}\\&\quad+\frac{2b(5A+7C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}\\&\quad+\frac{2b^3B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{6bB\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
-6/5*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*
x+c)^(1/2)+2/21*b*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/7*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(7/
2)+2/5*b^3*B*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/21*b^2*(5*A+7*C)*sin(d*x+
c)/d/(b*cos(d*x+c))^(3/2)+6/5*b*B*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.68

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$= \frac{2\sqrt{b\cos(c+dx)}\sec^3(c+dx)\left(-63B\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5A+7C)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\right.}{}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec
[c + d*x]^5,x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3*(-63*B*Cos[c + d*x]^(5/2)*EllipticE
[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2,
 2] + 21*B*Sin[c + d*x] + 63*B*Cos[c + d*x]^2*Sin[c + d*x] + (25*A*Sin[2*(
c + d*x)])/2 + (35*C*Sin[2*(c + d*x)])/2 + 15*A*Tan[c + d*x]))/(105*d)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.366$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5 \int \frac{C \sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2 + B \sin\left(\frac{1}{2}(2c+\pi)+dx\right) + A}{\left(b \sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 3500}$$

$$b^5 \left(\frac{2\int \frac{7Bb^2+(5A+7C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$b^5 \left(\frac{\int \frac{7Bb^2+(5A+7C)\cos(c+dx)b^2}{(b\cos(c+dx))^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^5 \left(\frac{\int \frac{7Bb^2+(5A+7C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3227}$$

$$b^5 \left(\frac{b(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx + 7b^2B\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^5 \left(\frac{b(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx + 7b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3116}$$

$$b^5 \left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right) + 7b^2B\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2} + \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^5\left(\frac{b(5A+7C)\left(\frac{\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\int\frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{7b^3}+\right.$$

$\downarrow$ 3116

$$b^5\left(\frac{b(5A+7C)\left(\frac{\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx)}\right.}{7b^3}\right.$$

$\downarrow$ 3042

$$b^5\left(\frac{b(5A+7C)\left(\frac{\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c}{5bd(b\cos(c}\right.}{7b^3}\right.$$

$\downarrow$ 3121

$$b^5\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}\right.}{7b^3}\right.$$

$\downarrow$ 3042

$$b^5\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\right.}}{b^2\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

$\downarrow$ 3119

$$b^5\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

$\downarrow$ 3120

$$b^5\left(\frac{b(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d
*x]^5,x]
```

output
```
b^5*((2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (b*(5*A + 7*C)*((
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*
x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + 7*b^2*B*((2*Sin[
c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*
d*Sqrt[b*Cos[c + d*x]])))/(5*b^2)))/(7*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 724 vs. $2(185) = 370$.

Time = 2.51 (sec) , antiderivative size = 725, normalized size of antiderivative = 3.45

method	result	size
default	Expression too large to display	725
parts	Expression too large to display	1000

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,me
thod=_RETURNVERBOSE)
```

output
```
-2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(A*(-1/56*
cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/2
1*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))+1/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1
/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+C*(-1/6*cos(1
/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(-1+2*co
s(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.10

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx =$$

$$-\frac{2\left(5\sqrt{\frac{1}{2}}(5i\,A+7i\,C)\sqrt{b}\cos\left(dx+c\right)^4\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\sqrt{}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
5,x, algorithm="fricas")
```

output
```
-2/105*(5*sqrt(1/2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(1/2)*(-5*I*A - 7*I*C)*
sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) + 63*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(1/2)*
B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - (63*B*cos(d*x + c)^3 + 5*(5*A + 7*C)*co
s(d*x + c)^2 + 21*B*cos(d*x + c) + 15*A)*sqrt(b*cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
5,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec
(d*x + c)^5, x)
```

Giac [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
5,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec
(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\int \frac{\sqrt{b\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^5}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^5,x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^5, x)
```

Reduce [F]

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$=\sqrt{b}\left(\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^5\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^5\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\sec(dx+c)^5\,dx\right)a\right)\right.$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

output
```
sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**5,x)*b + int(sq
rt(cos(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**5,x)*c + int(sqrt(cos(c + d
*x))*sec(c + d*x)**5,x)*a)
```

3.247 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+\right.$

Optimal result

Integrand size = 39, antiderivative size = 210

$$\begin{aligned}&\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)\right.\\&\left.+C\cos^2(c+dx)\right)\,dx=\frac{2b(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}\\&+\frac{10b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}\\&+\frac{10bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}\\&+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}\end{aligned}$$

output
```
2/15*b*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)/d/cos(d*x+c)^(1/2)+10/21*b^2*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+10/21*b*B*(b*cos(d*x+c))^(1/2)*sin(d
*x+c)/d+2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(b*cos(d*x+
c))^(5/2)*sin(d*x+c)/b/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.61

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{(b\cos(c+dx))^{5/2}\left(84(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+300B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[
c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(84*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 300*B*
EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A + 43*C)*Cos[c + d*
x] + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[c + d*x]
))/(630*b*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b}$$

$\downarrow$ 3502

$$\frac{\frac{2\int \frac{1}{2}(b\cos(c+dx))^{5/2}(b(9A+7C)+9bB\cos(c+dx))dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b}$$

$\downarrow$ 27

$$\frac{\frac{\int (b\cos(c+dx))^{5/2}(b(9A+7C)+9bB\cos(c+dx))dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b}$$

$\downarrow$ 3042

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(b(9A+7C)+9bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b}$$

$\downarrow$ 3227

$$\frac{\frac{b(9A+7C)\int (b\cos(c+dx))^{5/2}dx+9B\int (b\cos(c+dx))^{7/2}dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b}$$

$\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+9B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b}$$

$\downarrow$ 3115

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{9b}+2C\sin(c+d}{b}$$

$\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{9b}+2C}{b}$$

$\downarrow$ 3115

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)}{}\right.}{9b}}{b}$$

$\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c}{}\right.}{9b}}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{b(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)\right.}{9b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos}}{3d}\right.\right.}{9b}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+b(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+}}{5d\sqrt{\cos(c+dx)}}\right.}{9b}}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{b(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+d}}{3d}\right.\right.}{9b}}{b}$$

input

```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*
x]^2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + (b*(9*A + 7*C)*((6*b^
2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)) + 9*B*((2*b*(b*Cos[c +
 d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x
]]*Sin[c + d*x])/(3*d)))/7))/(9*b))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(185) = 370$.

Time = 2.58 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.83

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b^{2} (-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(720B+2240C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b^{2} (-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b(-1+2\text{c}}}$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/315*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-11
20*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos
(1/2*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(-126*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/
2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.93

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{2\left(75i\sqrt{\frac{1}{2}}Bb^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-75i\sqrt{\frac{1}{2}}Bb^{\frac{3}{2}}\text{weierstrassPInvers}\right.}{}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="fricas")
```

output
```
-2/315*(75*I*sqrt(1/2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) - 75*I*sqrt(1/2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(1/2)*(9*A + 7*C)*b^(3/2)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 21*I*sqrt(1/2)*(9*A + 7*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*C*b*cos(d*x + c)^3 + 45*
B*b*cos(d*x + c)^2 + 7*(9*A + 7*C)*b*cos(d*x + c) + 75*B*b)*sqrt(b*cos(d*x
 + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*c
os(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*c
os(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*
x)^2),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*
x)^2), x)
```

Reduce [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\sqrt{b}\,b\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{4}dx\right)c+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{3}dx\right)b+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{2}dx\right)a\right)$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c +
d*x))*cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a)
```

3.248 $\int (b\cos(c+dx))^{3/2} (A + B\cos(c+dx) + C\cos^2(c+$

Optimal result

Integrand size = 33, antiderivative size = 181

$$\begin{aligned}
&\int (b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \, dx = \frac{6bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} \\
&+ \frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} \\
&+ \frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} \\
&+ \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}
\end{aligned}$$

output

```
6/5*b*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d
*x+c)^(1/2)+2/21*b^2*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/
2*c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/21*b*(7*A+5*C)*(b*cos(d*x+c))^(1/2)*
sin(d*x+c)/d+2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*C*(b*cos(d*x+c))^
(5/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.60

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{(b\cos(c+dx))^{3/2}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\right.}{105d\cos^{\frac{3}{2}}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x
]
```

output
```
((b*Cos[c + d*x])^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*
EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c +
 d*x] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2\int \frac{1}{2}(b\cos(c+dx))^{3/2}(b(7A+5C)+7bB\cos(c+dx))dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (b\cos(c+dx))^{3/2}(b(7A+5C)+7bB\cos(c+dx))dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b(7A+5C)+7bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3227

$$\frac{b(7A+5C)\int (b\cos(c+dx))^{3/2}dx+7B\int (b\cos(c+dx))^{5/2}dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{b(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+7B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3115

$$\frac{b(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx}{5d}\right.}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{b(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)}{}\right.}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3121

$$\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx}{}\right.}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin(c+dx+\frac{\pi}{2})}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b}{}\right.}{7b}$$

$$\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3119

$$\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+}{}\right.}{7b}$$

$$\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 3120

$$\frac{b(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(}{}\right.}{7b}$$

$$\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + (b*(7*A + 5*C)*((2*b^2
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]])
+ (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + 7*B*((6*b^2*Sqrt[b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(b*Co
s[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(160) = 320$.

Time = 1.59 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.95

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b^{2}\left(240C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-168B-360C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(14\right.}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b^{2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\,s\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\, d}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/105*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(240
*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-360*C)*sin(1/2*d*x+1/2*
c)^6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.01

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}(7A+5C)b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{\frac{1}{2}}(7A+5C)b^{\frac{3}{2}}\text{w}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
-2/105*(5*I*sqrt(1/2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) - 5*I*sqrt(1/2)*(7*A + 5*C)*b^(3/2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(1/2)*B*b^(3/2)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + 63*I*sqrt(1/2)*B*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*C*b*cos(d*x + c)^2 + 21*B*
b*cos(d*x + c) + 5*(7*A + 5*C)*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\, dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2),
x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int {\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

output
```
int((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\sqrt{b}\,b\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{3}dx\right)c+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^{2}dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x
))*cos(c + d*x)**3,x)*c + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b)
```

3.249 $\int (b\cos(c+dx))^{3/2} (A + B\cos(c+dx) + C\cos^2(c+dx)) \sec(c+dx)\, dx$

Optimal result

Integrand size = 39, antiderivative size = 146

$$\int (b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec(c+dx)\, dx = \frac{2b(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/5*b*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
/d/cos(d*x+c)^(1/2)+2/3*b^2*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2
*c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/3*b*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)
/d+2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{2b\sqrt{b\cos(c+dx)}\Big(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5\cdots}{15d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x],x]
```

output
```
(2*b*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*E
llipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin
[c + d*x]))/(15*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 2030, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A\right)dx$$

$$\Big\downarrow \text{3502}$$

$$b\left(\frac{2\int \frac{1}{2}\sqrt{b\cos(c+dx)}(b(5A+3C)+5bB\cos(c+dx))dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \textbf{27}$$

$$b\left(\frac{\int \sqrt{b\cos(c+dx)}(b(5A+3C)+5bB\cos(c+dx))dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$b\left(\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(5A+3C)+5bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \textbf{3227}$$

$$b\left(\frac{b(5A+3C)\int \sqrt{b\cos(c+dx)}dx+5B\int (b\cos(c+dx))^{3/2}dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$b\left(\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\Big\downarrow \textbf{3115}$$

$$b\left(\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(\cdots}{5\cdots}\right.$$

$$\Big\downarrow \textbf{3042}$$

$$b\left(\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+\cdots}{\cdots}\right.$$

$$\Big\downarrow \textbf{3121}$$

$$b\left(\frac{\frac{b(5A+3C)\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}+5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx\right.$$

$\downarrow$ 3042

$$b\left(\frac{\frac{b(5A+3C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}+5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(\right.$$

$\downarrow$ 3119

$$b\left(\frac{5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{5b}+\frac{2C\sin(c+\right.$$

$\downarrow$ 3120

$$b\left(\frac{\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+5B\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c\right.$$

input

```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x],x]
```

output

```
b*((2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + ((2*b*(5*A + 3*C)*S
qrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + 5*
B*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
+ d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/(5*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(129) = 258$.

Time = 1.20 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.18

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,b^2\left(24C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-20B-24C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(10B+6C\right.}{}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,d} - \frac{2B\sqrt{b\left(-1+\right.}}{}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,meth
od=_RETURNVERBOSE)
```

output
```
2/15*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(24*C*
sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.13

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx =$$
$$-\frac{2\left(5i\sqrt{\frac{1}{2}}Bb^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})-5i\sqrt{\frac{1}{2}}Bb^{\frac{3}{2}}\text{weierstrassPInverse}(\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - 5*I*sqrt(1/2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) - 3*I*sqrt(1/2)*(5*A + 3*C)*b^(3/2)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I
*sqrt(1/2)*(5*A + 3*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*C*b*cos(d*x + c) + 5*B*b)*sqrt
(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*s
ec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*s
ec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x),x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\sqrt{b}\,b\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)\,dx\right)a+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*a + int(sqr
t(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x),x)*c + int(sqrt(cos(c + d*x))
*cos(c + d*x)**2*sec(c + d*x),x)*b)
```

3.250 $\int (b\cos(c+dx))^{3/2} (A + B\cos(c+dx) + C\cos^2(c+dx))\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 116

$$\int (b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{2bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*b*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x
+c)^(1/2)+2/3*b^2*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/3*b*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{b^2\left(6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\,\text{si}\right.}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^2,x]
```

output
```
(b^2*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)]))/(3*d*Sqrt[b
*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.268, Rules used = {3042, 2030, 3502, 27, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \; 3502$$

$$b^2\left(\frac{2\int \frac{b(3A+C)+3bB\cos(c+dx)}{2\sqrt{b\cos(c+dx)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \; 27$$

$$b^2\left(\frac{\int \frac{b(3A+C)+3bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{\int \frac{b(3A+C)+3bB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \; 3227$$

$$b^2\left(\frac{b(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+3B\int \sqrt{b\cos(c+dx)}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{b(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3B\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \; 3121$$

$$b^2\left(\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{3B\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{3B\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{6BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(\frac{\frac{2b(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{6BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^2,x]
```

output
```
b^2*(((6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c +
 d*x]]) + (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*
Sqrt[b*Cos[c + d*x]]))/(3*b) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*
b*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(105) = 210$.

Time = 0.90 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.46

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,b^2\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\cdots\right.\right.}{3\sqrt{-b\left(2\cdots\right.}}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,d}+\frac{2B\sqrt{b\left(-\cdots\right.}}{\cdots}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,me
thod=_RETURNVERBOSE)
```

output
```
-2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(4*C*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))
^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.28

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(3A+C)b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{\frac{1}{2}}(3A+C)b^{\frac{3}{2}}\text{weierst}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="fricas")
```

output
```
-2/3*(I*sqrt(1/2)*(3*A + C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - I*sqrt(1/2)*(3*A + C)*b^(3/2)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(1/2)*B*b^(3/2)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*
I*sqrt(1/2)*B*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*C*b*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*s
ec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*s
ec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \frac{(b\,\cos(c+d\,x))^{3/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^2}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^2,x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^2, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\sqrt{b}\,b\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^2\,dx\right)a+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^2\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*a + int(
sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**2,x)*c + int(sqrt(cos(c +
 d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x)*b)
```

3.251 $\int (b\cos(c+dx))^{3/2} (A + B\cos(c + dx) + C\cos^2(c + dx)) dx$

Optimal result

Integrand size = 41, antiderivative size = 114

$$\int (b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^3(c+dx)\, dx = -\frac{2b(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\, 2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
-2*b*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/co
s(d*x+c)^(1/2)+2*b^2*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1
/2))/d/(b*cos(d*x+c))^(1/2)+2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{2b^2\left(-\left((A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A\right.}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^3,x]
```

output
```
(2*b^2*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c
+ d*x]])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.268, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^3\left(\frac{2\int \frac{b^2B-b^2(A-C)\cos(c+dx)}{2\sqrt{b\cos(c+dx)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$b^3\left(\frac{\int \frac{b^2B-b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{\int \frac{b^2B-b^2(A-C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^3\left(\frac{b^2B\int \frac{1}{\sqrt{b\cos(c+dx)}}dx-b(A-C)\int \sqrt{b\cos(c+dx)}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{b^2B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b(A-C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^3\left(\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{b(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{b(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^3\left(\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^3\left(\frac{\frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^3,x]
```

output
```
b^3*(((-2*b*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqr
t[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/
(d*Sqrt[b*Cos[c + d*x]]))/b^3 + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x
]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(107) = 214$.

Time = 0.49 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.30

method	result
default	$\frac{2b^2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}\right.\right.}}$
parts	$-\frac{2A\,b^2\left(-2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{-2b\sin\left(\frac{dx}{2}+\right.}\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,me
thod=_RETURNVERBOSE)
```

output
```
2*b^2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-s
in(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)
^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.58

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}Bb^{\frac{3}{2}}\cos\left(dx+c\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-i\sqrt{\frac{1}{2}}Bb^{\frac{3}{2}}\cos\left(dx+c\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="fricas")
```

output
```
-2*(I*sqrt(1/2)*B*b^(3/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - I*sqrt(1/2)*B*b^(3/2)*cos(d*x + c)*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sqrt(1/2)*(A - C)*b^(3/2)*
cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) - I*sqrt(1/2)*(A - C)*b^(3/2)*cos(d*x + c)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) -
sqrt(b*cos(d*x + c))*A*b*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int (C\cos(dx+c)^2+B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*s
ec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int (C\cos(dx+c)^2+B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*s
ec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int\frac{(b\,\cos\left(c+d\,x\right))^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^3,x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^3, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\sqrt{b}\,b\left(\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^3dx\right)a+\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)^3dx\right)c+\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)^3dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*a + int(
sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**3,x)*c + int(sqrt(cos(c +
 d*x))*cos(c + d*x)**2*sec(c + d*x)**3,x)*b)
```

3.252 $\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 41, antiderivative size = 145

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = -\frac{2bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
-2*b*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*
x+c)^(1/2)+2/3*b^2*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/3*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2
)+2*b^2*B*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{2b^2\left(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(A$$
$$3d\sqrt{b\cos(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^4,x]
```

output
```
(2*b^2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c +
d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.317, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^4\left(\frac{2\int \frac{3Bb^2+(A+3C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$b^4\left(\frac{\int \frac{3Bb^2+(A+3C)\cos(c+dx)b^2}{(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{\int \frac{3Bb^2+(A+3C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^4\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+3b^2B\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^4\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^4\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right.$$

$\downarrow$ 3042

$$b^4\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+}{3bd(b\cos(c+d}\right.$$

$\downarrow$ 3119

$$b^4\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^3}\right.$$

$\downarrow$ 3120

$$b^4\left(\frac{\frac{2b(A+3C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx}{3bd(b\cos(c+dx)}\right.$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^4,x]
```

output
```
b^4*((2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + ((2*b*(A + 3*C)*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + 3*
b^2*B*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos
[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(3*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(132) = 264$.

Time = 0.53 (sec) , antiderivative size = 506, normalized size of antiderivative = 3.49

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-12 \cdots\right.}{\cdots}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(\cdots}{}}\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\cdots}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,me
thod=_RETURNVERBOSE)
```

output
```
2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*d
*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/
2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.38

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(A+3C)b^{\frac{3}{2}}\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{\frac{1}{2}}(A+3\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="fricas")
```

output
```
-2/3*(I*sqrt(1/2)*(A + 3*C)*b^(3/2)*cos(d*x + c)^2*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) - I*sqrt(1/2)*(A + 3*C)*b^(3/2)*cos(d*x
 + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sq
rt(1/2)*B*b^(3/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(1/2)*B*b^(3/2)*cos(d*x
 + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) - (3*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sin(d*x
+ c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*s
ec(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*s
ec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^4,x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^4, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \sqrt{b}\,b\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^4 dx\right)a + \left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)^4 dx\right)c + \left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)^4 dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**4,x)*a + int(
sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**4,x)*c + int(sqrt(cos(c +
 d*x))*cos(c + d*x)**2*sec(c + d*x)**4,x)*b)
```

3.253 $\int (b\cos(c+dx))^{3/2} (A+B\cos(c+dx)+C\cos^2(c+dx)) \sec^5(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 186

$$\begin{aligned}\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec^5(c+dx)\,dx = & -\frac{2b(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} \\ & +\frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} \\ & +\frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
-2/5*b*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)/d/cos(d*x+c)^(1/2)+2/3*b^2*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/
2*c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/5*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^
(5/2)+2/3*b^3*B*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/5*b^2*(3*A+5*C)*sin(d*
x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.66

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx =$$
$$-\frac{(b\cos(c+dx))^{3/2}\sec^3(c+dx)\left(6(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-10B\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\right.}{15d}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^5,x]
```

output
```
-1/15*((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*(6*(3*A + 5*C)*Cos[c + d*x]^(
3/2)*EllipticE[(c + d*x)/2, 2] - 10*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*
x)/2, 2] - 10*B*Sin[c + d*x] - 9*A*Sin[2*(c + d*x)] - 15*C*Sin[2*(c + d*x)
] - 6*A*Tan[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.317$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^5\left(\frac{2\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$b^5\left(\frac{\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{\int \frac{5Bb^2+(3A+5C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^5\left(\frac{b(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+5b^2B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{b(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+5b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^5\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A\sin(c+}{5bd(b\cos(c+}\right.$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A}{5bd(b}\right.$$

$$\Big\downarrow \text{3121}$$

$$b^5\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right.}{5b^3}\right.$$

$\downarrow$ 3042

$$b^5\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c}{3bd(b\cos(c}\right.}{5b^3}\right.$$

$\downarrow$ 3119

$$b^5\left(\frac{5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c-}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^3}\right.$$

$\downarrow$ 3120

$$b^5\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+d}{3bd(b\cos(c+d}\right.}{5b^3}\right.$$

input

```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^5,x]
```

output

```
b^5*((2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (5*b^2*B*((2*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) +
 (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + b*(3*A + 5*C)*((-2*Sqr
t[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) +
(2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(165) = 330$.

Time $= 0.56$ (sec) , antiderivative size $= 805$, normalized size of antiderivative $= 4.33$

method	result	size
default	Expression too large to display	805
parts	Expression too large to display	805

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,me
thod=_RETURNVERBOSE)
```

output
```
-2/15*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2
*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1
/2*d*x+1/2*c)^4+120*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-60*C*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^4+36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^2-120*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+60*C*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+30*C*sin(1/2*d*x+1/2*c)^2*cos(1/2...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.20

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}Bb^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5i\sqrt{\frac{1}{2}}Bb^{\frac{3}{2}}\cos\left(dx\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
5,x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*b^(3/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(1/2)*B*b^(3/2)*cos(d*x + c)^3*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(1/2)*(3*
A + 5*C)*b^(3/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(1/2)*(3*A + 5*C)*b^(3/2
)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) - (3*(3*A + 5*C)*b*cos(d*x + c)^2 + 5*B*b*cos(d*x
 + c) + 3*A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
5,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*s
ec(d*x + c)^5, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
5,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*s
ec(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^5}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^5,x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^5, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \sqrt{b}\,b\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^5\,dx\right)a + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^5\,dx\right)c + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^5\,dx\right)b\right)$$

input

```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

output

```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**5,x)*a + int(
sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**5,x)*c + int(sqrt(cos(c +
 d*x))*cos(c + d*x)**2*sec(c + d*x)**5,x)*b)
```

3.254 $\int (b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 41, antiderivative size = 215

$$\int (b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^6(c+dx)\, dx = -\frac{6bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^4B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6b^2B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output
```
-6/5*b*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(
d*x+c)^(1/2)+2/21*b^2*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/7*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))
^(7/2)+2/5*b^4*B*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/21*b^3*(5*A+7*C)*sin(
d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*b^2*B*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.62

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \frac{(b\cos(c+dx))^{3/2}\sec^5(c+dx)\left(-504B\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40(5A+7C)\cos^{\frac{7}{2}}(c+d\cdots\right.}{\cdots}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^6,x]
```

output
```
((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^5*(-504*B*Cos[c + d*x]^(7/2)*Elliptic
E[(c + d*x)/2, 2] + 40*(5*A + 7*C)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/
2, 2] + 2*(110*A + 70*C + 273*B*Cos[c + d*x] + 10*(5*A + 7*C)*Cos[2*(c + d
*x)] + 63*B*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.366$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{2030}$$

$$b^6\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^6\left(\frac{2\int \frac{7Bb^2+(5A+7C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$b^6\left(\frac{\int \frac{7Bb^2+(5A+7C)\cos(c+dx)b^2}{(b\cos(c+dx))^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{\int \frac{7Bb^2+(5A+7C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^6\left(\frac{b(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx+7b^2B\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{b(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+7b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^6\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{7b^3}+\frac{2A\sin}{7bd(b\cos}\right.$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{7b^3}+\right.$$

$$\Big\downarrow \text{3116}$$

$$b^6\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx)}\right.}{7b^3}\right.$$

$\downarrow$ 3042

$$b^6\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c}{5bd(b\cos(c}\right.}{7b^3}\right.$$

$\downarrow$ 3121

$$b^6\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}\right.}{7b^3}\right.$$

$\downarrow$ 3042

$$b^6\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\right.}}{b^2\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

$\downarrow$ 3119

$$b^6\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

$$\Big\downarrow \; 3120$$

$$b^6\left(\frac{b(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^6,x]
```

output
```
b^6*((2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (b*(5*A + 7*C)*((
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*
x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + 7*b^2*B*((2*Sin[
c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*
d*Sqrt[b*Cos[c + d*x]])))/(5*b^2)))/(7*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(190) = 380.

Time = 0.58 (sec) , antiderivative size = 727, normalized size of antiderivative = 3.38

method	result	size
default	Expression too large to display	727
parts	Expression too large to display	1005

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,me
thod=_RETURNVERBOSE)
```

output
```
-2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(A*(-1/5
6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5
/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+1/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin
(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+C*(-1/6*cos
(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(-1+2*
cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.09

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}(5A+7C)b^{\frac{3}{2}}\cos{(dx+c)}^4\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})-5i\sqrt{\frac{1}{2}}(5\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
6,x, algorithm="fricas")
```

output
```
-2/105*(5*I*sqrt(1/2)*(5*A + 7*C)*b^(3/2)*cos(d*x + c)^4*weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(1/2)*(5*A + 7*C)*b^(3/
2)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) + 63*I*sqrt(1/2)*B*b^(3/2)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(1/2)*B*b^(
3/2)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c))) - (63*B*b*cos(d*x + c)^3 + 5*(5*A + 7*C)*b*cos
(d*x + c)^2 + 21*B*b*cos(d*x + c) + 15*A*b)*sqrt(b*cos(d*x + c))*sin(d*x +
 c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^6\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
6,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*s
ec(d*x + c)^6, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^6\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
6,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*s
ec(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^6}\,dx$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^6,x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^6, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\sqrt{b}\,b\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^6dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3\sec\left(dx+c\right)^6dx\right)c+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^2\sec\left(dx+c\right)^6dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

output
```
sqrt(b)*b*(int(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**6,x)*a + int(
sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**6,x)*c + int(sqrt(cos(c +
 d*x))*cos(c + d*x)**2*sec(c + d*x)**6,x)*b)
```

3.255 $\int (b\cos(c+dx))^{5/2} (A + B\cos(c+dx) + C\cos^2(c+$

Optimal result

Integrand size = 33, antiderivative size = 212

$$\int (b\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx = \frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{10b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}$$

output

```
2/15*b^2*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))/d/cos(d*x+c)^(1/2)+10/21*b^3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*
x+1/2*c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+10/21*b^2*B*(b*cos(d*x+c))^(1/2)*s
in(d*x+c)/d+2/45*b*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(b*co
s(d*x+c))^(5/2)*sin(d*x+c)/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.59

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx = \frac{(b\cos(c+dx))^{5/2}\left(84(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+300B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\right.}{}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x
]
```

output
```
((b*Cos[c + d*x])^(5/2)*(84*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 300*B*
EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A + 43*C)*Cos[c + d*
x] + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[c + d*x]
))/(630*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \; 3502$$

$$\frac{2\int \frac{1}{2}(b\cos(c+dx))^{5/2}(b(9A+7C)+9bB\cos(c+dx))dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (b\cos(c+dx))^{5/2}(b(9A+7C)+9bB\cos(c+dx))dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(b(9A+7C)+9bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b(9A+7C)\int (b\cos(c+dx))^{5/2}dx+9B\int (b\cos(c+dx))^{7/2}dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(9A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+9B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b}{7}\right.}{9b}$$
$$\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\right.}{9b}$$
$$\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+}{}\right.\right.}{9b}$$
$$\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$\downarrow$ 3042

$$\frac{b(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin}{}\right.\right.}{9b}$$

$$\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$\downarrow$ 3121

$$\frac{b(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b}{}\right.\right.}{9b}$$

$$\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$\downarrow$ 3042

$$\frac{b(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}}{3\sqrt{b\cos(c+dx)}}\right.\right.}{9b}$$

$$\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$\downarrow$ 3119

$$\frac{9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+b(9A+7C)\left(\frac{6b}{}\right.}{9b}$$

$$\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

$\downarrow$ 3120

$$\frac{b(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}\right.\right.}{9b}$$

$$\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + (b*(9*A + 7*C)*((6*b^2
*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]])
+ (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)) + 9*B*((2*b*(b*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]
]*Sin[c + d*x])/(3*d)))/7))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(187) = 374$.

Time = 5.32 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.81

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b^{3}\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(720B+2240C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\cdots\right.}{\cdots}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b^{3}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\cdots\right.}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\cdots\right.}}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/315*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-11
20*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos
(1/2*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(-126*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/
2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.96

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = -\frac{2\left(75i\sqrt{\frac{1}{2}}Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-75i\sqrt{\frac{1}{2}}Bb^{\frac{5}{2}}\text{weierstrassPInvers}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
-2/315*(75*I*sqrt(1/2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) - 75*I*sqrt(1/2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(1/2)*(9*A + 7*C)*b^(5/2)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 21*I*sqrt(1/2)*(9*A + 7*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*C*b^2*cos(d*x + c)^3 + 4
5*B*b^2*cos(d*x + c)^2 + 7*(9*A + 7*C)*b^2*cos(d*x + c) + 75*B*b^2)*sqrt(b
*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2),
x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(C\,\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

output
```
int((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \sqrt{b}\,b^2\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c
 + d*x))*cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*
a)
```

3.256 $\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal result

Integrand size = 39, antiderivative size = 183

$$\begin{aligned}&\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{6b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}\\&+\frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}\\&+\frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}\\&+\frac{2bB(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}\end{aligned}$$

output

```
6/5*b^2*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos
(d*x+c)^(1/2)+2/21*b^3*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/21*b^2*(7*A+5*C)*(b*cos(d*x+c))^(1
/2)*sin(d*x+c)/d+2/5*b*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*C*(b*cos(d*
x+c))^(5/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.60

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{b(b\cos(c+dx))^{3/2}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+\right.}{105d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x],x]
```

output
```
(b*(b*Cos[c + d*x])^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C
)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c
 + d*x] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 2030, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A\right)dx$$

$$\Big\downarrow \text{3502}$$

$$b\left(\frac{2\int \frac{1}{2}(b\cos(c+dx))^{3/2}(b(7A+5C)+7bB\cos(c+dx))dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}\right)$$

$$\Big\downarrow \text{27}$$

$$b\left(\frac{\int (b\cos(c+dx))^{3/2}(b(7A+5C)+7bB\cos(c+dx))dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b(7A+5C)+7bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}\right)$$

$$\Big\downarrow \text{3227}$$

$$b\left(\frac{b(7A+5C)\int (b\cos(c+dx))^{3/2}dx+7B\int (b\cos(c+dx))^{5/2}dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{b(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+7B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}\right.$$

$$\Big\downarrow \text{3115}$$

$$b\left(\frac{b(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c}{5d}\right.}{7b}\right.$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{b(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+}{}\right.}{7b}\right.$$

$$\Big\downarrow \text{3121}$$

$$b\left(\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c\ldots}{}\right.}{7b}\right.$$

↓ 3042

$$b\left(\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\ldots\right.}{7b}\right.$$

↓ 3119

$$b\left(\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin\ldots}{}\right.}{7b}\right.$$

↓ 3120

$$b\left(\frac{b(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\ldots}{}\right.}{7b}\right.$$

input

```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x],x]
```

output

```
b*((2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + (b*(7*A + 5*C)*((2*
b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]
]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + 7*B*((6*b^2*Sqrt[b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/(7*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(162) = 324$.

Time = 9.23 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.93

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b^{3}\left(240C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-168B-360C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(14\cdots\right.}{\cdots}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,b^{3}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2s\cdots}\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d\cdots}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/105*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(240
*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-360*C)*sin(1/2*d*x+1/2*
c)^6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.03

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}(7A+5C)b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{\frac{1}{2}}(7A+5C)b^{\frac{5}{2}}\text{w}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="fricas")
```

output
```
-2/105*(5*I*sqrt(1/2)*(7*A + 5*C)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) - 5*I*sqrt(1/2)*(7*A + 5*C)*b^(5/2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(1/2)*B*b^(5/2)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + 63*I*sqrt(1/2)*B*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*C*b^2*cos(d*x + c)^2 + 21*
B*b^2*cos(d*x + c) + 5*(7*A + 5*C)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))
/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(b\cos\left(dx+c\right))^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(b\cos\left(dx+c\right))^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \frac{(b\cos(c+d\,x))^{5/2}\,\left(C\cos(c+d\,x)^2+B\cos(c+d\,x)+A\right)}{\cos(c+d\,x)}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x),x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\sqrt{b}\,b^2\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x),x)*c + i
nt(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x),x)*b + int(sqrt(cos(c +
 d*x))*cos(c + d*x)**2*sec(c + d*x),x)*a)
```

3.257 $\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 151

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{2b^2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/5*b^2*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))/d/cos(d*x+c)^(1/2)+2/3*b^3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1
/2*c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*B*(b*cos(d*x+c))^(1/2)*sin(d*
x+c)/d+2/5*b*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.64

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{2b^2\sqrt{b\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\right.}{15d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^2,x]
```

output
```
(2*b^2*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B
*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*S
in[c + d*x]))/(15*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.317$, Rules used = {3042, 2030, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A\right)dx$$

$$\downarrow \text{3502}$$

$$b^2\left(\frac{2\int \frac{1}{2}\sqrt{b\cos(c+dx)}(b(5A+3C)+5bB\cos(c+dx))dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\downarrow \text{27}$$

$$b^2\left(\frac{\int \sqrt{b\cos(c+dx)}(b(5A+3C)+5bB\cos(c+dx))dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\downarrow \text{3042}$$

$$b^2\left(\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(5A+3C)+5bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\downarrow \text{3227}$$

$$b^2\left(\frac{b(5A+3C)\int \sqrt{b\cos(c+dx)}dx+5B\int (b\cos(c+dx))^{3/2}dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\downarrow \text{3042}$$

$$b^2\left(\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}\right)$$

$$\downarrow \text{3115}$$

$$b^2\left(\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)}{5}\right.$$

$$\downarrow \text{3042}$$

$$b^2\left(\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+2C\sin(c+\right.$$

$$\downarrow \text{3121}$$

$$b^2\left(\frac{\frac{b(5A+3C)\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}+5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+d\ldots}{}\right.$$

$\downarrow$ 3042

$$b^2\left(\frac{\frac{b(5A+3C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}+5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin\ldots}{}\right.$$

$\downarrow$ 3119

$$b^2\left(\frac{5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{5b}+\frac{2C\sin(c+\ldots}{}\right.$$

$\downarrow$ 3120

$$b^2\left(\frac{\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+5B\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c\ldots}{}\right.$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^2,x]
```

output
```
b^2*((2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + ((2*b*(5*A + 3*C)
*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) +
5*B*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/(5*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(134) = 268$.

Time = 12.93 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.11

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b^{3}\left(24C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-20B-24C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(10B+6C\right.}{}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, b^{3}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}-\frac{2B\sqrt{b\left(-1+\right.}}{}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,me
thod=_RETURNVERBOSE)
```

output
```
2/15*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(24*C*
sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.12

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{\frac{1}{2}}Bb^{\frac{5}{2}}\text{weierstrassPInverse}(\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - 5*I*sqrt(1/2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) - 3*I*sqrt(1/2)*(5*A + 3*C)*b^(5/2)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I
*sqrt(1/2)*(5*A + 3*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*C*b^2*cos(d*x + c) + 5*B*b^2)*
sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \frac{(b\cos(c+d\,x))^{5/2}\,\left(C\cos(c+d\,x)^2+B\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^2}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^2,x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^2, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \sqrt{b}\,b^2\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^2\,dx\right)c + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^2\,dx\right)b + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^2\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**2,x)*c
+ int(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**2,x)*b + int(sqrt(c
os(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**2,x)*a)
```

3.258 $\int (b\cos(c+dx))^{5/2} (A + B\cos(c+dx) + C\cos^2(c+dx)) dx$

Optimal result

Integrand size = 41, antiderivative size = 120

$$\int (b\cos(c+dx))^{5/2} (A + B\cos(c+dx) + C\cos^2(c+dx)) \sec^3(c+dx)\, dx = \frac{2b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*b^2*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d
*x+c)^(1/2)+2/3*b^3*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)
/d
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\frac{2(b\cos(c+dx))^{5/2}\left(3BE\left(\frac{1}{2}(c+dx)\middle|2\right)+(3A+C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sqrt{\cos(c+dx)}\right.}{3d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^3,x]
```

output
```
(2*(b*Cos[c + d*x])^(5/2)*(3*B*EllipticE[(c + d*x)/2, 2] + (3*A + C)*Ellip
ticF[(c + d*x)/2, 2] + C*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d*Cos[c + d*
x]^(5/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.268$, Rules used = {3042, 2030, 3502, 27, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{3502}$$

$$b^3\left(\frac{2\int \frac{b(3A+C)+3bB\cos(c+dx)}{2\sqrt{b\cos(c+dx)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{27}$$

$$b^3\left(\frac{\int \frac{b(3A+C)+3bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{\int \frac{b(3A+C)+3bB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^3\left(\frac{b(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+3B\int \sqrt{b\cos(c+dx)}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{b(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3B\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^3\left(\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{3B\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{3B\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^3\left(\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{6BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^3\left(\frac{\frac{2b(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{6BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}\right)$$

input

```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^3,x]
```

output

```
b^3*(((6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c +
 d*x]]) + (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*
Sqrt[b*Cos[c + d*x]]))/(3*b) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*
b*d))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030

```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(109) = 218$.

Time = 68.94 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.38

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,b^3\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cdots\right.\right.}{3\sqrt{-b\left(2\cdots\right.}}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,d}+\frac{2B\sqrt{b\left(-\cdots\right.}}{\cdots}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,me
thod=_RETURNVERBOSE)
```

output
```
-2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(4*C*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))
^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(3A+C)b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{\frac{1}{2}}(3A+C)b^{\frac{5}{2}}\text{weierst}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="fricas")
```

output
```
-2/3*(I*sqrt(1/2)*(3*A + C)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - I*sqrt(1/2)*(3*A + C)*b^(5/2)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(1/2)*B*b^(5/2)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*
I*sqrt(1/2)*B*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*C*b^2*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\cos\left(c+d\,x\right)}^3}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^3,x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^3, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\sqrt{b}\,b^2\left(\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^3dx\right)c+\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^3dx\right)b+\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^3dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**3,x)*c
+ int(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**3,x)*b + int(sqrt(c
os(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**3,x)*a)
```

3.259 $\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 116

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = -\frac{2b^2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
-2*b^2*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/
cos(d*x+c)^(1/2)+2*b^3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^
(1/2))/d/(b*cos(d*x+c))^(1/2)+2*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{2b^3\left(-\left((A-C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+A}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^4,x]
```

output
```
(2*b^3*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c
+ d*x]])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.268, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^4\left(\frac{2\int \frac{b^2B-b^2(A-C)\cos(c+dx)}{2\sqrt{b\cos(c+dx)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \textbf{27}$$

$$b^4\left(\frac{\int \frac{b^2B-b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$b^4\left(\frac{\int \frac{b^2B-b^2(A-C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \textbf{3227}$$

$$b^4\left(\frac{b^2B\int \frac{1}{\sqrt{b\cos(c+dx)}}dx-b(A-C)\int \sqrt{b\cos(c+dx)}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$b^4\left(\frac{b^2B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b(A-C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \textbf{3121}$$

$$b^4\left(\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{b(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$b^4\left(\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{b(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \textbf{3119}$$

$$b^4\left(\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^4\left(\frac{\frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^4,x]
```

output
```
b^4*(((-2*b*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqr
t[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/
(d*Sqrt[b*Cos[c + d*x]]))/b^3 + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x
]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(109) = 218$.

Time = 219.99 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.26

method	result
default	$\frac{2b^3\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}\right.\right.}}$
parts	$-\frac{2A\,b^3\left(-2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{-2b\sin\left(\frac{dx}{2}+\right.}\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,me
thod=_RETURNVERBOSE)
```

output
```
2*b^3*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-s
in(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)
^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.57

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}Bb^{\frac{5}{2}}\cos\left(dx+c\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))-i\sqrt{\frac{1}{2}}Bb^{\frac{5}{2}}\cos\left(dx+c\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="fricas")
```

output
```
-2*(I*sqrt(1/2)*B*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - I*sqrt(1/2)*B*b^(5/2)*cos(d*x + c)*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sqrt(1/2)*(A - C)*b^(5/2)*
cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c))) - I*sqrt(1/2)*(A - C)*b^(5/2)*cos(d*x + c)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) -
sqrt(b*cos(d*x + c))*A*b^2*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^4,x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^4, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\sqrt{b}\,b^2\left(\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^4\,dx\right)c+\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^4\,dx\right)b+\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^4\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**4,x)*c
+ int(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**4,x)*b + int(sqrt(c
os(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**4,x)*a)
```

3.260 $\int (b\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 41, antiderivative size = 147

$$\int (b\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^5(c+dx)\, dx = -\frac{2b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
-2*b^2*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(
d*x+c)^(1/2)+2/3*b^3*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/3*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(3
/2)+2*b^3*B*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\frac{2b^3\left(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(A\right.}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^5,x]
```

output
```
(2*b^3*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c +
d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.317, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \; 2030$$

$$b^5\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3500$$

$$b^5\left(\frac{2\int \frac{3Bb^2+(A+3C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$b^5\left(\frac{\int \frac{3Bb^2+(A+3C)\cos(c+dx)b^2}{(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{\int \frac{3Bb^2+(A+3C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^5\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+3b^2B\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^5\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^5\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right.$$

$\downarrow$ 3042

$$b^5\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+}{3bd(b\cos(c+d}\right.$$

$\downarrow$ 3119

$$b^5\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^3}\right.$$

$\downarrow$ 3120

$$b^5\left(\frac{\frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx}{3bd(b\cos(c+dx)}\right.$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^5,x]
```

output
```
b^5*((2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + ((2*b*(A + 3*C)*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + 3*
b^2*B*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos
[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(3*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(134) = 268$.

Time = 0.19 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.46

$$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\, b^2\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

output
```
2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/2
*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*
b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+
1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.39

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}(A+3C)b^{\frac{5}{2}}\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})-i\sqrt{\frac{1}{2}}(A+3\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
5,x, algorithm="fricas")
```

output
```
-2/3*(I*sqrt(1/2)*(A + 3*C)*b^(5/2)*cos(d*x + c)^2*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) - I*sqrt(1/2)*(A + 3*C)*b^(5/2)*cos(d*x
 + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sq
rt(1/2)*B*b^(5/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(1/2)*B*b^(5/2)*cos(d*x
 + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) - (3*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
5,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c)^5, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
5,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\int \frac{(b\cos(c+d\,x))^{5/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^5}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^5,x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^5, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx=\sqrt{b}\,b^2\left(\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^5\,dx\right)c+\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^5\,dx\right)b+\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^5\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**5,x)*c
+ int(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**5,x)*b + int(sqrt(c
os(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**5,x)*a)
```

3.261 $\int (b\cos(c+dx))^{5/2} (A + B\cos(c + dx) + C\cos^2(c + dx)) dx$

Optimal result

Integrand size = 41, antiderivative size = 188

$$\int (b\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = -\frac{2b^2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output
```
-2/5*b^2*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))/d/cos(d*x+c)^(1/2)+2/3*b^3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+
1/2*c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/5*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c)
)^(5/2)+2/3*b^4*B*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/5*b^3*(3*A+5*C)*sin(
d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.64

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx =$$
$$-\frac{2b^4\left(3(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)-5B\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-5B\sin(c+ \right.}{15d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^6,x]
```

output
```
(-2*b^4*(3*(3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - 5*B*
Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] - 5*B*Sin[c + d*x] - (9*A*Sin
[2*(c + d*x)])/2 - (15*C*Sin[2*(c + d*x)])/2 - 3*A*Tan[c + d*x]))/(15*d*(b
*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.317$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$
$$\Big\downarrow \text{2030}$$
$$b^6\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \; 3500$$

$$b^6\left(\frac{2\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 27$$

$$b^6\left(\frac{\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^6\left(\frac{\int \frac{5Bb^2+(3A+5C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3227$$

$$b^6\left(\frac{b(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+5b^2B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^6\left(\frac{b(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+5b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3116$$

$$b^6\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A\sin(c+}{5bd(b\cos(c+}\right.$$

$$\Big\downarrow \; 3042$$

$$b^6\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A}{5bd(b}\right.$$

$$\Big\downarrow \; 3121$$

$$b^6\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right.}{5b^3}\right.$$

$\downarrow$ 3042

$$b^6\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c}{3bd(b\cos(c}\right.}{5b^3}\right.$$

$\downarrow$ 3119

$$b^6\left(\frac{5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c-}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^3}\right.$$

$\downarrow$ 3120

$$b^6\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+d}{3bd(b\cos(c+d}\right.}{5b^3}\right.$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^6,x]
```

output
```
b^6*((2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (5*b^2*B*((2*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) +
 (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + b*(3*A + 5*C)*((-2*Sqr
t[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) +
(2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(167) = 334$.

Time = 0.20 (sec) , antiderivative size = 807, normalized size of antiderivative = 4.29

Expression too large to display

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

output
```
-2/15*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin
(1/2*d*x+1/2*c)^4+120*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-60*C*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^4+36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^2-120*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+60*C*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+30*C*sin(1/2*d*x+1/2*c)^2*cos(1...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.22

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}Bb^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))-5i\sqrt{\frac{1}{2}}Bb^{\frac{5}{2}}\cos\left(dx\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
6,x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*b^(5/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(1/2)*B*b^(5/2)*cos(d*x + c)^3*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(1/2)*(3*
A + 5*C)*b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(1/2)*(3*A + 5*C)*b^(5/2
)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) - (3*(3*A + 5*C)*b^2*cos(d*x + c)^2 + 5*B*b^2*cos
(d*x + c) + 3*A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^6\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
6,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c)^6, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^6\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
6,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \int \frac{(b\,\cos(c+d\,x))^{5/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^6}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^6,x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^6, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = \sqrt{b}\,b^2\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^6\,dx\right)c + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^6\,dx\right)b + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^6\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**6,x)*c
+ int(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**6,x)*b + int(sqrt(c
os(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**6,x)*a)
```

3.262 $\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 41, antiderivative size = 217

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = -\frac{6b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^5B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6b^3B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output

```
-6/5*b^2*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/co
s(d*x+c)^(1/2)+2/21*b^3*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x
+1/2*c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/7*A*b^6*sin(d*x+c)/d/(b*cos(d*x+c
))^(7/2)+2/5*b^5*B*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/21*b^4*(5*A+7*C)*si
n(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*b^3*B*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2
)
```

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.62

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \frac{(b\cos(c+dx))^{5/2}\sec^6(c+dx)\left(-504B\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40(5A+7C)\cos^{\frac{7}{2}}(c+dx)\right.}{}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^7,x]
```

output
```
((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^6*(-504*B*Cos[c + d*x]^(7/2)*Elliptic
E[(c + d*x)/2, 2] + 40*(5*A + 7*C)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/
2, 2] + 2*(110*A + 70*C + 273*B*Cos[c + d*x] + 10*(5*A + 7*C)*Cos[2*(c + d
*x)] + 63*B*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.366$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\Big\downarrow \text{2030}$$

$$b^7\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^7\left(\frac{2\int \frac{7Bb^2+(5A+7C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$b^7\left(\frac{\int \frac{7Bb^2+(5A+7C)\cos(c+dx)b^2}{(b\cos(c+dx))^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^7\left(\frac{\int \frac{7Bb^2+(5A+7C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^7\left(\frac{b(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx+7b^2B\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^7\left(\frac{b(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+7b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^7\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^7\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^7\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx)}\right.}{7b^3}\right.$$

$\downarrow$ 3042

$$b^7\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c-}{5bd(b\cos(c-}\right.}{7b^3}\right.$$

$\downarrow$ 3121

$$b^7\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}\right.}{7b^3}\right.$$

$\downarrow$ 3042

$$b^7\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\right.}}{b^2\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

$\downarrow$ 3119

$$b^7 \left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right) + 7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

$$\Big\downarrow \ 3120$$

$$b^7 \left(\frac{b(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right) + 7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^7,x]
```

output
```
b^7*((2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (b*(5*A + 7*C)*((
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*
x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + 7*b^2*B*((2*Sin[
c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*
d*Sqrt[b*Cos[c + d*x]])))/(5*b^2)))/(7*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(192) = 384$.

Time = 0.20 (sec) , antiderivative size = 727, normalized size of antiderivative = 3.35

Expression too large to display

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

output
```
-2*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(A*(-1/5
6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5
/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+1/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin
(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+C*(-1/6*cos
(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(-1+2*
cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.12

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}(5A+7C)b^{\frac{5}{2}}\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{\frac{1}{2}}(5\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
7,x, algorithm="fricas")
```

output
```
-2/105*(5*I*sqrt(1/2)*(5*A + 7*C)*b^(5/2)*cos(d*x + c)^4*weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(1/2)*(5*A + 7*C)*b^(5/
2)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) + 63*I*sqrt(1/2)*B*b^(5/2)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(1/2)*B*b^(
5/2)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c))) - (63*B*b^2*cos(d*x + c)^3 + 5*(5*A + 7*C)*b^2
*cos(d*x + c)^2 + 21*B*b^2*cos(d*x + c) + 15*A*b^2)*sqrt(b*cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**7,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^7\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
7,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c)^7, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^7\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
7,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*s
ec(d*x + c)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\int \frac{(b\cos(c+dx))^{5/2}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)}{\cos(c+dx)^7}\,dx$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^7,x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^7, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx=\sqrt{b}\,b^2\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^4\sec(dx+c)^7\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\sec(dx+c)^7\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\sec(dx+c)^7\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

output
```
sqrt(b)*b**2*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4*sec(c + d*x)**7,x)*c
+ int(sqrt(cos(c + d*x))*cos(c + d*x)**3*sec(c + d*x)**7,x)*b + int(sqrt(c
os(c + d*x))*cos(c + d*x)**2*sec(c + d*x)**7,x)*a)
```

3.263 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 214

$$\begin{aligned}
&\int \frac{\cos^3(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx\\
&= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}}\\
&\quad+\frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}\\
&\quad+\frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d}\\
&\quad+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}
\end{aligned}$$

output
```
2/15*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b/d/cos(d*x+c)^(1/2)+10/21*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+10/21*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/
b/d+2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*B*(b*cos(d*x+
c))^(5/2)*sin(d*x+c)/b^3/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^4/d
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.59

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{168(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+600B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(7(36A+}{1260d\sqrt{b\cos(c+d}}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*
Cos[c + d*x]],x]
```

output
```
(168*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 600*B*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A + 43*C)*Cos[c + d*x] +
 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]
)/(1260*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.366$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^3}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{1}{2}(b\cos(c+dx))^{5/2}(b(9A+7C)+9bB\cos(c+dx))dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int (b\cos(c+dx))^{5/2}(b(9A+7C)+9bB\cos(c+dx))dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(b(9A+7C)+9bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^3}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{b(9A+7C)\int (b\cos(c+dx))^{5/2}dx+9B\int (b\cos(c+dx))^{7/2}dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(9A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+9B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{9b}+\frac{2C\sin(c+d}{}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{9b}+2C}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)}{}}{9b}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(\ldots}{}\right.}{9b}}{b^3}$$

$\Big\downarrow$ 3121

$$\frac{\frac{b(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)\right.}{9b}}{b^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos\ldots}}{3d}\right.\right.}{9b}}{b^3}$$

$\Big\downarrow$ 3119

$$\frac{\frac{9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+b(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+\ldots}}{5d\sqrt{\cos(c+dx)}}\right.}{9b}}{b^3}$$

$\Big\downarrow$ 3120

$$\frac{\frac{b(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+d\ldots}}{3d}\right.\right.}{9b}}{b^3}$$

input

```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c
+ d*x]],x]
```

output
```
((2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + (b*(9*A + 7*C)*((6*b^
2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)) + 9*B*((2*b*(b*Cos[c +
 d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x
]]*Sin[c + d*x])/(3*d)))/7))/(9*b))/b^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(189) = 378$.

Time = 1.49 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.78

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(720B+2240C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-\right.}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\right.\right.}{5\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\right.}}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-2/315*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c
)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-
126*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*
x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx =$$
$$-\frac{2\left(75i\sqrt{\frac{1}{2}}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-75i\sqrt{\frac{1}{2}}B\sqrt{b}\text{weierstrassPIn}\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2
),x, algorithm="fricas")
```

output
```
-2/315*(75*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) - 75*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 21*sqrt(1/2)*(-9*I*A - 7*I*C)*sqrt(b)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 21*sqrt(1/2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*C*cos(d*x + c)^3 +
45*B*cos(d*x + c)^2 + 7*(9*A + 7*C)*cos(d*x + c) + 75*B)*sqrt(b*cos(d*x +
c))*sin(d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*co
s(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*co
s(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(1/2),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b}\,\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4 dx\right)c+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^3 dx\right)b+\left(\int \sqrt{\cos\left(dx+c\right)}\right.\right.}{b}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c + d
*x))*cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a))/
b
```

3.264 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 185

$$\begin{aligned}\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} \\ &\quad+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} \\ &\quad+\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} \\ &\quad+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}+\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}\end{aligned}$$

output
```
6/5*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/cos(d
*x+c)^(1/2)+2/21*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*(b*cos(d*x+c))^(1/2)*sin(d*
x+c)/b/d+2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*C*(b*cos(d*x+c))^
(5/2)*sin(d*x+c)/b^3/d
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.58

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(70A+65\right.}{105d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*
Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*Elli
pticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x
] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.317$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{3/2}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{\frac{2\int \frac{1}{2}(b\cos(c+dx))^{3/2}(b(7A+5C)+7bB\cos(c+dx))dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int (b\cos(c+dx))^{3/2}(b(7A+5C)+7bB\cos(c+dx))dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b(7A+5C)+7bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^2}$$

$\Big\downarrow$ 3227

$$\frac{\frac{b(7A+5C)\int (b\cos(c+dx))^{3/2}dx+7B\int (b\cos(c+dx))^{5/2}dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+7B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^2}$$

$\Big\downarrow$ 3115

$$\frac{\frac{b(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+\frac{2C\sin(c+dx)(b\cos}{7bd}}{b^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+2C\sin(c+}{b^2}$$

$\Big\downarrow$ 3121

$$\frac{\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+2}{b^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right.}{7b}}{b^2}$$

$\downarrow$ 3119

$$\frac{\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+}{b^2}$$

$\downarrow$ 3120

$$\frac{\frac{b(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}}{b^2}$$

input

```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c
+ d*x]],x]
```

output

```
((2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + (b*(7*A + 5*C)*((2*b^
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]])
 + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + 7*B*((6*b^2*Sqrt[b*Cos
[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(b*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/(7*b))/b^2
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030

```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(164) = 328.

Time = 1.32 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.89

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(240C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-168B-360C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(140A\right.}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-2/105*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*s
in(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6
*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2
*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx =$$

$$-\frac{2\left(5\sqrt{\frac{1}{2}}(7iA+5iC)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{\frac{1}{2}}(-7iA-5i\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2
),x, algorithm="fricas")
```

output
```
-2/105*(5*sqrt(1/2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*sqrt(1/2)*(-7*I*A - 5*I*C)*sqrt(b)*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(1/2)*B*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + 63*I*sqrt(1/2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*C*cos(d*x + c)^2 + 21
*B*cos(d*x + c) + 35*A + 25*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*co
s(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*co
s(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(1/2),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{b}\left(\left(\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)a+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^3\,dx\right)c+\left(\int \sqrt{\cos(dx+c)}\,c\right.\right.\right.}{b}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
)*cos(c + d*x)**3,x)*c + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b))/b
```

3.265 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 150

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}$$

output
```
2/5*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b
/d/cos(d*x+c)^(1/2)+2/3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2
^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/3*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d+2
/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.65

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{b\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5B+3C\right.}{15bd\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Co
s[c + d*x]],x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Ell
ipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c
 + d*x]))/(15*b*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt{b\cos(c+dx)}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{1}{2}\sqrt{b\cos(c+dx)}(b(5A+3C)+5bB\cos(c+dx))dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \sqrt{b\cos(c+dx)}(b(5A+3C)+5bB\cos(c+dx))dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(5A+3C)+5bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$\Big\downarrow$ 3227

$$\frac{\frac{b(5A+3C)\int \sqrt{b\cos(c+dx)}dx+5B\int (b\cos(c+dx))^{3/2}dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$\Big\downarrow$ 3115

$$\frac{\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$\Big\downarrow$ 3121

$$\frac{\frac{\frac{b(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}+5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{b(5A+3C)\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}+5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$$\Big\downarrow \textbf{3119}$$

$$\frac{\frac{5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

$$\Big\downarrow \textbf{3120}$$

$$\frac{\frac{\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+5B\left(\frac{2b^2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c +
d*x]],x]
```

output
```
((2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + ((2*b*(5*A + 3*C)*Sqr
t[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + 5*B*
((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/(5*b))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(133) = 266$.

Time = 0.90 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.11

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(24C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-20B-24C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(10B+6C)\right.}{}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}-\frac{2B\sqrt{b\left(-1+2\right.}}{}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/15*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*C*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2
*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}B\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{\frac{1}{2}}B\sqrt{b}\,\text{weierstrassPInve}\right.}{}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),
x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - 5*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) + 3*sqrt(1/2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
3*sqrt(1/2)*(5*I*A + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*C*cos(d*x + c) + 5*B)*sqr
t(b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(
d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(
d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x)
)^(1/2),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x)
)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b}\left(\left(\int \sqrt{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)b+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)^2\,d\right.\right.}{b}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x
),x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*c))/b
```

3.266 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 117

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}$$

output
```
2*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/cos(d*x
+c)^(1/2)+2/3*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/
2))/d/(b*cos(d*x+c))^(1/2)+2/3*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+C\sin(2(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*d*Sqrt[b*Cos[c
 + d*x]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3502, 27, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2\int \frac{b(3A+C)+3bB\cos(c+dx)}{2\sqrt{b\cos(c+dx)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b(3A+C)+3bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{b(3A+C)+3bB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3227

$$\frac{b(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+3B\int \sqrt{b\cos(c+dx)}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{b(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3B\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3121

$$\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{3B\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{3B\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3119

$$\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{6BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

$\Big\downarrow$ 3120

$$\frac{\frac{2b(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{6BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
((6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]) + (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[
b*Cos[c + d*x]]))/(3*b) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(106) = 212$.

Time = 0.59 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.41

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \text{EllipticF}\left(\text{co}\right.\right.}{3\sqrt{-b\left(2\,\text{s}\right.}}$
parts	$\frac{2A\sqrt{\cos(dx+c)}\ \text{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\sqrt{2}\right)}{d\sqrt{b\cos(dx+c)}}+\frac{2B\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\right.}}$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/
2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.30

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A + i\,C)\sqrt{b}\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\,\sin(dx + c)) + \sqrt{\frac{1}{2}}(-3i\,A - i\,C)\sqrt{b}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-2/3*(sqrt(1/2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(1/2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(1/2)*B*sqrt(b)*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
 + 3*I*sqrt(1/2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{2\,C\,\sin\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{3\,b\,d}\\&\quad+\frac{2\,A\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{b\,\cos\left(c+d\,x\right)}}\\&\quad+\frac{2\,B\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{b\,\cos\left(c+d\,x\right)}}\\&\quad+\frac{2\,C\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d\,\sqrt{b\,\cos\left(c+d\,x\right)}}\end{aligned}$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/2),x)
```

output
```
(2*C*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*cos(c + d*x)^(1/2
)*ellipticF(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*B*cos(c + d
*x)^(1/2)*ellipticE(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*C*c
os(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1/2)
)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{b}\left(\left(\int\frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+\left(\int\sqrt{\cos(dx+c)}dx\right)b+\left(\int\sqrt{\cos(dx+c)}\,\cos(dx+c)\,dx\right)c\right)}{b}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
),x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x),x)*c))/b
```

3.267 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 110

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output

```
-2*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/co
s(d*x+c)^(1/2)+2*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
/d/(b*cos(d*x+c))^(1/2)+2*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.52 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.54

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{b\cos(c+dx)}(B+C\cos(c+dx)+A\sec(c+dx))\left(\frac{\csc(c)\left(-3(A-C)\cos(c-dx-\arctan(\tan(c)))\sec(c)-(A-C)\cos(c+\right.}{\sqrt{}}\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Co
s[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*((Csc[c]*(-3*(
A - C)*Cos[c - d*x - ArcTan[Tan[c]]]*Sec[c] - (A - C)*Cos[c + d*x + ArcTan
[Tan[c]]]*Sec[c] + 2*((2*A - C)*Cos[d*x] - C*Cos[2*c + d*x])*Sqrt[Sec[c]^2
]))/Sqrt[Sec[c]^2] - 4*B*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sq
rt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + (2*(A - C)*HypergeometricPFQ[{-1/2
, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c
]]])/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(b*d*(2*A + C +
2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)]))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.282$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$b\left(\frac{2\int \frac{b^2B-b^2(A-C)\cos(c+dx)}{2\sqrt{b\cos(c+dx)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$b\left(\frac{\int \frac{b^2B-b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{\int \frac{b^2B-b^2(A-C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b\left(\frac{b^2B\int \frac{1}{\sqrt{b\cos(c+dx)}}dx-b(A-C)\int \sqrt{b\cos(c+dx)}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{b^2B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b(A-C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b\left(\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{b(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{b(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b\left(\frac{\frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\right)$$

input
```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c +
d*x]],x]
```

output
```
b*(((-2*b*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[
Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d
*Sqrt[b*Cos[c + d*x]]))/b^3 + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(103) = 206$.

Time $= 0.90$ (sec) , antiderivative size $= 259$, normalized size of antiderivative $= 2.35$

method	result
default	$\frac{2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}\right.\right.}}$
parts	$-\frac{2A\left(-2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}}$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))
^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx =$$
$$-\frac{2\left(i\sqrt{\frac{1}{2}}B\sqrt{b}\cos(dx+c)\ \mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{\frac{1}{2}}B\sqrt{b}\cos(d\right.}{}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),
x, algorithm="fricas")
```

output
```
-2*(I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(1/2)*(I*A - I*C)*sqrt(b
)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + sqrt(1/2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) - sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/2
),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/sqrt(b*cos(
c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{(C\cos(dx + c)^2 + B\cos(dx + c) + A)\sec(dx + c)}{\sqrt{b\cos(dx + c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(
d*x + c)), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(
d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))
^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))
^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{b}\left(\left(\int\frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)}dx\right)a+\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)\sec\left(dx+c\right)dx\right)c+\left(\int\sqrt{\cos\left(dx+c\right)}\right.\right.}{b}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x))/cos(c + d*x),x)*a + int(sq
rt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x),x)*c + int(sqrt(cos(c + d*x))*s
ec(c + d*x),x)*b))/b
```

3.268 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 139

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
-2*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/cos(d*
x+c)^(1/2)+2/3*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1
/2))/d/(b*cos(d*x+c))^(1/2)+2/3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*B*
sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.16 (sec) , antiderivative size = 757, normalized size of antiderivative = 5.45

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{\cos^3(c+dx)\,(C+B\sec(c+dx)+A\sec^2(c+dx))\left(\frac{4B\csc(c)\sec(c)}{d}+\frac{4A\sec(c)\sec^2(c+dx)\sin(dx)}{3d}+\frac{4\sec(c)\sec(c+dx}{\ }\right.}{\sqrt{b\cos(c+dx)}(2A+C+2B\cos(c+dx)+C\cos(2c+2dx))}$$

$$-\frac{4A\cos^{\frac{5}{2}}(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)(C+B\sec(c+dx)+A\sec^2(c+dx))\,\mathrm{se}}{3d\sqrt{b\cos(c+dx)}(2A}$$

$$-\frac{4C\cos^{\frac{5}{2}}(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)(C+B\sec(c+dx)+A\sec^2(c+dx))\,\mathrm{se}}{d\sqrt{b\cos(c+dx)}(2A}$$

$$+\frac{2B\cos^{\frac{5}{2}}(c+dx)\csc(c)\,(C+B\sec(c+dx)+A\sec^2(c+dx))\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\mathrm{arcta}\right.}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\mathrm{ta}}}\right.}{d\sqrt{b\cos(c+dx)}(2A+C+2B}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*
Cos[c + d*x]],x]
```

output
```
(Cos[c + d*x]^3*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((4*B*Csc[c]*Sec[c
])/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c + d*x]
*(A*Sin[c] + 3*B*Sin[d*x]))/(3*d)))/(Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*C
os[c + d*x] + C*Cos[2*c + 2*d*x])) - (4*A*Cos[c + d*x]^(5/2)*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c +
 d*x] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[b*Cos[c + d*x]]*(2*A + C +
 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c +
 d*x]^(5/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[b*C
os[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Co
t[c]^2]) + (2*B*Cos[c + d*x]^(5/2)*Csc[c]*(C + B*Sec[c + d*x] + A*Sec[c +
d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 ...
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.317$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3500$$

$$b^2\left(\frac{2\int \frac{3Bb^2+(A+3C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 27$$

$$b^2\left(\frac{\int \frac{3Bb^2+(A+3C)\cos(c+dx)b^2}{(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{\int \frac{3Bb^2+(A+3C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3227$$

$$b^2\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+3b^2B\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3116$$

$$b^2\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^2\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(\frac{\frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

input
```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c
+ d*x]],x]
```

output
```
b^2*((2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + ((2*b*(A + 3*C)*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + 3*
b^2*B*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos
[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(3*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(126) = 252$.

Time = 0.92 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.65

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-12\right.}{}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\cos(\right.}{3\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin}$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d
*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/
2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.45

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}(iA+3iC)\sqrt{b}\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{\frac{1}{2}}(-\right.}{}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2
),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(1/2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-I*A - 3*I*C)*sqrt(b)*co
s(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3
*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(1/2)*B*sqrt(b)*co
s(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) - (3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x
 + c))/(b*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$
$$= \int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(
1/2),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)**2/sqrt(b*c
os(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$
$$= \int \frac{(C\cos(dx + c)^2 + B\cos(dx + c) + A)\sec(dx + c)^2}{\sqrt{b\cos(dx + c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*co
s(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sec\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*co
s(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2 + B\,\cos\left(c+d\,x\right) + A}{\cos\left(c+d\,x\right)^2\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x
))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x
))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)\sec\left(dx+c\right)^2dx\right)c+\left(\int \sqrt{\cos\left(dx+\right.}\right.\right.}{b}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**2)/cos(c + d*x),x)*a + int
(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**2,x)*c + int(sqrt(cos(c + d
*x))*sec(c + d*x)**2,x)*b))/b
```

3.269 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 180

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output
```
-2/5*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b/d/cos(d*x+c)^(1/2)+2/3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,
2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2
)+2/3*b*B*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/5*(3*A+5*C)*sin(d*x+c)/d/(b*
cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2\left(-3(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+9A\sin(c+\right.}{15d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*
Cos[c + d*x]],x]
```

output
```
(2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c +
 d*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[b*Co
s[c + d*x]])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.317, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 2030$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$\downarrow$ 3500

$$b^3 \left(\frac{2 \int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$\downarrow$ 27

$$b^3 \left(\frac{\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$\downarrow$ 3042

$$b^3 \left(\frac{\int \frac{5Bb^2+(3A+5C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$\downarrow$ 3227

$$b^3 \left(\frac{b(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+5b^2B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$\downarrow$ 3042

$$b^3 \left(\frac{b(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+5b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$\downarrow$ 3116

$$b^3 \left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A\sin(c+}{5bd(b\cos(c+}\right.$$

$\downarrow$ 3042

$$b^3 \left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A}{5bd(b\,c}\right.$$

$\downarrow$ 3121

$$b^3\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right.}{5b^3}\right.$$

$$\Big\downarrow \ 3042$$

$$b^3\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c}{3bd(b\cos(c}\right.}{5b^3}\right.$$

$$\Big\downarrow \ 3119$$

$$b^3\left(\frac{5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c-}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^3}\right.$$

$$\Big\downarrow \ 3120$$

$$b^3\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+d}{3bd(b\cos(c+d}\right.}{5b^3}\right.$$

input
```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*Cos[c
+ d*x]],x]
```

output
```
b^3*((2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (5*b^2*B*((2*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) +
 (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + b*(3*A + 5*C)*((-2*Sqr
t[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) +
(2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(159) = 318$.

Time = 1.22 (sec) , antiderivative size = 801, normalized size of antiderivative = 4.45

method	result	size
parts	Expression too large to display	801
default	Expression too large to display	807

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-2/5*A*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/
2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d
*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d-
2/3*B*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(b*(-1+2*cos(1/2*d*x+1
/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(-1+2*cos(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(b*(-1+
2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d-2*C*(-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*b*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.24

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\left(dx+c\right)^3\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\right.}{}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2
),x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^3*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(1/2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(1/2)*(-3*I*A - 5*I*C)*s
qrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))) - (3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*
x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt{b\cos(c + dx)}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(
1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt{b\cos(c + dx)}}\, dx = \int \frac{(C\cos(dx + c)^2 + B\cos(dx + c) + A)\sec(dx + c)^3}{\sqrt{b\cos(dx + c)}}\, dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*co
s(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{(C\cos(dx + c)^2 + B\cos(dx + c) + A)\sec(dx + c)^3}{\sqrt{b\cos(dx + c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*co
s(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{C\cos(c + dx)^2 + B\cos(c + dx) + A}{\cos(c + dx)^3\sqrt{b\cos(c + dx)}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x
))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x
))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx$$
$$= \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^3}{\cos(dx+c)}dx\right)a + \left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^3\,dx\right)c + \left(\int \sqrt{\cos(dx+\right.\right.}{b}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**3)/cos(c + d*x),x)*a + int
(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**3,x)*c + int(sqrt(cos(c + d
*x))*sec(c + d*x)**3,x)*b))/b
```

3.270 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 209

$$\begin{aligned}&\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx\\ &=-\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}\\ &\quad+\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}\\ &\quad+\frac{2b^2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{6B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
-6/5*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/cos(
d*x+c)^(1/2)+2/21*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2))/d/(b*cos(d*x+c))^(1/2)+2/7*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(7/
2)+2/5*b^2*B*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/21*b*(5*A+7*C)*sin(d*x+c)
/d/(b*cos(d*x+c))^(3/2)+6/5*B*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.64

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{2\left(-63B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+63B\sin(c}{105d\sqrt{}}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*
Cos[c + d*x]],x]
```

output
```
(2*(-63*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*B*Sin[c + d*x] + 25*A*Tan[c
 + d*x] + 35*C*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*A*Sec[c
+ d*x]^2*Tan[c + d*x]))/(105*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.366$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4 \int \frac{C \sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2 + B \sin\left(\frac{1}{2}(2c+\pi)+dx\right) + A}{\left(b \sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}} dx$$

$$\Big\downarrow \text{ 3500}$$

$$b^4 \left(\frac{2 \int \frac{7Bb^2+(5A+7C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$b^4 \left(\frac{\int \frac{7Bb^2+(5A+7C)\cos(c+dx)b^2}{(b\cos(c+dx))^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^4 \left(\frac{\int \frac{7Bb^2+(5A+7C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3227}$$

$$b^4 \left(\frac{b(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx + 7b^2B\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^4 \left(\frac{b(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx + 7b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3116}$$

$$b^4 \left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right) + 7b^2B\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2} + \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{7b^3} + \frac{2A\sin}{7bd(b\cos}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$b^4\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{7b^3}+\right.$$

$\downarrow$ 3116

$$b^4\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx)}\right.}{7b^3}\right.$$

$\downarrow$ 3042

$$b^4\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c}{5bd(b\cos(c}\right.}{7b^3}\right.$$

$\downarrow$ 3121

$$b^4\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}\right.}{7b^3}\right.$$

$\downarrow$ 3042

$$b^4\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\right.}}{b^2\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

$$\Big\downarrow \; 3119$$

$$b^4\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

$$\Big\downarrow \; 3120$$

$$b^4\left(\frac{b(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

input

```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c
+ d*x]],x]
```

output

```
b^4*((2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (b*(5*A + 7*C)*((
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*
x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + 7*b^2*B*((2*Sin[
c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*
d*Sqrt[b*Cos[c + d*x]])))/(5*b^2)))/(7*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(184) = 368$.

Time = 1.81 (sec) , antiderivative size = 726, normalized size of antiderivative = 3.47

method	result	size
default	Expression too large to display	726
parts	Expression too large to display	1001

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-1/56*co
s(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(
1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2)))+2/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2
*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x
+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*
c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+2*C*(-1/6*cos(1
/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(-1+2*co
s(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.12

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx =$$

$$-\frac{2\left(5\sqrt{\frac{1}{2}}(5i\,A+7i\,C)\sqrt{b}\cos\left(dx+c\right)^4\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\sqrt{}\right.}{}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2
),x, algorithm="fricas")
```

output
```
-2/105*(5*sqrt(1/2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(1/2)*(-5*I*A - 7*I*C)*
sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) + 63*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(1/2)*
B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - (63*B*cos(d*x + c)^3 + 5*(5*A + 7*C)*co
s(d*x + c)^2 + 21*B*cos(d*x + c) + 15*A)*sqrt(b*cos(d*x + c))*sin(d*x + c)
)/(b*d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^4(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(b*cos(d*x+c))**(
1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^4(c + dx)}{\sqrt{b\cos(c + dx)}}\,dx = \int \frac{(C\cos(dx + c)^2 + B\cos(dx + c) + A)\sec(dx + c)^4}{\sqrt{b\cos(dx + c)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^4/sqrt(b*co
s(d*x + c)), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^4/sqrt(b*co
s(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^4\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^4*(b*cos(c + d*x
))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^4*(b*cos(c + d*x
))^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^4}{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos(dx+c)}\,\cos(dx+c)\sec(dx+c)^4\,dx\right)c+\left(\int \sqrt{\cos(dx+\right.\right.}{b}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**4)/cos(c + d*x),x)*a + int
(sqrt(cos(c + d*x))*cos(c + d*x)*sec(c + d*x)**4,x)*c + int(sqrt(cos(c + d
*x))*sec(c + d*x)**4,x)*b))/b
```

3.271 $\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 217

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}}+\frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d}+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d}$$

output
```
2/15*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b^2/d/cos(d*x+c)^(1/2)+10/21*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/
2*c,2^(1/2))/b/d/(b*cos(d*x+c))^(1/2)+10/21*B*(b*cos(d*x+c))^(1/2)*sin(d*x
+c)/b^2/d+2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/7*B*(b*co
s(d*x+c))^(5/2)*sin(d*x+c)/b^4/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^5
/d
```

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.60

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{168(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(3/2),x]
```

output
```
(168*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 600*B*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A + 43*C)*Cos[c + d*x] +
 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]
)/(1260*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.366$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^4}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{1}{2}(b\cos(c+dx))^{5/2}(b(9A+7C)+9bB\cos(c+dx))dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^4}$$

$\downarrow$ 27

$$\frac{\frac{\int(b\cos(c+dx))^{5/2}(b(9A+7C)+9bB\cos(c+dx))dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^4}$$

$\downarrow$ 3042

$$\frac{\frac{\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(b(9A+7C)+9bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^4}$$

$\downarrow$ 3227

$$\frac{\frac{b(9A+7C)\int(b\cos(c+dx))^{5/2}dx+9B\int(b\cos(c+dx))^{7/2}dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^4}$$

$\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+9B\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^4}$$

$\downarrow$ 3115

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\int(b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{9b}+\frac{2C\sin(c+d\ldots}{}}{b^4}$$

$\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{9b}+2C\ldots}{b^4}$$

$\downarrow$ 3115

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)\ldots}{}\right.}{9b}}{b^4}$$

$\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(\ldots}{}\right.}{9b}}{b^4}$$

$\Big\downarrow$ 3121

$$\frac{\frac{b(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{9b}}{b^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos}}{3d}\right.\right.}{9b}}{b^4}$$

$\Big\downarrow$ 3119

$$\frac{\frac{9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+b(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+}}{5d\sqrt{\cos(c+dx)}}\right.}{9b}}{b^4}$$

$\Big\downarrow$ 3120

$$\frac{\frac{b(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+d}}{3d}\right.\right.}{9b}}{b^4}$$

input
```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(3/2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + (b*(9*A + 7*C)*((6*b^
2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)) + 9*B*((2*b*(b*Cos[c +
 d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x
]]*Sin[c + d*x])/(3*d)))/7))/(9*b))/b^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.77

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(720B+2240C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-\right.}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\right.\right.}{5b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\right.}}$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-2/315*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-1120
*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2
*c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1
/2*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-126*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*
d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.89

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(75i\sqrt{\frac{1}{2}}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-75i\sqrt{\frac{1}{2}}B\sqrt{b}\text{weierstrassPInve}\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2
),x, algorithm="fricas")
```

output
```
-2/315*(75*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) - 75*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 21*sqrt(1/2)*(-9*I*A - 7*I*C)*sqrt(b)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 21*sqrt(1/2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*C*cos(d*x + c)^3 +
45*B*cos(d*x + c)^2 + 7*(9*A + 7*C)*cos(d*x + c) + 75*B)*sqrt(b*cos(d*x +
c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*
x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*
x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4\,\left(C\,\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(3/2),x)
```

output
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4 dx\right)c+\left(\right.}{}$$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c + d
*x))*cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a))/
b**2
```

3.272 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 188

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}$$

output
```
6/5*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos
(d*x+c)^(1/2)+2/21*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))/b/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*(b*cos(d*x+c))^(1/2)*si
n(d*x+c)/b^2/d+2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/7*C*(b*cos(d*
x+c))^(5/2)*sin(d*x+c)/b^4/d
```

Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.57

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*Elli
pticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x
] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.317$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{3/2}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^3}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{1}{2}(b\cos(c+dx))^{3/2}(b(7A+5C)+7bB\cos(c+dx))dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^3}$$

$\downarrow$ 27

$$\frac{\frac{\int (b\cos(c+dx))^{3/2}(b(7A+5C)+7bB\cos(c+dx))dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^3}$$

$\downarrow$ 3042

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b(7A+5C)+7bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^3}$$

$\downarrow$ 3227

$$\frac{\frac{b(7A+5C)\int (b\cos(c+dx))^{3/2}dx+7B\int (b\cos(c+dx))^{5/2}dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^3}$$

$\downarrow$ 3042

$$\frac{\frac{b(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+7B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^3}$$

$\downarrow$ 3115

$$\frac{\frac{b(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+\frac{2C\sin(c+dx)(b\cos}{7bd}}{b^3}$$

$\downarrow$ 3042

$$\frac{\frac{b(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+2C\sin(c+}{b^3}$$

$\downarrow$ 3121

$$\frac{\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+2}{b^3}$$

$\downarrow$ 3042

$$\frac{\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right.}{7b}}{b^3}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+}{b^3}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{b(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}-}{b^3}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(3/2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + (b*(7*A + 5*C)*((2*b^
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]])
 + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + 7*B*((6*b^2*Sqrt[b*Cos
[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(b*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/(7*b))/b^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(167) = 334$.

Time = 1.78 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.88

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(240C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-168B-360C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(140A\right.}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\right.}{3b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}d}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-2/105*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(240*C
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-360*C)*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x
+1/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(5\sqrt{\frac{1}{2}}(7iA+5iC)\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{\frac{1}{2}}(-7iA-5iC)\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2
),x, algorithm="fricas")
```

output
```
-2/105*(5*sqrt(1/2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*sqrt(1/2)*(-7*I*A - 5*I*C)*sqrt(b)*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(1/2)*B*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + 63*I*sqrt(1/2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*C*cos(d*x + c)^2 + 21
*B*cos(d*x + c) + 35*A + 25*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*
x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*
x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(3/2),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int\sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)a+\left(\right.\right.}{}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
)*cos(c + d*x)**3,x)*c + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b))/b**
2
```

3.273 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 153

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}$$

output

```
2/5*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b
^2/d/cos(d*x+c)^(1/2)+2/3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2))/b/d/(b*cos(d*x+c))^(1/2)+2/3*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b
^2/d+2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d
```

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\cos^{\frac{3}{2}}(c+dx)\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(3/2),x]
```

output
```
(2*Cos[c + d*x]^(3/2)*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Ellip
ticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c +
 d*x]))/(15*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.317, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt{b\cos(c+dx)}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{1}{2}\sqrt{b\cos(c+dx)}(b(5A+3C)+5bB\cos(c+dx))dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{\frac{\int \sqrt{b\cos(c+dx)}(b(5A+3C)+5bB\cos(c+dx))dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \ \textbf{3042}$$

$$\frac{\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(5A+3C)+5bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \ \textbf{3227}$$

$$\frac{\frac{b(5A+3C)\int \sqrt{b\cos(c+dx)}dx+5B\int (b\cos(c+dx))^{3/2}dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \ \textbf{3042}$$

$$\frac{\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \ \textbf{3115}$$

$$\frac{\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \ \textbf{3042}$$

$$\frac{\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \ \textbf{3121}$$

$$\frac{\frac{\frac{b(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}+5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \ \textbf{3042}$$

$$\frac{\frac{\frac{b(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}+5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+5B\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(3/2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + ((2*b*(5*A + 3*C)*Sqr
t[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + 5*B*
((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/(5*b))/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(136) = 272$.

Time = 1.01 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.08

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(24C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-20B-24C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(10B+6C)\right.}{}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}-\frac{2B\sqrt{b\left(-1+2\right.}}{}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,me
thod=_RETURNVERBOSE)
```

output
```
2/15*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(24*C*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(5i\,\sqrt{\frac{1}{2}}B\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5i\,\sqrt{\frac{1}{2}}B\sqrt{b}\mathrm{weierstrassPInverse}\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2
),x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - 5*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) + 3*sqrt(1/2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
3*sqrt(1/2)*(5*I*A + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*C*cos(d*x + c) + 5*B)*sqr
t(b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*
x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\cos}{(b\cos(dx+c))^{\frac{3}{2}}}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*
x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^2\left(C\cos(c+d\,x)^2+B\,\cos(c+\right.}{(b\,\cos(c+d\,x))^{3/2}}$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(3/2),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int\sqrt{\cos(dx+c)}dx\right)a+\left(\int\sqrt{\cos(dx+\right.\right.}{}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x
),x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*c))/b**2
```

3.274 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 120

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}$$

output
```
2*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d
*x+c)^(1/2)+2/3*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(
1/2))/b/d/(b*cos(d*x+c))^(1/2)+2/3*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^2/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)}{3bd}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(3/2),x]
```

output
```
(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*b*d*Sqrt[b*Cos
[c + d*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.282$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{C\cos^2(c+dx)+B\cos(c+dx)+A}{\sqrt{b\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{b(3A+C)+3bB\cos(c+dx)}{2\sqrt{b\cos(c+dx)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{b(3A+C)+3bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{b(3A+C)+3bB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{b(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+3B\int \sqrt{b\cos(c+dx)}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3B\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{3B\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{3B\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{2b(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]
)^(3/2),x]
```

output
```
(((6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt
[b*Cos[c + d*x]]))/(3*b) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
)/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(109) = 218$.

Time = 0.63 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.38

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\mathrm{co}\right.\right.}{3b\sqrt{-b\left(2\mathrm{s}\right.}}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,d}+\frac{2B\sqrt{b\left(-1\right.}}{}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(4*C*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A+i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{\frac{1}{2}}(-3i\,A-i\,C)\sqrt{b}\mathrm{we}\right.}{}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),
x, algorithm="fricas")
```

output
```
-2/3*(sqrt(1/2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(1/2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(1/2)*B*sqrt(b)*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
 + 3*I*sqrt(1/2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x
+ c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x
+ c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)\ \left(C\cos\left(c+d\,x\right)^2+B\ \cos\left(c+\right.\right.}{\left(b\ \cos\left(c+d\,x\right)\right)^{3/2}}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x)
)^(3/2),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x)
)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}dx\right.\right.}{b^2}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
),x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x),x)*c))/b**2
```

3.275 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 116

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
-2*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/
cos(d*x+c)^(1/2)+2*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
))/b/d/(b*cos(d*x+c))^(1/2)+2*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\Big(-\Big((A-C)\sqrt{\cos(c+dx)}E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)\Big)+B\sqrt{\cos(c+}}{bd\sqrt{b\cos(c+dx}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x
]
```

output
```
(2*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(b*d*Sqrt[b*Cos[c +
d*x]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3500, 27, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{2\int \frac{b^2B-b^2(A-C)\cos(c+dx)}{2\sqrt{b\cos(c+dx)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{b^2B-b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{b^2B-b^2(A-C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

$\big\downarrow$ 3227

$$\frac{b^2B\int \frac{1}{\sqrt{b\cos(c+dx)}}dx-b(A-C)\int \sqrt{b\cos(c+dx)}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

$\big\downarrow$ 3042

$$\frac{b^2B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b(A-C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

$\big\downarrow$ 3121

$$\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{b(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

$\big\downarrow$ 3042

$$\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{b(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

$\big\downarrow$ 3119

$$\frac{\frac{b^2B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

$\big\downarrow$ 3120

$$\frac{\frac{2b^2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
((-2*b*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos
[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sq
rt[b*Cos[c + d*x]]))/b^3 + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(109) = 218$.

Time = 0.26 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.26

method	result
default	$\frac{2\sqrt{-2b\sin(\frac{dx}{2}+\frac{c}{2})^4+\sin(\frac{dx}{2}+\frac{c}{2})^2 b}\,(2A\cos(\frac{dx}{2}+\frac{c}{2})\sin(\frac{dx}{2}+\frac{c}{2})^2-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin(\frac{dx}{2}+\frac{c}{2})^2-1}\,\mathrm{EllipticE}(\cos(\frac{dx}{2}+\ldots}{b\sqrt{-b(2\sin(\frac{dx}{2}+\frac{c}{2})^4-\sin(\frac{dx}{2}\ldots}$
parts	$-\frac{2A(-2\sqrt{-2b\sin(\frac{dx}{2}+\frac{c}{2})^4+\sin(\frac{dx}{2}+\frac{c}{2})^2 b}\cos(\frac{dx}{2}+\frac{c}{2})\sin(\frac{dx}{2}+\frac{c}{2})^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin(\frac{dx}{2}+\frac{c}{2})^2-1}\,\sqrt{-2b\sin(\frac{dx}{2}+\frac{c}{2}\ldots}{b\sqrt{-b(2\sin(\frac{dx}{2}+\frac{c}{2})^4-\sin(\frac{dx}{2}+\frac{c}{2})^2)}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{b(-1+2\cos(\frac{dx}{2}+\frac{c}{2})^2)}\ldots}$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/b*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin
(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2
))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\left(dx+c\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\left(dx+\right.\right.}{}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
-2*(I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(1/2)*(I*A - I*C)*sqrt(b
)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + sqrt(1/2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) - sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^2*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2),
x)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos(c + d\,x)^2 + B\,\cos(c + d\,x) + A}{(b\,\cos(c + d\,x))^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a+\left(\int \sqrt{\cos\left(dx\right.}\right.\right.}{b^2}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*b + int(sqrt(cos(c + d*x)
)/cos(c + d*x)**2,x)*a + int(sqrt(cos(c + d*x)),x)*c))/b**2
```

3.276 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 144

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
-2*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(
d*x+c)^(1/2)+2/3*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^
(1/2))/b/d/(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*
B*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.28 (sec) , antiderivative size = 761, normalized size of antiderivative = 5.28

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^3(c+dx)(C+B\sec(c+dx)+A\sec^2(c+dx))\left(\frac{4B\csc(c)\sec(c)}{d}+4\ldots\right.}{\sqrt{b\cos(c+dx)}(2A+C+2B\,\mathrm{c}\ldots}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c
+ d*x])^(3/2),x]
```

output
```
((Cos[c + d*x]^3*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((4*B*Csc[c]*Sec[
c])/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c + d*x
]*(A*Sin[c] + 3*B*Sin[d*x]))/(3*d)))/(Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*
Cos[c + d*x] + C*Cos[2*c + 2*d*x])) - (4*A*Cos[c + d*x]^(5/2)*Csc[c]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c
+ d*x] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[b*Cos[c + d*x]]*(2*A + C
+ 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c
+ d*x]^(5/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*S
in[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[b*
Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + C
ot[c]^2]) + (2*B*Cos[c + d*x]^(5/2)*Csc[c]*(C + B*Sec[c + d*x] + A*Sec[c +
 d*x]^2)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]
]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1...
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$b\left(\frac{2\int \frac{3Bb^2+(A+3C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$b\left(\frac{\int \frac{3Bb^2+(A+3C)\cos(c+dx)b^2}{(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{\int \frac{3Bb^2+(A+3C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+3b^2B\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3116$$

$$b\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b\left(\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3121$$

$$b\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3119$$

$$b\left(\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3120$$

$$b\left(\frac{\frac{2b(A+3C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))}\right.$$

input
```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x]
)^(3/2),x]
```

output
```
b*((2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + ((2*b*(A + 3*C)*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + 3*b^
2*B*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c
 + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(3*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(131) = 262$.

Time = 0.44 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.53

method	result
default	$2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-12 \ldots$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(\ldots}{}}\ldots\right.}{3b\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\ldots}$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2
*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*
b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+
1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.40

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}(iA + 3iC)\sqrt{b}\cos(dx+c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + \sqrt{\frac{1}{2}}(-iA\right.}{}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),
x, algorithm="fricas")
```

output
```
-2/3*(sqrt(1/2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-I*A - 3*I*C)*sqrt(b)*co
s(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3
*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(1/2)*B*sqrt(b)*co
s(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) - (3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x
 + c))/(b^2*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c}{(b\cos(c + dx))^{\frac{3}{2}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(3/2
),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c +
d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{(C\cos(dx + c)^2 + B\cos(dx + c) + A)\sec(c}{(b\cos(dx + c))^{\frac{3}{2}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x
+ c))^(3/2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(c\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x
+ c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))
^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))
^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\left(\int \frac{\sqrt{\cos(dx+c)}\sec(dx+c)}{\cos(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx\right.}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x)
```

output

```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x))/cos(c + d*x),x)*b + int((s
qrt(cos(c + d*x))*sec(c + d*x))/cos(c + d*x)**2,x)*a + int(sqrt(cos(c + d*
x))*sec(c + d*x),x)*c))/b**2
```

3.277 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 183

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

output
```
-2/5*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b^2/d/cos(d*x+c)^(1/2)+2/3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))/b/d/(b*cos(d*x+c))^(1/2)+2/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5
/2)+2/3*B*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/5*(3*A+5*C)*sin(d*x+c)/b/d/(
b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.65

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(-3(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[
c + d*x])^(3/2),x]
```

output
```
(2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c +
 d*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b*d*Sqrt[b*
Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.317$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^2\left(\frac{2\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \textbf{27}$$

$$b^2\left(\frac{\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$b^2\left(\frac{\int \frac{5Bb^2+(3A+5C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \textbf{3227}$$

$$b^2\left(\frac{b(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+5b^2B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$b^2\left(\frac{b(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+5b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \textbf{3116}$$

$$b^2\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A\sin(c+}{5bd(b\cos(c+}\right.$$

$$\Big\downarrow \textbf{3042}$$

$$b^2\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A}{5bd(b}\right.$$

$$\Big\downarrow \textbf{3121}$$

$$b^2\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right.}{5b^3}\right.$$

$\downarrow$ 3042

$$b^2\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c}{3bd(b\cos(c}\right.}{5b^3}\right.$$

$\downarrow$ 3119

$$b^2\left(\frac{5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c-}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^3}\right.$$

$\downarrow$ 3120

$$b^2\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+d}{3bd(b\cos(c+d}\right.}{5b^3}\right.$$

input
```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*
x])^(3/2),x]
```

output
```
b^2*((2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (5*b^2*B*((2*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) +
 (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + b*(3*A + 5*C)*((-2*Sqr
t[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) +
(2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(162) = 324$.

Time $= 0.52$ (sec) , antiderivative size $= 807$, normalized size of antiderivative $= 4.41$

method	result	size
default	Expression too large to display	807
parts	Expression too large to display	807

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-2/15*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin
(1/2*d*x+1/2*c)^4+120*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-60*C*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^4+36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^2-120*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+60*C*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+30*C*sin(1/2*d*x+1/2*c)^2*cos(1...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.22

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{3/2}}\, dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\left(dx+c\right)^3 \text{weierstrassPInverse}(-4, 0, \cos\left(dx+c\right) + i\sin\left(dx+c\right)) - 5i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\left(d\right.\right.}{}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2
),x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^3*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(1/2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(1/2)*(-3*I*A - 5*I*C)*s
qrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))) - (3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*
x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^2}{(b\cos(c+dx))^{\frac{3}{2}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(
3/2),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c
 + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{(C\cos(dx+c)^2 + B\cos(dx+c) + A)\sec(}{(b\cos(dx+c))^{\frac{3}{2}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*
x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*
x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^2\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x
))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x
))^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+}}{\cos}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x)
```

output

```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**2)/cos(c + d*x),x)*b + int
((sqrt(cos(c + d*x))*sec(c + d*x)**2)/cos(c + d*x)**2,x)*a + int(sqrt(cos(
c + d*x))*sec(c + d*x)**2,x)*c))/b**2
```

3.278 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 212

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2bB\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{6B\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

output
```
-6/5*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/co
s(d*x+c)^(1/2)+2/21*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2
*c,2^(1/2))/b/d/(b*cos(d*x+c))^(1/2)+2/7*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))
^(7/2)+2/5*b*B*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/21*(5*A+7*C)*sin(d*x+c)
/d/(b*cos(d*x+c))^(3/2)+6/5*B*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.64

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(-63B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5A\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[
c + d*x])^(3/2),x]
```

output
```
(2*(-63*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*B*Sin[c + d*x] + 25*A*Tan[c
 + d*x] + 35*C*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*A*Sec[c
+ d*x]^2*Tan[c + d*x]))/(105*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.366$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\downarrow \text{3500}$$

$$b^3 \left(\frac{2 \int \frac{7Bb^2+(5A+7C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} \right)$$

$$\downarrow \text{27}$$

$$b^3 \left(\frac{\int \frac{7Bb^2+(5A+7C)\cos(c+dx)b^2}{(b\cos(c+dx))^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} \right)$$

$$\downarrow \text{3042}$$

$$b^3 \left(\frac{\int \frac{7Bb^2+(5A+7C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} \right)$$

$$\downarrow \text{3227}$$

$$b^3 \left(\frac{b(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx + 7b^2B\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} \right)$$

$$\downarrow \text{3042}$$

$$b^3 \left(\frac{b(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx + 7b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} \right)$$

$$\downarrow \text{3116}$$

$$b^3 \left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} \right) + 7b^2B\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2} + \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} \right)}{7b^3} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} \right)$$

$$\downarrow \text{3042}$$

$$b^3 \left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} \right) + 7b^2B\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2} + \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} \right)}{7b^3} + \right.$$

$$\downarrow \text{3116}$$

$$b^3\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx)}\right.}{7b^3}\right.$$

$\downarrow$ 3042

$$b^3\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c-}{5bd(b\cos(c-}\right.}{7b^3}\right.$$

$\downarrow$ 3121

$$b^3\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}\right.}{7b^3}\right.$$

$\downarrow$ 3042

$$b^3\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\right.}}{b^2\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

$\downarrow$ 3119

$$b^3\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

$\Big\downarrow$ 3120

$$b^3\left(\frac{b(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

input

```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*
x])^(3/2),x]
```

output

```
b^3*((2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (b*(5*A + 7*C)*((
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*
x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + 7*b^2*B*((2*Sin[
c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*
d*Sqrt[b*Cos[c + d*x]])))/(5*b^2)))/(7*b^3))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030

```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(187) = 374$.

Time = 0.58 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	729
parts	Expression too large to display	1007

input

```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x,me
thod=_RETURNVERBOSE)
```

output

```
-(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(2*A*(-1/56*
cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/2
1*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))+2/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1
/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+2*C*(-1/6*cos
(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(-1+2*
cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.10

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx =$$

$$-\frac{2\left(5\sqrt{\frac{1}{2}}(5i\,A + 7i\,C)\sqrt{b}\cos(dx + c)^4\,\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\,\sin(dx + c)) + 5\sqrt{\frac{1}{2}}(\cdots\right.}{\cdots}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2
),x, algorithm="fricas")
```

output
```
-2/105*(5*sqrt(1/2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(1/2)*(-5*I*A - 7*I*C)*
sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) + 63*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(1/2)*
B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - (63*B*cos(d*x + c)^3 + 5*(5*A + 7*C)*co
s(d*x + c)^2 + 21*B*cos(d*x + c) + 15*A)*sqrt(b*cos(d*x + c))*sin(d*x + c)
)/(b^2*d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(
3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A\right)\sec (}{\left(b\cos\left(dx + c\right)\right)^{\frac{3}{2}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*
x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A\right)\sec (}{\left(b\cos\left(dx + c\right)\right)^{\frac{3}{2}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*
x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^3(c + dx)}{(b\cos(c + dx))^{3/2}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + B\,\cos\left(c + d\,x\right) + A}{\cos\left(c + d\,x\right)^3\,\left(b\,\cos\left(c + d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x
))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x
))^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^3}{\cos(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+}}{\cos(}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**3)/cos(c + d*x),x)*b + int
((sqrt(cos(c + d*x))*sec(c + d*x)**3)/cos(c + d*x)**2,x)*a + int(sqrt(cos(
c + d*x))*sec(c + d*x)**3,x)*c))/b**2
```

3.279 $\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 217

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}}+\frac{10B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d}+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}+\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}$$

output
```
2/15*(9*A+7*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b^3/d/cos(d*x+c)^(1/2)+10/21*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/
2*c,2^(1/2))/b^2/d/(b*cos(d*x+c))^(1/2)+10/21*B*(b*cos(d*x+c))^(1/2)*sin(d
*x+c)/b^3/d+2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/7*B*(b*
cos(d*x+c))^(5/2)*sin(d*x+c)/b^5/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b
^6/d
```

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.60

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{168(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+}{}$$

input
```
Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(5/2),x]
```

output
```
(168*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 600*B*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A + 43*C)*Cos[c + d*x] +
 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]
)/(1260*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.366$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^5}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{1}{2}(b\cos(c+dx))^{5/2}(b(9A+7C)+9bB\cos(c+dx))dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^5}$$

$\big\downarrow$ 27

$$\frac{\frac{\int (b\cos(c+dx))^{5/2}(b(9A+7C)+9bB\cos(c+dx))dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^5}$$

$\big\downarrow$ 3042

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(b(9A+7C)+9bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^5}$$

$\big\downarrow$ 3227

$$\frac{\frac{b(9A+7C)\int (b\cos(c+dx))^{5/2}dx+9B\int (b\cos(c+dx))^{7/2}dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^5}$$

$\big\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+9B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{9b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}}{b^5}$$

$\big\downarrow$ 3115

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{9b}+\frac{2C\sin(c+d}{}}{b^5}$$

$\big\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)}{9b}+\frac{2C}{}}{b^5}$$

$\big\downarrow$ 3115

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)}{}}{9b}}{b^5}$$

$\big\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(}{}}{9b}}{b^5}$$

$\downarrow$ 3121

$$\frac{\frac{b(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)\right.}{9b}}{b^5}$$

$\downarrow$ 3042

$$\frac{\frac{b(9A+7C)\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos}}{3d}\right.\right.}{9b}}{b^5}$$

$\downarrow$ 3119

$$\frac{\frac{9B\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+b(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+}}{5d\sqrt{\cos(c+dx)}}\right.}{9b}}{b^5}$$

$\downarrow$ 3120

$$\frac{\frac{b(9A+7C)\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+9B\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+d}}{3d}\right.\right.}{9b}}{b^5}$$

input
```
Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(5/2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d) + (b*(9*A + 7*C)*((6*b^
2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)) + 9*B*((2*b*(b*Cos[c +
 d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*x
]]*Sin[c + d*x])/(3*d)))/7))/(9*b))/b^5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.77

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(720B+2240C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-\right.}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\right.\right.}{5b^{2}\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\,c\right.}}$

input
```
int(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-2/315*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-11
20*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos
(1/2*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(-126*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/
2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.89

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(75i\sqrt{\frac{1}{2}}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-75i\sqrt{\frac{1}{2}}B\sqrt{b}\text{weierstrassPInve}\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2
),x, algorithm="fricas")
```

output
```
-2/315*(75*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) - 75*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 21*sqrt(1/2)*(-9*I*A - 7*I*C)*sqrt(b)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 21*sqrt(1/2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*C*cos(d*x + c)^3 +
45*B*cos(d*x + c)^2 + 7*(9*A + 7*C)*cos(d*x + c) + 75*B)*sqrt(b*cos(d*x +
c))*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*
x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*
x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^5\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}$$

input
```
int((cos(c + d*x)^5*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(5/2),x)
```

output
```
int((cos(c + d*x)^5*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)^4dx\right)c+\left(\right.\right.}{}$$

input
```
int(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x)**4,x)*c + int(sqrt(cos(c + d
*x))*cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*a))/
b**3
```

3.280 $\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 188

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}$$

output
```
6/5*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d/cos
(d*x+c)^(1/2)+2/21*(7*A+5*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))/b^2/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*(b*cos(d*x+c))^(1/2)*
sin(d*x+c)/b^3/d+2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/7*C*(b*cos(
d*x+c))^(5/2)*sin(d*x+c)/b^5/d
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.59

$$\int \frac{\cos^4(c+dx) \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}} \, dx = \frac{\sqrt{\cos(c+dx)}\Big(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A$$

input
```
Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*Elli
pticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x
] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.317$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx) \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}} \, dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{3/2} \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2} \left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^4}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{1}{2}(b\cos(c+dx))^{3/2}(b(7A+5C)+7bB\cos(c+dx))dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int (b\cos(c+dx))^{3/2}(b(7A+5C)+7bB\cos(c+dx))dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b(7A+5C)+7bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^4}$$

$$\Big\downarrow \; 3227$$

$$\frac{\frac{b(7A+5C)\int (b\cos(c+dx))^{3/2}dx+7B\int (b\cos(c+dx))^{5/2}dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{b(7A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+7B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{7b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}}{b^4}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{b(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+\frac{2C\sin(c+dx)(b\,\mathrm{co}}{7bd}}{b^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{b(7A+5C)\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+2C\sin(c+}{b^4}$$

$$\Big\downarrow \; 3121$$

$$\frac{\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+2}{b^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right.}{7b}}{b^4}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{b(7A+5C)\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}+}{b^4}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{b(7A+5C)\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+7B\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)}{7b}-}{b^4}$$

input
```
Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(5/2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + (b*(7*A + 5*C)*((2*b^
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]])
 + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)) + 7*B*((6*b^2*Sqrt[b*Cos
[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(b*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/(7*b))/b^4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(167) = 334$.

Time = 1.58 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.88

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(240C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-168B-360C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(140A\right.}{}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin}\right.}{3b^{2}\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-2/105*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(240
*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-360*C)*sin(1/2*d*x+1/2*
c)^6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d
*x+1/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.96

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(5\sqrt{\frac{1}{2}}(7i\,A+5i\,C)\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{\frac{1}{2}}(-7i\,A-5i\,C)\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2
),x, algorithm="fricas")
```

output
```
-2/105*(5*sqrt(1/2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + 5*sqrt(1/2)*(-7*I*A - 5*I*C)*sqrt(b)*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(1/2)*B*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + 63*I*sqrt(1/2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*C*cos(d*x + c)^2 + 21
*B*cos(d*x + c) + 35*A + 25*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*
x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*
x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}$$

input
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(5/2),x)
```

output
```
int((cos(c + d*x)^4*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \sqrt{\cos\left(dx+c\right)}\,\cos\left(dx+c\right)dx\right)a+\left(\right.\right.}{}$$

input
```
int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))*cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
)*cos(c + d*x)**3,x)*c + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*b))/b**
3
```

3.281 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 153

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}$$

output

```
2/5*(5*A+3*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b
^3/d/cos(d*x+c)^(1/2)+2/3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c
,2^(1/2))/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)
/b^3/d+2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d
```

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\Big(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)-$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Ellip
ticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c +
 d*x]))/(15*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.317$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3115, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt{b\cos(c+dx)}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^3}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{1}{2}\sqrt{b\cos(c+dx)}(b(5A+3C)+5bB\cos(c+dx))dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \sqrt{b\cos(c+dx)}(b(5A+3C)+5bB\cos(c+dx))dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(5A+3C)+5bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{b(5A+3C)\int \sqrt{b\cos(c+dx)}dx+5B\int (b\cos(c+dx))^{3/2}dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(5A+3C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+5B\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{\frac{b(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}+5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{b(5A+3C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}+5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{5B\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+5B\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)}{5b}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}}{b^3}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(5/2),x]
```

output
```
((2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + ((2*b*(5*A + 3*C)*Sqr
t[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + 5*B*
((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/(5*b))/b^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(136) = 272$.

Time = 1.13 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.08

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(24C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-20B-24C)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(10B+6C)\right.}{}$
parts	$\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^2\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\,d}-\frac{2B\sqrt{b\left(-1+2\right.}}{}$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,me
thod=_RETURNVERBOSE)
```

output
```
2/15*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(24*C*
sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx =$$
$$-\frac{2\left(5i\sqrt{\frac{1}{2}}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{\frac{1}{2}}B\sqrt{b}\text{weierstrassPInverse}\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2
),x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - 5*I*sqrt(1/2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) + 3*sqrt(1/2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
3*sqrt(1/2)*(5*I*A + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*C*cos(d*x + c) + 5*B)*sqr
t(b*cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*
x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*
x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(5/2),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int\sqrt{\cos\left(dx+c\right)}dx\right)a+\left(\int\sqrt{\cos\left(dx+\right.\right.}\right.}{}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x)),x)*a + int(sqrt(cos(c + d*x))*cos(c + d*x
),x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x)**2,x)*c))/b**3
```

3.282 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 120

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d}$$

output
```
2*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d/cos(d
*x+c)^(1/2)+2/3*(3*A+C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(
1/2))/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/b^3
/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C}{3b^2c}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(5/2),x]
```

output
```
(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*b^2*d*Sqrt[b*C
os[c + d*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.268$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \frac{C\cos^2(c+dx)+B\cos(c+dx)+A}{\sqrt{b\cos(c+dx)}}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{\frac{2\int \frac{b(3A+C)+3bB\cos(c+dx)}{2\sqrt{b\cos(c+dx)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{b(3A+C)+3bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{b(3A+C)+3bB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{\frac{b(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+3B\int \sqrt{b\cos(c+dx)}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(3A+C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3B\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

$\Big\downarrow$ 3121

$$\frac{\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{3B\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{3B\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

$\Big\downarrow$ 3119

$$\frac{\frac{\frac{b(3A+C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

$\Big\downarrow$ 3120

$$\frac{\frac{\frac{2b(3A+C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{3b}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(5/2),x]
```

output
```
(((6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt
[b*Cos[c + d*x]]))/(3*b) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
)/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(109) = 218$.

Time = 0.64 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.38

method	result
default	$-\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(4C\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\ \text{EllipticF}\left(\text{co}\right.\right.}{3b^{2}\sqrt{-b\left(2\right.}}$
parts	$-\frac{2A\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^{2}\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\,d}+\frac{2B\sqrt{b\left(-1\right.}}{}$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(4*C*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))
^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}(3i\,A+i\,C)\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{\frac{1}{2}}(-3i\,A-i\,C)\sqrt{b}\text{we}\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2
),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(1/2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(1/2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(1/2)*B*sqrt(b)*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
 + 3*I*sqrt(1/2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*
x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*
x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(5/2),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)a+\left(\int \sqrt{\cos\left(dx+c\right)}dx\right.\right.}{b^3}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*a + int(sqrt(cos(c + d*x)
),x)*b + int(sqrt(cos(c + d*x))*cos(c + d*x),x)*c))/b**3
```

3.283 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 116

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2(A-C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

output

```
-2*(A-C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d/
cos(d*x+c)^(1/2)+2*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2
))/b^2/d/(b*cos(d*x+c))^(1/2)+2*A*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.20 (sec) , antiderivative size = 807, normalized size of antiderivative = 6.96

$$\int \frac{\cos(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(5/2),x]
```

output
```
((Cos[c + d*x]^2*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*((-2*(-2*A + C + C*
Cos[2*c])*Csc[c]*Sec[c])/d + (4*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/(Sqrt[
b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) - (4*B*
Cos[c + d*x]^(3/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - A
rcTan[Cot[c]]]^2]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqr
t[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1
 + Cot[c]^2]) + (2*A*Cos[c + d*x]^(3/2)*Csc[c]*(B + C*Cos[c + d*x] + A*Sec
[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*
Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2]]))/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x]
+ C*Cos[2*c + 2*d*x])) - (2*C*Cos[c + d*x]^(3/2)*Csc[c]*(B + C*Cos[c + d*x
] + A*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Arc
Tan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ...
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.282$, Rules used = {2030, 3042, 3500, 27, 3042, 3227, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{C\cos^2(c+dx)+B\cos(c+dx)+A}{(b\cos(c+dx))^{3/2}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}$$

$$\Big\downarrow \text{3500}$$

$$\frac{\frac{2\int \frac{b^2B-b^2(A-C)\cos(c+dx)}{2\sqrt{b\cos(c+dx)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{b^2B-b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{b^2B-b^2(A-C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{b^2B\int \frac{1}{\sqrt{b\cos(c+dx)}}dx-b(A-C)\int \sqrt{b\cos(c+dx)}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b^2 B\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b(A-C)\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{\frac{b^2 B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{b(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{b^2 B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{b(A-C)\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{\frac{b^2 B\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{2b^2 B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}}{b^3}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}}{b}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]
)^(5/2),x]
```

output
```
(((-2*b*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Co
s[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*S
qrt[b*Cos[c + d*x]]))/b^3 + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]))
/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(109) = 218$.

Time = 0.26 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.26

method	result
default	$\frac{2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{b^{2}\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}\right.\right.}}$
parts	$-\frac{2A\left(-2\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\sqrt{-2b\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.}{b^{2}\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}}$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/b^2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-s
in(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(-1+2*cos(1/2*d*x+1/2*c)
^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\left(dx+c\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))-i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\left(dx+\right.\right.}{}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x, algorithm="fricas")
```

output
```
-2*(I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(1/2)*(I*A - I*C)*sqrt(b
)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + sqrt(1/2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) - sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^3*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x
+ c))^(5/2), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x
+ c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x)
)^(5/2),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x)
)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}dx\right)a+\right.}{b^3}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*b + int(sqrt(cos(c + d*x)
)/cos(c + d*x)**2,x)*a + int(sqrt(cos(c + d*x)),x)*c))/b**3
```

3.284 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 147

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

output
```
-2*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d/cos(
d*x+c)^(1/2)+2/3*(A+3*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^
(1/2))/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2
)+2*B*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3C)\sqrt{\cos(c+\cdots}}{3b^2d\sqrt{b\cdots}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x
]
```

output
```
(2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*x]
))/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3500$$

$$\frac{2\int \frac{3Bb^2+(A+3C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3Bb^2+(A+3C)\cos(c+dx)b^2}{(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3Bb^2+(A+3C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+3b^2B\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(A+3C)\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3119

$$\frac{\frac{b(A+3C)\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$\Big\downarrow$ 3120

$$\frac{\frac{2b(A+3C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}+3b^2B\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{3b^3}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + ((2*b*(A + 3*C)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + 3*b^2*B
*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c +
d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(3*b^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(134) = 268$.

Time = 0.28 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.46

method	result
default	$\frac{2\sqrt{b\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-12\right.}{}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{3b^2\sqrt{-b\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}\left(-1+2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\sin}$

input

```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output

```
2/3*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/2
*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*
b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(-1+2*cos(1/2*d*x+
1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.37

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{1}{2}}(i\,A+3i\,C)\sqrt{b}\cos\left(dx+c\right)^2 \text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{\frac{1}{2}}(-i\,A\right.}{}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-2/3*(sqrt(1/2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(1/2)*(-I*A - 3*I*C)*sqrt(b)*co
s(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3
*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(1/2)*B*sqrt(b)*co
s(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) - (3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x
 + c))/(b^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)}dx\right)c + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)a + \left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}\right.\right.}{b^3}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x),x)*c + int(sqrt(cos(c + d*x)
)/cos(c + d*x)**3,x)*a + int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*b))/b**
3
```

3.285 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 185

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

output
```
-2/5*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b^3/d/cos(d*x+c)^(1/2)+2/3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))/b^2/d/(b*cos(d*x+c))^(1/2)+2/5*A*sin(d*x+c)/d/(b*cos(d*x+c))^(5
/2)+2/3*B*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+2/5*(3*A+5*C)*sin(d*x+c)/b^2
/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.64

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(-3(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c
+ d*x])^(5/2),x]
```

output
```
(2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c +
 d*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b^2*d*Sqrt[
b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3500}$$

$$b\left(\frac{2\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 27$$

$$b\left(\frac{\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{\int \frac{5Bb^2+(3A+5C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 3227$$

$$b\left(\frac{b(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+5b^2B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{b(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+5b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 3116$$

$$b\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A\sin(c+ }{5bd(b\cos(c+d}\right.$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A\,s}{5bd(b\,c}\right.$$

$$\Big\downarrow \ 3121$$

$$b\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right.}{5b^3}\right.$$

$\downarrow$ 3042

$$b\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+}{3bd(b\cos(c+}\right.}{5b^3}\right.$$

$\downarrow$ 3119

$$b\left(\frac{5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^3}\right.$$

$\downarrow$ 3120

$$b\left(\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx}{3bd(b\cos(c+dx}\right.}{5b^3}\right.$$

input
```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x]
)^(5/2),x]
```

output
```
b*((2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (5*b^2*B*((2*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (
2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + b*(3*A + 5*C)*((-2*Sqrt[
b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2
*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(164) = 328$.

Time = 0.47 (sec) , antiderivative size = 807, normalized size of antiderivative = 4.36

method	result	size
default	Expression too large to display	807
parts	Expression too large to display	807

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x,meth
od=_RETURNVERBOSE)
```

output
```
-2/15*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin
(1/2*d*x+1/2*c)^4+120*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-60*C*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^4+36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^2-120*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+60*C*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+30*C*sin(1/2*d*x+1/2*c)^2*cos(1...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.21

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}B\sqrt{b}\cos(dx+c)^3 \operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 5i\sqrt{\frac{1}{2}}B\sqrt{b}\cos(d\right.}{}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),
x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^3*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(1/2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(1/2)*(-3*I*A - 5*I*C)*s
qrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))) - (3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*
x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(5/2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A\right)\sec\left(c\right)}{\left(b\cos\left(dx + c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x
+ c))^(5/2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(c\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x
+ c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))
^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))
^(5/2)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)}{\cos(dx+c)}dx\right)c+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx\right.}\right.}{b^3}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x))/cos(c + d*x),x)*c + int((s
qrt(cos(c + d*x))*sec(c + d*x))/cos(c + d*x)**3,x)*a + int((sqrt(cos(c + d
*x))*sec(c + d*x))/cos(c + d*x)**2,x)*b))/b**3
```

3.286 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 212

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} + \frac{6B\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

output

```
-6/5*B*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d/co
s(d*x+c)^(1/2)+2/21*(5*A+7*C)*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2
*c,2^(1/2))/b^2/d/(b*cos(d*x+c))^(1/2)+2/7*A*b*sin(d*x+c)/d/(b*cos(d*x+c))
^(7/2)+2/5*B*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/21*(5*A+7*C)*sin(d*x+c)/b
/d/(b*cos(d*x+c))^(3/2)+6/5*B*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.64

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\Big(-63B\sqrt{\cos(c+dx)}E\big(\tfrac{1}{2}(c+dx)\big|\,2\big)+5(5A$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[
c + d*x])^(5/2),x]
```

output
```
(2*(-63*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*B*Sin[c + d*x] + 25*A*Tan[c
 + d*x] + 35*C*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*A*Sec[c
+ d*x]^2*Tan[c + d*x]))/(105*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.366$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \; 3500$$

$$b^2\left(\frac{2\int \frac{7Bb^2+(5A+7C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 27$$

$$b^2\left(\frac{\int \frac{7Bb^2+(5A+7C)\cos(c+dx)b^2}{(b\cos(c+dx))^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{\int \frac{7Bb^2+(5A+7C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 3227$$

$$b^2\left(\frac{b(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}dx+7b^2B\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{b(5A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+7b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 3116$$

$$b^2\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{7b^3}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{7b^3}+\ldots\right)$$

$$\Big\downarrow \; 3116$$

$$b^2\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx)}\right.}{7b^3}\right.$$

$\downarrow$ 3042

$$b^2\left(\frac{b(5A+7C)\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c-}{5bd(b\cos(c-}\right.}{7b^3}\right.$$

$\downarrow$ 3121

$$b^2\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}\right.}{7b^3}\right.$$

$\downarrow$ 3042

$$b^2\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\right.}}{b^2\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

$\downarrow$ 3119

$$b^2\left(\frac{b(5A+7C)\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(\frac{b(5A+7C)\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+7b^2B\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+}}{b^2d\sqrt{\cos(c+dx)}}\right.}{5b^2}\right.}{7b^3}\right.$$

input
```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*
x])^(5/2),x]
```

output
```
b^2*((2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (b*(5*A + 7*C)*((
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*
x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + 7*b^2*B*((2*Sin[
c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*
d*Sqrt[b*Cos[c + d*x]])))/(5*b^2)))/(7*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(187) = 374.

Time = 0.55 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	729
parts	Expression too large to display	1007

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(2*A*(-1/5
6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5
/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+2/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin
(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+2*C*(-1/6*c
os(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(-1+
2*cos(1/2*d*x+1/2*c)^2))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.10

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx =$$
$$-\frac{2\left(5\sqrt{\frac{1}{2}}(5i\,A+7i\,C)\sqrt{b}\cos{(dx+c)}^4\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5\sqrt{\frac{1}{2}}(\cdots\right.}{\cdots}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2
),x, algorithm="fricas")
```

output
```
-2/105*(5*sqrt(1/2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(1/2)*(-5*I*A - 7*I*C)*
sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) + 63*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(1/2)*
B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - (63*B*cos(d*x + c)^3 + 5*(5*A + 7*C)*co
s(d*x + c)^2 + 21*B*cos(d*x + c) + 15*A)*sqrt(b*cos(d*x + c))*sin(d*x + c)
)/(b^3*d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(
5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*
x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*
x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^2\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x
))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x
))^(5/2)), x)
```

Reduce [F]

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}\,\sec(dx+c)^2}{\cos(dx+c)}dx\right)c + \left(\int \frac{\sqrt{\cos(dx+}}{\cos}\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*(int((sqrt(cos(c + d*x))*sec(c + d*x)**2)/cos(c + d*x),x)*c + int
((sqrt(cos(c + d*x))*sec(c + d*x)**2)/cos(c + d*x)**3,x)*a + int((sqrt(cos
(c + d*x))*sec(c + d*x)**2)/cos(c + d*x)**2,x)*b))/b**3
```

3.287 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 188

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^3d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

output
```
-2/5*(3*A+5*C)*(b*cos(d*x+c))^(1/2)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/
b^4/d/cos(d*x+c)^(1/2)+2/3*B*cos(d*x+c)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*
c,2^(1/2))/b^3/d/(b*cos(d*x+c))^(1/2)+2/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^
(5/2)+2/3*B*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(3/2)+2/5*(3*A+5*C)*sin(d*x+c)
/b^3/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.63

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \frac{2\left(-3(3A+5C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\sqrt{\cos(c\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x
]
```

output
```
(2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c +
 d*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b^3*d*Sqrt[
b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3500, 27, 3042, 3227, 3042, 3116, 3042, 3121, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \; 3500$$

$$\frac{2\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{2(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{5Bb^2+(3A+5C)\cos(c+dx)b^2}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{5Bb^2+(3A+5C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$\Big\downarrow$ 3227

$$\frac{b(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}dx+5b^2B\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{b(3A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+5b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$\Big\downarrow$ 3116

$$\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)+5b^2B\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$\Big\downarrow$ 3121

$$\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$\Big\downarrow$ 3119

$$\frac{5b^2B\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)+b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$\Big\downarrow$ 3120

$$\frac{b(3A+5C)\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)+5b^2B\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{5b^3}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

output
```
(2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (5*b^2*B*((2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*S
in[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))) + b*(3*A + 5*C)*((-2*Sqrt[b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Si
n[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(167) = 334$.

Time $= 0.44$ (sec) , antiderivative size $= 807$, normalized size of antiderivative $= 4.29$

method	result	size
default	Expression too large to display	807
parts	Expression too large to display	807

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/15*(b*(-1+2*cos(1/2*d*x+1/2*c)^2)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin
(1/2*d*x+1/2*c)^4+120*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-60*C*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^4+36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^2-120*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+60*C*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+30*C*sin(1/2*d*x+1/2*c)^2*cos(1...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.19

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx =$$

$$-\frac{2\left(5i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\left(dx+c\right)^{3}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5i\sqrt{\frac{1}{2}}B\sqrt{b}\cos\left(d\right.\right.}{}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorith
m="fricas")
```

output
```
-2/15*(5*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(1/2)*B*sqrt(b)*cos(d*x + c)^3*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(1/2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(1/2)*(-3*I*A - 5*I*C)*s
qrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))) - (3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*
x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^4*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \int \frac{C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A}{\left(b\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx=\int \frac{C\cos{(dx+c)}^2+B\cos{(dx+c)}+A}{(b\cos{(dx+c)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx=\int \frac{C\cos{(c+d\,x)}^2+B\,\cos{(c+d\,x)}+A}{(b\,\cos{(c+d\,x)})^{7/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx=\frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^4}dx\right)a+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}dx\right)b+\left(\int \frac{\sqrt{\cos(dx+c)}}{\cos(dx+c)^2}\right.\right.}{b^4}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x)
```

output
```
(sqrt(b)*(int(sqrt(cos(c + d*x))/cos(c + d*x)**4,x)*a + int(sqrt(cos(c + d
*x))/cos(c + d*x)**3,x)*b + int(sqrt(cos(c + d*x))/cos(c + d*x)**2,x)*c))/
b**4
```

3.288 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C$

Optimal result

Integrand size = 43, antiderivative size = 223

$$
\begin{aligned}
&\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx \\
&= \frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{(5A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} \\
&\quad+\frac{3B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} \\
&\quad+\frac{B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} \\
&\quad+\frac{C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d} \\
&\quad-\frac{(5A+4C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{15d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

output

```
3/8*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/5*(5*A+4*C)*(b*cos(d*x+c))
^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+3/8*B*cos(d*x+c)^(1/2)*(b*cos(d*x+c))
^(1/2)*sin(d*x+c)/d+1/4*B*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)
/d+1/5*C*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d-1/15*(5*A+4*C)
*(b*cos(d*x+c))^(1/2)*sin(d*x+c)^3/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.49

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b\cos(c+dx)}(180Bc+180Bdx+60(6A+5C)\sin(c+dx)+120B\sin(2(c+dx))+40A\sin(3(c+dx$$
$$480d\sqrt{\cos(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*
Cos[c + d*x]^2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x] +
 120*B*Sin[2*(c + d*x)] + 40*A*Sin[3*(c + d*x)] + 50*C*Sin[3*(c + d*x)] +
15*B*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.62, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.279$, Rules used = {2031, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\int\cos^3(c+dx)(5A+4C+5B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\int\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(5A+4C+5B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3227

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left((5A+4C)\int\cos^3(c+dx)dx+5B\int\cos^4(c+dx)dx\right)+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left((5A+4C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^3dx+5B\int\sin\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3113

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\int\sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{(5A+4C)\int(1-\sin^2(c+dx))d(-\sin(c+dx))}{d}\right)+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 2009

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\int\sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3115

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\left(\frac{3}{4}\int\cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\,c}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\left(\frac{3}{4}\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c}{}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+ C}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin}{}}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c
+ d*x]^2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((C*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) + (-(((5*A +
4*C)*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d) + 5*B*((Cos[c + d*x]^3*Sin[c +
 d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/5))/Sqrt[
Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.49

method	result
default	$\frac{\left(45B(dx+c)+\left(40\cos(dx+c)^2+80\right)\sin(dx+c)A+\sin(dx+c)\cos(dx+c)\left(30\cos(dx+c)^2+45\right)B+\left(24\cos(dx+c)^4+32\cos(dx+c)^2+\cdots\right.}{120d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\left(2+\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}+\frac{B\left(2\cos(dx+c)^3\sin(dx+c)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)\sqrt{b\cos(dx+c)}}{8d\sqrt{\cos(dx+c)}}+\cdots$
risch	$\frac{3\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{i(dx+c)}Bx}{4\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{6i(dx+c)}C}{80\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{5i(dx+c)}B}{32\left(\mathrm{e}^{2i(dx+c)}+1\right)d}\cdots$

input
```
int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x,method=_RETURNVERBOSE)
```

output
```
1/120/d*(45*B*(d*x+c)+(40*cos(d*x+c)^2+80)*sin(d*x+c)*A+sin(d*x+c)*cos(d*x
+c)*(30*cos(d*x+c)^2+45)*B+(24*cos(d*x+c)^4+32*cos(d*x+c)^2+64)*sin(d*x+c)
*C)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.31

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\left[\frac{45\,B\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)+\right.$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="fricas")
```

output
```
[1/240*(45*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d
*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(24*C*cos(d*x +
 c)^4 + 30*B*cos(d*x + c)^3 + 8*(5*A + 4*C)*cos(d*x + c)^2 + 45*B*cos(d*x
+ c) + 80*A + 64*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)), 1/120*(45*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x
+ c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (24*C*cos(d*x + c)^4 + 3
0*B*cos(d*x + c)^3 + 8*(5*A + 4*C)*cos(d*x + c)^2 + 45*B*cos(d*x + c) + 80
*A + 64*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*
x+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.71

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)B\sqrt{b}+2\,C\sqrt{b}(3}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="maxima")
```

output
```
1/480*(15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 2*C*sqrt(b)*(3*sin(5*d*x + 5*c) +
25*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 150*sin(1/5*arct
an2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))) + 40*A*sqrt(b)*(sin(3*d*x + 3*c)
 + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.42

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{1}{480}\left(180\,Bx+\frac{6\,C\sin\left(5\,dx+5\,c\right)}{d}+\frac{15\,B\sin\left(4\,dx+4\,c\right)}{d}+\frac{10\,(4\,A+5\,C)\sin\left(3\,dx+3\,c\right)}{d}+\frac{120\,B\sin}{}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="giac")
```

output
```
1/480*(180*B*x + 6*C*sin(5*d*x + 5*c)/d + 15*B*sin(4*d*x + 4*c)/d + 10*(4*
A + 5*C)*sin(3*d*x + 3*c)/d + 120*B*sin(2*d*x + 2*c)/d + 60*(6*A + 5*C)*si
n(d*x + c)/d)*sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 43.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.63

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}\,\sqrt{b\,\cos(c+dx)}\,(120\,B\,\sin(c+dx)+400\,A\,\sin(2\,c+2\,dx)+40\,A\,\sin(4\,c+4\,dx)+1}{}$$

input
```
int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(
c + d*x)^2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(120*B*sin(c + d*x) + 400*A*sin
(2*c + 2*d*x) + 40*A*sin(4*c + 4*d*x) + 135*B*sin(3*c + 3*d*x) + 15*B*sin(
5*c + 5*d*x) + 350*C*sin(2*c + 2*d*x) + 56*C*sin(4*c + 4*d*x) + 6*C*sin(6*
c + 6*d*x) + 360*B*d*x*cos(c + d*x)))/(480*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.43

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b}\,(-30\cos(dx+c)\sin(dx+c)^3b+75\cos(dx+c)\sin(dx+c)\,b+24\sin(dx+c)^5c-40\sin(dx+c)}{120d}$$

input
```
int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x)
```

output
```
(sqrt(b)*( - 30*cos(c + d*x)*sin(c + d*x)**3*b + 75*cos(c + d*x)*sin(c + d
*x)*b + 24*sin(c + d*x)**5*c - 40*sin(c + d*x)**3*a - 80*sin(c + d*x)**3*c
 + 120*sin(c + d*x)*a + 120*sin(c + d*x)*c + 45*b*d*x))/(120*d)
```

3.289 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+$

Optimal result

Integrand size = 43, antiderivative size = 184

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$
$$+\frac{(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}$$
$$+\frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}-\frac{B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

output
```
1/8*(4*A+3*C)*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+B*(b*cos(d*x+c))^(1/
2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+1/8*(4*A+3*C)*cos(d*x+c)^(1/2)*(b*cos(d*x
+c))^(1/2)*sin(d*x+c)/d+1/4*C*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*sin(d*
x+c)/d-1/3*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)^3/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.50

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b\cos(c+dx)}(48Ac+36cC+48Adx+36Cdx+72B\sin(c+dx)+24(A+C)\sin(2(c+dx))+8B\,\mathrm{si}}{96d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*
Cos[c + d*x]^2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c
+ d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c
 + d*x)]))/(96*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.60, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2031, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 2031$$

$$\frac{\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3502$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\int \cos^2(c+dx)(4A+3C+4B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(4A+3C+4B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int \cos^2(c+dx)dx+4B\int \cos^3(c+dx)dx\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+4B\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3113}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4B\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c
+ d*x]^2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A + 3*C
)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (4*B*(-Sin[c + d*x] + Sin[c
+ d*x]^3/3))/d)/4))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.55

method	result
default	$\frac{\left(12A(dx+c)+9C(dx+c)+12A\cos(dx+c)\sin(dx+c)+\left(8\cos(dx+c)^2+16\right)\sin(dx+c)B+\sin(dx+c)\cos(dx+c)\left(6\cos(dx+c)^2+9\right)C\right.}{24d\sqrt{\cos(dx+c)}}$
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}+\frac{B\sin(dx+c)\left(2+\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}+\frac{C\left(2\cos(dx+c)^3\sin(dx+c)+3\cos\right.}{8}$
risch	$\frac{\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,e^{i(dx+c)}(8A+6C)x}{8\,e^{2i(dx+c)}+8}-\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,e^{5i(dx+c)}C}{32\left(e^{2i(dx+c)}+1\right)d}-\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,e^{4i(dx}}{12\left(e^{2i(dx+c)}+1\right)d}$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x,method=_RETURNVERBOSE)
```

output
```
1/24/d*(12*A*(d*x+c)+9*C*(d*x+c)+12*A*cos(d*x+c)*sin(d*x+c)+(8*cos(d*x+c)^
2+16)*sin(d*x+c)*B+sin(d*x+c)*cos(d*x+c)*(6*cos(d*x+c)^2+9)*C)*(b*cos(d*x+
c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.50

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$= \left[\frac{3\,(4\,A+3\,C)\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right.\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="fricas")
```

output
```
[1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt
(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*C*co
s(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqr
t(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*
(3*(4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*c
os(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^
2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*
x+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.63

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A\sqrt{b}+3\,\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin(4\,dx+4\right.\right.\right.}{9}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="maxima")
```

output
```
1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*sqrt(b) + 3*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*
C*sqrt(b) + 8*B*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))))/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.41

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{1}{96}\left(12\,(4\,A+3\,C)x+\frac{3\,C\sin(4\,dx+4\,c)}{d}+\frac{8\,B\sin(3\,dx+3\,c)}{d}+\frac{24\,(A+C)\sin(2\,dx+2\,c)}{d}+\frac{72\,B\,\mathrm{s}}{}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="giac")
```

output
```
1/96*(12*(4*A + 3*C)*x + 3*C*sin(4*d*x + 4*c)/d + 8*B*sin(3*d*x + 3*c)/d +
 24*(A + C)*sin(2*d*x + 2*c)/d + 72*B*sin(d*x + c)/d)*sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 41.79 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.74

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(24\,A\,\sin(c+d\,x)+24\,C\,\sin(c+d\,x)+24\,A\,\sin(3\,c+3\,d\,x)+80\,B\,\mathrm{s}}{}$$

input
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(
c + d*x)^2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(c
 + d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*
d*x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x
) + 72*C*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b}\,\left(-6\cos(dx+c)\sin(dx+c)^3c+12\cos(dx+c)\sin(dx+c)\,a+15\cos(dx+c)\sin(dx+c)\,c-8\sin}{24d}$$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x)
```

output
```
(sqrt(b)*( - 6*cos(c + d*x)*sin(c + d*x)**3*c + 12*cos(c + d*x)*sin(c + d*
x)*a + 15*cos(c + d*x)*sin(c + d*x)*c - 8*sin(c + d*x)**3*b + 24*sin(c + d
*x)*b + 12*a*d*x + 9*c*d*x))/(24*d)
```

3.290 $\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)$

Optimal result

Integrand size = 43, antiderivative size = 143

$$\begin{aligned}
&\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx \\
&= \frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{(3A+2C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} \\
&\quad+\frac{B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} \\
&\quad+\frac{C\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}
\end{aligned}$$

output

```
1/2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/3*(3*A+2*C)*(b*cos(d*x+c))
^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+1/2*B*cos(d*x+c)^(1/2)*(b*cos(d*x+c))
^(1/2)*sin(d*x+c)/d+1/3*C*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)
/d
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.52

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b\cos(c+dx)}(6Bc+6Bdx+3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+C\sin(3(c+dx)))}{12d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*
Cos[c + d*x]^2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*
Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2031, 3042, 3502, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 2031$$

$$\frac{\sqrt{b\cos(c+dx)}\int \cos(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3502$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\int\cos(c+dx)(3A+2C+3B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\int\sin\left(c+dx+\frac{\pi}{2}\right)\left(3A+2C+3B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3213}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(\frac{(3A+2C)\sin(c+dx)}{d}+\frac{3B\sin(c+dx)\cos(c+dx)}{2d}+\frac{3Bx}{2}\right)+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c
+ d*x]^2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) + ((3*B*x)/2
+ ((3*A + 2*C)*Sin[c + d*x])/d + (3*B*Cos[c + d*x]*Sin[c + d*x])/(2*d))/3)
)/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.54

method	result
default	$\frac{\left(3B(dx+c)+6A\sin(dx+c)+3B\sin(dx+c)\cos(dx+c)+\left(2\cos(dx+c)^{2}+4\right)\sin(dx+c)C\right)\sqrt{b\cos(dx+c)}}{6d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}+\frac{C\sin(dx+c)\left(2+\cos(dx+c)^{2}\right)\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}+\frac{B(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}$
risch	$\frac{\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{i(dx+c)}Bx}{\mathrm{e}^{2i(dx+c)}+1}-\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{4i(dx+c)}C}{12\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{i\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{3i(dx+c)}B}{4\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x,method=_RETURNVERBOSE)
```

output
```
1/6/d*(3*B*(d*x+c)+6*A*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)+(2*cos(d*x+c)^
2+4)*sin(d*x+c)*C)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.65

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$= \left[\frac{3\,B\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)+2}{12\,d\cos\left(dx+\right.}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="fricas")
```

output
```
[1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*cos(d*x + c)
^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x +
c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x
 + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [A] (verification not implemented)

Time = 29.47 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.69

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\begin{cases} x\sqrt{b\cos{(c)}}(A+B\cos{(c)}+C\cos^2{(c)})\sqrt{\cos{(c)}} \\ 0 \\ \frac{A\sqrt{b\cos{(c+dx)}}\sin{(c+dx)}}{d\sqrt{\cos{(c+dx)}}}+\frac{Bx\sqrt{b\cos{(c+dx)}}\sin^2{(c+dx)}}{2\sqrt{\cos{(c+dx)}}}+\frac{Bx\sqrt{b\cos{(c+dx)}}\cos^{\frac{3}{2}}{(c+dx)}}{2}+\frac{B\sqrt{b\cos{(c+dx)}}\sin{(c+dx)}\sqrt{\cos{(c+d}}}{2d} \end{cases}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*
x+c)**2),x)
```

output
```
Piecewise((x*sqrt(b*cos(c))*(A + B*cos(c) + C*cos(c)**2)*sqrt(cos(c)), Eq(
d, 0)), (0, Eq(c, -d*x + pi/2) | Eq(c, -d*x + 3*pi/2)), (A*sqrt(b*cos(c +
d*x))*sin(c + d*x)/(d*sqrt(cos(c + d*x))) + B*x*sqrt(b*cos(c + d*x))*sin(c
 + d*x)**2/(2*sqrt(cos(c + d*x))) + B*x*sqrt(b*cos(c + d*x))*cos(c + d*x)*
*(3/2)/2 + B*sqrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x))/(2*d) +
2*C*sqrt(b*cos(c + d*x))*sin(c + d*x)**3/(3*d*sqrt(cos(c + d*x))) + C*sqrt
(b*cos(c + d*x))*sin(c + d*x)*cos(c + d*x)**(3/2)/d, True))
```

Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3\,(2\,dx+2\,c+\sin{(2\,dx+2\,c)})B\sqrt{b}+C\sqrt{b}\big(\sin{(3\,dx+3\,c)}+9\,\sin{\left(\frac{1}{3}\,\arctan{(\sin{(3\,dx+3\,c)}\,,\cos{(3\,dx}}\right.}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="maxima")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*sqrt(b) + C*sqrt(b)*(sin(3*d*x
+ 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 12*A*sq
rt(b)*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.40

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{1}{12}\left(6\,Bx+\frac{C\sin{(3\,dx+3\,c)}}{d}+\frac{3\,B\sin{(2\,dx+2\,c)}}{d}+\frac{3\,(4\,A+3\,C)\sin{(dx+c)}}{d}\right)\sqrt{b}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="giac")
```

output
```
1/12*(6*B*x + C*sin(3*d*x + 3*c)/d + 3*B*sin(2*d*x + 2*c)/d + 3*(4*A + 3*C
)*sin(d*x + c)/d)*sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.73

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(3\,B\,\sin\left(c+d\,x\right)+12\,A\,\sin\left(2\,c+2\,d\,x\right)+3\,B\,\sin\left(3\,c+3\,d\,x\right)+10\,C}{12\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(
c + d*x)^2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*B*sin(c + d*x) + 12*A*sin(2*
c + 2*d*x) + 3*B*sin(3*c + 3*d*x) + 10*C*sin(2*c + 2*d*x) + C*sin(4*c + 4*
d*x) + 12*B*d*x*cos(c + d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.40

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{\sqrt{b}\left(3\cos\left(dx+c\right)\sin\left(dx+c\right)b-2\sin\left(dx+c\right)^3c+6\sin\left(dx+c\right)a+6\sin\left(dx+c\right)c+3bdx\right)}{6d}$$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x)
```

output
```
(sqrt(b)*(3*cos(c + d*x)*sin(c + d*x)*b - 2*sin(c + d*x)**3*c + 6*sin(c +
d*x)*a + 6*sin(c + d*x)*c + 3*b*d*x))/(6*d)
```

3.291 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 123

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
A*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*C*x*(b*cos(d*x+c))^(1/2)/cos
(d*x+c)^(1/2)+B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+1/2*C*c
os(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(c+dx)}(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/S
qrt[Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*
(c + d*x)]))/(4*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(Ax+\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Co
s[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(A*x + (C*x)/2 + (B*Sin[c + d*x])/d + (C*Cos[c + d*x
]*Sin[c + d*x])/(2*d)))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}$	63
risch	$\frac{\sqrt{b\cos(dx+c)}\,(4A+2C)x}{4\sqrt{\cos(dx+c)}}+\frac{B\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{d\sqrt{\cos(dx+c)}}+\frac{\sqrt{b\cos(dx+c)}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	92
parts	$\frac{C(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}+\frac{A(dx+c)\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}+\frac{B\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{d\sqrt{\cos(dx+c)}}$	101

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+C*(d*x+c))*(b*co
s(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.72

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \left[\frac{(2A+C)\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-\right.}{4d\cos(dx+c)}\right.$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(1/2),x, algorithm="fricas")
```

output
```
[1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*co
s(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(C*cos(d*x +
 c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(
sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [A] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \begin{cases} \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}\sin^2(c+dx)}{2\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}{2} + \frac{C\sqrt{b\cos(c+dx)}}{} \\ \frac{x\sqrt{b\cos(c)}(A+B\cos(c)+C\cos^2(c))}{\sqrt{\cos(c)}} \end{cases}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(1/2),x)
```

output
```
Piecewise((A*x*sqrt(b*cos(c + d*x))/sqrt(cos(c + d*x)) + B*sqrt(b*cos(c +
d*x))*sin(c + d*x)/(d*sqrt(cos(c + d*x))) + C*x*sqrt(b*cos(c + d*x))*sin(c
 + d*x)**2/(2*sqrt(cos(c + d*x))) + C*x*sqrt(b*cos(c + d*x))*cos(c + d*x)*
*(3/2)/2 + C*sqrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x))/(2*d), N
e(d, 0)), (x*sqrt(b*cos(c))*(A + B*cos(c) + C*cos(c)**2)/sqrt(cos(c)), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C\sqrt{b}+8\,A\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+4\,B\sqrt{b}\sin(dx+c)}{4\,d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(1/2),x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C*sqrt(b) + 8*A*sqrt(b)*arctan(sin(d
*x + c)/(cos(d*x + c) + 1)) + 4*B*sqrt(b)*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{1}{4}\left(2\,(2\,A+C)x+\frac{C\sin(2\,dx+2\,c)}{d}+\frac{4\,B\sin(dx+c)}{d}\right)\sqrt{b}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(1/2),x, algorithm="giac")
```

output
```
1/4*(2*(2*A + C)*x + C*sin(2*d*x + 2*c)/d + 4*B*sin(d*x + c)/d)*sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(4\,B\,\sin\left(c+d\,x\right)+C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(1/2),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(4*B*sin(c + d*x) + C*sin(2*c + 2*d*x) + 4*A*d*x +
 2*C*d*x))/(4*d*cos(c + d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b}\,(\cos(dx+c)\sin(dx+c)\,c+2\sin(dx+c)\,b+2adx+cdx)}{2d}$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),
x)
```

output
```
(sqrt(b)*(cos(c + d*x)*sin(c + d*x)*c + 2*sin(c + d*x)*b + 2*a*d*x + c*d*x
))/(2*d)
```

3.292 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 93

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{A\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*arctanh(sin(d*x+c))*(b*cos(d*x
+c))^(1/2)/d/cos(d*x+c)^(1/2)+C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+
c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{b\cos(c+dx)}\left(Bc+Bdx-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\right.\right.\right.}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/C
os[c + d*x]^(3/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*S
qrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2031, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}$$

$$\downarrow \text{3502}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\int (A+B\cos(c+dx))\sec(c+dx)dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\downarrow \text{3214}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \sec(c+dx)dx+Bx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+Bx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\downarrow \text{4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+Bx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
 d*x]^(3/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(B*x + (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x]
)/d))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

method	result
default	$-\frac{(2A\ \operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))-B(dx+c)-C\sin(dx+c))\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}$
parts	$\frac{C\sqrt{b\cos(dx+c)}\ \sin(dx+c)}{d\sqrt{\cos(dx+c)}}-\frac{2A\sqrt{b\cos(dx+c)}\ \operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))}{d\sqrt{\cos(dx+c)}}+\frac{B(dx+c)\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}$
risch	$\frac{Bx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}}-\frac{i\sqrt{b\cos(dx+c)}\,C\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}+\frac{i\sqrt{b\cos(dx+c)}\,C\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}-\frac{\sqrt{b\cos(dx+c)}\,A\ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}+\frac{\sqrt{b\cos(d}{}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),
x,method=_RETURNVERBOSE)
```

output
```
-1/d*(2*A*arctanh(-csc(d*x+c)+cot(d*x+c))-B*(d*x+c)-C*sin(d*x+c))*(b*cos(d
*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.27

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \left[-\frac{2\,A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c) - B\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2 - 2\right.}{2\,d\cos(}\right.$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c) - B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x
 + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
- b) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b
)*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d
*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c)
 - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(3/2),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)/cos
(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{A\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\right.\right.}{2\,d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(3/2),x, algorithm="maxima")
```

output
```
1/2*(A*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
- log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*B*sqrt(b)
*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 2*C*sqrt(b)*sin(d*x + c))/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\left(A\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-A\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+i\,B\log\left(i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-i\,B\log\right.}{d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(3/2),x, algorithm="giac")
```

output
```
(A*log(tan(1/2*d*x + 1/2*c) + 1) - A*log(tan(1/2*d*x + 1/2*c) - 1) + I*B*l
og(I*tan(1/2*d*x + 1/2*c) - 1) - I*B*log(-I*tan(1/2*d*x + 1/2*c) - 1) + 2*
C*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(3/2),x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a+\sin\left(dx+c\right)c+bdx\right)}{d}$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),
x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a + s
in(c + d*x)*c + b*d*x))/d
```

3.293 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 93

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
C*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+B*arctanh(sin(d*x+c))*(b*cos(d*x
+c))^(1/2)/d/cos(d*x+c)^(1/2)+A*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+
c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\left(Cdx\cos(c+dx)+B\coth^{-1}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx)\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/C
os[c + d*x]^(5/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + B*ArcCoth[Sin[c + d*x]]*Cos[c
+ d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2031, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\int (B+C\cos(c+dx))\sec(c+dx)dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\int \frac{B+C\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\int \sec(c+dx)dx+\frac{A\tan(c+dx)}{d}+Cx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)}{d}+Cx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+\frac{B\text{arctanh}(\sin(c+dx))}{d}+Cx\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
 d*x]^(5/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(C*x + (B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x]
)/d))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

method	result	si
default	$\frac{(-2B\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))\cos(dx+c)+C(dx+c)\cos(dx+c)+A\sin(dx+c))\sqrt{b\cos(dx+c)}}{d\cos(dx+c)^{\frac{3}{2}}}$	70
parts	$\frac{A\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}}-\frac{2B\sqrt{b\cos(dx+c)}\,\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))}{d\sqrt{\cos(dx+c)}}+\frac{C(dx+c)\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}$	99
risch	$\frac{Cx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}}+\frac{2i\sqrt{b\cos(dx+c)}\,A}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}+\frac{\sqrt{b\cos(dx+c)}\,B\ln\left(e^{i(dx+c)}+i\right)}{\sqrt{\cos(dx+c)}\,d}-\frac{\sqrt{b\cos(dx+c)}\,B\ln\left(e^{i(dx+c)}-i\right)}{\sqrt{\cos(dx+c)}\,d}$	13

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),
x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*B*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)+C*(d*x+c)*cos(d*x+c)+
A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 312, normalized size of antiderivative = 3.35

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left[-\frac{2\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^2 - C\sqrt{-b}\cos(dx+c)^2\log\left(2\,b\cos(dx+c)^2 - \right.}{2\,d\cos}\right.$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(5/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c)^2 - C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos
(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x +
 c) - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(
sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2*log
(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*si
n(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*
sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{B\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\right.\right.}{2\,d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(5/2),x, algorithm="maxima")
```

output
```
1/2*(B*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
- log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*C*sqrt(b)
*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*A*sqrt(b)*sin(2*d*x + 2*c)/(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\left(B\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-B\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+i\,C\log\left(i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-i\,C\log\right.}{d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(5/2),x, algorithm="giac")
```

output
```
(B*log(tan(1/2*d*x + 1/2*c) + 1) - B*log(tan(1/2*d*x + 1/2*c) - 1) + I*C*l
og(I*tan(1/2*d*x + 1/2*c) - 1) - I*C*log(-I*tan(1/2*d*x + 1/2*c) - 1) - 2*
A*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(5/2),x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{b}\left(-\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b+\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)b+\cos\left(dx+c\right)cdx+\mathrm{s}\right.}{\cos\left(dx+c\right)d}$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),
x)
```

output
```
(sqrt(b)*( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + cos(c + d*x)*log(t
an((c + d*x)/2) + 1)*b + cos(c + d*x)*c*d*x + sin(c + d*x)*a))/(cos(c + d*
x)*d)
```

3.294 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 111

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output

```
1/2*(A+2*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/
2*A*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+B*(b*cos(d*x+c))^(1
/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{b\cos(c+dx)}\left(2C\coth^{-1}(\sin(c+dx))\cos^2(c+dx)+A\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(A+2B\text{ c}\right.}{2d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/C
os[c + d*x]^(7/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(2*C*ArcCoth[Sin[c + d*x]]*Cos[c + d*x]^2 + A*ArcTan
h[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*
d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {2031, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\int(2B+(A+2C)\cos(c+dx))\sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\int\frac{2B+(A+2C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\sec(c+dx)dx+2B\int\sec^2(c+dx)dx\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+2B\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2B\int1d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2B\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left(\frac{(A+2C)\text{arctanh}(\sin(c+dx))}{d}+\frac{2B\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

input

```
Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
 d*x]^(7/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((A + 2*C)*A
rcTanh[Sin[c + d*x]])/d + (2*B*Tan[c + d*x])/d)/2))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

method	result
default	$\frac{\left(-A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+A\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-4C\,\operatorname{arctanh}(-\csc(dx+c)+\cot(d$ / $2d\cos(dx+c)^{\frac{5}{2}}$
parts	$-\frac{2C\sqrt{b\cos(dx+c)}\,\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))}{d\sqrt{\cos(dx+c)}}+\frac{A\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(c\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\left(A\,e^{3i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}-2B\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{\sqrt{b\cos(dx+c)}\,(A+}{2\sqrt{\cos(}}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),
x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(-A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^2+A*ln(-cot(d*x+c)+csc(d
*x+c)+1)*cos(d*x+c)^2-4*C*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)^2+2*B
*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$=\left[\frac{(A+2\,C)\sqrt{b}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2\,B\cos\left(c\right.\right.}{4\,d\cos\left(dx+c\right)^3}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-\left(2\,B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}}{2\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(7/2),x, algorithm="fricas")
```

output
```
[1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*c
os(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/c
os(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(
sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x
 + c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 780 vs. 2(95) = 190.

Time = 0.32 (sec) , antiderivative size = 780, normalized size of antiderivative = 7.03

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(7/2),x, algorithm="maxima")
```

output
```
1/4*(2*C*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1
) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) - (4*(sin(4
*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
+ 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*c
os(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x +
4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*co
s(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)
^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos...
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{\left((A+2C)\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-(A+2C)\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)+\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\left(\ldots\right.\right.}{\tan\left(\frac{1}{2}dx\ldots\right.}\right.}{2d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(7/2),x, algorithm="giac")
```

output
```
1/2*((A + 2*C)*log(tan(1/2*d*x + 1/2*c) + 1) - (A + 2*C)*log(tan(1/2*d*x +
 1/2*c) - 1) + 2*(A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 +
A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4
 - 2*tan(1/2*d*x + 1/2*c)^2 + 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(7/2),x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)b-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2a-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),
x)
```

output
```
(sqrt(b)*( - 2*cos(c + d*x)*sin(c + d*x)*b - log(tan((c + d*x)/2) - 1)*sin
(c + d*x)**2*a - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c + log(tan((
c + d*x)/2) - 1)*a + 2*log(tan((c + d*x)/2) - 1)*c + log(tan((c + d*x)/2)
+ 1)*sin(c + d*x)**2*a + 2*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*c - l
og(tan((c + d*x)/2) + 1)*a - 2*log(tan((c + d*x)/2) + 1)*c - sin(c + d*x)*
a))/(2*d*(sin(c + d*x)**2 - 1))
```

3.295 $\int \frac{\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 152

$$\begin{aligned}
&\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx \\
&= \frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} \\
&\quad +\frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

output

```
1/2*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/3*A*(b
*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+1/2*B*(b*cos(d*x+c))^(1/2
)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+1/3*(2*A+3*C)*(b*cos(d*x+c))^(1/2)*sin(d*x
+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{b\cos(c+dx)}(3B\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(4A+3C+3B\cos(c+dx)+(2A+3C)\cos(2(}{6d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/C
os[c + d*x]^(9/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3
*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*
Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.69, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {2031, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \ 2031$$

$$\frac{\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\int(3B+(2A+3C)\cos(c+dx))\sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\int\frac{3B+(2A+3C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left((2A+3C)\int\sec^2(c+dx)dx+3B\int\sec^3(c+dx)dx\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left((2A+3C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(3B\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(3B\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(\frac{(2A+3C)\tan(c+dx)}{d}+3B\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
 d*x]^(9/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (((2*A + 3*
C)*Tan[c + d*x])/d + 3*B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[
c + d*x])/(2*d)))/3))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

method	result
default	$\frac{\left(-3B\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^3+3B\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^3+\left(4\cos(dx+c)^2+2\right)\sin(dx+c)\right.}{6d\cos(dx+c)^{\frac{7}{2}}}$
parts	$\frac{C\sin(dx+c)\sqrt{b\cos(dx+c)}}{d\cos(dx+c)^{\frac{3}{2}}}+\frac{A\sin(dx+c)\left(2\cos(dx+c)^2+1\right)\sqrt{b\cos(dx+c)}}{3d\cos(dx+c)^{\frac{7}{2}}}+\frac{B\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\right.}{}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\left(3B\,\mathrm{e}^{5i(dx+c)}-6C\,\mathrm{e}^{4i(dx+c)}-12A\,\mathrm{e}^{2i(dx+c)}-12C\,\mathrm{e}^{2i(dx+c)}-3B\,\mathrm{e}^{i(dx+c)}-4A-6C\right)}{3\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{\sqrt{b\cos(dx+c)}\,B\ln(\mathrm{e}^{i(d}}{2\sqrt{\cos(dx+c)}\,d}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),
x,method=_RETURNVERBOSE)
```

output
```
1/6/d*(-3*B*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^3+3*B*ln(-cot(d*x+c)+c
sc(d*x+c)+1)*cos(d*x+c)^3+(4*cos(d*x+c)^2+2)*sin(d*x+c)*A+6*C*cos(d*x+c)^2
*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/
2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \left[\frac{3\,B\sqrt{b}\cos\left(dx+c\right)^4\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2\left(2\,A+3\,C\right)\mathrm{co}\right.}{12\,d\cos\left(dx+c\right)^4}\right.$$

$$-\frac{3\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^4-\left(2\left(2\,A+3\,C\right)\cos\left(dx+c\right)^2+3\,B\cos\left(dx+\right.\right.}{6\,d\cos\left(dx+c\right)^4}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(9/2),x, algorithm="fricas")
```

output
```
[1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*
x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*
x + c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt
(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1/6
*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(c
os(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*
x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos
(d*x + c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1009 vs. 2(128) = 256.

Time = 0.32 (sec) , antiderivative size = 1009, normalized size of antiderivative = 6.64

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(9/2),x, algorithm="maxima")
```

output
```
1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c
) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(s
in(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2
+ 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*
x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*
x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4
*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*
d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2...
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{\left(3\,B\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-3\,B\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(6\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,C\right.}{}\right.}{6\,d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(9/2),x, algorithm="giac")
```

output
```
1/6*(3*B*log(tan(1/2*d*x + 1/2*c) + 1) - 3*B*log(tan(1/2*d*x + 1/2*c) - 1)
 - 2*(6*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 6*C*tan(1/
2*d*x + 1/2*c)^5 - 4*A*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/2*d*x + 1/2*c)^
3 + 6*A*tan(1/2*d*x + 1/2*c) + 3*B*tan(1/2*d*x + 1/2*c) + 6*C*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^6 - 3*tan(1/2*d*x + 1/2*c)^4 + 3*tan(1/2*d
*x + 1/2*c)^2 - 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(9/2),x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{b}\left(-3\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2b+3\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b+3\,\mathrm{co}\right.}{}$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),
x)
```

output
```
(sqrt(b)*( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b +
3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 3*cos(c + d*x)*log(tan((c + d
*x)/2) + 1)*sin(c + d*x)**2*b - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b
 - 3*cos(c + d*x)*sin(c + d*x)*b + 4*sin(c + d*x)**3*a + 6*sin(c + d*x)**3
*c - 6*sin(c + d*x)*a - 6*sin(c + d*x)*c))/(6*cos(c + d*x)*d*(sin(c + d*x)
**2 - 1))
```

3.296 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 193

$$\begin{aligned}
&\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx \\
&= \frac{(3A+4C)\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}
\end{aligned}$$

output

```
1/8*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+
1/4*A*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)+1/8*(3*A+4*C)*(b*
cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+B*(b*cos(d*x+c))^(1/2)*sin
(d*x+c)/d/cos(d*x+c)^(3/2)+1/3*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)^3/d/cos(d
*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{b\cos(c+dx)}(3(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\left(6A+3(3A+4C)\cos^2(c+\right.}{24d\cos^{\frac{9}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/C
os[c + d*x]^(11/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4
+ Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 +
 8*B*Cos[c + d*x]*Sin[c + d*x]^2)))/(24*d*Cos[c + d*x]^(9/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {2031, 3042, 3500, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 2031$$

$$\frac{\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^5(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3500$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\int(4B+(3A+4C)\cos(c+dx))\sec^4(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\int\frac{4B+(3A+4C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3227$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\sec^3(c+dx)dx+4B\int\sec^4(c+dx)dx\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+4B\int\csc\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 4254$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4B\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 4255$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+\cdots}{\cdots}\right.}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+A\cdots\right.}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+\right.}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
 d*x]^(11/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A + 4*C
)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (4*B
*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d)/4))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.04

method	result
default	$-\frac{\left(9A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+12C\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-9A\ln(-\cot(dx+c)+\csc(dx+c)\right.}{}$
parts	$\frac{C\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+\sin(dx+c)\right)\sqrt{b\cos(dx+c)}}{2d\cos(dx+c)^{\frac{5}{2}}}-\frac{A\left(3\ln\right.}{}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\left(9A\,e^{7i(dx+c)}+12C\,e^{7i(dx+c)}+33A\,e^{5i(dx+c)}+12C\,e^{5i(dx+c)}-48B\,e^{4i(dx+c)}-33A\,e^{3i(dx+c)}-12C\,e^{3i(dx+c)}-64\right.}{12\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^4}$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2)
,x,method=_RETURNVERBOSE)
```

output
```
-1/24/d*(9*A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^4+12*C*ln(-cot(d*x+c)
+csc(d*x+c)-1)*cos(d*x+c)^4-9*A*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4-
12*C*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+(-9*cos(d*x+c)^2-6)*sin(d*x
+c)*A+sin(d*x+c)*cos(d*x+c)*(-16*cos(d*x+c)^2-8)*B-12*C*cos(d*x+c)^2*sin(d
*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$= \left[\frac{3(3A+4C)\sqrt{b}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2(16B\,c}{48\,d\,c}\right.$$
$$-\frac{3(3A+4C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\left(16B\cos(dx+c)^3+3(3A+4C\right.}{24\,d\cos(dx+c)^5}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(11/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqr
t(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x +
c))/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^
2 + 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*co
s(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 -
 (16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) +
6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. $2(165) = 330$.

Time = 0.39 (sec) , antiderivative size = 2611, normalized size of antiderivative = 13.53

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(11/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
 4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2
*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(si
n(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2
*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x
+ 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*
c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8
*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x
+ 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c
) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x
+ 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(
2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*...
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(3\,(3\,A+4\,C)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-3\,(3\,A+4\,C)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}\right.}{}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(11/2),x, algorithm="giac")
```

output
```
1/24*(3*(3*A + 4*C)*log(tan(1/2*d*x + 1/2*c) + 1) - 3*(3*A + 4*C)*log(tan(
1/2*d*x + 1/2*c) - 1) + 2*(15*A*tan(1/2*d*x + 1/2*c)^7 - 24*B*tan(1/2*d*x
+ 1/2*c)^7 + 12*C*tan(1/2*d*x + 1/2*c)^7 + 9*A*tan(1/2*d*x + 1/2*c)^5 + 40
*B*tan(1/2*d*x + 1/2*c)^5 - 12*C*tan(1/2*d*x + 1/2*c)^5 + 9*A*tan(1/2*d*x
+ 1/2*c)^3 - 40*B*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/2*d*x + 1/2*c)^3 + 1
5*A*tan(1/2*d*x + 1/2*c) + 24*B*tan(1/2*d*x + 1/2*c) + 12*C*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^8 - 4*tan(1/2*d*x + 1/2*c)^6 + 6*tan(1/2*d*x
 + 1/2*c)^4 - 4*tan(1/2*d*x + 1/2*c)^2 + 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\cos\left(c+d\,x\right)}^{11/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(11/2),x)
```

output
```
int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\left(-16\cos\left(dx+c\right)\sin\left(dx+c\right)^3 b+24\cos\left(dx+c\right)\sin\left(dx+c\right)b-9\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2)
,x)
```

output

```
(sqrt(b)*( - 16*cos(c + d*x)*sin(c + d*x)**3*b + 24*cos(c + d*x)*sin(c + d
*x)*b - 9*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 12*log(tan((c + d*
x)/2) - 1)*sin(c + d*x)**4*c + 18*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**
2*a + 24*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 9*log(tan((c + d*x)
/2) - 1)*a - 12*log(tan((c + d*x)/2) - 1)*c + 9*log(tan((c + d*x)/2) + 1)*
sin(c + d*x)**4*a + 12*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c - 18*lo
g(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - 24*log(tan((c + d*x)/2) + 1)*s
in(c + d*x)**2*c + 9*log(tan((c + d*x)/2) + 1)*a + 12*log(tan((c + d*x)/2)
 + 1)*c - 9*sin(c + d*x)**3*a - 12*sin(c + d*x)**3*c + 15*sin(c + d*x)*a +
 12*sin(c + d*x)*c))/(24*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.297 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+\right.$

Optimal result

Integrand size = 43, antiderivative size = 229

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b(5A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{3bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}+\frac{bB\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}+\frac{bC\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d}-\frac{b(5A+4C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{15d\sqrt{\cos(c+dx)}}$$

output
```
3/8*b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/5*b*(5*A+4*C)*(b*cos(d*x
+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+3/8*b*B*cos(d*x+c)^(1/2)*(b*cos(d
*x+c))^(1/2)*sin(d*x+c)/d+1/4*b*B*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*si
n(d*x+c)/d+1/5*b*C*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d-1/15
*b*(5*A+4*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)^3/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.48

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{(b\cos(c+dx))^{3/2}(180Bc+180Bdx+60(6A+5C)\sin(c+dx)+120B\sin(2(c+d}{480d\,c}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] +
C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x]
 + 120*B*Sin[2*(c + d*x)] + 40*A*Sin[3*(c + d*x)] + 50*C*Sin[3*(c + d*x)]
+ 15*B*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d*Cos[c + d*x]^(3/2)
)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.61, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.279, Rules used = {2031, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\int\cos^3(c+dx)(5A+4C+5B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\int\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(5A+4C+5B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left((5A+4C)\int\cos^3(c+dx)dx+5B\int\cos^4(c+dx)dx\right)+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left((5A+4C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^3dx+5B\int\sin\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\int\sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{(5A+4C)\int(1-\sin^2(c+dx))d(-\sin(c+dx))}{d}\right)+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\int\sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\left(\frac{3}{4}\int\cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)}{5}\right.}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\left(\frac{3}{4}\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin}{}\right.}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\right.}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\,\mathrm{si}}{}\right.}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[
c + d*x]^2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((C*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) + (-(((5*A
+ 4*C)*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d) + 5*B*((Cos[c + d*x]^3*Sin[c
 + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/5))/Sqr
t[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])    Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.48

method	result
default	$\frac{b\left(45B(dx+c)+\left(40\cos(dx+c)^2+80\right)\sin(dx+c)A+\sin(dx+c)\cos(dx+c)\left(30\cos(dx+c)^2+45\right)B+\left(24\cos(dx+c)^4+32\cos(dx+c)^2+\right.}{120d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\left(2+\cos(dx+c)^2\right)b\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}+\frac{B\left(2\cos(dx+c)^3\sin(dx+c)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)b\sqrt{b\cos(dx+c)}}{8d\sqrt{\cos(dx+c)}}+$
risch	$\frac{3b\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,e^{i(dx+c)}Bx}{4\left(e^{2i(dx+c)}+1\right)}-\frac{ib\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,e^{6i(dx+c)}C}{80\left(e^{2i(dx+c)}+1\right)d}-\frac{ib\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,e^{5i(dx+c)}}{32\left(e^{2i(dx+c)}+1\right)d}$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x,method=_RETURNVERBOSE)
```

output
```
1/120*b/d*(45*B*(d*x+c)+(40*cos(d*x+c)^2+80)*sin(d*x+c)*A+sin(d*x+c)*cos(d
*x+c)*(30*cos(d*x+c)^2+45)*B+(24*cos(d*x+c)^4+32*cos(d*x+c)^2+64)*sin(d*x+
c)*C)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.35

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \left[\frac{45\,B\sqrt{-b}b\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right.}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="fricas")
```

output
```
[1/240*(45*B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos
(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(24*C*b*cos(d
*x + c)^4 + 30*B*b*cos(d*x + c)^3 + 8*(5*A + 4*C)*b*cos(d*x + c)^2 + 45*B*
b*cos(d*x + c) + 16*(5*A + 4*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)), 1/120*(45*B*b^(3/2)*arctan(sqrt(b*cos(d*x
 + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (24*C*b*c
os(d*x + c)^4 + 30*B*b*cos(d*x + c)^3 + 8*(5*A + 4*C)*b*cos(d*x + c)^2 + 4
5*B*b*cos(d*x + c) + 16*(5*A + 4*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*
x+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.74

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{40\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)A\sqrt{b}+1}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="maxima")
```

output
```
1/480*(40*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))))*A*sqrt(b) + 15*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 2*(3*b*s
in(5*d*x + 5*c) + 25*b*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))
) + 150*b*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))*C*sqrt(b))
/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.41

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)\right.$$
$$\left.+C\cos^2(c+dx)\right)\,dx=\frac{1}{480}\left(180\,Bx+\frac{6\,C\sin\left(5\,dx+5\,c\right)}{d}+\frac{15\,B\sin\left(4\,dx+4\,c\right)}{d}+\frac{10\,(4\,A+5\,C)\sin\left(3\,d\right.}{d}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="giac")
```

output
```
1/480*(180*B*x + 6*C*sin(5*d*x + 5*c)/d + 15*B*sin(4*d*x + 4*c)/d + 10*(4*
A + 5*C)*sin(3*d*x + 3*c)/d + 120*B*sin(2*d*x + 2*c)/d + 60*(6*A + 5*C)*si
n(d*x + c)/d)*b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 42.80 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.62

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)\right.$$
$$\left.+C\cos^2(c+dx)\right)\,dx=\frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(120\,B\,\sin\left(c+d\,x\right)+400\,A\,\sin\left(2\,c+2\,d\,x\right)+40\right.}{}$$

input
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(
c + d*x)^2),x)
```

output
```
(b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(120*B*sin(c + d*x) + 400*A*s
in(2*c + 2*d*x) + 40*A*sin(4*c + 4*d*x) + 135*B*sin(3*c + 3*d*x) + 15*B*si
n(5*c + 5*d*x) + 350*C*sin(2*c + 2*d*x) + 56*C*sin(4*c + 4*d*x) + 6*C*sin(
6*c + 6*d*x) + 360*B*d*x*cos(c + d*x)))/(480*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.42

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b}\,b\left(-30\cos\left(dx+c\right)\sin\left(dx+c\right)^3b+75\cos\left(dx+c\right)\sin\left(dx+c\right)b+24\sin\left(dx+c\right.\right.}{}$$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x)
```

output
```
(sqrt(b)*b*( - 30*cos(c + d*x)*sin(c + d*x)**3*b + 75*cos(c + d*x)*sin(c +
 d*x)*b + 24*sin(c + d*x)**5*c - 40*sin(c + d*x)**3*a - 80*sin(c + d*x)**3
*c + 120*sin(c + d*x)*a + 120*sin(c + d*x)*c + 45*b*d*x))/(120*d)
```

3.298 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)\right.$

Optimal result

Integrand size = 43, antiderivative size = 189

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{b(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{b(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}+\frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}-\frac{bB\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

output
```
1/8*b*(4*A+3*C)*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b*B*(b*cos(d*x+c))
^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+1/8*b*(4*A+3*C)*cos(d*x+c)^(1/2)*(b*c
os(d*x+c))^(1/2)*sin(d*x+c)/d+1/4*b*C*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2
)*sin(d*x+c)/d-1/3*b*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)^3/d/cos(d*x+c)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.49

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{3/2}(48Ac+36cC+48Adx+36Cdx+72B\sin(c+dx)+24(A+C)}{96d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] +
C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[
c + d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*
(c + d*x)]))/(96*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.59, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2031, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\int \cos^2(c+dx)(4A+3C+4B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(4A+3C+4B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3227

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int \cos^2(c+dx)dx+4B\int \cos^3(c+dx)dx\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+4B\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3113

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4B\int(1-\sin^2(c+dx))d(-\sin(c+dx))}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 2009

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3115

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 24

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[
c + d*x]^2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A + 3
*C)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (4*B*(-Sin[c + d*x] + Sin[
c + d*x]^3/3))/d)/4))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.54

method	result
default	$\frac{b\left(12A(dx+c)+9C(dx+c)+12A\cos(dx+c)\sin(dx+c)+\left(8\cos(dx+c)^2+16\right)\sin(dx+c)B+\sin(dx+c)\cos(dx+c)\left(6\cos(dx+c)^2+9\right)C \cdots\right.}{24d\sqrt{\cos(dx+c)}}$
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)b\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}+\frac{B\sin(dx+c)\left(2+\cos(dx+c)^2\right)b\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}+\frac{C\left(2\cos(dx+c)^3\sin(dx+c)+\cdots\right.}{\cdots}$
risch	$\frac{b\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,e^{i(dx+c)}(8A+6C)x}{8\,e^{2i(dx+c)}+8}-\frac{ib\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,e^{5i(dx+c)}C}{32\left(e^{2i(dx+c)}+1\right)d}-\frac{ib\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,e^{4i\cdots}}{12\left(e^{2i(dx+c)}+1\right)d}$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x,method=_RETURNVERBOSE)
```

output
```
1/24*b/d*(12*A*(d*x+c)+9*C*(d*x+c)+12*A*cos(d*x+c)*sin(d*x+c)+(8*cos(d*x+c
)^2+16)*sin(d*x+c)*B+sin(d*x+c)*cos(d*x+c)*(6*cos(d*x+c)^2+9)*C)*(b*cos(d*
x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.51

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \left[\frac{3\,(4\,A+3\,C)\sqrt{-b}b\cos{(dx+c)}\log{\left(2\,b\cos{(dx+c)}^2-2\,\sqrt{b\cos{(dx+c)}}\sqrt{-b}\sqrt{\cos\right.}}}{}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="fricas")
```

output
```
[1/48*(3*(4*A + 3*C)*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sq
rt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*C*
b*cos(d*x + c)^3 + 8*B*b*cos(d*x + c)^2 + 3*(4*A + 3*C)*b*cos(d*x + c) + 1
6*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)), 1/24*(3*(4*A + 3*C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*b*cos(d*x + c)^3 + 8*B*b
*cos(d*x + c)^2 + 3*(4*A + 3*C)*b*cos(d*x + c) + 16*B*b)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*
x+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.67

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{24\,(2\,(dx+c)b+b\sin\left(2\,dx+2\,c\right))A\sqrt{b}+8\,\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\,\arctan\left(\mathrm{si}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="maxima")
```

output
```
1/96*(24*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*A*sqrt(b) + 8*(b*sin(3*d*x +
 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*sqrt(b
) + 3*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))))*C*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{1}{96}\left(12\,(4\,A+3\,C)x+\frac{3\,C\sin\left(4\,dx+4\,c\right)}{d}+\frac{8\,B\sin\left(3\,dx+3\,c\right)}{d}+\frac{24\,(A+C)\sin\left(\right.}{d}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="giac")
```

output
```
1/96*(12*(4*A + 3*C)*x + 3*C*sin(4*d*x + 4*c)/d + 8*B*sin(3*d*x + 3*c)/d +
 24*(A + C)*sin(2*d*x + 2*c)/d + 72*B*sin(d*x + c)/d)*b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 41.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \frac{b\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,(24A\sin(c+dx)+24C\sin(c+dx)+24A\sin$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(
c + d*x)^2),x)
```

output
```
(b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin
(c + d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c +
4*d*x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d
*x) + 72*C*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.46

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \frac{\sqrt{b}\,b\left(-6\cos(dx+c)\sin(dx+c)^3c+12\cos(dx+c)\sin(dx+c)a+15\cos(dx+c\right.}{24d}$$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x)
```

output
```
(sqrt(b)*b*( - 6*cos(c + d*x)*sin(c + d*x)**3*c + 12*cos(c + d*x)*sin(c +
d*x)*a + 15*cos(c + d*x)*sin(c + d*x)*c - 8*sin(c + d*x)**3*b + 24*sin(c +
 d*x)*b + 12*a*d*x + 9*c*d*x))/(24*d)
```

3.299 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 147

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b(3A+2C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{bC\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
1/2*b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/3*b*(3*A+2*C)*(b*cos(d*x
+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+1/2*b*B*cos(d*x+c)^(1/2)*(b*cos(d
*x+c))^(1/2)*sin(d*x+c)/d+1/3*b*C*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*si
n(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{b\sqrt{b\cos(c+dx)}(6Bc+6Bdx+3(4A+3}{1}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Sqrt[Cos[c + d*x]],x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*
B*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2031, 3042, 3502, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\int \cos(c+dx)(3A+2C+3B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(3A+2C+3B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3213}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(\frac{(3A+2C)\sin(c+dx)}{d}+\frac{3B\sin(c+dx)\cos(c+dx)}{2d}+\frac{3Bx}{2}\right)+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[
Cos[c + d*x]],x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) + ((3*B*x)/
2 + ((3*A + 2*C)*Sin[c + d*x])/d + (3*B*Cos[c + d*x]*Sin[c + d*x])/(2*d))/
3))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.53

method	result
default	$\frac{b\left(3B(dx+c)+6A\sin(dx+c)+3B\sin(dx+c)\cos(dx+c)+\left(2\cos(dx+c)^2+4\right)\sin(dx+c)C\right)\sqrt{b\cos(dx+c)}}{6d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)b\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}+\frac{B(\cos(dx+c)\sin(dx+c)+dx+c)b\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}+\frac{C\sin(dx+c)\left(2+\cos(dx+c)^2\right)b\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}$
risch	$\frac{bBx\sqrt{b\cos(dx+c)}}{2\sqrt{\cos(dx+c)}}+\frac{b\sqrt{b\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{b\cos(dx+c)}\,C\sin(3dx+3c)}{12\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{b\cos(dx+c)}\,B\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
1/6*b/d*(3*B*(d*x+c)+6*A*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)+(2*cos(d*x+c
)^2+4)*sin(d*x+c)*C)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.69

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{3\,B\sqrt{-b}b\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+\right.\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d
*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b*cos(d*x
+ c)^2 + 3*B*b*cos(d*x + c) + 2*(3*A + 2*C)*b)*sqrt(b*cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*B*b^(3/2)*arctan(sqrt(
b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) +
(2*C*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*(3*A + 2*C)*b)*sqrt(b*cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{12\,Ab^{\frac{3}{2}}\sin\left(dx+c\right)+3\left(2\left(dx+c\right)b+b\sin\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(1/2),x, algorithm="maxima")
```

output
```
1/12*(12*A*b^(3/2)*sin(d*x + c) + 3*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*B
*sqrt(b) + (b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))))*C*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{1}{12}\left(6\,Bx+\frac{C\sin\left(3\,dx+3\,c\right)}{d}+\frac{3\,B\sin\left(\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(1/2),x, algorithm="giac")
```

output
```
1/12*(6*B*x + C*sin(3*d*x + 3*c)/d + 3*B*sin(2*d*x + 2*c)/d + 3*(4*A + 3*C
)*sin(d*x + c)/d)*b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.48

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(c+d\,x\right)+9\right.}{}$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(1/2),x)
```

output
```
(b*(b*cos(c + d*x))^(1/2)*(12*A*sin(c + d*x) + 9*C*sin(c + d*x) + 3*B*sin(
2*c + 2*d*x) + C*sin(3*c + 3*d*x) + 6*B*d*x))/(12*d*cos(c + d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.39

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b}\,b\left(3\cos\left(dx+c\right)\sin\left(dx+c\right)b-2\sin\left(\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),
x)
```

output
```
(sqrt(b)*b*(3*cos(c + d*x)*sin(c + d*x)*b - 2*sin(c + d*x)**3*c + 6*sin(c
+ d*x)*a + 6*sin(c + d*x)*c + 3*b*d*x))/(6*d)
```

3.300 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 127

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
A*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b*C*x*(b*cos(d*x+c))^(1/2)
/cos(d*x+c)^(1/2)+b*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+1
/2*b*C*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.48

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}(2(2A+C)(c+dx)+4}{4d\cos^{\frac{3}{2}}(c+}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(3/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[
2*(c + d*x)]))/(4*d*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.51, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(Ax+\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(3/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(A*x + (C*x)/2 + (B*Sin[c + d*x])/d + (C*Cos[c + d
*x]*Sin[c + d*x])/(2*d)))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{b(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}$	64
risch	$\frac{b\sqrt{b\cos(dx+c)}\,(4A+2C)x}{4\sqrt{\cos(dx+c)}}+\frac{bB\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{d\sqrt{\cos(dx+c)}}+\frac{b\sqrt{b\cos(dx+c)}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	95
parts	$\frac{A(dx+c)b\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}+\frac{bB\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{d\sqrt{\cos(dx+c)}}+\frac{C(\cos(dx+c)\sin(dx+c)+dx+c)b\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}$	104

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),
x,method=_RETURNVERBOSE)
```

output
```
1/2*b/d*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+C*(d*x+c))*(b*
cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.71

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{(2\,A+C)\sqrt{-b}b\cos\left(dx+c\right)\log\left(2\,b\cos\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(3/2),x, algorithm="fricas")
```

output
```
[1/4*((2*A + C)*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*
cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(C*b*cos(d
*x + c) + 2*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)), 1/2*((2*A + C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x
+ c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*b*cos(d*x + c) + 2*B*
b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.53

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{8\,Ab^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+4\,Bb^{\frac{3}{2}}\sin\left(dx\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(3/2),x, algorithm="maxima")
```

output
```
1/4*(8*A*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*B*b^(3/2)*sin
(d*x + c) + (2*(d*x + c)*b + b*sin(2*d*x + 2*c))*C*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left((dx+c)(2\,A+C)+\frac{2\left(2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\right.}{}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(3/2),x, algorithm="giac")
```

output
```
1/2*((d*x + c)*(2*A + C) + 2*(2*B*tan(1/2*d*x + 1/2*c)^3 - C*tan(1/2*d*x +
 1/2*c)^3 + 2*B*tan(1/2*d*x + 1/2*c) + C*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^2)*b^(3/2)/d
```

Mupad [B] (verification not implemented)

Time = 39.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.43

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(4\,B\,\sin\left(c+d\,x\right)+C\right.}{4\,d\,\sqrt{\cos\left(c\right.}}$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(3/2),x)
```

output
```
(b*(b*cos(c + d*x))^(1/2)*(4*B*sin(c + d*x) + C*sin(2*c + 2*d*x) + 4*A*d*x
 + 2*C*d*x))/(4*d*cos(c + d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.32

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b(\cos\left(dx+c\right)\sin\left(dx+c\right)c+2\sin\left(dx\right.}{2d}$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),
x)
```

output
```
(sqrt(b)*b*(cos(c + d*x)*sin(c + d*x)*c + 2*sin(c + d*x)*b + 2*a*d*x + c*d
*x))/(2*d)
```

3.301 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 96

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bC\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*arctanh(sin(d*x+c))*(b*cos
(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+b*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/co
s(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{3/2}\left(Bc+Bdx-A\log(\mathrm{co}\right.}{}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(5/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d
*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2031, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\int (A+B\cos(c+dx))\sec(c+dx)dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int \sec(c+dx)dx+Bx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(A\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+Bx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+Bx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(5/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(B*x + (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*
x])/d))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

method	result
default	$-\frac{b(2A\ \operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))-B(dx+c)-C\sin(dx+c))\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}$
parts	$-\frac{2A\ \operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))\sqrt{b\cos(dx+c)}\,b}{d\sqrt{\cos(dx+c)}}+\frac{B(dx+c)b\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}+\frac{bC\sqrt{b\cos(dx+c)}\ \sin(dx+c)}{d\sqrt{\cos(dx+c)}}$
risch	$\frac{bBx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}}-\frac{ib\sqrt{b\cos(dx+c)}\,C\,e^{i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}+\frac{ib\sqrt{b\cos(dx+c)}\,C\,e^{-i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{b\cos(dx+c)}\,A\ln(e^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}-\frac{b\sqrt{b}}{}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),
x,method=_RETURNVERBOSE)
```

output
```
-b/d*(2*A*arctanh(-csc(d*x+c)+cot(d*x+c))-B*(d*x+c)-C*sin(d*x+c))*(b*cos(d
*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.21

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\left[-\frac{2\,A\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+}{b\sqrt{\cos(dx+c)}}\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(5/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c) - B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos
(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x +
 c) - b) - 2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)), 1/2*(2*B*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(
sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*b^(3/2)*cos(d*x + c)*log(-(b
*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*
x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*b*sq
rt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{4\,Bb^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+2\,Cb^{\frac{3}{2}}\sin\left(d\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(5/2),x, algorithm="maxima")
```

output
```
1/2*(4*B*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 2*C*b^(3/2)*sin
(d*x + c) + (b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
 b*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(A\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-A\log\left(\mathrm{ta}\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(5/2),x, algorithm="giac")
```

output
```
(A*log(tan(1/2*d*x + 1/2*c) + 1) - A*log(tan(1/2*d*x + 1/2*c) - 1) + I*B*l
og(I*tan(1/2*d*x + 1/2*c) - 1) - I*B*log(-I*tan(1/2*d*x + 1/2*c) - 1) + 2*
C*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))*b^(3/2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+\right.}{\cos\left(c+d\,x\right)^{5/2}}$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(5/2),x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.51

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\right.\right.}{d}$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),
x)
```

output
```
(sqrt(b)*b*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a +
 sin(c + d*x)*c + b*d*x))/d
```

3.302 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 96

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bB\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
b*C*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b*B*arctanh(sin(d*x+c))*(b*cos
(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+A*b*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/co
s(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{3/2}\left(Cdx\cos(c+dx)+B\,\mathrm{c}\right.}{d\,\mathrm{co}}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(7/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(C*d*x*Cos[c + d*x] + B*ArcCoth[Sin[c + d*x]]*Cos[
c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2031, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\int (B+C\cos(c+dx))\sec(c+dx)dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\int \frac{B+C\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\int \sec(c+dx)dx+\frac{A\tan(c+dx)}{d}+Cx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(B\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)}{d}+Cx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+\frac{B\text{arctanh}(\sin(c+dx))}{d}+Cx\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(7/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(C*x + (B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*
x])/d))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.74

method	result
default	$\frac{b(-2B\ \operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))\cos(dx+c)+C(dx+c)\cos(dx+c)+A\sin(dx+c))\sqrt{b\cos(dx+c)}}{d\cos(dx+c)^{\frac{3}{2}}}$
parts	$\frac{Ab\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}}-\frac{2B\ \operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))\sqrt{b\cos(dx+c)}\,b}{d\sqrt{\cos(dx+c)}}+\frac{C(dx+c)b\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}$
risch	$\frac{bCx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}}+\frac{2ib\sqrt{b\cos(dx+c)}\,A}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}+\frac{b\sqrt{b\cos(dx+c)}\,B\ln\left(e^{i(dx+c)}+i\right)}{\sqrt{\cos(dx+c)}\,d}-\frac{b\sqrt{b\cos(dx+c)}\,B\ln\left(e^{i(dx+c)}-i\right)}{\sqrt{\cos(dx+c)}\,d}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),
x,method=_RETURNVERBOSE)
```

output
```
b/d*(-2*B*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)+C*(d*x+c)*cos(d*x+c)+
A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.29

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left[-\frac{2B\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx-}{b\sqrt{\cos(dx+c)}}\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(7/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - C*sqrt(-b)*b*cos(d*x + c)^2*log(2*b
*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d
*x + c) - b) - 2*sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))
/(d*cos(d*x + c)^2), 1/2*(2*C*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x
+ c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*b^(3/2)*cos(d*x + c)
^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x +
c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x +
c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.53

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{4\,Cb^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\left(b\log\left(\cos\left(\right.\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(7/2),x, algorithm="maxima")
```

output
```
1/2*(4*C*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (b*log(cos(d*x
+ c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin
(d*x + c)^2 - 2*sin(d*x + c) + 1))*B*sqrt(b) + 4*A*b^(3/2)*sin(2*d*x + 2*c
)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(B\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-B\log\left(\text{ta}\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(7/2),x, algorithm="giac")
```

output
```
(B*log(tan(1/2*d*x + 1/2*c) + 1) - B*log(tan(1/2*d*x + 1/2*c) - 1) + I*C*l
og(I*tan(1/2*d*x + 1/2*c) - 1) - I*C*log(-I*tan(1/2*d*x + 1/2*c) - 1) - 2*
A*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))*b^(3/2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos(c+d\,x))^{3/2}\left(C\cos(c+d\,x)^2+\right.}{\cos(c+d\,x)^{7/2}}$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(7/2),x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b\left(-\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.\right.$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),
x)
```

output
```
(sqrt(b)*b*( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + cos(c + d*x)*log
(tan((c + d*x)/2) + 1)*b + cos(c + d*x)*c*d*x + sin(c + d*x)*a))/(cos(c +
d*x)*d)
```

3.303 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 114

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{b(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c}}{2d\sqrt{\cos(c+dx)}}$$
$$+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
1/2*b*(A+2*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+
1/2*A*b*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+b*B*(b*cos(d*x+
c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{3/2}\left(2C\coth^{-1}(\sin(c+dx)\right.}{}$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(9/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(2*C*ArcCoth[Sin[c + d*x]]*Cos[c + d*x]^2 + A*ArcT
anh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(
2*d*Cos[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {2031, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\int (2B+(A+2C)\cos(c+dx))\sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\int \frac{2B+(A+2C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int \sec(c+dx)dx+2B\int \sec^2(c+dx)dx\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+2B\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2B\int 1d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2B\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left(\frac{(A+2C)\text{arctanh}(\sin(c+dx))}{d}+\frac{2B\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(9/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((A + 2*C)
*ArcTanh[Sin[c + d*x]])/d + (2*B*Tan[c + d*x])/d)/2))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15

method	result
default	$\frac{b\left(-A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^{2}+A\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^{2}-4C\ \mathrm{arctanh}(-\csc(dx+c)+\cot(\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^{2}-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^{2}+\sin(dx+c)\right)\sqrt{b\cos(dx+c)}\,b}{2d\cos(dx+c)^{\frac{5}{2}}}+\frac{bB\sqrt{}}{}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\left(A\,e^{3i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}-2B\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^{2}}-\frac{b\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{b\cos(dx+c)}\,(}{2\sqrt{\mathrm{co}}}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),
x,method=_RETURNVERBOSE)
```

output
```
1/2*b/d*(-A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^2+A*ln(-cot(d*x+c)+csc
(d*x+c)+1)*cos(d*x+c)^2-4*C*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)^2+2
*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/
2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.11

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left[\frac{(A+2\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+}{}\right.}{}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{3}-\left(2\,Bb\cos\left(dx+c\right)+Ab\right)\sqrt{b\cos\left(dx+c\right)}}{2\,d\cos\left(dx+c\right)^{3}}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(9/2),x, algorithm="fricas")
```

output
```
[1/4*((A + 2*C)*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*c
os(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/c
os(d*x + c)^3) + 2*(2*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b*a
rctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*c
os(d*x + c)^3 - (2*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(98) = 196$.

Time = 0.35 (sec) , antiderivative size = 813, normalized size of antiderivative = 7.13

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(9/2),x, algorithm="maxima")
```

output
```
1/4*(2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*C*sqrt(b) + 8*B*b
^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1) - (4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b
*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
(b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*
b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(
2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*si
n(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x +
 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x +
2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(
2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(...
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.11

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left((A+2\,C)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(9/2),x, algorithm="giac")
```

output
```
1/2*((A + 2*C)*log(tan(1/2*d*x + 1/2*c) + 1) - (A + 2*C)*log(tan(1/2*d*x +
 1/2*c) - 1) + 2*(A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 +
A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4
 - 2*tan(1/2*d*x + 1/2*c)^2 + 1))*b^(3/2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+\right.}{\cos\left(c+d\,x\right)^{9/2}}$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(9/2),x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.71

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{\sqrt{b}\,b\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)b-\log\left(\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),
x)
```

output
```
(sqrt(b)*b*( - 2*cos(c + d*x)*sin(c + d*x)*b - log(tan((c + d*x)/2) - 1)*s
in(c + d*x)**2*a - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c + log(tan
((c + d*x)/2) - 1)*a + 2*log(tan((c + d*x)/2) - 1)*c + log(tan((c + d*x)/2
) + 1)*sin(c + d*x)**2*a + 2*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*c -
 log(tan((c + d*x)/2) + 1)*a - 2*log(tan((c + d*x)/2) + 1)*c - sin(c + d*x
)*a))/(2*d*(sin(c + d*x)**2 - 1))
```

3.304 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 156

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{bB\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{b(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
1/2*b*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/3*A*
b*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+1/2*b*B*(b*cos(d*x+c)
)^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+1/3*b*(2*A+3*C)*(b*cos(d*x+c))^(1/2)
*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.56

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\cos(c+dx)}(3B\operatorname{arctanh}(\sin(c+dx))$$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(11/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A +
 3*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*
d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {2031, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\int(3B+(2A+3C)\cos(c+dx))\sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\int\frac{3B+(2A+3C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left((2A+3C)\int\sec^2(c+dx)dx+3B\int\sec^3(c+dx)dx\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left((2A+3C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(3B\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(3B\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(\frac{(2A+3C)\tan(c+dx)}{d}+3B\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(11/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (((2*A +
3*C)*Tan[c + d*x])/d + 3*B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Ta
n[c + d*x])/(2*d)))/3))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86

method	result
default	$\frac{b\left(-3B\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^3+3B\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^3+\left(4\cos(dx+c)^2+2\right)\sin(dx+c\right.}{6d\cos(dx+c)^{\frac{7}{2}}}$
parts	$\frac{A\sin(dx+c)\left(2\cos(dx+c)^2+1\right)\sqrt{b\cos(dx+c)}\,b}{3d\cos(dx+c)^{\frac{7}{2}}}+\frac{B\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-1\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\left(3B\,e^{5i(dx+c)}-6C\,e^{4i(dx+c)}-12A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-3B\,e^{i(dx+c)}-4A-6C\right)}{3\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^3}+\frac{b\sqrt{b\cos(dx+c)}\,B\ln(e^{i}}{2\sqrt{\cos(dx+c)}}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2)
,x,method=_RETURNVERBOSE)
```

output
```
1/6*b/d*(-3*B*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^3+3*B*ln(-cot(d*x+c)
+csc(d*x+c)+1)*cos(d*x+c)^3+(4*cos(d*x+c)^2+2)*sin(d*x+c)*A+6*C*cos(d*x+c)
^2*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
7/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.74

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \left[\frac{3\,Bb^{\frac{3}{2}}\cos\left(dx+c\right)^4\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{\cdots}}{\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{3\,B\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^4-\left(2\left(2\,A+3\,C\right)b\cos\left(dx+c\right)^2+3\,Bb\cos\left(dx+\cdots\right.\right.}{6\,d\cos\left(dx+c\right)^4}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(11/2),x, algorithm="fricas")
```

output
```
[1/12*(3*B*b^(3/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*
x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*
x + c)^3) + 2*(2*(2*A + 3*C)*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*A*b
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
, -1/6*(3*B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(
b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*b*cos(d*x + c)^2 +
3*B*b*cos(d*x + c) + 2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(132) = 264$.

Time = 0.34 (sec) , antiderivative size = 1044, normalized size of antiderivative = 6.69

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(11/2),x, algorithm="maxima")
```

output

```
1/12*(24*C*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1) - 16*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) +
 9*b*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*
d*x + 6*c) - 3*(3*b*cos(2*d*x + 2*c) + b)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(
3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x
+ 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c
)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6
*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) - 3*
(4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)
^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*c
os(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 +
4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*co
s(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(co...
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.21

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(3\,B\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-3\,B\log\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(11/2),x, algorithm="giac")
```

output
```
1/6*(3*B*log(tan(1/2*d*x + 1/2*c) + 1) - 3*B*log(tan(1/2*d*x + 1/2*c) - 1)
 - 2*(6*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 6*C*tan(1/
2*d*x + 1/2*c)^5 - 4*A*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/2*d*x + 1/2*c)^
3 + 6*A*tan(1/2*d*x + 1/2*c) + 3*B*tan(1/2*d*x + 1/2*c) + 6*C*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^6 - 3*tan(1/2*d*x + 1/2*c)^4 + 3*tan(1/2*d
*x + 1/2*c)^2 - 1))*b^(3/2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(C\,\cos\left(c+d\,x\right)^2+\right.}{\cos\left(c+d\,x\right)^{11/2}}$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(11/2),x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.18

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b\left(-3\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.\right.}{}$$

input

```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2)
,x)
```

output

```
(sqrt(b)*b*( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b
+ 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 3*cos(c + d*x)*log(tan((c +
 d*x)/2) + 1)*sin(c + d*x)**2*b - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)
*b - 3*cos(c + d*x)*sin(c + d*x)*b + 4*sin(c + d*x)**3*a + 6*sin(c + d*x)*
*3*c - 6*sin(c + d*x)*a - 6*sin(c + d*x)*c))/(6*cos(c + d*x)*d*(sin(c + d*
x)**2 - 1))
```

3.305 $\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 198

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}$$
$$+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{b(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$
$$+\frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

output

```
1/8*b*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2
)+1/4*A*b*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)+1/8*b*(3*A+4*
C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+b*B*(b*cos(d*x+c))^(
1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+1/3*b*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)
^3/d/cos(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.56

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\cos(c+dx)}(3(3A+4C)\text{arctanh}(\sin($$

input
```
Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(13/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^
4 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3
 + 8*B*Cos[c + d*x]*Sin[c + d*x]^2)))/(24*d*Cos[c + d*x]^(9/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.61, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.256, Rules used = {2031, 3042, 3500, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^5(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\int(4B+(3A+4C)\cos(c+dx))\sec^4(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\int\frac{4B+(3A+4C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3227

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\sec^3(c+dx)dx+4B\int\sec^4(c+dx)dx\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+4B\int\csc\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 4254

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4B\int(\tan^2(c+dx)+1)d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 2009

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 4255

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+A\tan(c\cdots\right.}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\cdots\right.}{\sqrt{\cos(c+dx)}}$$

$\downarrow$ 4257

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c\cdot}{}\right.}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(13/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A + 4
*C)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (4
*B*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d)/4))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

method	result
default	$-\frac{b\left(9A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+12C\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-9A\ln(-\cot(dx+c)+\csc(dx\right.}{}$
parts	$-\frac{A\left(3\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-3\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^4-3\cos(dx+c)^2\sin(dx+c)-2\sin(\right.}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\,(9A\,e^{7i(dx+c)}+12C\,e^{7i(dx+c)}+33A\,e^{5i(dx+c)}+12C\,e^{5i(dx+c)}-48B\,e^{4i(dx+c)}-33A\,e^{3i(dx+c)}-12C\,e^{3i(dx+c)}-6}{12\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^4}$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2)
,x,method=_RETURNVERBOSE)
```

output
```
-1/24*b/d*(9*A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^4+12*C*ln(-cot(d*x+
c)+csc(d*x+c)-1)*cos(d*x+c)^4-9*A*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^
4-12*C*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+(-9*cos(d*x+c)^2-6)*sin(d
*x+c)*A+sin(d*x+c)*cos(d*x+c)*(-16*cos(d*x+c)^2-8)*B-12*C*cos(d*x+c)^2*sin
(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.56

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \left[\frac{3\,(3\,A+4\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos}{}\right.}{}\right.$$

$$-\frac{3\,(3\,A+4\,C)\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left(16\,Bb\cos\left(dx+c\right)^3+3\,(3\,A+4\,C)b\right.}{24\,d\cos\left(dx+c\right)^5}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(13/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(3*A + 4*C)*b^(3/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqr
t(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x +
c))/cos(d*x + c)^3) + 2*(16*B*b*cos(d*x + c)^3 + 3*(3*A + 4*C)*b*cos(d*x +
 c)^2 + 8*B*b*cos(d*x + c) + 6*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*b*arctan
(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*
x + c)^5 - (16*B*b*cos(d*x + c)^3 + 3*(3*A + 4*C)*b*cos(d*x + c)^2 + 8*B*b
*cos(d*x + c) + 6*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(13/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2732 vs. $2(170) = 340$.

Time = 0.41 (sec) , antiderivative size = 2732, normalized size of antiderivative = 13.80

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(13/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x +
4*c) + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x +
4*c) + 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x +
4*c) + 4*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x +
4*c) + 4*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d
*x + 4*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*
d*x + 6*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x
+ 4*c) + 4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4
*c) + 4*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c
) + b)*cos(4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) +
 3*b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*s
in(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) + 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 +...
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.36

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left(3\left(3A+4C\right)\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(13/2),x, algorithm="giac")
```

output
```
1/24*(3*(3*A + 4*C)*log(tan(1/2*d*x + 1/2*c) + 1) - 3*(3*A + 4*C)*log(tan(
1/2*d*x + 1/2*c) - 1) + 2*(15*A*tan(1/2*d*x + 1/2*c)^7 - 24*B*tan(1/2*d*x
+ 1/2*c)^7 + 12*C*tan(1/2*d*x + 1/2*c)^7 + 9*A*tan(1/2*d*x + 1/2*c)^5 + 40
*B*tan(1/2*d*x + 1/2*c)^5 - 12*C*tan(1/2*d*x + 1/2*c)^5 + 9*A*tan(1/2*d*x
+ 1/2*c)^3 - 40*B*tan(1/2*d*x + 1/2*c)^3 - 12*C*tan(1/2*d*x + 1/2*c)^3 + 1
5*A*tan(1/2*d*x + 1/2*c) + 24*B*tan(1/2*d*x + 1/2*c) + 12*C*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^8 - 4*tan(1/2*d*x + 1/2*c)^6 + 6*tan(1/2*d*x
 + 1/2*c)^4 - 4*tan(1/2*d*x + 1/2*c)^2 + 1))*b^(3/2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(C\,\cos\left(c+d\,x\right)^2+\right.}{\cos\left(c+d\,x\right)^{13/2}}$$

input
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(13/2),x)
```

output
```
int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(13/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.75

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b\big(-16\cos(dx+c)\sin(dx+c)^3 b+24$$

input
```
int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2)
,x)
```

output
```
(sqrt(b)*b*( - 16*cos(c + d*x)*sin(c + d*x)**3*b + 24*cos(c + d*x)*sin(c +
 d*x)*b - 9*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 12*log(tan((c +
d*x)/2) - 1)*sin(c + d*x)**4*c + 18*log(tan((c + d*x)/2) - 1)*sin(c + d*x)
**2*a + 24*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 9*log(tan((c + d*
x)/2) - 1)*a - 12*log(tan((c + d*x)/2) - 1)*c + 9*log(tan((c + d*x)/2) + 1
)*sin(c + d*x)**4*a + 12*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c - 18*
log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - 24*log(tan((c + d*x)/2) + 1)
*sin(c + d*x)**2*c + 9*log(tan((c + d*x)/2) + 1)*a + 12*log(tan((c + d*x)/
2) + 1)*c - 9*sin(c + d*x)**3*a - 12*sin(c + d*x)**3*c + 15*sin(c + d*x)*a
 + 12*sin(c + d*x)*c))/(24*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.306 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)\right.$

Optimal result

Integrand size = 43, antiderivative size = 241

$$\begin{aligned}&\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)\right.\\&\left.+C\cos^2(c+dx)\right)\,dx=\frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}\\&+\frac{b^2(5A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}\\&+\frac{3b^2B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}\\&+\frac{b^2B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}\\&+\frac{b^2C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{5d}\\&-\frac{b^2(5A+4C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{15d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
3/8*b^2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/5*b^2*(5*A+4*C)*(b*cos
(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+3/8*b^2*B*cos(d*x+c)^(1/2)*(b
*cos(d*x+c))^(1/2)*sin(d*x+c)/d+1/4*b^2*B*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^
(1/2)*sin(d*x+c)/d+1/5*b^2*C*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2)*sin(d*x
+c)/d-1/15*b^2*(5*A+4*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)^3/d/cos(d*x+c)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.45

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{5/2}(180Bc+180Bdx+60(6A+5C)\sin(c+dx)+120B\sin(2(c+d$$
$$480d\,c$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] +
C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x]
 + 120*B*Sin[2*(c + d*x)] + 40*A*Sin[3*(c + d*x)] + 50*C*Sin[3*(c + d*x)]
+ 15*B*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d*Cos[c + d*x]^(5/2)
)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.59, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.279$, Rules used = {2031, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\int \cos^3(c+dx)(5A+4C+5B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(5A+4C+5B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left((5A+4C)\int \cos^3(c+dx)dx+5B\int \cos^4(c+dx)dx\right)+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left((5A+4C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+5B\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3113}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{(5A+4C)\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{d}\right)+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^4(c+dx)}{5d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\left(\frac{3}{4}\int\cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx}{}\right.}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\left(\frac{3}{4}\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin}{}\right.}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3115

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\right.}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 24

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{5}\left(5B\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{(5A+4C)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C}{}\right.}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[
c + d*x]^2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((C*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) + (-(((5*
A + 4*C)*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d) + 5*B*((Cos[c + d*x]^3*Sin
[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/5))/S
qrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.46

method	result
default	$\frac{b^2\left(45B(dx+c)+\left(40\cos(dx+c)^2+80\right)\sin(dx+c)A+\sin(dx+c)\cos(dx+c)\left(30\cos(dx+c)^2+45\right)B+\left(24\cos(dx+c)^4+32\cos(dx+c)^2\right.}{120d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\left(2+\cos(dx+c)^2\right)b^2\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}+\frac{B\left(2\cos(dx+c)^3\sin(dx+c)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)b^2\sqrt{b\cos(dx+c)}}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{3b^2\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{i(dx+c)}Bx}{4(\mathrm{e}^{2i(dx+c)}+1)}-\frac{ib^2\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{6i(dx+c)}C}{80(\mathrm{e}^{2i(dx+c)}+1)d}-\frac{ib^2\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\mathrm{e}^{5i(dx}}{32(\mathrm{e}^{2i(dx+c)}+1)d}$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x,method=_RETURNVERBOSE)
```

output
```
1/120*b^2/d*(45*B*(d*x+c)+(40*cos(d*x+c)^2+80)*sin(d*x+c)*A+sin(d*x+c)*cos
(d*x+c)*(30*cos(d*x+c)^2+45)*B+(24*cos(d*x+c)^4+32*cos(d*x+c)^2+64)*sin(d*
x+c)*C)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.37

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\left[\frac{45\,B\sqrt{-b}b^2\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+\right.}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="fricas")
```

output
```
[1/240*(45*B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*c
os(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(24*C*b^2*c
os(d*x + c)^4 + 30*B*b^2*cos(d*x + c)^3 + 8*(5*A + 4*C)*b^2*cos(d*x + c)^2
 + 45*B*b^2*cos(d*x + c) + 16*(5*A + 4*C)*b^2)*sqrt(b*cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/120*(45*B*b^(5/2)*arctan(sq
rt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)
 + (24*C*b^2*cos(d*x + c)^4 + 30*B*b^2*cos(d*x + c)^3 + 8*(5*A + 4*C)*b^2*
cos(d*x + c)^2 + 45*B*b^2*cos(d*x + c) + 16*(5*A + 4*C)*b^2)*sqrt(b*cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*
x+c)**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.77

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)\right.$$
$$\left.+C\cos^2(c+dx)\right)\,dx=\frac{40\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)A\sqrt{b}+}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="maxima")
```

output
```
1/480*(40*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))))*A*sqrt(b) + 15*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c
) + 8*b^2*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b)
+ 2*(3*b^2*sin(5*d*x + 5*c) + 25*b^2*sin(3/5*arctan2(sin(5*d*x + 5*c), cos
(5*d*x + 5*c))) + 150*b^2*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c))))*C*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.49

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{1}{480}\left(180\,Bb^2x+\frac{6\,Cb^2\sin\left(5\,dx+5\,c\right)}{d}+\frac{15\,Bb^2\sin\left(4\,dx+4\,c\right)}{d}+\frac{120\,Bb^2\sin\left(2\,dx\right.}{d}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+
c)^2),x, algorithm="giac")
```

output
```
1/480*(180*B*b^2*x + 6*C*b^2*sin(5*d*x + 5*c)/d + 15*B*b^2*sin(4*d*x + 4*c
)/d + 120*B*b^2*sin(2*d*x + 2*c)/d + 10*(4*A*b^2 + 5*C*b^2)*sin(3*d*x + 3*
c)/d + 60*(6*A*b^2 + 5*C*b^2)*sin(d*x + c)/d)*sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 42.65 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.60

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(120\,B\,\sin\left(c+d\,x\right)+400\,A\,\sin\left(2\,c+2\,d\,x\right)+40\right.}{}$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(
c + d*x)^2),x)
```

output
```
(b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(120*B*sin(c + d*x) + 400*A
*sin(2*c + 2*d*x) + 40*A*sin(4*c + 4*d*x) + 135*B*sin(3*c + 3*d*x) + 15*B*
sin(5*c + 5*d*x) + 350*C*sin(2*c + 2*d*x) + 56*C*sin(4*c + 4*d*x) + 6*C*si
n(6*c + 6*d*x) + 360*B*d*x*cos(c + d*x)))/(480*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.41

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b}\,b^2\left(-30\cos\left(dx+c\right)\sin\left(dx+c\right)^3b+75\cos\left(dx+c\right)\sin\left(dx+c\right)b+24\sin\left(dx+\right.\right.}{}$$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x)
```

output
```
(sqrt(b)*b**2*( - 30*cos(c + d*x)*sin(c + d*x)**3*b + 75*cos(c + d*x)*sin(
c + d*x)*b + 24*sin(c + d*x)**5*c - 40*sin(c + d*x)**3*a - 80*sin(c + d*x)
**3*c + 120*sin(c + d*x)*a + 120*sin(c + d*x)*c + 45*b*d*x))/(120*d)
```

3.307 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 199

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx &= \frac{b^2(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}\\ &+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{b^2(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}\\ &+\frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}-\frac{b^2B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
1/8*b^2*(4*A+3*C)*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b^2*B*(b*cos(d*x
+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+1/8*b^2*(4*A+3*C)*cos(d*x+c)^(1/2
)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d+1/4*b^2*C*cos(d*x+c)^(5/2)*(b*cos(d*x+
c))^(1/2)*sin(d*x+c)/d-1/3*b^2*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)^3/d/cos(d
*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.46

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{(b\cos(c+dx))^{5/2}(48Ac+36cC+48Adx$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Sqrt[Cos[c + d*x]],x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[
c + d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*
(c + d*x)]))/(96*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.57, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2031, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\int\cos^2(c+dx)(4A+3C+4B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\int\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(4A+3C+4B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int\cos^2(c+dx)dx+4B\int\cos^3(c+dx)dx\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+4B\int\sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4B\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[
Cos[c + d*x]],x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A +
 3*C)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (4*B*(-Sin[c + d*x] + Si
n[c + d*x]^3/3))/d)/4))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

method	result
default	$\frac{b^2(12A(dx+c)+9C(dx+c)+12A\cos(dx+c)\sin(dx+c)+(8\cos(dx+c)^2+16)\sin(dx+c)B+\sin(dx+c)\cos(dx+c)(6\cos(dx+c)^2+9)}{24d\sqrt{\cos(dx+c)}}$
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)b^2\sqrt{b\cos(dx+c)}}{2d\sqrt{\cos(dx+c)}}+\frac{B\sin(dx+c)(2+\cos(dx+c)^2)b^2\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}+\frac{C(2\cos(dx+c)^3\sin(dx+c)}{}$
risch	$\frac{b^2\sqrt{b\cos(dx+c)}\,(8A+6C)x}{16\sqrt{\cos(dx+c)}}+\frac{3b^2B\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{4d\sqrt{\cos(dx+c)}}+\frac{b^2\sqrt{b\cos(dx+c)}\,C\sin(4dx+4c)}{32\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{b\cos(dx+c)}\,B\sin(3dx}{12\sqrt{\cos(dx+c)}\,d}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
1/24*b^2/d*(12*A*(d*x+c)+9*C*(d*x+c)+12*A*cos(d*x+c)*sin(d*x+c)+(8*cos(d*x
+c)^2+16)*sin(d*x+c)*B+sin(d*x+c)*cos(d*x+c)*(6*cos(d*x+c)^2+9)*C)*(b*cos(
d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.52

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{3\,(4\,A+3\,C)\sqrt{-b}b^2\cos\left(dx+c\right)\log\left(2\,b\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(4*A + 3*C)*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*
sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*
C*b^2*cos(d*x + c)^3 + 8*B*b^2*cos(d*x + c)^2 + 3*(4*A + 3*C)*b^2*cos(d*x
+ c) + 16*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)), 1/24*(3*(4*A + 3*C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin
(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*b^2*cos(d*x +
c)^3 + 8*B*b^2*cos(d*x + c)^2 + 3*(4*A + 3*C)*b^2*cos(d*x + c) + 16*B*b^2)
*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{24\left(2\left(dx+c\right)b^2+b^2\sin\left(2\,dx+2\,c\right)\right)A\sqrt{b}}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(1/2),x, algorithm="maxima")
```

output
```
1/96*(24*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*A*sqrt(b) + 8*(b^2*sin(3
*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*
B*sqrt(b) + 3*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*sqrt(b))/d
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{1}{96}\left(\frac{3\,Cb^2\sin\left(4\,dx+4\,c\right)}{d}+\frac{8\,Bb^2\sin\left(3\,c\right.}{d}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(1/2),x, algorithm="giac")
```

output
```
1/96*(3*C*b^2*sin(4*d*x + 4*c)/d + 8*B*b^2*sin(3*d*x + 3*c)/d + 72*B*b^2*s
in(d*x + c)/d + 12*(4*A*b^2 + 3*C*b^2)*x + 24*(A*b^2 + C*b^2)*sin(2*d*x +
2*c)/d)*sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.47

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(72\,B\,\sin\left(c+d\,x\right)+2\right.}{}$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(1/2),x)
```

output
```
(b^2*(b*cos(c + d*x))^(1/2)*(72*B*sin(c + d*x) + 24*A*sin(2*c + 2*d*x) + 8
*B*sin(3*c + 3*d*x) + 24*C*sin(2*c + 2*d*x) + 3*C*sin(4*c + 4*d*x) + 48*A*
d*x + 36*C*d*x))/(96*d*cos(c + d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.44

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b}\,b^2\left(-6\cos\left(dx+c\right)\sin\left(dx+c\right)^3c+12\right.}{}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),
x)
```

output
```
(sqrt(b)*b**2*( - 6*cos(c + d*x)*sin(c + d*x)**3*c + 12*cos(c + d*x)*sin(c
 + d*x)*a + 15*cos(c + d*x)*sin(c + d*x)*c - 8*sin(c + d*x)**3*b + 24*sin(
c + d*x)*b + 12*a*d*x + 9*c*d*x))/(24*d)
```

3.308 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 155

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2(3A+2C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{b^2C\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
1/2*b^2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/3*b^2*(3*A+2*C)*(b*cos
(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+1/2*b^2*B*cos(d*x+c)^(1/2)*(b
*cos(d*x+c))^(1/2)*sin(d*x+c)/d+1/3*b^2*C*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^
(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(6Bc+6Bdx+3(4A+$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(3/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*
B*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2031, 3042, 3502, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\int \cos(c+dx)(3A+2C+3B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(3A+2C+3B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3213}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(\frac{(3A+2C)\sin(c+dx)}{d}+\frac{3B\sin(c+dx)\cos(c+dx)}{2d}+\frac{3Bx}{2}\right)+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(3/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) + ((3*B*x
)/2 + ((3*A + 2*C)*Sin[c + d*x])/d + (3*B*Cos[c + d*x]*Sin[c + d*x])/(2*d)
)/3))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

method	result
default	$\frac{b^{2}\left(3B(dx+c)+6A\sin(dx+c)+3B\sin(dx+c)\cos(dx+c)+\left(2\cos(dx+c)^{2}+4\right)\sin(dx+c)C\right)\sqrt{b\cos(dx+c)}}{6d\sqrt{\cos(dx+c)}}$
parts	$\frac{C\sin(dx+c)\left(2+\cos(dx+c)^{2}\right)b^{2}\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}+\frac{A\sin(dx+c)b^{2}\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}+\frac{B(\cos(dx+c)\sin(dx+c)+dx+c)b^{2}\sqrt{b\cos(dx+}}{2d\sqrt{\cos(dx+c)}}$
risch	$\frac{b^{2}Bx\sqrt{b\cos(dx+c)}}{2\sqrt{\cos(dx+c)}}+\frac{b^{2}\sqrt{b\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4\sqrt{\cos(dx+c)}\,d}+\frac{b^{2}\sqrt{b\cos(dx+c)}\,C\sin(3dx+3c)}{12\sqrt{\cos(dx+c)}\,d}+\frac{b^{2}\sqrt{b\cos(dx+c)}\,B\sin(2dx+}{4\sqrt{\cos(dx+c)}\,d}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),
x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2/d*(3*B*(d*x+c)+6*A*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)+(2*cos(d*x
+c)^2+4)*sin(d*x+c)*C)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.70

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\left[\frac{3\,B\sqrt{-b}b^{2}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+\right.\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos
(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b^2*cos(
d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*(3*A + 2*C)*b^2)*sqrt(b*cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*B*b^(5/2)*arct
an(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
 + c) + (2*C*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*(3*A + 2*C)*b^2
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{12\,Ab^{\frac{5}{2}}\sin\left(dx+c\right)+3\left(2\left(dx+c\right)b^2+b^2\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(3/2),x, algorithm="maxima")
```

output
```
1/12*(12*A*b^(5/2)*sin(d*x + c) + 3*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c
))*B*sqrt(b) + (b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))))*C*sqrt(b))/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.44

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{\left(-3i\,Bb^2\log\left(i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+3i\,Bb^2\log\left(-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(6\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Bb^2\,\mathrm{t}\right.}{}\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(3/2),x, algorithm="giac")
```

output
```
-1/6*(-3*I*B*b^2*log(I*tan(1/2*d*x + 1/2*c) - 1) + 3*I*B*b^2*log(-I*tan(1/
2*d*x + 1/2*c) - 1) - 2*(6*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^2*tan(1/2*
d*x + 1/2*c)^5 + 6*C*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*A*b^2*tan(1/2*d*x + 1
/2*c)^3 + 4*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*b^2*tan(1/2*d*x + 1/2*c) +
3*B*b^2*tan(1/2*d*x + 1/2*c) + 6*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^6 + 3*tan(1/2*d*x + 1/2*c)^4 + 3*tan(1/2*d*x + 1/2*c)^2 + 1))*sqr
t(b)/d
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(c+d\,x\right)+9\right.}{}$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(3/2),x)
```

output
```
(b^2*(b*cos(c + d*x))^(1/2)*(12*A*sin(c + d*x) + 9*C*sin(c + d*x) + 3*B*si
n(2*c + 2*d*x) + C*sin(3*c + 3*d*x) + 6*B*d*x))/(12*d*cos(c + d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.39

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b^2\left(3\cos\left(dx+c\right)\sin\left(dx+c\right)b-2\sin\right.}{}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),
x)
```

output
```
(sqrt(b)*b**2*(3*cos(c + d*x)*sin(c + d*x)*b - 2*sin(c + d*x)**3*c + 6*sin
(c + d*x)*a + 6*sin(c + d*x)*c + 3*b*d*x))/(6*d)
```

3.309 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 135

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
A*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b^2*C*x*(b*cos(d*x+c))^(
1/2)/cos(d*x+c)^(1/2)+b^2*B*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(
1/2)+1/2*b^2*C*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}(2(2A+C)(c+dx)+4}{4d\cos^{\frac{5}{2}}(c+}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(5/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[
2*(c + d*x)]))/(4*d*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(Ax+\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(5/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(A*x + (C*x)/2 + (B*Sin[c + d*x])/d + (C*Cos[c +
 d*x]*Sin[c + d*x])/(2*d)))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{b^{2}\left(C\cos\left(dx+c\right)\sin\left(dx+c\right)+2A\left(dx+c\right)+2B\sin\left(dx+c\right)+C\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}}{2d\sqrt{\cos\left(dx+c\right)}}$	66
risch	$\frac{b^{2}\sqrt{b\cos\left(dx+c\right)}\,\left(4A+2C\right)x}{4\sqrt{\cos\left(dx+c\right)}}+\frac{b^{2}B\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}+\frac{b^{2}\sqrt{b\cos\left(dx+c\right)}\,C\sin\left(2dx+2c\right)}{4\sqrt{\cos\left(dx+c\right)}\,d}$	101
parts	$\frac{A\left(dx+c\right)b^{2}\sqrt{b\cos\left(dx+c\right)}}{d\sqrt{\cos\left(dx+c\right)}}+\frac{b^{2}B\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}+\frac{C\left(\cos\left(dx+c\right)\sin\left(dx+c\right)+dx+c\right)b^{2}\sqrt{b\cos\left(dx+c\right)}}{2d\sqrt{\cos\left(dx+c\right)}}$	110

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),
x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2/d*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+C*(d*x+c))*(
b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.68

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[\frac{(2A+C)\sqrt{-b}b^2\cos(dx+c)\log\left(2b\cos\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(5/2),x, algorithm="fricas")
```

output
```
[1/4*((2*A + C)*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(
b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(C*b^2*c
os(d*x + c) + 2*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)), 1/2*((2*A + C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*si
n(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*b^2*cos(d*x + c
) + 2*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{8\,Ab^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+4\,Bb^{\frac{5}{2}}\sin\left(d\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(5/2),x, algorithm="maxima")
```

output
```
1/4*(8*A*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*B*b^(5/2)*sin
(d*x + c) + (2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*C*sqrt(b))/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.16

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{\left(\left(-2i\,Ab^2-i\,Cb^2\right)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)-\left(-2i\,Ab^2-i\,Cb^2\right)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)-\frac{2\left(2\,Bb^2\tan\right.}{}\right.}{2\,d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(5/2),x, algorithm="giac")
```

output
```
-1/2*((-2*I*A*b^2 - I*C*b^2)*log(tan(1/2*d*x + 1/2*c) + I) - (-2*I*A*b^2 -
 I*C*b^2)*log(tan(1/2*d*x + 1/2*c) - I) - 2*(2*B*b^2*tan(1/2*d*x + 1/2*c)^
3 - C*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*B*b^2*tan(1/2*d*x + 1/2*c) + C*b^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 + 2*tan(1/2*d*x + 1/2*c)^2 + 1
))*sqrt(b)/d
```

Mupad [B] (verification not implemented)

Time = 44.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.42

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(4\,B\,\sin\left(c+d\,x\right)+C\right.}{4\,d\,\sqrt{\cos\left(c\right.}}$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(5/2),x)
```

output
```
(b^2*(b*cos(c + d*x))^(1/2)*(4*B*sin(c + d*x) + C*sin(2*c + 2*d*x) + 4*A*d
*x + 2*C*d*x))/(4*d*cos(c + d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b^2(\cos\left(dx+c\right)\sin\left(dx+c\right)c+2\sin\left(d\right.}{2d}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),
x)
```

output
```
(sqrt(b)*b**2*(cos(c + d*x)*sin(c + d*x)*c + 2*sin(c + d*x)*b + 2*a*d*x +
c*d*x))/(2*d)
```

3.310 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 102

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab^2\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
b^2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b^2*arctanh(sin(d*x+c))*(b
*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+b^2*C*(b*cos(d*x+c))^(1/2)*sin(d*x+c
)/d/cos(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{5/2}\left(Bc+Bdx-A\log(\text{co}\right.}{}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(7/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d
*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2031, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\int (A+B\cos(c+dx))\sec(c+dx)dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int \sec(c+dx)dx+Bx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(A\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+Bx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+Bx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(7/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(B*x + (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c +
d*x])/d))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

method	result
default	$-\frac{b^{2}\left(2A\,\operatorname{arctanh}\left(-\csc\left(dx+c\right)+\cot\left(dx+c\right)\right)-B\left(dx+c\right)-C\sin\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}}{d\sqrt{\cos\left(dx+c\right)}}$
parts	$-\frac{2A\sqrt{b\cos\left(dx+c\right)}\,\operatorname{arctanh}\left(-\csc\left(dx+c\right)+\cot\left(dx+c\right)\right)b^{2}}{d\sqrt{\cos\left(dx+c\right)}}+\frac{B\left(dx+c\right)b^{2}\sqrt{b\cos\left(dx+c\right)}}{d\sqrt{\cos\left(dx+c\right)}}+\frac{b^{2}C\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}$
risch	$\frac{b^{2}Bx\sqrt{b\cos\left(dx+c\right)}}{\sqrt{\cos\left(dx+c\right)}}-\frac{i b^{2}\sqrt{b\cos\left(dx+c\right)}\,C\,{\mathrm e}^{i\left(dx+c\right)}}{2\sqrt{\cos\left(dx+c\right)}\,d}+\frac{i b^{2}\sqrt{b\cos\left(dx+c\right)}\,C\,{\mathrm e}^{-i\left(dx+c\right)}}{2\sqrt{\cos\left(dx+c\right)}\,d}+\frac{b^{2}\sqrt{b\cos\left(dx+c\right)}\,A\ln\left({\mathrm e}^{i\left(dx+c\right)}+i\right)}{\sqrt{\cos\left(dx+c\right)}\,d}-$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),
x,method=_RETURNVERBOSE)
```

output
```
-b^2/d*(2*A*arctanh(-csc(d*x+c)+cot(d*x+c))-B*(d*x+c)-C*sin(d*x+c))*(b*cos
(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.10

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\left[-\frac{2\,A\sqrt{-b}b^{2}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx}{b\sqrt{\cos(dx+c)}}\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(7/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*A*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c))))*cos(d*x + c) - B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b
*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d
*x + c) - b) - 2*sqrt(b*cos(d*x + c))*C*b^2*sqrt(cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)), 1/2*(2*B*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x
+ c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*b^(5/2)*cos(d*x + c)*l
og(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*
sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*
C*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{4\,Bb^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+2\,Cb^{\frac{5}{2}}\sin\left(d\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(7/2),x, algorithm="maxima")
```

output
```
1/2*(4*B*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 2*C*b^(5/2)*sin
(d*x + c) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
 - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b
))/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(Ab^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-Ab^2\log\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(7/2),x, algorithm="giac")
```

output
```
(A*b^2*log(tan(1/2*d*x + 1/2*c) + 1) - A*b^2*log(tan(1/2*d*x + 1/2*c) - 1)
 + I*B*b^2*log(I*tan(1/2*d*x + 1/2*c) - 1) - I*B*b^2*log(-I*tan(1/2*d*x +
1/2*c) - 1) + 2*C*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))*s
qrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+\right.}{\cos\left(c+d\,x\right)^{7/2}}$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(7/2),x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b^2\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan \right.\right.}{d}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),
x)
```

output
```
(sqrt(b)*b**2*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*
a + sin(c + d*x)*c + b*d*x))/d
```

3.311 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 102

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2B\,\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
b^2*C*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b^2*B*arctanh(sin(d*x+c))*(b
*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+A*b^2*(b*cos(d*x+c))^(1/2)*sin(d*x+c
)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{5/2}\left(Cdx\cos(c+dx)+B\,\text{c}\right.}{d\,\text{co}}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(9/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(C*d*x*Cos[c + d*x] + B*ArcCoth[Sin[c + d*x]]*Cos[
c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2031, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\int (B+C\cos(c+dx))\sec(c+dx)dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\int \frac{B+C\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\int \sec(c+dx)dx+\frac{A\tan(c+dx)}{d}+Cx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(B\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)}{d}+Cx\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+\frac{B\text{arctanh}(\sin(c+dx))}{d}+Cx\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(9/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(C*x + (B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c +
d*x])/d))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

method	result
default	$\frac{b^2(-2B\ \mathrm{arctanh}(-\csc(dx+c)+\cot(dx+c))\cos(dx+c)+C(dx+c)\cos(dx+c)+A\sin(dx+c))\sqrt{b\cos(dx+c)}}{d\cos(dx+c)^{\frac{3}{2}}}$
parts	$\frac{A b^2\sqrt{b\cos(dx+c)}\ \sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}}-\frac{2B\ \mathrm{arctanh}(-\csc(dx+c)+\cot(dx+c))\sqrt{b\cos(dx+c)}\,b^2}{d\sqrt{\cos(dx+c)}}+\frac{C(dx+c)b^2\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}$
risch	$\frac{b^2Cx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}}+\frac{2ib^2\sqrt{b\cos(dx+c)}\,A}{\sqrt{\cos(dx+c)}\,d(\mathrm{e}^{2i(dx+c)}+1)}+\frac{b^2\sqrt{b\cos(dx+c)}\,B\ln(\mathrm{e}^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}-\frac{b^2\sqrt{b\cos(dx+c)}\,B\ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),
x,method=_RETURNVERBOSE)
```

output
```
b^2/d*(-2*B*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)+C*(d*x+c)*cos(d*x+c
)+A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.18

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left[-\frac{2\,B\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx}{b\sqrt{\cos(dx+c)}}\right.}{}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(9/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - C*sqrt(-b)*b^2*cos(d*x + c)^2*log
(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*s
in(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c)^2), 1/2*(2*C*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*si
n(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*b^(5/2)*cos(d*
x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(
d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(
d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.48

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{4\,Cb^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\frac{4}{\cos(2\,dx+2\,c)^2+}}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(9/2),x, algorithm="maxima")
```

output
```
1/2*(4*C*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*A*b^(5/2)*sin
(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*B*sqrt(b))/
d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(Bb^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-Bb^2\log\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(9/2),x, algorithm="giac")
```

output
```
(B*b^2*log(tan(1/2*d*x + 1/2*c) + 1) - B*b^2*log(tan(1/2*d*x + 1/2*c) - 1)
 + I*C*b^2*log(I*tan(1/2*d*x + 1/2*c) - 1) - I*C*b^2*log(-I*tan(1/2*d*x +
1/2*c) - 1) - 2*A*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))*s
qrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos(c+d\,x))^{5/2}\,\left(C\cos(c+d\,x)^2+\right.}{\cos(c+d\,x)^{9/2}}$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(9/2),x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{\sqrt{b}\,b^2\left(-\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),
x)
```

output
```
(sqrt(b)*b**2*( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + cos(c + d*x)*
log(tan((c + d*x)/2) + 1)*b + cos(c + d*x)*c*d*x + sin(c + d*x)*a))/(cos(c
 + d*x)*d)
```

3.312 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 120

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{b^2(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c}}{2d\sqrt{\cos(c+dx)}}$$
$$+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
1/2*b^2*(A+2*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2
)+1/2*A*b^2*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+b^2*B*(b*co
s(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\left(2C\coth^{-1}(\sin(c+dx)\right.}{}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(11/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(2*C*ArcCoth[Sin[c + d*x]]*Cos[c + d*x]^2 + A*ArcT
anh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(
2*d*Cos[c + d*x]^(9/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.209$, Rules used = {2031, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\int (2B+(A+2C)\cos(c+dx))\sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\int \frac{2B+(A+2C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int \sec(c+dx)dx+2B\int \sec^2(c+dx)dx\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+2B\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2B\int 1d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2B\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\left(\frac{(A+2C)\text{arctanh}(\sin(c+dx))}{d}+\frac{2B\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(11/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((A + 2*
C)*ArcTanh[Sin[c + d*x]])/d + (2*B*Tan[c + d*x])/d)/2))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11

method	result
default	$\frac{b^2\left(-A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+A\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-4C\operatorname{arctanh}(-\csc(dx+c)+\cot\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+\sin(dx+c)\right)\sqrt{b\cos(dx+c)}\,b^2}{2d\cos(dx+c)^{\frac{5}{2}}}+\frac{b^2 B}{}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\,\left(A\,\mathrm{e}^{3i(dx+c)}-2B\,\mathrm{e}^{2i(dx+c)}-A\,\mathrm{e}^{i(dx+c)}-2B\right)}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{b^2\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{b\cos(dx+}}{2}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2)
,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2/d*(-A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^2+A*ln(-cot(d*x+c)+c
sc(d*x+c)+1)*cos(d*x+c)^2-4*C*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)^2
+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.08

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \left[\frac{(A+2\,C)b^{\frac{5}{2}}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+}{}\right.}{}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-\left(2\,Bb^2\cos\left(dx+c\right)+Ab^2\right)\sqrt{b\cos\left(dx+\right.}}{2\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(11/2),x, algorithm="fricas")
```

output
```
[1/4*((A + 2*C)*b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*c
os(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/c
os(d*x + c)^3) + 2*(2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)
*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c
))))*cos(d*x + c)^3 - (2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(11/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(104) = 208$.

Time = 0.33 (sec) , antiderivative size = 873, normalized size of antiderivative = 7.28

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(11/2),x, algorithm="maxima")
```

output
```
1/4*(8*B*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1) + 2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 +
2*sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x
+ c) + 1))*C*sqrt(b) - (4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x +
4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4
*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x
+ 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)
*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2
 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) +
 b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*si
n(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*
c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), co...
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.28

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left((Ab^2+2\,Cb^2)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(11/2),x, algorithm="giac")
```

output
```
1/2*((A*b^2 + 2*C*b^2)*log(tan(1/2*d*x + 1/2*c) + 1) - (A*b^2 + 2*C*b^2)*l
og(tan(1/2*d*x + 1/2*c) - 1) + 2*(A*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*B*b^2*t
an(1/2*d*x + 1/2*c)^3 + A*b^2*tan(1/2*d*x + 1/2*c) + 2*B*b^2*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 2*tan(1/2*d*x + 1/2*c)^2 + 1))*sqrt(b)/
d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+\right.}{\cos\left(c+d\,x\right)^{11/2}}$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(11/2),x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.64

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b^2\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)b-\log\right.}{}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2)
,x)
```

output
```
(sqrt(b)*b**2*( - 2*cos(c + d*x)*sin(c + d*x)*b - log(tan((c + d*x)/2) - 1
)*sin(c + d*x)**2*a - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c + log(
tan((c + d*x)/2) - 1)*a + 2*log(tan((c + d*x)/2) - 1)*c + log(tan((c + d*x
)/2) + 1)*sin(c + d*x)**2*a + 2*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*
c - log(tan((c + d*x)/2) + 1)*a - 2*log(tan((c + d*x)/2) + 1)*c - sin(c +
d*x)*a))/(2*d*(sin(c + d*x)**2 - 1))
```

3.313 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 164

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx=\frac{b^2B\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2(2A+3C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

output

```
1/2*b^2*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/3*
A*b^2*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+1/2*b^2*B*(b*cos(
d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+1/3*b^2*(2*A+3*C)*(b*cos(d*x+c
))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\left(3B\text{arctanh}(\sin(c+dx\right.}{}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(13/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A +
 3*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*
d*Cos[c + d*x]^(9/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.66, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {2031, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\int(3B+(2A+3C)\cos(c+dx))\sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\int\frac{3B+(2A+3C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3227

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left((2A+3C)\int\sec^2(c+dx)dx+3B\int\sec^3(c+dx)dx\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left((2A+3C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 4254

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 24

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 4255

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(3B\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(3B\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+d}{3d}\right.}{\sqrt{\cos(c+dx)}}$$

$\Big\downarrow$ 4257

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{3}\left(\frac{(2A+3C)\tan(c+dx)}{d}+3B\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(13/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (((2*A
+ 3*C)*Tan[c + d*x])/d + 3*B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*
Tan[c + d*x])/(2*d)))/3))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

method	result
default	$\frac{b^2\left(-3B\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^3+3B\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^3+\left(4\cos(dx+c)^2+2\right)\sin(dx+\right.}{6d\cos(dx+c)^{\frac{7}{2}}}$
parts	$\frac{A\sin(dx+c)\left(2\cos(dx+c)^2+1\right)\sqrt{b\cos(dx+c)}\,b^2}{3d\cos(dx+c)^{\frac{7}{2}}}+\frac{B\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\left(3B\,e^{5i(dx+c)}-6C\,e^{4i(dx+c)}-12A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-3B\,e^{i(dx+c)}-4A-6C\right)}{3\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^3}+\frac{b^2\sqrt{b\cos(dx+c)}\,B\ln(}{2\sqrt{\cos(dx+}}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2)
,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2/d*(-3*B*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^3+3*B*ln(-cot(d*x+
c)+csc(d*x+c)+1)*cos(d*x+c)^3+(4*cos(d*x+c)^2+2)*sin(d*x+c)*A+6*C*cos(d*x+
c)^2*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.74

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \left[\frac{3\,Bb^{\frac{5}{2}}\cos\left(dx+c\right)^4\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{}}{}\right.}{}\right.$$

$$-\frac{3\,B\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^4-\left(2\left(2\,A+3\,C\right)b^2\cos\left(dx+c\right)^2+3\,Bb^2\cos\left(dx\right.\right.}{6\,d\cos\left(dx+c\right)^4}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(13/2),x, algorithm="fricas")
```

output
```
[1/12*(3*B*b^(5/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*
x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*
x + c)^3) + 2*(2*(2*A + 3*C)*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2
*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^4), -1/6*(3*B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*
x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*b^2*cos(d*x
 + c)^2 + 3*B*b^2*cos(d*x + c) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(13/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(140) = 280$.

Time = 0.33 (sec) , antiderivative size = 1112, normalized size of antiderivative = 6.78

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(13/2),x, algorithm="maxima")
```

output
```
1/12*(24*C*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1) - 16*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c)
 + 9*b^2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2
)*sin(6*d*x + 6*c) - 3*(3*b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*A*
sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos
(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x +
 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*
c) + 1) - 3*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^
2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)
^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 +
4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x
+ 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*co
s(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2...
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.34

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left(3\,Bb^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-3\,Bb^2\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(13/2),x, algorithm="giac")
```

output
```
1/6*(3*B*b^2*log(tan(1/2*d*x + 1/2*c) + 1) - 3*B*b^2*log(tan(1/2*d*x + 1/2
*c) - 1) - 2*(6*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^2*tan(1/2*d*x + 1/2*c
)^5 + 6*C*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 12
*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*b^2*tan(1/2*d*x + 1/2*c) + 3*B*b^2*tan
(1/2*d*x + 1/2*c) + 6*C*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^6
- 3*tan(1/2*d*x + 1/2*c)^4 + 3*tan(1/2*d*x + 1/2*c)^2 - 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(C\,\cos\left(c+d\,x\right)^2+\right.}{\cos\left(c+d\,x\right)^{13/2}}$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(13/2),x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(13/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.13

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b^2\left(-3\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.\right.}{}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2)
,x)
```

output
```
(sqrt(b)*b**2*( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2
*b + 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 3*cos(c + d*x)*log(tan((
c + d*x)/2) + 1)*sin(c + d*x)**2*b - 3*cos(c + d*x)*log(tan((c + d*x)/2) +
 1)*b - 3*cos(c + d*x)*sin(c + d*x)*b + 4*sin(c + d*x)**3*a + 6*sin(c + d*
x)**3*c - 6*sin(c + d*x)*a - 6*sin(c + d*x)*c))/(6*cos(c + d*x)*d*(sin(c +
 d*x)**2 - 1))
```

3.314 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 208

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{b^2(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b^2(3A+4C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
1/8*b^2*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1
/2)+1/4*A*b^2*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)+1/8*b^2*(
3*A+4*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+b^2*B*(b*cos(d
*x+c))^(1/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+1/3*b^2*B*(b*cos(d*x+c))^(1/2)*
sin(d*x+c)^3/d/cos(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.53

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{5/2}\left(3(3A+4C)\text{arctanh}(\text{si}\right.}{}$$

input
```
Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))
/Cos[c + d*x]^(15/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^
4 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3
 + 8*B*Cos[c + d*x]*Sin[c + d*x]^2)))/(24*d*Cos[c + d*x]^(13/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.59, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.256, Rules used = {2031, 3042, 3500, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^5(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\int(4B+(3A+4C)\cos(c+dx))\sec^4(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\int\frac{4B+(3A+4C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\sec^3(c+dx)dx+4B\int\sec^4(c+dx)dx\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+4B\int\csc\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4B\int(\tan^2(c+dx)+1)d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right.}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(\right.}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\right.}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(}{}\right.}{\sqrt{\cos(c+dx)}}$$

input
```
Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(15/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A +
 4*C)*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) -
(4*B*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d)/4))/Sqrt[Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.98

method	result
default	$-\frac{b^{2}\left(9A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^{4}+12C\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^{4}-9A\ln(-\cot(dx+c)+\csc(d\right.}{}$
parts	$\frac{A\left(-3\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^{4}+3\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^{4}+3\cos(dx+c)^{2}\sin(dx+c)+2\sin(\right.}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib^{2}\sqrt{b\cos(dx+c)}\,(9A\,e^{7i(dx+c)}+12C\,e^{7i(dx+c)}+33A\,e^{5i(dx+c)}+12C\,e^{5i(dx+c)}-48B\,e^{4i(dx+c)}-33A\,e^{3i(dx+c)}-12C\,e^{3i(dx+c)}-}{12\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^{4}}$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2)
,x,method=_RETURNVERBOSE)
```

output
```
-1/24*b^2/d*(9*A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^4+12*C*ln(-cot(d*
x+c)+csc(d*x+c)-1)*cos(d*x+c)^4-9*A*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c
)^4-12*C*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+(-9*cos(d*x+c)^2-6)*sin
(d*x+c)*A+sin(d*x+c)*cos(d*x+c)*(-16*cos(d*x+c)^2-8)*B-12*C*cos(d*x+c)^2*s
in(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.57

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \left[\frac{3\,(3\,A+4\,C)b^{\frac{5}{2}}\cos{(dx+c)}^5\log\left(-\frac{b\cos\right.}{}\right.}{}\right.$$

$$-\frac{3\,(3\,A+4\,C)\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos{(dx+c)}^5-\left(16\,Bb^2\cos{(dx+c)}^3+3\,(3\,A+4\,C\right.}{24\,d\cos{(dx+c)}}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(15/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(3*A + 4*C)*b^(5/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqr
t(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x +
c))/cos(d*x + c)^3) + 2*(16*B*b^2*cos(d*x + c)^3 + 3*(3*A + 4*C)*b^2*cos(d
*x + c)^2 + 8*B*b^2*cos(d*x + c) + 6*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*
b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c)
)))*cos(d*x + c)^5 - (16*B*b^2*cos(d*x + c)^3 + 3*(3*A + 4*C)*b^2*cos(d*x
+ c)^2 + 8*B*b^2*cos(d*x + c) + 6*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c
)**(15/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2972 vs. $2(180) = 360$.

Time = 0.38 (sec) , antiderivative size = 2972, normalized size of antiderivative = 14.29

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(15/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*
d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^
2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c
) + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d
*x + 6*c) + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b
^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c
)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*
x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x +
 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2
*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*
b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12
*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*
c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 1
6*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2...
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.53

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{\Big(3\,(3\,Ab^2+4\,Cb^2)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.-$$

input
```
integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^
(15/2),x, algorithm="giac")
```

output
```
1/24*(3*(3*A*b^2 + 4*C*b^2)*log(tan(1/2*d*x + 1/2*c) + 1) - 3*(3*A*b^2 + 4
*C*b^2)*log(tan(1/2*d*x + 1/2*c) - 1) + 2*(15*A*b^2*tan(1/2*d*x + 1/2*c)^7
 - 24*B*b^2*tan(1/2*d*x + 1/2*c)^7 + 12*C*b^2*tan(1/2*d*x + 1/2*c)^7 + 9*A
*b^2*tan(1/2*d*x + 1/2*c)^5 + 40*B*b^2*tan(1/2*d*x + 1/2*c)^5 - 12*C*b^2*t
an(1/2*d*x + 1/2*c)^5 + 9*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 40*B*b^2*tan(1/2*
d*x + 1/2*c)^3 - 12*C*b^2*tan(1/2*d*x + 1/2*c)^3 + 15*A*b^2*tan(1/2*d*x +
1/2*c) + 24*B*b^2*tan(1/2*d*x + 1/2*c) + 12*C*b^2*tan(1/2*d*x + 1/2*c))/(t
an(1/2*d*x + 1/2*c)^8 - 4*tan(1/2*d*x + 1/2*c)^6 + 6*tan(1/2*d*x + 1/2*c)^
4 - 4*tan(1/2*d*x + 1/2*c)^2 + 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+\right.}{\cos\left(c+d\,x\right)^{15/2}}$$

input
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(15/2),x)
```

output
```
int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^(15/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.68

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\,b^2\left(-16\cos\left(dx+c\right)\sin\left(dx+c\right)^3 b+2\right.}{}$$

input
```
int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2)
,x)
```

output
```
(sqrt(b)*b**2*( - 16*cos(c + d*x)*sin(c + d*x)**3*b + 24*cos(c + d*x)*sin(
c + d*x)*b - 9*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 12*log(tan((c
 + d*x)/2) - 1)*sin(c + d*x)**4*c + 18*log(tan((c + d*x)/2) - 1)*sin(c + d
*x)**2*a + 24*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 9*log(tan((c +
 d*x)/2) - 1)*a - 12*log(tan((c + d*x)/2) - 1)*c + 9*log(tan((c + d*x)/2)
+ 1)*sin(c + d*x)**4*a + 12*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c -
18*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - 24*log(tan((c + d*x)/2) +
 1)*sin(c + d*x)**2*c + 9*log(tan((c + d*x)/2) + 1)*a + 12*log(tan((c + d*
x)/2) + 1)*c - 9*sin(c + d*x)**3*a - 12*sin(c + d*x)**3*c + 15*sin(c + d*x
)*a + 12*sin(c + d*x)*c))/(24*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.315 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 184

$$
\begin{aligned}
&\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} \\
&\quad+\frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}} \\
&\quad+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}-\frac{B\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

output
```
1/8*(4*A+3*C)*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*cos(d*x+c)^(1/2)*s
in(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)
/d/(b*cos(d*x+c))^(1/2)+1/4*C*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(b*cos(d*x+c))
^(1/2)-1/3*B*cos(d*x+c)^(1/2)*sin(d*x+c)^3/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.50

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}(48Ac+36cC+48Adx+36Cdx+72B\sin(c+dx)+24(A+C)\sin(2(c+dx))+8B\sin}{96d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqr
t[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c +
d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c +
 d*x)]))/(96*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.60, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2031, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$\Big\downarrow \text{2031}$$
$$\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{b\cos(c+dx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\int\cos^2(c+dx)(4A+3C+4B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\int\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(4A+3C+4B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int\cos^2(c+dx)dx+4B\int\cos^3(c+dx)dx\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+4B\int\sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4B\int(1-\sin^2(c+dx))d(-\sin(c+dx))}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Co
s[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*((C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A + 3*C)*
(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (4*B*(-Sin[c + d*x] + Sin[c +
d*x]^3/3))/d)/4))/Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.55

method	result
default	$\frac{\left(12A(dx+c)+9C(dx+c)+12A\cos(dx+c)\sin(dx+c)+\left(8\cos(dx+c)^2+16\right)\sin(dx+c)B+\sin(dx+c)\cos(dx+c)\left(6\cos(dx+c)^2+9\right)C\right.}{24d\sqrt{b\cos(dx+c)}}$
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}}+\frac{B\sin(dx+c)\left(2+\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}}{3d\sqrt{b\cos(dx+c)}}+\frac{C\left(2\cos(dx+c)^3\sin(dx+c)+3\cos\right.}{8d\sqrt{}}$
risch	$\frac{\sqrt{\cos(dx+c)}\,(8A+6C)x}{16\sqrt{b\cos(dx+c)}}+\frac{3B\sqrt{\cos(dx+c)}\,\sin(dx+c)}{4d\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,C\sin(4dx+4c)}{32\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,B\sin(3dx+3c)}{12\sqrt{b\cos(dx+c)}\,d}+\sqrt{\cos}$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
1/24/d*(12*A*(d*x+c)+9*C*(d*x+c)+12*A*cos(d*x+c)*sin(d*x+c)+(8*cos(d*x+c)^
2+16)*sin(d*x+c)*B+sin(d*x+c)*cos(d*x+c)*(6*cos(d*x+c)^2+9)*C)*cos(d*x+c)^
(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.53

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\left[-\frac{3\,(4\,A+3\,C)\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+\right.\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqr
t(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(6*C*c
os(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)), 1/
24*(3*(4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b
)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x +
c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c)
)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.63

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{\sqrt{b}}+\frac{3\,\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)\right)C}{\sqrt{b}}+\frac{8\,B(\sin(3\,dx+3\,c)+9\,\sin}{}}{96\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(1/2),x, algorithm="maxima")
```

output
```
1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/sqrt(b) + 3*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*
C/sqrt(b) + 8*B*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))))/sqrt(b))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 46.54 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.76

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,(24\,A\,\sin(c+dx)+24\,C\,\sin(c+dx)+24\,A\,\sin(3\,c+3\,d\,x)+80\,B\,\mathrm{s}}{}$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(c
 + d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*
d*x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x
) + 72*C*d*x*cos(c + d*x)))/(96*b*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.48

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b}\,\left(-6\cos(dx+c)\sin(dx+c)^3c+12\cos(dx+c)\sin(dx+c)a+15\cos(dx+c)\sin(dx+c)c-8\sin\right.}{24bd}$$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),
x)
```

output
```
(sqrt(b)*( - 6*cos(c + d*x)*sin(c + d*x)**3*c + 12*cos(c + d*x)*sin(c + d*
x)*a + 15*cos(c + d*x)*sin(c + d*x)*c - 8*sin(c + d*x)**3*b + 24*sin(c + d
*x)*b + 12*a*d*x + 9*c*d*x))/(24*b*d)
```

3.316 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 143

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{(3A+2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{b\cos(c+dx)}}\\ &\quad+\frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{b\cos(c+dx)}}\end{aligned}$$

output
```
1/2*B*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/3*(3*A+2*C)*cos(d*x+c)^(1/
2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(
b*cos(d*x+c))^(1/2)+1/3*C*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/
2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.52

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}(6Bc+6Bdx+3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+C\sin(3(c+dx)))}{12d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqr
t[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Si
n[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2031, 3042, 3502, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$\Big\downarrow \text{2031}$$
$$\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt{b\cos(c+dx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt{b\cos(c+dx)}}$$
$$\Big\downarrow \text{3502}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\int\cos(c+dx)(3A+2C+3B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\int\sin\left(c+dx+\frac{\pi}{2}\right)\left(3A+2C+3B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(\frac{(3A+2C)\sin(c+dx)}{d}+\frac{3B\sin(c+dx)\cos(c+dx)}{2d}+\frac{3Bx}{2}\right)+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Co
s[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*((C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) + ((3*B*x)/2 +
((3*A + 2*C)*Sin[c + d*x])/d + (3*B*Cos[c + d*x]*Sin[c + d*x])/(2*d))/3))/
Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\left(3B(dx+c)+6A\sin(dx+c)+3B\sin(dx+c)\cos(dx+c)+\left(2\cos(dx+c)^{2}+4\right)\sin(dx+c)C\right)\sqrt{\cos(dx+c)}}{6d\sqrt{b\cos(dx+c)}}$	77
parts	$\frac{A\sin(dx+c)\sqrt{\cos(dx+c)}}{d\sqrt{b\cos(dx+c)}}+\frac{B(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}}+\frac{C\sin(dx+c)\left(2+\cos(dx+c)^{2}\right)\sqrt{\cos(dx+c)}}{3d\sqrt{b\cos(dx+c)}}$	113
risch	$\frac{Bx\sqrt{\cos(dx+c)}}{2\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,C\sin(3dx+3c)}{12\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,B\sin(2dx+2c)}{4\sqrt{b\cos(dx+c)}\,d}$	126

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
1/6/d*(3*B*(d*x+c)+6*A*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)+(2*cos(d*x+c)^
2+4)*sin(d*x+c)*C)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.69

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \left[-\frac{3\,B\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)-}{12\,bd\cos\left(d\right.}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*C*cos(d*x + c
)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(b*d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x
 + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(
d*x + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c)
)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{\sqrt{b}}+\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{\sqrt{b}}+\frac{12\,A\sin(dx+c)}{\sqrt{b}}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(1/2),x, algorithm="maxima")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B/sqrt(b) + C*(sin(3*d*x + 3*c) +
 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/sqrt(b) + 12*A*si
n(d*x + c)/sqrt(b))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,\left(3\,B\,\sin(c+d\,x)+12\,A\,\sin(2\,c+2\,d\,x)+3\,B\,\sin(3\,c+3\,d\,x)+10\,C\right.}{12\,b\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*B*sin(c + d*x) + 12*A*sin(2*
c + 2*d*x) + 3*B*sin(3*c + 3*d*x) + 10*C*sin(2*c + 2*d*x) + C*sin(4*c + 4*
d*x) + 12*B*d*x*cos(c + d*x)))/(12*b*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{b}\left(3\cos\left(dx+c\right)\sin\left(dx+c\right)b-2\sin\left(dx+c\right)^3c+6\sin\left(dx+c\right)a+6\sin\left(dx+c\right)c+3bdx\right)}{6bd}$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),
x)
```

output
```
(sqrt(b)*(3*cos(c + d*x)*sin(c + d*x)*b - 2*sin(c + d*x)**3*c + 6*sin(c +
d*x)*a + 6*sin(c + d*x)*c + 3*b*d*x))/(6*b*d)
```

3.317 $\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 123

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

output
```
A*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/(b*cos(
d*x+c))^(1/2)+B*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/2*C*c
os(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqr
t[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c
 + d*x)]))/(4*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$\Big\downarrow \text{2031}$$
$$\frac{\sqrt{\cos(c+dx)}\int (C\cos^2(c+dx)+B\cos(c+dx)+A)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{\cos(c+dx)}\left(Ax+\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Co
s[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*x + (C*x)/2 + (B*Sin[c + d*x])/d + (C*Cos[c + d*x]*
Sin[c + d*x])/(2*d)))/Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}}$	63
risch	$\frac{\sqrt{\cos(dx+c)}\,(4A+2C)x}{4\sqrt{b\cos(dx+c)}}+\frac{B\sqrt{\cos(dx+c)}\,\sin(dx+c)}{d\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,C\sin(2dx+2c)}{4\sqrt{b\cos(dx+c)}\,d}$	92
parts	$\frac{A(dx+c)\sqrt{\cos(dx+c)}}{d\sqrt{b\cos(dx+c)}}+\frac{B\sqrt{\cos(dx+c)}\,\sin(dx+c)}{d\sqrt{b\cos(dx+c)}}+\frac{C(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}}$	101

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+C*(d*x+c))*cos(d
*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[-\frac{(2A+C)\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)\right.}{4bd\cos(dx+c)}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*c
os(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(C*cos(d*x
+ c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos
(d*x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c
)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt
(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [A] (verification not implemented)

Time = 16.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \begin{cases} \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{Cx\sin^2(c+dx)\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{Cx\cos^{\frac{5}{2}}(c+dx)}{2\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}} & \text{for } d \\ \frac{x(A+B\cos(c)+C\cos^2(c))\sqrt{\cos(c)}}{\sqrt{b\cos(c)}} & \text{othe} \end{cases}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c)
)**(1/2),x)
```

output
```
Piecewise((A*x*sqrt(cos(c + d*x))/sqrt(b*cos(c + d*x)) + B*sin(c + d*x)*sq
rt(cos(c + d*x))/(d*sqrt(b*cos(c + d*x))) + C*x*sin(c + d*x)**2*sqrt(cos(c
 + d*x))/(2*sqrt(b*cos(c + d*x))) + C*x*cos(c + d*x)**(5/2)/(2*sqrt(b*cos(
c + d*x))) + C*sin(c + d*x)*cos(c + d*x)**(3/2)/(2*d*sqrt(b*cos(c + d*x)))
, Ne(d, 0)), (x*(A + B*cos(c) + C*cos(c)**2)*sqrt(cos(c))/sqrt(b*cos(c)),
True))
```

Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{\sqrt{b}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}+\frac{4\,B\sin(dx+c)}{\sqrt{b}}}{4\,d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(1/2),x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/sqrt(b) + 8*A*arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/sqrt(b) + 4*B*sin(d*x + c)/sqrt(b))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(c+d\,x\right)+4\,B\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(3\,c+3\,d\,x\right)+8\,A\,d\,x\,\mathrm{co}\right.}{4\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + 4*B*sin(2*c +
 2*d*x) + C*sin(3*c + 3*d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)
))/(4*b*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b}\,(\cos(dx+c)\sin(dx+c)\,c+2\sin(dx+c)\,b+2adx+cdx)}{2bd}$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),
x)
```

output
```
(sqrt(b)*(cos(c + d*x)*sin(c + d*x)*c + 2*sin(c + d*x)*b + 2*a*d*x + c*d*x
))/(2*b*d)
```

3.318 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 93

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{A\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
B*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+c)
^(1/2)/d/(b*cos(d*x+c))^(1/2)+C*cos(d*x+c)^(1/2)*sin(d*x+c)/d/(b*cos(d*x+c
))^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\left(Bc+Bdx-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}\right.\right.\right.}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt
[b*Cos[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*Sqr
t[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2032, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\int (A+B\cos(c+dx))\sec(c+dx)dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \sec(c+dx)dx+Bx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+Bx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+Bx+\frac{C\sin(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos
[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(B*x + (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/
d))/Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\left(2 A \,\operatorname{arctanh}\left(-\csc \left(d x +c \right)+\cot \left(d x +c \right)\right)-B \left(d x +c \right)-C \sin \left(d x +c \right)\right) \sqrt{\cos \left(d x +c \right)}}{d \sqrt{b \cos \left(d x +c \right)}}$	61
parts	$\frac{C \sqrt{\cos \left(d x +c \right)}\, \sin \left(d x +c \right)}{d \sqrt{b \cos \left(d x +c \right)}}-\frac{2 A \,\operatorname{arctanh}\left(-\csc \left(d x +c \right)+\cot \left(d x +c \right)\right) \sqrt{\cos \left(d x +c \right)}}{d \sqrt{b \cos \left(d x +c \right)}}+\frac{B \left(d x +c \right) \sqrt{\cos \left(d x +c \right)}}{d \sqrt{b \cos \left(d x +c \right)}}$	99
risch	$\frac{B x \sqrt{\cos \left(d x +c \right)}}{\sqrt{b \cos \left(d x +c \right)}}+\frac{\sqrt{\cos \left(d x +c \right)}\, A \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{\sqrt{b \cos \left(d x +c \right)}\, d}-\frac{\sqrt{\cos \left(d x +c \right)}\, A \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{\sqrt{b \cos \left(d x +c \right)}\, d}+\frac{C \sin \left(2 d x +2 c \right)}{2 d \sqrt{\cos \left(d x +c \right)}\, \sqrt{b \cos \left(d x +c \right)}}$	129

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
-1/d*(2*A*arctanh(-csc(d*x+c)+cot(d*x+c))-B*(d*x+c)-C*sin(d*x+c))*cos(d*x+
c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.32

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[-\frac{2\,A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)+B\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\right.}{2\,bd\cos}\right.$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c) + B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x
 + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
- b) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(
d*x + c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt
(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*cos
(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x +
c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos
(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(b*cos(d*x+c)
)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(sqrt(b*cos(c + d*x))*sq
rt(cos(c + d*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)}{\sqrt{b}}+\frac{4B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}+\frac{2C\sin}{\sqrt{}}}{2d}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="maxima")
```

output
```
1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*B*arctan(
sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + 2*C*sin(d*x + c)/sqrt(b))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
 d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
 d*x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.55

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a+\sin\left(dx+c\right)c+bdx\right)}{bd}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),
x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a + s
in(c + d*x)*c + b*d*x))/(b*d)
```

3.319 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 93

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{B\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
C*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*cos(d*x+c)
^(1/2)/d/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c
))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{Cdx\cos(c+dx)+B\coth^{-1}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt
[b*Cos[c + d*x]]),x]
```

output
```
(C*d*x*Cos[c + d*x] + B*ArcCoth[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x
])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2032, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int (C\cos^2(c+dx)+B\cos(c+dx)+A)\sec^2(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\int (B+C\cos(c+dx))\sec(c+dx)dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\int \frac{B+C\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{A\tan(c+dx)}{d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \sec(c+dx)dx+\frac{A\tan(c+dx)}{d}+Cx\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)}{d}+Cx\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+\frac{B\text{arctanh}(\sin(c+dx))}{d}+Cx\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos
[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(C*x + (B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x])/
d))/Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])    Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{-2B\ \operatorname{arctanh}\left(-\csc\left(dx+c\right)+\cot\left(dx+c\right)\right)\cos\left(dx+c\right)+C\left(dx+c\right)\cos\left(dx+c\right)+A\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}\,\sqrt{b\cos\left(dx+c\right)}}$	70
parts	$\frac{A\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}\,\sqrt{b\cos\left(dx+c\right)}}-\frac{2B\ \operatorname{arctanh}\left(-\csc\left(dx+c\right)+\cot\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{d\sqrt{b\cos\left(dx+c\right)}}+\frac{C\left(dx+c\right)\sqrt{\cos\left(dx+c\right)}}{d\sqrt{b\cos\left(dx+c\right)}}$	99
risch	$\frac{Cx\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)}}+\frac{i\,\mathrm{e}^{-i\left(dx+c\right)}A}{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,d}+\frac{\sqrt{\cos\left(dx+c\right)}\,B\ln\left(\mathrm{e}^{i\left(dx+c\right)}+i\right)}{\sqrt{b\cos\left(dx+c\right)}\,d}-\frac{\sqrt{\cos\left(dx+c\right)}\,B\ln\left(\mathrm{e}^{i\left(dx+c\right)}-i\right)}{\sqrt{b\cos\left(dx+c\right)}\,d}$	130

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*B*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)+C*(d*x+c)*cos(d*x+c)+
A*sin(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.41

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[-\frac{2\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^2+C\sqrt{-b}\cos\left(dx+c\right)^2\log\left(2\,b\cos\left(dx+c\right)^2+\right.}{2\,bd\cos}\right.$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c)^2 + C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos
(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x +
 c) - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*
cos(d*x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)
/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2*l
og(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*
sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*
A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)}{\sqrt{b\cos\left(c+dx\right)}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c)
)**(1/2),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(sqrt(b*cos(c + d*x))*co
s(c + d*x)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.60

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\frac{B\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)}{\sqrt{b}} + \frac{4C\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}} + \frac{\cdots}{b\cos(2\cdots}}{2d}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="maxima")
```

output
```
1/2*(B*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*C*arctan(
sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + 4*A*sqrt(b)*sin(2*d*x + 2*c)/(b
*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*x + 2*c) + b))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos{(c+d\,x)}^2+B\,\cos{(c+d\,x)}+A}{\cos{(c+d\,x)}^{3/2}\,\sqrt{b\,\cos{(c+d\,x)}}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
 d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
 d*x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b}\left(-\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b+\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)b+\cos(dx+c)\,cdx+\mathrm{s}\right.}{\cos(dx+c)\,bd}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),
x)
```

output
```
(sqrt(b)*( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + cos(c + d*x)*log(t
an((c + d*x)/2) + 1)*b + cos(c + d*x)*c*d*x + sin(c + d*x)*a))/(cos(c + d*
x)*b*d)
```

3.320 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 111

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
1/2*(A+2*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+1/
2*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/d/cos(
d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{2C\coth^{-1}(\sin(c+dx))\cos^2(c+dx)+A\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+dx))\sin(}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt
[b*Cos[c + d*x]]),x]
```

output
```
(2*C*ArcCoth[Sin[c + d*x]]*Cos[c + d*x]^2 + A*ArcTanh[Sin[c + d*x]]*Cos[c
+ d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sq
rt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {2032, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^3(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int\frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\int(2B+(A+2C)\cos(c+dx))\sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\int\frac{2B+(A+2C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\sec(c+dx)dx+2B\int\sec^2(c+dx)dx\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+2B\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2B\int 1d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2B\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left(\frac{(A+2C)\text{arctanh}(\sin(c+dx))}{d}+\frac{2B\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

input

```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos
[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((A + 2*C)*Arc
Tanh[Sin[c + d*x]])/d + (2*B*Tan[c + d*x])/d)/2))/Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.18

method	result
default	$-\frac{A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2-A\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2+4C\ \mathrm{arctanh}(-\csc(dx+c)+\cot(d}{2d\cos(dx+c)^{\frac{3}{2}}\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i\left(A\,\mathrm{e}^{2i(dx+c)}-A-4B\cos(dx+c)\right)}{2\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{\sqrt{\cos(dx+c)}\,(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2\sqrt{b\cos(dx+c)}\,d}$
parts	$\frac{A\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+\sin(dx+c)\right)}{2d\cos(dx+c)^{\frac{3}{2}}\sqrt{b\cos(dx+c)}}+\frac{B\sin(dx+c)}{d\sqrt{\cos(dx+c)}\,\sqrt{b\cos}}$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^2-A*ln(-cot(d*x+c)+csc(d
*x+c)+1)*cos(d*x+c)^2+4*C*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)^2-2*B
*sin(d*x+c)*cos(d*x+c)-A*sin(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.15

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$=\left[\frac{(A+2\,C)\sqrt{b}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,(2\,B\cos\,(d}{4\,bd\cos\left(dx+c\right)^3}\right.$$
$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-(2\,B\cos\left(dx+c\right)+A)\sqrt{b\cos\left(dx+c\right)}}{2\,bd\cos\left(dx+c\right)^3}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="fricas")
```

output
```
[1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*c
os(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/c
os(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arcta
n(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d
*x + c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(b*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c)
)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. 2(95) = 190.

Time = 0.38 (sec) , antiderivative size = 785, normalized size of antiderivative = 7.07

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*C*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(
cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 8*B*sqrt(
b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos
(2*d*x + 2*c) + b) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x
+ 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*si
n(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
 (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos
(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c
) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((2*(2*cos(
2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x +...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
 d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
 d*x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.77

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{b}\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)b-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2a-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),
x)
```

output
```
(sqrt(b)*( - 2*cos(c + d*x)*sin(c + d*x)*b - log(tan((c + d*x)/2) - 1)*sin
(c + d*x)**2*a - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c + log(tan((
c + d*x)/2) - 1)*a + 2*log(tan((c + d*x)/2) - 1)*c + log(tan((c + d*x)/2)
+ 1)*sin(c + d*x)**2*a + 2*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*c - l
og(tan((c + d*x)/2) + 1)*a - 2*log(tan((c + d*x)/2) + 1)*c - sin(c + d*x)*
a))/(2*b*d*(sin(c + d*x)**2 - 1))
```

3.321 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 152

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\frac{B\,\mathrm{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(2A+3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+1/3*A*si
n(d*x+c)/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/2*B*sin(d*x+c)/d/cos(d*
x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*sin(d*x+c)/d/cos(d*x+c)^(1/2
)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.57

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{3B\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(4A+3C+3B\cos(c+dx)+(2A+3C)\cos(2(c+dx)))\tan(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt
[b*Cos[c + d*x]]),x]
```

output
```
(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C + 3*B*Cos[c + d*x]
+ (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x])/(6*d*Cos[c + d*x]^(3/2)*Sqrt
[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.69, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.256, Rules used = {2032, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int (C\cos^2(c+dx)+B\cos(c+dx)+A)\sec^4(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\int (3B+(2A+3C)\cos(c+dx))\sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\int \frac{3B+(2A+3C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left((2A+3C)\int \sec^2(c+dx)dx+3B\int \sec^3(c+dx)dx\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left((2A+3C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(3B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(3B\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(3B\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(3B\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(\frac{(2A+3C)\tan(c+dx)}{d}+3B\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos
[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (((2*A + 3*C)
*Tan[c + d*x])/d + 3*B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
+ d*x])/(2*d)))/3))/Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

method	result
default	$\frac{-3B\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^3+3B\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^3+\left(4\cos(dx+c)^2+2\right)\sin(dx+c)A}{6d\cos(dx+c)^{\frac{5}{2}}\sqrt{b\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\left(2\cos(dx+c)^2+1\right)}{3d\cos(dx+c)^{\frac{5}{2}}\sqrt{b\cos(dx+c)}}+\frac{B\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+\right.}{2d\cos(dx+c)^{\frac{3}{2}}\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i\left(3B\,e^{4i(dx+c)}-6C\,e^{3i(dx+c)}-3B+(-16A-18C)\cos(dx+c)+i(-8A-6C)\sin(dx+c)\right)}{6\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)^2 d}+\frac{\sqrt{\cos(dx+c)}\,B\ln\left(e^{i(dx+c)}+i\right)}{2\sqrt{b\cos(dx+c)}\,d}-$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
1/6/d*(-3*B*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^3+3*B*ln(-cot(d*x+c)+c
sc(d*x+c)+1)*cos(d*x+c)^3+(4*cos(d*x+c)^2+2)*sin(d*x+c)*A+6*C*cos(d*x+c)^2
*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.78

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[\frac{3\,B\sqrt{b}\cos\left(dx+c\right)^4\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2\left(2\,A+3\,C\right)\cos\ldots\right.}{12\,bd\cos\left(dx+c\right)^4}\right.$$
$$-\frac{3\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^4-\left(2\left(2\,A+3\,C\right)\cos\left(dx+c\right)^2+3\,B\cos\left(dx+\ldots\right.\right.}{6\,bd\cos\left(dx+c\right)^4}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*
x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*
x + c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt
(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^4), -1
/6*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt
(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(
d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d
*cos(d*x + c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c)
)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(128) = 256$.

Time = 0.35 (sec) , antiderivative size = 1014, normalized size of antiderivative = 6.67

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)\sqrt{b\cos(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="maxima")
```

output
```
1/12*(24*C*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x +
2*c)^2 + 2*b*cos(2*d*x + 2*c) + b) + 16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*
x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c
)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((2*(3*cos(4*d
*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2
+ 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*c
os(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*
c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(b)) - 3*
(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)
*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
+ 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{7/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c +
 d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c +
 d*x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{b}\left(-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2 b+3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b+3\,\text{co}\right.}{}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),
x)
```

output
```
(sqrt(b)*( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b +
3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 3*cos(c + d*x)*log(tan((c + d
*x)/2) + 1)*sin(c + d*x)**2*b - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b
 - 3*cos(c + d*x)*sin(c + d*x)*b + 4*sin(c + d*x)**3*a + 6*sin(c + d*x)**3
*c - 6*sin(c + d*x)*a - 6*sin(c + d*x)*c))/(6*cos(c + d*x)*b*d*(sin(c + d*
x)**2 - 1))
```

3.322 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 193

$$\begin{aligned}
&\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{B\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}
\end{aligned}$$

output
```
1/8*(3*A+4*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+
1/4*A*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*sin
(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/d/cos(d*x+c)^
(1/2)/(b*cos(d*x+c))^(1/2)+1/3*B*sin(d*x+c)^3/d/cos(d*x+c)^(5/2)/(b*cos(d*
x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.57

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{3(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\left(6A+3(3A+4C)\cos^2(c+dx)+24B\cos^3\right.}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt
[b*Cos[c + d*x]]),x]
```

output
```
(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A +
3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[
c + d*x]^2))/(24*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {2032, 3042, 3500, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^5(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\int (4B+(3A+4C)\cos(c+dx))\sec^4(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\int \frac{4B+(3A+4C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int \sec^3(c+dx)dx+4B\int \sec^4(c+dx)dx\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+4B\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4B\int (\tan^2(c+dx)+1)d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+d}{}\right.}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\,\text{ta}}{}\right.}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+dx)}{}\right.}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos
[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A + 4*C)*
(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (4*B*(
-Tan[c + d*x] - Tan[c + d*x]^3/3))/d)/4))/Sqrt[b*Cos[c + d*x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.04

method	result
default	$\frac{-9A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-12C\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+9A\ln(-\cot(dx+c)+\csc(dx+c}{}$
parts	$-\frac{A\left(3\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-3\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^4-3\cos(dx+c)^2\sin(dx+c)-2\sin(\right.}{8d\cos(dx+c)^{\frac{7}{2}}\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i(9A\,e^{6i(dx+c)}+12C\,e^{6i(dx+c)}+33A\,e^{4i(dx+c)}+12C\,e^{4i(dx+c)}-48B\,e^{3i(dx+c)}-33A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-9A-12C-80B\,c}{24\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)^3d}$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
1/24/d*(-9*A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^4-12*C*ln(-cot(d*x+c)
+csc(d*x+c)-1)*cos(d*x+c)^4+9*A*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+
12*C*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+(9*cos(d*x+c)^2+6)*sin(d*x+
c)*A+sin(d*x+c)*cos(d*x+c)*(16*cos(d*x+c)^2+8)*B+12*C*cos(d*x+c)^2*sin(d*x
+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.58

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[\frac{3\,(3\,A+4\,C)\sqrt{b}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,\left(16\,B\,\mathrm{c}\right.}{48\,bd}\right.$$
$$-\frac{3\,(3\,A+4\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left(16\,B\cos\left(dx+c\right)^3+3\,(3\,A+4\,C\right.}{24\,bd\cos\left(dx+c\right)^5}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqr
t(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x +
c))/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^
2 + 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(b*d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*
cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5
 - (16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c)
+ 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x
+ c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(b*cos(d*x+c)
)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. $2(165) = 330$.

Time $= 0.35$ (sec) , antiderivative size $= 2611$, normalized size of antiderivative $= 13.53$

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
 4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2
*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(si
n(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2
*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x
+ 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*
c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8
*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x
+ 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c
) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x
+ 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(
2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^
(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{9/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(9/2)*(b*cos(c +
 d*x))^(1/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(9/2)*(b*cos(c +
 d*x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.81

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{\sqrt{b}\,\left(-16\cos\left(dx+c\right)\sin\left(dx+c\right)^3b+24\cos\left(dx+c\right)\sin\left(dx+c\right)b-9\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)\right.}{}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),
x)
```

output
```
(sqrt(b)*( - 16*cos(c + d*x)*sin(c + d*x)**3*b + 24*cos(c + d*x)*sin(c + d
*x)*b - 9*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 12*log(tan((c + d*
x)/2) - 1)*sin(c + d*x)**4*c + 18*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**
2*a + 24*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 9*log(tan((c + d*x)
/2) - 1)*a - 12*log(tan((c + d*x)/2) - 1)*c + 9*log(tan((c + d*x)/2) + 1)*
sin(c + d*x)**4*a + 12*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c - 18*lo
g(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - 24*log(tan((c + d*x)/2) + 1)*s
in(c + d*x)**2*c + 9*log(tan((c + d*x)/2) + 1)*a + 12*log(tan((c + d*x)/2)
 + 1)*c - 9*sin(c + d*x)**3*a - 12*sin(c + d*x)**3*c + 15*sin(c + d*x)*a +
 12*sin(c + d*x)*c))/(24*b*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.323 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 199

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}} - \frac{B\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

output

```
1/8*(4*A+3*C)*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+B*cos(d*x+c)^(1/2)
*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*
x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/4*C*cos(d*x+c)^(7/2)*sin(d*x+c)/b/d/(b*cos
(d*x+c))^(1/2)-1/3*B*cos(d*x+c)^(1/2)*sin(d*x+c)^3/b/d/(b*cos(d*x+c))^(1/2
)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.46

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(48Ac+36cC+48Adx+36Cdx}{}$$

input
```
Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*
Cos[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c +
d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c +
 d*x)]))/(96*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.57, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2031, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\int \cos^2(c+dx)(4A+3C+4B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(4A+3C+4B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int \cos^2(c+dx)dx+4B\int \cos^3(c+dx)dx\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+4B\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3113}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4B\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A + 3*C)*
(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (4*B*(-Sin[c + d*x] + Sin[c +
d*x]^3/3))/d)/4))/(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

method	result
default	$\frac{\left(12A(dx+c)+9C(dx+c)+12A\cos(dx+c)\sin(dx+c)+\left(8\cos(dx+c)^{2}+16\right)\sin(dx+c)B+\sin(dx+c)\cos(dx+c)\left(6\cos(dx+c)^{2}+9\right)C}{24bd\sqrt{b\cos(dx+c)}}$
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}\,b}+\frac{B\sin(dx+c)\left(2+\cos(dx+c)^{2}\right)\sqrt{\cos(dx+c)}}{3d\sqrt{b\cos(dx+c)}\,b}+\frac{C\left(2\cos(dx+c)^{3}\sin(dx+c)+3\cos}{8d\sqrt{b}}$
risch	$\frac{\sqrt{\cos(dx+c)}\,(8A+6C)x}{16b\sqrt{b\cos(dx+c)}}+\frac{3B\sqrt{\cos(dx+c)}\,\sin(dx+c)}{4bd\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,C\sin(4dx+4c)}{32b\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,B\sin(3dx+3c)}{12b\sqrt{b\cos(dx+c)}\,d}+\sqrt{\cos}$

input
```
int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),
x,method=_RETURNVERBOSE)
```

output
```
1/24/b/d*(12*A*(d*x+c)+9*C*(d*x+c)+12*A*cos(d*x+c)*sin(d*x+c)+(8*cos(d*x+c
)^2+16)*sin(d*x+c)*B+sin(d*x+c)*cos(d*x+c)*(6*cos(d*x+c)^2+9)*C)*cos(d*x+c
)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.42

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{3\,(4\,A+3\,C)\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqr
t(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(6*C*c
os(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)),
1/24*(3*(4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt
(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x
+ c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c)
)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.58

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{3}{2}}}+\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)}{}}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(3/2),x, algorithm="maxima")
```

output
```
1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(3/2) + 3*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*
C/b^(3/2) + 8*B*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))))/b^(3/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 46.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,A\,\sin\left(c+\right.\right.}{}$$

input
```
int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(c
 + d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*
d*x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x
) + 72*C*d*x*cos(c + d*x)))/(96*b^2*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.44

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\,\left(-6\cos\left(dx+c\right)\sin\left(dx+c\right)^3c+12\cos\left(d\right.\right.}{}$$

input
```
int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),
x)
```

output
```
(sqrt(b)*( - 6*cos(c + d*x)*sin(c + d*x)**3*c + 12*cos(c + d*x)*sin(c + d*
x)*a + 15*cos(c + d*x)*sin(c + d*x)*c - 8*sin(c + d*x)**3*b + 24*sin(c + d
*x)*b + 12*a*d*x + 9*c*d*x))/(24*b**2*d)
```

3.324 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 155

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{(3A+2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

output
```
1/2*B*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+1/3*(3*A+2*C)*cos(d*x+c)^(
1/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)
/b/d/(b*cos(d*x+c))^(1/2)+1/3*C*cos(d*x+c)^(5/2)*sin(d*x+c)/b/d/(b*cos(d*x
+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(6Bc+6Bdx+3(4A+3C)\sin(\cdots}{12d(b\cos\cdots}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*
Cos[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Si
n[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2031, 3042, 3502, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\int \cos(c+dx)(3A+2C+3B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(3A+2C+3B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(\frac{(3A+2C)\sin(c+dx)}{d}+\frac{3B\sin(c+dx)\cos(c+dx)}{2d}+\frac{3Bx}{2}\right)+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) + ((3*B*x)/2 +
((3*A + 2*C)*Sin[c + d*x])/d + (3*B*Cos[c + d*x]*Sin[c + d*x])/(2*d))/3))/
(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{\left(3B(dx+c)+6A\sin(dx+c)+3B\sin(dx+c)\cos(dx+c)+\left(2\cos(dx+c)^{2}+4\right)\sin(dx+c)C\right)\sqrt{\cos(dx+c)}}{6bd\sqrt{b\cos(dx+c)}}$	80
parts	$\frac{A\sin(dx+c)\sqrt{\cos(dx+c)}}{d\sqrt{b\cos(dx+c)}\,b}+\frac{B(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}\,b}+\frac{C\sin(dx+c)\left(2+\cos(dx+c)^{2}\right)\sqrt{\cos(dx+c)}}{3d\sqrt{b\cos(dx+c)}\,b}$	122
risch	$\frac{Bx\sqrt{\cos(dx+c)}}{2b\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4b\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,C\sin(3dx+3c)}{12b\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,B\sin(2dx+2c)}{4b\sqrt{b\cos(dx+c)}\,d}$	138

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),
x,method=_RETURNVERBOSE)
```

output
```
1/6/b/d*(3*B*(d*x+c)+6*A*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)+(2*cos(d*x+c
)^2+4)*sin(d*x+c)*C)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.56

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{3\,B\sqrt{-b}\cos(dx+c)\log\left(2\,b\cos(dx+c)^{2}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*C*cos(d*x + c
)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(b^2*d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d
*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*co
s(d*x + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c)
)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{b^{\frac{3}{2}}} + \frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\mathrm{arc}\right.\right.}{}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(3/2),x, algorithm="maxima")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(3/2) + C*(sin(3*d*x + 3*c) +
 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(3/2) + 12*A*si
n(d*x + c)/b^(3/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(3\,B\,\sin(c+$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*B*sin(c + d*x) + 12*A*sin(2*
c + 2*d*x) + 3*B*sin(3*c + 3*d*x) + 10*C*sin(2*c + 2*d*x) + C*sin(4*c + 4*
d*x) + 12*B*d*x*cos(c + d*x)))/(12*b^2*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.39

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(3\cos\left(dx+c\right)\sin\left(dx+c\right)b-2\sin\left(dx+\right.\right.}{6}$$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),
x)
```

output
```
(sqrt(b)*(3*cos(c + d*x)*sin(c + d*x)*b - 2*sin(c + d*x)**3*c + 6*sin(c +
d*x)*a + 6*sin(c + d*x)*c + 3*b*d*x))/(6*b**2*d)
```

3.325 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 135

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

output
```
A*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/b/(b*
cos(d*x+c))^(1/2)+B*cos(d*x+c)^(1/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1
/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{\cos^{\frac{3}{2}}(c+dx)(2(2A+C)(c+dx)+4B\sin(c+}{4d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*
Cos[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c
 + d*x)]))/(4*d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(Ax+\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*x + (C*x)/2 + (B*Sin[c + d*x])/d + (C*Cos[c + d*x]*
Sin[c + d*x])/(2*d)))/(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{\left(C\cos\left(dx+c\right)\sin\left(dx+c\right)+2A\left(dx+c\right)+2B\sin\left(dx+c\right)+C\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{2bd\sqrt{b\cos\left(dx+c\right)}}$	66
risch	$\frac{\sqrt{\cos\left(dx+c\right)}\,\left(4A+2C\right)x}{4b\sqrt{b\cos\left(dx+c\right)}}+\frac{B\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{bd\sqrt{b\cos\left(dx+c\right)}}+\frac{\sqrt{\cos\left(dx+c\right)}\,C\sin\left(2dx+2c\right)}{4b\sqrt{b\cos\left(dx+c\right)}\,d}$	101
parts	$\frac{A\left(dx+c\right)\sqrt{\cos\left(dx+c\right)}}{db\sqrt{b\cos\left(dx+c\right)}}+\frac{B\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{bd\sqrt{b\cos\left(dx+c\right)}}+\frac{C\left(\cos\left(dx+c\right)\sin\left(dx+c\right)+dx+c\right)\sqrt{\cos\left(dx+c\right)}}{2d\sqrt{b\cos\left(dx+c\right)}\,b}$	110

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),
x,method=_RETURNVERBOSE)
```

output
```
1/2/b/d*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+C*(d*x+c))*cos
(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.61

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{(2\,A+C)\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx\right.\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*c
os(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(C*cos(d*x
+ c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*c
os(d*x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x +
 c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c)
)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{b^{\frac{3}{2}}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}+4}{4\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(3/2),x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(3/2) + 8*A*arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/b^(3/2) + 4*B*sin(d*x + c)/b^(3/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.69

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(C\,\sin(c+d$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + 4*B*sin(2*c +
 2*d*x) + C*sin(3*c + 3*d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)
))/(4*b^2*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\,(\cos\left(dx+c\right)\sin\left(dx+c\right)c+2\sin\left(dx+c\right)}{2b^2d}$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),
x)
```

output
```
(sqrt(b)*(cos(c + d*x)*sin(c + d*x)*c + 2*sin(c + d*x)*b + 2*a*d*x + c*d*x
))/(2*b**2*d)
```

3.326 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 102

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{A\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
B*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+
c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+C*cos(d*x+c)^(1/2)*sin(d*x+c)/b/d/(b*cos
(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\left(Bdx+A\coth^{-1}(\sin(c+dx)) -\right.}{d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*
Cos[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(B*d*x + A*ArcCoth[Sin[c + d*x]] + C*Sin[c + d*x]))/(d
*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2031, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int\frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\int(A+B\cos(c+dx))\sec(c+dx)dx+\frac{C\sin(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\int\frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{C\sin(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3214}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int\sec(c+dx)dx+Bx+\frac{C\sin(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+Bx+\frac{C\sin(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+Bx+\frac{C\sin(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(B*x + (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/
d))/(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

method	result
default	$-\frac{(2A\,\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))-B(dx+c)-C\sin(dx+c))\sqrt{\cos(dx+c)}}{bd\sqrt{b\cos(dx+c)}}$
parts	$-\frac{2A\,\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))\sqrt{\cos(dx+c)}}{db\sqrt{b\cos(dx+c)}}+\frac{B(dx+c)\sqrt{\cos(dx+c)}}{db\sqrt{b\cos(dx+c)}}+\frac{C\sqrt{\cos(dx+c)}\,\sin(dx+c)}{bd\sqrt{b\cos(dx+c)}}$
risch	$\frac{Bx\sqrt{\cos(dx+c)}}{b\sqrt{b\cos(dx+c)}}-\frac{i\sqrt{\cos(dx+c)}\,C\,\mathrm{e}^{i(dx+c)}}{2b\sqrt{b\cos(dx+c)}\,d}+\frac{i\sqrt{\cos(dx+c)}\,C\,\mathrm{e}^{-i(dx+c)}}{2b\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,A\ln(\mathrm{e}^{i(dx+c)}+i)}{b\sqrt{b\cos(dx+c)}\,d}-\frac{\sqrt{\cos(dx+c)}\,A}{b\sqrt{b\cos}}$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),
x,method=_RETURNVERBOSE)
```

output
```
-1/b/d*(2*A*arctanh(-csc(d*x+c)+cot(d*x+c))-B*(d*x+c)-C*sin(d*x+c))*cos(d*
x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.03

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{2\,A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\mathrm{c}}{}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c) + B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x
 + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
- b) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*co
s(d*x + c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sq
rt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*c
os(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x
+ c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(c
os(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c)
)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(d\right.\right.}{b^{\frac{3}{2}}}}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(3/2),x, algorithm="maxima")
```

output
```
1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2) + 4*B*arctan(
sin(d*x + c)/(cos(d*x + c) + 1))/b^(3/2) + 2*C*sin(d*x + c)/b^(3/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}\,\left(C\cos(c+d\,x)^2+B\,\cos(c\right.}{(b\,\cos(c+d\,x))^{3/2}}$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(3/2),x)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\left(\frac{dx}{2}\right.\right.\right.}{b^2d}$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),
x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a + s
in(c + d*x)*c + b*d*x))/(b**2*d)
```

3.327 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 102

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{B\,\mathrm{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
C*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*cos(d*x+
c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos
(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(Cdx\cos(c+dx)+B\coth^{-1}(\sin(c+dx))\cos(\right.}{d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*C
os[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + B*ArcCoth[Sin[c + d*x]]*Cos[c +
d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2032, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^2(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\int (B+C\cos(c+dx))\sec(c+dx)dx+\frac{A\tan(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\int \frac{B+C\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{A\tan(c+dx)}{d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \sec(c+dx)dx+\frac{A\tan(c+dx)}{d}+Cx\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)}{d}+Cx\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+\frac{B\text{arctanh}(\sin(c+dx))}{d}+Cx\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c +
 d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(C*x + (B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x])/
d))/(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{-2B\,\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))\cos(dx+c)+C(dx+c)\cos(dx+c)+A\sin(dx+c)}{bd\sqrt{\cos(dx+c)}\,\sqrt{b\cos(dx+c)}}$	73
parts	$\frac{A\sin(dx+c)}{bd\sqrt{\cos(dx+c)}\,\sqrt{b\cos(dx+c)}}-\frac{2B\,\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))\sqrt{\cos(dx+c)}}{db\sqrt{b\cos(dx+c)}}+\frac{C(dx+c)\sqrt{\cos(dx+c)}}{db\sqrt{b\cos(dx+c)}}$	108
risch	$\frac{Cx\sqrt{\cos(dx+c)}}{b\sqrt{b\cos(dx+c)}}+\frac{i\mathrm{e}^{-i(dx+c)}A}{b\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,B\ln(\mathrm{e}^{i(dx+c)}+i)}{b\sqrt{b\cos(dx+c)}\,d}-\frac{\sqrt{\cos(dx+c)}\,B\ln(\mathrm{e}^{i(dx+c)}-i)}{b\sqrt{b\cos(dx+c)}\,d}$	142

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),
x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-2*B*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)+C*(d*x+c)*cos(d*x+c
)+A*sin(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.11

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{2\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^2+C\right.$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^
(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c)^2 + C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos
(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x +
 c) - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*
d*cos(d*x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x +
c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2
*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c)
)*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c)
)*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b\cos(c + dx))^{3/2}}\,dx = \int \frac{A + B\cos\left(c + dx\right) + C\cos^2\left(c + dx\right)}{\left(b\cos\left(c + dx\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(c + dx\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(b*cos(d*x+c)
)**(3/2),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/((b*cos(c + d*x))**(3/2)
*sqrt(cos(c + d*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.54

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b\cos(c + dx))^{3/2}}\,dx = \frac{\frac{4\,A\sqrt{b}\sin(2\,dx+2\,c)}{b^2\cos(2\,dx+2\,c)^2+b^2\sin(2\,dx+2\,c)^2+2\,b^2\cos(2\,dx+2\,c)+b^2} + \frac{B\left(\log\left(\cos(dx+c)^2\right.\right.}{}}{}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^
(3/2),x, algorithm="maxima")
```

output
```
1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x
+ 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + B*(log(cos(d*x + c)^2 + sin(d*x
 + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*si
n(d*x + c) + 1))/b^(3/2) + 4*C*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(
3/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^
(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
 d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
 d*x))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(-\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b+\cos\left(dx+c\right)\log\right.}{\cos\left(dx+\right.}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),
x)
```

output
```
(sqrt(b)*( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + cos(c + d*x)*log(t
an((c + d*x)/2) + 1)*b + cos(c + d*x)*c*d*x + sin(c + d*x)*a))/(cos(c + d*
x)*b**2*d)
```

3.328 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 120

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
1/2*(A+2*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+
1/2*A*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b/
d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\frac{2C\coth^{-1}(\sin(c+dx))\cos^2(c+dx)+A\text{arctanh}(\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(b\cos(}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*C
os[c + d*x])^(3/2)),x]
```

output
```
(2*C*ArcCoth[Sin[c + d*x]]*Cos[c + d*x]^2 + A*ArcTanh[Sin[c + d*x]]*Cos[c
+ d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(b
*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.209$, Rules used = {2032, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^3(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\int (2B+(A+2C)\cos(c+dx))\sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\int \frac{2B+(A+2C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int \sec(c+dx)dx+2B\int \sec^2(c+dx)dx\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+2B\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2B\int 1d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2B\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left(\frac{(A+2C)\text{arctanh}(\sin(c+dx))}{d}+\frac{2B\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c +
 d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((A + 2*C)*Arc
Tanh[Sin[c + d*x]])/d + (2*B*Tan[c + d*x])/d)/2))/(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.12

method	result
default	$-\frac{A \ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2 - A\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2 + 4C \operatorname{arctanh}(-\csc(dx+c)+\cot(d}{2bd\cos(dx+c)^{\frac{3}{2}}\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i\left(A\,\mathrm{e}^{2i(dx+c)}-A-4B\cos(dx+c)\right)}{2b\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)d} - \frac{\sqrt{\cos(dx+c)}\,(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2b\sqrt{b\cos(dx+c)}\,d} + \frac{\sqrt{\cos(dx+c)}\,(A+2C)\ln(\mathrm{e}^{i(dx+c)}+}{2b\sqrt{b\cos(dx+c)}\,d}$
parts	$\frac{A\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+\sin(dx+c)\right)}{2d\cos(dx+c)^{\frac{3}{2}}b\sqrt{b\cos(dx+c)}} + \frac{B\sin(dx+c}{bd\sqrt{\cos(dx+c)}\,\sqrt{b\,c}}$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),
x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/d*(A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^2-A*ln(-cot(d*x+c)+csc
(d*x+c)+1)*cos(d*x+c)^2+4*C*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)^2-2
*B*sin(d*x+c)*cos(d*x+c)-A*sin(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.99

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{(A+2\,C)\sqrt{b}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cdots}}{\cos(d\cdots}\right.}{\cdots}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-\left(2\,B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cdots}}{2\,b^2d\cos\left(dx+c\right)^3}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^
(3/2),x, algorithm="fricas")
```

output
```
[1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*c
os(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/c
os(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arc
tan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos
(d*x + c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c)
)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(104) = 208$.

Time = 0.38 (sec) , antiderivative size = 802, normalized size of antiderivative = 6.68

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^
(3/2),x, algorithm="maxima")
```

output
```
1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x
+ 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) - (4*(sin(4*d*x + 4*c) + 2*sin(2*
d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(
4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo
g(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*
c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*A/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x +
 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 +
 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) +...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^
(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{3/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
 d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
 d*x))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.64

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{b}\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)b-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\cdots\right.}{\cdots}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),
x)
```

output
```
(sqrt(b)*( - 2*cos(c + d*x)*sin(c + d*x)*b - log(tan((c + d*x)/2) - 1)*sin
(c + d*x)**2*a - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c + log(tan((
c + d*x)/2) - 1)*a + 2*log(tan((c + d*x)/2) - 1)*c + log(tan((c + d*x)/2)
+ 1)*sin(c + d*x)**2*a + 2*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*c - l
og(tan((c + d*x)/2) + 1)*a - 2*log(tan((c + d*x)/2) + 1)*c - sin(c + d*x)*
a))/(2*b**2*d*(sin(c + d*x)**2 - 1))
```

3.329 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 164

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{B\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+1/3*A*
sin(d*x+c)/b/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/2*B*sin(d*x+c)/b/d/
cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*sin(d*x+c)/b/d/cos(d*x
+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{3B\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(4A+3C+3B\cos(c}{6d\sqrt{\cos(c+dx)}(b\cos(c}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*C
os[c + d*x])^(3/2)),x]
```

output
```
(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C + 3*B*Cos[c + d*x]
+ (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(b*C
os[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.66, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {2032, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^4(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\int(3B+(2A+3C)\cos(c+dx))\sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\int\frac{3B+(2A+3C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left((2A+3C)\int\sec^2(c+dx)dx+3B\int\sec^3(c+dx)dx\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left((2A+3C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(3B\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(3B\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(\frac{(2A+3C)\tan(c+dx)}{d}+3B\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c +
 d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (((2*A + 3*C)
*Tan[c + d*x])/d + 3*B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
+ d*x])/(2*d)))/3))/(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

method	result
default	$\frac{-3B\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^3+3B\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^3+\left(4\cos(dx+c)^2+2\right)\sin(dx+c)A}{6bd\cos(dx+c)^{\frac{5}{2}}\sqrt{b\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\left(2\cos(dx+c)^2+1\right)}{3d\cos(dx+c)^{\frac{5}{2}}b\sqrt{b\cos(dx+c)}}+\frac{B\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+\right.}{2d\cos(dx+c)^{\frac{3}{2}}b\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i\left(3B\,e^{4i(dx+c)}-6C\,e^{3i(dx+c)}-3B+(-16A-18C)\cos(dx+c)+i(-8A-6C)\sin(dx+c)\right)}{6b\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\left(e^{2i(dx+c)}+1\right)^2d}+\frac{\sqrt{\cos(dx+c)}\,B\ln\left(e^{i(dx+c)}+i\right)}{2b\sqrt{b\cos(dx+c)}\,d}-$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),
x,method=_RETURNVERBOSE)
```

output
```
1/6/b/d*(-3*B*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^3+3*B*ln(-cot(d*x+c)
+csc(d*x+c)+1)*cos(d*x+c)^3+(4*cos(d*x+c)^2+2)*sin(d*x+c)*A+6*C*cos(d*x+c)
^2*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.65

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{3\,B\sqrt{b}\cos\left(dx+c\right)^4\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+}}{\cos(dx+c)^3}\right.}{}\right.$$

$$-\frac{3\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^4-\left(2\left(2\,A+3\,C\right)\cos\left(dx+c\right)^2+3\,B\cos\left(dx+c\right)\right.}{6\,b^2d\cos\left(dx+c\right)^4}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^
(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*
x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*
x + c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt
(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^4),
-1/6*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*co
s(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b
^2*d*cos(d*x + c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c)
)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. $2(140) = 280$.

Time = 0.39 (sec) , antiderivative size = 1048, normalized size of antiderivative = 6.39

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^
(3/2),x, algorithm="maxima")
```

output
```
1/12*(24*C*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*
x + 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + 16*((3*cos(2*d*x + 2*c) + 1)*
sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d
*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b*co
s(6*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*cos(2*d*x + 2*c)^2 + b*sin
(6*d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x + 4*c) + 3*b*cos(2*d*x
 + 2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4
*c) + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) + b*sin(2*d*x + 2*c))*s
in(6*d*x + 6*c) + b)*sqrt(b)) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4
*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4
*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^
(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A}{\cos(c+d\,x)^{5/2}\,(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
 d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
 d*x))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.13

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(-3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin(dx+c)^2b+3\right.}{}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),
x)
```

output

```
(sqrt(b)*( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b +
3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 3*cos(c + d*x)*log(tan((c + d
*x)/2) + 1)*sin(c + d*x)**2*b - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b
 - 3*cos(c + d*x)*sin(c + d*x)*b + 4*sin(c + d*x)**3*a + 6*sin(c + d*x)**3
*c - 6*sin(c + d*x)*a - 6*sin(c + d*x)*c))/(6*cos(c + d*x)*b**2*d*(sin(c +
 d*x)**2 - 1))
```

3.330 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 208

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{B\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/8*(3*A+4*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2
)+1/4*A*sin(d*x+c)/b/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)
*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b/d/cos
(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/3*B*sin(d*x+c)^3/b/d/cos(d*x+c)^(5/2)
/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.53

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{3(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\,(}{24d\,\text{co}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*C
os[c + d*x])^(3/2)),x]
```

output
```
(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A +
3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[
c + d*x]^2))/(24*d*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.59, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.256, Rules used = {2032, 3042, 3500, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^5(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\int(4B+(3A+4C)\cos(c+dx))\sec^4(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\int\frac{4B+(3A+4C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\sec^3(c+dx)dx+4B\int\sec^4(c+dx)dx\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+4B\int\csc\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4B\int(\tan^2(c+dx)+1)d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+d}{}\right.}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\,\mathrm{ta}}{}\right.}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+dx)}{}\right.}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Cos[c +
 d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A + 4*C)*
(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (4*B*(
-Tan[c + d*x] - Tan[c + d*x]^3/3))/d)/4))/(b*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.98

method	result
default	$\frac{-9A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-12C\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+9A\ln(-\cot(dx+c)+\csc(dx+c}{}$
parts	$-\frac{A\left(3\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-3\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^4-3\cos(dx+c)^2\sin(dx+c)-2\sin(\right.}{8d\cos(dx+c)^{\frac{7}{2}}b\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i(9A\,e^{6i(dx+c)}+12C\,e^{6i(dx+c)}+33A\,e^{4i(dx+c)}+12C\,e^{4i(dx+c)}-48B\,e^{3i(dx+c)}-33A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-9A-12C-80B\,c}{24b\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)^3d}$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),
x,method=_RETURNVERBOSE)
```

output
```
1/24/b/d*(-9*A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^4-12*C*ln(-cot(d*x+
c)+csc(d*x+c)-1)*cos(d*x+c)^4+9*A*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^
4+12*C*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+(9*cos(d*x+c)^2+6)*sin(d*
x+c)*A+sin(d*x+c)*cos(d*x+c)*(16*cos(d*x+c)^2+8)*B+12*C*cos(d*x+c)^2*sin(d
*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.47

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \Bigg[\frac{3\,(3\,A+4\,C)\sqrt{b}\cos{(dx+c)}^5\log\Big(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\cdots}{c\cdots}}{\cdots}$$

$$-\frac{3\,(3\,A+4\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos{(dx+c)}^5-\left(16\,B\cos{(dx+c)}^3+3\,(3\,A+4\,C)\,\mathrm{co}\cdots\right.}{24\,b^2d\cos{(dx+c)}^5}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^
(3/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqr
t(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x +
c))/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^
2 + 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(b^2*d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(
b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)
^5 - (16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c
) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(
d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c)
)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2660 vs. $2(180) = 360$.

Time = 0.33 (sec) , antiderivative size = 2660, normalized size of antiderivative = 12.79

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(b\cos(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^
(3/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
 4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2
*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(si
n(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2
*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x
+ 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*
c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8
*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x
+ 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c
) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x
+ 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(
2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^
(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{7/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c +
 d*x))^(3/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c +
 d*x))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.68

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\,(-16\cos(dx+c)\sin(dx+c)^3 b+24\cos(dx+c)\sin(dx$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),
x)
```

output
```
(sqrt(b)*( - 16*cos(c + d*x)*sin(c + d*x)**3*b + 24*cos(c + d*x)*sin(c + d
*x)*b - 9*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 12*log(tan((c + d*
x)/2) - 1)*sin(c + d*x)**4*c + 18*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**
2*a + 24*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 9*log(tan((c + d*x)
/2) - 1)*a - 12*log(tan((c + d*x)/2) - 1)*c + 9*log(tan((c + d*x)/2) + 1)*
sin(c + d*x)**4*a + 12*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c - 18*lo
g(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - 24*log(tan((c + d*x)/2) + 1)*s
in(c + d*x)**2*c + 9*log(tan((c + d*x)/2) + 1)*a + 12*log(tan((c + d*x)/2)
 + 1)*c - 9*sin(c + d*x)**3*a - 12*sin(c + d*x)**3*c + 15*sin(c + d*x)*a +
 12*sin(c + d*x)*c))/(24*b**2*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.331 $\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 199

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}} - \frac{B\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

output
```
1/8*(4*A+3*C)*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+B*cos(d*x+c)^(1/
2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*si
n(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/4*C*cos(d*x+c)^(7/2)*sin(d*x+c)/b^2/
d/(b*cos(d*x+c))^(1/2)-1/3*B*cos(d*x+c)^(1/2)*sin(d*x+c)^3/b^2/d/(b*cos(d*
x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.48

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(48Ac+36cC+48Adx+36Cd$$

input
```
Integrate[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*
Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c +
d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c +
 d*x)]))/(96*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.57, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2031, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\int\cos^2(c+dx)(4A+3C+4B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\int\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(4A+3C+4B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 3227$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int\cos^2(c+dx)dx+4B\int\cos^3(c+dx)dx\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+4B\int\sin\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 3113$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4B\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 3115$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 24$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((4A+3C)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{4B\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((C*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((4*A + 3*C)*
(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (4*B*(-Sin[c + d*x] + Sin[c +
d*x]^3/3))/d)/4))/(b^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.53

method	result
default	$\frac{\left(12A(dx+c)+9C(dx+c)+12A\cos(dx+c)\sin(dx+c)+\left(8\cos(dx+c)^2+16\right)\sin(dx+c)B+\sin(dx+c)\cos(dx+c)\left(6\cos(dx+c)^2+9\right)C\right.}{24b^2d\sqrt{b\cos(dx+c)}}$
parts	$\frac{A(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}\,b^2}+\frac{B\sin(dx+c)\left(2+\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}}{3d\sqrt{b\cos(dx+c)}\,b^2}+\frac{C\left(2\cos(dx+c)^3\sin(dx+c)+3\cos\right.}{8d\sqrt{b}}$
risch	$\frac{\sqrt{\cos(dx+c)}\,(8A+6C)x}{16b^2\sqrt{b\cos(dx+c)}}+\frac{3B\sqrt{\cos(dx+c)}\,\sin(dx+c)}{4b^2d\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,C\sin(4dx+4c)}{32b^2\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,B\sin(3dx+3c)}{12b^2\sqrt{b\cos(dx+c)}\,d}+\sqrt{\cos}$

input
```
int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x,method=_RETURNVERBOSE)
```

output
```
1/24/b^2/d*(12*A*(d*x+c)+9*C*(d*x+c)+12*A*cos(d*x+c)*sin(d*x+c)+(8*cos(d*x
+c)^2+16)*sin(d*x+c)*B+sin(d*x+c)*cos(d*x+c)*(6*cos(d*x+c)^2+9)*C)*cos(d*x
+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.42

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \left[-\frac{3\,(4\,A+3\,C)\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqr
t(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(6*C*c
os(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)),
1/24*(3*(4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt
(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x
+ c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c)
)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.58

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{5}{2}}}+\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)}{}}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="maxima")
```

output
```
1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(5/2) + 3*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*
C/b^(5/2) + 8*B*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))))/b^(5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 41.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,A\,\sin\left(c+\right.\right.}{}$$

input
```
int((cos(c + d*x)^(9/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(c
 + d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*
d*x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x
) + 72*C*d*x*cos(c + d*x)))/(96*b^3*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.44

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,\left(-6\cos\left(dx+c\right)\sin\left(dx+c\right)^3c+12\cos\left(d\right.\right.}{}$$

input
```
int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x)
```

output
```
(sqrt(b)*( - 6*cos(c + d*x)*sin(c + d*x)**3*c + 12*cos(c + d*x)*sin(c + d*
x)*a + 15*cos(c + d*x)*sin(c + d*x)*c - 8*sin(c + d*x)**3*b + 24*sin(c + d
*x)*b + 12*a*d*x + 9*c*d*x))/(24*b**3*d)
```

3.332 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 155

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{(3A+2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

output

```
1/2*B*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+1/3*(3*A+2*C)*cos(d*x+c)
^(1/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/2*B*cos(d*x+c)^(3/2)*sin(d*
x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/3*C*cos(d*x+c)^(5/2)*sin(d*x+c)/b^2/d/(b
*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.50

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(6Bc+6Bdx+3(4A+3C)\sin}{12b^2d\sqrt{b}}$$

input
```
Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*
Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Si
n[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2031, 3042, 3502, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\int \cos(c+dx)(3A+2C+3B\cos(c+dx))dx+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(3A+2C+3B\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(\frac{(3A+2C)\sin(c+dx)}{d}+\frac{3B\sin(c+dx)\cos(c+dx)}{2d}+\frac{3Bx}{2}\right)+\frac{C\sin(c+dx)\cos^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((C*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) + ((3*B*x)/2 +
((3*A + 2*C)*Sin[c + d*x])/d + (3*B*Cos[c + d*x]*Sin[c + d*x])/(2*d))/3))/
(b^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{\left(3B(dx+c)+6A\sin(dx+c)+3B\sin(dx+c)\cos(dx+c)+\left(2\cos(dx+c)^{2}+4\right)\sin(dx+c)C\right)\sqrt{\cos(dx+c)}}{6b^{2}d\sqrt{b\cos(dx+c)}}$	80
parts	$\frac{A\sin(dx+c)\sqrt{\cos(dx+c)}}{d\sqrt{b\cos(dx+c)}\,b^{2}}+\frac{B(\cos(dx+c)\sin(dx+c)+dx+c)\sqrt{\cos(dx+c)}}{2d\sqrt{b\cos(dx+c)}\,b^{2}}+\frac{C\sin(dx+c)\left(2+\cos(dx+c)^{2}\right)\sqrt{\cos(dx+c)}}{3d\sqrt{b\cos(dx+c)}\,b^{2}}$	122
risch	$\frac{Bx\sqrt{\cos(dx+c)}}{2b^{2}\sqrt{b\cos(dx+c)}}+\frac{\sqrt{\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4b^{2}\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,C\sin(3dx+3c)}{12b^{2}\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,B\sin(2dx+2c)}{4b^{2}\sqrt{b\cos(dx+c)}\,d}$	138

input
```
int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x,method=_RETURNVERBOSE)
```

output
```
1/6/b^2/d*(3*B*(d*x+c)+6*A*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c)+(2*cos(d*x
+c)^2+4)*sin(d*x+c)*C)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.56

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\left[-\frac{3\,B\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*C*cos(d*x + c
)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(b^3*d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d
*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*co
s(d*x + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c)
)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{b^{\frac{5}{2}}}+\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\ \mathrm{arc}\right.\right.}{}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="maxima")
```

output
```
1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(5/2) + C*(sin(3*d*x + 3*c) +
 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(5/2) + 12*A*si
n(d*x + c)/b^(5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(3\,B\,\sin\left(c+\right.\right.}{}$$

input
```
int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*B*sin(c + d*x) + 12*A*sin(2*
c + 2*d*x) + 3*B*sin(3*c + 3*d*x) + 10*C*sin(2*c + 2*d*x) + C*sin(4*c + 4*
d*x) + 12*B*d*x*cos(c + d*x)))/(12*b^3*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.39

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(3\cos\left(dx+c\right)\sin\left(dx+c\right)b-2\sin\left(dx+\right.\right.}{6}$$

input
```
int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x)
```

output
```
(sqrt(b)*(3*cos(c + d*x)*sin(c + d*x)*b - 2*sin(c + d*x)**3*c + 6*sin(c +
d*x)*a + 6*sin(c + d*x)*c + 3*b*d*x))/(6*b**3*d)
```

3.333 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 135

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

output
```
A*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/b^2
/(b*cos(d*x+c))^(1/2)+B*cos(d*x+c)^(1/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(
1/2)+1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(2(2A+C)(c+dx)+4B\sin(c}{4b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*
Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c
 + d*x)]))/(4*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2031, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\cos(c+dx)}\left(Ax+\frac{B\sin(c+dx)}{d}+\frac{C\sin(c+dx)\cos(c+dx)}{2d}+\frac{Cx}{2}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(A*x + (C*x)/2 + (B*Sin[c + d*x])/d + (C*Cos[c + d*x]*
Sin[c + d*x])/(2*d)))/(b^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{\left(C\cos\left(dx+c\right)\sin\left(dx+c\right)+2A\left(dx+c\right)+2B\sin\left(dx+c\right)+C\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{2b^{2}d\sqrt{b\cos\left(dx+c\right)}}$	66
risch	$\frac{\sqrt{\cos\left(dx+c\right)}\,\left(4A+2C\right)x}{4b^{2}\sqrt{b\cos\left(dx+c\right)}}+\frac{B\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{b^{2}d\sqrt{b\cos\left(dx+c\right)}}+\frac{\sqrt{\cos\left(dx+c\right)}\,C\sin\left(2dx+2c\right)}{4b^{2}\sqrt{b\cos\left(dx+c\right)}\,d}$	101
parts	$\frac{A\left(dx+c\right)\sqrt{\cos\left(dx+c\right)}}{d\,b^{2}\sqrt{b\cos\left(dx+c\right)}}+\frac{B\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{b^{2}d\sqrt{b\cos\left(dx+c\right)}}+\frac{C\left(\cos\left(dx+c\right)\sin\left(dx+c\right)+dx+c\right)\sqrt{\cos\left(dx+c\right)}}{2d\sqrt{b\cos\left(dx+c\right)}\,b^{2}}$	110

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2/d*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+C*(d*x+c))*c
os(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.61

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\left[-\frac{(2\,A+C)\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx\right.\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*c
os(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(C*cos(d*x
+ c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*c
os(d*x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x +
 c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c)
)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{b^{\frac{5}{2}}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}+}{4\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(5/2) + 8*A*arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/b^(5/2) + 4*B*sin(d*x + c)/b^(5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.69

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(c+d\right.\right.}{}$$

input
```
int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + 4*B*sin(2*c +
 2*d*x) + C*sin(3*c + 3*d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)
))/(4*b^3*d*(cos(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\,(\cos\left(dx+c\right)\sin\left(dx+c\right)c+2\sin\left(dx+c\right)}{2b^3d}$$

input
```
int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x)
```

output
```
(sqrt(b)*(cos(c + d*x)*sin(c + d*x)*c + 2*sin(c + d*x)*b + 2*a*d*x + c*d*x
))/(2*b**3*d)
```

3.334 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 102

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{A\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

output
```
B*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*
x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+C*cos(d*x+c)^(1/2)*sin(d*x+c)/b^2/d/
(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(Bx+\frac{A\coth^{-1}(\sin(c+dx))}{d}+\frac{C\sin(c}{d}\right.}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*
Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(B*x + (A*ArcCoth[Sin[c + d*x]])/d + (C*Sin[c + d*x])/
d))/(b^2*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2031, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int\frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\int(A+B\cos(c+dx))\sec(c+dx)dx+\frac{C\sin(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\int\frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{C\sin(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int\sec(c+dx)dx+Bx+\frac{C\sin(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(A\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+Bx+\frac{C\sin(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\text{arctanh}(\sin(c+dx))}{d}+Bx+\frac{C\sin(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(B*x + (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/
d))/(b^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

method	result
default	$-\frac{(2A\,\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))-B(dx+c)-C\sin(dx+c))\sqrt{\cos(dx+c)}}{b^{2}d\sqrt{b\cos(dx+c)}}$
parts	$-\frac{2A\,\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))\sqrt{\cos(dx+c)}}{d\,b^{2}\sqrt{b\cos(dx+c)}}+\frac{B(dx+c)\sqrt{\cos(dx+c)}}{d\,b^{2}\sqrt{b\cos(dx+c)}}+\frac{C\sqrt{\cos(dx+c)}\,\sin(dx+c)}{b^{2}d\sqrt{b\cos(dx+c)}}$
risch	$\frac{Bx\sqrt{\cos(dx+c)}}{b^{2}\sqrt{b\cos(dx+c)}}-\frac{i\sqrt{\cos(dx+c)}\,C\,e^{i(dx+c)}}{2b^{2}\sqrt{b\cos(dx+c)}\,d}+\frac{i\sqrt{\cos(dx+c)}\,C\,e^{-i(dx+c)}}{2b^{2}\sqrt{b\cos(dx+c)}\,d}-\frac{\sqrt{\cos(dx+c)}\,A\ln(e^{i(dx+c)}-i)}{b^{2}\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,A}{b^{2}\sqrt{b\cos}}$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x,method=_RETURNVERBOSE)
```

output
```
-1/b^2/d*(2*A*arctanh(-csc(d*x+c)+cot(d*x+c))-B*(d*x+c)-C*sin(d*x+c))*cos(
d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.03

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx=\left[-\frac{2\,A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\mathrm{co}}{}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c) + B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x
 + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
- b) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*co
s(d*x + c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sq
rt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*c
os(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x
+ c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(c
os(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c)
)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(d\right.\right.}{b^{\frac{5}{2}}}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="maxima")
```

output
```
1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2) + 4*B*arctan(
sin(d*x + c)/(cos(d*x + c) + 1))/b^(5/2) + 2*C*sin(d*x + c)/b^(5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^{3/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c\right.}{(b\,\cos(c+d\,x))^{5/2}}$$

input
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\log\left(\tan\left(\frac{dx}{2}\right.\right.\right.}{b^3d}$$

input
```
int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x)
```

output
```
(sqrt(b)*( - log(tan((c + d*x)/2) - 1)*a + log(tan((c + d*x)/2) + 1)*a + s
in(c + d*x)*c + b*d*x))/(b**3*d)
```

3.335 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 102

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
C*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*cos(d*
x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/
(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\left(Cdx\cos(c+dx)+B\coth^{-1}(\mathrm{si}\ldots\right.}{d(b\cos(c+\ldots}$$

input
```
Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*
Cos[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(C*d*x*Cos[c + d*x] + B*ArcCoth[Sin[c + d*x]]*Cos[c +
d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2031, 3042, 3500, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^2(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\int (B+C\cos(c+dx))\sec(c+dx)dx+\frac{A\tan(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\int \frac{B+C\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{A\tan(c+dx)}{d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \sec(c+dx)dx+\frac{A\tan(c+dx)}{d}+Cx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(B\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{A\tan(c+dx)}{d}+Cx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{A\tan(c+dx)}{d}+\frac{B\text{arctanh}(\sin(c+dx))}{d}+Cx\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(C*x + (B*ArcTanh[Sin[c + d*x]])/d + (A*Tan[c + d*x])/
d))/(b^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

method	result	si
default	$\frac{-2B\,\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))\cos(dx+c)+C(dx+c)\cos(dx+c)+A\sin(dx+c)}{b^2 d\sqrt{\cos(dx+c)}\,\sqrt{b\cos(dx+c)}}$	73
parts	$\frac{A\sin(dx+c)}{b^2 d\sqrt{\cos(dx+c)}\,\sqrt{b\cos(dx+c)}}-\frac{2B\,\operatorname{arctanh}(-\csc(dx+c)+\cot(dx+c))\sqrt{\cos(dx+c)}}{d\,b^2\sqrt{b\cos(dx+c)}}+\frac{C(dx+c)\sqrt{\cos(dx+c)}}{d\,b^2\sqrt{b\cos(dx+c)}}$	10
risch	$\frac{Cx\sqrt{\cos(dx+c)}}{b^2\sqrt{b\cos(dx+c)}}+\frac{2i\sqrt{\cos(dx+c)}\,A}{b^2\sqrt{b\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}+\frac{\sqrt{\cos(dx+c)}\,B\ln\left(e^{i(dx+c)}+i\right)}{b^2\sqrt{b\cos(dx+c)}\,d}-\frac{\sqrt{\cos(dx+c)}\,B\ln\left(e^{i(dx+c)}-i\right)}{b^2\sqrt{b\cos(dx+c)}\,d}$	14

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x,method=_RETURNVERBOSE)
```

output
```
1/b^2/d*(-2*B*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)+C*(d*x+c)*cos(d*x
+c)+A*sin(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.11

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx=\left[-\frac{2\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)c\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c)^2 + C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos
(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x +
 c) - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*
d*cos(d*x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x +
c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2
*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c)
)*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c)
)*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c)
)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\frac{4A\sqrt{b}\sin(2dx+2c)}{b^3\cos(2dx+2c)^2+b^3\sin(2dx+2c)^2+2b^3\cos(2dx+2c)+b^3} +$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="maxima")
```

output
```
1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x
+ 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + B*(log(cos(d*x + c)^2 + sin(d*x
 + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*si
n(d*x + c) + 1))/b^(5/2) + 4*C*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(
5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^
(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(c+dx)}\,\left(C\cos(c+dx)^2+B\cos(c+dx)\right.}{(b\cos(c+dx))^{5/2}}$$

input
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(5/2),x)
```

output
```
int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c
+ d*x))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(-\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b+\right.}{}$$

input
```
int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),
x)
```

output
```
(sqrt(b)*( - cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + cos(c + d*x)*log(t
an((c + d*x)/2) + 1)*b + cos(c + d*x)*c*d*x + sin(c + d*x)*a))/(cos(c + d*
x)*b**3*d)
```

3.336 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 120

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \frac{(A+2C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
1/2*(A+2*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2
)+1/2*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c
)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(2C\coth^{-1}(\sin(c+dx))\cos^2(c+dx)+A\text{arcta}\right.}{2d(b\cos}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*C
os[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(2*C*ArcCoth[Sin[c + d*x]]*Cos[c + d*x]^2 + A*ArcTanh[
Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*
(b*Cos[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.209$, Rules used = {2032, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^3(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\int (2B+(A+2C)\cos(c+dx))\sec^2(c+dx)dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\int \frac{2B+(A+2C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int \sec(c+dx)dx+2B\int \sec^2(c+dx)dx\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+2B\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2B\int 1d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left((A+2C)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2B\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\left(\frac{(A+2C)\text{arctanh}(\sin(c+dx))}{d}+\frac{2B\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c +
 d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (((A + 2*C)*Arc
Tanh[Sin[c + d*x]])/d + (2*B*Tan[c + d*x])/d)/2))/(b^2*Sqrt[b*Cos[c + d*x]
])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.12

method	result
default	$-\frac{A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2-A\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2+4C\ \operatorname{arctanh}(-\csc(dx+c)+\cot(d}{2b^2 d\cos(dx+c)^{\frac{3}{2}}\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i\left(A\,e^{2i(dx+c)}-A-4B\cos(dx+c)\right)}{2b^2\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)d}-\frac{\sqrt{\cos(dx+c)}\,(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2b^2\sqrt{b\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)}\,(A+2C)\ln\left(e^{i(dx+c)}+\right.}{2b^2\sqrt{b\cos(dx+c)}\,d}$
parts	$\frac{A\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+\sin(dx+c)\right)}{2d\cos(dx+c)^{\frac{3}{2}}b^2\sqrt{b\cos(dx+c)}}+\frac{B\sin(dx+c}{b^2 d\sqrt{\cos(dx+c)}\,\sqrt{b}}$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),
x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2/d*(A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^2-A*ln(-cot(d*x+c)+c
sc(d*x+c)+1)*cos(d*x+c)^2+4*C*arctanh(-csc(d*x+c)+cot(d*x+c))*cos(d*x+c)^2
-2*B*sin(d*x+c)*cos(d*x+c)-A*sin(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.99

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{(A+2\,C)\sqrt{b}\cos\left(dx+c\right)^3\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{}}{\cos(d}\right.}{}\right.$$

$$-\frac{(A+2\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^3-(2\,B\cos\left(dx+c\right)+A)\sqrt{b\cos\left(dx+c\right)}\sqrt{c}}{2\,b^3d\cos\left(dx+c\right)^3}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^
(5/2),x, algorithm="fricas")
```

output
```
[1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*c
os(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/c
os(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arc
tan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos
(d*x + c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b\cos(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(b*cos(d*x+c)
)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(104) = 208$.

Time = 0.32 (sec) , antiderivative size = 820, normalized size of antiderivative = 6.83

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b\cos(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^
(5/2),x, algorithm="maxima")
```

output
```
1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x
+ 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) - (4*(sin(4*d*x + 4*c) + 2*sin(2*
d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(
4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo
g(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*
c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*A/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4
*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x
+ 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b\cos(c + dx))^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^
(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\sqrt{\cos\left(c+d\,x\right)}\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
 d*x))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
 d*x))^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.64

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{b}\,\left(-2\cos\left(dx+c\right)\sin\left(dx+c\right)b-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),
x)
```

output
```
(sqrt(b)*( - 2*cos(c + d*x)*sin(c + d*x)*b - log(tan((c + d*x)/2) - 1)*sin
(c + d*x)**2*a - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c + log(tan((
c + d*x)/2) - 1)*a + 2*log(tan((c + d*x)/2) - 1)*c + log(tan((c + d*x)/2)
+ 1)*sin(c + d*x)**2*a + 2*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*c - l
og(tan((c + d*x)/2) + 1)*a - 2*log(tan((c + d*x)/2) + 1)*c - sin(c + d*x)*
a))/(2*b**3*d*(sin(c + d*x)**2 - 1))
```

3.337 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 164

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{B\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+1/3*
A*sin(d*x+c)/b^2/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/2*B*sin(d*x+c)/
b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*sin(d*x+c)/b^2/d
/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(3B\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+(4A+}{6d(b\cos(}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*C
os[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C
 + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*(b
*Cos[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.66, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {2032, 3042, 3500, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^4(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3500}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\int(3B+(2A+3C)\cos(c+dx))\sec^3(c+dx)dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\int\frac{3B+(2A+3C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left((2A+3C)\int\sec^2(c+dx)dx+3B\int\sec^3(c+dx)dx\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left((2A+3C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{(2A+3C)\int 1d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(3B\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(3B\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(3B\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{(2A+3C)\tan(c+dx)}{d}\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{3}\left(\frac{(2A+3C)\tan(c+dx)}{d}+3B\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)\right)+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c +
 d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (((2*A + 3*C)
*Tan[c + d*x])/d + 3*B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
+ d*x])/(2*d)))/3))/(b^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

method	result
default	$\frac{-3B\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^3+3B\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^3+\left(4\cos(dx+c)^2+2\right)\sin(dx+c)A}{6b^2d\cos(dx+c)^{\frac{5}{2}}\sqrt{b\cos(dx+c)}}$
parts	$\frac{A\sin(dx+c)\left(2\cos(dx+c)^2+1\right)}{3d\cos(dx+c)^{\frac{5}{2}}b^2\sqrt{b\cos(dx+c)}}+\frac{B\left(\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^2-\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^2+\right.}{2d\cos(dx+c)^{\frac{3}{2}}b^2\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i\left(3B\,e^{4i(dx+c)}-6C\,e^{3i(dx+c)}-3B+(-16A-18C)\cos(dx+c)+i(-8A-6C)\sin(dx+c)\right)}{6b^2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\left(e^{2i(dx+c)}+1\right)^2d}+\frac{\sqrt{\cos(dx+c)}\,B\ln\left(e^{i(dx+c)}+i\right)}{2b^2\sqrt{b\cos(dx+c)}\,d}-$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),
x,method=_RETURNVERBOSE)
```

output
```
1/6/b^2/d*(-3*B*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^3+3*B*ln(-cot(d*x+
c)+csc(d*x+c)+1)*cos(d*x+c)^3+(4*cos(d*x+c)^2+2)*sin(d*x+c)*A+6*C*cos(d*x+
c)^2*sin(d*x+c)+3*B*sin(d*x+c)*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.65

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{3\,B\sqrt{b}\cos\left(dx+c\right)^4\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+}}{\cos(dx+c)^3}\right.}{}\right.$$

$$-\frac{3\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^4-\left(2\left(2\,A+3\,C\right)\cos\left(dx+c\right)^2+3\,B\cos\left(dx+c\right)\right.}{6\,b^3d\cos\left(dx+c\right)^4}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^
(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*
x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*
x + c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt
(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^4),
-1/6*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*co
s(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b
^3*d*cos(d*x + c)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c)
)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(140) = 280$.

Time = 0.32 (sec) , antiderivative size = 1098, normalized size of antiderivative = 6.70

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b\cos(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^
(5/2),x, algorithm="maxima")
```

output
```
1/12*(24*C*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*
x + 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + 16*((3*cos(2*d*x + 2*c) + 1)*
sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d
*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b^2*
cos(6*d*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2 + 9*b^2*cos(2*d*x + 2*c)^2 +
 b^2*sin(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*c)^2 + 18*b^2*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2 + 6*b^2*cos(2*d*x + 2*c) +
b^2 + 2*(3*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x
+ 6*c) + 6*(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 6*(b^2*sin(4*
d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b)) - 3*(4*(sin(
4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d
*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*
x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*
cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x +
 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^
(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{3/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
 d*x))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
 d*x))^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.13

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(-3\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sin\left(dx+c\right)^2 b+3}{}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),
x)
```

output
```
(sqrt(b)*( - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b +
3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b + 3*cos(c + d*x)*log(tan((c + d
*x)/2) + 1)*sin(c + d*x)**2*b - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b
 - 3*cos(c + d*x)*sin(c + d*x)*b + 4*sin(c + d*x)**3*a + 6*sin(c + d*x)**3
*c - 6*sin(c + d*x)*a - 6*sin(c + d*x)*c))/(6*cos(c + d*x)*b**3*d*(sin(c +
 d*x)**2 - 1))
```

3.338 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 208

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx=\frac{(3A+4C)\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/8*(3*A+4*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1
/2)+1/4*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+
4*C)*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b
^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/3*B*sin(d*x+c)^3/b^2/d/cos(d*
x+c)^(5/2)/(b*cos(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.53

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{3(3A+4C)\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\,(}{24d\,\text{co}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*C
os[c + d*x])^(5/2)),x]
```

output
```
(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A +
3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[
c + d*x]^2))/(24*d*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.59, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.256, Rules used = {2032, 3042, 3500, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)\sec^5(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\int(4B+(3A+4C)\cos(c+dx))\sec^4(c+dx)dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\int\frac{4B+(3A+4C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$\downarrow$ 3227

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\sec^3(c+dx)dx+4B\int\sec^4(c+dx)dx\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+4B\int\csc\left(c+dx+\frac{\pi}{2}\right)^4dx\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$\downarrow$ 4254

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4B\int(\tan^2(c+dx)+1)d(-\tan(c+dx))}{d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$\downarrow$ 2009

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$\downarrow$ 4255

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+dx\ldots}{\ldots}\right.}{b^2\sqrt{b\cos(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\,\mathrm{ta}\ldots}{\ldots}\right.}{b^2\sqrt{b\cos(c+dx)}}$$

$\downarrow$ 4257

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{4}\left((3A+4C)\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{4B\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}\right)+\frac{A\tan(c+dx)}{}\right.}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c +
 d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((3*A + 4*C)*
(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (4*B*(
-Tan[c + d*x] - Tan[c + d*x]^3/3))/d)/4))/(b^2*Sqrt[b*Cos[c + d*x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.98

method	result
default	$\frac{-9A\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-12C\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4+9A\ln(-\cot(dx+c)+\csc(dx+c}{}$
parts	$-\frac{A\left(3\ln(-\cot(dx+c)+\csc(dx+c)-1)\cos(dx+c)^4-3\ln(-\cot(dx+c)+\csc(dx+c)+1)\cos(dx+c)^4-3\cos(dx+c)^2\sin(dx+c)-2\sin(\right.}{8d\cos(dx+c)^{\frac{7}{2}}b^2\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i(9A\,e^{6i(dx+c)}+12C\,e^{6i(dx+c)}+33A\,e^{4i(dx+c)}+12C\,e^{4i(dx+c)}-48B\,e^{3i(dx+c)}-33A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-9A-12C-80B\,c}{24b^2\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)^3d}$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),
x,method=_RETURNVERBOSE)
```

output
```
1/24/b^2/d*(-9*A*ln(-cot(d*x+c)+csc(d*x+c)-1)*cos(d*x+c)^4-12*C*ln(-cot(d*
x+c)+csc(d*x+c)-1)*cos(d*x+c)^4+9*A*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c
)^4+12*C*ln(-cot(d*x+c)+csc(d*x+c)+1)*cos(d*x+c)^4+(9*cos(d*x+c)^2+6)*sin(
d*x+c)*A+sin(d*x+c)*cos(d*x+c)*(16*cos(d*x+c)^2+8)*B+12*C*cos(d*x+c)^2*sin
(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.47

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{3\,(3\,A+4\,C)\sqrt{b}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\cdots}{c\cdots}\right)\cdots}{\cdots}\right.$$
$$-\frac{3\,(3\,A+4\,C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left(16\,B\cos\left(dx+c\right)^3+3\,(3\,A+4\,C)\cos\cdots\right.}{24\,b^3d\cos\left(dx+c\right)^5}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^
(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqr
t(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x +
c))/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^
2 + 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(b^3*d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(
b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)
^5 - (16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c
) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(
d*x + c)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c)
)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2760 vs. $2(180) = 360$.

Time $= 0.41$ (sec), antiderivative size $= 2760$, normalized size of antiderivative $= 13.27$

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^
(5/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
 4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2
*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(si
n(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2
*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x
+ 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*
c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8
*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x
+ 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c
) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x
+ 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(
2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^
(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{5/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
 d*x))^(5/2)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
 d*x))^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.68

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(-16\cos\left(dx+c\right)\sin\left(dx+c\right)^3b+24\cos\left(dx+c\right)\sin\left(dx\right.\right.}{}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),
x)
```

output

```
(sqrt(b)*( - 16*cos(c + d*x)*sin(c + d*x)**3*b + 24*cos(c + d*x)*sin(c + d
*x)*b - 9*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**4*a - 12*log(tan((c + d*
x)/2) - 1)*sin(c + d*x)**4*c + 18*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**
2*a + 24*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*c - 9*log(tan((c + d*x)
/2) - 1)*a - 12*log(tan((c + d*x)/2) - 1)*c + 9*log(tan((c + d*x)/2) + 1)*
sin(c + d*x)**4*a + 12*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**4*c - 18*lo
g(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a - 24*log(tan((c + d*x)/2) + 1)*s
in(c + d*x)**2*c + 9*log(tan((c + d*x)/2) + 1)*a + 12*log(tan((c + d*x)/2)
 + 1)*c - 9*sin(c + d*x)**3*a - 12*sin(c + d*x)**3*c + 15*sin(c + d*x)*a +
 12*sin(c + d*x)*c))/(24*b**3*d*(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1))
```

3.339 $\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+\right.$

Optimal result

Integrand size = 39, antiderivative size = 154

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d}-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{11b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^2/d-3/88*(11*A+8*C)*(b*cos(d*x+c)
)^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x
+c)^2)^(1/2)-3/11*B*(b*cos(d*x+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos
(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.76

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3(b\cos(c+dx))^{5/3}\cot(c+dx)\left(-8C\sin^2(c+dx)+(11A+8C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+d\right.\right.}{88bd}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[
c + d*x]^2),x]
```

output
```
(-3*(b*Cos[c + d*x])^(5/3)*Cot[c + d*x]*(-8*C*Sin[c + d*x]^2 + (11*A + 8*C
)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] +
8*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2]))/(88*b*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/3}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{3\int \frac{1}{3}(b\cos(c+dx))^{5/3}(b(11A+8C)+11bB\cos(c+dx))dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\int (b\cos(c+dx))^{5/3}(b(11A+8C)+11bB\cos(c+dx))dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}\left(b(11A+8C)+11bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b}$$

$$\Big\downarrow \textbf{3227}$$

$$\frac{\frac{b(11A+8C)\int (b\cos(c+dx))^{5/3}dx+11B\int (b\cos(c+dx))^{8/3}dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{b(11A+8C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx+11B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{8/3}dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b}$$

$$\Big\downarrow \textbf{3122}$$

$$\frac{\frac{-\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{11/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}}{11b}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*
x]^2),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b*d) + ((-3*(11*A + 8*C)*(b
*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[
c + d*x])/(8*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hyperg
eometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c
 + d*x]^2]))/(11*b))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos(dx+c)\,(b\cos(dx+c))^{\frac{2}{3}}\left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)dx$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
 + c))^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*c
os(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*c
os(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*
x)^2),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*
x)^2), x)
```

Reduce [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = b^{\frac{2}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{11}{3}}dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{8}{3}}dx\right)b\right)$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**(2/3)*cos(c + d*x),x)*a + int(cos(c + d*x)**(2
/3)*cos(c + d*x)**3,x)*c + int(cos(c + d*x)**(2/3)*cos(c + d*x)**2,x)*b)
```

3.340 $\int (b\cos(c+dx))^{2/3} (A + B\cos(c+dx) + C\cos^2(c+$

Optimal result

Integrand size = 33, antiderivative size = 154

$$\int (b\cos(c+dx))^{2/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \, dx = \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40bd\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(5
/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2
)^(1/2)-3/8*B*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2
)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3(b\cos(c+dx))^{2/3}\left(2(8A+5C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}\right.}{}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x
]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*(2*(8*A + 5*C)*Cot[c + d*x]*Hypergeometric2F1[1
/2, 5/6, 11/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 10*B*Cos[c + d*x]*Co
t[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d
*x]^2] - 5*C*Sin[2*(c + d*x)]))/(80*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{3\int \frac{1}{3}(b\cos(c+dx))^{2/3}(b(8A+5C)+8bB\cos(c+dx))dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (b\cos(c+dx))^{2/3}(b(8A+5C)+8bB\cos(c+dx))dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}$$

$\Big\downarrow$ 3042

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(b(8A+5C)+8bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}$$

$\Big\downarrow$ 3227

$$\frac{b(8A+5C)\int (b\cos(c+dx))^{2/3}dx+8B\int (b\cos(c+dx))^{5/3}dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}$$

$\Big\downarrow$ 3042

$$\frac{b(8A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx+8B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}$$

$\Big\downarrow$ 3122

$$\frac{-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\right.}{bd\sqrt{\sin^2(c+dx)}}}{8b}$$
$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}$$

input
```
Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((-3*(8*A + 5*C)*(b*Co
s[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(5*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hypergeom
etric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*
x]^2]))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{2}{3}} \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right) dx$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3), x
)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3),
x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \int \left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \int (b\cos(c+dx))^{2/3} \left(C\cos(c+dx)^2 + B\cos(c+dx) + A\right) dx$$

input
```
int((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

output
```
int((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = b^{\frac{2}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}dx\right)b+\left(\int \cos\left(dx+c\right)^{\frac{8}{3}}dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**(2/3),x)*a + int(cos(c + d*x)**(2/3)*cos(c + d
*x),x)*b + int(cos(c + d*x)**(2/3)*cos(c + d*x)**2,x)*c)
```

3.341 $\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 39, antiderivative size = 148

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

output

```
3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2/3
)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/
2)-3/5*B*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*si
n(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\frac{b\Big(-3(5A+2C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}-6B\cos}{10d\sqrt[3]{b}}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x],x]
```

output
```
(b*(-3*(5*A + 2*C)*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d
*x]^2]*Sqrt[Sin[c + d*x]^2] - 6*B*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric
2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 3*C*Sin[2*(c +
d*x)]))/(10*d*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.205$, Rules used = {3042, 2030, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\sqrt[3]{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$\Big\downarrow$ 3502

$$b\left(\frac{3\int \frac{b(5A+2C)+5bB\cos(c+dx)}{3\sqrt[3]{b\cos(c+dx)}}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}\right)$$

$\Big\downarrow$ 27

$$b\left(\frac{\int \frac{b(5A+2C)+5bB\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}\right)$$

$\Big\downarrow$ 3042

$$b\left(\frac{\int \frac{b(5A+2C)+5bB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}\right)$$

$\Big\downarrow$ 3227

$$b\left(\frac{b(5A+2C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx+5B\int (b\cos(c+dx))^{2/3}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}\right)$$

$\Big\downarrow$ 3042

$$b\left(\frac{b(5A+2C)\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+5B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}\right.$$

$\Big\downarrow$ 3122

$$b\left(\frac{-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{bd\sqrt{\sin^2(c+dx)}}}{5b}\right.$$

input
```
Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x],x]
```

output
```
b*((3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + ((-3*(5*A + 2*C)*(b
*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[
c + d*x])/(2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hyperge
ometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c +
 d*x]^2]))/(5*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{2}{3}} \left(A+B\cos(dx+c)+C\cos(dx+c)^{2}\right)\sec(dx+c)\,dx$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos(dx+c)^{2}+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*s
ec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*s
ec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \frac{(b\,\cos(c+d\,x))^{2/3}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)}\,dx$$

input
```
int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x),x)
```

output
```
int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=b^{\frac{2}{3}}\left(\left(\int \cos(dx+c)^{\frac{5}{3}}\sec(dx+c)\,dx\right)b+\left(\int \cos(dx+c)^{\frac{8}{3}}\sec(dx+c)\,dx\right)c+\left(\int \cos(dx+c)^{\frac{2}{3}}\sec(dx+c)\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**(2/3)*cos(c + d*x)*sec(c + d*x),x)*b + int(cos
(c + d*x)**(2/3)*cos(c + d*x)**2*sec(c + d*x),x)*c + int(cos(c + d*x)**(2/
3)*sec(c + d*x),x)*a)
```

3.342 $\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 147

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}-\frac{3B(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}+\frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

output
```
3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)-3/2*B*(b*cos(d*x+c))^(2/3)*hyperge
om([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)+3/5*(2
*A-C)*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d
*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$
$$-\frac{3b\left(-10A\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)+\cot(c+dx)\left(5B\,\text{Hypergeometric2F}\ldots\right)\right)}{10d\sqrt[3]{b\cos\ldots}}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^2,x]
```

output
```
(-3*b*(-10*A*Csc[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2
] + Cot[c + d*x]*(5*B*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2
*C*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]))*Sqrt[S
in[c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^2\left(\frac{3\int \frac{b^2B-b^2(2A-C)\cos(c+dx)}{3\sqrt[3]{b\cos(c+dx)}}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$b^2\left(\frac{\int \frac{b^2B-b^2(2A-C)\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\int \frac{b^2B-b^2(2A-C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(\frac{b^2B\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx-b(2A-C)\int (b\cos(c+dx))^{2/3}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{b^2B\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b(2A-C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{\frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}}-\frac{3bB\sin(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3}\right.}{2d\sqrt{\sin^2(c+dx)}}}{b^3}\right.$$

input
```
Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^2,x]
```

output
```
b^2*((3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) + ((-3*b*B*(b*Cos[c +
 d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x]
)/(2*d*Sqrt[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hyperge
ometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*Sqrt[Sin[c +
 d*x]^2]))/b^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{2}{3}} \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)\sec(dx+c)^2\,dx$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*s
ec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*s
ec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \frac{(b\cos(c+dx))^{2/3}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)}{\cos(c+dx)^2}\,dx$$

input
```
int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^2,x)
```

output
```
int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^2, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=b^{\frac{2}{3}}\left(\left(\int \cos(dx+c)^{\frac{5}{3}}\sec(dx+c)^2\,dx\right)b+\left(\int \cos(dx+c)^{\frac{8}{3}}\sec(dx+c)^2\,dx\right)c+\left(\int \cos(dx+c)^{\frac{2}{3}}\sec(dx+c)^2\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**(2/3)*cos(c + d*x)*sec(c + d*x)**2,x)*b + int(
cos(c + d*x)**(2/3)*cos(c + d*x)**2*sec(c + d*x)**2,x)*c + int(cos(c + d*x
)**(2/3)*sec(c + d*x)**2,x)*a)
```

3.343 $\int (b\cos(c+dx))^{2/3} (A + B\cos(c+dx) + C\cos^2(c+dx)) dx$

Optimal result

Integrand size = 41, antiderivative size = 145

$$\int (b\cos(c+dx))^{2/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3bB\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)+3*b*B*hypergeom([-1/6, 1/2],[5
/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3
/8*(A+4*C)*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*s
in(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$
$$-\frac{3(b\cos(c+dx))^{2/3}\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+2\cos(c+dx)\left(-2B\,\text{H}\right.\right.}{}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^3,x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2,
1/3, Cos[c + d*x]^2]) + 2*Cos[c + d*x]*(-2*B*Hypergeometric2F1[-1/6, 1/2,
5/6, Cos[c + d*x]^2] + C*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos
[c + d*x]^2]))*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/3}}dx$$

$\downarrow$ 3500

$$b^3 \left(\frac{3 \int \frac{4Bb^2+(A+4C)\cos(c+dx)b^2}{3(b\cos(c+dx))^{4/3}}dx}{4b^3} + \frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}} \right)$$

$\downarrow$ 27

$$b^3 \left(\frac{\int \frac{4Bb^2+(A+4C)\cos(c+dx)b^2}{(b\cos(c+dx))^{4/3}}dx}{4b^3} + \frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}} \right)$$

$\downarrow$ 3042

$$b^3 \left(\frac{\int \frac{4Bb^2+(A+4C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{4b^3} + \frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}} \right)$$

$\downarrow$ 3227

$$b^3 \left(\frac{b(A+4C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx + 4b^2B\int \frac{1}{(b\cos(c+dx))^{4/3}}dx}{4b^3} + \frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}} \right)$$

$\downarrow$ 3042

$$b^3 \left(\frac{b(A+4C)\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx + 4b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{4b^3} + \frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}} \right)$$

$\downarrow$ 3122

$$b^3 \left(\frac{\frac{12bB\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}} - \frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx\right.}{2d\sqrt{\sin^2(c+dx)}}}{4b^3} \right.$$

input
```
Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^3,x]
```

output
```
b^3*((3*A*Sin[c + d*x])/(4*b*d*(b*Cos[c + d*x])^(4/3)) + ((12*b*B*Hypergeo
metric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x]
)^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hyperg
eometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*Sqrt[Sin[c +
 d*x]^2]))/(4*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{2}{3}} \left(A + B\cos(dx+c) + C\cos(dx+c)^2\right)\sec(dx+c)^3\,dx$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*s
ec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*s
ec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^3,x)
```

output
```
int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^3, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=b^{\frac{2}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}\sec\left(dx+c\right)^3dx\right)b+\left(\int \cos\left(dx+c\right)^{\frac{8}{3}}\sec\left(dx+c\right)^3dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}\sec\left(dx+c\right)^3dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**(2/3)*cos(c + d*x)*sec(c + d*x)**3,x)*b + int(
cos(c + d*x)**(2/3)*cos(c + d*x)**2*sec(c + d*x)**3,x)*c + int(cos(c + d*x
)**(2/3)*sec(c + d*x)**3,x)*a)
```

3.344 $\int (b\cos(c+dx))^{2/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^4(c+dx)\, dx$

Optimal result

Integrand size = 41, antiderivative size = 152

$$\int (b\cos(c+dx))^{2/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^4(c+dx)\, dx = \frac{3Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3b^2B\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}} + \frac{3b(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3/7*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/4*b^2*B*hypergeom([-2/3, 1/2
],[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/
2)+3/7*b*(4*A+7*C)*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/
(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \frac{3(b\cos(c+dx))^{2/3}\csc(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)+7\cos(c+d\ldots\right.}{}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^4,x]
```

output
```
(3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -
1/6, Cos[c + d*x]^2] + 7*Cos[c + d*x]*(B*Hypergeometric2F1[-2/3, 1/2, 1/3,
 Cos[c + d*x]^2] + 4*C*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[
c + d*x]^2]))*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]^2])/(28*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \; 2030$$

$$b^4\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{10/3}}dx$$

$$\Big\downarrow \; 3500$$

$$b^4\left(\frac{3\int \frac{7Bb^2+(4A+7C)\cos(c+dx)b^2}{3(b\cos(c+dx))^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \; 27$$

$$b^4\left(\frac{\int \frac{7Bb^2+(4A+7C)\cos(c+dx)b^2}{(b\cos(c+dx))^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^4\left(\frac{\int \frac{7Bb^2+(4A+7C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \; 3227$$

$$b^4\left(\frac{b(4A+7C)\int \frac{1}{(b\cos(c+dx))^{4/3}}dx+7b^2B\int \frac{1}{(b\cos(c+dx))^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^4\left(\frac{b(4A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx+7b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \; 3122$$

$$b^4\left(\frac{\frac{3(4A+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}+\frac{21bB\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}}{7b^3}+\frac{3A}{7bd(b}\right.$$

input

```
Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^4,x]
```

output
```
b^4*((3*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/3)) + ((21*b*B*Hypergeo
metric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*
x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1
/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[
c + d*x]^2]))/(7*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{2}{3}} \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)\sec(dx+c)^4\,dx$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*s
ec(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*s
ec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^4,x)
```

output
```
int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^4, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=b^{\frac{2}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}\sec\left(dx+c\right)^4dx\right)b+\left(\int \cos\left(dx+c\right)^{\frac{8}{3}}\sec\left(dx+c\right)^4dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}\sec\left(dx+c\right)^4dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**(2/3)*cos(c + d*x)*sec(c + d*x)**4,x)*b + int(
cos(c + d*x)**(2/3)*cos(c + d*x)**2*sec(c + d*x)**4,x)*c + int(cos(c + d*x
)**(2/3)*sec(c + d*x)**4,x)*a)
```

3.345 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+\right.$

Optimal result

Integrand size = 39, antiderivative size = 154

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d}-\frac{3(13A+10C)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{130b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos(c+dx))^{13/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sin(c+dx)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
3/13*C*(b*cos(d*x+c))^(10/3)*sin(d*x+c)/b^2/d-3/130*(13*A+10*C)*(b*cos(d*x
+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin
(d*x+c)^2)^(1/2)-3/13*B*(b*cos(d*x+c))^(13/3)*hypergeom([1/2, 13/6],[19/6]
,cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.76

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3(b\cos(c+dx))^{7/3}\cot(c+dx)\left(-10C\sin^2(c+dx)+(13A+10C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+\right.\right.}{130b}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[
c + d*x]^2),x]
```

output
```
(-3*(b*Cos[c + d*x])^(7/3)*Cot[c + d*x]*(-10*C*Sin[c + d*x]^2 + (13*A + 10
*C)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]
+ 10*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sqr
t[Sin[c + d*x]^2]))/(130*b*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.205, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{7/3}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{3\int \frac{1}{3}(b\cos(c+dx))^{7/3}(b(13A+10C)+13bB\cos(c+dx))dx}{13b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13bd}}{b}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\int (b\cos(c+dx))^{7/3}(b(13A+10C)+13bB\cos(c+dx))dx}{13b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13bd}}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}\left(b(13A+10C)+13bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{13b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13bd}}{b}$$

$$\Big\downarrow \textbf{3227}$$

$$\frac{\frac{b(13A+10C)\int (b\cos(c+dx))^{7/3}dx+13B\int (b\cos(c+dx))^{10/3}dx}{13b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13bd}}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{b(13A+10C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx+13B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{10/3}dx}{13b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13bd}}{b}$$

$$\Big\downarrow \textbf{3122}$$

$$\frac{\frac{-\frac{3(13A+10C)\sin(c+dx)(b\cos(c+dx))^{10/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx\right.}{bd\sqrt{\sin^2(c+dx)}}}{13b}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*
x]^2),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b*d) + ((-3*(13*A + 10*C)*
(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*S
in[c + d*x])/(10*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(13/3)*Hy
pergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[S
in[c + d*x]^2]))/(13*b))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos(dx+c)\,(b\cos(dx+c))^{\frac{4}{3}}\left(A+B\cos(dx+c)+C\cos(dx+c)^{2}\right)dx$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\,dx=\int \left(C\cos(dx+c)^{2}+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="fricas")
```

output
```
integral((C*b*cos(d*x + c)^4 + B*b*cos(d*x + c)^3 + A*b*cos(d*x + c)^2)*(b
*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^{2}(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*c
os(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*c
os(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*
x)^2),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*
x)^2), x)
```

Reduce [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=b^{\frac{4}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{13}{3}}dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{10}{3}}dx\right)b+\left(\int \cos\left(dx+c\right)^{\frac{7}{3}}dx\right)a\right)$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**(1/3)*cos(c + d*x)**4,x)*c + int(cos(c + d*x
)**(1/3)*cos(c + d*x)**3,x)*b + int(cos(c + d*x)**(1/3)*cos(c + d*x)**2,x)
*a)
```

3.346 $\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+\right.$

Optimal result

Integrand size = 33, antiderivative size = 154

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd}-\frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{70bd\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3/10*C*(b*cos(d*x+c))^(7/3)*sin(d*x+c)/b/d-3/70*(10*A+7*C)*(b*cos(d*x+c))^
(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)
^2)^(1/2)-3/10*B*(b*cos(d*x+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+
c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.74

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\left(-7C\sin^2(c+dx)+(10A+7C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+\right.\right.}{70d}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x
]
```

output
```
(-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(-7*C*Sin[c + d*x]^2 + (10*A + 7*C
)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] +
 7*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2]))/(70*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{3\int \frac{1}{3}(b\cos(c+dx))^{4/3}(b(10A+7C)+10bB\cos(c+dx))dx}{10b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (b\cos(c+dx))^{4/3}(b(10A+7C)+10bB\cos(c+dx))dx}{10b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}$$

$\Big\downarrow$ 3042

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(b(10A+7C)+10bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{10b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}$$

$\Big\downarrow$ 3227

$$\frac{b(10A+7C)\int (b\cos(c+dx))^{4/3}dx+10B\int (b\cos(c+dx))^{7/3}dx}{10b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}$$

$\Big\downarrow$ 3042

$$\frac{b(10A+7C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx+10B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx}{10b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}$$

$\Big\downarrow$ 3122

$$\frac{-\frac{3(10A+7C)\sin(c+dx)(b\cos(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{bd\sqrt{\sin^2(c+dx)}}}{10b}$$
$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b*d) + ((-3*(10*A + 7*C)*(b*
Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[
c + d*x])/(7*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(10/3)*Hyperg
eometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c +
 d*x]^2]))/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos (dx+c))^{\frac{4}{3}} \left(A+B\cos (dx+c)+C\cos (dx+c)^2\right) dx$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="fricas")
```

output
```
integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*c
os(d*x + c))^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3),
x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\, dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\, dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3),
x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\, dx = \int {\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\, dx$$

input
```
int((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

output
```
int((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = b^{\frac{4}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{4}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{10}{3}}dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{7}{3}}dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**(1/3)*cos(c + d*x),x)*a + int(cos(c + d*x)**
(1/3)*cos(c + d*x)**3,x)*c + int(cos(c + d*x)**(1/3)*cos(c + d*x)**2,x)*b)
```

3.347 $\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal result

Integrand size = 39, antiderivative size = 148

$$\begin{aligned}\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c\\+dx)\,dx &= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d}\\&-\frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{28d\sqrt{\sin^2(c+dx)}}\\&-\frac{3B(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
3/7*C*(b*cos(d*x+c))^(4/3)*sin(d*x+c)/d-3/28*(7*A+4*C)*(b*cos(d*x+c))^(4/3
)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/
2)-3/7*B*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*si
n(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx =$$

$$-\frac{3b\sqrt[3]{b\cos(c+dx)}\Big((7A+4C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}+4\ldots}{2\ldots}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x],x]
```

output
```
(-3*b*(b*Cos[c + d*x])^(1/3)*((7*A + 4*C)*Cot[c + d*x]*Hypergeometric2F1[1
/2, 2/3, 5/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 4*B*Cos[c + d*x]*Cot[
c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*
x]^2] - 2*C*Sin[2*(c + d*x)]))/(28*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {3042, 2030, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \sqrt[3]{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A\right)dx$$

$$\Big\downarrow \ 3502$$

$$b\left(\frac{3\int \frac{1}{3}\sqrt[3]{b\cos(c+dx)}(b(7A+4C)+7bB\cos(c+dx))dx}{7b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}\right)$$

$$\Big\downarrow \ 27$$

$$b\left(\frac{\int \sqrt[3]{b\cos(c+dx)}(b(7A+4C)+7bB\cos(c+dx))dx}{7b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}\right)$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(7A+4C)+7bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}\right)$$

$$\Big\downarrow \ 3227$$

$$b\left(\frac{b(7A+4C)\int \sqrt[3]{b\cos(c+dx)}dx+7B\int(b\cos(c+dx))^{4/3}dx}{7b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}\right)$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{b(7A+4C)\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+7B\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx}{7b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}\right.$$

$$\Big\downarrow \ 3122$$

$$b\left(\frac{-\frac{3(7A+4C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{bd\sqrt{\sin^2(c+dx)}}}{7b}\right.$$

input

```
Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + 
d*x],x]
```

output
```
b*((3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) + ((-3*(7*A + 4*C)*(b
*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[
c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/3)*Hyperge
ometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c +
 d*x]^2]))/(7*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{4}{3}} \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right) \sec(dx+c)\,dx$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="fricas")
```

output
```
integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*c
os(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*s
ec(d*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*s
ec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=\int \frac{(b\,\cos(c+d\,x))^{4/3}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)}\,dx$$

input
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x),x)
```

output
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx=b^{\frac{4}{3}}\left(\left(\int \cos(dx+c)^{\frac{4}{3}}\sec(dx+c)\,dx\right)a+\left(\int \cos(dx+c)^{\frac{10}{3}}\sec(dx+c)\,dx\right)c+\left(\int \cos(dx+c)^{\frac{7}{3}}\sec(dx+c)\,dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**(1/3)*cos(c + d*x)*sec(c + d*x),x)*a + int(c
os(c + d*x)**(1/3)*cos(c + d*x)**3*sec(c + d*x),x)*c + int(cos(c + d*x)**(
1/3)*cos(c + d*x)**2*sec(c + d*x),x)*b)
```

3.348 $\int (b\cos(c+dx))^{4/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^2(c+dx)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 145

$$\int (b\cos(c+dx))^{4/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^2(c+dx)\,dx = \frac{3bC\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{4d} - \frac{3b(4A+C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

output

```
3/4*b*C*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d-3/4*b*(4*A+C)*(b*cos(d*x+c))^(1/
3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1
/2)-3/4*B*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*si
n(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.84

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\frac{b^2\left(-6(4A+C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}-6B\cos\right.}{8d(b\cos}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^2,x]
```

output
```
(b^2*(-6*(4*A + C)*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d
*x]^2]*Sqrt[Sin[c + d*x]^2] - 6*B*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric
2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 3*C*Sin[2*(c + d
*x)]))/(8*d*(b*Cos[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3042, 2030, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3502}$$

$$b^2\left(\frac{3\int \frac{b(4A+C)+4bB\cos(c+dx)}{3(b\cos(c+dx))^{2/3}}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}\right)$$

$$\Big\downarrow \text{27}$$

$$b^2\left(\frac{\int \frac{b(4A+C)+4bB\cos(c+dx)}{(b\cos(c+dx))^{2/3}}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\int \frac{b(4A+C)+4bB\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(\frac{b(4A+C)\int \frac{1}{(b\cos(c+dx))^{2/3}}dx+4B\int \sqrt[3]{b\cos(c+dx)}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{b(4A+C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx+4B\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{-\frac{3(4A+C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\text{Hypergeometric2F1}}{bd\sqrt{\sin^2(c+dx)}}}{4b}\right.$$

input

```
Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^2,x]
```

output
```
b^2*((3*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) + ((-3*(4*A + C)*(b
*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[
c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*Hypergeom
etric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*
x]^2]))/(4*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{4}{3}} \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)\sec(dx+c)^2\,dx$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="fricas")
```

output
```
integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*c
os(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*s
ec(d*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
2,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*s
ec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \frac{(b\cos(c+d\,x))^{4/3}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^2}\,dx$$

input
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^2,x)
```

output
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^2, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{4/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^{\frac{4}{3}}\left(\left(\int \cos(dx+c)^{\frac{4}{3}}\sec(dx+c)^2\,dx\right)a + \left(\int \cos(dx+c)^{\frac{10}{3}}\sec(dx+c)^2\,dx\right)c + \left(\int \cos(dx+c)^{\frac{7}{3}}\sec(dx+c)^2\,dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**(1/3)*cos(c + d*x)*sec(c + d*x)**2,x)*a + in
t(cos(c + d*x)**(1/3)*cos(c + d*x)**3*sec(c + d*x)**2,x)*c + int(cos(c + d
*x)**(1/3)*cos(c + d*x)**2*sec(c + d*x)**2,x)*b)
```

3.349 $\int (b\cos(c+dx))^{4/3} (A + B\cos(c+dx) + C\cos^2(c+dx)) dx$

Optimal result

Integrand size = 41, antiderivative size = 145

$$\int (b\cos(c+dx))^{4/3} (A+B\cos(c+dx)+C\cos^2(c+dx)) \sec^3(c+dx)\,dx = \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} - \frac{3bB\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}} + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}$$

output
```
3/2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)-3*b*B*(b*cos(d*x+c))^(1/3)*hyp
ergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)+3/
8*(A-2*C)*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*si
n(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.81

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx =$$
$$-\frac{3b^2\csc(c+dx)\left(-2A\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)+\cos(c+dx)\left(4B\,\text{Hypergeometric2F}\right.\right.}{4d(b\cos(c+}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^3,x]
```

output
```
(-3*b^2*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^
2] + Cos[c + d*x]*(4*B*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] +
C*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]))*Sqrt[Sin
[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/3}}dx$$

$$\Big\downarrow \; 3500$$

$$b^3\left(\frac{3\int \frac{2b^2B-b^2(A-2C)\cos(c+dx)}{3(b\cos(c+dx))^{2/3}}dx}{2b^3}+\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}\right)$$

$$\Big\downarrow \; 27$$

$$b^3\left(\frac{\int \frac{2b^2B-b^2(A-2C)\cos(c+dx)}{(b\cos(c+dx))^{2/3}}dx}{2b^3}+\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^3\left(\frac{\int \frac{2b^2B-b^2(A-2C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{2b^3}+\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}\right)$$

$$\Big\downarrow \; 3227$$

$$b^3\left(\frac{2b^2B\int \frac{1}{(b\cos(c+dx))^{2/3}}dx-b(A-2C)\int \sqrt[3]{b\cos(c+dx)}dx}{2b^3}+\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^3\left(\frac{2b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx-b(A-2C)\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b^3}+\frac{3A\sin(c+dx)}{2bd(b\cos(c+dx))^{2/3}}\right)$$

$$\Big\downarrow \; 3122$$

$$b^3\left(\frac{\frac{3(A-2C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}-\frac{6bB\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}(}{d\sqrt{\sin^2(c+dx)}}}{2b^3}\right.$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^3,x]
```

output
```
b^3*((3*A*Sin[c + d*x])/(2*b*d*(b*Cos[c + d*x])^(2/3)) + ((-6*b*B*(b*Cos[c
 + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(d*Sqrt[Sin[c + d*x]^2]) + (3*(A - 2*C)*(b*Cos[c + d*x])^(4/3)*Hyperge
ometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c +
d*x]^2]))/(2*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{4}{3}} \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)\sec(dx+c)^3\,dx$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="fricas")
```

output
```
integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*c
os(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*s
ec(d*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
3,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*s
ec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{4/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^3,x)
```

output
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^3, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=b^{\frac{4}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{4}{3}}\sec\left(dx+c\right)^3dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{10}{3}}\sec\left(dx+c\right)^3dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{7}{3}}\sec\left(dx+c\right)^3dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**(1/3)*cos(c + d*x)*sec(c + d*x)**3,x)*a + in
t(cos(c + d*x)**(1/3)*cos(c + d*x)**3*sec(c + d*x)**3,x)*c + int(cos(c + d
*x)**(1/3)*cos(c + d*x)**2*sec(c + d*x)**3,x)*b)
```

3.350 $\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$

Optimal result

Integrand size = 41, antiderivative size = 152

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\frac{3Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}}+\frac{3b^2B\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}-\frac{3b(2A+5C)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d\sqrt{\sin^2(c+dx)}}$$

output
```
3/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)+3/2*b^2*B*hypergeom([-1/3, 1/2
],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/
2)-3/5*b*(2*A+5*C)*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x
+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.82

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx =$$
$$-\frac{3(b\cos(c+dx))^{4/3}\csc(c+dx)\left(-2A\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)+5\cos(c+dx)\left(-B\,\text{H}\right.\right.}{}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*S
ec[c + d*x]^4,x]
```

output
```
(-3*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-5/6, 1/2,
 1/6, Cos[c + d*x]^2] + 5*Cos[c + d*x]*(-(B*Hypergeometric2F1[-1/3, 1/2, 2
/3, Cos[c + d*x]^2]) + 2*C*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, C
os[c + d*x]^2]))*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]^2])/(10*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{8/3}}dx$$

$$\Big\downarrow \; 3500$$

$$b^4 \left(\frac{3 \int \frac{5Bb^2+(2A+5C)\cos(c+dx)b^2}{3(b\cos(c+dx))^{5/3}}dx}{5b^3} + \frac{3A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/3}} \right)$$

$$\Big\downarrow \; 27$$

$$b^4 \left(\frac{\int \frac{5Bb^2+(2A+5C)\cos(c+dx)b^2}{(b\cos(c+dx))^{5/3}}dx}{5b^3} + \frac{3A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/3}} \right)$$

$$\Big\downarrow \; 3042$$

$$b^4 \left(\frac{\int \frac{5Bb^2+(2A+5C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx}{5b^3} + \frac{3A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/3}} \right)$$

$$\Big\downarrow \; 3227$$

$$b^4 \left(\frac{b(2A+5C)\int \frac{1}{(b\cos(c+dx))^{2/3}}dx + 5b^2B\int \frac{1}{(b\cos(c+dx))^{5/3}}dx}{5b^3} + \frac{3A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/3}} \right)$$

$$\Big\downarrow \; 3042$$

$$b^4 \left(\frac{b(2A+5C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx + 5b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx}{5b^3} + \frac{3A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/3}} \right)$$

$$\Big\downarrow \; 3122$$

$$b^4 \left(\frac{\frac{15bB\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}} - \frac{3(2A+5C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(\right.}{d\sqrt{\sin^2(c+dx)}}}{5b^3} \right.$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
 d*x]^4,x]
```

output
```
b^4*((3*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/3)) + ((15*b*B*Hypergeo
metric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*
x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*(2*A + 5*C)*(b*Cos[c + d*x])^(1/3)*Hy
pergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c
 + d*x]^2]))/(5*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (b\cos(dx+c))^{\frac{4}{3}} \left(A + B\cos(dx+c) + C\cos(dx+c)^2\right) \sec(dx+c)^4 \, dx$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^{4/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^4(c+dx) \, dx = \int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(b\cos(dx+c))^{\frac{4}{3}} \sec(dx+c)^4 \, dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="fricas")
```

output
```
integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*c
os(d*x + c))^(1/3)*sec(d*x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) \sec^4(c+dx) \, dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c
)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*s
ec(d*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^
4,x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*s
ec(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=\int \frac{(b\,\cos\left(c+d\,x\right))^{4/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^4,x)
```

output
```
int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
 + d*x)^4, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx=b^{\frac{4}{3}}\left(\left(\int \cos\left(dx+c\right)^{\frac{4}{3}}\sec\left(dx+c\right)^4dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{10}{3}}\sec\left(dx+c\right)^4dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{7}{3}}\sec\left(dx+c\right)^4dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**(1/3)*cos(c + d*x)*sec(c + d*x)**4,x)*a + in
t(cos(c + d*x)**(1/3)*cos(c + d*x)**3*sec(c + d*x)**4,x)*c + int(cos(c + d
*x)**(1/3)*cos(c + d*x)**2*sec(c + d*x)**4,x)*b)
```

3.351 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 154

$$\begin{aligned}
&\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx \\
&= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d} \\
&\quad - \frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^3d\sqrt{\sin^2(c+dx)}} \\
&\quad - \frac{3B(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{11b^4d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

output
```
3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^3/d-3/88*(11*A+8*C)*(b*cos(d*x+c)
)^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x
+c)^2)^(1/2)-3/11*B*(b*cos(d*x+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos
(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.86

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx =$$
$$-\frac{3(b\cos(c+dx))^{2/3}\sin(c+dx)\left((11A+8C)\cot^2(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(1/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x]*((11*A + 8*C)*Cot[c + d*x]^2*Hyper
geometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 8*Cos[c
 + d*x]*(-(C*Cos[c + d*x]) + B*Cot[c + d*x]^2*Hypergeometric2F1[1/2, 11/6,
 17/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(88*b*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$\Big\downarrow \text{2030}$$
$$\frac{\int (b\cos(c+dx))^{5/3}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{3\int \frac{1}{3}(b\cos(c+dx))^{5/3}(b(11A+8C)+11bB\cos(c+dx))dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int (b\cos(c+dx))^{5/3}(b(11A+8C)+11bB\cos(c+dx))dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}\left(b(11A+8C)+11bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{b(11A+8C)\int (b\cos(c+dx))^{5/3}dx+11B\int (b\cos(c+dx))^{8/3}dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(11A+8C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx+11B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{8/3}dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{-\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{11/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}}{11b}}{b^2}$$

input

```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(1/3),x]
```

output

```
((3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b*d) + ((-3*(11*A + 8*C)*(b
*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[
c + d*x])/(8*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hyperg
eometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c
 + d*x]^2]))/(11*b))/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos(dx+c)^2 \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)}{\left(b\cos(dx+c)\right)^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\cos(dx+c)^2}{\left(b\cos(dx+c)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
 + c))^(2/3)/b, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
1/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*
x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*
x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(1/3),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(1/3), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{11}{3}}dx\right)c+\left(\int \cos\left(dx+c\right)^{\frac{8}{3}}dx\right)b}{b^{\frac{1}{3}}}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(cos(c + d*x)**4/cos(c + d*x)**(1/3),x)*c + int(cos(c + d*x)**3/cos(c
+ d*x)**(1/3),x)*b + int(cos(c + d*x)**2/cos(c + d*x)**(1/3),x)*a)/b**(1/3
)
```

3.352 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 154

$$\begin{aligned}
&\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx \\
&= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d} \\
&\quad - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40b^2d\sqrt{\sin^2(c+dx)}} \\
&\quad - \frac{3B(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^3d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

output
```
3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b^2/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^
(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+
c)^2)^(1/2)-3/8*B*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+
c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx =$$
$$-\frac{3(b\cos(c+dx))^{2/3}\left(2(8A+5C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+d}\right.}{}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(1/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*(2*(8*A + 5*C)*Cot[c + d*x]*Hypergeometric2F1[1
/2, 5/6, 11/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 10*B*Cos[c + d*x]*Co
t[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d
*x]^2] - 5*C*Sin[2*(c + d*x)]))/(80*b*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \ 2030$$

$$\frac{\int (b\cos(c+dx))^{2/3}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b}$$

$$\Big\downarrow \; 3502$$

$$\frac{\frac{3\int \frac{1}{3}(b\cos(c+dx))^{2/3}(b(8A+5C)+8bB\cos(c+dx))dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int (b\cos(c+dx))^{2/3}(b(8A+5C)+8bB\cos(c+dx))dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(b(8A+5C)+8bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b}$$

$$\Big\downarrow \; 3227$$

$$\frac{\frac{b(8A+5C)\int (b\cos(c+dx))^{2/3}dx+8B\int (b\cos(c+dx))^{5/3}dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{b(8A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx+8B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b}$$

$$\Big\downarrow \; 3122$$

$$\frac{\frac{-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}}{8b}+}{b}$$

input

```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]
)^(1/3),x]
```

output

```
((3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((-3*(8*A + 5*C)*(b*C
os[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(5*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hypergeo
metric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d
*x]^2]))/(8*b))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos (dx+c) \left(A+B\cos (dx+c)+C\cos (dx+c)^{2}\right)}{\left(b\cos (dx+c)\right)^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos (dx+c)^{2}+B\cos (dx+c)+A\right)\cos (dx+c)}{\left(b\cos (dx+c)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/b,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x
+ c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x
+ c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x)
)^(1/3),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x)
)^(1/3), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\frac{\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}dx\right)b+\left(\int \cos\left(dx+c\right)^{\frac{8}{3}}dx\right)c}{b^{\frac{1}{3}}}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(cos(c + d*x)/cos(c + d*x)**(1/3),x)*a + int(cos(c + d*x)**3/cos(c + d
*x)**(1/3),x)*c + int(cos(c + d*x)**2/cos(c + d*x)**(1/3),x)*b)/b**(1/3)
```

3.353 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 154

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10bd\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

output

```
3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2
/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)
^(1/2)-3/5*B*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2
)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= \frac{-3(5A+2C)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}-6B\cos(c+dx)\,\text{co}}{10d\sqrt[3]{b\cos(c+d}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x
]
```

output
```
(-3*(5*A + 2*C)*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]
^2]*Sqrt[Sin[c + d*x]^2] - 6*B*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F1
[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 3*C*Sin[2*(c + d*x
)])/(10*d*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{3\int \frac{b(5A+2C)+5bB\cos(c+dx)}{3\sqrt[3]{b\cos(c+dx)}}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b(5A+2C)+5bB\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(5A+2C)+5bB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b(5A+2C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx+5B\int (b\cos(c+dx))^{2/3}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(5A+2C)\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+5B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \text{3122}$$

$$\frac{-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{1}{6}\right)}{bd\sqrt{\sin^2(c+dx)}}}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + ((-3*(5*A + 2*C)*(b*Co
s[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c +
 d*x])/(2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeome
tric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*
x]^2]))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{A + B\cos(dx + c) + C\cos(dx + c)^2}{(b\cos(dx + c))^{\frac{1}{3}}}dx$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b
*cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(1/3),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(1/3),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/3),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\frac{\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}dx\right)b+\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}dx\right)c+\left(\int \frac{1}{\cos(dx+c)^{\frac{1}{3}}}dx\right)a}{b^{\frac{1}{3}}}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(cos(c + d*x)/cos(c + d*x)**(1/3),x)*b + int(cos(c + d*x)**2/cos(c + d
*x)**(1/3),x)*c + int(1/cos(c + d*x)**(1/3),x)*a)/b**(1/3)
```

3.354 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 149

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} - \frac{3B(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)-3/2*B*(b*cos(d*x+c))^(2/3)*hypergeom
([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)+3/5*(2
*A-C)*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d
*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= \frac{3\left(10A\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)-\cot(c+dx)\left(5B\,\text{Hypergeometric2F1}\left(\right.\right.\right.}{10d\sqrt[3]{b\cos(}}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c
+ d*x])^(1/3),x]
```

output
```
(3*(10*A*Csc[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] -
Cot[c + d*x]*(5*B*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*C*C
os[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]))*Sqrt[Sin[c
 + d*x]^2])/(10*d*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{4/3}}dx$$

$$\Big\downarrow \ 3500$$

$$b\left(\frac{3\int \frac{b^2B-b^2(2A-C)\cos(c+dx)}{3\sqrt[3]{b\cos(c+dx)}}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \ 27$$

$$b\left(\frac{\int \frac{b^2B-b^2(2A-C)\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{\int \frac{b^2B-b^2(2A-C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \ 3227$$

$$b\left(\frac{b^2B\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx-b(2A-C)\int (b\cos(c+dx))^{2/3}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{b^2B\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b(2A-C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}\right)$$

$$\Big\downarrow \ 3122$$

$$b\left(\frac{\frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}}-\frac{3bB\sin(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2}\right.}{2d\sqrt{\sin^2(c+dx)}}}{b^3}\right.$$

input
```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x]
)^(1/3),x]
```

output
```
b*((3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) + ((-3*b*B*(b*Cos[c + d
*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/
(2*d*Sqrt[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeom
etric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*Sqrt[Sin[c + d
*x]^2]))/b^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\left(A + B\cos\left(dx + c\right) + C\cos\left(dx + c\right)^{2}\right)\sec\left(dx + c\right)}{\left(b\cos\left(dx + c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

output
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^{2} + B\cos\left(dx + c\right) + A\right)\sec\left(dx + c\right)}{\left(b\cos\left(dx + c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)/(b*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx$$
$$= \int \frac{(A + B\cos{(c + dx)} + C\cos^2{(c + dx)})\sec{(c + dx)}}{\sqrt[3]{b\cos{(c + dx)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/3
),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c +
d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx$$
$$= \int \frac{\left(C\cos{(dx + c)}^2 + B\cos{(dx + c)} + A\right)\sec{(dx + c)}}{(b\cos{(dx + c)})^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x
+ c))^(1/3), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x
+ c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))
^(1/3)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))
^(1/3)), x)
```

Reduce [F]

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= \frac{\left(\int \frac{\sec(dx+c)}{\cos(dx+c)^{\frac{1}{3}}}dx\right)a + \left(\int \cos(dx+c)^{\frac{2}{3}}\sec(dx+c)\,dx\right)b + \left(\int \cos(dx+c)^{\frac{5}{3}}\sec(dx+c)\,dx\right)c}{b^{\frac{1}{3}}}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(sec(c + d*x)/cos(c + d*x)**(1/3),x)*a + int((cos(c + d*x)*sec(c + d*x
))/cos(c + d*x)**(1/3),x)*b + int((cos(c + d*x)**2*sec(c + d*x))/cos(c + d
*x)**(1/3),x)*c)/b**(1/3)
```

3.355 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 145

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3B\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)+3*B*hypergeom([-1/6, 1/2],[5/6],
cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3/8*(
A+4*C)*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d
*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.80

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx =$$
$$-\frac{3b\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+2\cos(c+dx)\left(-2B\,\text{Hypergeometr}\right.\right.}{4d(b\,\text{co}}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[
c + d*x])^(1/3),x]
```

output
```
(-3*b*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2])
 + 2*Cos[c + d*x]*(-2*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]
+ C*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]))*Sqrt[S
in[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/3}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^2\left(\frac{3\int \frac{4Bb^2+(A+4C)\cos(c+dx)b^2}{3(b\cos(c+dx))^{4/3}}dx}{4b^3}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \text{27}$$

$$b^2\left(\frac{\int \frac{4Bb^2+(A+4C)\cos(c+dx)b^2}{(b\cos(c+dx))^{4/3}}dx}{4b^3}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\int \frac{4Bb^2+(A+4C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{4b^3}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(\frac{b(A+4C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx+4b^2B\int \frac{1}{(b\cos(c+dx))^{4/3}}dx}{4b^3}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{b(A+4C)\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+4b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{4b^3}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{\frac{12bB\sin(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}}{4b^3}\right.$$

input
```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*
x])^(1/3),x]
```

output
```
b^2*((3*A*Sin[c + d*x])/(4*b*d*(b*Cos[c + d*x])^(4/3)) + ((12*b*B*Hypergeo
metric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x]
)^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hyperg
eometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*Sqrt[Sin[c +
 d*x]^2]))/(4*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\left(A+B\cos\left(dx+c\right)+C\cos\left(dx+c\right)^{2}\right)\sec\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

output
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^2/(b*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(A+B\cos\left(c+dx\right)+C\cos^{2}\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)}{\sqrt[3]{b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(
1/3),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c
 + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*
x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*
x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x
))^(1/3)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x
))^(1/3)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\frac{\left(\int \frac{\sec(dx+c)^2}{\cos(dx+c)^{\frac{1}{3}}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}\sec\left(dx+c\right)^2dx\right)b+\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}\sec\left(dx+c\right)^2dx\right)c}{b^{\frac{1}{3}}}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(sec(c + d*x)**2/cos(c + d*x)**(1/3),x)*a + int((cos(c + d*x)*sec(c +
d*x)**2)/cos(c + d*x)**(1/3),x)*b + int((cos(c + d*x)**2*sec(c + d*x)**2)/
cos(c + d*x)**(1/3),x)*c)/b**(1/3)
```

3.356 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 149

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3bB\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}} + \frac{3(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3/7*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/4*b*B*hypergeom([-2/3, 1/2],
[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/2)
+3/7*(4*A+7*C)*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*c
os(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= \frac{3b^2\csc(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)+7\cos(c+dx)\left(B\,\text{Hypergeometric2}\right.\right.}{28d(b\cos}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[
c + d*x])^(1/3),x]
```

output
```
(3*b^2*Csc[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2
] + 7*Cos[c + d*x]*(B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] +
4*C*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]))*Sqrt[
Sin[c + d*x]^2])/(28*d*(b*Cos[c + d*x])^(7/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^3\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{10/3}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^3\left(\frac{3\int \frac{7Bb^2+(4A+7C)\cos(c+dx)b^2}{3(b\cos(c+dx))^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \text{27}$$

$$b^3\left(\frac{\int \frac{7Bb^2+(4A+7C)\cos(c+dx)b^2}{(b\cos(c+dx))^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{\int \frac{7Bb^2+(4A+7C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \text{3227}$$

$$b^3\left(\frac{b(4A+7C)\int \frac{1}{(b\cos(c+dx))^{4/3}}dx+7b^2B\int \frac{1}{(b\cos(c+dx))^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{b(4A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx+7b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$$\Big\downarrow \text{3122}$$

$$b^3\left(\frac{\frac{3(4A+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}+\frac{21bB\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}}{7b^3}+\frac{3A}{7bd(b}\right.$$

input
```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*
x])^(1/3),x]
```

output
```
b^3*((3*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/3)) + ((21*b*B*Hypergeo
metric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*
x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1
/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[
c + d*x]^2]))/(7*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\left(A+B\cos\left(dx+c\right)+C\cos\left(dx+c\right)^{2}\right)\sec\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

output
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^3/(b*cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(
1/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*
x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{(A + B\cos(c+dx) + C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2 + B\cos\left(dx+c\right) + A\right)\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*
x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x
))^(1/3)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x
))^(1/3)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{\left(\int \frac{\sec(dx+c)^3}{\cos(dx+c)^{\frac{1}{3}}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}\sec\left(dx+c\right)^3dx\right)b+\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}\sec\left(dx+c\right)^3dx\right)c}{b^{\frac{1}{3}}}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(sec(c + d*x)**3/cos(c + d*x)**(1/3),x)*a + int((cos(c + d*x)*sec(c +
d*x)**3)/cos(c + d*x)**(1/3),x)*b + int((cos(c + d*x)**2*sec(c + d*x)**3)/
cos(c + d*x)**(1/3),x)*c)/b**(1/3)
```

3.357 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 154

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^4d}-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{88b^4d\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{11b^5d\sqrt{\sin^2(c+dx)}}$$

output

```
3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^4/d-3/88*(11*A+8*C)*(b*cos(d*x+c)
)^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x
+c)^2)^(1/2)-3/11*B*(b*cos(d*x+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos
(d*x+c)^2)*sin(d*x+c)/b^5/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.79

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx =$$

$$-\frac{3\cos^3(c+dx)\cot(c+dx)\left(-8C\sin^2(c+dx)+(11A+8C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\cdots\right.}{88d(b\cos(c+dx\cdots}$$

input
```
Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(4/3),x]
```

output
```
(-3*Cos[c + d*x]^3*Cot[c + d*x]*(-8*C*Sin[c + d*x]^2 + (11*A + 8*C)*Hyperg
eometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 8*B*Cos[
c + d*x]*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d
*x]^2]))/(88*d*(b*Cos[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/3}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^3}$$

$$\Big\downarrow \; 3502$$

$$\frac{\frac{3\int \frac{1}{3}(b\cos(c+dx))^{5/3}(b(11A+8C)+11bB\cos(c+dx))dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int (b\cos(c+dx))^{5/3}(b(11A+8C)+11bB\cos(c+dx))dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}\left(b(11A+8C)+11bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b^3}$$

$$\Big\downarrow \; 3227$$

$$\frac{\frac{b(11A+8C)\int (b\cos(c+dx))^{5/3}dx+11B\int (b\cos(c+dx))^{8/3}dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{b(11A+8C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx+11B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{8/3}dx}{11b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11bd}}{b^3}$$

$$\Big\downarrow \; 3122$$

$$\frac{\frac{-\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{11/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}}{11b}}{b^3}$$

input
```
Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(4/3),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b*d) + ((-3*(11*A + 8*C)*(b
*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[
c + d*x])/(8*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hyperg
eometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c
 + d*x]^2]))/(11*b))/b^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos(dx+c)^3 \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)}{(b\cos(dx+c))^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\cos}{(b\cos(dx+c))^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
 + c))^(2/3)/b^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*
x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*
x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}$$

input
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(4/3),x)
```

output
```
int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(4/3), x)
```

Reduce [F]

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{11}{3}}dx\right.}{b^{\frac{4}{3}}}$$

input
```
int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(cos(c + d*x)**4/cos(c + d*x)**(1/3),x)*c + int(cos(c + d*x)**3/cos(c
+ d*x)**(1/3),x)*b + int(cos(c + d*x)**2/cos(c + d*x)**(1/3),x)*a)/(b**(1/
3)*b)
```

3.358 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 154

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d}$$
$$-\frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{40b^3d\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3B(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^4d\sqrt{\sin^2(c+dx)}}$$

output
```
3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b^3/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^
(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+
c)^2)^(1/2)-3/8*B*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+
c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.79

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx =$$

$$-\frac{3\cos^2(c+dx)\cot(c+dx)\left(-5C\sin^2(c+dx)+(8A+5C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sqrt{}\right.}{40d(b\cos(c+dx)}$$

input
```
Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(4/3),x]
```

output
```
(-3*Cos[c + d*x]^2*Cot[c + d*x]*(-5*C*Sin[c + d*x]^2 + (8*A + 5*C)*Hyperge
ometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 5*B*Cos[
c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x
]^2]))/(40*d*(b*Cos[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{2/3}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b^2}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{3\int \frac{1}{3}(b\cos(c+dx))^{2/3}(b(8A+5C)+8bB\cos(c+dx))dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int (b\cos(c+dx))^{2/3}(b(8A+5C)+8bB\cos(c+dx))dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(b(8A+5C)+8bB\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{b(8A+5C)\int (b\cos(c+dx))^{2/3}dx+8B\int (b\cos(c+dx))^{5/3}dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(8A+5C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx+8B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{8b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}}{8b}+}{b^2}$$

input
```
Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(4/3),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((-3*(8*A + 5*C)*(b*C
os[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(5*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hypergeo
metric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d
*x]^2]))/(8*b))/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos(dx+c)^2 \left(A + B\cos(dx+c) + C\cos(dx+c)^2\right)}{(b\cos(dx+c))^{\frac{4}{3}}} dx$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)\cos}{(b\cos(dx+c))^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/b^
2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*
x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*
x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2\,\left(C\,\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}$$

input
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(4/3),x)
```

output
```
int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(4/3), x)
```

Reduce [F]

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}dx\right)}{b^{\frac{4}{3}}}$$

input
```
int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(cos(c + d*x)/cos(c + d*x)**(1/3),x)*a + int(cos(c + d*x)**3/cos(c + d
*x)**(1/3),x)*c + int(cos(c + d*x)**2/cos(c + d*x)**(1/3),x)*b)/(b**(1/3)*
b)
```

3.359 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 154

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

output

```
3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/b^2/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^
(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c
)^2)^(1/2)-3/5*B*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+
c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{-3(5A+2C)\cot(c+dx)\,\text{Hypergeometric2F1}}{}$$

input
```
Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(4/3),x]
```

output
```
(-3*(5*A + 2*C)*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]
^2]*Sqrt[Sin[c + d*x]^2] - 6*B*Cos[c + d*x]*Cot[c + d*x]*Hypergeometric2F1
[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 3*C*Sin[2*(c + d*x
)])/(10*b*d*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {2030, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$$

$$\Big\downarrow \ 2030$$

$$\frac{\int \frac{C\cos^2(c+dx)+B\cos(c+dx)+A}{\sqrt[3]{b\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \ 3502$$

$$\frac{\frac{3\int \frac{b(5A+2C)+5bB\cos(c+dx)}{3\sqrt[3]{b\cos(c+dx)}}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}}{b}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{b(5A+2C)+5bB\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{b(5A+2C)+5bB\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}}{b}$$

$\Big\downarrow$ 3227

$$\frac{\frac{b(5A+2C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx+5B\int (b\cos(c+dx))^{2/3}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}}{b}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(5A+2C)\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+5B\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{5b}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}}{b}$$

$\Big\downarrow$ 3122

$$\frac{\frac{-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}}{5b}+}{b}$$

input
```
Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]
)^(4/3),x]
```

output
```
((3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + ((-3*(5*A + 2*C)*(b*C
os[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c
+ d*x])/(2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeom
etric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d
*x]^2]))/(5*b))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos(dx+c)\left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)}{(b\cos(dx+c))^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)\cos(c}{(b\cos(dx+c))^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b
^2*cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x
+ c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x
+ c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}$$

input
```
int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x)
)^(4/3),x)
```

output
```
int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x)
)^(4/3), x)
```

Reduce [F]

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}dx\right)b+\left(\int \cos\left(dx+c\right)^{\frac{5}{3}}dx\right)c}{b^{\frac{4}{3}}}$$

input
```
int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(cos(c + d*x)/cos(c + d*x)**(1/3),x)*b + int(cos(c + d*x)**2/cos(c + d
*x)**(1/3),x)*c + int(1/cos(c + d*x)**(1/3),x)*a)/(b**(1/3)*b)
```

3.360 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 152

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}-\frac{3B(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}}+\frac{3(2A-C)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

output

```
3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)-3/2*B*(b*cos(d*x+c))^(2/3)*hyperge
om([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)+3/
5*(2*A-C)*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*s
in(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx =$$
$$-\frac{3\cot(c+dx)\left(-10A\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)+\cos(c+dx)\left(5B\,\text{Hypergeometric2F}\right.\right.}{10d(b\cos(c}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x
]
```

output
```
(-3*Cot[c + d*x]*(-10*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]
+ Cos[c + d*x]*(5*B*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*C
*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]))*Sqrt[Sin
[c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3042, 3500, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 3500}$$

$$\frac{3\int \frac{b^2B-b^2(2A-C)\cos(c+dx)}{3\sqrt[3]{b\cos(c+dx)}}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^2B-b^2(2A-C)\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b^2B-b^2(2A-C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b^2B\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx-b(2A-C)\int (b\cos(c+dx))^{2/3}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2B\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-b(2A-C)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b^3}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}}-\frac{3bB\sin(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\right.}{2d\sqrt{\sin^2(c+dx)}}}{b^3}$$

$$\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) + ((-3*b*B*(b*Cos[c + d*x]
)^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*
d*Sqrt[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometr
ic2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*Sqrt[Sin[c + d*x]
^2]))/b^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{A + B\cos(dx+c) + C\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}dx$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{C\cos{(dx+c)}^2+B\cos{(dx+c)}+A}{(b\cos{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorith
m="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b
^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{C\cos{(dx+c)}^2+B\cos{(dx+c)}+A}{(b\cos{(dx+c)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorith
m="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3),
x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorith
m="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(4/3),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{\left(\int \cos\left(dx+c\right)^{\frac{2}{3}}dx\right)c+\left(\int \frac{1}{\cos(dx+c)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{1}{\cos(dx+c)^{\frac{4}{3}}}\right.}{b^{\frac{4}{3}}}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(cos(c + d*x)/cos(c + d*x)**(1/3),x)*c + int(1/cos(c + d*x)**(1/3),x)*
b + int(1/(cos(c + d*x)**(1/3)*cos(c + d*x)),x)*a)/(b**(1/3)*b)
```

3.361 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 147

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3B\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*A*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)+3*B*hypergeom([-1/6, 1/2],[5/6],co
s(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3/8*(
A+4*C)*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d
*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\csc(c+dx)\left(-A\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)+2\cos(c+dx)\left(-2B\,\text{Hypergeometric2F1}\right.\right.}{4d(b\cos(c}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c
+ d*x])^(4/3),x]
```

output
```
(-3*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]) +
 2*Cos[c + d*x]*(-2*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] +
C*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]))*Sqrt[Sin
[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/3}}dx$$

$$\Big\downarrow \ 3500$$

$$b\left(\frac{3\int \frac{4Bb^2+(A+4C)\cos(c+dx)b^2}{3(b\cos(c+dx))^{4/3}}dx}{4b^3}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \ 27$$

$$b\left(\frac{\int \frac{4Bb^2+(A+4C)\cos(c+dx)b^2}{(b\cos(c+dx))^{4/3}}dx}{4b^3}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{\int \frac{4Bb^2+(A+4C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{4b^3}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \ 3227$$

$$b\left(\frac{b(A+4C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx+4b^2B\int \frac{1}{(b\cos(c+dx))^{4/3}}dx}{4b^3}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{b(A+4C)\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+4b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx}{4b^3}+\frac{3A\sin(c+dx)}{4bd(b\cos(c+dx))^{4/3}}\right)$$

$$\Big\downarrow \ 3122$$

$$b\left(\frac{\frac{12bB\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}}{4b^3}\right.$$

input
```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x]
)^(4/3),x]
```

output
```
b*((3*A*Sin[c + d*x])/(4*b*d*(b*Cos[c + d*x])^(4/3)) + ((12*b*B*Hypergeome
tric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^
(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeo
metric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*Sqrt[Sin[c + d
*x]^2]))/(4*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\left(A+B\cos\left(dx+c\right)+C\cos\left(dx+c\right)^{2}\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}dx$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

output
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(c\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),
x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(4/3
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A\right)\sec\left(c\right.}{\left(b\cos\left(dx + c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),
x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x
+ c))^(4/3), x)
```

Giac [F]

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A\right)\sec\left(c\right.}{\left(b\cos\left(dx + c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),
x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x
+ c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{4/3}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + B\,\cos\left(c + d\,x\right) + A}{\cos\left(c + d\,x\right)\,\left(b\,\cos\left(c + d\,x\right)\right)^{4/3}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))
^(4/3)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))
^(4/3)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\left(\int \frac{\sec(dx+c)}{\cos(dx+c)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{\sec(dx+c)}{\cos(dx+c)^{\frac{4}{3}}}dx\right)a+\left(\int\right.}{b^{\frac{4}{3}}}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(sec(c + d*x)/cos(c + d*x)**(1/3),x)*b + int(sec(c + d*x)/(cos(c + d*x
)**(1/3)*cos(c + d*x)),x)*a + int((cos(c + d*x)*sec(c + d*x))/cos(c + d*x)
**(1/3),x)*c)/(b**(1/3)*b)
```

3.362 $\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 149

$$\begin{aligned}\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}\\ &+ \frac{3B\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}\\ &+ \frac{3(4A+7C)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

output
```
3/7*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/4*B*hypergeom([-2/3, 1/2],[1/3
],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/2)+3/7
*(4*A+7*C)*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos
(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3b^2\cot(c+dx)\left(4A\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\right.\right.}{}$$

input
```
Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[
c + d*x])^(4/3),x]
```

output
```
(3*b^2*Cot[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2
] + 7*Cos[c + d*x]*(B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] +
4*C*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]))*Sqrt[
Sin[c + d*x]^2])/(28*d*(b*Cos[c + d*x])^(10/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {3042, 2030, 3500, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{10/3}}dx$$

$$\Big\downarrow \text{3500}$$

$$b^2\left(\frac{3\int \frac{7Bb^2+(4A+7C)\cos(c+dx)b^2}{3(b\cos(c+dx))^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$\Big\downarrow$ 27

$$b^2\left(\frac{\int \frac{7Bb^2+(4A+7C)\cos(c+dx)b^2}{(b\cos(c+dx))^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$\Big\downarrow$ 3042

$$b^2\left(\frac{\int \frac{7Bb^2+(4A+7C)\sin\left(c+dx+\frac{\pi}{2}\right)b^2}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$\Big\downarrow$ 3227

$$b^2\left(\frac{b(4A+7C)\int \frac{1}{(b\cos(c+dx))^{4/3}}dx+7b^2B\int \frac{1}{(b\cos(c+dx))^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$\Big\downarrow$ 3042

$$b^2\left(\frac{b(4A+7C)\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx+7b^2B\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}}dx}{7b^3}+\frac{3A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/3}}\right)$$

$\Big\downarrow$ 3122

$$b^2\left(\frac{\frac{3(4A+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}+\frac{21bB\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}}{7b^3}+\frac{3A}{7bd(b}\right.$$

input
```
Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*
x])^(4/3),x]
```

output
```
b^2*((3*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/3)) + ((21*b*B*Hypergeo
metric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*
x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1
/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[
c + d*x]^2]))/(7*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [F]

$$\int \frac{\left(A+B\cos\left(dx+c\right)+C\cos\left(dx+c\right)^{2}\right)\sec\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}dx$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

output
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\sec\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^2/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(
4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*
x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\sec\left(\right.}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*
x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^2\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x
))^(4/3)),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x
))^(4/3)), x)
```

Reduce [F]

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\left(\int \frac{\sec(dx+c)^2}{\cos(dx+c)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{\sec(dx+c)^2}{\cos(dx+c)^{\frac{4}{3}}}dx\right)a+\left(\int\right.}{b^{\frac{4}{3}}}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(sec(c + d*x)**2/cos(c + d*x)**(1/3),x)*b + int(sec(c + d*x)**2/(cos(c
 + d*x)**(1/3)*cos(c + d*x)),x)*a + int((cos(c + d*x)*sec(c + d*x)**2)/cos
(c + d*x)**(1/3),x)*c)/(b**(1/3)*b)
```

3.363 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+\right.$

Optimal result

Integrand size = 41, antiderivative size = 232

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)\right.$$
$$\left.+C\cos^2(c+dx)\right)\,dx=\frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(10+3m)}$$
$$-\frac{3b(C(7+3m)+A(10+3m))\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+\right.}{d(7+3m)(10+3m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3bB\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(10+3m),\frac{1}{6}(16+3m),\cos^2(c+dx)\right)\sin(c}{d(10+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
3*b*C*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d/(10+3*m)-3*b*(C*(
7+3*m)+A*(10+3*m))*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7
/6+1/2*m],[13/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(10+3*m)/(sin(d*
x+c)^2)^(1/2)-3*b*B*cos(d*x+c)^(3+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2,
5/3+1/2*m],[8/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(10+3*m)/(sin(d*x+c)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.77

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{4/3}\csc(c+dx)\left(C(7+3m)\sin^2(c+dx)-B(7+3r\right.}{}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Co
s[c + d*x]^2),x]
```

output
```
(3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(C*(7 + 3*m)*S
in[c + d*x]^2 - B*(7 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3 + m/2,
 8/3 + m/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - (C*(7 + 3*m) + A*(10 +
3*m))*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sq
rt[Sin[c + d*x]^2]))/(d*(7 + 3*m)*(10 + 3*m))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{4/3}\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\int \cos^{m+\frac{4}{3}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(\frac{3\int \frac{1}{3}\cos^{m+\frac{4}{3}}(c+dx)\left(3C\left(m+\frac{7}{3}\right)+3A\left(m+\frac{10}{3}\right)+B(3m+10)\cos(c+dx)\right)dx}{3m+10}+\frac{3C\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)}{d(3m+10)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(\frac{\int \cos^{m+\frac{4}{3}}(c+dx)(C(3m+7)+A(3m+10)+B(3m+10)\cos(c+dx))dx}{3m+10}+\frac{3C\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)}{d(3m+10)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}\left(C(3m+7)+A(3m+10)+B(3m+10)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{3m+10}+\frac{3C\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)}{d(3m+10)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(\frac{(A(3m+10)+C(3m+7))\int \cos^{m+\frac{4}{3}}(c+dx)dx+B(3m+10)\int \cos^{m+\frac{7}{3}}(c+dx)dx}{3m+10}+\frac{3C\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)}{d(3m+10)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(\frac{(A(3m+10)+C(3m+7))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}dx+B(3m+10)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{7}{3}}dx}{3m+10}+\frac{3C\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)}{d(3m+10)}\right.}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\left(\frac{-\frac{3(A(3m+10)+C(3m+7))\sin(c+dx)\cos^{m+\frac{7}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+7),\frac{1}{6}(3m+13),\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m-}}{}}{3m+10}\right.}{\sqrt[3]{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c +
d*x]^2),x]
```

output
```
(b*(b*Cos[c + d*x])^(1/3)*((3*C*Cos[c + d*x]^(7/3 + m)*Sin[c + d*x])/(d*(1
0 + 3*m)) + ((-3*(C*(7 + 3*m) + A*(10 + 3*m))*Cos[c + d*x]^(7/3 + m)*Hyper
geometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])
/(d*(7 + 3*m)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(10/3 + m)*Hyperge
ometric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/
(d*Sqrt[Sin[c + d*x]^2]))/(10 + 3*m)))/Cos[c + d*x]^(1/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos (dx+c)^m (b\cos (dx+c))^{\frac{4}{3}} \left(A+B\cos (dx+c)+C\cos (dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3} (A+B\cos(c+dx) +C\cos^2(c+dx))\, dx = \int \left(C\cos (dx+c)^2+B\cos (dx+c)+A\right)(b\cos (dx+c))^{\frac{4}{3}}\cos (dx+c)^m\, dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="fricas")
```

output
```
integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*c
os(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)
**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*c
os(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*c
os(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2), x)
```

Reduce [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=b^{\frac{4}{3}}\left(\left(\int \cos\left(dx+c\right)^{m+\frac{1}{3}}\cos\left(dx+c\right)dx\right)a+\left(\int \cos\left(dx+c\right)^{m+\frac{1}{3}}\cos\left(dx+c\right)^{3}dx\right)c+\left(\int \cos\left(dx+c\right)^{m+\frac{1}{3}}\cos\left(dx+c\right)^{2}dx\right)b\right)$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*b*(int(cos(c + d*x)**((3*m + 1)/3)*cos(c + d*x),x)*a + int(cos(c
+ d*x)**((3*m + 1)/3)*cos(c + d*x)**3,x)*c + int(cos(c + d*x)**((3*m + 1)/
3)*cos(c + d*x)**2,x)*b)
```

3.364 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+\right.$

Optimal result

Integrand size = 41, antiderivative size = 229

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)\right.$$
$$\left.+C\cos^2(c+dx)\right)\,dx=\frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+dx)}{d(8+3m)}$$
$$-\frac{3(C(5+3m)+A(8+3m))\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+\right.}{d(5+3m)(8+3m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3B\cos^{2+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(8+3m),\frac{1}{6}(14+3m),\cos^2(c+dx)\right)\sin(c}{d(8+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
3*C*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/d/(8+3*m)-3*(C*(5+3*m
)+A*(8+3*m))*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 5/6+1/2
*m],[11/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(5+3*m)/(8+3*m)/(sin(d*x+c)^2)
^(1/2)-3*B*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 4/3+1/2*m
],[7/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(8+3*m)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.76

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\csc(c+dx)\left(-\left((C(5+3m)+A(8+3m))\,\mathrm{Hype}\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Co
s[c + d*x]^2),x]
```

output
```
(3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(-((C*(5 + 3*m
) + A*(8 + 3*m))*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c +
 d*x]^2]*Sqrt[Sin[c + d*x]^2]) + (5 + 3*m)*(C*Sin[c + d*x]^2 - B*Cos[c + d
*x]*Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])))/(d*(5 + 3*m)*(8 + 3*m))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{2/3}\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{(b\cos(c+dx))^{2/3}\int \cos^{m+\frac{2}{3}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(b\cos(c+dx))^{2/3}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 3502$$

$$\frac{(b\cos(c+dx))^{2/3}\left(\frac{3\int \frac{1}{3}\cos^{m+\frac{2}{3}}(c+dx)\left(3C\left(m+\frac{5}{3}\right)+3A\left(m+\frac{8}{3}\right)+B(3m+8)\cos(c+dx)\right)dx}{3m+8}+\frac{3C\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)}{d(3m+8)}\right)}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{(b\cos(c+dx))^{2/3}\left(\frac{\int \cos^{m+\frac{2}{3}}(c+dx)(C(3m+5)+A(3m+8)+B(3m+8)\cos(c+dx))dx}{3m+8}+\frac{3C\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)}{d(3m+8)}\right)}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{(b\cos(c+dx))^{2/3}\left(\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}\left(C(3m+5)+A(3m+8)+B(3m+8)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{3m+8}+\frac{3C\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)}{d(3m+8)}\right)}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 3227$$

$$\frac{(b\cos(c+dx))^{2/3}\left(\frac{(A(3m+8)+C(3m+5))\int \cos^{m+\frac{2}{3}}(c+dx)dx+B(3m+8)\int \cos^{m+\frac{5}{3}}(c+dx)dx}{3m+8}+\frac{3C\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)}{d(3m+8)}\right)}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{(b\cos(c+dx))^{2/3}\left(\frac{(A(3m+8)+C(3m+5))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}dx+B(3m+8)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{5}{3}}dx}{3m+8}+\frac{3C\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)}{d(3m+8)}\right)}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \; 3122$$

$$\frac{(b\cos(c+dx))^{2/3}\left(\frac{-\frac{3(A(3m+8)+C(3m+5))\sin(c+dx)\cos^{m+\frac{5}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+5),\frac{1}{6}(3m+11),\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m}}{}}{3m+8}\right)}{\cos^{\frac{2}{3}}(c+dx)}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c +
d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(2/3)*((3*C*Cos[c + d*x]^(5/3 + m)*Sin[c + d*x])/(d*(8 +
 3*m)) + ((-3*(C*(5 + 3*m) + A*(8 + 3*m))*Cos[c + d*x]^(5/3 + m)*Hypergeom
etric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*
(5 + 3*m)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(8/3 + m)*Hypergeometr
ic2F1[1/2, (8 + 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqr
t[Sin[c + d*x]^2]))/(8 + 3*m)))/Cos[c + d*x]^(2/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos (dx+c)^m (b\cos (dx+c))^{\frac{2}{3}} \left(A+B\cos (dx+c)+C\cos (dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\, dx = \int \left(C\cos (dx+c)^2+B\cos (dx+c)+A\right)(b\cos (dx+c))^{\frac{2}{3}}\cos (dx+c)^m\, dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*co
s(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)
**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*c
os(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*c
os(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \cos\left(c+d\,x\right)^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2), x)
```

Reduce [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=b^{\frac{2}{3}}\left(\left(\int \cos\left(dx+c\right)^{m+\frac{2}{3}}dx\right)a+\left(\int \cos\left(dx+c\right)^{m+\frac{2}{3}}\cos\left(dx+c\right)dx\right)b+\left(\int \cos\left(dx+c\right)^{m+\frac{2}{3}}\cos\left(dx+c\right)^{2}dx\right)c\right)$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**(2/3)*(int(cos(c + d*x)**((3*m + 2)/3),x)*a + int(cos(c + d*x)**((3*m +
 2)/3)*cos(c + d*x),x)*b + int(cos(c + d*x)**((3*m + 2)/3)*cos(c + d*x)**2
,x)*c)
```

3.365 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx)+$

Optimal result

Integrand size = 41, antiderivative size = 229

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(7+3m)}$$
$$-\frac{3(C(4+3m)+A(7+3m))\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+\right.}{d(4+3m)(7+3m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{3B\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)\sin(\right.}{d(7+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
3*C*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d/(7+3*m)-3*(C*(4+3*m
)+A*(7+3*m))*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 2/3+1/2
*m],[5/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(4+3*m)/(7+3*m)/(sin(d*x+c)^2)^
(1/2)-3*B*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6+1/2*m]
,[13/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.76

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\left(-\left((C(4+3m)+A(7+3m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4\right.\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Co
s[c + d*x]^2),x]
```

output
```
(3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(-((C*(4 + 3*m
) + A*(7 + 3*m))*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*
x]^2]*Sqrt[Sin[c + d*x]^2]) + (4 + 3*m)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]
*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])))/(d*(4 + 3*m)*(7 + 3*m))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\cos(c+dx)}\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{m+\frac{1}{3}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(\frac{3\int \frac{1}{3}\cos^{m+\frac{1}{3}}(c+dx)\left(3C\left(m+\frac{4}{3}\right)+3A\left(m+\frac{7}{3}\right)+B(3m+7)\cos(c+dx)\right)dx}{3m+7}+\frac{3C\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)}{d(3m+7)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(\frac{\int \cos^{m+\frac{1}{3}}(c+dx)(C(3m+4)+A(3m+7)+B(3m+7)\cos(c+dx))dx}{3m+7}+\frac{3C\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)}{d(3m+7)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}\left(C(3m+4)+A(3m+7)+B(3m+7)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{3m+7}+\frac{3C\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)}{d(3m+7)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(\frac{(A(3m+7)+C(3m+4))\int \cos^{m+\frac{1}{3}}(c+dx)dx+B(3m+7)\int \cos^{m+\frac{4}{3}}(c+dx)dx}{3m+7}+\frac{3C\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)}{d(3m+7)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(\frac{(A(3m+7)+C(3m+4))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}dx+B(3m+7)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}dx}{3m+7}+\frac{3C\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)}{d(3m+7)}\right)}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\left(\frac{-\frac{3(A(3m+7)+C(3m+4))\sin(c+dx)\cos^{m+\frac{4}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+4),\frac{1}{6}(3m+10),\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+\frac{7}{3}}}{}}{3m+7}\right.}{\sqrt[3]{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c +
d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^(1/3)*((3*C*Cos[c + d*x]^(4/3 + m)*Sin[c + d*x])/(d*(7 +
 3*m)) + ((-3*(C*(4 + 3*m) + A*(7 + 3*m))*Cos[c + d*x]^(4/3 + m)*Hypergeom
etric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*
(4 + 3*m)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(7/3 + m)*Hypergeometr
ic2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqr
t[Sin[c + d*x]^2]))/(7 + 3*m)))/Cos[c + d*x]^(1/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos(dx+c)^m (b\cos(dx+c))^{\frac{1}{3}} \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*co
s(d*x + c)^m, x)
```

Sympy [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\cos^m(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)
**2),x)
```

output
```
Integral((b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)*
cos(c + d*x)**m, x)
```

Maxima [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*c
os(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\cos\left(dx+c\right)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*c
os(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)^m\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2), x)
```

Reduce [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=b^{\frac{1}{3}}\left(\left(\int \cos(dx+c)^{m+\frac{1}{3}}\,dx\right)a+\left(\int \cos(dx+c)^{m+\frac{1}{3}}\cos(dx+c)\,dx\right)b+\left(\int \cos(dx+c)^{m+\frac{1}{3}}\cos(dx+c)^2\,dx\right)c\right)$$

input
```
int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**(1/3)*(int(cos(c + d*x)**((3*m + 1)/3),x)*a + int(cos(c + d*x)**((3*m +
 1)/3)*cos(c + d*x),x)*b + int(cos(c + d*x)**((3*m + 1)/3)*cos(c + d*x)**2
,x)*c)
```

3.366 $\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 229

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} - \frac{3(C(2+3m)+A(5+3m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+d \ldots}{d(2+3m)(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3*C*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)-3*(C*(2+3*m
)+A*(5+3*m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d
*x+c)^2)*sin(d*x+c)/d/(2+3*m)/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^
(1/2)-3*B*cos(d*x+c)^(2+m)*hypergeom([1/2, 5/6+1/2*m],[11/6+1/2*m],cos(d*x
+c)^2)*sin(d*x+c)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.76

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= \frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(-\left((C(2+3m)+A(5+3m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3\right.\right.\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(1/3),x]
```

output
```
(3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-((C*(2 + 3*m) + A*(5 + 3*m))*Hyperg
eometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*
x]^2]) + (2 + 3*m)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/
2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(
2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$\Big\downarrow \text{2034}$$
$$\frac{\sqrt[3]{\cos(c+dx)}\int \cos^{m-\frac{1}{3}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{\sqrt[3]{b\cos(c+dx)}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3\int \frac{1}{3}\cos^{m-\frac{1}{3}}(c+dx)\left(3C\left(m+\frac{2}{3}\right)+3A\left(m+\frac{5}{3}\right)+B(3m+5)\cos(c+dx)\right)dx}{3m+5}+\frac{3C\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)}{d(3m+5)}\right)}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int \cos^{m-\frac{1}{3}}(c+dx)(C(3m+2)+A(3m+5)+B(3m+5)\cos(c+dx))dx}{3m+5}+\frac{3C\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)}{d(3m+5)}\right)}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}\left(C(3m+2)+A(3m+5)+B(3m+5)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{3m+5}+\frac{3C\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)}{d(3m+5)}\right)}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{(A(3m+5)+C(3m+2))\int \cos^{m-\frac{1}{3}}(c+dx)dx+B(3m+5)\int \cos^{m+\frac{2}{3}}(c+dx)dx}{3m+5}+\frac{3C\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)}{d(3m+5)}\right)}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{(A(3m+5)+C(3m+2))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}dx+B(3m+5)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}dx}{3m+5}+\frac{3C\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)}{d(3m+5)}\right)}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{-\frac{3(A(3m+5)+C(3m+2))\sin(c+dx)\cos^{m+\frac{2}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+2),\frac{1}{6}(3m+8),\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+\frac{5}{3}}(c\cdots}{\cdots}}{3m+5}\cdots\right.}{\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(1/3),x]
```

output
```
(Cos[c + d*x]^(1/3)*((3*C*Cos[c + d*x]^(2/3 + m)*Sin[c + d*x])/(d*(5 + 3*m
)) + ((-3*(C*(2 + 3*m) + A*(5 + 3*m))*Cos[c + d*x]^(2/3 + m)*Hypergeometri
c2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 +
3*m)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(5/3 + m)*Hypergeometric2F1
[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin
[c + d*x]^2]))/(5 + 3*m)))/(b*Cos[c + d*x])^(1/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos \left(dx+c\right)^{m} \left(A+B\cos \left(dx+c\right)+C\cos \left(dx+c\right)^{2}\right)}{\left(b\cos \left(dx+c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{m}}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*co
s(d*x + c)^m/(b*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{(A+B\cos{(c+dx)}+C\cos^2{(c+dx)})\cos^m{(c+dx)}}{\sqrt[3]{b\cos{(c+dx)}}}\,dx$$

input
```
integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
1/3),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c
 + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{(C\cos{(dx+c)}^2+B\cos{(dx+c)}+A)\cos{(dx+c)}^m}{(b\cos{(dx+c)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*
x + c))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^m}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*
x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(1/3),x)
```

output
```
int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(1/3), x)
```

Reduce [F]

$$\int \frac{\cos^m(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$=\frac{\left(\int \frac{\cos(dx+c)^m}{\cos(dx+c)^{\frac{1}{3}}}dx\right)a+\left(\int \cos\left(dx+c\right)^m\cos\left(dx+c\right)^{\frac{2}{3}}dx\right)b+\left(\int \cos\left(dx+c\right)^m\cos\left(dx+c\right)^{\frac{5}{3}}dx\right)c}{b^{\frac{1}{3}}}$$

input
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

output
```
(int(cos(c + d*x)**m/cos(c + d*x)**(1/3),x)*a + int((cos(c + d*x)**m*cos(c
 + d*x))/cos(c + d*x)**(1/3),x)*b + int((cos(c + d*x)**m*cos(c + d*x)**2)/
cos(c + d*x)**(1/3),x)*c)/b**(1/3)
```

3.367 $\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 227

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}} -\frac{3(C+3Cm+A(4+3m))\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1+3m),\frac{1}{6}(7+3m),\cos^2(c+dx)\right)}{d(1+3m)(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} -\frac{3B\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output

```
3*C*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(4+3*m)/(b*cos(d*x+c))^(2/3)-3*(C+3*C*m+
A*(4+3*m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/6+1/2*m],[7/6+1/2*m],cos(d*x
+c)^2)*sin(d*x+c)/d/(1+3*m)/(4+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1
/2)-3*B*cos(d*x+c)^(2+m)*hypergeom([1/2, 2/3+1/2*m],[5/3+1/2*m],cos(d*x+c)
^2)*sin(d*x+c)/d/(4+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.76

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(-\left((C+3Cm\right.\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(2/3),x]
```

output
```
(3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-((C + 3*C*m + A*(4 + 3*m))*Hypergeo
metric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]
^2]) + (1 + 3*m)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2,
 (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(1 + 3
*m)*(4 + 3*m)*(b*Cos[c + d*x])^(2/3))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{m-\frac{2}{3}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{2}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(\frac{3\int \frac{1}{3}\cos^{m-\frac{2}{3}}(c+dx)(3mC+C+A(3m+4)+B(3m+4)\cos(c+dx))dx}{3m+4}+\frac{3C\sin(c+dx)\cos^{m+\frac{1}{3}}(c+dx)}{d(3m+4)}\right)}{(b\cos(c+dx))^{2/3}}$$

$\Big\downarrow$ 27

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(\frac{\int \cos^{m-\frac{2}{3}}(c+dx)(3mC+C+A(3m+4)+B(3m+4)\cos(c+dx))dx}{3m+4}+\frac{3C\sin(c+dx)\cos^{m+\frac{1}{3}}(c+dx)}{d(3m+4)}\right)}{(b\cos(c+dx))^{2/3}}$$

$\Big\downarrow$ 3042

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{2}{3}}\left(3mC+C+A(3m+4)+B(3m+4)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{3m+4}+\frac{3C\sin(c+dx)\cos^{m+\frac{1}{3}}(c+dx)}{d(3m+4)}\right)}{(b\cos(c+dx))^{2/3}}$$

$\Big\downarrow$ 3227

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(\frac{(A(3m+4)+3Cm+C)\int \cos^{m-\frac{2}{3}}(c+dx)dx+B(3m+4)\int \cos^{m+\frac{1}{3}}(c+dx)dx}{3m+4}+\frac{3C\sin(c+dx)\cos^{m+\frac{1}{3}}(c+dx)}{d(3m+4)}\right)}{(b\cos(c+dx))^{2/3}}$$

$\Big\downarrow$ 3042

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(\frac{(A(3m+4)+3Cm+C)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{2}{3}}dx+B(3m+4)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}dx}{3m+4}+\frac{3C\sin(c+dx)\cos^{m+\frac{1}{3}}(c+dx)}{d(3m+4)}\right)}{(b\cos(c+dx))^{2/3}}$$

$\Big\downarrow$ 3122

$$\frac{\cos^{\frac{2}{3}}(c+dx)\left(\frac{-\frac{3(A(3m+4)+3Cm+C)\sin(c+dx)\cos^{m+\frac{1}{3}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+1),\frac{1}{6}(3m+7),\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+\frac{4}{3}}(c+d}{}}{3m+4}\right.}{(b\cos(c+dx))^{2/3}}$$

input
```
Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(2/3),x]
```

output
```
(Cos[c + d*x]^(2/3)*((3*C*Cos[c + d*x]^(1/3 + m)*Sin[c + d*x])/(d*(4 + 3*m
)) + ((-3*(C + 3*C*m + A*(4 + 3*m))*Cos[c + d*x]^(1/3 + m)*Hypergeometric2
F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*
m)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(4/3 + m)*Hypergeometric2F1[1
/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c
 + d*x]^2]))/(4 + 3*m)))/(b*Cos[c + d*x])^(2/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos (dx+c)^{m} \left(A+B\cos (dx+c)+C\cos (dx+c)^{2}\right)}{(b\cos (dx+c))^{\frac{2}{3}}}dx$$

input
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

output
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\left(C\cos (dx+c)^{2}+B\cos (dx+c)+A\right)\cos}{(b\cos (dx+c))^{\frac{2}{3}}}$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*co
s(d*x + c)^m/(b*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)\right)\cos}{\left(b\cos\left(c+dx\right)\right)^{\frac{2}{3}}}$$

input
```
integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
2/3),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c
 + d*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*
x + c))^(2/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*
x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}$$

input
```
int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(2/3),x)
```

output
```
int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(2/3), x)
```

Reduce [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx=\frac{\left(\int \frac{\cos(dx+c)^m}{\cos(dx+c)^{\frac{2}{3}}}dx\right)a+\left(\int \cos\left(dx+c\right)^m\cos\left(d\right.\right.}{}$$

input
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

output
```
(int(cos(c + d*x)**m/cos(c + d*x)**(2/3),x)*a + int((cos(c + d*x)**m*cos(c
 + d*x))/cos(c + d*x)**(2/3),x)*b + int((cos(c + d*x)**m*cos(c + d*x)**2)/
cos(c + d*x)**(2/3),x)*c)/b**(2/3)
```

3.368 $\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 235

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}} - \frac{3(C(1-3m)-A(2+3m))\cos^m(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m),\cos^2(c+dx)\right)}{bd(1-3m)(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{3B\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3*C*cos(d*x+c)^m*sin(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)-3*(C*(1-3*m)-
A*(2+3*m))*cos(d*x+c)^m*hypergeom([1/2, -1/6+1/2*m],[5/6+1/2*m],cos(d*x+c)
^2)*sin(d*x+c)/b/d/(1-3*m)/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/
2)-3*B*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x+c)^
2)*sin(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.74

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx =$$

$$-\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left((C(-1+3m)+A(2+3m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3\right.\right.}{}$$

input
```
Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[
c + d*x])^(4/3),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*((C*(-1 + 3*m) + A*(2 + 3*m))*Hyperg
eometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d
*x]^2] - (-1 + 3*m)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1
/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(
-1 + 3*m)*(2 + 3*m)*(b*Cos[c + d*x])^(4/3))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \cos^{m-\frac{4}{3}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{4}{3}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3\int -\frac{1}{3}\cos^{m-\frac{4}{3}}(c+dx)\left(3C\left(\frac{1}{3}-m\right)-3A\left(m+\frac{2}{3}\right)-B(3m+2)\cos(c+dx)\right)dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int -\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int-\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int-\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int-\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int-\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int-\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{\int-\cos^{m-\frac{4}{3}}(c+dx)(C(1-3m)-A(3m+2)-B(3m+2)\cos(c+dx))dx}{3m+2}+\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[3]{\cos(c+dx)}\left(\frac{3C\sin(c+dx)\cos^{m-\frac{1}{3}}(c+dx)}{d(3m+2)}-\frac{\int-\cos^{m-\frac{4}{3}}(c+dx)(3mA+2A-C+3Cm+B(3m+2)\cos(c+dx))dx}{3m+2}\right)}{b\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*
x])^(4/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cos{(dx+c)}^m \left(A+B\cos{(dx+c)}+C\cos{(dx+c)}^2\right)}{(b\cos{(dx+c)})^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos{(dx+c)}^2+B\cos{(dx+c)}+A\right)\cos}{(b\cos{(dx+c)})^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*co
s(d*x + c)^m/(b^2*cos(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(A+B\cos{(c+dx)}+C\cos^2{(c+dx)}\right)\cos}{(b\cos{(c+dx)})^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(
4/3),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c
 + d*x))**(4/3), x)
```

Maxima [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*
x + c))^(4/3), x)
```

Giac [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\cos}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}$$

input
```
integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*
x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c\right.\right.}{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}$$

input
```
int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(4/3),x)
```

output
```
int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*
x))^(4/3), x)
```

Reduce [F]

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx=\frac{\left(\int \frac{\cos(dx+c)^m}{\cos(dx+c)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{\cos(dx+c)^m}{\cos(dx+c)^{\frac{4}{3}}}dx\right)a+\left(\right.}{b^{\frac{4}{3}}}$$

input
```
int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

output
```
(int(cos(c + d*x)**m/cos(c + d*x)**(1/3),x)*b + int(cos(c + d*x)**m/(cos(c
 + d*x)**(1/3)*cos(c + d*x)),x)*a + int((cos(c + d*x)**m*cos(c + d*x))/cos
(c + d*x)**(1/3),x)*c)/(b**(1/3)*b)
```

3.369 $\int (a\cos(c+dx))^m (b\cos(c+dx))^n (A+B\cos(c+dx)$

Optimal result

Integrand size = 41, antiderivative size = 227

$$\int (a\cos(c+dx))^m (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+dx)}{ad(2+m+n)}$$
$$-\frac{(C(1+m+n)+A(2+m+n))(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m\right.}{ad(1+m+n)(2+m+n)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{B(a\cos(c+dx))^{2+m}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2+m+n),\frac{1}{2}(4+m+n),\cos^2(c+d\right.}{a^2d(2+m+n)\sqrt{\sin^2(c+dx)}}$$

output
```
C*(a*cos(d*x+c))^(1+m)*(b*cos(d*x+c))^n*sin(d*x+c)/a/d/(2+m+n)-(C*(1+m+n)+
A*(2+m+n))*(a*cos(d*x+c))^(1+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2+1/2*m
+1/2*n],[3/2+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a/d/(1+m+n)/(2+m+n)/(si
n(d*x+c)^2)^(1/2)-B*(a*cos(d*x+c))^(2+m)*(b*cos(d*x+c))^n*hypergeom([1/2,
1+1/2*m+1/2*n],[2+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a^2/d/(2+m+n)/(sin
(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.69

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{(a\cos(c+dx))^m(b\cos(c+dx))^n\cot(c+dx)\left(C\sin^2(c+dx)-\frac{(C(1+m+n)+A(2+m+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1\right.}{1+m+r}\right.}{}$$

input
```
Integrate[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Co
s[c + d*x]^2),x]
```

output
```
((a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*Cot[c + d*x]*(C*Sin[c + d*x]^2 - ((
C*(1 + m + n) + A*(2 + m + n))*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 +
m + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(1 + m + n) - B*Cos[c + d*
x]*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sq
rt[Sin[c + d*x]^2]))/(d*(2 + m + n))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2034, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 2034$$

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\int (a\cos(c+dx))^{m+n}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx$$

$$\Big\downarrow \; 3042$$

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\int\left(a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{m+n}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx$$

$\Big\downarrow$ 3502

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\left(\frac{\int(a\cos(c+dx))^{m+n}(a(C(m+n+1)+A(m+n+2))+aB(m+n+2)\cos(c+dx))dx}{a(m+n+2)}+\frac{C\sin(c+d}{a}\right.$$

$\Big\downarrow$ 3042

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\left(\frac{\int\left(a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{m+n}\left(a(C(m+n+1)+A(m+n+2))+aB(m+n+2)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a(m+n+2)}+\right.$$

$\Big\downarrow$ 3227

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\left(\frac{a(A(m+n+2)+C(m+n+1))\int(a\cos(c+dx))^{m+n}dx+B(m+n+2)\int(a\cos(c+dx))^{m+n+1}dx}{a(m+n+2)}+\right.$$

$\Big\downarrow$ 3042

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\left(\frac{a(A(m+n+2)+C(m+n+1))\int\left(a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{m+n}dx+B(m+n+2)\int\left(a\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{m}}{a(m+n+2)}\right.$$

$\Big\downarrow$ 3122

$$(a\cos(c+dx))^{-n}(b\cos(c+dx))^n\left(\frac{-\frac{(A(m+n+2)+C(m+n+1))\sin(c+dx)(a\cos(c+dx))^{m+n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),\cos^2(c+dx)\right)}{d(m+n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(}{}}{a(m+n+2)}\right.$$

input
```
Int[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c +
d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^n*((C*(a*Cos[c + d*x])^(1 + m + n)*Sin[c + d*x])/(a*d*(2
 + m + n)) + (-(((C*(1 + m + n) + A*(2 + m + n))*(a*Cos[c + d*x])^(1 + m +
 n)*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2]*S
in[c + d*x])/(d*(1 + m + n)*Sqrt[Sin[c + d*x]^2])) - (B*(a*Cos[c + d*x])^(
2 + m + n)*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*
x]^2]*Sin[c + d*x])/(a*d*Sqrt[Sin[c + d*x]^2]))/(a*(2 + m + n))))/(a*Cos[c
 + d*x])^n
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] &&
 !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 && !LtQ[m, -1]
```

Maple [F]

$$\int (a\cos (dx+c))^m (b\cos (dx+c))^n \left(A+B\cos (dx+c)+C\cos (dx+c)^2\right) dx$$

input
```
int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \int \left(C\cos (dx+c)^2+B\cos (dx+c)+A\right)(a\cos (dx+c))^m (b\cos (dx+c))^n \, dx$$

input
```
integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c))^m*(b*cos
(d*x + c))^n, x)
```

Sympy [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \int (a\cos (c+dx))^m (b\cos (c+dx))^n \left(A+B\cos (c+dx)+C\cos^2 (c+dx)\right) dx$$

input
```
integrate((a*cos(d*x+c))**m*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)
**2),x)
```

output
```
Integral((a*cos(c + d*x))**m*(b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*c
os(c + d*x)**2), x)
```

Maxima [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int (C\cos(dx+c)^2+B\cos(dx+c)+A)(a\cos(dx+c))^m(b\cos(dx+c))^n dx$$

input
```
integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c))^m*(b*co
s(d*x + c))^n, x)
```

Giac [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int (C\cos(dx+c)^2+B\cos(dx+c)+A)(a\cos(dx+c))^m(b\cos(dx+c))^n dx$$

input
```
integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c))^m*(b*co
s(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$=\int (a\cos(c+dx))^m (b\cos(c+dx))^n \left(C\cos(c+dx)^2+B\cos(c+dx)+A\right) dx$$

input
```
int((a*cos(c + d*x))^m*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2),x)
```

output
```
int((a*cos(c + d*x))^m*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2), x)
```

Reduce [F]

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$=b^n a^m\left(\left(\int\cos(dx+c)^{m+n}\,dx\right)a+\left(\int\cos(dx+c)^{m+n}\cos(dx+c)\,dx\right)b+\left(\int\cos(dx+c)^{m+n}\cos(dx+c)^2\,dx\right)c\right)$$

input
```
int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*a**m*(int(cos(c + d*x)**(m + n),x)*a + int(cos(c + d*x)**(m + n)*cos(
c + d*x),x)*b + int(cos(c + d*x)**(m + n)*cos(c + d*x)**2,x)*c)
```

3.370 $\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\right.$

Optimal result

Integrand size = 39, antiderivative size = 187

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)} - \frac{(C(3+n)+A(4+n))(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)(4+n)\sqrt{\sin^2(c+dx)}} - \frac{B(b\cos(c+dx))^{4+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{2},\frac{6+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^4d(4+n)\sqrt{\sin^2(c+dx)}}$$

output
```
C*(b*cos(d*x+c))^(3+n)*sin(d*x+c)/b^3/d/(4+n)-(C*(3+n)+A*(4+n))*(b*cos(d*x
+c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)
/b^3/d/(3+n)/(4+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(4+n)*hypergeom([
1/2, 2+1/2*n],[3+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(4+n)/(sin(d*x+c)^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.82

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx =$$
$$-\frac{\cos^2(c+dx)(b\cos(c+dx))^n \cot(c+dx) \left((C(3+n)+A(4+n)) \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\text{co}\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c
+ d*x]^2),x]
```

output
```
-((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*((C*(3 + n) + A*(4 + n))
*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c +
 d*x]^2] - (3 + n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/
2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(3 + n
)*(4 + n)))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2030, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{n+2} \left(C\cos^2(c+dx)+B\cos(c+dx)+A\right) dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2} \left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right) dx}{b^2}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{\int (b\cos(c+dx))^{n+2}(b(C(n+3)+A(n+4))+bB(n+4)\cos(c+dx))dx}{b(n+4)}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}}{bd(n+4)}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}\left(b(C(n+3)+A(n+4))+bB(n+4)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b(n+4)}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}}{bd(n+4)}}{b^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{b(A(n+4)+C(n+3))\int (b\cos(c+dx))^{n+2}dx+B(n+4)\int (b\cos(c+dx))^{n+3}dx}{b(n+4)}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}}{bd(n+4)}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(A(n+4)+C(n+3))\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}dx+B(n+4)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+3}dx}{b(n+4)}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}}{bd(n+4)}}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{-\frac{(A(n+4)+C(n+3))\sin(c+dx)(b\cos(c+dx))^{n+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{n+5}{2},\cos^2(c+dx)\right)}{d(n+3)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+4}{2},\frac{n}{}\right.}{bd\sqrt{\sin^2(c+dx)}}}{b(n+4)}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]
^2),x]
```

output
```
((C*(b*Cos[c + d*x])^(3 + n)*Sin[c + d*x])/(b*d*(4 + n)) + (-(((C*(3 + n)
+ A*(4 + n))*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5
 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + n)*Sqrt[Sin[c + d*x]^2])) -
 (B*(b*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]))/(b*(4 + n)))/b^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos(dx+c)^2 (b\cos(dx+c))^n \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right) dx$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d
*x + c))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d
*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$
$$= \int \left(C \cos\left(dx+c\right)^2 + B \cos\left(dx+c\right) + A\right) \left(b \cos\left(dx+c\right)\right)^n \cos\left(dx+c\right)^2 \, dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
 algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d
*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$
$$= \int \cos\left(c+d\,x\right)^2 \left(b\,\cos\left(c+d\,x\right)\right)^n \left(C\cos\left(c+d\,x\right)^2 + B\,\cos\left(c+d\,x\right) + A\right) \, dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)
^2),x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)
^2), x)
```

Reduce [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$
$$= b^n\left(\left(\int \cos(dx+c)^n \cos(dx+c)^4 \, dx\right)c+\left(\int \cos(dx+c)^n \cos(dx+c)^3 \, dx\right)b + \left(\int \cos(dx+c)^n \cos(dx+c)^2 \, dx\right)a\right)$$

input
```
int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)**4,x)*c + int(cos(c + d*x)**n*cos(c
 + d*x)**3,x)*b + int(cos(c + d*x)**n*cos(c + d*x)**2,x)*a)
```

3.371 $\int \cos(c+dx)(b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\right.$

Optimal result

Integrand size = 37, antiderivative size = 187

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)}$$
$$-\frac{(C(2+n)+A(3+n))(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)(3+n)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{B(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}$$

output
```
C*(b*cos(d*x+c))^(2+n)*sin(d*x+c)/b^2/d/(3+n)-(C*(2+n)+A*(3+n))*(b*cos(d*x
+c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2
/d/(2+n)/(3+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(3+n)*hypergeom([1/2,
 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+c)^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$
$$-\frac{\cos(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(((C(2+n)+A(3+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c +
d*x]^2),x]
```

output
```
-((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*((C*(2 + n) + A*(3 + n))*H
ypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d
*x]^2] - (2 + n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2,
 (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(2 + n)*
(3 + n)))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {2030, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{n+1}\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx}{b}$$

$$\Big\downarrow \; 3502$$

$$\frac{\frac{\int (b\cos(c+dx))^{n+1}(b(C(n+2)+A(n+3))+bB(n+3)\cos(c+dx))dx}{b(n+3)}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}}{bd(n+3)}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}\left(b(C(n+2)+A(n+3))+bB(n+3)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b(n+3)}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}}{bd(n+3)}}{b}$$

$$\Big\downarrow \; 3227$$

$$\frac{\frac{b(A(n+3)+C(n+2))\int (b\cos(c+dx))^{n+1}dx+B(n+3)\int (b\cos(c+dx))^{n+2}dx}{b(n+3)}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}}{bd(n+3)}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{b(A(n+3)+C(n+2))\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}dx+B(n+3)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}dx}{b(n+3)}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}}{bd(n+3)}}{b}$$

$$\Big\downarrow \; 3122$$

$$\frac{\frac{-\frac{(A(n+3)+C(n+2))\sin(c+dx)(b\cos(c+dx))^{n+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\cos^2(c+dx)\right)}{d(n+2)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\ldots\right.}{bd\sqrt{\sin^2(c+dx)}}}{b(n+3)}}{b}$$

input

```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
```

output

```
((C*(b*Cos[c + d*x])^(2 + n)*Sin[c + d*x])/(b*d*(3 + n)) + (-(((C*(2 + n)
+ A*(3 + n))*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4
 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + n)*Sqrt[Sin[c + d*x]^2])) -
 (B*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]))/(b*(3 + n)))/b
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos(dx+c)\,(b\cos(dx+c))^n\left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)dx$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right) dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, a
lgorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
 + c))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right) dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, a
lgorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d
*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, a
lgorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d
*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2
),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2
), x)
```

Reduce [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$
$$= b^n\left(\left(\int \cos(dx+c)^n \cos(dx+c)\,dx\right)a+\left(\int \cos(dx+c)^n \cos(dx+c)^3\,dx\right)c + \left(\int \cos(dx+c)^n \cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x),x)*a + int(cos(c + d*x)**n*cos(c +
d*x)**3,x)*c + int(cos(c + d*x)**n*cos(c + d*x)**2,x)*b)
```

3.372 $\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+d\right.$

Optimal result

Integrand size = 31, antiderivative size = 187

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$
$$= \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)}$$
$$- \frac{(C(1+n)+A(2+n))(b\cos(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)(2+n)\sqrt{\sin^2(c+dx)}}$$
$$- \frac{B(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}$$

output
```
C*(b*cos(d*x+c))^(1+n)*sin(d*x+c)/b/d/(2+n)-(C*(1+n)+A*(2+n))*(b*cos(d*x+c
))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b
/d/(1+n)/(2+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(2+n)*hypergeom([1/2,
 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.77

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{(b\cos(c+dx))^n\cot(c+dx)\left(-\left((C(1+n)+A(2+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\right.\right.}{}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^n*Cot[c + d*x]*(-((C*(1 + n) + A*(2 + n))*Hypergeometric
2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]) + (1
+ n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2,
(4 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(1 + n)*(2 + n))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int (b\cos(c+dx))^n(b(C(n+1)+A(n+2))+bB(n+2)\cos(c+dx))dx}{b(n+2)}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}}{bd(n+2)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n \left(b(C(n+1)+A(n+2))+bB(n+2)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b(n+2)}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}}{bd(n+2)}$$

$$\Big\downarrow \textbf{3227}$$

$$\frac{b(A(n+2)+C(n+1))\int(b\cos(c+dx))^n dx+B(n+2)\int(b\cos(c+dx))^{n+1}dx}{b(n+2)}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}}{bd(n+2)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{b(A(n+2)+C(n+1))\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n dx+B(n+2)\int\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}dx}{b(n+2)}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}}{bd(n+2)}$$

$$\Big\downarrow \textbf{3122}$$

$$\frac{-\frac{(A(n+2)+C(n+1))\sin(c+dx)(b\cos(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(c+dx)\right)}{d(n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+2}\,\text{Hypergeom}}{bd\sqrt{\sin^2(c+}}}{b(n+2)}$$
$$\frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}}{bd(n+2)}$$

input
```
Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(C*(b*Cos[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(2 + n)) + (-(((C*(1 + n) +
 A*(2 + n))*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3
+ n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + n)*Sqrt[Sin[c + d*x]^2])) -
(B*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, C
os[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]))/(b*(2 + n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^n \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right) dx$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="f
ricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(b\cos\left(c+dx\right)\right)^n\left(A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)\right)\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*cos(c + d*x)**2), x)
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="m
axima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (C\cos(dx+c)^2+B\cos(dx+c)+A)(b\cos(dx+c))^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="g
iac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)\,dx$$

input
```
int((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

output
```
int((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= b^n\left(\left(\int \cos(dx+c)^n\,dx\right)a+\left(\int \cos(dx+c)^n\cos(dx+c)\,dx\right)b + \left(\int \cos(dx+c)^n\cos(dx+c)^2\,dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**n,x)*a + int(cos(c + d*x)**n*cos(c + d*x),x)*b + i
nt(cos(c + d*x)**n*cos(c + d*x)**2,x)*c)
```

3.373 $\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \sec(c+dx)\, dx$

Optimal result

Integrand size = 37, antiderivative size = 170

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\, dx$$
$$= \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)}$$
$$- \frac{(A+An+Cn)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn(1+n)\sqrt{\sin^2(c+dx)}}$$
$$- \frac{B(b\cos(c+dx))^{1+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

output
```
C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1+n)-(A*n+C*n+A)*(b*cos(d*x+c))^n*hyperge
om([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/n/(1+n)/(sin(d*x+c)^2
)^(1/2)-B*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(
d*x+c)^2)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.81

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \frac{b(b\cos(c+dx))^{-1+n}\cot(c+dx)\left(-\left((A+An+Cn)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sqrt{\sin\right.\right.}{}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
 + d*x],x]
```

output
```
(b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(-((A + A*n + C*n)*Hypergeometri
c2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]) + n*(C*Sin
[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*n*(1 + n))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3042, 2030, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-1}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A\right)dx$$

$$\Big\downarrow \text{3502}$$

$$b\left(\frac{\int (b\cos(c+dx))^{n-1}(b(nA+A+Cn)+bB(n+1)\cos(c+dx))dx}{b(n+1)}+\frac{C\sin(c+dx)(b\cos(c+dx))^n}{bd(n+1)}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-1}\left(b(nA+A+Cn)+bB(n+1)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b(n+1)}+\frac{C\sin(c+dx)(b\cos(c+dx))}{bd(n+1)}\right.$$

$$\Big\downarrow \text{3227}$$

$$b\left(\frac{b(An+A+Cn)\int (b\cos(c+dx))^{n-1}dx+B(n+1)\int (b\cos(c+dx))^n dx}{b(n+1)}+\frac{C\sin(c+dx)(b\cos(c+dx))^n}{bd(n+1)}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{b(An+A+Cn)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-1}dx+B(n+1)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n dx}{b(n+1)}+\frac{C\sin(c+dx)(b\cos(c}{bd(n+1)}\right.$$

$$\Big\downarrow \text{3122}$$

$$b\left(\frac{-\frac{(An+A+Cn)\sin(c+dx)(b\cos(c+dx))^n\text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+1}\text{ Hypergeometric2F}}{bd\sqrt{\sin^2(c+dx)}}}{b(n+1)}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x
],x]
```

output
```
b*((C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(b*d*(1 + n)) + (-(((A + A*n + C*n)
*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])) - (B*(b*Cos[c + d*x])^(1 + n)*H
ypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/
(b*d*Sqrt[Sin[c + d*x]^2]))/(b*(1 + n)))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos (dx+c))^n \left(A+B\cos (dx+c)+C\cos (dx+c)^2\right)\sec (dx+c)\, dx$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, a
lgorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c), x)
```

Sympy [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(b\cos\left(c+dx\right)\right)^n \left(A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(
c + d*x), x)
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, a
lgorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d
*x + c), x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(b\cos\left(dx+c\right))^n\sec\left(dx+c\right)\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, a
lgorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d
*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= \int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x),x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x), x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= b^n\left(\left(\int \cos(dx+c)^n \cos(dx+c)\sec(dx+c)\,dx\right)b + \left(\int \cos(dx+c)^n \cos(dx+c)^2 \sec(dx+c)\,dx\right)c + \left(\int \cos(dx+c)^n \sec(dx+c)\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)*sec(c + d*x),x)*b + int(cos(c + d*x
)**n*cos(c + d*x)**2*sec(c + d*x),x)*c + int(cos(c + d*x)**n*sec(c + d*x),
x)*a)
```

3.374 $\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+d\right.$ $\left. dx\right) dx$

Optimal result

Integrand size = 39, antiderivative size = 173

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$= \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn}$$
$$-\frac{b(C(1-n)-An)(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+d}{d(1-n)n\sqrt{\sin^2(c+dx)}}$$
$$-\frac{B(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}$$

output
```
b*C*(b*cos(d*x+c))^(-1+n)*sin(d*x+c)/d/n-b*(C*(1-n)-A*n)*(b*cos(d*x+c))^(-
1+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1
-n)/n/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2*n],[1+1/
2*n],cos(d*x+c)^2)*sin(d*x+c)/d/n/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.75

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx =$$
$$-\frac{(b\cos(c+dx))^n \left((C(-1+n)+An)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)+(-1+\right.}{d(-1+n)n\sqrt{\text{s}}}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
 + d*x]^2,x]
```

output
```
-(((b*Cos[c + d*x])^n*((C*(-1 + n) + A*n)*Hypergeometric2F1[1/2, (-1 + n)/
2, (1 + n)/2, Cos[c + d*x]^2] + (-1 + n)*(B*Cos[c + d*x]*Hypergeometric2F1
[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))*Tan[c + d
*x])/(d*(-1 + n)*n*Sqrt[Sin[c + d*x]^2]))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.205, Rules used = {3042, 2030, 3502, 25, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n \left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-2}\left(C\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2+B\sin\left(\frac{1}{2}(2c+\pi)+dx\right)+A\right)dx$$

$$\Big\downarrow \text{3502}$$

$$b^2\left(\frac{\int -(b\cos(c+dx))^{n-2}(b(C(1-n)-An)-bBn\cos(c+dx))dx}{bn}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n-1}}{bdn}\right)$$

$$\Big\downarrow \text{25}$$

$$b^2\left(\frac{C\sin(c+dx)(b\cos(c+dx))^{n-1}}{bdn}-\frac{\int (b\cos(c+dx))^{n-2}(b(C(1-n)-An)-bBn\cos(c+dx))dx}{bn}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{C\sin(c+dx)(b\cos(c+dx))^{n-1}}{bdn}-\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-2}\left(b(C(1-n)-An)-bBn\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{bn}\right.$$

$$\Big\downarrow \text{3227}$$

$$b^2\left(\frac{C\sin(c+dx)(b\cos(c+dx))^{n-1}}{bdn}-\frac{b(C(1-n)-An)\int (b\cos(c+dx))^{n-2}dx-Bn\int (b\cos(c+dx))^{n-1}dx}{bn}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{C\sin(c+dx)(b\cos(c+dx))^{n-1}}{bdn}-\frac{b(C(1-n)-An)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-2}dx-Bn\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right.\right.}{bn}\right.$$

$$\Big\downarrow \text{3122}$$

$$b^2\left(\frac{C\sin(c+dx)(b\cos(c+dx))^{n-1}}{bdn}-\frac{\frac{(C(1-n)-An)\sin(c+dx)(b\cos(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}+}{bn}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x
]^2,x]
```

output
```
b^2*((C*(b*Cos[c + d*x])^(-1 + n)*Sin[c + d*x])/(b*d*n) - (((C*(1 - n) - A
*n)*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2
, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]) + (B*(b*C
os[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[
c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]))/(b*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^n \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)\sec(dx+c)^2\,dx$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\sec\left(dx+c\right)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
 algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int (C\cos{(dx+c)}^2+B\cos{(dx+c)}+A)(b\cos{(dx+c)})^n\sec{(dx+c)}^2\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
 algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d
*x + c)^2, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$$
$$=\int (C\cos{(dx+c)}^2+B\cos{(dx+c)}+A)(b\cos{(dx+c)})^n\sec{(dx+c)}^2\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
 algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d
*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = \int \frac{(b\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^2}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^2,x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^2, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^n \left(\left(\int \cos(dx+c)^n \cos(dx+c)\sec(dx+c)^2\,dx\right) b + \left(\int \cos(dx+c)^n \cos(dx+c)^2 \sec(dx+c)^2\,dx\right) c + \left(\int \cos(dx+c)^n \sec(dx+c)^2\,dx\right) a\right)$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)*sec(c + d*x)**2,x)*b + int(cos(c +
d*x)**n*cos(c + d*x)**2*sec(c + d*x)**2,x)*c + int(cos(c + d*x)**n*sec(c +
 d*x)**2,x)*a)
```

3.375 $\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+d\right.$ $dx)\,dx$

Optimal result

Integrand size = 39, antiderivative size = 194

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)}$$
$$+ \frac{b^2(A(1-n)+C(2-n))(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}}$$
$$+ \frac{bB(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

output
```
-b^2*C*(b*cos(d*x+c))^(-2+n)*sin(d*x+c)/d/(1-n)+b^2*(A*(1-n)+C*(2-n))*(b*c
os(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*sin(d*x+
c)/d/(1-n)/(2-n)/(sin(d*x+c)^2)^(1/2)+b*B*(b*cos(d*x+c))^(-1+n)*hypergeom(
[1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)
^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\frac{(b\cos(c+dx))^n\csc(c+dx)\sec^2(c+dx)\left(-\left(\left(C(-2+n)+A(-1+n)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+\right.\right.\right.}{}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
 + d*x]^3,x]
```

output
```
((b*Cos[c + d*x])^n*Csc[c + d*x]*Sec[c + d*x]^2*(-((C*(-2 + n) + A*(-1 + n
))*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*
x]^2]) + (-2 + n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2
, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(-2 +
n)*(-1 + n))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {3042, 2030, 3502, 25, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3 \int \left(b \sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-3} \left(C \sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2 + B \sin\left(\frac{1}{2}(2c+\pi)+dx\right) + A\right) dx$$

$\big\downarrow$ 3502

$$b^3 \left(-\frac{\int -(b\cos(c+dx))^{n-3}(b(A(1-n)+C(2-n))+bB(1-n)\cos(c+dx))dx}{b(1-n)} - \frac{C\sin(c+dx)(b\cos(c+dx)}{bd(1-n)}\right.$$

$\big\downarrow$ 25

$$b^3 \left(\frac{\int (b\cos(c+dx))^{n-3}(b(A(1-n)+C(2-n))+bB(1-n)\cos(c+dx))dx}{b(1-n)} - \frac{C\sin(c+dx)(b\cos(c+dx))^{n-2}}{bd(1-n)}\right.$$

$\big\downarrow$ 3042

$$b^3 \left(\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-3}\left(b(A(1-n)+C(2-n))+bB(1-n)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b(1-n)} - \frac{C\sin(c+dx)(b\cos(}{bd(1-n}\right.$$

$\big\downarrow$ 3227

$$b^3 \left(\frac{b(A(1-n)+C(2-n))\int (b\cos(c+dx))^{n-3}dx + B(1-n)\int (b\cos(c+dx))^{n-2}dx}{b(1-n)} - \frac{C\sin(c+dx)(b\cos(c+}{bd(1-n)}\right.$$

$\big\downarrow$ 3042

$$b^3 \left(\frac{b(A(1-n)+C(2-n))\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-3}dx + B(1-n)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-2}dx}{b(1-n)} - \frac{C\sin(c+d}{}\right.$$

$\big\downarrow$ 3122

$$b^3 \left(\frac{\frac{(A(1-n)+C(2-n))\sin(c+dx)(b\cos(c+dx))^{n-2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-2}{2},\frac{n}{2},\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}} + \frac{B\sin(c+dx)(b\cos(c+dx))^{n-1}\text{Hypergeom}}{bd\sqrt{\sin^2(c-}}}{b(1-n)}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x
]^3,x]
```

output
```
b^3*(-((C*(b*Cos[c + d*x])^(-2 + n)*Sin[c + d*x])/(b*d*(1 - n))) + (((A*(1
 - n) + C*(2 - n))*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 +
n)/2, n/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
+ (B*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]))/(b*(1 - n)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^n \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)\sec(dx+c)^3\,dx$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
 algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(b\cos\left(dx+c\right))^n\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
 algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d
*x + c)^3, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)(b\cos\left(dx+c\right))^n\sec\left(dx+c\right)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
 algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d
*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^3,x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^3, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^n\left(\left(\int \cos\left(dx+c\right)^n\cos\left(dx+c\right)\sec\left(dx+c\right)^3dx\right)b + \left(\int \cos\left(dx+c\right)^n\cos\left(dx+c\right)^2\sec\left(dx+c\right)^3dx\right)c + \left(\int \cos\left(dx+c\right)^n\sec\left(dx+c\right)^3dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)*sec(c + d*x)**3,x)*b + int(cos(c +
d*x)**n*cos(c + d*x)**2*sec(c + d*x)**3,x)*c + int(cos(c + d*x)**n*sec(c +
 d*x)**3,x)*a)
```

3.376 $\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+d\right.$ $\left. dx\right) dx$

Optimal result

Integrand size = 39, antiderivative size = 196

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)}$$
$$+\frac{b^3(A(2-n)+C(3-n))(b\cos(c+dx))^{-3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+\right.}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}}$$
$$+\frac{b^2B(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

output

```
-b^3*C*(b*cos(d*x+c))^(-3+n)*sin(d*x+c)/d/(2-n)+b^3*(A*(2-n)+C*(3-n))*(b*c
os(d*x+c))^(-3+n)*hypergeom([1/2, -3/2+1/2*n],[-1/2+1/2*n],cos(d*x+c)^2)*s
in(d*x+c)/d/(2-n)/(3-n)/(sin(d*x+c)^2)^(1/2)+b^2*B*(b*cos(d*x+c))^(-2+n)*h
ypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(2-n)/(sin(d*x
+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \frac{(b\cos(c+dx))^n\csc(c+dx)\sec^3(c+dx)\left(-\left(\left(C(-3+n)+A(-2+n)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+\right.\right.\right.}{}$$

input
```
Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
 + d*x]^4,x]
```

output
```
((b*Cos[c + d*x])^n*Csc[c + d*x]*Sec[c + d*x]^3*(-(((C*(-3 + n) + A*(-2 + n
))*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin
[c + d*x]^2]) + (-3 + n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric
2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(-3 +
 n)*(-2 + n))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {3042, 2030, 3502, 25, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4 \int \left(b \sin \left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-4} \left(C \sin \left(\frac{1}{2}(2c+\pi)+dx\right)^2 + B \sin \left(\frac{1}{2}(2c+\pi)+dx\right)+A\right)dx$$

$\big\downarrow$ 3502

$$b^4 \left(-\frac{\int -(b\cos(c+dx))^{n-4}(b(A(2-n)+C(3-n))+bB(2-n)\cos(c+dx))dx}{b(2-n)} - \frac{C\sin(c+dx)(b\cos(c+dx)}{bd(2-n)}\right.$$

$\big\downarrow$ 25

$$b^4 \left(\frac{\int (b\cos(c+dx))^{n-4}(b(A(2-n)+C(3-n))+bB(2-n)\cos(c+dx))dx}{b(2-n)} - \frac{C\sin(c+dx)(b\cos(c+dx))^{n-3}}{bd(2-n)}\right.$$

$\big\downarrow$ 3042

$$b^4 \left(\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-4}\left(b(A(2-n)+C(3-n))+bB(2-n)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{b(2-n)} - \frac{C\sin(c+dx)(b\cos(}{bd(2-n}\right.$$

$\big\downarrow$ 3227

$$b^4 \left(\frac{b(A(2-n)+C(3-n))\int (b\cos(c+dx))^{n-4}dx + B(2-n)\int (b\cos(c+dx))^{n-3}dx}{b(2-n)} - \frac{C\sin(c+dx)(b\cos(c+}{bd(2-n)}\right.$$

$\big\downarrow$ 3042

$$b^4 \left(\frac{b(A(2-n)+C(3-n))\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-4}dx + B(2-n)\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n-3}dx}{b(2-n)} - \frac{C\sin(c+d}{}\right.$$

$\big\downarrow$ 3122

$$b^4 \left(\frac{\frac{(A(2-n)+C(3-n))\sin(c+dx)(b\cos(c+dx))^{n-3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-3}{2},\frac{n-1}{2},\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}} + \frac{B\sin(c+dx)(b\cos(c+dx))^{n-2}\,\text{Hyperge}}{bd\sqrt{\sin^2(}}}{b(2-n)}\right.$$

input
```
Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x
]^4,x]
```

output
```
b^4*(-((C*(b*Cos[c + d*x])^(-3 + n)*Sin[c + d*x])/(b*d*(2 - n))) + (((A*(2
 - n) + C*(3 - n))*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 +
n)/2, (-1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*
x]^2]) + (B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n
/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]))/(b*(2 - n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (b\cos(dx+c))^n \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)\sec(dx+c)^4\,dx$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Fricas [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
 algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int (C\cos(dx+c)^2+B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
 algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d
*x + c)^4, x)
```

Giac [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$=\int (C\cos(dx+c)^2+B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
 algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d
*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = \int \frac{(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^4}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^4,x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^4, x)
```

Reduce [F]

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = b^n\left(\left(\int \cos(dx+c)^n\cos(dx+c)\sec(dx+c)^4\,dx\right)b + \left(\int \cos(dx+c)^n\cos(dx+c)^2\sec(dx+c)^4\,dx\right)c + \left(\int \cos(dx+c)^n\sec(dx+c)^4\,dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

output
```
b**n*(int(cos(c + d*x)**n*cos(c + d*x)*sec(c + d*x)**4,x)*b + int(cos(c +
d*x)**n*cos(c + d*x)**2*sec(c + d*x)**4,x)*c + int(cos(c + d*x)**n*sec(c +
 d*x)**4,x)*a)
```

3.377 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\right.$

Optimal result

Integrand size = 41, antiderivative size = 223

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$=\frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)}$$

$$-\frac{2(C(5+2n)+A(7+2n))\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n)\right.}{d(5+2n)(7+2n)\sqrt{\sin^2(c+dx)}}$$

$$-\frac{2B\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\sin(c+}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
2*C*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(7+2*n)-2*(C*(5+2*n)+A*
(7+2*n))*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4
+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(7+2*n)/(sin(d*x+c)^2)^(1/2)-2*
B*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2*n
],cos(d*x+c)^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left((C(5+2n)+A(7+2n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Co
s[c + d*x]^2),x]
```

output
```
(-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*((C*(5 + 2*n) + A*(
7 + 2*n))*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]
*Sqrt[Sin[c + d*x]^2] - (5 + 2*n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hyper
geometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c +
d*x]^2])))/(d*(5 + 2*n)*(7 + 2*n))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n+\frac{3}{2}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx$$

$$\Big\downarrow \; 3502$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2\int \frac{1}{2}\cos^{n+\frac{3}{2}}(c+dx)\left(2C\left(n+\frac{5}{2}\right)+2A\left(n+\frac{7}{2}\right)+B(2n+7)\cos(c+dx)\right)dx}{2n+7}+\frac{2C\sin(c+dx)\cos^{n+\frac{5}{2}}(c}{d(2n+7)}\right.$$

$$\Big\downarrow \; 27$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\int \cos^{n+\frac{3}{2}}(c+dx)(C(2n+5)+A(2n+7)+B(2n+7)\cos(c+dx))dx}{2n+7}+\frac{2C\sin(c+dx)\cos^{n+\frac{5}{2}}(c+dx)}{d(2n+7)}\right.$$

$$\Big\downarrow \; 3042$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}\left(C(2n+5)+A(2n+7)+B(2n+7)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2n+7}+\frac{2C\sin(c+dx)\cos^{n}}{d(2n+7}\right.$$

$$\Big\downarrow \; 3227$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(2n+7)+C(2n+5))\int \cos^{n+\frac{3}{2}}(c+dx)dx+B(2n+7)\int \cos^{n+\frac{5}{2}}(c+dx)dx}{2n+7}+\frac{2C\sin(c+dx)\cos^{n+\frac{5}{2}}}{d(2n+7)}\right.$$

$$\Big\downarrow \; 3042$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(2n+7)+C(2n+5))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}dx+B(2n+7)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{5}{2}}dx}{2n+7}+\frac{2C\sin(c+}{}\right.$$

$$\Big\downarrow \; 3122$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{-\frac{2(A(2n+7)+C(2n+5))\sin(c+dx)\cos^{n+\frac{5}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+5),\frac{1}{4}(2n+9),\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{n+\frac{7}{2}}}{}}{2n+7}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c +
d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*C*Cos[c + d*x]^(5/2 + n)*Sin[c + d*x])/(d*(7 + 2*n
)) + ((-2*(C*(5 + 2*n) + A*(7 + 2*n))*Cos[c + d*x]^(5/2 + n)*Hypergeometri
c2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 +
2*n)*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(7/2 + n)*Hypergeometric2F1
[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin
[c + d*x]^2]))/(7 + 2*n)))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \cos \left(dx+c\right)^{\frac{3}{2}} \left(b\cos \left(dx+c\right)\right)^{n} \left(A+B\cos \left(dx+c\right)+C\cos \left(dx+c\right)^{2}\right)dx$$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos \left(dx+c\right)^{2}+B\cos \left(dx+c\right)+A\right)\left(b\cos \left(dx+c\right)\right)^{n}\cos \left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
 + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)
**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d
*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d
*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \cos(c+d\,x)^{3/2}\,(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=b^n\left(\left(\int \cos(dx+c)^{n+\frac{1}{2}}\cos(dx+c)\,dx\right)a+\left(\int \cos(dx+c)^{n+\frac{1}{2}}\cos(dx+c)^3\,dx\right)c+\left(\int \cos(dx+c)^{n+\frac{1}{2}}\cos(dx+c)^2\,dx\right)b\right)$$

input
```
int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)*cos(c + d*x),x)*a + int(cos(c + d*x)
**((2*n + 1)/2)*cos(c + d*x)**3,x)*c + int(cos(c + d*x)**((2*n + 1)/2)*cos
(c + d*x)**2,x)*b)
```

3.378 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\left(A+B\cos(c+dx)\right.$ -

Optimal result

Integrand size = 41, antiderivative size = 223

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)}$$
$$-\frac{2(C(3+2n)+A(5+2n))\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\right.}{d(3+2n)(5+2n)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{2B\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin(c+d}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
2*C*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(5+2*n)-2*(C*(3+2*n)+A*
(5+2*n))*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4
+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(5+2*n)/(sin(d*x+c)^2)^(1/2)-2*
B*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*n]
,cos(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx =$$
$$-\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \csc(c+dx)\left((C(3+2n)+A(5+2n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2\right.\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Co
s[c + d*x]^2),x]
```

output
```
(-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*((C*(3 + 2*n) + A*(
5 + 2*n))*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]
*Sqrt[Sin[c + d*x]^2] - (3 + 2*n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hyper
geometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d
*x]^2])))/(d*(3 + 2*n)*(5 + 2*n))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) dx$$

$$\Big\downarrow \ 2034$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \cos^{n+\frac{1}{2}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right) dx$$

$$\Big\downarrow \ 3042$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n \int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right) dx$$

$$\Big\downarrow \text{3502}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2\int\frac{1}{2}\cos^{n+\frac{1}{2}}(c+dx)\left(2C\left(n+\frac{3}{2}\right)+2A\left(n+\frac{5}{2}\right)+B(2n+5)\cos(c+dx)\right)dx}{2n+5}+\frac{2C\sin(c+dx)\cos^{n+\frac{3}{2}}(c}{d(2n+5)}\right.$$

$$\Big\downarrow \text{27}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\int\cos^{n+\frac{1}{2}}(c+dx)(C(2n+3)+A(2n+5)+B(2n+5)\cos(c+dx))dx}{2n+5}+\frac{2C\sin(c+dx)\cos^{n+\frac{3}{2}}(c+dx)}{d(2n+5)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}\left(C(2n+3)+A(2n+5)+B(2n+5)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2n+5}+\frac{2C\sin(c+dx)\cos^{n}}{d(2n+5}\right.$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(2n+5)+C(2n+3))\int\cos^{n+\frac{1}{2}}(c+dx)dx+B(2n+5)\int\cos^{n+\frac{3}{2}}(c+dx)dx}{2n+5}+\frac{2C\sin(c+dx)\cos^{n+\frac{3}{2}}}{d(2n+5)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(2n+5)+C(2n+3))\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}dx+B(2n+5)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}dx}{2n+5}+\frac{2C\sin(c+}{}\right.$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{-\frac{2(A(2n+5)+C(2n+3))\sin(c+dx)\cos^{n+\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{n+\frac{5}{2}}}{}}{2n+5}\right.$$

input

```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c +
d*x]^2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*C*Cos[c + d*x]^(3/2 + n)*Sin[c + d*x])/(d*(5 + 2*n
)) + ((-2*(C*(3 + 2*n) + A*(5 + 2*n))*Cos[c + d*x]^(3/2 + n)*Hypergeometri
c2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 +
2*n)*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(5/2 + n)*Hypergeometric2F1
[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[
c + d*x]^2]))/(5 + 2*n)))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \sqrt{\cos \left(dx+c\right)}\, \left(b\cos \left(dx+c\right)\right)^{n} \left(A+B\cos \left(dx+c\right)+C \cos \left(dx+c\right)^{2}\right) dx$$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos \left(dx+c\right)^{2}+B\cos \left(dx+c\right)+A\right) \left(b\cos \left(dx+c\right)\right)^{n} \sqrt{\cos \left(dx+c\right)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(c
os(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)
**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(
cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$= \int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(
cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$
$$= \int \sqrt{\cos\left(c+d\,x\right)}\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right) \, dx$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c +
d*x)^2), x)
```

Reduce [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right) \, dx$$
$$= b^n \left(\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}dx\right)a+\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}\cos\left(dx+c\right)dx\right)b + \left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}\cos\left(dx+c\right)^2 dx\right)c\right)$$

input
```
int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2),x)*a + int(cos(c + d*x)**((2*n + 1)/
2)*cos(c + d*x),x)*b + int(cos(c + d*x)**((2*n + 1)/2)*cos(c + d*x)**2,x)*
c)
```

3.379 $\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 221

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2C\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)}$$
$$- \frac{2(C+2Cn+A(3+2n))\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\right.}{d(1+2n)(3+2n)\sqrt{\sin^2(c+dx)}}$$
$$- \frac{2B\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin(c+d}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
2*C*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(3+2*n)-2*(C+2*C*n+A*(3
+2*n))*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1
/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1+2*n)/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*B*
cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2*n],c
os(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.77

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$
$$=\frac{2\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\csc(c+dx)\left(-\left((C+2Cn+A(3+2n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2\right.\right.\right.}{}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqr
t[Cos[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(-((C + 2*C*n + A*(3
 + 2*n))*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*
Sqrt[Sin[c + d*x]^2]) + (1 + 2*n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hyper
geometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d
*x]^2])))/(d*(1 + 2*n)*(3 + 2*n))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{1}{2}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx$$

$\downarrow$ 3502

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2\int\frac{1}{2}\cos^{n-\frac{1}{2}}(c+dx)(2nC+C+A(2n+3)+B(2n+3)\cos(c+dx))dx}{2n+3}+\frac{2C\sin(c+dx)\cos^{n+\frac{1}{2}}(c+d}{d(2n+3)}\right.$$

$\downarrow$ 27

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\int\cos^{n-\frac{1}{2}}(c+dx)(2nC+C+A(2n+3)+B(2n+3)\cos(c+dx))dx}{2n+3}+\frac{2C\sin(c+dx)\cos^{n+\frac{1}{2}}(c+dx)}{d(2n+3)}\right)$$

$\downarrow$ 3042

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}\left(2nC+C+A(2n+3)+B(2n+3)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2n+3}+\frac{2C\sin(c+dx)\cos^{n+}}{d(2n+3)}\right.$$

$\downarrow$ 3227

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(2n+3)+2Cn+C)\int\cos^{n-\frac{1}{2}}(c+dx)dx+B(2n+3)\int\cos^{n+\frac{1}{2}}(c+dx)dx}{2n+3}+\frac{2C\sin(c+dx)\cos^{n+\frac{1}{2}}(}{d(2n+3)}\right.$$

$\downarrow$ 3042

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(2n+3)+2Cn+C)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}dx+B(2n+3)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}dx}{2n+3}+\frac{2C\sin(c+}{d}\right.$$

$\downarrow$ 3122

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{-\frac{2(A(2n+3)+2Cn+C)\sin(c+dx)\cos^{n+\frac{1}{2}}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+1),\frac{1}{4}(2n+5),\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{n+\frac{3}{2}}(}{}}{2n+3}\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[
c + d*x]],x]
```

output
```
((b*Cos[c + d*x])^n*((2*C*Cos[c + d*x]^(1/2 + n)*Sin[c + d*x])/(d*(3 + 2*n
)) + ((-2*(C + 2*C*n + A*(3 + 2*n))*Cos[c + d*x]^(1/2 + n)*Hypergeometric2
F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*
n)*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(3/2 + n)*Hypergeometric2F1[1
/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c
+ d*x]^2]))/(3 + 2*n)))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{(b\cos(dx+c))^n \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)}{\sqrt{\cos(dx+c)}}dx$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

output
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$
$$=\int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(c
os(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$=\int \frac{(b\cos{(c+dx)})^n\,(A+B\cos{(c+dx)}+C\cos^2{(c+dx)})}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(
1/2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)/sqrt
(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$
$$=\int \frac{(C\cos{(dx+c)}^2+B\cos{(dx+c)}+A)(b\cos{(dx+c)})^n}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(
cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$
$$=\int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(
cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$
$$=\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^(1/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= b^n\left(\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}dx\right)b+\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)}dx\right)a + \left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}\cos\left(dx+c\right)dx\right)c\right)$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2),x)*b + int(cos(c + d*x)**((2*n + 1)/
2)/cos(c + d*x),x)*a + int(cos(c + d*x)**((2*n + 1)/2)*cos(c + d*x),x)*c)
```

3.380 $\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 217

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}}$$
$$+ \frac{2(A-C(1-2n)+2An)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\ldots}{d\left(1-4n^2\right)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$
$$- \frac{2B\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos^2(c+dx)\right)\sin(c+\ldots}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1+2*n)/cos(d*x+c)^(1/2)+2*(A-C*(1-2*n)+
2*A*n)*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)
^2)*sin(d*x+c)/d/(-4*n^2+1)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)-2*B*cos(
d*x+c)^(1/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d
*x+c)^2)*sin(d*x+c)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.76

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2(b\cos(c+dx))^n \csc(c+dx)\left(-\left((A+2An+C(-1+2n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+\right.\right.\right.}{}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos
[c + d*x]^(3/2),x]
```

output
```
(2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(-((A + 2*A*n + C*(-1 + 2*n))*Hypergeom
etric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]
^2]) + (-1 + 2*n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[1/2
, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d*(-1
 + 4*n^2)*Sqrt[Cos[c + d*x]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{3}{2}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx$$

$\big\downarrow$ 3502

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{2\int\frac{1}{2}\cos^{n-\frac{3}{2}}(c+dx)(2nA+A-C(1-2n)+B(2n+1)\cos(c+dx))dx}{2n+1}+\frac{2C\sin(c+dx)\cos^{n-\frac{1}{2}}(c+d}{d(2n+1)}\right.$$

$\big\downarrow$ 27

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\int\cos^{n-\frac{3}{2}}(c+dx)(2nA+A-C(1-2n)+B(2n+1)\cos(c+dx))dx}{2n+1}+\frac{2C\sin(c+dx)\cos^{n-\frac{1}{2}}(c+dx)}{d(2n+1)}\right)$$

$\big\downarrow$ 3042

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}\left(2nA+A-C(1-2n)+B(2n+1)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2n+1}+\frac{2C\sin(c+dx)\cos^{n-}}{d(2n+1)}\right.$$

$\big\downarrow$ 3227

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(2An+A-C(1-2n))\int\cos^{n-\frac{3}{2}}(c+dx)dx+B(2n+1)\int\cos^{n-\frac{1}{2}}(c+dx)dx}{2n+1}+\frac{2C\sin(c+dx)\cos^{n-\frac{1}{2}}(}{d(2n+1)}\right.$$

$\big\downarrow$ 3042

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(2An+A-C(1-2n))\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}dx+B(2n+1)\int\sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}dx}{2n+1}+\frac{2C\sin(c+}{d}\right.$$

$\big\downarrow$ 3122

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\frac{2(2An+A-C(1-2n))\sin(c+dx)\cos^{n-\frac{1}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{n+\frac{1}{2}}(c+}{}}{2n+1}\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d
*x]^(3/2),x]
```

output
```
((b*Cos[c + d*x])^n*((2*C*Cos[c + d*x]^(-1/2 + n)*Sin[c + d*x])/(d*(1 + 2*
n)) + ((2*(A - C*(1 - 2*n) + 2*A*n)*Cos[c + d*x]^(-1/2 + n)*Hypergeometric
2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 -
2*n)*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(1/2 + n)*Hypergeometric2F1
[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[
c + d*x]^2]))/(1 + 2*n)))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{(b\cos(dx+c))^n \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)}{\cos(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

output
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(3/2), x)
```

Sympy [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(
3/2),x)
```

output
```
Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)/cos(
c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d
*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d
*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^(3/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= b^n\left(\left(\int \cos\left(dx+c\right)^{n+\frac{1}{2}}dx\right)c+\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)}dx\right)b+\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^2}dx\right)a\right)$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2),x)*c + int(cos(c + d*x)**((2*n + 1)/
2)/cos(c + d*x),x)*b + int(cos(c + d*x)**((2*n + 1)/2)/cos(c + d*x)**2,x)*
a)
```

3.381 $\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 221

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)}$$
$$+ \frac{2(A+C(3-2n)-2An)(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx\right.}{d(1-2n)(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$
$$+ \frac{2B(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
-2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(3/2)+2*(A+C*(3-2*n)
-2*A*n)*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c
)^2)*sin(d*x+c)/d/(1-2*n)/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)+2*
B*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*s
in(d*x+c)/d/(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2(b\cos(c+dx))^n\csc(c+dx)\left(-\left(\left(C(-3+2n)+A(-1+2n)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1\right.\right.\right.}{}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos
[c + d*x]^(5/2),x]
```

output
```
(2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(-((C*(-3 + 2*n) + A*(-1 + 2*n))*Hyperg
eometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d
*x]^2]) + (-3 + 2*n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1[
1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(d
*(-3 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{5}{2}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx$$

$\Big\downarrow$ 3502

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(-\frac{2\int -\frac{1}{2}\cos^{n-\frac{5}{2}}(c+dx)\left(2A\left(\frac{1}{2}-n\right)+2C\left(\frac{3}{2}-n\right)+B(1-2n)\cos(c+dx)\right)dx}{1-2n}-\frac{2C\sin(c+dx)\cos^{n-}}{d(1-2n)}\right.$$

$\Big\downarrow$ 27

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\int \cos^{n-\frac{5}{2}}(c+dx)(-2nA+A+C(3-2n)+B(1-2n)\cos(c+dx))dx}{1-2n}-\frac{2C\sin(c+dx)\cos^{n-\frac{3}{2}}(c+dx)}{d(1-2n)}\right.$$

$\Big\downarrow$ 3042

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}\left(-2nA+A+C(3-2n)+B(1-2n)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{1-2n}-\frac{2C\sin(c+dx)\cos}{d(1-2n)}\right.$$

$\Big\downarrow$ 3227

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(-2An+A+C(3-2n))\int \cos^{n-\frac{5}{2}}(c+dx)dx+B(1-2n)\int \cos^{n-\frac{3}{2}}(c+dx)dx}{1-2n}-\frac{2C\sin(c+dx)\cos^{n-}}{d(1-2n)}\right.$$

$\Big\downarrow$ 3042

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(-2An+A+C(3-2n))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}dx+B(1-2n)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}dx}{1-2n}-\frac{2C\sin(c+}{}\right.$$

$\Big\downarrow$ 3122

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\frac{2(-2An+A+C(3-2n))\sin(c+dx)\cos^{n-\frac{3}{2}}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-3),\frac{1}{4}(2n+1),\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}}+\frac{2B\sin(c+dx)\cos^{n-\frac{1}{2}}(}{}}{1-2n}\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d
*x]^(5/2),x]
```

output
```
((b*Cos[c + d*x])^n*((-2*C*Cos[c + d*x]^(-3/2 + n)*Sin[c + d*x])/(d*(1 - 2
*n)) + ((2*(A + C*(3 - 2*n) - 2*A*n)*Cos[c + d*x]^(-3/2 + n)*Hypergeometri
c2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 -
 2*n)*Sqrt[Sin[c + d*x]^2]) + (2*B*Cos[c + d*x]^(-1/2 + n)*Hypergeometric2
F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[S
in[c + d*x]^2]))/(1 - 2*n)))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{(b\cos (dx+c))^n \left(A+B\cos (dx+c)+C\cos (dx+c)^2\right)}{\cos (dx+c)^{\frac{5}{2}}}dx$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

output
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos (dx+c)^2+B\cos (dx+c)+A\right)(b\cos (dx+c))^n}{\cos (dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(
5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d
*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d
*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^(5/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = b^n\left(\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)}dx\right)c+\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^3}dx\right)a + \left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^2}dx\right)b\right)$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

output
```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)/cos(c + d*x),x)*c + int(cos(c + d*x)
**((2*n + 1)/2)/cos(c + d*x)**3,x)*a + int(cos(c + d*x)**((2*n + 1)/2)/cos
(c + d*x)**2,x)*b)
```

3.382 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 223

$$\begin{aligned}
&\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx \\
&= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} \\
&\quad+\frac{2(A(3-2n)+C(5-2n))(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+\right.}{d(3-2n)(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}} \\
&\quad+\frac{2B(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

output
```
-2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(5/2)+2*(A*(3-2*n)+C
*(5-2*n))*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*
x+c)^2)*sin(d*x+c)/d/(3-2*n)/(5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)
+2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2(b\cos(c+dx))^n\csc(c+dx)\left(-\left(\left(C(-5+2n)+A(-3+2n)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-\right.\right.\right.}{}$$

input
```
Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos
[c + d*x]^(7/2),x]
```

output
```
(2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(-((C*(-5 + 2*n) + A*(-3 + 2*n))*Hyperg
eometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c +
d*x]^2]) + (-5 + 2*n)*(C*Sin[c + d*x]^2 - B*Cos[c + d*x]*Hypergeometric2F1
[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))/(
d*(-5 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 3502, 27, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{7}{2}}(c+dx)\left(C\cos^2(c+dx)+B\cos(c+dx)+A\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{7}{2}}\left(C\sin\left(c+dx+\frac{\pi}{2}\right)^2+B\sin\left(c+dx+\frac{\pi}{2}\right)+A\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(-\frac{2\int -\frac{1}{2}\cos^{n-\frac{7}{2}}(c+dx)\left(2A\left(\frac{3}{2}-n\right)+2C\left(\frac{5}{2}-n\right)+B(3-2n)\cos(c+dx)\right)dx}{3-2n}-\frac{2C\sin(c+dx)\cos^{n-}}{d(3-2n)}\right.$$

$$\Big\downarrow \text{27}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\int \cos^{n-\frac{7}{2}}(c+dx)(A(3-2n)+B\cos(c+dx)(3-2n)+C(5-2n))dx}{3-2n}-\frac{2C\sin(c+dx)\cos^{n-\frac{5}{2}}(c+dx)}{d(3-2n)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{7}{2}}\left(A(3-2n)+B\sin\left(c+dx+\frac{\pi}{2}\right)(3-2n)+C(5-2n)\right)dx}{3-2n}-\frac{2C\sin(c+dx)\cos^{n}}{d(3-2n}\right.$$

$$\Big\downarrow \text{3227}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(3-2n)+C(5-2n))\int \cos^{n-\frac{7}{2}}(c+dx)dx+B(3-2n)\int \cos^{n-\frac{5}{2}}(c+dx)dx}{3-2n}-\frac{2C\sin(c+dx)\cos^{n-\frac{5}{2}}}{d(3-2n)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{(A(3-2n)+C(5-2n))\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{7}{2}}dx+B(3-2n)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}dx}{3-2n}-\frac{2C\sin(c+}{}\right.$$

$$\Big\downarrow \text{3122}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\left(\frac{\frac{2(A(3-2n)+C(5-2n))\sin(c+dx)\cos^{n-\frac{5}{2}}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-5),\frac{1}{4}(2n-1),\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}}+\frac{2B\sin(c+dx)\cos^{n-\frac{3}{2}}(c}{}}{3-2n}\right.$$

input
```
Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d
*x]^(7/2),x]
```

output
```
((b*Cos[c + d*x])^n*((-2*C*Cos[c + d*x]^(-5/2 + n)*Sin[c + d*x])/(d*(3 - 2
*n)) + ((2*(A*(3 - 2*n) + C*(5 - 2*n))*Cos[c + d*x]^(-5/2 + n)*Hypergeomet
ric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(
5 - 2*n)*Sqrt[Sin[c + d*x]^2]) + (2*B*Cos[c + d*x]^(-3/2 + n)*Hypergeometr
ic2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqr
t[Sin[c + d*x]^2]))/(3 - 2*n)))/Cos[c + d*x]^n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{(b\cos(dx+c))^n \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right)}{\cos(dx+c)^{\frac{7}{2}}}dx$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

output
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2
),x, algorithm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(7/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(
7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2
),x, algorithm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d
*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^n}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2
),x, algorithm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d
*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

input
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^(7/2),x)
```

output
```
int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d
*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = b^n \left(\left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^4}dx \right) a + \left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^3}dx \right) b + \left(\int \frac{\cos\left(dx+c\right)^{n+\frac{1}{2}}}{\cos\left(dx+c\right)^2}dx \right) c \right)$$

input
```
int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

output

```
b**n*(int(cos(c + d*x)**((2*n + 1)/2)/cos(c + d*x)**4,x)*a + int(cos(c + d
*x)**((2*n + 1)/2)/cos(c + d*x)**3,x)*b + int(cos(c + d*x)**((2*n + 1)/2)/
cos(c + d*x)**2,x)*c)
```

3.383 $\int (a+a\cos(e+fx))^m (A+B\cos(e+fx)+C\cos^2(e$

Optimal result

Integrand size = 33, antiderivative size = 183

$$\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e+fx)}{f(1+m)(2+m)} + \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{2^{\frac{1}{2}+m}(Bm(2+m)+C(1+m+m^2)+A(2+3m+m^2))\,(1+\cos(e+fx))^{-\frac{1}{2}-m}(a+a\cos(e+fx))^{m}}{f(1+m)(2+m)}$$

output
```
-(C-B*(2+m))*(a+a*cos(f*x+e))^m*sin(f*x+e)/f/(1+m)/(2+m)+C*(a+a*cos(f*x+e)
)^(1+m)*sin(f*x+e)/a/f/(2+m)+2^(1/2+m)*(B*m*(2+m)+C*(m^2+m+1)+A*(m^2+3*m+2
))*(1+cos(f*x+e))^(-1/2-m)*(a+a*cos(f*x+e))^m*hypergeom([1/2, 1/2-m],[3/2]
,1/2-1/2*cos(f*x+e))*sin(f*x+e)/f/(1+m)/(2+m)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.12 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.05

$$\int (a + a\cos(e + fx))^m \left(A + B\cos(e + fx) + C\cos^2(e + fx)\right) dx$$
$$= \frac{i4^{-1-m}e^{ifmx}\left(1 + e^{i(e+fx)}\right)^{-2m}\left(e^{-\frac{1}{2}i(e+fx)}\left(1 + e^{i(e+fx)}\right)\right)^{2m}\cos^{-2m}\left(\frac{1}{2}(e + fx)\right)(a(1 + \cos(e + fx)))^m \left(\cdots\right.}{\cdots}$$

input
```
Integrate[(a + a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x
]
```

output
```
(I*4^(-1 - m)*E^(I*f*m*x)*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f*x)))^(2*m
)*(a*(1 + Cos[e + f*x]))^m*((C*Hypergeometric2F1[-2 - m, -2*m, -1 - m, -E^
(I*(e + f*x))])/(E^(I*(2*e + f*(2 + m)*x))*(2 + m)) + (2*B*Hypergeometric2
F1[-1 - m, -2*m, -m, -E^(I*(e + f*x))])/(E^(I*(e + f*(1 + m)*x))*(1 + m))
+ (2*B*E^(I*(e - f*(-1 + m)*x))*Hypergeometric2F1[1 - m, -2*m, 2 - m, -E^(
I*(e + f*x))])/(-1 + m) + (C*E^((2*I)*e - I*f*(-2 + m)*x)*Hypergeometric2F
1[2 - m, -2*m, 3 - m, -E^(I*(e + f*x))])/(-2 + m) + (4*A*Hypergeometric2F1
[-2*m, -m, 1 - m, -E^(I*(e + f*x))])/(E^(I*f*m*x)*m) + (2*C*Hypergeometric
2F1[-2*m, -m, 1 - m, -E^(I*(e + f*x))])/(E^(I*f*m*x)*m)))/((1 + E^(I*(e +
f*x)))^(2*m)*f*Cos[(e + f*x)/2]^(2*m))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3502, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(e + fx) + a)^m \left(A + B\cos(e + fx) + C\cos^2(e + fx)\right) dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sin \left(e+fx+\frac{\pi}{2}\right)+a\right)^m \left(A+B \sin \left(e+fx+\frac{\pi}{2}\right)+C \sin \left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{\int (\cos(e+fx)a+a)^m(a(C(m+1)+A(m+2))-a(C-B(m+2))\cos(e+fx))dx}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(\sin \left(e+fx+\frac{\pi}{2}\right)a+a\right)^m \left(a(C(m+1)+A(m+2))-a(C-B(m+2))\sin \left(e+fx+\frac{\pi}{2}\right)\right)dx}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{\frac{a\left(A(m^2+3m+2)+Bm(m+2)+C\left(m^2+m+1\right)\right)\int (\cos(e+fx)a+a)^m dx}{m+1}-\frac{a(C-B(m+2))\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{a\left(A(m^2+3m+2)+Bm(m+2)+C\left(m^2+m+1\right)\right)\int \left(\sin\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx}{m+1}-\frac{a(C-B(m+2))\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{ 3131}$$

$$\frac{\frac{a\left(A(m^2+3m+2)+Bm(m+2)+C\left(m^2+m+1\right)\right)(\cos(e+fx)+1)^{-m}(a\cos(e+fx)+a)^m \int (\cos(e+fx)+1)^m dx}{m+1}-\frac{a(C-B(m+2))\sin(e+fx)(a\cos}{f(m+1)}}{a(m+2)}$$
$$\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{a\left(A\left(m^2+3m+2\right)+Bm(m+2)+C\left(m^2+m+1\right)\right)(\cos(e+fx)+1)^{-m}(a\cos(e+fx)+a)^m\int\left(\sin\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx}{m+1}-\frac{a(C-B(m+2))\sin(e+fx)(a}{f(m+1)}}{a(m+2)}$$
$$\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

$\big\downarrow$ 3130

$$\frac{\frac{a2^{m+\frac{1}{2}}\left(A\left(m^2+3m+2\right)+Bm(m+2)+C\left(m^2+m+1\right)\right)\sin(e+fx)(\cos(e+fx)+1)^{-m-\frac{1}{2}}(a\cos(e+fx)+a)^m\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\right.}{f(m+1)}}{a(m+2)}$$
$$\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

input
```
Int[(a + a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

output
```
(C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (-((a*(C - B
*(2 + m))*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(1 + m))) + (2^(1/2 + m)
*a*(B*m*(2 + m) + C*(1 + m + m^2) + A*(2 + 3*m + m^2))*(1 + Cos[e + f*x])^
(-1/2 - m)*(a + a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1
- Cos[e + f*x])/2]*Sin[e + f*x])/(f*(1 + m)))/(a*(2 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a + a\cos(fx+e))^m \left(A + B\cos(fx+e) + C\cos(fx+e)^2\right) dx$$

input
```
int((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
int((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a + a\cos(e+fx))^m \left(A + B\cos(e+fx) + C\cos^2(e+fx)\right) dx$$
$$= \int \left(C\cos(fx+e)^2 + B\cos(fx+e) + A\right)(a\cos(fx+e) + a)^m dx$$

input
```
integrate((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm=
"fricas")
```

output
```
integral((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e) + a)^m, x
)
```

Sympy [F]

$$\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(a(\cos{(e+fx)}+1)\right)^m \left(A+B\cos{(e+fx)}+C\cos^2{(e+fx)}\right)\,dx$$

input
```
integrate((a+a*cos(f*x+e))**m*(A+B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

output
```
Integral((a*(cos(e + f*x) + 1))**m*(A + B*cos(e + f*x) + C*cos(e + f*x)**2
), x)
```

Maxima [F]

$$\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos{(fx+e)}^2+B\cos{(fx+e)}+A\right)(a\cos{(fx+e)}+a)^m\,dx$$

input
```
integrate((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e) + a)^m,
x)
```

Giac [F]

$$\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int \left(C\cos{(fx+e)}^2+B\cos{(fx+e)}+A\right)(a\cos{(fx+e)}+a)^m\,dx$$

input
```
integrate((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e) + a)^m,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int (a+a\,\cos{(e+f\,x)})^m\,\left(C\cos{(e+f\,x)}^2+B\,\cos{(e+f\,x)}+A\right)\,dx$$

input
```
int((a + a*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2),x)
```

output
```
int((a + a*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2), x)
```

Reduce [F]

$$\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\left(\int (\cos{(fx+e)}\,a+a)^m\,dx\right)a+\left(\int (\cos{(fx+e)}\,a+a)^m\cos{(fx+e)}\,dx\right)b$$
$$+\left(\int (\cos{(fx+e)}\,a+a)^m\cos{(fx+e)}^2\,dx\right)c$$

input
```
int((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
int((cos(e + f*x)*a + a)**m,x)*a + int((cos(e + f*x)*a + a)**m*cos(e + f*x
),x)*b + int((cos(e + f*x)*a + a)**m*cos(e + f*x)**2,x)*c
```

3.384 $\int (a+a\cos(c+dx))^{2/3} \left(A+B\cos(c+dx)+C\cos^2(c\right.$

Optimal result

Integrand size = 35, antiderivative size = 144

$$\int (a+a\cos(c+dx))^{2/3} \left(A+B\cos(c+dx) + C\cos^2(c+dx)\right)\,dx = \frac{3(8B-3C)(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d} + \frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad} + \frac{(40A+16B+19C)(a+a\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx}{10\ 2^{5/6}d(1+\cos(c+dx))^{7/6}}$$

output
```
3/40*(8*B-3*C)*(a+a*cos(d*x+c))^(2/3)*sin(d*x+c)/d+3/8*C*(a+a*cos(d*x+c))^
(5/3)*sin(d*x+c)/a/d+1/20*(40*A+16*B+19*C)*(a+a*cos(d*x+c))^(2/3)*hypergeo
m([-1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c)
)^(7/6)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

$$\int (a+a\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3(a(1+\cos(c+dx)))^{2/3}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-2i(40A+16B+19C)\text{Hypergeometric}\right.}{}$$

input
```
Integrate[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

output
```
(3*(a*(1 + Cos[c + d*x]))^(2/3)*Sec[(c + d*x)/2]^2*((-2*I)*(40*A + 16*B +
19*C)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^(I*(c + d*x))]*(1 + Cos[c + d*x]
 + I*Sin[c + d*x])^(2/3) + 2*(40*A + 32*B + 28*C + 2*(8*B + 7*C)*Cos[c + d
*x] + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(320*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {3042, 3502, 27, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+a)^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{3\int \frac{1}{3}(\cos(c+dx)a+a)^{2/3}(a(8A+5C)+a(8B-3C)\cos(c+dx))dx}{8a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int(\cos(c+dx)a+a)^{2/3}(a(8A+5C)+a(8B-3C)\cos(c+dx))dx}{8a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}\left(a(8A+5C)+a(8B-3C)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{8a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{1}{5}a(40A+16B+19C)\int(\cos(c+dx)a+a)^{2/3}dx+\frac{3a(8B-3C)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}a(40A+16B+19C)\int\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}dx+\frac{3a(8B-3C)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \text{3131}$$

$$\frac{\frac{a(40A+16B+19C)(a\cos(c+dx)+a)^{2/3}\int(\cos(c+dx)+1)^{2/3}dx}{5(\cos(c+dx)+1)^{2/3}}+\frac{3a(8B-3C)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(40A+16B+19C)(a\cos(c+dx)+a)^{2/3}\int\left(\sin\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{5(\cos(c+dx)+1)^{2/3}}+\frac{3a(8B-3C)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \text{3130}$$

$$\frac{\frac{2\sqrt[6]{2}a(40A+16B+19C)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{5d(\cos(c+dx)+1)^{7/6}}+\frac{3a(8B-3C)\sin(c+dx)(a\cos(c+dx)}{5d}}{8a}$$

$$\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}$$

input
```
Int[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(a + a*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*a*d) + ((3*a*(8*B - 3*C)*
(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) + (2*2^(1/6)*a*(40*A + 16*B
 + 19*C)*(a + a*Cos[c + d*x])^(2/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 -
 Cos[c + d*x])/2]*Sin[c + d*x])/(5*d*(1 + Cos[c + d*x])^(7/6)))/(8*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a + a\cos(dx+c))^{\frac{2}{3}} \left(A + B\cos(dx+c) + C\cos(dx+c)^2\right) dx$$

input
```
int((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a + a\cos(c+dx))^{2/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx = \int \left(C\cos(dx+c)^2 + B\cos(dx+c) + A\right)(a\cos(dx+c) + a)^{\frac{2}{3}} dx$$

input
```
integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algori
thm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3
), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algori
thm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/
3), x)
```

Giac [F]

$$\int (a+a\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algori
thm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/
3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int (a+a\,\cos\left(c+d\,x\right))^{2/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int((a + a*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

output
```
int((a + a*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

Reduce [F]

$$\int (a+a\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=a^{\frac{2}{3}}\left(\left(\int (\cos(dx+c)+1)^{\frac{2}{3}}dx\right)a+\left(\int (\cos(dx+c)+1)^{\frac{2}{3}}\cos(dx+c)\,dx\right)b+\left(\int (\cos(dx+c)+1)^{\frac{2}{3}}\cos(dx+c)^2\,dx\right)c\right)$$

input
```
int((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
a**(2/3)*(int((cos(c + d*x) + 1)**(2/3),x)*a + int((cos(c + d*x) + 1)**(2/
3)*cos(c + d*x),x)*b + int((cos(c + d*x) + 1)**(2/3)*cos(c + d*x)**2,x)*c)
```

3.385 $\int \sqrt[3]{a+a\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2($

Optimal result

Integrand size = 35, antiderivative size = 144

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3(7B-3C)\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{28d}+\frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad}+\frac{(28A+7B+13C)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{14\sqrt[6]{2}d(1+\cos(c+dx))^{5/6}}$$

output

```
3/28*(7*B-3*C)*(a+a*cos(d*x+c))^(1/3)*sin(d*x+c)/d+3/7*C*(a+a*cos(d*x+c))^
(4/3)*sin(d*x+c)/a/d+1/28*(28*A+7*B+13*C)*(a+a*cos(d*x+c))^(1/3)*hypergeom
([1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/d/(1+cos(d*x+c))^
(5/6)
```

Mathematica [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

input
```
Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

output
```
Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2), x]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3502, 27, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a\cos(c+dx)+a}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{3\int \frac{1}{3}\sqrt[3]{\cos(c+dx)a+a}(a(7A+4C)+a(7B-3C)\cos(c+dx))dx}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt[3]{\cos(c+dx)a+a}(a(7A+4C)+a(7B-3C)\cos(c+dx))dx}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}\left(a(7A+4C)+a(7B-3C)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{1}{4}a(28A+7B+13C)\int \sqrt[3]{\cos(c+dx)a+a}dx+\frac{3a(7B-3C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}a(28A+7B+13C)\int \sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{3a(7B-3C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{3131}$$

$$\frac{\frac{a(28A+7B+13C)\sqrt[3]{a\cos(c+dx)+a}\int \sqrt[3]{\cos(c+dx)+1}dx}{4\sqrt[3]{\cos(c+dx)+1}}+\frac{3a(7B-3C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(28A+7B+13C)\sqrt[3]{a\cos(c+dx)+a}\int \sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx}{4\sqrt[3]{\cos(c+dx)+1}}+\frac{3a(7B-3C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}}{7a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

$$\Big\downarrow \text{3130}$$

$$\frac{\frac{a(28A+7B+13C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{2\sqrt[6]{2}d(\cos(c+dx)+1)^{5/6}}+\frac{3a(7B-3C)\sin(c+dx)\sqrt[3]{a\cos(c+}{4d}}{7a}$$
$$\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

input
```
Int[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(a + a*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*a*d) + ((3*a*(7*B - 3*C)*
(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) + (a*(28*A + 7*B + 13*C)*(a
 + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x
])/2]*Sin[c + d*x])/(2*2^(1/6)*d*(1 + Cos[c + d*x])^(5/6)))/(7*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a+a\cos(dx+c))^{\frac{1}{3}}\left(A+B\cos(dx+c)+C\cos(dx+c)^{2}\right)dx$$

input
```
int((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^{2}+B\cos(dx+c)+A\right)(a\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algori
thm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3
), x)
```

Sympy [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \sqrt[3]{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)\right)\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x
)**2), x)
```

Maxima [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algori
thm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/
3), x)
```

Giac [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(a\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algori
thm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/
3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+a\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))\,dx = \int (a+a\cos(c+d\,x))^{1/3}\,\left(C\cos(c+d\,x)^2+B\cos(c+d\,x)+A\right)\,dx$$

input
```
int((a + a*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

output
```
int((a + a*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

Reduce [F]

$$\int \sqrt[3]{a+a\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))\,dx = a^{\frac{1}{3}}\left(\left(\int (\cos(dx+c)+1)^{\frac{1}{3}}\,dx\right)a+\left(\int (\cos(dx+c)+1)^{\frac{1}{3}}\cos(dx+c)\,dx\right)b + \left(\int (\cos(dx+c)+1)^{\frac{1}{3}}\cos(dx+c)^2\,dx\right)c\right)$$

input
```
int((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
a**(1/3)*(int((cos(c + d*x) + 1)**(1/3),x)*a + int((cos(c + d*x) + 1)**(1/
3)*cos(c + d*x),x)*b + int((cos(c + d*x) + 1)**(1/3)*cos(c + d*x)**2,x)*c)
```

3.386 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 144

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{3(5B-3C)\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}}+\frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad} + \frac{(10A-5B+7C)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{5\ 2^{5/6}d\sqrt[6]{1+\cos(c+dx)}\sqrt[3]{a+a\cos(c+dx)}}$$

output
```
3/10*(5*B-3*C)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/3)+3/5*C*(a+a*cos(d*x+c))^
(2/3)*sin(d*x+c)/a/d+1/10*(10*A-5*B+7*C)*hypergeom([1/2, 5/6],[3/2],1/2-1/
2*cos(d*x+c))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(1/6)/(a+a*cos(d*x+c))^(
1/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.73

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{-3i(10A-5B+7C)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-e^{i(c+dx)}\right)(1+\cos(c+dx)+i\sin(c+dx))^{2/3}+3(5...}{10d\sqrt[3]{a(1+\cos(c+dx))}}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/
3),x]
```

output
```
((-3*I)*(10*A - 5*B + 7*C)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^(I*(c + d*x
))]*(1 + Cos[c + d*x] + I*Sin[c + d*x])^(2/3) + 3*(5*B - C + 2*C*Cos[c + d
*x])*Sin[c + d*x])/(10*d*(a*(1 + Cos[c + d*x]))^(1/3))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3502, 27, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a\cos(c+dx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt[3]{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{3\int \frac{a(5A+2C)+a(5B-3C)\cos(c+dx)}{3\sqrt[3]{\cos(c+dx)a+a}}dx}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$\big\downarrow$ 27

$$\frac{\int \frac{a(5A+2C)+a(5B-3C)\cos(c+dx)}{\sqrt[3]{\cos(c+dx)a+a}}dx}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{a(5A+2C)+a(5B-3C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$\big\downarrow$ 3230

$$\frac{\frac{1}{2}a(10A-5B+7C)\int \frac{1}{\sqrt[3]{\cos(c+dx)a+a}}dx+\frac{3a(5B-3C)\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$\big\downarrow$ 3042

$$\frac{\frac{1}{2}a(10A-5B+7C)\int \frac{1}{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{3a(5B-3C)\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$\big\downarrow$ 3131

$$\frac{\frac{a(10A-5B+7C)\sqrt[3]{\cos(c+dx)+1}\int \frac{1}{\sqrt[3]{\cos(c+dx)+1}}dx}{2\sqrt[3]{a\cos(c+dx)+a}}+\frac{3a(5B-3C)\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$\big\downarrow$ 3042

$$\frac{\frac{a(10A-5B+7C)\sqrt[3]{\cos(c+dx)+1}\int \frac{1}{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{2\sqrt[3]{a\cos(c+dx)+a}}+\frac{3a(5B-3C)\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \text{3130}$$

$$\frac{\frac{a(10A-5B+7C)\sin(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{2^{5/6}d\sqrt[6]{\cos(c+dx)+1}\sqrt[3]{a\cos(c+dx)+a}}+\frac{3a(5B-3C)\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}}{5a}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3),x]
```

output
```
(3*C*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*a*d) + ((3*a*(5*B - 3*C)*
Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(1/3)) + (a*(10*A - 5*B + 7*C)*Hyp
ergeometric2F1[1/2, 5/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2^(5/6)
*d*(1 + Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3)))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{A+B\cos(dx+c)+C\cos(dx+c)^2}{(a+a\cos(dx+c))^{\frac{1}{3}}}dx$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x)
```

output
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx=\int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algori
thm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3
), x)
```

Sympy [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt[3]{a + a\cos(c + dx)}}\,dx = \int \frac{A + B\cos{\left(c + dx \right)} + C\cos^{2}{\left(c + dx \right)}}{\sqrt[3]{a \left(\cos{\left(c + dx \right)} + 1\right)}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/3),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(a*(cos(c + d*x) + 1))**
(1/3), x)
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt[3]{a + a\cos(c + dx)}}\,dx = \int \frac{C\cos\left(dx + c\right)^{2} + B\cos\left(dx + c\right) + A}{\left(a\cos\left(dx + c\right) + a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algori
thm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/
3), x)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt[3]{a + a\cos(c + dx)}}\,dx = \int \frac{C\cos\left(dx + c\right)^{2} + B\cos\left(dx + c\right) + A}{\left(a\cos\left(dx + c\right) + a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algori
thm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/
3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(1/3),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{\left(\int \frac{\cos(dx+c)}{(\cos(dx+c)+1)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{\cos(dx+c)^2}{(\cos(dx+c)+1)^{\frac{1}{3}}}dx\right)c+\left(\int \frac{1}{(\cos(dx+c)+1)^{\frac{1}{3}}}dx\right)a}{a^{\frac{1}{3}}}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x)
```

output
```
(int(cos(c + d*x)/(cos(c + d*x) + 1)**(1/3),x)*b + int(cos(c + d*x)**2/(co
s(c + d*x) + 1)**(1/3),x)*c + int(1/(cos(c + d*x) + 1)**(1/3),x)*a)/a**(1/
3)
```

3.387 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 144

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \frac{3(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} + \frac{3C\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4ad} - \frac{(4A-8B+7C)\sqrt[3]{a+a\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{2\sqrt[6]{2}ad(1+\cos(c+dx))^{5/6}}$$

output
```
3*(A-B+C)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(2/3)+3/4*C*(a+a*cos(d*x+c))^(1/3)
*sin(d*x+c)/a/d-1/4*(4*A-8*B+7*C)*(a+a*cos(d*x+c))^(1/3)*hypergeom([1/6, 1
/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/a/d/(1+cos(d*x+c))^(5/6)
```

Mathematica [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/
3),x]
```

output
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/
3), x]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3502, 27, 3042, 3229, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a\cos(c+dx)+a)^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{3\int \frac{a(4A+C)+a(4B-3C)\cos(c+dx)}{3(\cos(c+dx)a+a)^{2/3}}dx}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(4A+C)+a(4B-3C)\cos(c+dx)}{(\cos(c+dx)a+a)^{2/3}}dx}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(4A+C)+a(4B-3C)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(\sin\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}}dx}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$$\Big\downarrow \text{3229}$$

$$\frac{\frac{12a(A-B+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-(4A-8B+7C)\int \sqrt[3]{\cos(c+dx)a+a}dx}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{12a(A-B+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-(4A-8B+7C)\int\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$\Big\downarrow$ 3131

$$\frac{\frac{12a(A-B+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{(4A-8B+7C)\sqrt[3]{a\cos(c+dx)+a}\int\sqrt[3]{\cos(c+dx)+1}dx}{\sqrt[3]{\cos(c+dx)+1}}}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$\Big\downarrow$ 3042

$$\frac{\frac{12a(A-B+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{(4A-8B+7C)\sqrt[3]{a\cos(c+dx)+a}\int\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx}{\sqrt[3]{\cos(c+dx)+1}}}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

$\Big\downarrow$ 3130

$$\frac{\frac{12a(A-B+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{2^{5/6}(4A-8B+7C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{5/6}}}{4a}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

input

```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3),x]
```

output

```
(3*C*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*a*d) + ((12*a*(A - B + C)
*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) - (2^(5/6)*(4*A - 8*B + 7*C)
*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c +
d*x])/2]*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(5/6)))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{A+B\cos(dx+c)+C\cos(dx+c)^2}{(a+a\cos(dx+c))^{\frac{2}{3}}}dx$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x)
```

output
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algori
thm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3
), x)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a(\cos(c+dx)+1))^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(a*(cos(c + d*x) + 1))**
(2/3), x)
```

Maxima [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(a + a\cos(c + dx))^{2/3}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A}{\left(a\cos\left(dx + c\right) + a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algori
thm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/
3), x)
```

Giac [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(a + a\cos(c + dx))^{2/3}}\,dx = \int \frac{C\cos\left(dx + c\right)^2 + B\cos\left(dx + c\right) + A}{\left(a\cos\left(dx + c\right) + a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algori
thm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/
3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{(a + a\cos(c + dx))^{2/3}}\,dx = \int \frac{C\cos\left(c + d\,x\right)^2 + B\,\cos\left(c + d\,x\right) + A}{\left(a + a\,\cos\left(c + d\,x\right)\right)^{2/3}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(2/3),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \frac{\left(\int \frac{\cos(dx+c)}{(\cos(dx+c)+1)^{\frac{2}{3}}}dx\right)b + \left(\int \frac{\cos(dx+c)^2}{(\cos(dx+c)+1)^{\frac{2}{3}}}dx\right)c + \left(\int \frac{1}{(\cos(dx+}\right.}{a^{\frac{2}{3}}}$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x)
```

output
```
(int(cos(c + d*x)/(cos(c + d*x) + 1)**(2/3),x)*b + int(cos(c + d*x)**2/(co
s(c + d*x) + 1)**(2/3),x)*c + int(1/(cos(c + d*x) + 1)**(2/3),x)*a)/a**(2/
3)
```

3.388 $\int (a+b\cos(c+dx))^{2/3} (A + B\cos(c + dx) + C\cos^2(c$

Optimal result

Integrand size = 35, antiderivative size = 287

$$\int (a+b\cos(c+dx))^{2/3} (A+B\cos(c+dx) + C\cos^2(c+dx))\,dx = \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{(8bB-3aC)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{5/3}} + \frac{(8Ab^2-8abB+3a^2C+5b^2C)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+d}{4\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

output
```
3/8*C*(a+b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d+1/8*(8*B*b-3*C*a)*AppellF1(1/2
,-5/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^
(5/3)*sin(d*x+c)*2^(1/2)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b
))^(5/3)+1/8*(8*A*b^2-8*B*a*b+3*C*a^2+5*C*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b
*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c
)*2^(1/2)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b))^(2/3)
```

Mathematica [A] (warning: unable to verify)

Time = 5.36 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.03

$$\int (a+b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(20(-a^2+b^2)(8bB-3aC)\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right.\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

output
```
(-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(20*(-a^2 + b^2)*(8*b*B - 3*a*
C)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c
 + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 +
Cos[c + d*x]))/(a - b))] + 4*(40*A*b^2 + 16*a*b*B - 6*a^2*C + 25*b^2*C)*Ap
pellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*
x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c +
 d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) - 20*b^2*(8*b*B + 2*a*C + 5*b*C*Cos
[c + d*x])*Sin[c + d*x]^2))/(800*b^3*d)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3502, 27, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{3\int \frac{1}{3}(a+b\cos(c+dx))^{2/3}(b(8A+5C)+(8bB-3aC)\cos(c+dx))dx}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (a+b\cos(c+dx))^{2/3}(b(8A+5C)+(8bB-3aC)\cos(c+dx))dx}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(b(8A+5C)+(8bB-3aC)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; 3235$$

$$\frac{\frac{(3a^2C-8abB+8Ab^2+5b^2C)\int (a+b\cos(c+dx))^{2/3}dx}{b}+\frac{(8bB-3aC)\int (a+b\cos(c+dx))^{5/3}dx}{b}}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(3a^2C-8abB+8Ab^2+5b^2C)\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}+\frac{(8bB-3aC)\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{b}}{8b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; 3144$$

$$\frac{-\frac{\sin(c+dx)(3a^2C-8abB+8Ab^2+5b^2C)\int \frac{(a+b\cos(c+dx))^{2/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{(8bB-3aC)\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{5/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\,\mathrm{co}}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}}{8b}$$
$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; 156$$

$$-\frac{\sin(c+dx)(3a^2C-8abB+8Ab^2+5b^2C)(a+b\cos(c+dx))^{2/3}\int\frac{\left(\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}-\frac{(a+b)(8bB-3aC)\sin(c+dx)(a+b\cos}{bd\sqrt{1-\cos(c+dx)}}$$

$$8b$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; 155$$

$$\frac{\sqrt{2}\sin(c+dx)(3a^2C-8abB+8Ab^2+5b^2C)(a+b\cos(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{\sqrt{2}(a+b)(8bB-3aC)}{}$$

$$8b$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

input
```
Int[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((Sqrt[2]*(a + b)*
(8*b*B - 3*a*C)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1
- Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(8*A
*b^2 - 8*a*b*B + 3*a^2*C + 5*b^2*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos
[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*S
in[c + d*x])/(b*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2
/3)))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a+b\cos(dx+c))^{\frac{2}{3}} \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right) dx$$

input
```
int((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int (a+b\cos(c+dx))^{2/3} (A+B\cos(c+dx)+C\cos^2(c+dx)) \, dx = \int (C\cos(dx+c)^2+B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{2}{3}} \, dx$$

input
```
integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algori
thm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3
), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algori
thm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/
3), x)
```

Giac [F]

$$\int (a+b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algori
thm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/
3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\int {\left(a+b\,\cos\left(c+d\,x\right)\right)}^{2/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int((a + b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

output
```
int((a + b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

Reduce [F]

$$\int (a+b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\left(\int (\cos(dx+c)\,b+a)^{\frac{2}{3}}dx\right)a+\left(\int (\cos(dx+c)\,b+a)^{\frac{2}{3}}\cos(dx+c)\,dx\right)b+\left(\int (\cos(dx+c)\,b+a)^{\frac{2}{3}}\cos(dx+c)^{2}dx\right)c$$

input
```
int((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((cos(c + d*x)*b + a)**(2/3),x)*a + int((cos(c + d*x)*b + a)**(2/3)*cos
(c + d*x),x)*b + int((cos(c + d*x)*b + a)**(2/3)*cos(c + d*x)**2,x)*c
```

3.389 $\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2($

Optimal result

Integrand size = 35, antiderivative size = 287

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd}$$
$$+\frac{\sqrt{2}(7bB-3aC)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{4/3}\sin(c}{7b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{4/3}}$$
$$+\frac{\sqrt{2}(7Ab^2-7abB+3a^2C+4b^2C)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos}}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

output
```
3/7*C*(a+b*cos(d*x+c))^(4/3)*sin(d*x+c)/b/d+1/7*2^(1/2)*(7*B*b-3*C*a)*Appe
llF1(1/2,-4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(
d*x+c))^(4/3)*sin(d*x+c)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b
))^(4/3)+1/7*2^(1/2)*(7*A*b^2-7*B*a*b+3*C*a^2+4*C*b^2)*AppellF1(1/2,-1/3,1
/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*s
in(d*x+c)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b))^(1/3)
```

Mathematica [A] (warning: unable to verify)

Time = 5.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.02

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx =$$

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(-a^2+b^2)\left(7bB-3aC\right)\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c}{a+b}\right.\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

output
```
(-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*(7*b*B - 3*a*C
)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c
+ d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + C
os[c + d*x]))/(a - b))] + (28*A*b^2 + 7*a*b*B - 3*a^2*C + 16*b^2*C)*Appell
F1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/
(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x
]))/(-a + b)]*(a + b*Cos[c + d*x]) - 4*b^2*(7*b*B + a*C + 4*b*C*Cos[c + d*
x])*Sin[c + d*x]^2))/(112*b^3*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3502, 27, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \; 3502$$

$$\frac{3\int \frac{1}{3}\sqrt[3]{a+b\cos(c+dx)}(b(7A+4C)+(7bB-3aC)\cos(c+dx))dx}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sqrt[3]{a+b\cos(c+dx)}(b(7A+4C)+(7bB-3aC)\cos(c+dx))dx}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b(7A+4C)+(7bB-3aC)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \; 3235$$

$$\frac{\frac{(3a^2C-7abB+7Ab^2+4b^2C)\int \sqrt[3]{a+b\cos(c+dx)}dx}{b}+\frac{(7bB-3aC)\int (a+b\cos(c+dx))^{4/3}dx}{b}}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(3a^2C-7abB+7Ab^2+4b^2C)\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}+\frac{(7bB-3aC)\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx}{b}}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \; 3144$$

$$\frac{-\frac{\sin(c+dx)(3a^2C-7abB+7Ab^2+4b^2C)\int \frac{\sqrt[3]{a+b\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{(7bB-3aC)\sin(c+dx)\int \frac{(a+b\cos(c+dx))^{4/3}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}}{7b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$$\Big\downarrow \; 156$$

$$\frac{-\frac{\sin(c+dx)(3a^2C-7abB+7Ab^2+4b^2C)\sqrt[3]{a+b\cos(c+dx)}\int\frac{\sqrt[3]{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(a+b)(7bB-3aC)\sin(c+d}{bd\sqrt{1-c}}}{7b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

$\big\downarrow$ 155

$$\frac{\frac{\sqrt{2}\sin(c+dx)(3a^2C-7abB+7Ab^2+4b^2C)\sqrt[3]{a+b\cos(c+dx)}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\sqrt{2}(a+b)(7bB-}{}}{7b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

input
```
Int[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

output
```
(3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) + ((Sqrt[2]*(a + b)*
(7*b*B - 3*a*C)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1
- Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(7*A
*b^2 - 7*a*b*B + 3*a^2*C + 4*b^2*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos
[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*S
in[c + d*x])/(b*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1
/3)))/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a+b\cos(dx+c))^{\frac{1}{3}} \left(A+B\cos(dx+c)+C\cos(dx+c)^2\right) dx$$

input
```
int((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Fricas [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int \left(C\cos(dx+c)^2+B\cos(dx+c)+A\right)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algori
thm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3
), x)
```

Sympy [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \sqrt[3]{a+b\cos\left(c+dx\right)}\left(A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)\right)\,dx$$

input
```
integrate((a+b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

output
```
Integral((a + b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)*
*2), x)
```

Maxima [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algori
thm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/
3), x)
```

Giac [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \int \left(C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algori
thm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/
3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\int (a+b\,\cos\left(c+d\,x\right))^{1/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

input
```
int((a + b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

output
```
int((a + b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

Reduce [F]

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$
$$=\left(\int (\cos(dx+c)\,b+a)^{\frac{1}{3}}\,dx\right)a+\left(\int (\cos(dx+c)\,b+a)^{\frac{1}{3}}\cos(dx+c)\,dx\right)b$$
$$+\left(\int (\cos(dx+c)\,b+a)^{\frac{1}{3}}\cos(dx+c)^2\,dx\right)c$$

input
```
int((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

output
```
int((cos(c + d*x)*b + a)**(1/3),x)*a + int((cos(c + d*x)*b + a)**(1/3)*cos
(c + d*x),x)*b + int((cos(c + d*x)*b + a)**(1/3)*cos(c + d*x)**2,x)*c
```

3.390 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 287

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}$$
$$+\frac{\sqrt{2}(5bB-3aC)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c \ldots}{5b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$
$$+\frac{\sqrt{2}(5Ab^2-5abB+3a^2C+2b^2C)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\cos \ldots}{a+ \ldots}}}{5b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{a+b\cos(c+dx)}}$$

output
```
3/5*C*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d+1/5*2^(1/2)*(5*B*b-3*C*a)*Appe
llF1(1/2,-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(
d*x+c))^(2/3)*sin(d*x+c)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b
))^(2/3)+1/5*2^(1/2)*(5*A*b^2-5*B*a*b+3*C*a^2+2*C*b^2)*AppellF1(1/2,1/3,1/
2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^
(1/3)*sin(d*x+c)/b^2/d/(1+cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/3)
```

Mathematica [A] (warning: unable to verify)

Time = 3.91 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.93

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx =$$

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(5(5Ab^2-5abB+3a^2C+2b^2C)\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\cos(c+dx)}{a-b}\right.\right.}{}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/
3),x]
```

output
```
(-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(5*A*b^2 - 5*a*b*B + 3*a^2*
C + 2*b^2*C)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a
 + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[
(b*(1 + Cos[c + d*x]))/(-a + b)] + 2*(5*b*B - 3*a*C)*AppellF1[5/3, 1/2, 1/
2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-
((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(
a + b*Cos[c + d*x]) - 10*b^2*C*Sin[c + d*x]^2))/(50*b^3*d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3502, 27, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 3502$$

$$\frac{3\int \frac{b(5A+2C)+(5bB-3aC)\cos(c+dx)}{3\sqrt[3]{a+b\cos(c+dx)}}dx}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{b(5A+2C)+(5bB-3aC)\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}dx}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{b(5A+2C)+(5bB-3aC)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 3235$$

$$\frac{\frac{\left(3a^2C-5abB+5Ab^2+2b^2C\right)\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}dx}{b}+\frac{(5bB-3aC)\int (a+b\cos(c+dx))^{2/3}dx}{b}}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\left(3a^2C-5abB+5Ab^2+2b^2C\right)\int \frac{1}{\sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}+\frac{(5bB-3aC)\int \left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}}{5b}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 3144$$

$$\frac{-\frac{\sin(c+dx)\left(3a^2C-5abB+5Ab^2+2b^2C\right)\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{(5bB-3aC)\sin(c+dx)\int \frac{(\text{c}}{\sqrt{1-\text{co}}}}{bd\sqrt{1-\cos(c+d}}}{5b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 156$$

$$-\frac{-\frac{\sin(c+dx)\left(3a^2C-5abB+5Ab^2+2b^2C\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}} - \cdots}{5b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

$\big\downarrow$ 155

$$\frac{\frac{\sqrt{2}\sin(c+dx)\left(3a^2C-5abB+5Ab^2+2b^2C\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}+\frac{\sqrt{2}(5bB-3aC)\operatorname{si}\cdots}{\cdots}}{5b}$$

$$\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

input

```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

output

```
(3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + ((Sqrt[2]*(5*b*B -
 3*a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c
+ d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 + C
os[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(5*A*b^2 - 5
*a*b*B + 3*a^2*C + 2*b^2*C)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x]
)/2, (b*(1 - Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*
Sin[c + d*x])/(b*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3)))/(5*
b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{A + B\cos(dx + c) + C\cos(dx + c)^2}{(a + b\cos(dx + c))^{\frac{1}{3}}}dx$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

output
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt[3]{a + b\cos(c + dx)}}\,dx = \int \frac{C\cos(dx + c)^2 + B\cos(dx + c) + A}{(b\cos(dx + c) + a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algori
thm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3
), x)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{A+B\cos{\left(c+dx \right)}+C\cos^{2}{\left(c+dx \right)}}{\sqrt[3]{a+b\cos{\left(c+dx \right)}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/3),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(a + b*cos(c + d*x))**(1
/3), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algori
thm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/
3), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{C\cos\left(dx+c\right)^{2}+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algori
thm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/
3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx=\left(\int \frac{\cos\left(dx+c\right)}{\left(\cos\left(dx+c\right)b+a\right)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{\cos\left(dx+c\right)^2}{\left(\cos\left(dx+c\right)b+a\right)^{\frac{1}{3}}}dx\right)c+\left(\int \frac{1}{\left(\cos\left(dx+c\right)b+a\right)^{\frac{1}{3}}}dx\right)a$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

output
```
int(cos(c + d*x)/(cos(c + d*x)*b + a)**(1/3),x)*b + int(cos(c + d*x)**2/(c
os(c + d*x)*b + a)**(1/3),x)*c + int(1/(cos(c + d*x)*b + a)**(1/3),x)*a
```

3.391 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 286

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{4bd}$$
$$+\frac{(4bB-3aC)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$+\frac{(4Ab^2-4abB+3a^2C+b^2C)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\text{si}}{2\sqrt{2}b^2d\sqrt{1+\cos(c+dx)}(a+b\cos(c+dx))^{2/3}}$$

output
```
3/4*C*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)/b/d+1/4*(4*B*b-3*C*a)*AppellF1(1/2
,-1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^
(1/3)*sin(d*x+c)*2^(1/2)/b^2/d/(1+cos(d*x+c))^(1/2)/((a+b*cos(d*x+c))/(a+b
))^(1/3)+1/4*(4*A*b^2-4*B*a*b+3*C*a^2+C*b^2)*AppellF1(1/2,2/3,1/2,3/2,b*(1
-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(
d*x+c)*2^(1/2)/b^2/d/(1+cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(2/3)
```

Mathematica [A] (warning: unable to verify)

Time = 3.88 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.93

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx =$$

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(4Ab^2-4abB+3a^2C+b^2C)\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos\ldots}{\ldots}\right.\right.}{\ldots}$$

input
```
Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/
3),x]
```

output
```
(-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(4*A*b^2 - 4*a*b*B + 3*a^2*
C + b^2*C)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a +
 b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b
*(1 + Cos[c + d*x]))/(-a + b)] + (4*b*B - 3*a*C)*AppellF1[4/3, 1/2, 1/2, 7
/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*
(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a +
b*Cos[c + d*x]) - 4*b^2*C*Sin[c + d*x]^2))/(16*b^3*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3502, 27, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin\left(c+dx+\frac{\pi}{2}\right)+C\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \; 3502$$

$$\frac{3\int \frac{b(4A+C)+(4bB-3aC)\cos(c+dx)}{3(a+b\cos(c+dx))^{2/3}}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b(4A+C)+(4bB-3aC)\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(4A+C)+(4bB-3aC)\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$$\Big\downarrow \text{3235}$$

$$\frac{\frac{(3a^2C-4abB+4Ab^2+b^2C)\int \frac{1}{(a+b\cos(c+dx))^{2/3}}dx}{b}+\frac{(4bB-3aC)\int \sqrt[3]{a+b\cos(c+dx)}dx}{b}}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(3a^2C-4abB+4Ab^2+b^2C)\int \frac{1}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}+\frac{(4bB-3aC)\int \sqrt[3]{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{4b}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$$\Big\downarrow \text{3144}$$

$$\frac{-\frac{\sin(c+dx)(3a^2C-4abB+4Ab^2+b^2C)\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{(4bB-3aC)\sin(c+dx)\int \frac{\sqrt[3]{a+b\cos}}{\sqrt{1-\cos(c+dx)}\sqrt{}}}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(}}}{4b}$$

$$\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$$\Big\downarrow \text{156}$$

$$\frac{-\frac{\sin(c+dx)(3a^2C-4abB+4Ab^2+b^2C)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}\right)^{2/3}}d\cos(c+dx)}{bd\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}-\frac{(4bB-3aC)\sin}{}}{4b}$$

$$\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

$$\Big\downarrow \; 155$$

$$\frac{\frac{\sqrt{2}\sin(c+dx)(3a^2C-4abB+4Ab^2+b^2C)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}+\frac{\sqrt{2}(4bB-3aC)\sin(c+dx)}{}}{4b}$$

$$\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

input
```
Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) + ((Sqrt[2]*(4*b*B -
 3*a*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c
+ d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 + C
os[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(4*A*b^2 - 4
*a*b*B + 3*a^2*C + b^2*C)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/
2, (b*(1 - Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Si
n[c + d*x])/(b*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3)))/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
 - e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int \frac{A+B\cos(dx+c)+C\cos(dx+c)^2}{(a+b\cos(dx+c))^{\frac{2}{3}}}dx$$

input
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

output
```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{C\cos\left(dx+c\right)^2+B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algori
thm="fricas")
```

output
```
integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3
), x)
```

Sympy [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx=\int \frac{A+B\cos\left(c+dx\right)+C\cos^2\left(c+dx\right)}{\left(a+b\cos\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(a + b*cos(c + d*x))**(2
/3), x)
```

Maxima [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algori
thm="maxima")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/
3), x)
```

Giac [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos(dx+c)^2+B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algori
thm="giac")
```

output
```
integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/
3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \int \frac{C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A}{(a+b\,\cos(c+d\,x))^{2/3}}\,dx$$

input
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3),x)
```

output
```
int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \left(\int \frac{\cos(dx+c)}{(\cos(dx+c)\,b+a)^{\frac{2}{3}}}dx\right)b + \left(\int \frac{\cos(dx+c)^2}{(\cos(dx+c)\,b+a)^{\frac{2}{3}}}dx\right)c + \left(\int \frac{1}{(\cos(dx+c)\,b+a)^{\frac{2}{3}}}dx\right)a$$

input

```
int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

output

```
int(cos(c + d*x)/(cos(c + d*x)*b + a)**(2/3),x)*b + int(cos(c + d*x)**2/(c
os(c + d*x)*b + a)**(2/3),x)*c + int(1/(cos(c + d*x)*b + a)**(2/3),x)*a
```

3.392 $\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\right.$

Optimal result

Integrand size = 35, antiderivative size = 215

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$

$$= \frac{4\sqrt{2}C\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-}}{f\sqrt{1+\cos(e+fx)}}$$

$$+\frac{2\sqrt{2}(A-C)\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos}{a-}\right.}{f\sqrt{1+\cos(e+fx)}}$$

output
```
4*2^(1/2)*C*AppellF1(1/2,-m,-3/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*
x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)/f/(1+cos(f*x+e))^(1/2)/(((a+b*cos(f*x+
e))/(a+b))^m)+2*2^(1/2)*(A-C)*AppellF1(1/2,-m,-1/2,3/2,b*(1-cos(f*x+e))/(a
+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)/f/(1+cos(f*x+e))^(1/
2)/(((a+b*cos(f*x+e))/(a+b))^m)
```

Mathematica [F]

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$

input
```
Integrate[(a + b*Cos[e + f*x])^m*(A + (A + C)*Cos[e + f*x] + C*Cos[e + f*x
]^2),x]
```

output
```
Integrate[(a + b*Cos[e + f*x])^m*(A + (A + C)*Cos[e + f*x] + C*Cos[e + f*x
]^2), x]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3496, 3042, 3234, 156, 155, 3263, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left((A+C)\cos(e+fx)+A+C\cos^2(e+fx)\right)(a+b\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left((A+C)\sin\left(e+fx+\frac{\pi}{2}\right)+A+C\sin\left(e+fx+\frac{\pi}{2}\right)^2\right)\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3496}$$

$$(A-C)\int (\cos(e+fx)+1)(a+b\cos(e+fx))^m dx+C\int (\cos(e+fx)+1)^2(a+b\cos(e+fx))^m dx$$

$$\Big\downarrow \text{3042}$$

$$(A-C)\int \left(\sin \left(e+fx+\frac{\pi}{2}\right)+1\right) \left(a+b\sin \left(e+fx+\frac{\pi}{2}\right)\right)^m dx+ C\int \left(\sin \left(e+fx+\frac{\pi}{2}\right)+1\right)^2 \left(a+b\sin \left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$\downarrow$ 3234

$$C\int \left(\sin \left(e+fx+\frac{\pi}{2}\right)+1\right)^2 \left(a+b\sin \left(e+fx+\frac{\pi}{2}\right)\right)^m dx-\frac{(A-C)\sin(e+fx)\int \frac{\sqrt{\cos(e+fx)+1}(a+b\cos(e+fx))^m}{\sqrt{1-\cos(e+fx)}}d\cos(e+fx)}{f\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}$$

$\downarrow$ 156

$$C\int \left(\sin \left(e+fx+\frac{\pi}{2}\right)+1\right)^2 \left(a+b\sin \left(e+fx+\frac{\pi}{2}\right)\right)^m dx-\frac{(A-C)\sin(e+fx)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\int \frac{\sqrt{\cos(e+fx)+1}\left(\frac{a}{a+b}+\frac{b\cos(e+fx)}{a+b}\right)^m}{\sqrt{1-\cos(e+fx)}}d\cos(e+fx)}{f\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}$$

$\downarrow$ 155

$$C\int \left(\sin \left(e+fx+\frac{\pi}{2}\right)+1\right)^2 \left(a+b\sin \left(e+fx+\frac{\pi}{2}\right)\right)^m dx+\frac{2\sqrt{2}(A-C)\sin(e+fx)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(}{}\right.}{f\sqrt{\cos(e+fx)+1}}$$

$\downarrow$ 3263

$$\frac{2\sqrt{2}(A-C)\sin(e+fx)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(}{}\right.}{f\sqrt{\cos(e+fx)+1}}$$
$$\frac{C\sin(e+fx)\int \frac{(\cos(e+fx)+1)^{3/2}(a+b\cos(e+fx))^m}{\sqrt{1-\cos(e+fx)}}d\cos(e+fx)}{f\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}$$

$\downarrow$ 156

$$\frac{2\sqrt{2}(A-C)\sin(e+fx)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(}{}\right.}{f\sqrt{\cos(e+fx)+1}}$$
$$\frac{C\sin(e+fx)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\int \frac{(\cos(e+fx)+1)^{3/2}\left(\frac{a}{a+b}+\frac{b\cos(e+fx)}{a+b}\right)^m}{\sqrt{1-\cos(e+fx)}}d\cos(e+fx)}{f\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}$$

$\downarrow$ 155

$$\frac{2\sqrt{2}(A-C)\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1}{}\right.}{f\sqrt{\cos(e+fx)+1}}$$
$$\frac{4\sqrt{2}C\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e}{a+b}\right.}{f\sqrt{\cos(e+fx)+1}}$$

input
```
Int[(a + b*Cos[e + f*x])^m*(A + (A + C)*Cos[e + f*x] + C*Cos[e + f*x]^2),x
]
```

output
```
(4*Sqrt[2]*C*AppellF1[1/2, -3/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Co
s[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos
[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (2*Sqrt[2]*(A - C)*AppellF1
[1/2, -1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]
*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]]*((a + b*Co
s[e + f*x])/(a + b))^m)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3234
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]]))   Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 -
(d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2,
0]
```

rule 3263
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]]))   Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d
*x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& IntegerQ[m]
```

rule 3496
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A - C)
  Int[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]), x], x] + Simp[C   Int[(a +
 b*Sin[e + f*x])^m*(1 + Sin[e + f*x])^2, x], x] /; FreeQ[{a, b, e, f, A, B,
 C, m}, x] && EqQ[A - B + C, 0] &&  !IntegerQ[2*m]
```

Maple [F]

$$\int (a+b\cos (fx+e))^m \left(A+(A+C)\cos (fx+e)+C\cos (fx+e)^2\right) dx$$

input
```
int((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
int((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \int \left(C\cos(fx+e)^2+(A+C)\cos(fx+e)+A\right)(b\cos(fx+e)+a)^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x, algori
thm="fricas")
```

output
```
integral((C*cos(f*x + e)^2 + (A + C)*cos(f*x + e) + A)*(b*cos(f*x + e) + a
)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(f*x+e))**m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \int \left(C\cos(fx+e)^2+(A+C)\cos(fx+e)+A\right)(b\cos(fx+e)+a)^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x, algori
thm="maxima")
```

output
```
integrate((C*cos(f*x + e)^2 + (A + C)*cos(f*x + e) + A)*(b*cos(f*x + e) +
a)^m, x)
```

Giac [F]

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int (C\cos(fx+e)^2+(A+C)\cos(fx+e)+A)(b\cos(fx+e)+a)^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x, algori
thm="giac")
```

output
```
integrate((C*cos(f*x + e)^2 + (A + C)*cos(f*x + e) + A)*(b*cos(f*x + e) +
a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\int (a+b\,\cos(e+f\,x))^m\,\left(C\cos(e+f\,x)^2+(A+C)\,\cos(e+f\,x)+A\right)\,dx$$

input
```
int((a + b*cos(e + f*x))^m*(A + C*cos(e + f*x)^2 + cos(e + f*x)*(A + C)),x
)
```

output
```
int((a + b*cos(e + f*x))^m*(A + C*cos(e + f*x)^2 + cos(e + f*x)*(A + C)),
x)
```

Reduce [F]

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \left(\int (\cos(fx+e)b+a)^m dx\right)a + \left(\int (\cos(fx+e)b+a)^m \cos(fx+e)\,dx\right)a$$
$$+ \left(\int (\cos(fx+e)b+a)^m \cos(fx+e)\,dx\right)c$$
$$+ \left(\int (\cos(fx+e)b+a)^m \cos(fx+e)^2\,dx\right)c$$

input
```
int((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
int((cos(e + f*x)*b + a)**m,x)*a + int((cos(e + f*x)*b + a)**m*cos(e + f*x
),x)*a + int((cos(e + f*x)*b + a)**m*cos(e + f*x),x)*c + int((cos(e + f*x)
*b + a)**m*cos(e + f*x)**2,x)*c
```

3.393 $\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e\right.$

Optimal result

Integrand size = 33, antiderivative size = 304

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$=\frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$
$$-\frac{\sqrt{2}(aC-bB(2+m))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+f}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$
$$+\frac{\sqrt{2}(a^2C+b^2C(1+m)+Ab^2(2+m)-abB(2+m))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-}{b^2f(2+m)\sqrt{1+\cos(e+fx)}}$$

output
```
C*(a+b*cos(f*x+e))^(1+m)*sin(f*x+e)/b/f/(2+m)-2^(1/2)*(a*C-b*B*(2+m))*Appe
llF1(1/2,-1-m,1/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(
f*x+e))^(1+m)*((a+b*cos(f*x+e))/(a+b))^(-1-m)*sin(f*x+e)/b^2/f/(2+m)/(1+co
s(f*x+e))^(1/2)+2^(1/2)*(a^2*C+b^2*C*(1+m)+A*b^2*(2+m)-a*b*B*(2+m))*Appell
F1(1/2,-m,1/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+
e))^m*sin(f*x+e)/b^2/f/(2+m)/(1+cos(f*x+e))^(1/2)/(((a+b*cos(f*x+e))/(a+b)
)^m)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 16142 vs. $2(304) = 608$.

Time = 28.78 (sec) , antiderivative size = 16142, normalized size of antiderivative = 53.10

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$
$$= \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x
]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3502, 3042, 3235, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m \left(A+B\sin\left(e+fx+\frac{\pi}{2}\right)+C\sin\left(e+fx+\frac{\pi}{2}\right)^2\right) dx$$

$$\Big\downarrow \; 3502$$

$$\frac{\int (a+b\cos(e+fx))^m(b(C(m+1)+A(m+2))-(aC-bB(m+2))\cos(e+fx))dx}{b(m+2)}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m \left(b(C(m+1)+A(m+2))+(bB(m+2)-aC)\sin\left(e+fx+\frac{\pi}{2}\right)\right)dx}{b(m+2)}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \ 3235$$

$$\frac{\frac{(a^2C-abB(m+2)+Ab^2(m+2)+b^2C(m+1))\int(a+b\cos(e+fx))^m dx}{b}-\frac{(aC-bB(m+2))\int(a+b\cos(e+fx))^{m+1}dx}{b}}{b(m+2)}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(a^2C-abB(m+2)+Ab^2(m+2)+b^2C(m+1))\int\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx}{b}-\frac{(aC-bB(m+2))\int\left(a+b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}dx}{b}}{b(m+2)}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \ 3144$$

$$\frac{\frac{\sin(e+fx)(aC-bB(m+2))\int\frac{(a+b\cos(e+fx))^{m+1}}{\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}d\cos(e+fx)}{bf\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}-\frac{\sin(e+fx)(a^2C-abB(m+2)+Ab^2(m+2)+b^2C(m+1))\int\frac{(a+b\cos}{\sqrt{1-\cos(e+f}}}{bf\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}}{b(m+2)}$$
$$\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \ 156$$

$$\frac{\frac{(a+b)\sin(e+fx)(aC-bB(m+2))(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\int\frac{\left(\frac{a}{a+b}+\frac{b\cos(e+fx)}{a+b}\right)^{m+1}}{\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}d\cos(e+fx)}{bf\sqrt{1-\cos(e+fx)}\sqrt{\cos(e+fx)+1}}-\frac{\sin(e+fx)(a^2C-abB(m}{}}{b(m+2)}$$
$$\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \ 155$$

$$\frac{\frac{\sqrt{2}\sin(e+fx)(a^2C-abB(m+2)+Ab^2(m+2)+b^2C(m+1))(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-c}{}\right.}{bf\sqrt{\cos(e+fx)+1}}}{b(}$$
$$\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

input
```
Int[(a + b*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

output
```
(C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) + (-((Sqrt[2]*
(a + b)*(a*C - b*B*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f
*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x
])/(b*f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m)) + (Sqrt[
2]*(a^2*C + b^2*C*(1 + m) + A*b^2*(2 + m) - a*b*B*(2 + m))*AppellF1[1/2, 1
/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*
Cos[e + f*x])^m*Sin[e + f*x])/(b*f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e +
f*x])/(a + b))^m))/(b*(2 + m))
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3235
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)/b   Int[(a + b*Sin[e + f*x])^m,
 x], x] + Simp[d/b   Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a+b\cos (fx+e))^m \left(A+B\cos (fx+e)+C\cos (fx+e)^2\right) dx$$

input
```
int((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
int((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Fricas [F]

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx = \int (C\cos (fx+e)^2+B\cos (fx+e)+A)(b\cos (fx+e)+a)^m dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm=
"fricas")
```

output
```
integral((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x
)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(f*x+e))**m*(A+B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$
$$= \int \left(C\cos(fx+e)^2+B\cos(fx+e)+A\right)(b\cos(fx+e)+a)^m\,dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm=
"maxima")
```

output
```
integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m,
x)
```

Giac [F]

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx = \int (C\cos(fx+e)^2 + B\cos(fx+e) + A)(b\cos(fx+e)+a)^m\, dx$$

input
```
integrate((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm=
"giac")
```

output
```
integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx = \int (a+b\,\cos(e+f\,x))^m \left(C\cos(e+f\,x)^2 + B\,\cos(e+f\,x) + A\right) dx$$

input
```
int((a + b*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2),x)
```

output
```
int((a + b*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2), x)
```

Reduce [F]

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right) dx = \left(\int (\cos(fx+e)\,b+a)^m\,dx\right) a + \left(\int (\cos(fx+e)\,b+a)^m \cos(fx+e)\,dx\right) b + \left(\int (\cos(fx+e)\,b+a)^m \cos(fx+e)^2\,dx\right) c$$

input
```
int((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

output
```
int((cos(e + f*x)*b + a)**m,x)*a + int((cos(e + f*x)*b + a)**m*cos(e + f*x
),x)*b + int((cos(e + f*x)*b + a)**m*cos(e + f*x)**2,x)*c
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*       is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*       antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```python
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])     #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file