Computer Algebra
Independent Integration Tests

Summer 2024

4-Trig-functions/4.2-Cosine/210-4.2.7.1

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Contents

1 Introduction **4**

1.1 Listing of CAS systems tested . . . 5
1.2 Results . . . 6
1.3 Time and leaf size Performance . . . 10
1.4 Performance based on number of rules Rubi used . . . 12
1.5 Performance based on number of steps Rubi used . . . 13
1.6 Solved integrals histogram based on leaf size of result . . . 14
1.7 Solved integrals histogram based on CPU time used . . . 15
1.8 Leaf size vs. CPU time used . . . 16
1.9 list of integrals with no known antiderivative . . . 17
1.10 List of integrals solved by CAS but has no known antiderivative . . . 17
1.11 list of integrals solved by CAS but failed verification . . . 17
1.12 Timing . . . 18
1.13 Verification . . . 18
1.14 Important notes about some of the results . . . 19
1.15 Current tree layout of integration tests . . . 22
1.16 Design of the test system . . . 23

2 detailed summary tables of results **24**

2.1 List of integrals sorted by grade for each CAS . . . 25
2.2 Detailed conclusion table per each integral for all CAS systems . . . 29
2.3 Detailed conclusion table specific for Rubi results . . . 48

3 Listing of integrals **51**

3.1 $\int \frac{1}{1-\cos^2(x)}\,dx$ . . . 54
3.2 $\int \frac{1}{1-\cos^4(x)}\,dx$ . . . 59
3.3 $\int \frac{1}{1-\cos^6(x)}\,dx$ . . . 65
3.4 $\int \frac{1}{1-\cos^8(x)}\,dx$ . . . 75
3.5 $\int \frac{1}{1-\cos(x)}\,dx$ . . . 84
3.6 $\int \frac{1}{1-\cos^3(x)}\,dx$ . . . 89

3.7 $\int \frac{1}{1-\cos^5(x)}\,dx$ . . . 97
3.8 $\int \frac{1}{1+\cos^2(x)}\,dx$ . . . 106
3.9 $\int \frac{1}{1+\cos^4(x)}\,dx$ . . . 111
3.10 $\int \frac{1}{1+\cos^6(x)}\,dx$ . . . 121
3.11 $\int \frac{1}{1+\cos^8(x)}\,dx$ . . . 128
3.12 $\int \frac{1}{1+\cos(x)}\,dx$ . . . 137
3.13 $\int \frac{1}{1+\cos^3(x)}\,dx$ . . . 142
3.14 $\int \frac{1}{1+\cos^5(x)}\,dx$ . . . 151
3.15 $\int \frac{1}{a-b\cos^2(x)}\,dx$ . . . 160
3.16 $\int \frac{1}{a-b\cos^4(x)}\,dx$ . . . 166
3.17 $\int \frac{1}{a-b\cos^6(x)}\,dx$ . . . 174
3.18 $\int \frac{1}{a-b\cos^8(x)}\,dx$ . . . 181
3.19 $\int \frac{1}{a-b\cos(x)}\,dx$ . . . 188
3.20 $\int \frac{1}{a-b\cos^3(x)}\,dx$ . . . 194
3.21 $\int \frac{1}{a-b\cos^5(x)}\,dx$ . . . 201
3.22 $\int \frac{1}{a+b\cos^2(x)}\,dx$ . . . 209
3.23 $\int \frac{1}{a+b\cos^4(x)}\,dx$ . . . 215
3.24 $\int \frac{1}{a+b\cos^6(x)}\,dx$ . . . 226
3.25 $\int \frac{1}{a+b\cos^8(x)}\,dx$ . . . 233
3.26 $\int \frac{1}{a+b\cos(x)}\,dx$ . . . 240
3.27 $\int \frac{1}{a+b\cos^3(x)}\,dx$ . . . 246
3.28 $\int \frac{1}{a+b\cos^5(x)}\,dx$ . . . 253
3.29 $\int \frac{1}{a-a\cos^2(x)}\,dx$ . . . 261
3.30 $\int \frac{1}{(a-a\cos^2(x))^2}\,dx$ . . . 266
3.31 $\int \frac{1}{(a-a\cos^2(x))^3}\,dx$ . . . 271
3.32 $\int \frac{1}{a+a\cos^2(x)}\,dx$ . . . 277
3.33 $\int \frac{1}{(a+a\cos^2(x))^2}\,dx$ . . . 282
3.34 $\int \frac{1}{(a+a\cos^2(x))^3}\,dx$ . . . 289
3.35 $\int (1-\cos^2(x))^{5/2}\,dx$ . . . 297
3.36 $\int (1-\cos^2(x))^{3/2}\,dx$ . . . 303
3.37 $\int \sqrt{1-\cos^2(x)}\,dx$ . . . 309
3.38 $\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx$ . . . 314
3.39 $\int \frac{1}{(1-\cos^2(x))^{3/2}}\,dx$ . . . 320
3.40 $\int \frac{1}{(1-\cos^2(x))^{5/2}}\,dx$ . . . 327
3.41 $\int (a-a\cos^2(x))^{5/2}\,dx$ . . . 334
3.42 $\int (a-a\cos^2(x))^{3/2}\,dx$ . . . 340

3.43 $\int \sqrt{a-a\cos^2(x)}\,dx$ . . . 346
3.44 $\int \frac{1}{\sqrt{a-a\cos^2(x)}}\,dx$ . . . 351
3.45 $\int \frac{1}{(a-a\cos^2(x))^{3/2}}\,dx$ . . . 357
3.46 $\int \frac{1}{(a-a\cos^2(x))^{5/2}}\,dx$ . . . 364
3.47 $\int (1+\cos^2(x))^{5/2}\,dx$ . . . 371
3.48 $\int (1+\cos^2(x))^{3/2}\,dx$ . . . 378
3.49 $\int \sqrt{1+\cos^2(x)}\,dx$ . . . 384
3.50 $\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx$ . . . 389
3.51 $\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx$ . . . 394
3.52 $\int \frac{1}{(1+\cos^2(x))^{5/2}}\,dx$ . . . 400
3.53 $\int (a+a\cos^2(x))^{5/2}\,dx$ . . . 407
3.54 $\int (a+a\cos^2(x))^{3/2}\,dx$ . . . 415
3.55 $\int \sqrt{a+a\cos^2(x)}\,dx$ . . . 422
3.56 $\int \frac{1}{\sqrt{a+a\cos^2(x)}}\,dx$ . . . 427
3.57 $\int \frac{1}{(a+a\cos^2(x))^{3/2}}\,dx$ . . . 433
3.58 $\int \frac{1}{(a+a\cos^2(x))^{5/2}}\,dx$ . . . 439
3.59 $\int (a+b\cos^2(x))^4\,dx$ . . . 447
3.60 $\int (a+b\cos^2(x))^3\,dx$ . . . 456
3.61 $\int (a+b\cos^2(x))^2\,dx$ . . . 463
3.62 $\int (a+b\cos^2(x))\,dx$ . . . 469
3.63 $\int \frac{1}{a+b\cos^2(x)}\,dx$ . . . 474
3.64 $\int \frac{1}{(a+b\cos^2(x))^2}\,dx$ . . . 480
3.65 $\int \frac{1}{(a+b\cos^2(x))^3}\,dx$ . . . 487
3.66 $\int \frac{1}{(a+b\cos^2(x))^4}\,dx$ . . . 495
3.67 $\int (a+b\cos^2(x))^{5/2}\,dx$ . . . 505
3.68 $\int (a+b\cos^2(x))^{3/2}\,dx$ . . . 514
3.69 $\int \sqrt{a+b\cos^2(x)}\,dx$ . . . 521
3.70 $\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx$ . . . 526
3.71 $\int \frac{1}{(a+b\cos^2(x))^{3/2}}\,dx$ . . . 532
3.72 $\int \frac{1}{(a+b\cos^2(x))^{5/2}}\,dx$ . . . 539

4 Appendix **549**
4.1 Listing of Grading functions . . . 549
4.2 Links to plain text integration problems used in this report for each CAS 567

CHAPTER 1

Introduction

1.1 Listing of CAS systems tested 5
1.2 Results 6
1.3 Time and leaf size Performance 10
1.4 Performance based on number of rules Rubi used 12
1.5 Performance based on number of steps Rubi used 13
1.6 Solved integrals histogram based on leaf size of result 14
1.7 Solved integrals histogram based on CPU time used 15
1.8 Leaf size vs. CPU time used 16
1.9 list of integrals with no known antiderivative 17
1.10 List of integrals solved by CAS but has no known antiderivative 17
1.11 list of integrals solved by CAS but failed verification 17
1.12 Timing 18
1.13 Verification 18
1.14 Important notes about some of the results 19
1.15 Current tree layout of integration tests 22
1.16 Design of the test system 23

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**72**]. This is test number [210].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (72)	0.00 (0)
Mathematica	100.00 (72)	0.00 (0)
Maple	100.00 (72)	0.00 (0)
Fricas	84.72 (61)	15.28 (11)
Mupad	62.50 (45)	37.50 (27)
Giac	62.50 (45)	37.50 (27)
Maxima	45.83 (33)	54.17 (39)
Reduce	36.11 (26)	63.89 (46)
Sympy	31.94 (23)	68.06 (49)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

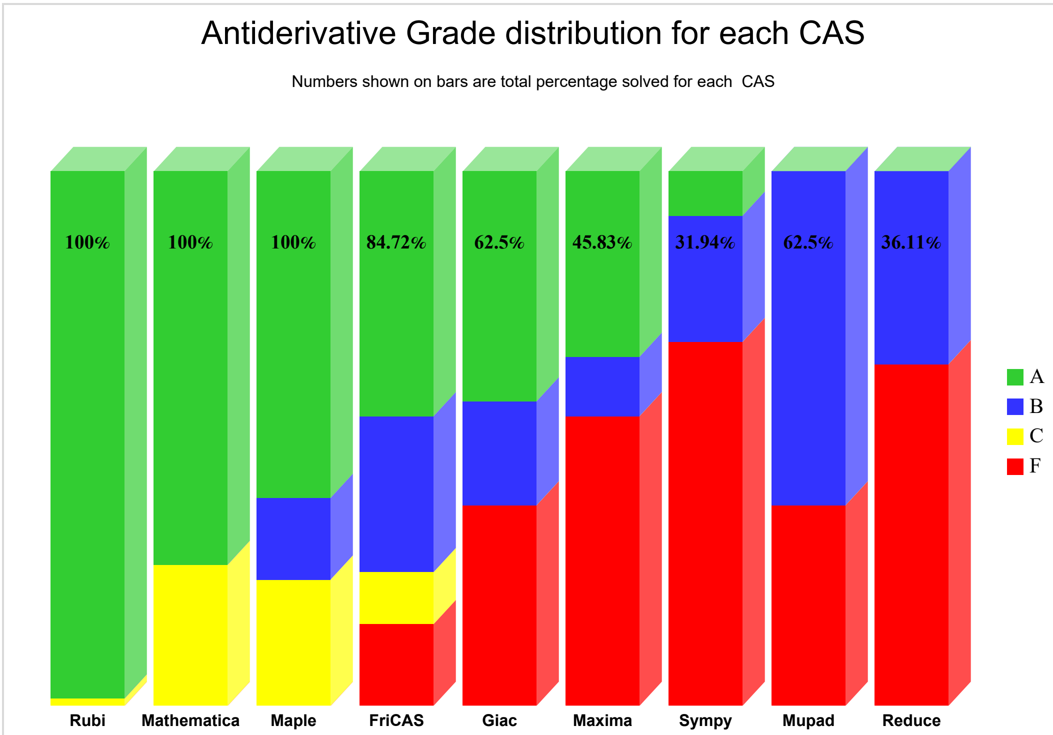
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

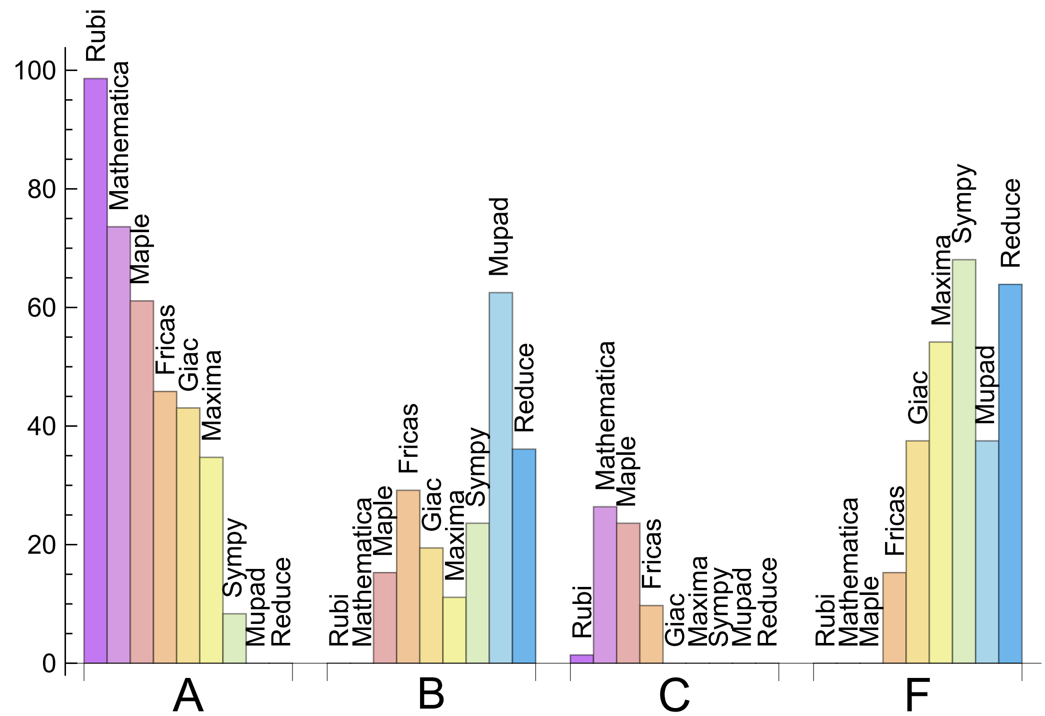
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.611	0.000	1.389	0.000
Mathematica	73.611	0.000	26.389	0.000
Maple	61.111	15.278	23.611	0.000
Fricas	45.833	29.167	9.722	15.278
Giac	43.056	19.444	0.000	37.500
Maxima	34.722	11.111	0.000	54.167
Sympy	8.333	23.611	0.000	68.056
Mupad	0.000	62.500	0.000	37.500
Reduce	0.000	36.111	0.000	63.889

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Fricas	11	81.82	0.00	18.18
Mupad	27	0.00	100.00	0.00
Giac	27	88.89	7.41	3.70
Maxima	39	92.31	0.00	7.69
Reduce	46	100.00	0.00	0.00
Sympy	49	57.14	42.86	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.11
Reduce	0.17
Giac	0.17
Rubi	0.41
Fricas	0.45
Maple	0.69
Mupad	1.29
Mathematica	1.70
Sympy	3.93

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	72.12	1.03	59.00	0.80
Mathematica	74.65	0.87	48.50	0.89
Rubi	98.53	0.96	59.00	1.00
Maxima	122.73	2.43	26.00	0.83
Giac	133.47	1.57	65.00	1.40
Reduce	219.42	2.60	53.00	1.38
Mupad	241.33	1.34	95.00	0.90
Sympy	1567.96	47.42	138.00	2.83
Fricas	22812.15	101.80	138.00	2.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

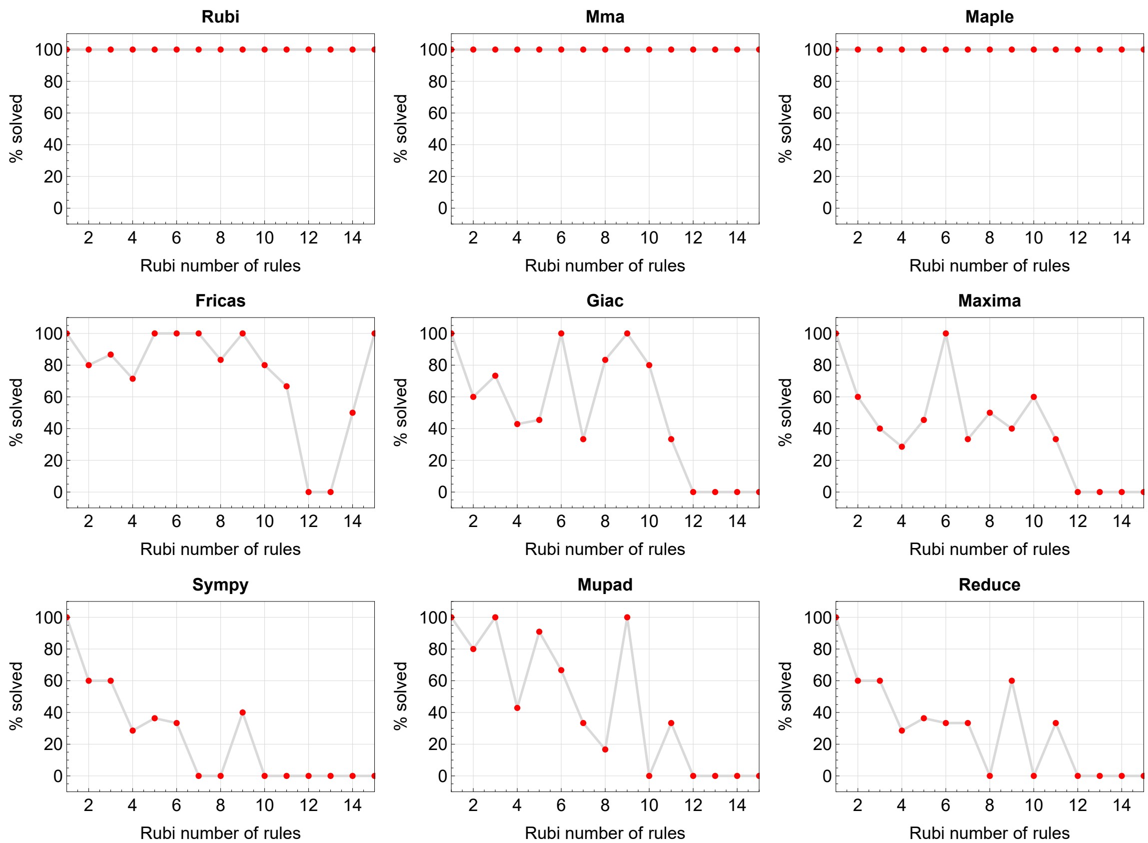


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

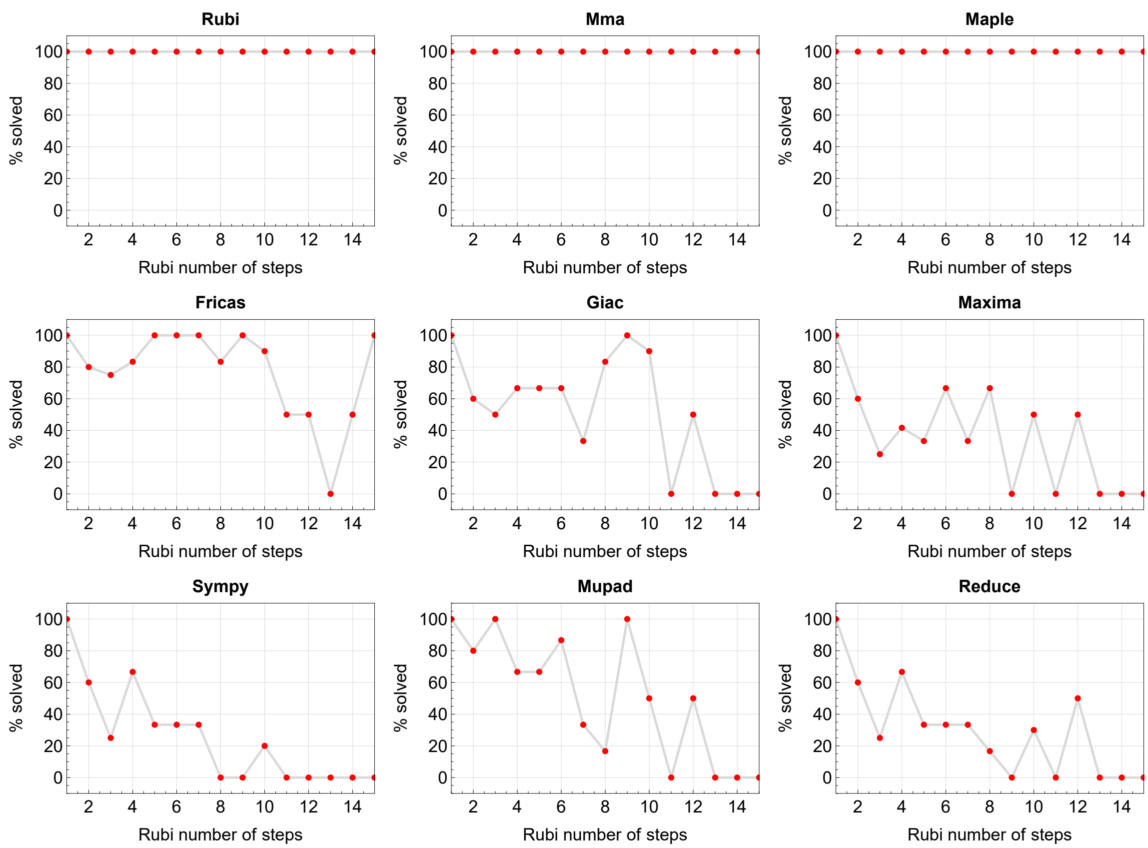


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

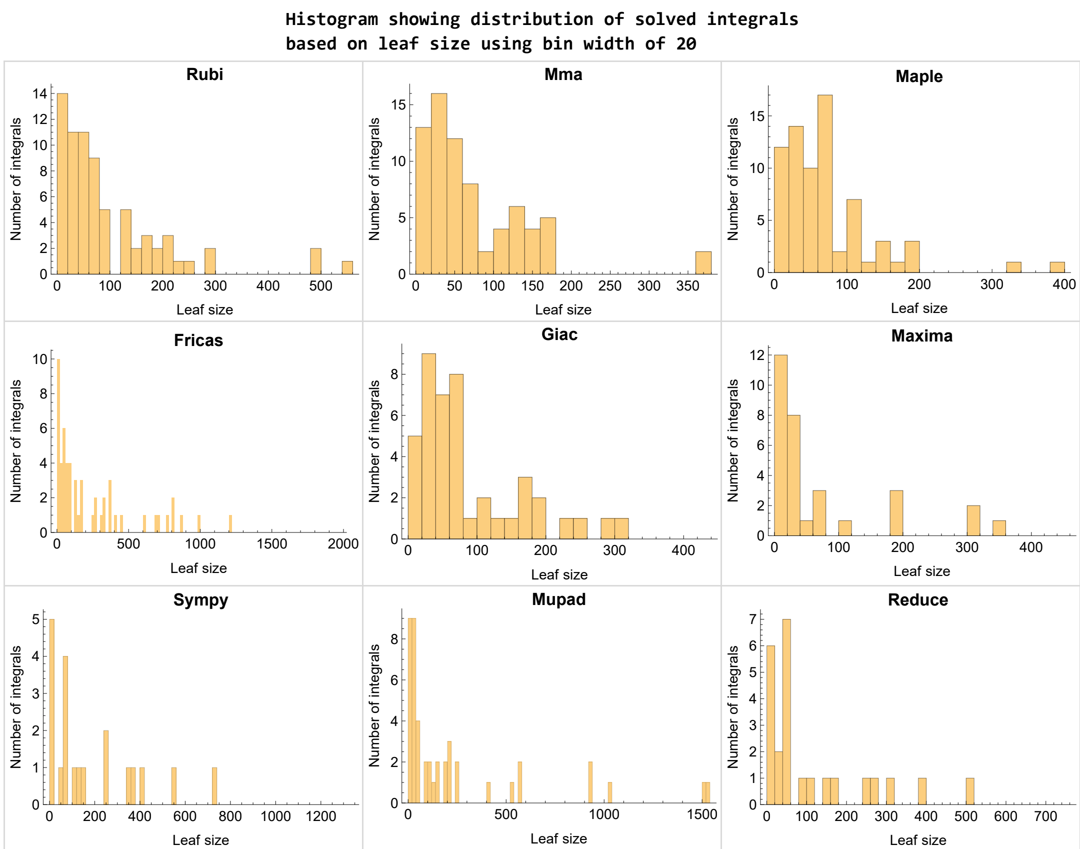


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

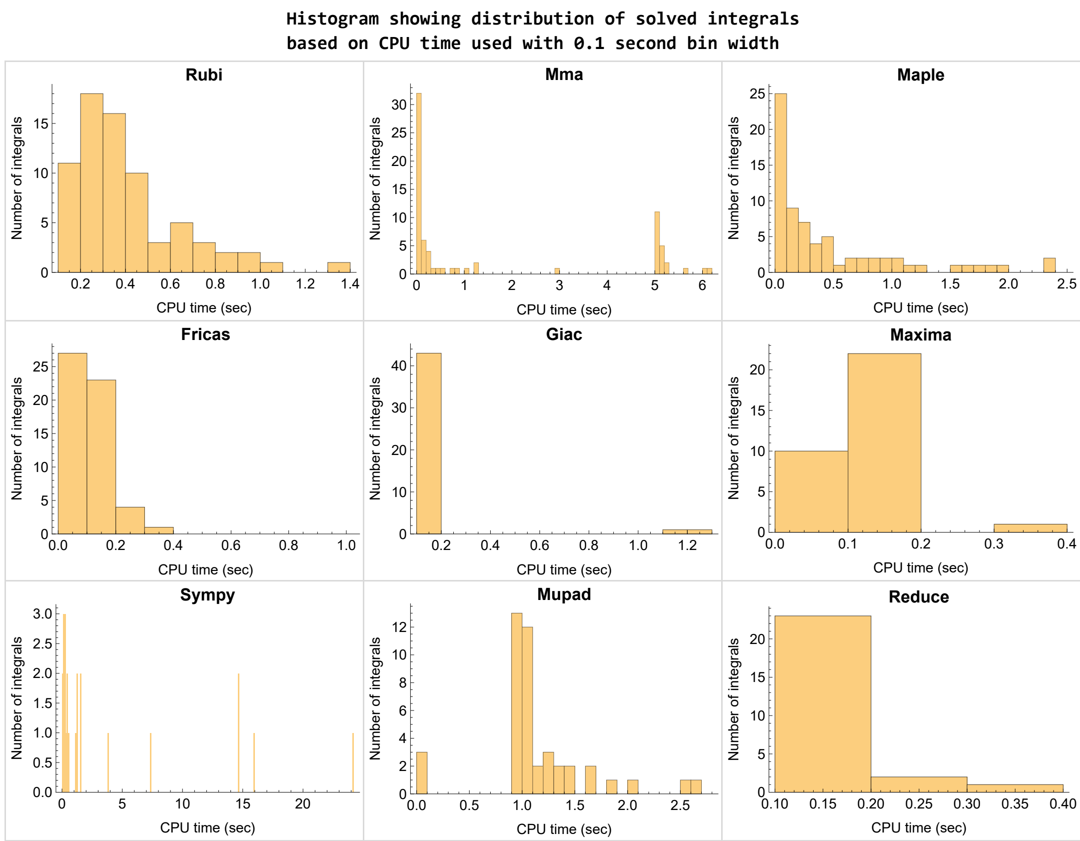


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

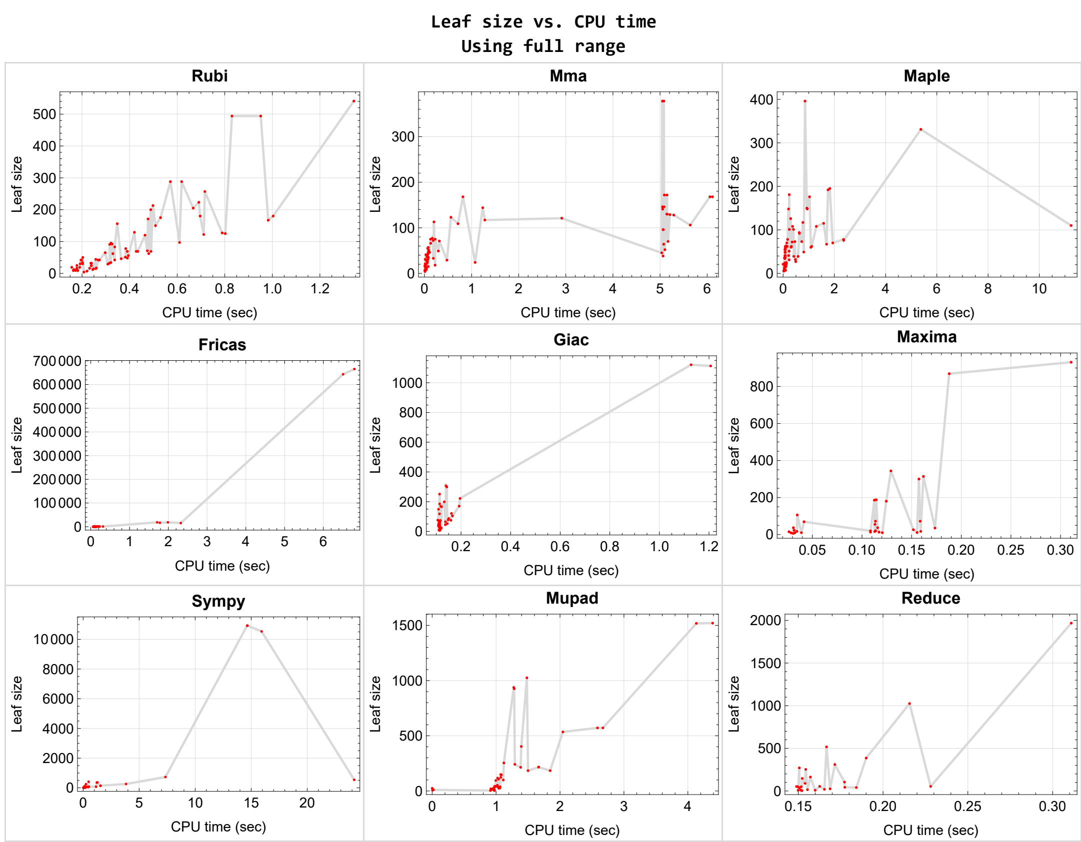


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{18, 21, 25, 28\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

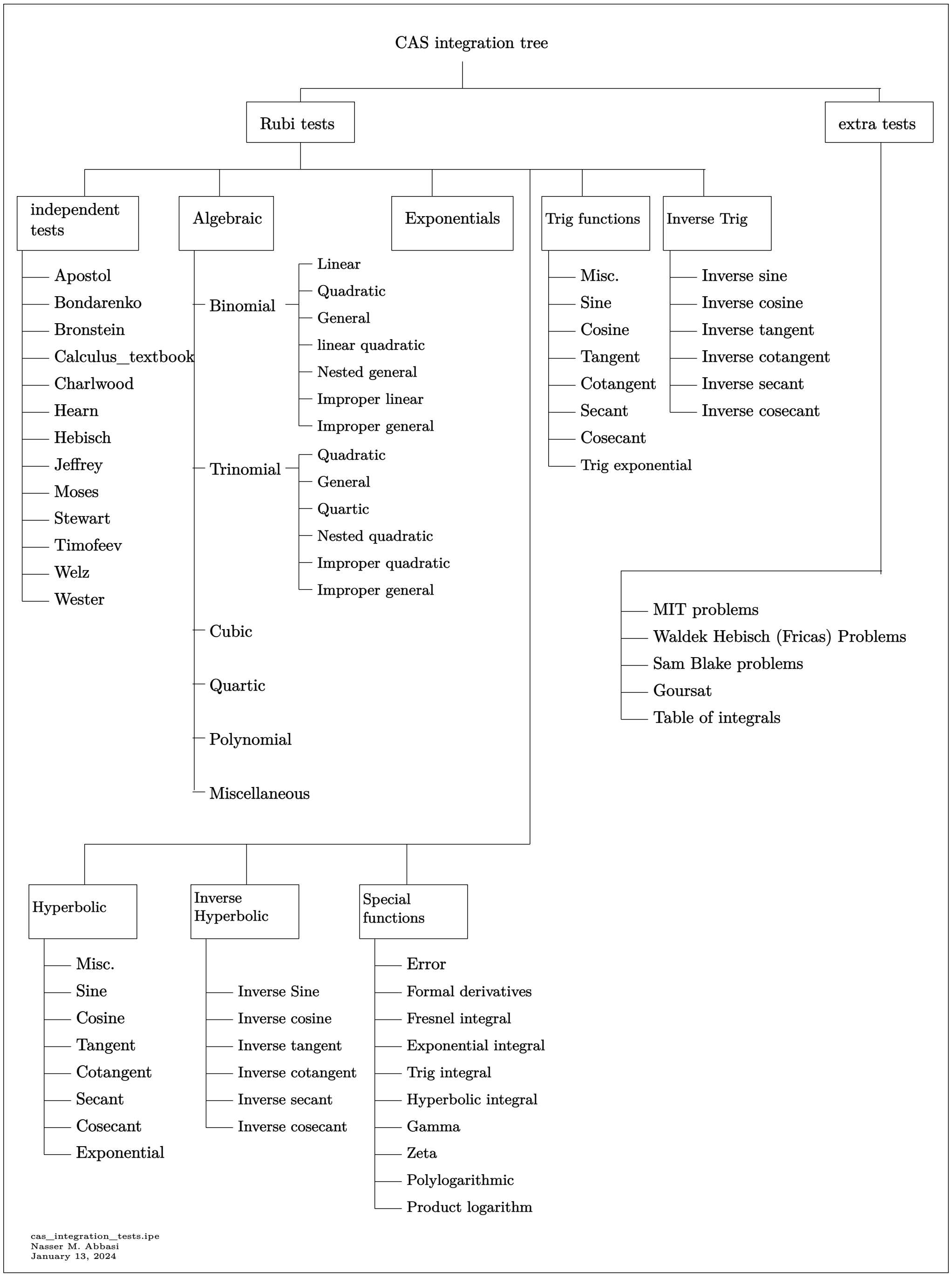
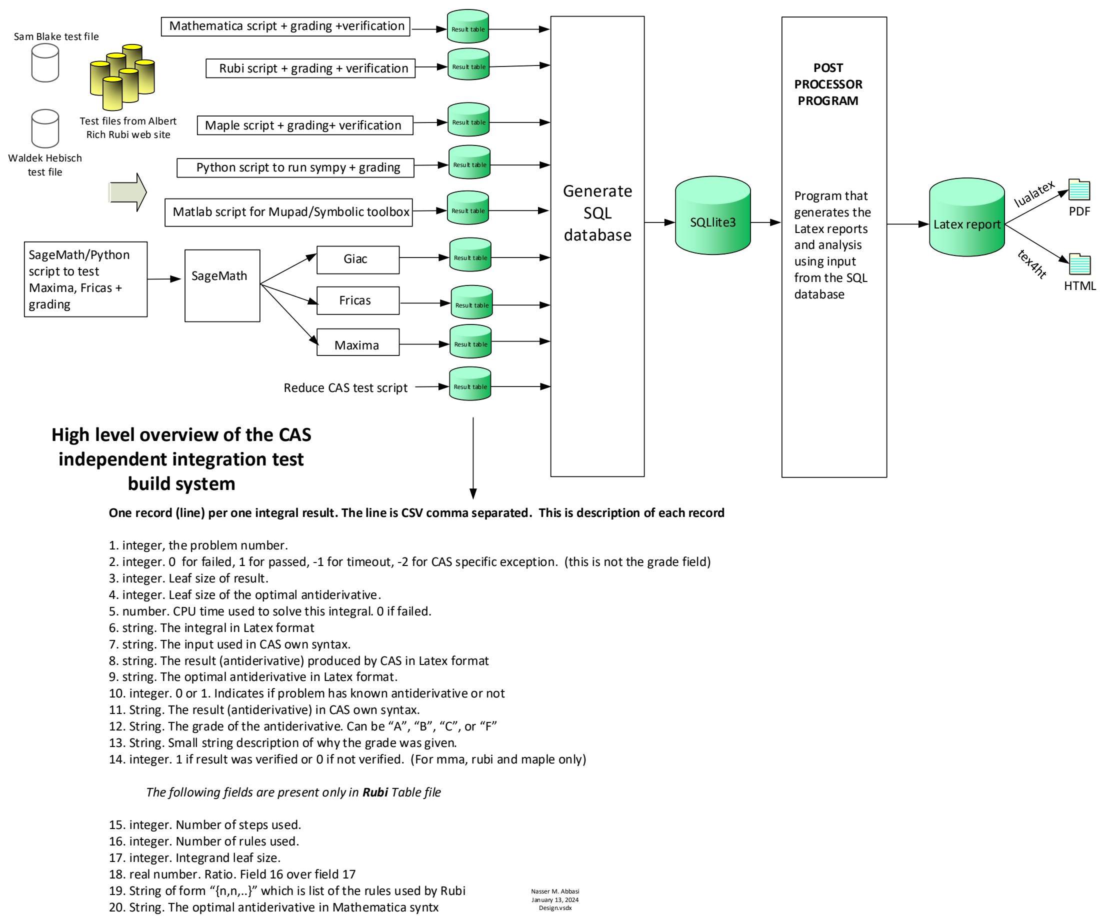


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 25
2.2 Detailed conclusion table per each integral for all CAS systems 29
2.3 Detailed conclusion table specific for Rubi results 48

2.1 List of integrals sorted by grade for each CAS

Rubi 25
Mma 25
Maple 26
Fricas 26
Maxima 26
Giac 27
Mupad 27
Sympy 27
Reduce 28

Rubi

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72 }

B grade { }

C grade { 4 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 5, 8, 12, 15, 16, 19, 22, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72 }

B grade { }

C grade { 3, 4, 6, 7, 9, 10, 11, 13, 14, 17, 18, 20, 21, 23, 24, 25, 27, 28, 60 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 5, 8, 10, 12, 15, 16, 19, 22, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71 }

B grade { 44, 47, 48, 49, 50, 51, 52, 55, 56, 67, 72 }

C grade { 3, 4, 6, 7, 9, 11, 13, 14, 17, 18, 20, 21, 23, 24, 25, 27, 28 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 15, 19, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 45, 46, 59, 60, 61, 62 }

B grade { 6, 7, 13, 14, 16, 18, 22, 23, 25, 40, 44, 50, 51, 52, 56, 57, 58, 63, 64, 65, 66 }

C grade { 17, 20, 24, 27, 70, 71, 72 }

F normal fail { 47, 48, 49, 53, 54, 55, 67, 68, 69 }

F(-1) timedout fail { }

F(-2) exception fail { 21, 28 }

Maxima

A grade { 1, 2, 5, 8, 10, 12, 22, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 59, 60, 61, 62, 63, 64, 65 }

B grade { 6, 13, 38, 39, 40, 45, 46, 66 }

C grade { }

F normal fail { 3, 4, 7, 9, 11, 14, 16, 17, 18, 20, 21, 23, 24, 25, 27, 28, 41, 42, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72 }

F(-1) timedout fail { }

F(-2) exception fail { 15, 19, 26 }

Giac

A grade { 1, 2, 3, 4, 5, 8, 9, 10, 15, 19, 22, 23, 26, 29, 30, 31, 32, 33, 34, 38, 41, 42, 44, 59, 60, 61, 62, 63, 64, 65, 66 }

B grade { 6, 7, 12, 13, 14, 16, 35, 36, 37, 39, 40, 43, 45, 46 }

C grade { }

F normal fail { 18, 20, 21, 25, 27, 28, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72 }

F(-1) timedout fail { 17, 24 }

F(-2) exception fail { 11 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 43, 49, 59, 60, 61, 62, 63, 64, 65, 66 }

C grade { }

F normal fail { }

F(-1) timedout fail { 35, 36, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72 }

F(-2) exception fail { }

Sympy

A grade { 2, 5, 8, 12, 32, 62 }

B grade { 1, 3, 6, 13, 15, 19, 22, 26, 29, 30, 31, 33, 34, 59, 60, 61, 63 }

C grade { }

F normal fail { 17, 18, 21, 24, 25, 28, 36, 37, 38, 39, 40, 43, 44, 45, 46, 48, 49, 50, 51, 52, 55, 56, 57, 58, 69, 70, 71, 72 }

F(-1) timedout fail { 4, 7, 9, 10, 11, 14, 16, 20, 23, 27, 35, 41, 42, 47, 53, 54, 64, 65, 66, 67, 68 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 5, 6, 8, 12, 13, 15, 19, 22, 26, 29, 30, 31, 32, 33, 34, 59, 60, 61, 62, 63, 64, 65, 66 }

C grade { }

F normal fail { 4, 7, 9, 10, 11, 14, 16, 17, 18, 20, 21, 23, 24, 25, 27, 28, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	7	6	8	14	6	8	4
N.S.	1	1.00	1.00	1.75	1.50	2.00	3.50	1.50	2.00	1.00
time (sec)	N/A	0.208	0.011	0.098	0.031	0.075	0.191	0.113	0.160	0.911

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	25	24	21	20	43	78	53	52	20
N.S.	1	0.56	0.53	0.47	0.44	0.96	1.73	1.18	1.16	0.44
time (sec)	N/A	0.178	1.074	0.127	0.109	0.087	0.519	0.114	0.149	0.919

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	71	117	51	0	442	728	199	517	95
N.S.	1	0.22	0.36	0.16	0.00	1.36	2.25	0.61	1.60	0.29
time (sec)	N/A	0.475	1.281	0.234	0.000	0.189	7.346	0.132	0.167	0.996

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	69	64	176	0	414	0	222	135	241
N.S.	1	0.23	0.21	0.58	0.00	1.35	0.00	0.73	0.44	0.79
time (sec)	N/A	0.428	5.078	1.031	0.000	0.199	0.000	0.196	0.181	1.293

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	7	10	10	7	8	8	6
N.S.	1	1.00	0.67	0.58	0.83	0.83	0.58	0.67	0.67	0.50
time (sec)	N/A	0.167	0.038	0.031	0.039	0.072	0.159	0.115	0.150	0.977

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	129	55	180	364	367	168	271	107
N.S.	1	1.00	1.36	0.58	1.89	3.83	3.86	1.77	2.85	1.13
time (sec)	N/A	0.322	5.208	0.088	0.124	0.121	1.279	0.119	0.151	1.039

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	378	62	0	688	0	1121	83	403
N.S.	1	1.00	1.84	0.30	0.00	3.36	0.00	5.47	0.40	1.97
time (sec)	N/A	0.667	5.050	0.158	0.000	0.207	0.000	1.128	0.167	1.395

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	16	15	14	13	31	63	46	37	26
N.S.	1	0.47	0.44	0.41	0.38	0.91	1.85	1.35	1.09	0.76
time (sec)	N/A	0.178	0.080	0.078	0.109	0.094	0.236	0.109	0.151	1.041

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	200	45	126	0	364	0	170	10	214
N.S.	1	0.76	0.17	0.48	0.00	1.39	0.00	0.65	0.04	0.82
time (sec)	N/A	0.489	5.035	0.295	0.000	0.136	0.000	0.192	0.157	1.384

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	83	168	73	72	138	0	185	10	99
N.S.	1	0.67	1.35	0.59	0.58	1.11	0.00	1.49	0.08	0.80
time (sec)	N/A	0.337	6.059	0.724	0.114	0.140	0.000	0.114	0.161	1.112

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	777	129	141	67	0	877	0	0	10	1025
N.S.	1	0.17	0.18	0.09	0.00	1.13	0.00	0.00	0.01	1.32
time (sec)	N/A	0.420	5.056	1.697	0.000	0.315	0.000	0.000	0.153	1.482

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	3	30	4	4
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.33	3.33	0.44	0.44
time (sec)	N/A	0.163	0.007	0.026	0.029	0.066	0.078	0.112	0.152	0.973

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	128	55	188	379	354	166	253	149
N.S.	1	1.00	1.41	0.60	2.07	4.16	3.89	1.82	2.78	1.64
time (sec)	N/A	0.316	5.291	0.068	0.114	0.130	1.222	0.122	0.154	1.072

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	378	62	0	705	0	1113	72	535
N.S.	1	1.00	1.70	0.28	0.00	3.16	0.00	4.99	0.32	2.40
time (sec)	N/A	0.691	5.081	0.148	0.000	0.205	0.000	1.207	0.158	2.045

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	25	0	168	10533	39	310	26
N.S.	1	1.00	0.97	0.74	0.00	4.94	309.79	1.15	9.12	0.76
time (sec)	N/A	0.202	0.188	0.121	0.000	0.102	15.930	0.114	0.172	1.060

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	156	109	72	0	817	0	299	16	938
N.S.	1	1.54	1.08	0.71	0.00	8.09	0.00	2.96	0.16	9.29
time (sec)	N/A	0.348	0.711	0.352	0.000	0.228	0.000	0.143	0.159	1.276

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	146	62	0	16679	0	0	16	184
N.S.	1	1.00	0.83	0.35	0.00	95.31	0.00	0.00	0.09	1.05
time (sec)	N/A	0.530	5.085	1.117	0.000	1.782	0.000	0.000	0.200	1.846

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	172	78	0	643291	0	0	16	216
N.S.	1	1.00	0.81	0.37	0.00	3020.15	0.00	0.00	0.08	1.01
time (sec)	N/A	0.498	5.147	2.356	0.000	6.509	0.000	0.000	0.201	1.665

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	39	34	0	141	138	59	50	38
N.S.	1	1.00	0.93	0.81	0.00	3.36	3.29	1.40	1.19	0.90
time (sec)	N/A	0.198	0.051	0.069	0.000	0.095	1.544	0.112	0.152	1.025

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	96	92	0	18595	0	0	16	571
N.S.	1	1.00	0.33	0.32	0.00	64.57	0.00	0.00	0.06	1.98
time (sec)	N/A	0.619	5.066	0.637	0.000	1.989	0.000	0.000	0.163	2.585

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	130	148	0	0	0	0	16	1518
N.S.	1	1.00	0.26	0.30	0.00	0.00	0.00	0.00	0.03	3.07
time (sec)	N/A	0.951	5.144	0.942	0.000	0.000	0.000	0.000	0.161	4.133

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	21	20	163	10924	37	54	24
N.S.	1	1.00	0.97	0.70	0.67	5.43	364.13	1.23	1.80	0.80
time (sec)	N/A	0.203	0.477	0.099	0.114	0.116	14.659	0.110	0.228	1.045

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	541	121	101	0	809	0	307	12	926
N.S.	1	1.55	0.35	0.29	0.00	2.31	0.00	0.88	0.03	2.65
time (sec)	N/A	1.343	2.914	0.404	0.000	0.187	0.000	0.139	0.242	1.283

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	146	60	0	15483	0	0	12	184
N.S.	1	1.00	0.85	0.35	0.00	90.54	0.00	0.00	0.07	1.08
time (sec)	N/A	0.478	5.052	1.083	0.000	2.322	0.000	0.000	0.293	1.500

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	257	172	76	0	665467	0	0	12	216
N.S.	1	1.05	0.70	0.31	0.00	2716.19	0.00	0.00	0.05	0.88
time (sec)	N/A	0.716	5.091	2.366	0.000	6.800	0.000	0.000	0.187	1.666

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	41	36	0	137	144	61	51	38
N.S.	1	1.00	0.98	0.86	0.00	3.26	3.43	1.45	1.21	0.90
time (sec)	N/A	0.195	0.029	0.053	0.000	0.088	1.558	0.114	0.150	1.062

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	96	94	0	18599	0	0	12	571
N.S.	1	1.00	0.33	0.33	0.00	64.58	0.00	0.00	0.04	1.98
time (sec)	N/A	0.571	5.068	0.629	0.000	1.712	0.000	0.000	0.177	2.671

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	130	150	0	0	0	0	12	1520
N.S.	1	1.00	0.26	0.30	0.00	0.00	0.00	0.00	0.02	3.08
time (sec)	N/A	0.830	5.151	0.927	0.000	0.000	0.000	0.000	0.151	4.387

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	10	9	11	17	9	11	7
N.S.	1	1.00	1.00	1.43	1.29	1.57	2.43	1.29	1.57	1.00
time (sec)	N/A	0.221	0.010	0.055	0.032	0.065	0.217	0.111	0.151	0.969

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	16	21	18	17	33	48	17	19	13
N.S.	1	0.84	1.11	0.95	0.89	1.74	2.53	0.89	1.00	0.68
time (sec)	N/A	0.232	0.015	0.086	0.034	0.065	0.464	0.114	0.165	0.944

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	24	31	24	23	46	75	23	25	21
N.S.	1	0.80	1.03	0.80	0.77	1.53	2.50	0.77	0.83	0.70
time (sec)	N/A	0.239	0.016	0.131	0.032	0.066	1.150	0.114	0.169	0.952

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	19	18	17	16	34	66	49	40	16
N.S.	1	0.48	0.45	0.42	0.40	0.85	1.65	1.22	1.00	0.40
time (sec)	N/A	0.190	0.224	0.088	0.113	0.081	0.391	0.111	0.184	0.949

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	44	38	31	37	63	258	65	104	32
N.S.	1	0.66	0.57	0.46	0.55	0.94	3.85	0.97	1.55	0.48
time (sec)	N/A	0.258	5.063	0.234	0.116	0.083	3.829	0.112	0.177	0.970

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	69	52	39	55	90	541	74	164	41
N.S.	1	0.78	0.59	0.44	0.62	1.02	6.15	0.84	1.86	0.47
time (sec)	N/A	0.390	5.103	0.584	0.113	0.095	24.194	0.117	0.157	0.994

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	48	31	27	17	17	0	65	46	0
N.S.	1	1.12	0.72	0.63	0.40	0.40	0.00	1.51	1.07	0.00
time (sec)	N/A	0.391	0.074	0.492	0.159	0.071	0.000	0.137	0.185	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	19	11	11	0	45	30	0
N.S.	1	1.00	0.79	0.66	0.38	0.38	0.00	1.55	1.03	0.00
time (sec)	N/A	0.308	0.057	0.057	0.155	0.076	0.000	0.137	0.171	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	10	4	0	24	11	10
N.S.	1	1.00	1.00	1.08	0.83	0.33	0.00	2.00	0.92	0.83
time (sec)	N/A	0.245	0.024	0.049	0.121	0.117	0.000	0.120	0.169	0.020

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	35	19	0	21	22	0
N.S.	1	1.00	1.00	0.93	2.33	1.27	0.00	1.40	1.47	0.00
time (sec)	N/A	0.259	0.011	0.070	0.174	0.090	0.000	0.120	0.162	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	51	37	300	44	0	78	26	0
N.S.	1	1.00	1.59	1.16	9.38	1.38	0.00	2.44	0.81	0.00
time (sec)	N/A	0.315	0.089	0.080	0.157	0.111	0.000	0.148	0.168	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	72	58	869	69	0	104	34	0
N.S.	1	1.11	1.57	1.26	18.89	1.50	0.00	2.26	0.74	0.00
time (sec)	N/A	0.382	0.185	0.092	0.188	0.099	0.000	0.166	0.173	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	56	36	32	0	43	0	75	52	0
N.S.	1	1.06	0.68	0.60	0.00	0.81	0.00	1.42	0.98	0.00
time (sec)	N/A	0.395	0.053	0.478	0.000	0.072	0.000	0.159	0.166	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	0	29	0	52	34	0
N.S.	1	1.00	0.76	0.71	0.00	0.85	0.00	1.53	1.00	0.00
time (sec)	N/A	0.321	0.063	0.064	0.000	0.076	0.000	0.145	0.167	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	16	13	19	0	27	14	40
N.S.	1	1.00	1.00	1.14	0.93	1.36	0.00	1.93	1.00	2.86
time (sec)	N/A	0.248	0.024	0.071	0.117	0.096	0.000	0.117	0.165	1.028

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	49	26	70	0	24	27	0
N.S.	1	1.00	1.00	2.88	1.53	4.12	0.00	1.41	1.59	0.00
time (sec)	N/A	0.260	0.009	0.077	0.152	0.084	0.000	0.118	0.161	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	55	41	314	58	0	87	32	0
N.S.	1	1.00	1.31	0.98	7.48	1.38	0.00	2.07	0.76	0.00
time (sec)	N/A	0.337	0.067	0.086	0.162	0.082	0.000	0.151	0.158	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	69	77	63	931	78	0	121	39	0
N.S.	1	1.13	1.26	1.03	15.26	1.28	0.00	1.98	0.64	0.00
time (sec)	N/A	0.434	0.165	0.100	0.311	0.126	0.000	0.161	0.168	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	69	46	108	0	0	0	0	40	0
N.S.	1	1.10	0.73	1.71	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.490	0.109	1.298	0.000	0.000	0.000	0.000	0.188	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	46	39	101	0	0	0	0	24	0
N.S.	1	1.07	0.91	2.35	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.364	0.062	0.243	0.000	0.000	0.000	0.000	0.178	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	41	0	0	0	0	9	7
N.S.	1	1.00	1.22	4.56	0.00	0.00	0.00	0.00	1.00	0.78
time (sec)	N/A	0.172	0.029	0.216	0.000	0.000	0.000	0.000	0.163	0.005

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	41	0	87	0	0	18	0
N.S.	1	1.00	1.22	4.56	0.00	9.67	0.00	0.00	2.00	0.00
time (sec)	N/A	0.182	0.043	0.070	0.000	0.111	0.000	0.000	0.155	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	35	70	0	247	0	0	24	0
N.S.	1	1.00	1.09	2.19	0.00	7.72	0.00	0.00	0.75	0.00
time (sec)	N/A	0.238	0.083	0.145	0.000	0.114	0.000	0.000	0.163	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	62	49	148	0	316	0	0	30	0
N.S.	1	0.98	0.78	2.35	0.00	5.02	0.00	0.00	0.48	0.00
time (sec)	N/A	0.481	0.294	0.212	0.000	0.150	0.000	0.000	0.153	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	127	66	115	0	0	0	0	46	0
N.S.	1	1.05	0.55	0.95	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.789	0.132	1.581	0.000	0.000	0.000	0.000	0.167	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	97	57	108	0	0	0	0	28	0
N.S.	1	1.03	0.61	1.15	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.610	0.079	0.368	0.000	0.000	0.000	0.000	0.170	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	62	0	0	0	0	12	0
N.S.	1	1.00	0.88	1.94	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.237	0.051	0.324	0.000	0.000	0.000	0.000	0.195	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	61	0	97	0	0	24	0
N.S.	1	1.00	0.88	1.91	0.00	3.03	0.00	0.00	0.75	0.00
time (sec)	N/A	0.240	0.057	0.083	0.000	0.102	0.000	0.000	0.240	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	48	78	0	267	0	0	30	0
N.S.	1	1.00	0.77	1.26	0.00	4.31	0.00	0.00	0.48	0.00
time (sec)	N/A	0.330	0.089	0.176	0.000	0.108	0.000	0.000	0.233	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	125	71	181	0	339	0	0	36	0
N.S.	1	1.02	0.58	1.47	0.00	2.76	0.00	0.00	0.29	0.00
time (sec)	N/A	0.802	0.315	0.240	0.000	0.125	0.000	0.000	0.246	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	150	113	110	106	123	410	118	147	147
N.S.	1	1.07	0.81	0.79	0.76	0.88	2.93	0.84	1.05	1.05
time (sec)	N/A	0.509	0.200	11.240	0.035	0.086	0.491	0.113	0.152	1.083

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	92	75	70	69	81	246	76	89	117
N.S.	1	1.06	0.86	0.80	0.79	0.93	2.83	0.87	1.02	1.34
time (sec)	N/A	0.326	0.128	1.937	0.042	0.085	0.253	0.106	0.154	1.024

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	43	39	37	46	110	42	43	43
N.S.	1	1.00	0.86	0.78	0.74	0.92	2.20	0.84	0.86	0.86
time (sec)	N/A	0.203	0.074	0.416	0.031	0.084	0.121	0.114	0.177	0.979

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	16	15	15	15	15
N.S.	1	1.00	1.00	0.84	0.79	0.84	0.79	0.79	0.79	0.79
time (sec)	N/A	0.157	0.032	0.115	0.026	0.073	0.019	0.116	0.155	0.966

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	21	20	163	10924	37	54	24
N.S.	1	1.00	0.97	0.70	0.67	5.43	364.13	1.23	1.80	0.80
time (sec)	N/A	0.192	0.031	0.000	0.114	0.107	14.637	0.109	0.163	0.001

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	70	60	72	326	0	69	386	52
N.S.	1	1.00	1.08	0.92	1.11	5.02	0.00	1.06	5.94	0.80
time (sec)	N/A	0.297	5.169	0.310	0.159	0.115	0.000	0.112	0.190	1.020

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	120	106	117	186	616	0	149	1025	123
N.S.	1	1.12	0.99	1.09	1.74	5.76	0.00	1.39	9.58	1.15
time (sec)	N/A	0.465	5.641	0.773	0.112	0.127	0.000	0.109	0.216	1.073

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	180	168	195	344	994	0	251	1968	254
N.S.	1	1.17	1.09	1.27	2.23	6.45	0.00	1.63	12.78	1.65
time (sec)	N/A	0.696	6.113	1.822	0.129	0.175	0.000	0.114	0.311	1.122

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	167	168	331	0	0	0	0	56	0
N.S.	1	1.02	1.02	2.02	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.983	0.816	5.376	0.000	0.000	0.000	0.000	0.178	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	122	123	192	0	0	0	0	32	0
N.S.	1	0.99	1.00	1.56	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.712	0.561	1.753	0.000	0.000	0.000	0.000	0.169	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	46	49	0	0	0	0	11	0
N.S.	1	0.98	1.07	1.14	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.268	0.082	0.806	0.000	0.000	0.000	0.000	0.152	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	46	48	0	276	0	0	22	0
N.S.	1	0.98	1.07	1.12	0.00	6.42	0.00	0.00	0.51	0.00
time (sec)	N/A	0.273	0.081	0.082	0.000	0.096	0.000	0.000	0.164	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	78	75	73	0	775	0	0	34	0
N.S.	1	0.99	0.95	0.92	0.00	9.81	0.00	0.00	0.43	0.00
time (sec)	N/A	0.384	0.226	0.460	0.000	0.145	0.000	0.000	0.152	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	180	144	396	0	1213	0	0	46	0
N.S.	1	1.02	0.81	2.24	0.00	6.85	0.00	0.00	0.26	0.00
time (sec)	N/A	1.003	1.237	0.855	0.000	0.221	0.000	0.000	0.152	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [58] had the largest ratio of [1.2500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	10	0.500
2	A	5	4	0.56	10	0.400
3	A	10	9	0.22	10	0.900
4	C	10	9	0.23	10	0.900
5	A	2	2	1.00	8	0.250
6	A	3	3	1.00	10	0.300
7	A	3	3	1.00	10	0.300
8	A	4	3	0.47	8	0.375
9	A	9	8	0.76	8	1.000
10	A	6	5	0.67	8	0.625
11	A	6	5	0.17	8	0.625
12	A	2	2	1.00	6	0.333
13	A	3	3	1.00	8	0.375
14	A	3	3	1.00	8	0.375
15	A	4	3	1.00	11	0.273
16	A	5	4	1.54	11	0.364
17	A	6	5	1.00	11	0.455
18	A	6	5	1.00	11	0.455
19	A	4	3	1.00	9	0.333
20	A	3	3	1.00	11	0.273
21	A	3	3	1.00	11	0.273

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	4	3	1.00	10	0.300
23	A	10	9	1.55	10	0.900
24	A	6	5	1.00	10	0.500
25	A	6	5	1.05	10	0.500
26	A	4	3	1.00	8	0.375
27	A	3	3	1.00	10	0.300
28	A	3	3	1.00	10	0.300
29	A	6	5	1.00	11	0.455
30	A	6	5	0.84	11	0.455
31	A	6	5	0.80	11	0.455
32	A	4	3	0.48	10	0.300
33	A	7	6	0.66	10	0.600
34	A	10	9	0.78	10	0.900
35	A	10	10	1.12	12	0.833
36	A	8	8	1.00	12	0.667
37	A	6	6	1.00	12	0.500
38	A	6	6	1.00	12	0.500
39	A	8	8	1.00	12	0.667
40	A	10	10	1.11	12	0.833
41	A	10	10	1.06	13	0.769
42	A	8	8	1.00	13	0.615
43	A	6	6	1.00	13	0.462
44	A	6	6	1.00	13	0.462
45	A	8	8	1.00	13	0.615
46	A	10	10	1.13	13	0.769
47	A	10	10	1.10	10	1.000
48	A	8	8	1.07	10	0.800
49	A	2	2	1.00	10	0.200
50	A	2	2	1.00	10	0.200
51	A	5	5	1.00	10	0.500
52	A	11	11	0.98	10	1.100
53	A	14	14	1.05	12	1.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	12	12	1.03	12	1.000
55	A	4	4	1.00	12	0.333
56	A	4	4	1.00	12	0.333
57	A	7	7	1.00	12	0.583
58	A	15	15	1.02	12	1.250
59	A	6	6	1.07	10	0.600
60	A	4	4	1.06	10	0.400
61	A	2	2	1.00	10	0.200
62	A	1	1	1.00	8	0.125
63	A	4	3	1.00	10	0.300
64	A	8	7	1.00	10	0.700
65	A	10	9	1.12	10	0.900
66	A	12	11	1.17	10	1.100
67	A	13	13	1.02	12	1.083
68	A	11	11	0.99	12	0.917
69	A	4	4	0.98	12	0.333
70	A	4	4	0.98	12	0.333
71	A	7	7	0.99	12	0.583
72	A	14	14	1.02	12	1.167

CHAPTER 3

Listing of integrals

3.1 $\int \frac{1}{1-\cos^2(x)}\,dx$ 54
3.2 $\int \frac{1}{1-\cos^4(x)}\,dx$ 59
3.3 $\int \frac{1}{1-\cos^6(x)}\,dx$ 65
3.4 $\int \frac{1}{1-\cos^8(x)}\,dx$ 75
3.5 $\int \frac{1}{1-\cos(x)}\,dx$ 84
3.6 $\int \frac{1}{1-\cos^3(x)}\,dx$ 89
3.7 $\int \frac{1}{1-\cos^5(x)}\,dx$ 97
3.8 $\int \frac{1}{1+\cos^2(x)}\,dx$ 106
3.9 $\int \frac{1}{1+\cos^4(x)}\,dx$ 111
3.10 $\int \frac{1}{1+\cos^6(x)}\,dx$ 121
3.11 $\int \frac{1}{1+\cos^8(x)}\,dx$ 128
3.12 $\int \frac{1}{1+\cos(x)}\,dx$ 137
3.13 $\int \frac{1}{1+\cos^3(x)}\,dx$ 142
3.14 $\int \frac{1}{1+\cos^5(x)}\,dx$ 151
3.15 $\int \frac{1}{a-b\cos^2(x)}\,dx$ 160
3.16 $\int \frac{1}{a-b\cos^4(x)}\,dx$ 166
3.17 $\int \frac{1}{a-b\cos^6(x)}\,dx$ 174
3.18 $\int \frac{1}{a-b\cos^8(x)}\,dx$ 181
3.19 $\int \frac{1}{a-b\cos(x)}\,dx$ 188
3.20 $\int \frac{1}{a-b\cos^3(x)}\,dx$ 194
3.21 $\int \frac{1}{a-b\cos^5(x)}\,dx$ 201
3.22 $\int \frac{1}{a+b\cos^2(x)}\,dx$ 209
3.23 $\int \frac{1}{a+b\cos^4(x)}\,dx$ 215
3.24 $\int \frac{1}{a+b\cos^6(x)}\,dx$ 226
3.25 $\int \frac{1}{a+b\cos^8(x)}\,dx$ 233

3.26 $\int \frac{1}{a+b\cos(x)}\,dx$ . . . 240
3.27 $\int \frac{1}{a+b\cos^3(x)}\,dx$ . . . 246
3.28 $\int \frac{1}{a+b\cos^5(x)}\,dx$ . . . 253
3.29 $\int \frac{1}{a-a\cos^2(x)}\,dx$ . . . 261
3.30 $\int \frac{1}{(a-a\cos^2(x))^2}\,dx$ . . . 266
3.31 $\int \frac{1}{(a-a\cos^2(x))^3}\,dx$ . . . 271
3.32 $\int \frac{1}{a+a\cos^2(x)}\,dx$ . . . 277
3.33 $\int \frac{1}{(a+a\cos^2(x))^2}\,dx$ . . . 282
3.34 $\int \frac{1}{(a+a\cos^2(x))^3}\,dx$ . . . 289
3.35 $\int \left(1-\cos^2(x)\right)^{5/2}\,dx$ . . . 297
3.36 $\int \left(1-\cos^2(x)\right)^{3/2}\,dx$ . . . 303
3.37 $\int \sqrt{1-\cos^2(x)}\,dx$ . . . 309
3.38 $\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx$ . . . 314
3.39 $\int \frac{1}{(1-\cos^2(x))^{3/2}}\,dx$ . . . 320
3.40 $\int \frac{1}{(1-\cos^2(x))^{5/2}}\,dx$ . . . 327
3.41 $\int \left(a-a\cos^2(x)\right)^{5/2}\,dx$ . . . 334
3.42 $\int \left(a-a\cos^2(x)\right)^{3/2}\,dx$ . . . 340
3.43 $\int \sqrt{a-a\cos^2(x)}\,dx$ . . . 346
3.44 $\int \frac{1}{\sqrt{a-a\cos^2(x)}}\,dx$ . . . 351
3.45 $\int \frac{1}{(a-a\cos^2(x))^{3/2}}\,dx$ . . . 357
3.46 $\int \frac{1}{(a-a\cos^2(x))^{5/2}}\,dx$ . . . 364
3.47 $\int \left(1+\cos^2(x)\right)^{5/2}\,dx$ . . . 371
3.48 $\int \left(1+\cos^2(x)\right)^{3/2}\,dx$ . . . 378
3.49 $\int \sqrt{1+\cos^2(x)}\,dx$ . . . 384
3.50 $\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx$ . . . 389
3.51 $\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx$ . . . 394
3.52 $\int \frac{1}{(1+\cos^2(x))^{5/2}}\,dx$ . . . 400
3.53 $\int \left(a+a\cos^2(x)\right)^{5/2}\,dx$ . . . 407
3.54 $\int \left(a+a\cos^2(x)\right)^{3/2}\,dx$ . . . 415
3.55 $\int \sqrt{a+a\cos^2(x)}\,dx$ . . . 422
3.56 $\int \frac{1}{\sqrt{a+a\cos^2(x)}}\,dx$ . . . 427
3.57 $\int \frac{1}{(a+a\cos^2(x))^{3/2}}\,dx$ . . . 433
3.58 $\int \frac{1}{(a+a\cos^2(x))^{5/2}}\,dx$ . . . 439
3.59 $\int \left(a+b\cos^2(x)\right)^4\,dx$ . . . 447
3.60 $\int \left(a+b\cos^2(x)\right)^3\,dx$ . . . 456

3.61 $\int \left(a+b\cos^2(x)\right)^2 \, dx$ 463
3.62 $\int \left(a+b\cos^2(x)\right) \, dx$ 469
3.63 $\int \frac{1}{a+b\cos^2(x)} \, dx$ 474
3.64 $\int \frac{1}{\left(a+b\cos^2(x)\right)^2} \, dx$ 480
3.65 $\int \frac{1}{\left(a+b\cos^2(x)\right)^3} \, dx$ 487
3.66 $\int \frac{1}{\left(a+b\cos^2(x)\right)^4} \, dx$ 495
3.67 $\int \left(a+b\cos^2(x)\right)^{5/2} \, dx$ 505
3.68 $\int \left(a+b\cos^2(x)\right)^{3/2} \, dx$ 514
3.69 $\int \sqrt{a+b\cos^2(x)} \, dx$ 521
3.70 $\int \frac{1}{\sqrt{a+b\cos^2(x)}} \, dx$ 526
3.71 $\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}} \, dx$ 532
3.72 $\int \frac{1}{\left(a+b\cos^2(x)\right)^{5/2}} \, dx$ 539

3.1 $\int \frac{1}{1-\cos^2(x)}\,dx$

Optimal result 54
Mathematica [A] (verified) 54
Rubi [A] (verified) 55
Maple [A] (verified) 56
Fricas [A] (verification not implemented) 57
Sympy [B] (verification not implemented) 57
Maxima [A] (verification not implemented) 57
Giac [A] (verification not implemented) 58
Mupad [B] (verification not implemented) 58
Reduce [B] (verification not implemented) 58

Optimal result

Integrand size = 10, antiderivative size = 4

$$\int \frac{1}{1-\cos^2(x)}\,dx = -\cot(x)$$

output
```
-cot(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-\cos^2(x)}\,dx = -\cot(x)$$

input
```
Integrate[(1 - Cos[x]^2)^(-1),x]
```

output
```
-Cot[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\int \csc^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^2dx$$

$$\Big\downarrow \text{4254}$$

$$-\int 1d\cot(x)$$

$$\Big\downarrow \text{24}$$

$$-\cot(x)$$

input
```
Int[(1 - Cos[x]^2)^(-1),x]
```

output
```
-Cot[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.75

method	result	size
default	$-\frac{1}{\tan(x)}$	7
risch	$-\frac{2i}{e^{2ix}-1}$	13
parallelrisch	$-\frac{\cot(\frac{x}{2})}{2}+\frac{\tan(\frac{x}{2})}{2}$	14
norman	$\frac{-\frac{1}{2}+\frac{\tan(\frac{x}{2})^2}{2}}{\tan(\frac{x}{2})}$	18

input
```
int(1/(1-cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/tan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int \frac{1}{1-\cos^2(x)}\,dx = -\frac{\cos(x)}{\sin(x)}$$

input
```
integrate(1/(1-cos(x)^2),x, algorithm="fricas")
```

output
```
-cos(x)/sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(3) = 6.

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \frac{1}{1-\cos^2(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cos(x)**2),x)
```

output
```
tan(x/2)/2 - 1/(2*tan(x/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \frac{1}{1-\cos^2(x)}\,dx = -\frac{1}{\tan(x)}$$

input
```
integrate(1/(1-cos(x)^2),x, algorithm="maxima")
```

output
```
-1/tan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \frac{1}{1-\cos^2(x)}\,dx = -\frac{1}{\tan(x)}$$

input
```
integrate(1/(1-cos(x)^2),x, algorithm="giac")
```

output
```
-1/tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-\cos^2(x)}\,dx = -\cot(x)$$

input
```
int(-1/(cos(x)^2 - 1),x)
```

output
```
-cot(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int \frac{1}{1-\cos^2(x)}\,dx = -\frac{\cos(x)}{\sin(x)}$$

input
```
int(1/(1-cos(x)^2),x)
```

output
```
( - cos(x))/sin(x)
```

3.2 $\int \frac{1}{1-\cos^4(x)}\,dx$

Optimal result 59
Mathematica [A] (verified) 59
Rubi [A] (verified) 60
Maple [A] (verified) 61
Fricas [A] (verification not implemented) 62
Sympy [A] (verification not implemented) 62
Maxima [A] (verification not implemented) 63
Giac [A] (verification not implemented) 63
Mupad [B] (verification not implemented) 63
Reduce [B] (verification not implemented) 64

Optimal result

Integrand size = 10, antiderivative size = 45

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{x}{2\sqrt{2}} - \frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{2\sqrt{2}} - \frac{\cot(x)}{2}$$

output
```
1/4*x*2^(1/2)-1/4*arctan(cos(x)*sin(x)/(1+2^(1/2)+cos(x)^2))*2^(1/2)-1/2*c
ot(x)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.53

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{1}{4}\left(\sqrt{2}\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right) - 2\cot(x)\right)$$

input
```
Integrate[(1 - Cos[x]^4)^(-1),x]
```

output
```
(Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]] - 2*Cot[x])/4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.56, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3688, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos^4(x)} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3688}$$

$$-\int \frac{\cot^2(x)+1}{2\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{299}$$

$$-\frac{1}{2}\int \frac{1}{2\cot^2(x)+1}d\cot(x)-\frac{\cot(x)}{2}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\sqrt{2}\cot(x)\right)}{2\sqrt{2}}-\frac{\cot(x)}{2}$$

input
```
Int[(1 - Cos[x]^4)^(-1),x]
```

output
```
-1/2*ArcTan[Sqrt[2]*Cot[x]]/Sqrt[2] - Cot[x]/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{1}{2\tan\left(x\right)}+\frac{\sqrt{2}\,\arctan\left(\frac{\tan\left(x\right)\sqrt{2}}{2}\right)}{4}$	21
risch	$-\frac{i}{{\mathrm e}^{2 i x}-1}+\frac{i\sqrt{2}\,\ln\left({\mathrm e}^{2 i x}+2\sqrt{2}+3\right)}{8}-\frac{i\sqrt{2}\,\ln\left({\mathrm e}^{2 i x}-2\sqrt{2}+3\right)}{8}$	52

input
```
int(1/(1-cos(x)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/2/tan(x)+1/4*2^(1/2)*arctan(1/2*tan(x)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{1}{1-\cos^4(x)}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)\sin(x)+4\cos(x)}{8\sin(x)}$$

input
```
integrate(1/(1-cos(x)^4),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))*s
in(x) + 4*cos(x))/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4} + \frac{\tan\left(\frac{x}{2}\right)}{4} - \frac{1}{4\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cos(x)**4),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/4 + sqrt(
2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/4 + tan(x/2)/4
 - 1/(4*tan(x/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.44

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right) - \frac{1}{2\tan(x)}$$

input
```
integrate(1/(1-cos(x)^4),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/2/tan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{1}{4}\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x) - \sin(2x)}{\sqrt{2}\cos(2x) + \sqrt{2} - \cos(2x) + 1}\right)\right) - \frac{1}{2\tan(x)}$$

input
```
integrate(1/(1-cos(x)^4),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1))) - 1/2/tan(x)
```

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.44

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{4} - \frac{1}{2\tan(x)}$$

input
```
int(-1/(cos(x)^4 - 1),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*tan(x))/2))/4 - 1/(2*tan(x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \frac{1}{1-\cos^4(x)}\,dx = \frac{-\sqrt{2}\,atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\sin(x)+\sqrt{2}\,atan\left(\frac{\sqrt{2}+2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\sin(x)-2\cos(x)}{4\sin(x)}$$

input
```
int(1/(1-cos(x)^4),x)
```

output
```
( - sqrt(2)*atan((sqrt(2) - 2*tan(x/2))/sqrt(2))*sin(x) + sqrt(2)*atan((sq
rt(2) + 2*tan(x/2))/sqrt(2))*sin(x) - 2*cos(x))/(4*sin(x))
```

3.3 $\int \frac{1}{1-\cos^6(x)}\,dx$

Optimal result 65
Mathematica [C] (verified) 66
Rubi [A] (verified) 66
Maple [C] (verified) 69
Fricas [A] (verification not implemented) 69
Sympy [B] (verification not implemented) 70
Maxima [F] 71
Giac [A] (verification not implemented) 72
Mupad [B] (verification not implemented) 73
Reduce [B] (verification not implemented) 74

Optimal result

Integrand size = 10, antiderivative size = 324

$$\int \frac{1}{1-\cos^6(x)}\,dx = \frac{1}{3}\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}x$$
$$-\frac{1}{6}\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\arctan\left(\frac{2(5-3\sqrt{3})\cos(x)\sin(x)-\sqrt{-45+26\sqrt{3}}(1-2\sin^2(x))}{6-4\sqrt{3}-\sqrt{-3+2\sqrt{3}}+2\sqrt{-45+26\sqrt{3}}\cos(x)\sin(x)-2\left(5-3\sqrt{3}\right)\sin\right.$$
$$-\frac{1}{6}\sqrt{\frac{1}{3}\left(3+2\sqrt{3}\right)}\arctan\left(\frac{2(5-3\sqrt{3})\cos(x)\sin(x)+\sqrt{-45+26\sqrt{3}}(1-2\sin^2(x))}{6-4\sqrt{3}-\sqrt{-3+2\sqrt{3}}-2\sqrt{-45+26\sqrt{3}}\cos(x)\sin(x)-2\left(5-3\sqrt{3}\right)\sin\right.$$
$$-\frac{1}{6}\sqrt{\frac{1}{3}\left(-3+2\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\cot(x)}{1+\sqrt{3}\cot^2(x)}\right)-\frac{\cot(x)}{3}$$

output
```
1/9*(9+6*3^(1/2))^(1/2)*x-1/18*(9+6*3^(1/2))^(1/2)*arctan((2*(5-3*3^(1/2))
*cos(x)*sin(x)-(-45+26*3^(1/2))^(1/2)*(1-2*sin(x)^2))/(6-4*3^(1/2)-(-3+2*3
^(1/2))^(1/2)+2*(-45+26*3^(1/2))^(1/2)*cos(x)*sin(x)-2*(5-3*3^(1/2))*sin(x
)^2))-1/18*(9+6*3^(1/2))^(1/2)*arctan((2*(5-3*3^(1/2))*cos(x)*sin(x)+(-45+
26*3^(1/2))^(1/2)*(1-2*sin(x)^2))/(6-4*3^(1/2)-(-3+2*3^(1/2))^(1/2)-2*(-45
+26*3^(1/2))^(1/2)*cos(x)*sin(x)-2*(5-3*3^(1/2))*sin(x)^2))-1/18*(-9+6*3^(
1/2))^(1/2)*arctanh((-3+2*3^(1/2))^(1/2)*cot(x)/(1+3^(1/2)*cot(x)^2))-1/3*
cot(x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.36

$$\int \frac{1}{1-\cos^6(x)}\,dx = \frac{(15+8\cos(2x)+\cos(4x))\sin(x)\left(6\cos(x)+i\sqrt[4]{-3}(3i+\sqrt{3})\arctan\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}(-i+\sqrt{3})\tan(x)\right)\sin(x)\right.}{144\left(-1+\cos^6(x)\right)}$$

input
```
Integrate[(1 - Cos[x]^6)^(-1),x]
```

output
```
((15 + 8*Cos[2*x] + Cos[4*x])*Sin[x]*(6*Cos[x] + I*(-3)^(1/4)*(3*I + Sqrt[
3])*ArcTan[((-1/3)^(1/4)*(-I + Sqrt[3])*Tan[x])/2]*Sin[x] + (-3)^(1/4)*(-3
*I + Sqrt[3])*ArcTan[((-1)^(3/4)*(I + Sqrt[3])*Tan[x])/(2*3^(1/4))]*Sin[x]
))/(144*(-1 + Cos[x]^6))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3690, 3042, 3654, 3042, 3660, 216, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos^6(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{3}\int \frac{1}{1-\cos^2(x)}dx+\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\cos^2(x)+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{1-\sin \left(x+\frac{\pi }{2}\right)^2}dx+\frac{1}{3}\int \frac{1}{\sqrt[3]{-1} \sin \left(x+\frac{\pi }{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3} \sin \left(x+\frac{\pi }{2}\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{-1} \sin \left(x+\frac{\pi }{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3} \sin \left(x+\frac{\pi }{2}\right)^2}dx+\frac{1}{3}\int \csc^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{-1} \sin \left(x+\frac{\pi }{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3} \sin \left(x+\frac{\pi }{2}\right)^2}dx+\frac{1}{3}\int \csc(x)^2dx$$

$$\Big\downarrow \text{3660}$$

$$-\frac{1}{3}\int \frac{1}{\left(1+\sqrt[3]{-1}\right) \cot^2(x)+1}d\cot(x)-\frac{1}{3}\int \frac{1}{\left(1-(-1)^{2/3}\right) \cot^2(x)+1}d\cot(x)+\frac{1}{3}\int \csc(x)^2dx$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\int \csc(x)^2dx-\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}} \cot(x)\right)}{3 \sqrt{1+\sqrt[3]{-1}}}-\frac{\arctan\left(\sqrt{1-(-1)^{2/3}} \cot(x)\right)}{3 \sqrt{1-(-1)^{2/3}}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d\cot(x)}{3}-\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}} \cot(x)\right)}{3 \sqrt{1+\sqrt[3]{-1}}}-\frac{\arctan\left(\sqrt{1-(-1)^{2/3}} \cot(x)\right)}{3 \sqrt{1-(-1)^{2/3}}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\arctan\left(\sqrt{1+\sqrt[3]{-1}} \cot(x)\right)}{3 \sqrt{1+\sqrt[3]{-1}}}-\frac{\arctan\left(\sqrt{1-(-1)^{2/3}} \cot(x)\right)}{3 \sqrt{1-(-1)^{2/3}}}-\frac{\cot(x)}{3}$$

input
```
Int[(1 - Cos[x]^6)^(-1),x]
```

output
```
-1/3*ArcTan[Sqrt[1 + (-1)^(1/3)]*Cot[x]]/Sqrt[1 + (-1)^(1/3)] - ArcTan[Sqr
t[1 - (-1)^(2/3)]*Cot[x]]/(3*Sqrt[1 - (-1)^(2/3)]) - Cot[x]/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.16

method	result
risch	$-\frac{2i}{3(e^{2ix}-1)}+\left(\sum_{\_R=\mathrm{RootOf}(3888\_Z^4+108\_Z^2+1)}\_R\ln\left(e^{2ix}+1296i\_R^3-216\_R^2-1\right)\right)$
default	$-\frac{1}{3\tan(x)}-\frac{\sqrt{3}\left(-\frac{\sqrt{-3+2\sqrt{3}}\,\ln\left(\tan(x)^2-\tan(x)\sqrt{-3+2\sqrt{3}}+\sqrt{3}\right)}{2}+\frac{2\left(-\frac{3}{2}-\sqrt{3}\right)\arctan\left(\frac{2\tan(x)-\sqrt{-3+2\sqrt{3}}}{\sqrt{2\sqrt{3}+3}}\right)}{\sqrt{2\sqrt{3}+3}}\right)}{18}-\frac{\sqrt{3}\left(\frac{\sqrt{-3+2\sqrt{3}}}{}\right.}{}$

input
```
int(1/(1-cos(x)^6),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/(exp(2*I*x)-1)+sum(_R*ln(exp(2*I*x)+1296*I*_R^3-216*_R^2-1),_R=Root
Of(3888*_Z^4+108*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.36

$$\int \frac{1}{1-\cos^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-cos(x)^6),x, algorithm="fricas")
```

output
```
1/24*(2*sqrt(2/3*sqrt(3) + 1)*arctan((4*(4*(sqrt(3) - 1)*cos(x)^7 - 4*cos(
x)^5 - 5*(sqrt(3) - 1)*cos(x)^3 + sqrt(3)*cos(x))*sin(x) + (8*(2*sqrt(3) -
 3)*cos(x)^6 - 4*(2*sqrt(3) - 3)*cos(x)^4 - 2*(4*sqrt(3) - 3)*cos(x)^2 - 3
)*sqrt(2/3*sqrt(3) - 1))*sqrt(2/3*sqrt(3) + 1)/(16*cos(x)^8 - 32*cos(x)^6
- 8*cos(x)^4 + 24*cos(x)^2 - 3))*sin(x) - 2*sqrt(2/3*sqrt(3) + 1)*arctan(-
(4*(4*(sqrt(3) - 1)*cos(x)^7 - 4*cos(x)^5 - 5*(sqrt(3) - 1)*cos(x)^3 + sqr
t(3)*cos(x))*sin(x) - (8*(2*sqrt(3) - 3)*cos(x)^6 - 4*(2*sqrt(3) - 3)*cos(
x)^4 - 2*(4*sqrt(3) - 3)*cos(x)^2 - 3)*sqrt(2/3*sqrt(3) - 1))*sqrt(2/3*sqr
t(3) + 1)/(16*cos(x)^8 - 32*cos(x)^6 - 8*cos(x)^4 + 24*cos(x)^2 - 3))*sin(
x) + sqrt(2/3*sqrt(3) - 1)*log(-(4*sqrt(3) - 7)*cos(x)^4 + (4*sqrt(3) - 5)
*cos(x)^2 + 2*((sqrt(3) - 3)*cos(x)^3 - sqrt(3)*cos(x))*sqrt(2/3*sqrt(3) -
 1)*sin(x) + 1)*sin(x) - sqrt(2/3*sqrt(3) - 1)*log(-(4*sqrt(3) - 7)*cos(x)
^4 + (4*sqrt(3) - 5)*cos(x)^2 - 2*((sqrt(3) - 3)*cos(x)^3 - sqrt(3)*cos(x)
)*sqrt(2/3*sqrt(3) - 1)*sin(x) + 1)*sin(x) - 8*cos(x))/sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(272) = 544$.

Time = 7.35 (sec) , antiderivative size = 728, normalized size of antiderivative = 2.25

$$\int \frac{1}{1-\cos^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-cos(x)**6),x)
```

output
```
sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(1/4)*tan(x/2) - 1) + pi*floor((x/2 - pi
/2)/pi))/36 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3**(1/4)*tan(x/2) - 1) + pi*f
loor((x/2 - pi/2)/pi))/12 + sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(1/4)*tan(x/
2) + 1) + pi*floor((x/2 - pi/2)/pi))/36 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3
**(1/4)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/12 + sqrt(2)*3**(3/4)*(
atan(sqrt(2)*3**(3/4)*tan(x/2)/3 - 1) + pi*floor((x/2 - pi/2)/pi))/36 + sq
rt(2)*3**(1/4)*(atan(sqrt(2)*3**(3/4)*tan(x/2)/3 - 1) + pi*floor((x/2 - pi
/2)/pi))/12 + sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(3/4)*tan(x/2)/3 + 1) + pi
*floor((x/2 - pi/2)/pi))/36 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3**(3/4)*tan(
x/2)/3 + 1) + pi*floor((x/2 - pi/2)/pi))/12 - sqrt(2)*3**(1/4)*log(4*tan(x
/2)**2 - 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/24 + sqrt(2)*3**(3/4)*lo
g(4*tan(x/2)**2 - 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/72 - sqrt(2)*3*
*(3/4)*log(4*tan(x/2)**2 + 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/72 + s
qrt(2)*3**(1/4)*log(4*tan(x/2)**2 + 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3
))/24 - sqrt(2)*3**(3/4)*log(36*tan(x/2)**2 - 12*sqrt(2)*3**(3/4)*tan(x/2)
 + 12*sqrt(3))/72 + sqrt(2)*3**(1/4)*log(36*tan(x/2)**2 - 12*sqrt(2)*3**(3
/4)*tan(x/2) + 12*sqrt(3))/24 - sqrt(2)*3**(1/4)*log(36*tan(x/2)**2 + 12*s
qrt(2)*3**(3/4)*tan(x/2) + 12*sqrt(3))/24 + sqrt(2)*3**(3/4)*log(36*tan(x/
2)**2 + 12*sqrt(2)*3**(3/4)*tan(x/2) + 12*sqrt(3))/72 + tan(x/2)/6 - 1/(6*
tan(x/2))
```

Maxima [F]

$$\int \frac{1}{1-\cos^6(x)}\,dx = \int -\frac{1}{\cos{(x)}^6-1}\,dx$$

input
```
integrate(1/(1-cos(x)^6),x, algorithm="maxima")
```

output
```
1/3*(3*(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*integrate(1/3*((cos(3*x)
 + 4*cos(2*x) + cos(x))*cos(4*x) + (14*cos(2*x) + 4*cos(x) + 1)*cos(3*x) +
 2*cos(3*x)^2 + 2*(7*cos(x) + 2)*cos(2*x) + 24*cos(2*x)^2 + 2*cos(x)^2 + (
sin(3*x) + 4*sin(2*x) + sin(x))*sin(4*x) + 2*(7*sin(2*x) + 2*sin(x))*sin(3
*x) + 2*sin(3*x)^2 + 24*sin(2*x)^2 + 14*sin(2*x)*sin(x) + 2*sin(x)^2 + cos
(x))/(2*(2*cos(3*x) + 6*cos(2*x) + 2*cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 4
*(6*cos(2*x) + 2*cos(x) + 1)*cos(3*x) + 4*cos(3*x)^2 + 12*(2*cos(x) + 1)*c
os(2*x) + 36*cos(2*x)^2 + 4*cos(x)^2 + 4*(sin(3*x) + 3*sin(2*x) + sin(x))*
sin(4*x) + sin(4*x)^2 + 8*(3*sin(2*x) + sin(x))*sin(3*x) + 4*sin(3*x)^2 +
36*sin(2*x)^2 + 24*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1), x) - 3*(c
os(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*integrate(-1/3*((cos(3*x) - 4*cos
(2*x) + cos(x))*cos(4*x) + (14*cos(2*x) - 4*cos(x) + 1)*cos(3*x) - 2*cos(3
*x)^2 + 2*(7*cos(x) - 2)*cos(2*x) - 24*cos(2*x)^2 - 2*cos(x)^2 + (sin(3*x)
 - 4*sin(2*x) + sin(x))*sin(4*x) + 2*(7*sin(2*x) - 2*sin(x))*sin(3*x) - 2*
sin(3*x)^2 - 24*sin(2*x)^2 + 14*sin(2*x)*sin(x) - 2*sin(x)^2 + cos(x))/(2*
(2*cos(3*x) - 6*cos(2*x) + 2*cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 4*(6*cos(
2*x) - 2*cos(x) + 1)*cos(3*x) - 4*cos(3*x)^2 + 12*(2*cos(x) - 1)*cos(2*x)
- 36*cos(2*x)^2 - 4*cos(x)^2 + 4*(sin(3*x) - 3*sin(2*x) + sin(x))*sin(4*x)
 - sin(4*x)^2 + 8*(3*sin(2*x) - sin(x))*sin(3*x) - 4*sin(3*x)^2 - 36*sin(2
*x)^2 + 24*sin(2*x)*sin(x) - 4*sin(x)^2 + 4*cos(x) - 1), x) - 2*sin(2*x...
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.61

$$\int \frac{1}{1-\cos^6(x)}\,dx$$
$$= \frac{1}{18}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor - \arctan\left(-\frac{3^{\frac{3}{4}}\left(3^{\frac{1}{4}}(\sqrt{6}-\sqrt{2})+4\tan(x)\right)}{3(\sqrt{6}+\sqrt{2})}\right)\right)\sqrt{6\sqrt{3}+9}$$
$$+\frac{1}{18}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor + \arctan\left(-\frac{3^{\frac{3}{4}}\left(3^{\frac{1}{4}}(\sqrt{6}-\sqrt{2})-4\tan(x)\right)}{3(\sqrt{6}+\sqrt{2})}\right)\right)\sqrt{6\sqrt{3}+9}$$
$$-\frac{1}{36}\sqrt{6\sqrt{3}-9}\log\left(\frac{1}{2}\left(\sqrt{6}3^{\frac{1}{4}}-3^{\frac{1}{4}}\sqrt{2}\right)\tan(x)+\tan(x)^2+\sqrt{3}\right)$$
$$+\frac{1}{36}\sqrt{6\sqrt{3}-9}\log\left(-\frac{1}{2}\left(\sqrt{6}3^{\frac{1}{4}}-3^{\frac{1}{4}}\sqrt{2}\right)\tan(x)+\tan(x)^2+\sqrt{3}\right)-\frac{1}{3\tan(x)}$$

input
```
integrate(1/(1-cos(x)^6),x, algorithm="giac")
```

output
```
1/18*(pi*floor(x/pi + 1/2) - arctan(-1/3*3^(3/4)*(3^(1/4)*(sqrt(6) - sqrt(
2)) + 4*tan(x))/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) + 1/18*(pi*floor
(x/pi + 1/2) + arctan(-1/3*3^(3/4)*(3^(1/4)*(sqrt(6) - sqrt(2)) - 4*tan(x)
)/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) - 1/36*sqrt(6*sqrt(3) - 9)*log
(1/2*(sqrt(6)*3^(1/4) - 3^(1/4)*sqrt(2))*tan(x) + tan(x)^2 + sqrt(3)) + 1/
36*sqrt(6*sqrt(3) - 9)*log(-1/2*(sqrt(6)*3^(1/4) - 3^(1/4)*sqrt(2))*tan(x)
 + tan(x)^2 + sqrt(3)) - 1/3/tan(x)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.29

$$\int \frac{1}{1-\cos^6(x)}\,dx = -\frac{1}{3\tan(x)} + \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{3^{1/4}\sqrt{6}\tan(x)\left(\frac{1}{27}-\frac{1}{27}\mathrm{i}\right)}{-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}}\right)\left(3^{1/4}\left(1+1\mathrm{i}\right)+3^{3/4}\left(-1+1\mathrm{i}\right)\right)1\mathrm{i}}{36} + \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{3^{1/4}\sqrt{6}\tan(x)\left(\frac{1}{27}+\frac{1}{27}\mathrm{i}\right)}{\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}}\right)\left(3^{1/4}\left(1-\mathrm{i}\right)+3^{3/4}\left(-1-\mathrm{i}\right)\right)1\mathrm{i}}{36}$$

input
```
int(-1/(cos(x)^6 - 1),x)
```

output
```
(6^(1/2)*atan((3^(1/4)*6^(1/2)*tan(x)*(1/27 - 1i/27))/((3^(1/2)*1i)/9 - 1/
9))*(3^(1/4)*(1 + 1i) - 3^(3/4)*(1 - 1i))*1i)/36 - 1/(3*tan(x)) + (6^(1/2)
*atan((3^(1/4)*6^(1/2)*tan(x)*(1/27 + 1i/27))/((3^(1/2)*1i)/9 + 1/9))*(3^(
1/4)*(1 - 1i) - 3^(3/4)*(1 + 1i))*1i)/36
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.60

$$\int \frac{1}{1-\cos^6(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(1-cos(x)^6),x)
```

output
```
( - 2*sqrt(6)*3**(1/4)*atan((sqrt(2)*3**(1/4) - 2*sqrt(3)*tan(x/2))/(sqrt(
2)*3**(1/4)))*sin(x) - 6*sqrt(2)*3**(1/4)*atan((sqrt(2)*3**(1/4) - 2*sqrt(
3)*tan(x/2))/(sqrt(2)*3**(1/4)))*sin(x) - 2*sqrt(6)*3**(1/4)*atan((sqrt(2)
*3**(1/4) - 2*tan(x/2))/(sqrt(2)*3**(1/4)))*sin(x) - 6*sqrt(2)*3**(1/4)*at
an((sqrt(2)*3**(1/4) - 2*tan(x/2))/(sqrt(2)*3**(1/4)))*sin(x) + 2*sqrt(6)*
3**(1/4)*atan((sqrt(2)*3**(1/4) + 2*sqrt(3)*tan(x/2))/(sqrt(2)*3**(1/4)))*
sin(x) + 6*sqrt(2)*3**(1/4)*atan((sqrt(2)*3**(1/4) + 2*sqrt(3)*tan(x/2))/(
sqrt(2)*3**(1/4)))*sin(x) + 2*sqrt(6)*3**(1/4)*atan((sqrt(2)*3**(1/4) + 2*
tan(x/2))/(sqrt(2)*3**(1/4)))*sin(x) + 6*sqrt(2)*3**(1/4)*atan((sqrt(2)*3*
*(1/4) + 2*tan(x/2))/(sqrt(2)*3**(1/4)))*sin(x) - 24*cos(x) + sqrt(6)*3**(
1/4)*log( - sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3) + tan(x/2)**2)*sin(x) - sq
rt(6)*3**(1/4)*log( - sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3)*tan(x/2)**2 + 1)
*sin(x) - sqrt(6)*3**(1/4)*log(sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3) + tan(x
/2)**2)*sin(x) + sqrt(6)*3**(1/4)*log(sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3)*
tan(x/2)**2 + 1)*sin(x) - 3*sqrt(2)*3**(1/4)*log( - sqrt(2)*3**(1/4)*tan(x
/2) + sqrt(3) + tan(x/2)**2)*sin(x) + 3*sqrt(2)*3**(1/4)*log( - sqrt(2)*3*
*(1/4)*tan(x/2) + sqrt(3)*tan(x/2)**2 + 1)*sin(x) + 3*sqrt(2)*3**(1/4)*log
(sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3) + tan(x/2)**2)*sin(x) - 3*sqrt(2)*3**
(1/4)*log(sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3)*tan(x/2)**2 + 1)*sin(x))/(72
*sin(x))
```

3.4 $\int \frac{1}{1-\cos^8(x)}\,dx$

Optimal result 75
Mathematica [C] (verified) 76
Rubi [C] (verified) 76
Maple [C] (verified) 79
Fricas [A] (verification not implemented) 79
Sympy [F(-1)] 80
Maxima [F] 80
Giac [A] (verification not implemented) 81
Mupad [B] (verification not implemented) 82
Reduce [F] 83

Optimal result

Integrand size = 10, antiderivative size = 306

$$\begin{aligned}\int \frac{1}{1-\cos^8(x)}\,dx &= \frac{x}{4\sqrt{2}}+\frac{1}{4}\sqrt{1+\sqrt{2}}x-\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}}\\ &-\frac{1}{8}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\left(2-\sqrt{2}\right)\cos(x)\sin(x)-\sqrt{-1+\sqrt{2}}\left(1-2\sin^2(x)\right)}{2+\sqrt{1+\sqrt{2}}+2\sqrt{-1+\sqrt{2}}\cos(x)\sin(x)-\left(2-\sqrt{2}\right)\sin^2(x)}\right)\\ &-\frac{1}{8}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\left(2-\sqrt{2}\right)\cos(x)\sin(x)+\sqrt{-1+\sqrt{2}}\left(1-2\sin^2(x)\right)}{2+\sqrt{1+\sqrt{2}}-2\sqrt{-1+\sqrt{2}}\cos(x)\sin(x)-\left(2-\sqrt{2}\right)\sin^2(x)}\right)\\ &-\frac{1}{8}\sqrt{-1+\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\cot(x)}{1+\sqrt{2}\cot^2(x)}\right)-\frac{\cot(x)}{4}\end{aligned}$$

output
```
1/8*x*2^(1/2)+1/4*(1+2^(1/2))^(1/2)*x-1/8*arctan(cos(x)*sin(x)/(1+2^(1/2)+
cos(x)^2))*2^(1/2)-1/8*(1+2^(1/2))^(1/2)*arctan(((2-2^(1/2))*cos(x)*sin(x)
-(2^(1/2)-1)^(1/2)*(1-2*sin(x)^2))/(2+(1+2^(1/2))^(1/2)+2*(2^(1/2)-1)^(1/2
)*cos(x)*sin(x)-(2-2^(1/2))*sin(x)^2))-1/8*(1+2^(1/2))^(1/2)*arctan(((2-2^
(1/2))*cos(x)*sin(x)+(2^(1/2)-1)^(1/2)*(1-2*sin(x)^2))/(2+(1+2^(1/2))^(1/2
)-2*(2^(1/2)-1)^(1/2)*cos(x)*sin(x)-(2-2^(1/2))*sin(x)^2))-1/8*(2^(1/2)-1)
^(1/2)*arctanh((-2+2*2^(1/2))^(1/2)*cot(x)/(1+2^(1/2)*cot(x)^2))-1/4*cot(x
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.21

$$\int \frac{1}{1-\cos^8(x)}\,dx = \frac{1}{8}\left(\frac{2\arctan\left(\frac{\tan(x)}{\sqrt{1-i}}\right)}{\sqrt{1-i}} + \frac{2\arctan\left(\frac{\tan(x)}{\sqrt{1+i}}\right)}{\sqrt{1+i}} + \sqrt{2}\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right) - 2\cot(x)\right)$$

input
```
Integrate[(1 - Cos[x]^8)^(-1),x]
```

output
```
((2*ArcTan[Tan[x]/Sqrt[1 - I]])/Sqrt[1 - I] + (2*ArcTan[Tan[x]/Sqrt[1 + I]
])/Sqrt[1 + I] + Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]] - 2*Cot[x])/8
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3690, 3042, 3654, 3042, 3660, 216, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos^8(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^8}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{4}\int \frac{1}{1-\cos^2(x)}dx + \frac{1}{4}\int \frac{1}{1-i\cos^2(x)}dx + \frac{1}{4}\int \frac{1}{i\cos^2(x)+1}dx + \frac{1}{4}\int \frac{1}{\cos^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{1}{1-\sin \left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{1-i \sin \left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{i \sin \left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin \left(x+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{1}{4}\int \frac{1}{1-i \sin \left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{i \sin \left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin \left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \csc^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{1}{1-i \sin \left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{i \sin \left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin \left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \csc(x)^2dx$$

$$\Big\downarrow \text{3660}$$

$$-\frac{1}{4}\int \frac{1}{(1-i) \cot^2(x)+1}d\cot(x)-\frac{1}{4}\int \frac{1}{(1+i) \cot^2(x)+1}d\cot(x)-\frac{1}{4}\int \frac{1}{2 \cot^2(x)+1}d\cot(x)+\frac{1}{4}\int \csc(x)^2dx$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\int \csc(x)^2dx-\frac{\arctan\left(\sqrt{1-i} \cot(x)\right)}{4 \sqrt{1-i}}-\frac{\arctan\left(\sqrt{1+i} \cot(x)\right)}{4 \sqrt{1+i}}-\frac{\arctan\left(\sqrt{2} \cot(x)\right)}{4 \sqrt{2}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d\cot(x)}{4}-\frac{\arctan\left(\sqrt{1-i} \cot(x)\right)}{4 \sqrt{1-i}}-\frac{\arctan\left(\sqrt{1+i} \cot(x)\right)}{4 \sqrt{1+i}}-\frac{\arctan\left(\sqrt{2} \cot(x)\right)}{4 \sqrt{2}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\arctan\left(\sqrt{1-i} \cot(x)\right)}{4 \sqrt{1-i}}-\frac{\arctan\left(\sqrt{1+i} \cot(x)\right)}{4 \sqrt{1+i}}-\frac{\arctan\left(\sqrt{2} \cot(x)\right)}{4 \sqrt{2}}-\frac{\cot(x)}{4}$$

input
```
Int[(1 - Cos[x]^8)^(-1),x]
```

output
```
-1/4*ArcTan[Sqrt[1 - I]*Cot[x]]/Sqrt[1 - I] - ArcTan[Sqrt[1 + I]*Cot[x]]/(
4*Sqrt[1 + I]) - ArcTan[Sqrt[2]*Cot[x]]/(4*Sqrt[2]) - Cot[x]/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{i}{2(e^{2ix}-1)}+\frac{\sqrt{-2-2i}\,\ln(e^{2ix}-i\sqrt{-2-2i}+\sqrt{-2-2i}+1+2i)}{16}-\frac{\sqrt{-2-2i}\,\ln(e^{2ix}+i\sqrt{-2-2i}-\sqrt{-2-2i}+1+2i)}{16}+\frac{i\sqrt{2}\,\ln\left(e^{2}\right.}{}$
default	$-\frac{1}{4\tan(x)}+\frac{\sqrt{2}\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{8}-\frac{\sqrt{2}\left(\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(\tan(x)^2+\tan(x)\sqrt{-2+2\sqrt{2}}+\sqrt{2}\right)}{2}+\frac{2\left(-1-\sqrt{2}\right)\arctan\left(\frac{2\tan(x)+\sqrt{-2+2\sqrt{2}}}{\sqrt{2\sqrt{2}+2}}\right.}{\sqrt{2\sqrt{2}+2}}\right.}{16}$

input
```
int(1/(1-cos(x)^8),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/(exp(2*I*x)-1)+1/16*(-2-2*I)^(1/2)*ln(exp(2*I*x)-I*(-2-2*I)^(1/2)+(
-2-2*I)^(1/2)+1+2*I)-1/16*(-2-2*I)^(1/2)*ln(exp(2*I*x)+I*(-2-2*I)^(1/2)-(-
2-2*I)^(1/2)+1+2*I)+1/16*I*2^(1/2)*ln(exp(2*I*x)+2*2^(1/2)+3)-1/16*I*2^(1/
2)*ln(exp(2*I*x)-2*2^(1/2)+3)+1/16*(-2+2*I)^(1/2)*ln(exp(2*I*x)-I*(-2+2*I)
^(1/2)-(-2+2*I)^(1/2)+1-2*I)-1/16*(-2+2*I)^(1/2)*ln(exp(2*I*x)+I*(-2+2*I)^
(1/2)+(-2+2*I)^(1/2)+1-2*I)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.35

$$\int \frac{1}{1-\cos^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-cos(x)^8),x, algorithm="fricas")
```

output
```
1/32*(2*sqrt(sqrt(2) + 1)*arctan(((2*sqrt(2)*cos(x)^7 - 4*cos(x)^5 - (3*sq
rt(2) - 2)*cos(x)^3 + sqrt(2)*cos(x))*sin(x) + (2*(sqrt(2) - 1)*cos(x)^6 -
 2*(sqrt(2) - 1)*cos(x)^4 - cos(x)^2)*sqrt(sqrt(2) - 1))*sqrt(sqrt(2) + 1)
/(4*cos(x)^8 - 8*cos(x)^6 + 4*cos(x)^2 - 1))*sin(x) - 2*sqrt(sqrt(2) + 1)*
arctan(-((2*sqrt(2)*cos(x)^7 - 4*cos(x)^5 - (3*sqrt(2) - 2)*cos(x)^3 + sqr
t(2)*cos(x))*sin(x) - (2*(sqrt(2) - 1)*cos(x)^6 - 2*(sqrt(2) - 1)*cos(x)^4
 - cos(x)^2)*sqrt(sqrt(2) - 1))*sqrt(sqrt(2) + 1)/(4*cos(x)^8 - 8*cos(x)^6
 + 4*cos(x)^2 - 1))*sin(x) - 2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sq
rt(2))/(cos(x)*sin(x)))*sin(x) + sqrt(sqrt(2) - 1)*log(-1/2*(4*sqrt(2) - 5
)*cos(x)^4 + 2*(sqrt(2) - 1)*cos(x)^2 + ((sqrt(2) - 2)*cos(x)^3 - sqrt(2)*
cos(x))*sqrt(sqrt(2) - 1)*sin(x) + 1/2)*sin(x) - sqrt(sqrt(2) - 1)*log(-1/
2*(4*sqrt(2) - 5)*cos(x)^4 + 2*(sqrt(2) - 1)*cos(x)^2 - ((sqrt(2) - 2)*cos
(x)^3 - sqrt(2)*cos(x))*sqrt(sqrt(2) - 1)*sin(x) + 1/2)*sin(x) - 8*cos(x))
/sin(x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1-\cos^8(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1-cos(x)**8),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{1-\cos^8(x)}\,dx = \int -\frac{1}{\cos{(x)}^8-1}\,dx$$

input
```
integrate(1/(1-cos(x)^8),x, algorithm="maxima")
```

output
```
1/8*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*arctan2(4*sqrt(2)*sin(2*x)
/(2*(2*sqrt(2) + 3)*cos(2*x) + cos(2*x)^2 + sin(2*x)^2 + 12*sqrt(2) + 17),
 (cos(2*x)^2 + sin(2*x)^2 + 6*cos(2*x) + 1)/(2*(2*sqrt(2) + 3)*cos(2*x) +
cos(2*x)^2 + sin(2*x)^2 + 12*sqrt(2) + 17)) + 64*(sqrt(2)*cos(2*x)^2 + sqr
t(2)*sin(2*x)^2 - 2*sqrt(2)*cos(2*x) + sqrt(2))*integrate(((4*cos(2*x) + 1
)*cos(4*x) + cos(8*x)*cos(4*x) + 4*cos(6*x)*cos(4*x) + 22*cos(4*x)^2 + sin
(8*x)*sin(4*x) + 4*sin(6*x)*sin(4*x) + 22*sin(4*x)^2 + 4*sin(4*x)*sin(2*x)
)/(2*(4*cos(6*x) + 22*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2 + 8
*(22*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 44*(4*cos(2*x)
+ 1)*cos(4*x) + 484*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 11*sin(4*
x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(11*sin(4*x) + 2*sin(2*x))*sin
(6*x) + 16*sin(6*x)^2 + 484*sin(4*x)^2 + 176*sin(4*x)*sin(2*x) + 16*sin(2*
x)^2 + 8*cos(2*x) + 1), x) - 4*sqrt(2)*sin(2*x))/(sqrt(2)*cos(2*x)^2 + sqr
t(2)*sin(2*x)^2 - 2*sqrt(2)*cos(2*x) + sqrt(2))
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.73

$$\int \frac{1}{1-\cos^8(x)}\,dx$$
$$= \frac{1}{8}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)$$
$$+\frac{1}{8}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}$$
$$+\frac{1}{8}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\tan(x)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}$$
$$-\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2+2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{2}\right)$$
$$+\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2-2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{2}\right)-\frac{1}{4\tan(x)}$$

input
```
integrate(1/(1-cos(x)^8),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1))) + 1/8*(pi*floor(x/pi + 1/2) + arctan(1/2*2^(3/4
)*(2^(1/4)*sqrt(-sqrt(2) + 2) + 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2)
 + 1) + 1/8*(pi*floor(x/pi + 1/2) + arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(-sqr
t(2) + 2) - 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) - 1/16*sqrt(sq
rt(2) - 1)*log(tan(x)^2 + 2^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2)) + 1
/16*sqrt(sqrt(2) - 1)*log(tan(x)^2 - 2^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + s
qrt(2)) - 1/4/tan(x)
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.79

$$\int \frac{1}{1-\cos^8(x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{8} - \mathrm{atan}\left(\frac{\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,1\mathrm{i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}-\frac{1}{16}\right)} + \frac{\sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,1\mathrm{i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}-\frac{1}{16}\right)}\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i} - \sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}\right) + \mathrm{atan}\left(\frac{\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,1\mathrm{i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}+\frac{1}{16}\right)} - \frac{\sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,1\mathrm{i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}+\frac{1}{16}\right)}\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i} + \sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}\right) - \frac{1}{4\tan(x)}$$

input
```
int(-1/(cos(x)^8 - 1),x)
```

output
```
atan((2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256)^(1/2)*1i)/(2*(16*(2^(1/2)/256
 - 1/256)^(1/2)*(- 2^(1/2)/256 - 1/256)^(1/2) + 1/16)) - (2^(1/2)*tan(x)*(
2^(1/2)/256 - 1/256)^(1/2)*1i)/(2*(16*(2^(1/2)/256 - 1/256)^(1/2)*(- 2^(1/
2)/256 - 1/256)^(1/2) + 1/16)))*((- 2^(1/2)/256 - 1/256)^(1/2)*2i + (2^(1/
2)/256 - 1/256)^(1/2)*2i) - atan((2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256)^(
1/2)*1i)/(2*(16*(2^(1/2)/256 - 1/256)^(1/2)*(- 2^(1/2)/256 - 1/256)^(1/2)
- 1/16)) + (2^(1/2)*tan(x)*(2^(1/2)/256 - 1/256)^(1/2)*1i)/(2*(16*(2^(1/2)
/256 - 1/256)^(1/2)*(- 2^(1/2)/256 - 1/256)^(1/2) - 1/16)))*((- 2^(1/2)/25
6 - 1/256)^(1/2)*2i - (2^(1/2)/256 - 1/256)^(1/2)*2i) - 1/(4*tan(x)) + (2^
(1/2)*atan((2^(1/2)*tan(x))/2))/8
```

Reduce [F]

$$\int \frac{1}{1-\cos^8(x)}\,dx$$
$$= \frac{-21\sqrt{2}\,atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\sin(x)+21\sqrt{2}\,atan\left(\frac{\sqrt{2}+2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\sin(x)+62\cos(x)+192\left(\int\frac{\tan(\frac{x}{2}}{\tan\left(\frac{x}{2}\right)^{12}+7\tan\left(\frac{x}{2}\right.}\right.}{8\sin(x)}$$

input
```
int(1/(1-cos(x)^8),x)
```

output
```
( - 21*sqrt(2)*atan((sqrt(2) - 2*tan(x/2))/sqrt(2))*sin(x) + 21*sqrt(2)*at
an((sqrt(2) + 2*tan(x/2))/sqrt(2))*sin(x) + 62*cos(x) + 192*int(tan(x/2)**
2/(tan(x/2)**12 + 7*tan(x/2)**8 + 7*tan(x/2)**4 + 1),x)*sin(x) + 32*int(1/
(tan(x/2)**14 + 7*tan(x/2)**10 + 7*tan(x/2)**6 + tan(x/2)**2),x)*sin(x) +
14*sin(x)*x + 64)/(8*sin(x))
```

3.5 $\int \frac{1}{1-\cos(x)} dx$

Optimal result 84
Mathematica [A] (verified) 84
Rubi [A] (verified) 85
Maple [A] (verified) 86
Fricas [A] (verification not implemented) 86
Sympy [A] (verification not implemented) 86
Maxima [A] (verification not implemented) 87
Giac [A] (verification not implemented) 87
Mupad [B] (verification not implemented) 87
Reduce [B] (verification not implemented) 88

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

output
```
-sin(x)/(1-cos(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 - Cos[x])^(-1),x]
```

output
```
-Cot[x/2]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3127}$$

$$-\frac{\sin(x)}{1-\cos(x)}$$

input
```
Int[(1 - Cos[x])^(-1),x]
```

output
```
-(Sin[x]/(1 - Cos[x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

method	result	size
parallelrisch	$-\cot\left(\frac{x}{2}\right)$	7
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
norman	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

input
```
int(1/(1-cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-cot(1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="fricas")
```

output
```
-(cos(x) + 1)/sin(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cos(x)),x)
```

output
```
-1/tan(x/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="maxima")
```

output
```
-(cos(x) + 1)/sin(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(1/(1-cos(x)),x, algorithm="giac")
```

output
```
-1/tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{1-\cos(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

input
```
int(-1/(cos(x) - 1),x)
```

output
```
-cot(x/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{1}{\tan\left(\frac{x}{2}\right)}$$

input
```
int(1/(1-cos(x)),x)
```

output
```
( - 1)/tan(x/2)
```

3.6 $\int \frac{1}{1-\cos^3(x)}\,dx$

Optimal result 89
Mathematica [C] (verified) 89
Rubi [A] (verified) 90
Maple [C] (verified) 91
Fricas [B] (verification not implemented) 92
Sympy [B] (verification not implemented) 93
Maxima [B] (verification not implemented) 94
Giac [B] (verification not implemented) 95
Mupad [B] (verification not implemented) 95
Reduce [B] (verification not implemented) 96

Optimal result

Integrand size = 10, antiderivative size = 95

$$\int \frac{1}{1-\cos^3(x)}\,dx=-\frac{2\sqrt[4]{-1}\arctan\left(\frac{(-1)^{3/4}\tan\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1+\sqrt[3]{-1}\right)}-\frac{2\sqrt[4]{-1}\text{arctanh}\left(\frac{(-1)^{3/4}\tan\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1-(-1)^{2/3}\right)}-\frac{\sin(x)}{3(1-\cos(x))}$$

output
```
-2/3*(-1)^(1/4)*arctan(1/3*(-1)^(3/4)*tan(1/2*x)*3^(3/4))*3^(1/4)/(1+(-1)^
(1/3))-2/3*(-1)^(1/4)*arctanh(1/3*(-1)^(3/4)*tan(1/2*x)*3^(3/4))*3^(1/4)/(
1-(-1)^(2/3))-sin(x)/(3-3*cos(x))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \frac{1}{1-\cos^3(x)}\,dx = \frac{\sqrt{2}(i+\sqrt{3})\arctan\left(\frac{(-i+\sqrt{3})\tan\left(\frac{x}{2}\right)}{\sqrt{6+2i\sqrt{3}}}\right)+2\sqrt{2}\arctan\left(\frac{(i+\sqrt{3})\tan\left(\frac{x}{2}\right)}{\sqrt{6-2i\sqrt{3}}}\right)-\sqrt{3+i\sqrt{3}}\cot\left(\frac{x}{2}\right)}{3\sqrt{3+i\sqrt{3}}}$$

input
```
Integrate[(1 - Cos[x]^3)^(-1),x]
```

output
```
(Sqrt[2]*(I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*Tan[x/2])/Sqrt[6 + (2*I)*Sqr
t[3]]] + 2*Sqrt[2]*ArcTan[((I + Sqrt[3])*Tan[x/2])/Sqrt[6 - (2*I)*Sqrt[3]]
] - Sqrt[3 + I*Sqrt[3]]*Cot[x/2])/(3*Sqrt[3 + I*Sqrt[3]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos^3(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \; 3692$$

$$\int \left(\frac{1}{3\left(\sqrt[3]{-1}\cos(x)+1\right)}+\frac{1}{3\left(1-(-1)^{2/3}\cos(x)\right)}+\frac{1}{3(1-\cos(x))}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\sqrt[4]{-1}\arctan\left(\frac{(-1)^{3/4}\tan\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1+\sqrt[3]{-1}\right)}-\frac{2\sqrt[4]{-1}\text{arctanh}\left(\frac{(-1)^{3/4}\tan\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1-(-1)^{2/3}\right)}-\frac{\sin(x)}{3(1-\cos(x))}$$

input
```
Int[(1 - Cos[x]^3)^(-1),x]
```

output
```
(-2*(-1)^(1/4)*ArcTan[((-1)^(3/4)*Tan[x/2])/3^(1/4)])/(3^(3/4)*(1 + (-1)^(
1/3))) - (2*(-1)^(1/4)*ArcTanh[((-1)^(3/4)*Tan[x/2])/3^(1/4)])/(3^(3/4)*(1
 - (-1)^(2/3))) - Sin[x]/(3*(1 - Cos[x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{2i}{3(\mathrm{e}^{ix}-1)}+\left(\sum_{\_R=\mathrm{RootOf}(243\_Z^4+27\_Z^2+1)}\_R\ln\left(\mathrm{e}^{ix}-162i\_R^3+27\_R^2-9i\_R+2\right)\right)$
default	$\frac{3^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\tan\left(\frac{x}{2}\right)^2+3^{\frac{1}{4}}\tan\left(\frac{x}{2}\right)\sqrt{2}+\sqrt{3}}{\tan\left(\frac{x}{2}\right)^2-3^{\frac{1}{4}}\tan\left(\frac{x}{2}\right)\sqrt{2}+\sqrt{3}}\right)+2\arctan\left(\frac{\sqrt{2}\,3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)}{3}+1\right)+2\arctan\left(\frac{\sqrt{2}\,3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)}{3}-1\right)\right)}{12}+\frac{3^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{\tan\left(\frac{x}{2}\right.}{\tan\left(\frac{x}{2}\right.}\right.\right.}{}$

input
```
int(1/(1-cos(x)^3),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/(exp(I*x)-1)+sum(_R*ln(exp(I*x)-162*I*_R^3+27*_R^2-9*I*_R+2),_R=Roo
tOf(243*_Z^4+27*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(67) = 134$.

Time = 0.12 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.83

$$\int \frac{1}{1-\cos^3(x)}\,dx =$$

$$-\frac{2\sqrt{\frac{2}{3}\sqrt{3}+1}\arctan\left(\frac{\left(2\left(2\left(\sqrt{3}-2\right)\cos(x)^2-2\cos(x)^3+\left(\sqrt{3}+2\right)\cos(x)+1\right)\sin(x)+\left(4\sqrt{3}\cos(x)^3+2\sqrt{3}\cos(x)^2-2\left(2\sqrt{3}+3\right)c\right.\right.}{4\cos(x)^4-8\cos(x)^2+1}\right.}{}$$

input
```
integrate(1/(1-cos(x)^3),x, algorithm="fricas")
```

output
```
-1/12*(2*sqrt(2/3*sqrt(3) + 1)*arctan((2*(2*(sqrt(3) - 2)*cos(x)^2 - 2*cos
(x)^3 + (sqrt(3) + 2)*cos(x) + 1)*sin(x) + (4*sqrt(3)*cos(x)^3 + 2*sqrt(3)
*cos(x)^2 - 2*(2*sqrt(3) + 3)*cos(x) - 2*sqrt(3) - 3)*sqrt(2/3*sqrt(3) - 1
))*sqrt(2/3*sqrt(3) + 1)/(4*cos(x)^4 - 8*cos(x)^2 + 1))*sin(x) - 2*sqrt(2/
3*sqrt(3) + 1)*arctan(-(2*(2*(sqrt(3) - 2)*cos(x)^2 - 2*cos(x)^3 + (sqrt(3
) + 2)*cos(x) + 1)*sin(x) - (4*sqrt(3)*cos(x)^3 + 2*sqrt(3)*cos(x)^2 - 2*(
2*sqrt(3) + 3)*cos(x) - 2*sqrt(3) - 3)*sqrt(2/3*sqrt(3) - 1))*sqrt(2/3*sqr
t(3) + 1)/(4*cos(x)^4 - 8*cos(x)^2 + 1))*sin(x) - sqrt(2/3*sqrt(3) - 1)*lo
g(-(sqrt(3) - 1)*cos(x)^2 + (sqrt(3)*cos(x) + 2*sqrt(3) + 3)*sqrt(2/3*sqrt
(3) - 1)*sin(x) + sqrt(3) + cos(x) + 1)*sin(x) + sqrt(2/3*sqrt(3) - 1)*log
(-(sqrt(3) - 1)*cos(x)^2 - (sqrt(3)*cos(x) + 2*sqrt(3) + 3)*sqrt(2/3*sqrt(
3) - 1)*sin(x) + sqrt(3) + cos(x) + 1)*sin(x) + 4*cos(x) + 4)/sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(78) = 156.

Time = 1.28 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.86

$$\int \frac{1}{1-\cos^3(x)}\,dx = \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\left(\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)}{3}-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{18} + \frac{\sqrt{2}\cdot\sqrt[4]{3}\left(\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)}{3}-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6} + \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\left(\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)}{3}+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{18} + \frac{\sqrt{2}\cdot\sqrt[4]{3}\left(\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)}{3}+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6} - \frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(4\tan^2\left(\frac{x}{2}\right)-4\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{12} + \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(4\tan^2\left(\frac{x}{2}\right)-4\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{36} - \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(4\tan^2\left(\frac{x}{2}\right)+4\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{36} + \frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(4\tan^2\left(\frac{x}{2}\right)+4\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{12} - \frac{1}{3\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cos(x)**3),x)
```

output
```
sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(3/4)*tan(x/2)/3 - 1) + pi*floor((x/2 -
pi/2)/pi))/18 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3**(3/4)*tan(x/2)/3 - 1) +
pi*floor((x/2 - pi/2)/pi))/6 + sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(3/4)*tan
(x/2)/3 + 1) + pi*floor((x/2 - pi/2)/pi))/18 + sqrt(2)*3**(1/4)*(atan(sqrt
(2)*3**(3/4)*tan(x/2)/3 + 1) + pi*floor((x/2 - pi/2)/pi))/6 - sqrt(2)*3**(
1/4)*log(4*tan(x/2)**2 - 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/12 + sqr
t(2)*3**(3/4)*log(4*tan(x/2)**2 - 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))
/36 - sqrt(2)*3**(3/4)*log(4*tan(x/2)**2 + 4*sqrt(2)*3**(1/4)*tan(x/2) + 4
*sqrt(3))/36 + sqrt(2)*3**(1/4)*log(4*tan(x/2)**2 + 4*sqrt(2)*3**(1/4)*tan
(x/2) + 4*sqrt(3))/12 - 1/(3*tan(x/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(67) = 134$.

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.89

$$\begin{aligned}\int \frac{1}{1-\cos^3(x)}\,dx =& \frac{1}{18}\cdot 3^{\frac{1}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+\frac{2\sin(x)}{\cos(x)+1}\right)\right)\\&+\frac{1}{18}\cdot 3^{\frac{1}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-\frac{2\sin(x)}{\cos(x)+1}\right)\right)\\&-\frac{1}{36}\cdot 3^{\frac{1}{4}}\sqrt{2}\left(\sqrt{3}-3\right)\log\left(\frac{3^{\frac{1}{4}}\sqrt{2}\sin(x)}{\cos(x)+1}+\sqrt{3}+\frac{\sin(x)^2}{(\cos(x)+1)^2}\right)\\&+\frac{1}{36}\cdot 3^{\frac{1}{4}}\sqrt{2}\left(\sqrt{3}-3\right)\log\left(-\frac{3^{\frac{1}{4}}\sqrt{2}\sin(x)}{\cos(x)+1}+\sqrt{3}+\frac{\sin(x)^2}{(\cos(x)+1)^2}\right)\\&-\frac{\cos(x)+1}{3\sin(x)}\end{aligned}$$

input
```
integrate(1/(1-cos(x)^3),x, algorithm="maxima")
```

output
```
1/18*3^(1/4)*sqrt(2)*(sqrt(3) + 3)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqr
t(2) + 2*sin(x)/(cos(x) + 1))) + 1/18*3^(1/4)*sqrt(2)*(sqrt(3) + 3)*arctan
(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) - 2*sin(x)/(cos(x) + 1))) - 1/36*3^
(1/4)*sqrt(2)*(sqrt(3) - 3)*log(3^(1/4)*sqrt(2)*sin(x)/(cos(x) + 1) + sqrt
(3) + sin(x)^2/(cos(x) + 1)^2) + 1/36*3^(1/4)*sqrt(2)*(sqrt(3) - 3)*log(-3
^(1/4)*sqrt(2)*sin(x)/(cos(x) + 1) + sqrt(3) + sin(x)^2/(cos(x) + 1)^2) -
1/3*(cos(x) + 1)/sin(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(67) = 134$.

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.77

$$\int \frac{1}{1-\cos^3(x)}\,dx$$
$$= \frac{1}{9}\left(\pi\left\lfloor \frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor + \arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+2\,\tan\left(\frac{1}{2}\,x\right)\right)\right)\right)\sqrt{6\,\sqrt{3}+9}$$
$$+\frac{1}{9}\left(\pi\left\lfloor \frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor + \arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-2\,\tan\left(\frac{1}{2}\,x\right)\right)\right)\right)\sqrt{6\,\sqrt{3}+9}$$
$$+\frac{1}{18}\sqrt{6\,\sqrt{3}-9}\log\left(\tan\left(\frac{1}{2}\,x\right)^2+3^{\frac{1}{4}}\sqrt{2}\tan\left(\frac{1}{2}\,x\right)+\sqrt{3}\right)$$
$$-\frac{1}{18}\sqrt{6\,\sqrt{3}-9}\log\left(\tan\left(\frac{1}{2}\,x\right)^2-3^{\frac{1}{4}}\sqrt{2}\tan\left(\frac{1}{2}\,x\right)+\sqrt{3}\right)-\frac{1}{3\,\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(1/(1-cos(x)^3),x, algorithm="giac")
```

output
```
1/9*(pi*floor(1/2*x/pi + 1/2) + arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2
) + 2*tan(1/2*x))))*sqrt(6*sqrt(3) + 9) + 1/9*(pi*floor(1/2*x/pi + 1/2) +
arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) - 2*tan(1/2*x))))*sqrt(6*sqrt
(3) + 9) + 1/18*sqrt(6*sqrt(3) - 9)*log(tan(1/2*x)^2 + 3^(1/4)*sqrt(2)*tan
(1/2*x) + sqrt(3)) - 1/18*sqrt(6*sqrt(3) - 9)*log(tan(1/2*x)^2 - 3^(1/4)*s
qrt(2)*tan(1/2*x) + sqrt(3)) - 1/3/tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int \frac{1}{1-\cos^3(x)}\,dx = -\frac{\cot\left(\frac{x}{2}\right)}{3}+3^{1/4}\sqrt{6}\,\mathrm{atan}\left(3^{1/4}\sqrt{6}\tan\left(\frac{x}{2}\right)\left(\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)\right)\left(\frac{1}{18}-\frac{1}{18}\mathrm{i}\right)$$
$$+3^{1/4}\sqrt{6}\,\mathrm{atan}\left(3^{1/4}\sqrt{6}\tan\left(\frac{x}{2}\right)\left(\frac{1}{6}+\frac{1}{6}\mathrm{i}\right)\right)\left(\frac{1}{18}+\frac{1}{18}\mathrm{i}\right)+3^{3/4}\sqrt{6}\,\mathrm{atan}\left(3^{1/4}\sqrt{6}\tan\left(\frac{x}{2}\right)\left(\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)\right)\left(\right.$$

input
```
int(-1/(cos(x)^3 - 1),x)
```

output
```
3^(1/4)*6^(1/2)*atan(3^(1/4)*6^(1/2)*tan(x/2)*(1/6 - 1i/6))*(1/18 - 1i/18)
 - cot(x/2)/3 + 3^(1/4)*6^(1/2)*atan(3^(1/4)*6^(1/2)*tan(x/2)*(1/6 + 1i/6)
)*(1/18 + 1i/18) + 3^(3/4)*6^(1/2)*atan(3^(1/4)*6^(1/2)*tan(x/2)*(1/6 - 1i
/6))*(1/18 + 1i/18) + 3^(3/4)*6^(1/2)*atan(3^(1/4)*6^(1/2)*tan(x/2)*(1/6 +
 1i/6))*(1/18 - 1i/18)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.85

$$\int \frac{1}{1-\cos^3(x)}\,dx = \frac{-2\sqrt{6}\,3^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,3^{\frac{1}{4}}-2\tan\left(\frac{x}{2}\right)\right)3^{\frac{3}{4}}}{3\sqrt{2}}\right)\tan\left(\frac{x}{2}\right)-6\sqrt{2}\,3^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,3^{\frac{1}{4}}-2\tan\left(\frac{x}{2}\right)\right)3^{\frac{3}{4}}}{3\sqrt{2}}\right)\tan\left(\frac{x}{2}\right)+2\sqrt{6}\,3^{\frac{1}{4}}\,atan\left(\right.}{}$$

input
```
int(1/(1-cos(x)^3),x)
```

output
```
( - 2*sqrt(6)*3**(1/4)*atan((sqrt(2)*3**(1/4) - 2*tan(x/2))/(sqrt(2)*3**(1
/4)))*tan(x/2) - 6*sqrt(2)*3**(1/4)*atan((sqrt(2)*3**(1/4) - 2*tan(x/2))/(
sqrt(2)*3**(1/4)))*tan(x/2) + 2*sqrt(6)*3**(1/4)*atan((sqrt(2)*3**(1/4) +
2*tan(x/2))/(sqrt(2)*3**(1/4)))*tan(x/2) + 6*sqrt(2)*3**(1/4)*atan((sqrt(2
)*3**(1/4) + 2*tan(x/2))/(sqrt(2)*3**(1/4)))*tan(x/2) + sqrt(6)*3**(1/4)*l
og( - sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3) + tan(x/2)**2)*tan(x/2) - sqrt(6
)*3**(1/4)*log(sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3) + tan(x/2)**2)*tan(x/2)
 - 3*sqrt(2)*3**(1/4)*log( - sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3) + tan(x/2
)**2)*tan(x/2) + 3*sqrt(2)*3**(1/4)*log(sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3
) + tan(x/2)**2)*tan(x/2) - 12)/(36*tan(x/2))
```

3.7 $\int \frac{1}{1-\cos^5(x)}\,dx$

Optimal result . . . 97
Mathematica [C] (verified) . . . 98
Rubi [A] (verified) . . . 98
Maple [C] (verified) . . . 100
Fricas [B] (verification not implemented) . . . 100
Sympy [F(-1)] . . . 101
Maxima [F] . . . 102
Giac [B] (verification not implemented) . . . 102
Mupad [B] (verification not implemented) . . . 104
Reduce [F] . . . 105

Optimal result

Integrand size = 10, antiderivative size = 205

$$\int \frac{1}{1-\cos^5(x)}\,dx = \frac{2\arctan\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}} + \frac{2\arctan\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}} - \frac{2\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{-1+(-1)^{4/5}}} + \frac{2\text{arctanh}\left(\sqrt{-\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{-1-(-1)^{3/5}}} - \frac{\sin(x)}{5(1-\cos(x))}$$

output
```
2/5*arctan(((1-(-1)^(1/5))/(1+(-1)^(1/5)))^(1/2)*tan(1/2*x))/(1-(-1)^(2/5)
)^(1/2)+2/5*arctan(((1-(-1)^(3/5))/(1+(-1)^(3/5)))^(1/2)*tan(1/2*x))/(1+(-
1)^(1/5))^(1/2)-2/5*arctanh(tan(1/2*x)/(-(1-(-1)^(2/5))/(1+(-1)^(2/5)))^(1
/2))/(-1+(-1)^(4/5))^(1/2)+2/5*arctanh((-(1+(-1)^(4/5))/(1-(-1)^(4/5)))^(1
/2)*tan(1/2*x))/(-1-(-1)^(3/5))^(1/2)-sin(x)/(5-5*cos(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.05 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.84

$$\int \frac{1}{1-\cos^5(x)}\,dx$$
$$= -\frac{1}{5}\cot\left(\frac{x}{2}\right) + \frac{1}{10}\text{RootSum}\Big[1+2\#1+8\#1^2+14\#1^3+30\#1^4+14\#1^5+8\#1^6+2\#1^7$$
$$+\#1^8\&, \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right) - i\log\left(1-2\cos(x)\#1+\#1^2\right) + 8\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1 - 4i\log\left(1-\right.}{}$$

input
```
Integrate[(1 - Cos[x]^5)^(-1),x]
```

output
```
-1/5*Cot[x/2] + RootSum[1 + 2*#1 + 8*#1^2 + 14*#1^3 + 30*#1^4 + 14*#1^5 +
8*#1^6 + 2*#1^7 + #1^8 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)] - I*Log[1 - 2*C
os[x]*#1 + #1^2] + 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - (4*I)*Log[1 - 2*Cos
[x]*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log[1 -
2*Cos[x]*#1 + #1^2]*#1^2 + 80*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - (40*I)*L
og[1 - 2*Cos[x]*#1 + #1^2]*#1^3 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 - (
15*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 + 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1
^5 - (4*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] - #1
)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(1 + 8*#1 + 21*#1^2 + 60*#1^
3 + 35*#1^4 + 24*#1^5 + 7*#1^6 + 4*#1^7) & ]/10
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cos^5(x)}\,dx$$

$\big\downarrow$ 3042

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^5}dx$$

$\big\downarrow$ 3692

$$\int \left(\frac{1}{5\left(\sqrt[5]{-1}\cos(x)+1\right)}+\frac{1}{5\left(1-(-1)^{2/5}\cos(x)\right)}+\frac{1}{5\left((-1)^{3/5}\cos(x)+1\right)}+\frac{1}{5\left(1-(-1)^{4/5}\cos(x)\right)}+\frac{1}{5(1-\cos$$

$\big\downarrow$ 2009

$$\frac{2\arctan\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\arctan\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{(-1)^{4/5}-1}}+\frac{2\text{arctanh}\left(\sqrt{-\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{-1-(-1)^{3/5}}}-\frac{\sin(x)}{5(1-\cos(x))}$$

input
```
Int[(1 - Cos[x]^5)^(-1),x]
```

output
```
(2*ArcTan[Sqrt[(1 - (-1)^(1/5))/(1 + (-1)^(1/5))]*Tan[x/2]])/(5*Sqrt[1 - (
-1)^(2/5)]) + (2*ArcTan[Sqrt[(1 - (-1)^(3/5))/(1 + (-1)^(3/5))]*Tan[x/2]])
/(5*Sqrt[1 + (-1)^(1/5)]) - (2*ArcTanh[Tan[x/2]/Sqrt[-((1 - (-1)^(2/5))/(1
 + (-1)^(2/5)))]])/(5*Sqrt[-1 + (-1)^(4/5)]) + (2*ArcTanh[Sqrt[-((1 + (-1)
^(4/5))/(1 - (-1)^(4/5)))]*Tan[x/2]])/(5*Sqrt[-1 - (-1)^(3/5)]) - Sin[x]/(
5*(1 - Cos[x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.30

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+10\_Z^4+5\right)}\frac{\left(\_R^6+5\_R^4+5\_R^2+5\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^7+5\_R^3}\right)}{10}-\frac{1}{5\tan\left(\frac{x}{2}\right)}$
risch	$-\frac{2i}{5(e^{ix}-1)}+\left(\sum_{\_R=\mathrm{RootOf}(1953125\_Z^8+156250\_Z^6+6250\_Z^4+125\_Z^2+1)}\_R\ln\left(e^{ix}+2343750i\_R^7-234\right.\right.$

input
```
int(1/(1-cos(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/10*sum((_R^6+5*_R^4+5*_R^2+5)/(_R^7+5*_R^3)*ln(tan(1/2*x)-_R),_R=RootOf(
_Z^8+10*_Z^4+5))-1/5/tan(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(137) = 274.

Time = 0.21 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.36

$$\int \frac{1}{1-\cos^5(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-cos(x)^5),x, algorithm="fricas")
```

output
```
1/20*(sqrt(2*sqrt(2/5*sqrt(5) - 1) - 2)*log((3*sqrt(5) + 5)*sqrt(2/5*sqrt(
5) - 1)*sqrt(2*sqrt(2/5*sqrt(5) - 1) - 2)*sin(x) - (3*sqrt(5) + 5)*sqrt(2/
5*sqrt(5) - 1)*cos(x) - (sqrt(5) - 1)*cos(x) + 4)*sin(x) - sqrt(2*sqrt(2/5
*sqrt(5) - 1) - 2)*log(-(3*sqrt(5) + 5)*sqrt(2/5*sqrt(5) - 1)*sqrt(2*sqrt(
2/5*sqrt(5) - 1) - 2)*sin(x) - (3*sqrt(5) + 5)*sqrt(2/5*sqrt(5) - 1)*cos(x
) - (sqrt(5) - 1)*cos(x) + 4)*sin(x) + sqrt(-2*sqrt(2/5*sqrt(5) - 1) - 2)*
log((3*sqrt(5) + 5)*sqrt(2/5*sqrt(5) - 1)*sqrt(-2*sqrt(2/5*sqrt(5) - 1) -
2)*sin(x) - (3*sqrt(5) + 5)*sqrt(2/5*sqrt(5) - 1)*cos(x) + (sqrt(5) - 1)*c
os(x) - 4)*sin(x) - sqrt(-2*sqrt(2/5*sqrt(5) - 1) - 2)*log(-(3*sqrt(5) + 5
)*sqrt(2/5*sqrt(5) - 1)*sqrt(-2*sqrt(2/5*sqrt(5) - 1) - 2)*sin(x) - (3*sqr
t(5) + 5)*sqrt(2/5*sqrt(5) - 1)*cos(x) + (sqrt(5) - 1)*cos(x) - 4)*sin(x)
+ sqrt(2*sqrt(-2/5*sqrt(5) - 1) - 2)*log((3*sqrt(5) - 5)*sqrt(-2/5*sqrt(5)
 - 1)*sqrt(2*sqrt(-2/5*sqrt(5) - 1) - 2)*sin(x) - (3*sqrt(5) - 5)*sqrt(-2/
5*sqrt(5) - 1)*cos(x) - (sqrt(5) + 1)*cos(x) - 4)*sin(x) - sqrt(2*sqrt(-2/
5*sqrt(5) - 1) - 2)*log(-(3*sqrt(5) - 5)*sqrt(-2/5*sqrt(5) - 1)*sqrt(2*sqr
t(-2/5*sqrt(5) - 1) - 2)*sin(x) - (3*sqrt(5) - 5)*sqrt(-2/5*sqrt(5) - 1)*c
os(x) - (sqrt(5) + 1)*cos(x) - 4)*sin(x) + sqrt(-2*sqrt(-2/5*sqrt(5) - 1)
- 2)*log((3*sqrt(5) - 5)*sqrt(-2/5*sqrt(5) - 1)*sqrt(-2*sqrt(-2/5*sqrt(5)
- 1) - 2)*sin(x) - (3*sqrt(5) - 5)*sqrt(-2/5*sqrt(5) - 1)*cos(x) + (sqrt(5
) + 1)*cos(x) + 4)*sin(x) - sqrt(-2*sqrt(-2/5*sqrt(5) - 1) - 2)*log(-(3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1-\cos^5(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1-cos(x)**5),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{1-\cos^5(x)}\,dx = \int -\frac{1}{\cos{(x)}^5 - 1}\,dx$$

input
```
integrate(1/(1-cos(x)^5),x, algorithm="maxima")
```

output
```
1/5*(5*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(2/5*((cos(7*x) + 4*c
os(6*x) + 15*cos(5*x) + 40*cos(4*x) + 15*cos(3*x) + 4*cos(2*x) + cos(x))*c
os(8*x) + (16*cos(6*x) + 44*cos(5*x) + 110*cos(4*x) + 44*cos(3*x) + 16*cos
(2*x) + 4*cos(x) + 1)*cos(7*x) + 2*cos(7*x)^2 + 4*(44*cos(5*x) + 110*cos(4
*x) + 44*cos(3*x) + 16*cos(2*x) + 4*cos(x) + 1)*cos(6*x) + 32*cos(6*x)^2 +
 (1010*cos(4*x) + 420*cos(3*x) + 176*cos(2*x) + 44*cos(x) + 15)*cos(5*x) +
 210*cos(5*x)^2 + 10*(101*cos(3*x) + 44*cos(2*x) + 11*cos(x) + 4)*cos(4*x)
 + 1200*cos(4*x)^2 + (176*cos(2*x) + 44*cos(x) + 15)*cos(3*x) + 210*cos(3*
x)^2 + 4*(4*cos(x) + 1)*cos(2*x) + 32*cos(2*x)^2 + 2*cos(x)^2 + (sin(7*x)
+ 4*sin(6*x) + 15*sin(5*x) + 40*sin(4*x) + 15*sin(3*x) + 4*sin(2*x) + sin(
x))*sin(8*x) + 2*(8*sin(6*x) + 22*sin(5*x) + 55*sin(4*x) + 22*sin(3*x) + 8
*sin(2*x) + 2*sin(x))*sin(7*x) + 2*sin(7*x)^2 + 8*(22*sin(5*x) + 55*sin(4*
x) + 22*sin(3*x) + 8*sin(2*x) + 2*sin(x))*sin(6*x) + 32*sin(6*x)^2 + 2*(50
5*sin(4*x) + 210*sin(3*x) + 88*sin(2*x) + 22*sin(x))*sin(5*x) + 210*sin(5*
x)^2 + 10*(101*sin(3*x) + 44*sin(2*x) + 11*sin(x))*sin(4*x) + 1200*sin(4*x
)^2 + 44*(4*sin(2*x) + sin(x))*sin(3*x) + 210*sin(3*x)^2 + 32*sin(2*x)^2 +
 16*sin(2*x)*sin(x) + 2*sin(x)^2 + cos(x))/(2*(2*cos(7*x) + 8*cos(6*x) + 1
4*cos(5*x) + 30*cos(4*x) + 14*cos(3*x) + 8*cos(2*x) + 2*cos(x) + 1)*cos(8*
x) + cos(8*x)^2 + 4*(8*cos(6*x) + 14*cos(5*x) + 30*cos(4*x) + 14*cos(3*x)
+ 8*cos(2*x) + 2*cos(x) + 1)*cos(7*x) + 4*cos(7*x)^2 + 16*(14*cos(5*x) ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1121 vs. $2(137) = 274$.

Time = 1.13 (sec) , antiderivative size = 1121, normalized size of antiderivative = 5.47

$$\int \frac{1}{1-\cos^5(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-cos(x)^5),x, algorithm="giac")
```

output
```
1/100*sqrt(1/5)*(pi + 4*arctan(1/992900017033721756840 0*(35029730487952382
38665*sqrt(5)*(sqrt(5) + 5) + 2978700051101165270 52*sqrt(5)*sqrt(10*sqrt(5
) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) + 198580003406744351368 0*sqrt(5
)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 496450008516860878420*sqrt(10*sqrt(
5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 175148652439761911933 25*sqrt
(5) - 3971600068134887027360*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 17514865
243976191193325)*tan(1/2*x) - 1))*sqrt(2*sqrt(5) + 5)*sqrt(5*sqrt(10*sqrt(
5) + 50) - 25)/(2*sqrt(1/10)*sqrt(sqrt(5) + 5) - 1) - 1/100*sqrt(1/5)*(pi
+ 4*arctan(-1/9929000170337217568400*(35029730487952382 38665*sqrt(5)*(sqrt
(5) + 5) + 2978700051101165270 52*sqrt(5)*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt
(10*sqrt(5) + 50) - 25) + 1985800034067443513680*sqrt(5)*sqrt(5*sqrt(10*sq
rt(5) + 50) - 25) - 496450008516860878420*sqrt(10*sqrt(5) + 50)*sqrt(5*sqr
t(10*sqrt(5) + 50) - 25) - 17514865243976191193325*sqrt(5) - 3971600068134
887027360*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 17514865243976191193325)*ta
n(1/2*x) - 1))*sqrt(2*sqrt(5) + 5)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25)/(2*s
qrt(1/10)*sqrt(sqrt(5) + 5) - 1) + 1/50*sqrt(5*sqrt(10*sqrt(5) + 50) - 25)
*log(403804341790951405519072825116416445147381760000*(3*sqrt(5)*sqrt(10*s
qrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) + 20*sqrt(5)*sqrt(5*sqrt(1
0*sqrt(5) + 50) - 25) - 5*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 5
0) - 25) + 100*tan(1/2*x))^2 + 403804341790951405519072825116416445147 3...
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.97

$$\int \frac{1}{1-\cos^5(x)}\,dx$$
$$= 2\,\mathrm{atanh}\left(\frac{50\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}-20\sqrt{5}\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\sqrt{5}\sqrt{-\frac{2\sqrt{5}}{5}-1}+2\sqrt{5}-10\sqrt{-\frac{2\sqrt{5}}{5}-1}-5}\right)\sqrt{\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}$$
$$-2\,\mathrm{atanh}\left(\frac{50\tan\left(\frac{x}{2}\right)\sqrt{-\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}-20\sqrt{5}\tan\left(\frac{x}{2}\right)\sqrt{-\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\sqrt{5}\sqrt{-\frac{2\sqrt{5}}{5}-1}-2\sqrt{5}-10\sqrt{-\frac{2\sqrt{5}}{5}-1}+5}\right)\sqrt{-\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}$$
$$-\frac{\cot\left(\frac{x}{2}\right)}{5}$$
$$+2\,\mathrm{atanh}\left(\frac{50\tan\left(\frac{x}{2}\right)\sqrt{-\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}+20\sqrt{5}\tan\left(\frac{x}{2}\right)\sqrt{-\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\sqrt{5}\sqrt{\frac{2\sqrt{5}}{5}-1}-2\sqrt{5}+10\sqrt{\frac{2\sqrt{5}}{5}-1}-5}\right)\sqrt{-\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}$$
$$-2\,\mathrm{atanh}\left(\frac{50\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}+20\sqrt{5}\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\sqrt{5}\sqrt{\frac{2\sqrt{5}}{5}-1}+2\sqrt{5}+10\sqrt{\frac{2\sqrt{5}}{5}-1}+5}\right)\sqrt{\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}$$

input
```
int(-1/(cos(x)^5 - 1),x)
```

output
```
2*atanh((50*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 20*5^
(1/2)*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(1/2)*(
- (2*5^(1/2))/5 - 1)^(1/2) + 2*5^(1/2) - 10*(- (2*5^(1/2))/5 - 1)^(1/2) -
5))*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((50*tan(x/2)*(
- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 20*5^(1/2)*tan(x/2)*(- (-
 (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(1/2)*(- (2*5^(1/2))/5 -
1)^(1/2) - 2*5^(1/2) - 10*(- (2*5^(1/2))/5 - 1)^(1/2) + 5))*(- (- (2*5^(1/
2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - cot(x/2)/5 + 2*atanh((50*tan(x/2)*(- (
(2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) + 20*5^(1/2)*tan(x/2)*(- ((2*5^(
1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2)
- 2*5^(1/2) + 10*((2*5^(1/2))/5 - 1)^(1/2) - 5))*(- ((2*5^(1/2))/5 - 1)^(1
/2)/50 - 1/50)^(1/2) - 2*atanh((50*tan(x/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50
- 1/50)^(1/2) + 20*5^(1/2)*tan(x/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^
(1/2))/(5*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2) + 2*5^(1/2) + 10*((2*5^(1/2))/
5 - 1)^(1/2) + 5))*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2)
```

Reduce [F]

$$\int \frac{1}{1-\cos^5(x)}\,dx = \frac{12\left(\int \frac{\tan\left(\frac{x}{2}\right)^6}{\tan\left(\frac{x}{2}\right)^8+10\tan\left(\frac{x}{2}\right)^4+5}dx\right)\tan\left(\frac{x}{2}\right)+20\left(\int \frac{\tan\left(\frac{x}{2}\right)^2}{\tan\left(\frac{x}{2}\right)^8+10\tan\left(\frac{x}{2}\right)^4+5}dx\right)\tan\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)x-1}{5\tan\left(\frac{x}{2}\right)}$$

input
```
int(1/(1-cos(x)^5),x)
```

output
```
(12*int(tan(x/2)**6/(tan(x/2)**8 + 10*tan(x/2)**4 + 5),x)*tan(x/2) + 20*in
t(tan(x/2)**2/(tan(x/2)**8 + 10*tan(x/2)**4 + 5),x)*tan(x/2) + 2*tan(x/2)*
x - 1)/(5*tan(x/2))
```

3.8 $\int \frac{1}{1+\cos^2(x)}\,dx$

Optimal result 106
Mathematica [A] (verified) 106
Rubi [A] (verified) 107
Maple [A] (verified) 108
Fricas [A] (verification not implemented) 108
Sympy [A] (verification not implemented) 109
Maxima [A] (verification not implemented) 109
Giac [A] (verification not implemented) 109
Mupad [B] (verification not implemented) 110
Reduce [B] (verification not implemented) 110

Optimal result

Integrand size = 8, antiderivative size = 34

$$\int \frac{1}{1+\cos^2(x)}\,dx=\frac{x}{\sqrt{2}}-\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}$$

output
```
1/2*x*2^(1/2)-1/2*arctan(cos(x)*sin(x)/(1+2^(1/2)+cos(x)^2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos^2(x)}\,dx=\frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[(1 + Cos[x]^2)^(-1),x]
```

output
```
ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^2(x)+1} \, dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx$$
$$\Big\downarrow \ 3660$$
$$-\int \frac{1}{2\cot^2(x)+1}d\cot(x)$$
$$\Big\downarrow \ 216$$
$$-\frac{\arctan\left(\sqrt{2}\cot(x)\right)}{\sqrt{2}}$$

input
```
Int[(1 + Cos[x]^2)^(-1),x]
```

output
```
-(ArcTan[Sqrt[2]*Cot[x]]/Sqrt[2])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{2}$	14
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)}{4}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)}{4}$	40

input
```
int(1/(1+cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*arctan(1/2*tan(x)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{1+\cos^2(x)}\,dx = -\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos{(x)}^2-\sqrt{2}}{4\,\cos{(x)}\sin{(x)}}\right)$$

input
```
integrate(1/(1+cos(x)^2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2}$$

input
```
integrate(1/(1+cos(x)**2),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/2 + sqrt(
2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.38

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)$$

input
```
integrate(1/(1+cos(x)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)$$

input
```
integrate(1/(1+cos(x)^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1)))
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{\sqrt{2}\,(x-\operatorname{atan}(\tan(x)))}{2} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{2}$$

input
```
int(1/(cos(x)^2 + 1),x)
```

output
```
(2^(1/2)*(x - atan(tan(x))))/2 + (2^(1/2)*atan((2^(1/2)*tan(x))/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{\sqrt{2}\left(-atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+atan\left(\frac{\sqrt{2}+2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\right)}{2}$$

input
```
int(1/(1+cos(x)^2),x)
```

output
```
(sqrt(2)*( - atan((sqrt(2) - 2*tan(x/2))/sqrt(2)) + atan((sqrt(2) + 2*tan(
x/2))/sqrt(2))))/2
```

3.9 $\int \frac{1}{1+\cos^4(x)}\,dx$

Optimal result . . . 111
Mathematica [C] (verified) . . . 112
Rubi [A] (verified) . . . 112
Maple [C] (verified) . . . 115
Fricas [A] (verification not implemented) . . . 116
Sympy [F(-1)] . . . 117
Maxima [F] . . . 117
Giac [A] (verification not implemented) . . . 118
Mupad [B] (verification not implemented) . . . 119
Reduce [F] . . . 120

Optimal result

Integrand size = 8, antiderivative size = 262

$$\begin{aligned}\int \frac{1}{1+\cos^4(x)}\,dx =& \frac{1}{2}\sqrt{1+\sqrt{2}}x \\ &-\frac{1}{4}\sqrt{1+\sqrt{2}}\arctan\left(\frac{(2-\sqrt{2})\cos(x)\sin(x)-\sqrt{-1+\sqrt{2}}(1-2\sin^2(x))}{2+\sqrt{1+\sqrt{2}}+2\sqrt{-1+\sqrt{2}}\cos(x)\sin(x)-(2-\sqrt{2})\sin^2(x)}\right) \\ &-\frac{1}{4}\sqrt{1+\sqrt{2}}\arctan\left(\frac{(2-\sqrt{2})\cos(x)\sin(x)+\sqrt{-1+\sqrt{2}}(1-2\sin^2(x))}{2+\sqrt{1+\sqrt{2}}-2\sqrt{-1+\sqrt{2}}\cos(x)\sin(x)-(2-\sqrt{2})\sin^2(x)}\right) \\ &-\frac{1}{4}\sqrt{-1+\sqrt{2}}\text{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\cot(x)}{1+\sqrt{2}\cot^2(x)}\right)\end{aligned}$$

output
```
1/2*(1+2^(1/2))^(1/2)*x-1/4*(1+2^(1/2))^(1/2)*arctan(((2-2^(1/2))*cos(x)*s
in(x)-(2^(1/2)-1)^(1/2)*(1-2*sin(x)^2))/(2+(1+2^(1/2))^(1/2)+2*(2^(1/2)-1)
^(1/2)*cos(x)*sin(x)-(2-2^(1/2))*sin(x)^2))-1/4*(1+2^(1/2))^(1/2)*arctan((
(2-2^(1/2))*cos(x)*sin(x)+(2^(1/2)-1)^(1/2)*(1-2*sin(x)^2))/(2+(1+2^(1/2))
^(1/2)-2*(2^(1/2)-1)^(1/2)*cos(x)*sin(x)-(2-2^(1/2))*sin(x)^2))-1/4*(2^(1/
2)-1)^(1/2)*arctanh((-2+2*2^(1/2))^(1/2)*cot(x)/(1+2^(1/2)*cot(x)^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.17

$$\int \frac{1}{1+\cos^4(x)}\,dx=\frac{\arctan\left(\frac{\tan(x)}{\sqrt{1-i}}\right)}{2\sqrt{1-i}}+\frac{\arctan\left(\frac{\tan(x)}{\sqrt{1+i}}\right)}{2\sqrt{1+i}}$$

input
```
Integrate[(1 + Cos[x]^4)^(-1),x]
```

output
```
ArcTan[Tan[x]/Sqrt[1 - I]]/(2*Sqrt[1 - I]) + ArcTan[Tan[x]/Sqrt[1 + I]]/(2
*Sqrt[1 + I])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3688, 1483, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^4(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^4+1}dx$$

$$\Big\downarrow \text{3688}$$

$$-\int \frac{\cot^2(x)+1}{2\cot^4(x)+2\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{1483}$$

$$-\frac{\int \frac{2\sqrt{-1+\sqrt{2}}-\left(2-\sqrt{2}\right)\cot(x)}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\int \frac{\left(2-\sqrt{2}\right)\cot(x)+2\sqrt{-1+\sqrt{2}}}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\Big\downarrow \ 1142$$

$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)-\frac{1}{4}\left(2-\sqrt{2}\right)\int -\frac{2\left(\sqrt{-1+\sqrt{2}}-2\cot(x)\right)}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)+\frac{1}{4}\left(2-\sqrt{2}\right)\int \frac{2\left(2\cot(x)+\sqrt{-1+\sqrt{2}}\right)}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)+\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{\sqrt{-1+\sqrt{2}}-2\cot(x)}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)+\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{2\cot(x)+\sqrt{-1+\sqrt{2}}}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\Big\downarrow \ 1083$$

$$-\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{\sqrt{-1+\sqrt{2}}-2\cot(x)}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-\left(4\cot(x)-2\sqrt{-1+\sqrt{2}}\right)^2-4\left(1+\sqrt{2}\right)}d\big(4\cot(x)-}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{2\cot(x)+\sqrt{-1+\sqrt{2}}}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-\left(4\cot(x)+2\sqrt{-1+\sqrt{2}}\right)^2-4\left(1+\sqrt{2}\right)}d\big(4\cot(x)+2}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\Big\downarrow \ 217$$

$$-\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{\sqrt{-1+\sqrt{2}}-2\cot(x)}{2\cot^2(x)-2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)+\frac{\arctan\left(\frac{4\cot(x)-2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}}{2\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\frac{1}{2}\left(2-\sqrt{2}\right)\int \frac{2\cot(x)+\sqrt{-1+\sqrt{2}}}{2\cot^2(x)+2\sqrt{-1+\sqrt{2}}\cot(x)+\sqrt{2}}d\cot(x)+\frac{\arctan\left(\frac{4\cot(x)+2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{\frac{\arctan\left(\frac{4\cot(x)-2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}-\frac{1}{4}(2-\sqrt{2})\log\left(2\cot^2(x)-2\sqrt{\sqrt{2}-1}\cot(x)+\sqrt{2}\right)}{2\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\frac{\arctan\left(\frac{4\cot(x)+2\sqrt{\sqrt{2}-1}}{2\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}+\frac{1}{4}(2-\sqrt{2})\log\left(\sqrt{2}\cot^2(x)+\sqrt{2\left(\sqrt{2}-1\right)}\cot(x)+1\right)}{2\sqrt{2\left(\sqrt{2}-1\right)}}$$

input
```
Int[(1 + Cos[x]^4)^(-1),x]
```

output
```
-1/2*(ArcTan[(-2*Sqrt[-1 + Sqrt[2]] + 4*Cot[x])/(2*Sqrt[1 + Sqrt[2]])]/Sqr
t[2] - ((2 - Sqrt[2])*Log[Sqrt[2] - 2*Sqrt[-1 + Sqrt[2]]*Cot[x] + 2*Cot[x]
^2])/4)/Sqrt[2*(-1 + Sqrt[2])] - (ArcTan[(2*Sqrt[-1 + Sqrt[2]] + 4*Cot[x])
/(2*Sqrt[1 + Sqrt[2]])]/Sqrt[2] + ((2 - Sqrt[2])*Log[1 + Sqrt[2*(-1 + Sqrt
[2])]*Cot[x] + Sqrt[2]*Cot[x]^2])/4)/(2*Sqrt[2*(-1 + Sqrt[2])])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.48

method	result
risch	$\frac{\sqrt{-2-2i}\,\ln\left(e^{2ix}-i\sqrt{-2-2i}+\sqrt{-2-2i}+1+2i\right)}{8}-\frac{\sqrt{-2-2i}\,\ln\left(e^{2ix}+i\sqrt{-2-2i}-\sqrt{-2-2i}+1+2i\right)}{8}+\frac{\sqrt{-2+2i}\,\ln\left(e^{2ix}-i\sqrt{-2+2\right.}{8}$
default	$-\frac{\sqrt{2}\left(\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(\tan(x)^2+\tan(x)\sqrt{-2+2\sqrt{2}}+\sqrt{2}\right)}{2}+\frac{2\left(-1-\sqrt{2}\right)\arctan\left(\frac{2\tan(x)+\sqrt{-2+2\sqrt{2}}}{\sqrt{2\sqrt{2}+2}}\right)}{\sqrt{2\sqrt{2}+2}}\right)}{8}-\frac{\sqrt{2}\left(-\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(\tan(x)^2-\right.}{2}\right.}{}$

input
```
int(1/(1+cos(x)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-2-2*I)^(1/2)*ln(exp(2*I*x)-I*(-2-2*I)^(1/2)+(-2-2*I)^(1/2)+1+2*I)-1/
8*(-2-2*I)^(1/2)*ln(exp(2*I*x)+I*(-2-2*I)^(1/2)-(-2-2*I)^(1/2)+1+2*I)+1/8*
(-2+2*I)^(1/2)*ln(exp(2*I*x)-I*(-2+2*I)^(1/2)-(-2+2*I)^(1/2)+1-2*I)-1/8*(-
2+2*I)^(1/2)*ln(exp(2*I*x)+I*(-2+2*I)^(1/2)+(-2+2*I)^(1/2)+1-2*I)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.39

$$\int \frac{1}{1+\cos^4(x)}\,dx$$
$$= \frac{1}{8}\sqrt{\sqrt{2}+1}\arctan\left(\frac{\left(\left(2\sqrt{2}\cos(x)^7-4\cos(x)^5-\left(3\sqrt{2}-2\right)\cos(x)^3+\sqrt{2}\cos(x)\right)\sin(x)+\left(2\left(\sqrt{2}\right.\right.}{4\cos(x)^8-8\cos(x)^6+4}\right.$$
$$-\frac{1}{8}\sqrt{\sqrt{2}+1}\arctan\left(-\frac{\left(\left(2\sqrt{2}\cos(x)^7-4\cos(x)^5-\left(3\sqrt{2}-2\right)\cos(x)^3+\sqrt{2}\cos(x)\right)\sin(x)-\left(2\left(\right.\right.}{4\cos(x)^8-8\cos(x)^6}\right.$$
$$+\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(-\frac{1}{2}\left(4\sqrt{2}-5\right)\cos(x)^4+2\left(\sqrt{2}-1\right)\cos(x)^2+\left(\left(\sqrt{2}-2\right)\cos(x)^3-\sqrt{2}\cos(x)\right)\sqrt{\sqrt{2}-1}\sin(x)+\frac{1}{2}\right)$$
$$-\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(-\frac{1}{2}\left(4\sqrt{2}-5\right)\cos(x)^4+2\left(\sqrt{2}-1\right)\cos(x)^2-\left(\left(\sqrt{2}-2\right)\cos(x)^3-\sqrt{2}\cos(x)\right)\sqrt{\sqrt{2}-1}\sin(x)+\frac{1}{2}\right)$$

input
```
integrate(1/(1+cos(x)^4),x, algorithm="fricas")
```

output
```
1/8*sqrt(sqrt(2) + 1)*arctan(((2*sqrt(2)*cos(x)^7 - 4*cos(x)^5 - (3*sqrt(2
) - 2)*cos(x)^3 + sqrt(2)*cos(x))*sin(x) + (2*(sqrt(2) - 1)*cos(x)^6 - 2*(
sqrt(2) - 1)*cos(x)^4 - cos(x)^2)*sqrt(sqrt(2) - 1))*sqrt(sqrt(2) + 1)/(4*
cos(x)^8 - 8*cos(x)^6 + 4*cos(x)^2 - 1)) - 1/8*sqrt(sqrt(2) + 1)*arctan(-(
(2*sqrt(2)*cos(x)^7 - 4*cos(x)^5 - (3*sqrt(2) - 2)*cos(x)^3 + sqrt(2)*cos(
x))*sin(x) - (2*(sqrt(2) - 1)*cos(x)^6 - 2*(sqrt(2) - 1)*cos(x)^4 - cos(x)
^2)*sqrt(sqrt(2) - 1))*sqrt(sqrt(2) + 1)/(4*cos(x)^8 - 8*cos(x)^6 + 4*cos(
x)^2 - 1)) + 1/16*sqrt(sqrt(2) - 1)*log(-1/2*(4*sqrt(2) - 5)*cos(x)^4 + 2*
(sqrt(2) - 1)*cos(x)^2 + ((sqrt(2) - 2)*cos(x)^3 - sqrt(2)*cos(x))*sqrt(sq
rt(2) - 1)*sin(x) + 1/2) - 1/16*sqrt(sqrt(2) - 1)*log(-1/2*(4*sqrt(2) - 5)
*cos(x)^4 + 2*(sqrt(2) - 1)*cos(x)^2 - ((sqrt(2) - 2)*cos(x)^3 - sqrt(2)*c
os(x))*sqrt(sqrt(2) - 1)*sin(x) + 1/2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\cos^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+cos(x)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{1+\cos^4(x)}\,dx = \int \frac{1}{\cos{(x)}^4+1}\,dx$$

input
```
integrate(1/(1+cos(x)^4),x, algorithm="maxima")
```

output
```
integrate(1/(cos(x)^4 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.65

$$\int \frac{1}{1+\cos^4(x)}\,dx$$
$$= \frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\,\tan(x)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}$$
$$+\frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\,\tan(x)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}$$
$$-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2+2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{2}\right)$$
$$+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^2-2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{2}\right)$$

input
```
integrate(1/(1+cos(x)^4),x, algorithm="giac")
```

output
```
1/4*(pi*floor(x/pi + 1/2) + arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(-sqrt(2) + 2)
 + 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) + 1/4*(pi*floor(x/pi +
1/2) + arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(-sqrt(2) + 2) - 2*tan(x))/sqrt(sq
rt(2) + 2)))*sqrt(sqrt(2) + 1) - 1/8*sqrt(sqrt(2) - 1)*log(tan(x)^2 + 2^(1
/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2)) + 1/8*sqrt(sqrt(2) - 1)*log(tan(x
)^2 - 2^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2))
```

Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\cos^4(x)}\,dx = \operatorname{atanh}\left(\frac{4\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1} + \frac{4\sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}} - 2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right) - \operatorname{atanh}\left(\frac{4\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1} - \frac{4\sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}} + 2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)$$

input
```
int(1/(cos(x)^4 + 1),x)
```

output
```
atanh((4*2^(1/2)*tan(x)*(- 2^(1/2)/64 - 1/64)^(1/2))/(64*(2^(1/2)/64 - 1/6
4)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) - 1) + (4*2^(1/2)*tan(x)*(2^(1/2)/64
- 1/64)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) -
 1))*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*(2^(1/2)/64 - 1/64)^(1/2)) - atanh
((4*2^(1/2)*tan(x)*(- 2^(1/2)/64 - 1/64)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1
/2)*(- 2^(1/2)/64 - 1/64)^(1/2) + 1) - (4*2^(1/2)*tan(x)*(2^(1/2)/64 - 1/6
4)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) + 1))*
(2*(- 2^(1/2)/64 - 1/64)^(1/2) + 2*(2^(1/2)/64 - 1/64)^(1/2))
```

Reduce [F]

$$\int \frac{1}{1+\cos^4(x)}\,dx = \int \frac{1}{\cos{(x)}^4+1}dx$$

input
```
int(1/(1+cos(x)^4),x)
```

output
```
int(1/(cos(x)**4 + 1),x)
```

3.10 $\int \frac{1}{1+\cos^6(x)}\,dx$

Optimal result . . . 121
Mathematica [C] (verified) . . . 121
Rubi [A] (verified) . . . 122
Maple [A] (verified) . . . 124
Fricas [A] (verification not implemented) . . . 124
Sympy [F(-1)] . . . 125
Maxima [A] (verification not implemented) . . . 125
Giac [A] (verification not implemented) . . . 126
Mupad [B] (verification not implemented) . . . 127
Reduce [F] . . . 127

Optimal result

Integrand size = 8, antiderivative size = 124

$$\int \frac{1}{1+\cos^6(x)}\,dx = \frac{x}{3\sqrt{2}}+\frac{x}{\sqrt{3}}-\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{3\sqrt{2}}-\frac{\arctan\left(\frac{1-2\sin^2(x)}{2+\sqrt{3}-2\cos(x)\sin(x)}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1-2\sin^2(x)}{2+\sqrt{3}+2\cos(x)\sin(x)}\right)}{2\sqrt{3}}-\frac{1}{6}\text{arctanh}(\cos(x)\sin(x))$$

output
```
1/6*x*2^(1/2)+1/3*x*3^(1/2)-1/6*arctan(cos(x)*sin(x)/(1+2^(1/2)+cos(x)^2))
*2^(1/2)-1/6*arctan((1-2*sin(x)^2)/(2+3^(1/2)-2*cos(x)*sin(x)))*3^(1/2)+1/
6*arctan((1-2*sin(x)^2)/(2+3^(1/2)+2*cos(x)*sin(x)))*3^(1/2)-1/6*arctanh(c
os(x)*sin(x))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.35

$$\int \frac{1}{1+\cos^6(x)}\,dx = \frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{3\sqrt{2}} - \frac{\left(\sqrt{1-i\sqrt{3}}\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)\tan(x)\right)+\sqrt{1+i\sqrt{3}}\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)\tan(x)\right)\right)(-3+\cos(2x))}{3\left(\sqrt{3-3i\sqrt{3}}+\sqrt{3+3i\sqrt{3}}-i\left(\sqrt{1-i\sqrt{3}}-\sqrt{1+i\sqrt{3}}\right)\cos(2x)\right)}$$

input
```
Integrate[(1 + Cos[x]^6)^(-1),x]
```

output
```
ArcTan[Tan[x]/Sqrt[2]]/(3*Sqrt[2]) - ((Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + S
qrt[3])*Tan[x])/2] + Sqrt[1 + I*Sqrt[3]]*ArcTan[((I + Sqrt[3])*Tan[x])/2])
*(-3 + Cos[2*x]))/(3*(Sqrt[3 - (3*I)*Sqrt[3]] + Sqrt[3 + (3*I)*Sqrt[3]] -
I*(Sqrt[1 - I*Sqrt[3]] - Sqrt[1 + I*Sqrt[3]])*Cos[2*x]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^6(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^6+1}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{3}\int \frac{1}{\cos^2(x)+1}dx+\frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\cos^2(x)}dx+\frac{1}{3}\int \frac{1}{(-1)^{2/3}\cos^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\int \frac{1}{(-1)^{2/3}\sin\left(x+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \; 3660$$

$$-\frac{1}{3}\int \frac{1}{2\cot^2(x)+1}d\cot(x)-\frac{1}{3}\int \frac{1}{\left(1-\sqrt[3]{-1}\right)\cot^2(x)+1}d\cot(x)-\frac{1}{3}\int \frac{1}{\left(1+(-1)^{2/3}\right)\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \; 216$$

$$-\frac{\arctan\left(\sqrt{2}\cot(x)\right)}{3\sqrt{2}}-\frac{\arctan\left(\sqrt{1-\sqrt[3]{-1}}\cot(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}-\frac{\arctan\left(\sqrt{1+(-1)^{2/3}}\cot(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

input
```
Int[(1 + Cos[x]^6)^(-1),x]
```

output
```
-1/3*ArcTan[Sqrt[2]*Cot[x]]/Sqrt[2] - ArcTan[Sqrt[1 - (-1)^(1/3)]*Cot[x]]/
(3*Sqrt[1 - (-1)^(1/3)]) - ArcTan[Sqrt[1 + (-1)^(2/3)]*Cot[x]]/(3*Sqrt[1 +
 (-1)^(2/3)])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

method	result
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{6}+\frac{\ln\left(\tan(x)^2-\tan(x)+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2\tan(x)-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln\left(\tan(x)^2+\tan(x)+1\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\right.}{}$
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}+3\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}+3\right)}{12}-\frac{\ln\left(e^{2ix}+2i+i\sqrt{3}\right)}{12}+\frac{i\ln\left(e^{2ix}+2i+i\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(e^{2ix}+2i-i\sqrt{3}\right)}{12}-\frac{i}{}$

input
```
int(1/(1+cos(x)^6),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(1/2*tan(x)*2^(1/2))+1/12*ln(tan(x)^2-tan(x)+1)+1/6*3^(1
/2)*arctan(1/3*(2*tan(x)-1)*3^(1/2))-1/12*ln(tan(x)^2+tan(x)+1)+1/6*3^(1/2
)*arctan(1/3*(2*tan(x)+1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{1}{1+\cos^6(x)}\,dx &= \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{4\,\sqrt{3}\cos(x)\sin(x)+\sqrt{3}}{3\,\left(2\,\cos(x)^2-1\right)}\right)\\ &\quad+\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{4\,\sqrt{3}\cos(x)\sin(x)-\sqrt{3}}{3\,\left(2\,\cos(x)^2-1\right)}\right)\\ &\quad-\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos(x)^2-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)\\ &\quad-\frac{1}{24}\,\log\left(-\cos(x)^4+\cos(x)^2+2\,\cos(x)\sin(x)+1\right)\\ &\quad+\frac{1}{24}\,\log\left(-\cos(x)^4+\cos(x)^2-2\,\cos(x)\sin(x)+1\right)\end{aligned}$$

input
```
integrate(1/(1+cos(x)^6),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 -
1)) + 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x
)^2 - 1)) - 1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)
*sin(x))) - 1/24*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) + 1/24*lo
g(-cos(x)^4 + cos(x)^2 - 2*cos(x)*sin(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\cos^6(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+cos(x)**6),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\begin{aligned}\int \frac{1}{1+\cos^6(x)}\,dx &= \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)+1)\right) \\ &\quad+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)-1)\right) \\ &\quad+\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)-\frac{1}{12}\log\left(\tan(x)^2+\tan(x)+1\right) \\ &\quad+\frac{1}{12}\log\left(\tan(x)^2-\tan(x)+1\right)\end{aligned}$$

input
```
integrate(1/(1+cos(x)^6),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) + 1)) + 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*tan(x) - 1)) + 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/12*log(
tan(x)^2 + tan(x) + 1) + 1/12*log(tan(x)^2 - tan(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.49

$$\int \frac{1}{1+\cos^6(x)}\,dx = \frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)+\cos(2x)-2\sin(2x)+1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)-\sin(2x)+2}\right)\right) + \frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-\cos(2x)-2\sin(2x)-1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)+\sin(2x)+2}\right)\right) + \frac{1}{6}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right) - \frac{1}{12}\log\left(\tan(x)^2+\tan(x)+1\right) + \frac{1}{12}\log\left(\tan(x)^2-\tan(x)+1\right)$$

input
```
integrate(1/(1+cos(x)^6),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) + cos(2*x) - 2*sin(2*x) + 1)/(s
qrt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) - sin(2*x) + 2))) + 1/6*sqrt(3)*(x
+ arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(sqrt(3)*cos(2*x)
 + sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) + 1/6*sqrt(2)*(x + arctan(-(sqrt
(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - cos(2*x) + 1))) - 1
/12*log(tan(x)^2 + tan(x) + 1) + 1/12*log(tan(x)^2 - tan(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{1}{1+\cos^6(x)} dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{6} + \mathrm{atan}\left(\frac{\sqrt{3}\tan(x)}{2} + \frac{\tan(x)\ 1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6} + \frac{1}{6}\mathrm{i}\right) - \mathrm{atan}\left(-\frac{\sqrt{3}\tan(x)}{2} + \frac{\tan(x)\ 1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6} - \frac{1}{6}\mathrm{i}\right) + \frac{(x - \mathrm{atan}(\tan(x)))\left(\frac{\pi\sqrt{2}}{6} + \pi\left(\frac{\sqrt{3}}{6} - \frac{1}{6}\mathrm{i}\right) + \pi\left(\frac{\sqrt{3}}{6} + \frac{1}{6}\mathrm{i}\right)\right)}{\pi}$$

input
```
int(1/(cos(x)^6 + 1),x)
```

output
```
atan((tan(x)*1i)/2 + (3^(1/2)*tan(x))/2)*(3^(1/2)/6 + 1i/6) - atan((tan(x)
*1i)/2 - (3^(1/2)*tan(x))/2)*(3^(1/2)/6 - 1i/6) + (2^(1/2)*atan((2^(1/2)*t
an(x))/2))/6 + ((x - atan(tan(x)))*((2^(1/2)*pi)/6 + pi*(3^(1/2)/6 - 1i/6)
 + pi*(3^(1/2)/6 + 1i/6)))/pi
```

Reduce [F]

$$\int \frac{1}{1+\cos^6(x)} dx = \int \frac{1}{\cos(x)^6 + 1} dx$$

input
```
int(1/(1+cos(x)^6),x)
```

output
```
int(1/(cos(x)**6 + 1),x)
```

3.11 $\int \frac{1}{1+\cos^8(x)} \, dx$

Optimal result . . . 129
Mathematica [C] (verified) . . . 130
Rubi [A] (verified) . . . 131
Maple [C] (verified) . . . 133
Fricas [A] (verification not implemented) . . . 133
Sympy [F(-1)] . . . 134
Maxima [F] . . . 135
Giac [F(-2)] . . . 135
Mupad [B] (verification not implemented) . . . 135
Reduce [F] . . . 136

Optimal result

Integrand size = 8, antiderivative size = 777

$$\int \frac{1}{1+\cos^8(x)}\,dx = \frac{1}{4}\sqrt{1+\sqrt{4-2\sqrt{2}}}x+\frac{1}{4}\sqrt{1+\sqrt{2\left(2+\sqrt{2}\right)}}x$$

$$-\frac{1}{8}\sqrt{1+\sqrt{4-2\sqrt{2}}}\arctan\left(\frac{\left(2-\sqrt{4-2\sqrt{2}}\right)\cos(x)\sin(x)-\sqrt{-1+\sqrt{4-2\sqrt{2}}}(1-2\sin^2}{2+\sqrt{1+\sqrt{4-2\sqrt{2}}}+2\sqrt{-1+\sqrt{4-2\sqrt{2}}}\cos(x)\sin(x)-\left(2-\sqrt{4-}\right.}\right.$$

$$-\frac{1}{8}\sqrt{1+\sqrt{4-2\sqrt{2}}}\arctan\left(\frac{\left(2-\sqrt{4-2\sqrt{2}}\right)\cos(x)\sin(x)+\sqrt{-1+\sqrt{4-2\sqrt{2}}}(1-2\sin^2}{2+\sqrt{1+\sqrt{4-2\sqrt{2}}}-2\sqrt{-1+\sqrt{4-2\sqrt{2}}}\cos(x)\sin(x)-\left(2-\sqrt{4-}\right.}\right.$$

$$-\frac{1}{8}\sqrt{1+\sqrt{2\left(2+\sqrt{2}\right)}}\arctan\left(\frac{\left(2-\sqrt{2\left(2+\sqrt{2}\right)}\right)\cos(x)\sin(x)-\sqrt{-1+\sqrt{2\left(2+\sqrt{2}\right)}}}{2+\sqrt{1+\sqrt{2\left(2+\sqrt{2}\right)}}+2\sqrt{-1+\sqrt{2\left(2+\sqrt{2}\right)}}\cos(x)\sin(x)-\left(2\right.}\right.$$

$$-\frac{1}{8}\sqrt{1+\sqrt{2\left(2+\sqrt{2}\right)}}\arctan\left(\frac{\left(2-\sqrt{2\left(2+\sqrt{2}\right)}\right)\cos(x)\sin(x)+\sqrt{-1+\sqrt{2\left(2+\sqrt{2}\right)}}}{2+\sqrt{1+\sqrt{2\left(2+\sqrt{2}\right)}}-2\sqrt{-1+\sqrt{2\left(2+\sqrt{2}\right)}}\cos(x)\sin(x)-\left(2\right.}\right.$$

$$-\frac{1}{8}\sqrt{-1+\sqrt{4-2\sqrt{2}}}\operatorname{arctanh}\left(\frac{\sqrt{-1+\sqrt{4-2\sqrt{2}}}\cot(x)}{\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)+\cot^2(x)}}\right)$$

$$-\frac{1}{8}\sqrt{-1+\sqrt{2\left(2+\sqrt{2}\right)}}\operatorname{arctanh}\left(\frac{\sqrt{-1+\sqrt{2\left(2+\sqrt{2}\right)}}\cot(x)}{\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)+\cot^2(x)}}\right)$$

output
```
1/4*(1+(4-2*2^(1/2))^(1/2))^(1/2)*x+1/4*(1+(4+2*2^(1/2))^(1/2))^(1/2)*x-1/
8*(1+(4-2*2^(1/2))^(1/2))^(1/2)*arctan(((2-(4-2*2^(1/2))^(1/2))*cos(x)*sin
(x)-(-1+(4-2*2^(1/2))^(1/2))^(1/2)*(1-2*sin(x)^2))/(2+(1+(4-2*2^(1/2))^(1/
2))^(1/2)+2*(-1+(4-2*2^(1/2))^(1/2))^(1/2)*cos(x)*sin(x)-(2-(4-2*2^(1/2))^
(1/2))*sin(x)^2))-1/8*(1+(4-2*2^(1/2))^(1/2))^(1/2)*arctan(((2-(4-2*2^(1/2
))^(1/2))*cos(x)*sin(x)+(-1+(4-2*2^(1/2))^(1/2))^(1/2)*(1-2*sin(x)^2))/(2+
(1+(4-2*2^(1/2))^(1/2))^(1/2)-2*(-1+(4-2*2^(1/2))^(1/2))^(1/2)*cos(x)*sin(
x)-(2-(4-2*2^(1/2))^(1/2))*sin(x)^2))-1/8*(1+(4+2*2^(1/2))^(1/2))^(1/2)*ar
ctan(((2-(4+2*2^(1/2))^(1/2))*cos(x)*sin(x)-(-1+(4+2*2^(1/2))^(1/2))^(1/2)
*(1-2*sin(x)^2))/(2+(1+(4+2*2^(1/2))^(1/2))^(1/2)+2*(-1+(4+2*2^(1/2))^(1/2
))^(1/2)*cos(x)*sin(x)-(2-(4+2*2^(1/2))^(1/2))*sin(x)^2))-1/8*(1+(4+2*2^(1
/2))^(1/2))^(1/2)*arctan(((2-(4+2*2^(1/2))^(1/2))*cos(x)*sin(x)+(-1+(4+2*2
^(1/2))^(1/2))^(1/2)*(1-2*sin(x)^2))/(2+(1+(4+2*2^(1/2))^(1/2))^(1/2)-2*(-
1+(4+2*2^(1/2))^(1/2))^(1/2)*cos(x)*sin(x)-(2-(4+2*2^(1/2))^(1/2))*sin(x)^
2))-1/8*(-1+(4-2*2^(1/2))^(1/2))^(1/2)*arctanh((-1+(4-2*2^(1/2))^(1/2))^(1
/2)*cot(x)/(1/2*(4-2*2^(1/2))^(1/2)+cot(x)^2))-1/8*(-1+(4+2*2^(1/2))^(1/2)
)^(1/2)*arctanh((-1+(4+2*2^(1/2))^(1/2))^(1/2)*cot(x)/(1/2*(4+2*2^(1/2))^(
1/2)+cot(x)^2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.18

$$\int \frac{1}{1+\cos^8(x)}\,dx$$
$$= 8\text{RootSum}\left[1+8\#1+28\#1^2+56\#1^3+326\#1^4+56\#1^5+28\#1^6+8\#1^7 + \#1^8\&, \frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{1+7\#1+21\#1^2+163\#1^3+35\#1^4+21\#1^5+7\#1^6+\#1^7}\&\right]$$

input
```
Integrate[(1 + Cos[x]^8)^(-1),x]
```

output

```
8*RootSum[1 + 8*#1 + 28*#1^2 + 56*#1^3 + 326*#1^4 + 56*#1^5 + 28*#1^6 + 8*
#1^7 + #1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^3 - I*Log[1 - 2*Cos
[2*x]*#1 + #1^2]*#1^3)/(1 + 7*#1 + 21*#1^2 + 163*#1^3 + 35*#1^4 + 21*#1^5
+ 7*#1^6 + #1^7) & ]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^8(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^8+1}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\cos^2(x)}dx+\frac{1}{4}\int \frac{1}{\sqrt[4]{-1}\cos^2(x)+1}dx+\frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\cos^2(x)}dx+\frac{1}{4}\int \frac{1}{(-1)^{3/4}\cos^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{\sqrt[4]{-1}\sin\left(x+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{(-1)^{3/4}\sin\left(x+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \text{3660}$$

$$-\frac{1}{4}\int \frac{1}{\left(1-\sqrt[4]{-1}\right)\cot^2(x)+1}d\cot(x)-\frac{1}{4}\int \frac{1}{\left(1+\sqrt[4]{-1}\right)\cot^2(x)+1}d\cot(x)-\frac{1}{4}\int \frac{1}{\left(1-(-1)^{3/4}\right)\cot^2(x)+1}d\cot(x)-\frac{1}{4}\int \frac{1}{\left(1+(-1)^{3/4}\right)\cot^2(x)+1}d\cot(x)$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\sqrt{1-\sqrt[4]{-1}}\cot(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}-\frac{\arctan\left(\sqrt{1+\sqrt[4]{-1}}\cot(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}-\frac{\arctan\left(\sqrt{1-(-1)^{3/4}}\cot(x)\right)}{4\sqrt{1-(-1)^{3/4}}}-\frac{\arctan\left(\sqrt{1+(-1)^{3/4}}\cot(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

input
```
Int[(1 + Cos[x]^8)^(-1),x]
```

output
```
-1/4*ArcTan[Sqrt[1 - (-1)^(1/4)]*Cot[x]]/Sqrt[1 - (-1)^(1/4)] - ArcTan[Sqr
t[1 + (-1)^(1/4)]*Cot[x]]/(4*Sqrt[1 + (-1)^(1/4)]) - ArcTan[Sqrt[1 - (-1)^
(3/4)]*Cot[x]]/(4*Sqrt[1 - (-1)^(3/4)]) - ArcTan[Sqrt[1 + (-1)^(3/4)]*Cot[
x]]/(4*Sqrt[1 + (-1)^(3/4)])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.70 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.09

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+4\_Z^{6}+6\_Z^{4}+4\_Z^{2}+2\right)}\frac{\left(\_R^{6}+3\_R^{4}+3\_R^{2}+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^{7}+3\_R^{5}+3\_R^{3}+\_R}\right)}{8}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(8192\_Z^{4}+(128-128i)\_Z^{2}+1-i\right)}\_R\ln\left(\mathrm{e}^{2ix}+(1024+1024i)\_R^{3}+(-128+128i)\_R^{2}+\right.\right.$

input
```
int(1/(1+cos(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=Ro
otOf(_Z^8+4*_Z^6+6*_Z^4+4*_Z^2+2))
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.13

$$\int \frac{1}{1+\cos^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+cos(x)^8),x, algorithm="fricas")
```

output
```
-1/16*sqrt(-1/2*sqrt(2*sqrt(2) - 3) - 1/2)*log(2*(sqrt(2) + 1)*cos(x)^2 +
(2*(sqrt(2) + 2)*cos(x)^2 - sqrt(2) - 2)*sqrt(2*sqrt(2) - 3) + 2*(sqrt(2*s
qrt(2) - 3)*(sqrt(2) + 2)*cos(x)*sin(x) + (sqrt(2) + 2)*cos(x)*sin(x))*sqr
t(-1/2*sqrt(2*sqrt(2) - 3) - 1/2) - sqrt(2)) + 1/16*sqrt(-1/2*sqrt(2*sqrt(
2) - 3) - 1/2)*log(2*(sqrt(2) + 1)*cos(x)^2 + (2*(sqrt(2) + 2)*cos(x)^2 -
sqrt(2) - 2)*sqrt(2*sqrt(2) - 3) - 2*(sqrt(2*sqrt(2) - 3)*(sqrt(2) + 2)*co
s(x)*sin(x) + (sqrt(2) + 2)*cos(x)*sin(x))*sqrt(-1/2*sqrt(2*sqrt(2) - 3) -
 1/2) - sqrt(2)) - 1/16*sqrt(1/2*sqrt(2*sqrt(2) - 3) - 1/2)*log(-2*(sqrt(2
) + 1)*cos(x)^2 + (2*(sqrt(2) + 2)*cos(x)^2 - sqrt(2) - 2)*sqrt(2*sqrt(2)
- 3) + 2*(sqrt(2*sqrt(2) - 3)*(sqrt(2) + 2)*cos(x)*sin(x) - (sqrt(2) + 2)*
cos(x)*sin(x))*sqrt(1/2*sqrt(2*sqrt(2) - 3) - 1/2) + sqrt(2)) + 1/16*sqrt(
1/2*sqrt(2*sqrt(2) - 3) - 1/2)*log(-2*(sqrt(2) + 1)*cos(x)^2 + (2*(sqrt(2)
 + 2)*cos(x)^2 - sqrt(2) - 2)*sqrt(2*sqrt(2) - 3) - 2*(sqrt(2*sqrt(2) - 3)
*(sqrt(2) + 2)*cos(x)*sin(x) - (sqrt(2) + 2)*cos(x)*sin(x))*sqrt(1/2*sqrt(
2*sqrt(2) - 3) - 1/2) + sqrt(2)) - 1/16*sqrt(-1/2*sqrt(-2*sqrt(2) - 3) - 1
/2)*log(2*(sqrt(2) - 1)*cos(x)^2 + (2*(sqrt(2) - 2)*cos(x)^2 - sqrt(2) + 2
)*sqrt(-2*sqrt(2) - 3) + 2*((sqrt(2) - 2)*sqrt(-2*sqrt(2) - 3)*cos(x)*sin(
x) + (sqrt(2) - 2)*cos(x)*sin(x))*sqrt(-1/2*sqrt(-2*sqrt(2) - 3) - 1/2) -
sqrt(2)) + 1/16*sqrt(-1/2*sqrt(-2*sqrt(2) - 3) - 1/2)*log(2*(sqrt(2) - 1)*
cos(x)^2 + (2*(sqrt(2) - 2)*cos(x)^2 - sqrt(2) + 2)*sqrt(-2*sqrt(2) - 3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\cos^8(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+cos(x)**8),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{1+\cos^8(x)}\,dx = \int \frac{1}{\cos(x)^8+1}\,dx$$

input
```
integrate(1/(1+cos(x)^8),x, algorithm="maxima")
```

output
```
integrate(1/(cos(x)^8 + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{1+\cos^8(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1+cos(x)^8),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to find common minimal polyn
omial Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 1025, normalized size of antiderivative = 1.32

$$\int \frac{1}{1+\cos^8(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(x)^8 + 1),x)
```

output
```
atan((tan(x)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^
(1/2) - 3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1) - (2^(1/2)*ta
n(x)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(2*2^(1/2) -
3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1) - (tan(x)*(2*2^(1/2)
- 3)^(1/2)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(1
/2) - 3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1) + (2^(1/2)*tan(
x)*(2*2^(1/2) - 3)^(1/2)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2
^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1))*
((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*2i - atan((tan(x)*(- (2*2^(1/2)
- 3)^(1/2)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2^(
1/2)/2 - (2*2^(1/2) - 3)^(1/2) - 1) - (2^(1/2)*tan(x)*(- (2*2^(1/2) - 3)^(
1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2
 - (2*2^(1/2) - 3)^(1/2) - 1) + (tan(x)*(2*2^(1/2) - 3)^(1/2)*(- (2*2^(1/2
) - 3)^(1/2)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2
^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) - 1) - (2^(1/2)*tan(x)*(2*2^(1/2) - 3)^(1
/2)*(- (2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(2*2^(1/2) -
 3)^(1/2))/2 + 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) - 1))*(- (2*2^(1/2) - 3)^
(1/2)/128 - 1/128)^(1/2)*2i + atan((tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/128
- 1/128)^(1/2)*8i)/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 + (- 2
*2^(1/2) - 3)^(1/2) + 1) + (2^(1/2)*tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/1...
```

Reduce [F]

$$\int \frac{1}{1+\cos^8(x)}\,dx = \int \frac{1}{\cos\left(x\right)^{8}+1}dx$$

input
```
int(1/(1+cos(x)^8),x)
```

output
```
int(1/(cos(x)**8 + 1),x)
```

3.12 $\int \frac{1}{1+\cos(x)}\,dx$

Optimal result 137
Mathematica [A] (verified) 137
Rubi [A] (verified) 138
Maple [A] (verified) 139
Fricas [A] (verification not implemented) 139
Sympy [A] (verification not implemented) 139
Maxima [A] (verification not implemented) 140
Giac [B] (verification not implemented) 140
Mupad [B] (verification not implemented) 141
Reduce [B] (verification not implemented) 141

Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{1+\cos(x)}$$

output
```
sin(x)/(1+cos(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 + Cos[x])^(-1),x]
```

output
```
Tan[x/2]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos(x)+1}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \; 3127$$

$$\frac{\sin(x)}{\cos(x)+1}$$

input
```
Int[(1 + Cos[x])^(-1),x]
```

output
```
Sin[x]/(1 + Cos[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
norman	$\tan\left(\frac{x}{2}\right)$	5
parallelrisch	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{e^{ix}+1}$	13

input
```
int(1/(1+cos(x)),x,method=_RETURNVERBOSE)
```

output
```
tan(1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="fricas")
```

output
```
sin(x)/(cos(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.33

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
integrate(1/(1+cos(x)),x)
```

output
```
tan(x/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{\cos(x)+1}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="maxima")
```

output
```
sin(x)/(cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(9) = 18.

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 3.33

$$\int \frac{1}{1+\cos(x)}\,dx = -\frac{2\tan\left(\frac{1}{2}x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

input
```
integrate(1/(1+cos(x)),x, algorithm="giac")
```

output
```
-2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
int(1/(cos(x) + 1),x)
```

output
```
tan(x/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{1}{1+\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
int(1/(1+cos(x)),x)
```

output
```
tan(x/2)
```

3.13 $\int \frac{1}{1+\cos^3(x)}\,dx$

Optimal result 142
Mathematica [C] (verified) 142
Rubi [A] (verified) 143
Maple [C] (verified) 144
Fricas [B] (verification not implemented) 145
Sympy [B] (verification not implemented) 146
Maxima [B] (verification not implemented) 147
Giac [B] (verification not implemented) 147
Mupad [B] (verification not implemented) 148
Reduce [B] (verification not implemented) 149

Optimal result

Integrand size = 8, antiderivative size = 91

$$\int \frac{1}{1+\cos^3(x)}\,dx = -\frac{2\sqrt[4]{-\frac{1}{3}}\arctan\left((-1)^{3/4}\sqrt[4]{3}\tan\left(\frac{x}{2}\right)\right)}{3\left(1+(-1)^{2/3}\right)} - \frac{2\sqrt[4]{-\frac{1}{3}}\operatorname{arctanh}\left((-1)^{3/4}\sqrt[4]{3}\tan\left(\frac{x}{2}\right)\right)}{3\left(1-\sqrt[3]{-1}\right)} + \frac{\sin(x)}{3(1+\cos(x))}$$

output
```
-2/3*(-1)^(1/4)*3^(3/4)*arctan((-1)^(3/4)*3^(1/4)*tan(1/2*x))/(3+3*(-1)^(2
/3))-2/3*(-1)^(1/4)*3^(3/4)*arctanh((-1)^(3/4)*3^(1/4)*tan(1/2*x))/(3-3*(-
1)^(1/3))+sin(x)/(3+3*cos(x))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.41

$$\int \frac{1}{1+\cos^3(x)}\,dx = \frac{\sqrt{2}(-i+\sqrt{3})\arctan\left(\frac{\left(-3i+\sqrt{3}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{6-2i\sqrt{3}}}\right)+2\sqrt{2}\arctan\left(\frac{\left(3i+\sqrt{3}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{6+2i\sqrt{3}}}\right)+\sqrt{3-i\sqrt{3}}\tan\left(\frac{x}{2}\right)}{3\sqrt{3-i\sqrt{3}}}$$

input
```
Integrate[(1 + Cos[x]^3)^(-1),x]
```

output
```
(Sqrt[2]*(-I + Sqrt[3])*ArcTan[((-3*I + Sqrt[3])*Tan[x/2])/Sqrt[6 - (2*I)*
Sqrt[3]]] + 2*Sqrt[2]*ArcTan[((3*I + Sqrt[3])*Tan[x/2])/Sqrt[6 + (2*I)*Sqr
t[3]]] + Sqrt[3 - I*Sqrt[3]]*Tan[x/2])/(3*Sqrt[3 - I*Sqrt[3]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^3(x)+1}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^3+1}dx$$
$$\Big\downarrow \text{3692}$$
$$\int \left(-\frac{1}{3\left(\sqrt[3]{-1}\cos(x)-1\right)}-\frac{1}{3\left(-(-1)^{2/3}\cos(x)-1\right)}-\frac{1}{3(-\cos(x)-1)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt[4]{-\frac{1}{3}}\arctan\left((-1)^{3/4}\sqrt[4]{3}\tan\left(\frac{x}{2}\right)\right)}{3\left(1+(-1)^{2/3}\right)}-\frac{2\sqrt[4]{-\frac{1}{3}}\operatorname{arctanh}\left((-1)^{3/4}\sqrt[4]{3}\tan\left(\frac{x}{2}\right)\right)}{3\left(1-\sqrt[3]{-1}\right)}+\frac{\sin(x)}{3(\cos(x)+1)}$$

input
```
Int[(1 + Cos[x]^3)^(-1),x]
```

output
```
(-2*(-1/3)^(1/4)*ArcTan[(-1)^(3/4)*3^(1/4)*Tan[x/2]])/(3*(1 + (-1)^(2/3)))
 - (2*(-1/3)^(1/4)*ArcTanh[(-1)^(3/4)*3^(1/4)*Tan[x/2]])/(3*(1 - (-1)^(1/3
))) + Sin[x]/(3*(1 + Cos[x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

method	result
risch	$\frac{2i}{3(e^{ix}+1)}+\left(\sum_{\_R=\operatorname{RootOf}(243\_Z^4+27\_Z^2+1)}\_R\ln\left(e^{ix}+162i\_R^3-27\_R^2+9i\_R-2\right)\right)$
default	$\frac{\tan(\frac{x}{2})}{3}+\frac{3^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{\tan(\frac{x}{2})^2+\frac{\sqrt{2}\,3^{\frac{3}{4}}\tan(\frac{x}{2})}{3}+\frac{\sqrt{3}}{3}}{\tan(\frac{x}{2})^2-\frac{\sqrt{2}\,3^{\frac{3}{4}}\tan(\frac{x}{2})}{3}+\frac{\sqrt{3}}{3}}\right)+2\arctan\left(3^{\frac{1}{4}}\tan(\frac{x}{2})\sqrt{2}+1\right)+2\arctan\left(3^{\frac{1}{4}}\tan(\frac{x}{2})\sqrt{2}-1\right)\right)}{36}+\frac{3^{\frac{1}{4}}\sqrt{2}\,(}{}$

input
```
int(1/(1+cos(x)^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/(exp(I*x)+1)+sum(_R*ln(exp(I*x)+162*I*_R^3-27*_R^2+9*I*_R-2),_R=Root
Of(243*_Z^4+27*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(65) = 130$.

Time = 0.13 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.16

$$\int \frac{1}{1+\cos^3(x)}\,dx$$
$$= \frac{2\sqrt{\frac{2}{3}\sqrt{3}+1}(\cos(x)+1)\arctan\left(\frac{\left(2\left(2\left(\sqrt{3}-2\right)\cos(x)^2+2\cos(x)^3-\left(\sqrt{3}+2\right)\cos(x)+1\right)\sin(x)+\left(4\sqrt{3}\cos(x)^3-2\sqrt{3}\cos(x)^2-\right.}{4\cos(x)^4-8\cos(x)^2+1}\right.}{}$$

input
```
integrate(1/(1+cos(x)^3),x, algorithm="fricas")
```

output
```
1/12*(2*sqrt(2/3*sqrt(3) + 1)*(cos(x) + 1)*arctan((2*(2*(sqrt(3) - 2)*cos(
x)^2 + 2*cos(x)^3 - (sqrt(3) + 2)*cos(x) + 1)*sin(x) + (4*sqrt(3)*cos(x)^3
 - 2*sqrt(3)*cos(x)^2 - 2*(2*sqrt(3) + 3)*cos(x) + 2*sqrt(3) + 3)*sqrt(2/3
*sqrt(3) - 1))*sqrt(2/3*sqrt(3) + 1)/(4*cos(x)^4 - 8*cos(x)^2 + 1)) - 2*sq
rt(2/3*sqrt(3) + 1)*(cos(x) + 1)*arctan(-(2*(2*(sqrt(3) - 2)*cos(x)^2 + 2*
cos(x)^3 - (sqrt(3) + 2)*cos(x) + 1)*sin(x) - (4*sqrt(3)*cos(x)^3 - 2*sqrt
(3)*cos(x)^2 - 2*(2*sqrt(3) + 3)*cos(x) + 2*sqrt(3) + 3)*sqrt(2/3*sqrt(3)
- 1))*sqrt(2/3*sqrt(3) + 1)/(4*cos(x)^4 - 8*cos(x)^2 + 1)) + sqrt(2/3*sqrt
(3) - 1)*(cos(x) + 1)*log(-(sqrt(3) - 1)*cos(x)^2 + (sqrt(3)*cos(x) - 2*sq
rt(3) - 3)*sqrt(2/3*sqrt(3) - 1)*sin(x) + sqrt(3) - cos(x) + 1) - sqrt(2/3
*sqrt(3) - 1)*(cos(x) + 1)*log(-(sqrt(3) - 1)*cos(x)^2 - (sqrt(3)*cos(x) -
 2*sqrt(3) - 3)*sqrt(2/3*sqrt(3) - 1)*sin(x) + sqrt(3) - cos(x) + 1) + 4*s
in(x))/(cos(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(73) = 146$.

Time = 1.22 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.89

$$\int \frac{1}{1+\cos^3(x)}\,dx = \frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}\left(\operatorname{atan}\left(\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{-18+18\sqrt{3}} + \frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}\left(\operatorname{atan}\left(\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{-18+18\sqrt{3}} - \frac{3\sqrt{2}\cdot\sqrt[4]{3}\log\left(36\tan^2\left(\frac{x}{2}\right)-12\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{-18+18\sqrt{3}} + \frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(36\tan^2\left(\frac{x}{2}\right)-12\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{-18+18\sqrt{3}} - \frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(36\tan^2\left(\frac{x}{2}\right)+12\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{-18+18\sqrt{3}} + \frac{3\sqrt{2}\cdot\sqrt[4]{3}\log\left(36\tan^2\left(\frac{x}{2}\right)+12\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{-18+18\sqrt{3}} - \frac{6\tan\left(\frac{x}{2}\right)}{-18+18\sqrt{3}} + \frac{6\sqrt{3}\tan\left(\frac{x}{2}\right)}{-18+18\sqrt{3}}$$

input
```
integrate(1/(1+cos(x)**3),x)
```

output
```
2*sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(1/4)*tan(x/2) - 1) + pi*floor((x/2 -
pi/2)/pi))/(-18 + 18*sqrt(3)) + 2*sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(1/4)*
tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/(-18 + 18*sqrt(3)) - 3*sqrt(2)*
3**(1/4)*log(36*tan(x/2)**2 - 12*sqrt(2)*3**(3/4)*tan(x/2) + 12*sqrt(3))/(
-18 + 18*sqrt(3)) + 2*sqrt(2)*3**(3/4)*log(36*tan(x/2)**2 - 12*sqrt(2)*3**
(3/4)*tan(x/2) + 12*sqrt(3))/(-18 + 18*sqrt(3)) - 2*sqrt(2)*3**(3/4)*log(3
6*tan(x/2)**2 + 12*sqrt(2)*3**(3/4)*tan(x/2) + 12*sqrt(3))/(-18 + 18*sqrt(
3)) + 3*sqrt(2)*3**(1/4)*log(36*tan(x/2)**2 + 12*sqrt(2)*3**(3/4)*tan(x/2)
 + 12*sqrt(3))/(-18 + 18*sqrt(3)) - 6*tan(x/2)/(-18 + 18*sqrt(3)) + 6*sqrt
(3)*tan(x/2)/(-18 + 18*sqrt(3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(65) = 130$.

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.07

$$\int \frac{1}{1+\cos^3(x)}\,dx = \frac{1}{18}\cdot 3^{\frac{1}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}+\frac{2\sqrt{3}\sin(x)}{\cos(x)+1}\right)\right) + \frac{1}{18}\cdot 3^{\frac{1}{4}}\sqrt{2}\left(\sqrt{3}+3\right)\arctan\left(-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(3^{\frac{1}{4}}\sqrt{2}-\frac{2\sqrt{3}\sin(x)}{\cos(x)+1}\right)\right) + \frac{1}{36}\cdot 3^{\frac{1}{4}}\sqrt{2}\left(\sqrt{3}-3\right)\log\left(\frac{3^{\frac{1}{4}}\sqrt{2}\sin(x)}{\cos(x)+1}+\frac{\sqrt{3}\sin(x)^2}{(\cos(x)+1)^2}+1\right) - \frac{1}{36}\cdot 3^{\frac{1}{4}}\sqrt{2}\left(\sqrt{3}-3\right)\log\left(-\frac{3^{\frac{1}{4}}\sqrt{2}\sin(x)}{\cos(x)+1}+\frac{\sqrt{3}\sin(x)^2}{(\cos(x)+1)^2}+1\right) + \frac{\sin(x)}{3(\cos(x)+1)}$$

input
```
integrate(1/(1+cos(x)^3),x, algorithm="maxima")
```

output
```
1/18*3^(1/4)*sqrt(2)*(sqrt(3) + 3)*arctan(1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqr
t(2) + 2*sqrt(3)*sin(x)/(cos(x) + 1))) + 1/18*3^(1/4)*sqrt(2)*(sqrt(3) + 3
)*arctan(-1/6*3^(3/4)*sqrt(2)*(3^(1/4)*sqrt(2) - 2*sqrt(3)*sin(x)/(cos(x)
+ 1))) + 1/36*3^(1/4)*sqrt(2)*(sqrt(3) - 3)*log(3^(1/4)*sqrt(2)*sin(x)/(co
s(x) + 1) + sqrt(3)*sin(x)^2/(cos(x) + 1)^2 + 1) - 1/36*3^(1/4)*sqrt(2)*(s
qrt(3) - 3)*log(-3^(1/4)*sqrt(2)*sin(x)/(cos(x) + 1) + sqrt(3)*sin(x)^2/(c
os(x) + 1)^2 + 1) + 1/3*sin(x)/(cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(65) = 130$.

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.82

$$\int \frac{1}{1+\cos^3(x)}\,dx$$
$$= \frac{1}{9}\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{3}{2}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}+2\tan\left(\frac{1}{2}x\right)\right)\right)\right)\sqrt{6\sqrt{3}+9}$$
$$+\frac{1}{9}\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{3}{2}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}-2\tan\left(\frac{1}{2}x\right)\right)\right)\right)\sqrt{6\sqrt{3}+9}$$
$$-\frac{1}{18}\sqrt{6\sqrt{3}-9}\log\left(\tan\left(\frac{1}{2}x\right)^2+\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\tan\left(\frac{1}{2}x\right)+\sqrt{\frac{1}{3}}\right)$$
$$+\frac{1}{18}\sqrt{6\sqrt{3}-9}\log\left(\tan\left(\frac{1}{2}x\right)^2-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\tan\left(\frac{1}{2}x\right)+\sqrt{\frac{1}{3}}\right)+\frac{1}{3}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(1/(1+cos(x)^3),x, algorithm="giac")
```

output
```
1/9*(pi*floor(1/2*x/pi + 1/2) + arctan(3/2*sqrt(2)*(1/3)^(3/4)*(sqrt(2)*(1
/3)^(1/4) + 2*tan(1/2*x))))*sqrt(6*sqrt(3) + 9) + 1/9*(pi*floor(1/2*x/pi +
 1/2) + arctan(-3/2*sqrt(2)*(1/3)^(3/4)*(sqrt(2)*(1/3)^(1/4) - 2*tan(1/2*x
))))*sqrt(6*sqrt(3) + 9) - 1/18*sqrt(6*sqrt(3) - 9)*log(tan(1/2*x)^2 + sqr
t(2)*(1/3)^(1/4)*tan(1/2*x) + sqrt(1/3)) + 1/18*sqrt(6*sqrt(3) - 9)*log(ta
n(1/2*x)^2 - sqrt(2)*(1/3)^(1/4)*tan(1/2*x) + sqrt(1/3)) + 1/3*tan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.64

$$\int \frac{1}{1+\cos^3(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{3}$$
$$+\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{3^{1/4}\sqrt{6}\tan\left(\frac{x}{2}\right)\left(\frac{16}{729}-\frac{16}{729}\mathrm{i}\right)}{-\frac{32}{243}+\frac{\sqrt{3}\,32\mathrm{i}}{729}}+\frac{3^{3/4}\sqrt{6}\tan\left(\frac{x}{2}\right)\left(\frac{16}{729}+\frac{16}{729}\mathrm{i}\right)}{-\frac{32}{243}+\frac{\sqrt{3}\,32\mathrm{i}}{729}}\right)\left(3^{1/4}\,(1+1\mathrm{i})+3^{3/4}\,(-1+1\mathrm{i})\right)\,1\mathrm{i}}{18}$$
$$+\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{3^{1/4}\sqrt{6}\tan\left(\frac{x}{2}\right)\left(\frac{16}{729}+\frac{16}{729}\mathrm{i}\right)}{\frac{32}{243}+\frac{\sqrt{3}\,32\mathrm{i}}{729}}+\frac{3^{3/4}\sqrt{6}\tan\left(\frac{x}{2}\right)\left(\frac{16}{729}-\frac{16}{729}\mathrm{i}\right)}{\frac{32}{243}+\frac{\sqrt{3}\,32\mathrm{i}}{729}}\right)\left(3^{1/4}\,(1-\mathrm{i})+3^{3/4}\,(-1-\mathrm{i})\right)\,1\mathrm{i}}{18}$$

input
```
int(1/(cos(x)^3 + 1),x)
```

output
```
tan(x/2)/3 + (6^(1/2)*atan((3^(1/4)*6^(1/2)*tan(x/2)*(16/729 - 16i/729))/(
(3^(1/2)*32i)/729 - 32/243) + (3^(3/4)*6^(1/2)*tan(x/2)*(16/729 + 16i/729)
)/((3^(1/2)*32i)/729 - 32/243))*(3^(1/4)*(1 + 1i) - 3^(3/4)*(1 - 1i))*1i)/
18 + (6^(1/2)*atan((3^(1/4)*6^(1/2)*tan(x/2)*(16/729 + 16i/729))/((3^(1/2)
*32i)/729 + 32/243) + (3^(3/4)*6^(1/2)*tan(x/2)*(16/729 - 16i/729))/((3^(1
/2)*32i)/729 + 32/243))*(3^(1/4)*(1 - 1i) - 3^(3/4)*(1 + 1i))*1i)/18
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.78

$$\int \frac{1}{1+\cos^3(x)}\,dx = -\frac{\sqrt{6}\,3^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,3^{\frac{1}{4}}-2\sqrt{3}\,\tan\left(\frac{x}{2}\right)\right)3^{\frac{3}{4}}}{3\sqrt{2}}\right)}{18} - \frac{\sqrt{2}\,3^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,3^{\frac{1}{4}}-2\sqrt{3}\,\tan\left(\frac{x}{2}\right)\right)3^{\frac{3}{4}}}{3\sqrt{2}}\right)}{6} + \frac{\sqrt{6}\,3^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,3^{\frac{1}{4}}+2\sqrt{3}\,\tan\left(\frac{x}{2}\right)\right)3^{\frac{3}{4}}}{3\sqrt{2}}\right)}{18} + \frac{\sqrt{2}\,3^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,3^{\frac{1}{4}}+2\sqrt{3}\,\tan\left(\frac{x}{2}\right)\right)3^{\frac{3}{4}}}{3\sqrt{2}}\right)}{6} - \frac{\sqrt{6}\,3^{\frac{1}{4}}\log\left(-\sqrt{2}\,3^{\frac{1}{4}}\tan\left(\frac{x}{2}\right)+\sqrt{3}\tan\left(\frac{x}{2}\right)^{2}+1\right)}{36} + \frac{\sqrt{6}\,3^{\frac{1}{4}}\log\left(\sqrt{2}\,3^{\frac{1}{4}}\tan\left(\frac{x}{2}\right)+\sqrt{3}\tan\left(\frac{x}{2}\right)^{2}+1\right)}{36} + \frac{\sqrt{2}\,3^{\frac{1}{4}}\log\left(-\sqrt{2}\,3^{\frac{1}{4}}\tan\left(\frac{x}{2}\right)+\sqrt{3}\tan\left(\frac{x}{2}\right)^{2}+1\right)}{12} - \frac{\sqrt{2}\,3^{\frac{1}{4}}\log\left(\sqrt{2}\,3^{\frac{1}{4}}\tan\left(\frac{x}{2}\right)+\sqrt{3}\tan\left(\frac{x}{2}\right)^{2}+1\right)}{12} + \frac{\tan\left(\frac{x}{2}\right)}{3}$$

input
```
int(1/(1+cos(x)^3),x)
```

output
```
( - 2*sqrt(6)*3**(1/4)*atan((sqrt(2)*3**(1/4) - 2*sqrt(3)*tan(x/2))/(sqrt(
2)*3**(1/4))) - 6*sqrt(2)*3**(1/4)*atan((sqrt(2)*3**(1/4) - 2*sqrt(3)*tan(
x/2))/(sqrt(2)*3**(1/4))) + 2*sqrt(6)*3**(1/4)*atan((sqrt(2)*3**(1/4) + 2*
sqrt(3)*tan(x/2))/(sqrt(2)*3**(1/4))) + 6*sqrt(2)*3**(1/4)*atan((sqrt(2)*3
**(1/4) + 2*sqrt(3)*tan(x/2))/(sqrt(2)*3**(1/4))) - sqrt(6)*3**(1/4)*log(
- sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3)*tan(x/2)**2 + 1) + sqrt(6)*3**(1/4)*
log(sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3)*tan(x/2)**2 + 1) + 3*sqrt(2)*3**(1
/4)*log( - sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3)*tan(x/2)**2 + 1) - 3*sqrt(2
)*3**(1/4)*log(sqrt(2)*3**(1/4)*tan(x/2) + sqrt(3)*tan(x/2)**2 + 1) + 12*t
an(x/2))/36
```

3.14 $\int \frac{1}{1+\cos^5(x)}\,dx$

Optimal result 151
Mathematica [C] (verified) 152
Rubi [A] (verified) 152
Maple [C] (verified) 154
Fricas [B] (verification not implemented) 155
Sympy [F(-1)] 156
Maxima [F] 156
Giac [B] (verification not implemented) 157
Mupad [B] (verification not implemented) 158
Reduce [F] 159

Optimal result

Integrand size = 8, antiderivative size = 223

$$\int \frac{1}{1+\cos^5(x)}\,dx = \frac{2\arctan\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}} + \frac{2\arctan\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}} - \frac{2\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{-1+(-1)^{2/5}}} - \frac{2\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\text{arctanh}\left(\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\left(1+(-1)^{3/5}\right)} + \frac{\sin(x)}{5(1+\cos(x))}$$

output

```
2/5*arctan(((1-(-1)^(2/5))/(1+(-1)^(2/5)))^(1/2)*tan(1/2*x))/(1-(-1)^(4/5)
)^(1/2)+2/5*arctan(((1-(-1)^(4/5))/(1+(-1)^(4/5)))^(1/2)*tan(1/2*x))/(1+(-
1)^(3/5))^(1/2)-2/5*arctanh(tan(1/2*x)/(-(1-(-1)^(1/5))/(1+(-1)^(1/5)))^(1
/2))/(-1+(-1)^(2/5))^(1/2)-2*(-(1+(-1)^(3/5))/(1-(-1)^(3/5)))^(1/2)*arctan
h((-(1+(-1)^(3/5))/(1-(-1)^(3/5)))^(1/2)*tan(1/2*x))/(5+5*(-1)^(3/5))+sin(
x)/(5+5*cos(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.08 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.70

$$\int \frac{1}{1+\cos^5(x)}\,dx$$
$$= -\frac{1}{10}\text{RootSum}\Bigg[1-2\#1+8\#1^2-14\#1^3+30\#1^4-14\#1^5+8\#1^6-2\#1^7$$
$$+\#1^8\&,\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\log\left(1-2\cos(x)\#1+\#1^2\right)-8\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1+4i\log\left(1-\right.}{}$$
$$+\frac{1}{5}\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 + Cos[x]^5)^(-1),x]
```

output
```
-1/10*RootSum[1 - 2*#1 + 8*#1^2 - 14*#1^3 + 30*#1^4 - 14*#1^5 + 8*#1^6 - 2
*#1^7 + #1^8 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)] - I*Log[1 - 2*Cos[x]*#1 +
 #1^2] - 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 + (4*I)*Log[1 - 2*Cos[x]*#1 + #
1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log[1 - 2*Cos[x]*#
1 + #1^2]*#1^2 - 80*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 + (40*I)*Log[1 - 2*C
os[x]*#1 + #1^2]*#1^3 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 - (15*I)*Log[
1 - 2*Cos[x]*#1 + #1^2]*#1^4 - 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^5 + (4*I)
*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^6 -
I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(-1 + 8*#1 - 21*#1^2 + 60*#1^3 - 35*#1
^4 + 24*#1^5 - 7*#1^6 + 4*#1^7) & ] + Tan[x/2]/5
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^5(x)+1}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^5+1}dx$$

$$\Big\downarrow \text{ 3692}$$

$$\int \left(-\frac{1}{5\left(\sqrt[5]{-1}\cos(x)-1\right)}-\frac{1}{5\left(-(-1)^{2/5}\cos(x)-1\right)}-\frac{1}{5\left((-1)^{3/5}\cos(x)-1\right)}-\frac{1}{5\left(-(-1)^{4/5}\cos(x)-1\right)}-\frac{1}{5(\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\arctan\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\arctan\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{(-1)^{2/5}-1}}-\frac{2\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\text{arctanh}\left(\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\left(1+(-1)^{3/5}\right)}+\frac{\sin(x)}{5(\cos(x)+1)}$$

input
```
Int[(1 + Cos[x]^5)^(-1),x]
```

output
```
(2*ArcTan[Sqrt[(1 - (-1)^(2/5))/(1 + (-1)^(2/5))]*Tan[x/2]])/(5*Sqrt[1 - (
-1)^(4/5)]) + (2*ArcTan[Sqrt[(1 - (-1)^(4/5))/(1 + (-1)^(4/5))]*Tan[x/2]])
/(5*Sqrt[1 + (-1)^(3/5)]) - (2*ArcTanh[Tan[x/2]/Sqrt[-((1 - (-1)^(1/5))/(1
 + (-1)^(1/5)))]])/(5*Sqrt[-1 + (-1)^(2/5)]) - (2*Sqrt[-((1 + (-1)^(3/5))/
(1 - (-1)^(3/5)))]*ArcTanh[Sqrt[-((1 + (-1)^(3/5))/(1 - (-1)^(3/5)))]*Tan[
x/2]])/(5*(1 + (-1)^(3/5))) + Sin[x]/(5*(1 + Cos[x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.28

method	result
default	$\frac{\tan\left(\frac{x}{2}\right)}{5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(5\_Z^8+10\_Z^4+1\right)}\frac{\left(5\_R^6+5\_R^4+5\_R^2+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^7+\_R^3}\right)}{50}$
risch	$\frac{2i}{5(\mathrm{e}^{ix}+1)}+\left(\sum_{\_R=\mathrm{RootOf}\left(1953125\_Z^8+156250\_Z^6+6250\_Z^4+125\_Z^2+1\right)}\_R\ln\left(\mathrm{e}^{ix}-2343750i\_R^7+23437\right.\right.$

input
```
int(1/(1+cos(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*tan(1/2*x)+1/50*sum((5*_R^6+5*_R^4+5*_R^2+1)/(_R^7+_R^3)*ln(tan(1/2*x)
-_R),_R=RootOf(5*_Z^8+10*_Z^4+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(150) = 300$.

Time = 0.20 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.16

$$\int \frac{1}{1+\cos^5(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+cos(x)^5),x, algorithm="fricas")
```

output
```
1/20*(sqrt(2*sqrt(2/5*sqrt(5) - 1) - 2)*(cos(x) + 1)*log((3*sqrt(5) + 5)*s
qrt(2/5*sqrt(5) - 1)*sqrt(2*sqrt(2/5*sqrt(5) - 1) - 2)*sin(x) - (3*sqrt(5)
 + 5)*sqrt(2/5*sqrt(5) - 1)*cos(x) - (sqrt(5) - 1)*cos(x) - 4) - sqrt(2*sq
rt(2/5*sqrt(5) - 1) - 2)*(cos(x) + 1)*log(-(3*sqrt(5) + 5)*sqrt(2/5*sqrt(5
) - 1)*sqrt(2*sqrt(2/5*sqrt(5) - 1) - 2)*sin(x) - (3*sqrt(5) + 5)*sqrt(2/5
*sqrt(5) - 1)*cos(x) - (sqrt(5) - 1)*cos(x) - 4) + sqrt(-2*sqrt(2/5*sqrt(5
) - 1) - 2)*(cos(x) + 1)*log((3*sqrt(5) + 5)*sqrt(2/5*sqrt(5) - 1)*sqrt(-2
*sqrt(2/5*sqrt(5) - 1) - 2)*sin(x) - (3*sqrt(5) + 5)*sqrt(2/5*sqrt(5) - 1)
*cos(x) + (sqrt(5) - 1)*cos(x) + 4) - sqrt(-2*sqrt(2/5*sqrt(5) - 1) - 2)*(
cos(x) + 1)*log(-(3*sqrt(5) + 5)*sqrt(2/5*sqrt(5) - 1)*sqrt(-2*sqrt(2/5*sq
rt(5) - 1) - 2)*sin(x) - (3*sqrt(5) + 5)*sqrt(2/5*sqrt(5) - 1)*cos(x) + (s
qrt(5) - 1)*cos(x) + 4) + sqrt(2*sqrt(-2/5*sqrt(5) - 1) - 2)*(cos(x) + 1)*
log((3*sqrt(5) - 5)*sqrt(-2/5*sqrt(5) - 1)*sqrt(2*sqrt(-2/5*sqrt(5) - 1) -
 2)*sin(x) - (3*sqrt(5) - 5)*sqrt(-2/5*sqrt(5) - 1)*cos(x) - (sqrt(5) + 1)
*cos(x) + 4) - sqrt(2*sqrt(-2/5*sqrt(5) - 1) - 2)*(cos(x) + 1)*log(-(3*sqr
t(5) - 5)*sqrt(-2/5*sqrt(5) - 1)*sqrt(2*sqrt(-2/5*sqrt(5) - 1) - 2)*sin(x)
 - (3*sqrt(5) - 5)*sqrt(-2/5*sqrt(5) - 1)*cos(x) - (sqrt(5) + 1)*cos(x) +
4) + sqrt(-2*sqrt(-2/5*sqrt(5) - 1) - 2)*(cos(x) + 1)*log((3*sqrt(5) - 5)*
sqrt(-2/5*sqrt(5) - 1)*sqrt(-2*sqrt(-2/5*sqrt(5) - 1) - 2)*sin(x) - (3*sqr
t(5) - 5)*sqrt(-2/5*sqrt(5) - 1)*cos(x) + (sqrt(5) + 1)*cos(x) - 4) - s...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\cos^5(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+cos(x)**5),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{1+\cos^5(x)}\,dx = \int \frac{1}{\cos\left(x\right)^5+1}\,dx$$

input
```
integrate(1/(1+cos(x)^5),x, algorithm="maxima")
```

output
```
-1/5*(5*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(-2/5*((cos(7*x) - 4
*cos(6*x) + 15*cos(5*x) - 40*cos(4*x) + 15*cos(3*x) - 4*cos(2*x) + cos(x))
*cos(8*x) + (16*cos(6*x) - 44*cos(5*x) + 110*cos(4*x) - 44*cos(3*x) + 16*c
os(2*x) - 4*cos(x) + 1)*cos(7*x) - 2*cos(7*x)^2 + 4*(44*cos(5*x) - 110*cos
(4*x) + 44*cos(3*x) - 16*cos(2*x) + 4*cos(x) - 1)*cos(6*x) - 32*cos(6*x)^2
 + (1010*cos(4*x) - 420*cos(3*x) + 176*cos(2*x) - 44*cos(x) + 15)*cos(5*x)
 - 210*cos(5*x)^2 + 10*(101*cos(3*x) - 44*cos(2*x) + 11*cos(x) - 4)*cos(4*
x) - 1200*cos(4*x)^2 + (176*cos(2*x) - 44*cos(x) + 15)*cos(3*x) - 210*cos(
3*x)^2 + 4*(4*cos(x) - 1)*cos(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 + (sin(7*x
) - 4*sin(6*x) + 15*sin(5*x) - 40*sin(4*x) + 15*sin(3*x) - 4*sin(2*x) + si
n(x))*sin(8*x) + 2*(8*sin(6*x) - 22*sin(5*x) + 55*sin(4*x) - 22*sin(3*x) +
 8*sin(2*x) - 2*sin(x))*sin(7*x) - 2*sin(7*x)^2 + 8*(22*sin(5*x) - 55*sin(
4*x) + 22*sin(3*x) - 8*sin(2*x) + 2*sin(x))*sin(6*x) - 32*sin(6*x)^2 + 2*(
505*sin(4*x) - 210*sin(3*x) + 88*sin(2*x) - 22*sin(x))*sin(5*x) - 210*sin(
5*x)^2 + 10*(101*sin(3*x) - 44*sin(2*x) + 11*sin(x))*sin(4*x) - 1200*sin(4
*x)^2 + 44*(4*sin(2*x) - sin(x))*sin(3*x) - 210*sin(3*x)^2 - 32*sin(2*x)^2
 + 16*sin(2*x)*sin(x) - 2*sin(x)^2 + cos(x))/(2*(2*cos(7*x) - 8*cos(6*x) +
 14*cos(5*x) - 30*cos(4*x) + 14*cos(3*x) - 8*cos(2*x) + 2*cos(x) - 1)*cos(
8*x) - cos(8*x)^2 + 4*(8*cos(6*x) - 14*cos(5*x) + 30*cos(4*x) - 14*cos(3*x
) + 8*cos(2*x) - 2*cos(x) + 1)*cos(7*x) - 4*cos(7*x)^2 + 16*(14*cos(5*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1113 vs. $2(150) = 300$.

Time = 1.21 (sec) , antiderivative size = 1113, normalized size of antiderivative = 4.99

$$\int \frac{1}{1+\cos^5(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+cos(x)^5),x, algorithm="giac")
```

output
```
1/100*sqrt(1/5)*(pi + 4*arctan(1/9929000170337217568400*(35881763176905408
15475*sqrt(5)*(sqrt(5) + 5) + 297870005110116527052*sqrt(5)*sqrt(10*sqrt(5
) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) + 1985800034067443513680*sqrt(5
)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 496450008516860878420*sqrt(10*sqrt(
5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 17940881588452704077375*sqrt
(5) - 17940881588452704077375)*tan(1/2*x) + 1))*sqrt(2*sqrt(5) + 5)*sqrt(5
*sqrt(10*sqrt(5) + 50) - 25)/(sqrt(2/5)*sqrt(sqrt(5) + 5) - 1) - 1/100*sqr
t(1/5)*(pi + 4*arctan(-1/9929000170337217568400*(3588176317690540815475*sq
rt(5)*(sqrt(5) + 5) + 297870005110116527052*sqrt(5)*sqrt(10*sqrt(5) + 50)*
sqrt(5*sqrt(10*sqrt(5) + 50) - 25) + 1985800034067443513680*sqrt(5)*sqrt(5
*sqrt(10*sqrt(5) + 50) - 25) - 496450008516860878420*sqrt(10*sqrt(5) + 50)
*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 17940881588452704077375*sqrt(5) - 17
940881588452704077375)*tan(1/2*x) + 1))*sqrt(2*sqrt(5) + 5)*sqrt(5*sqrt(10
*sqrt(5) + 50) - 25)/(sqrt(2/5)*sqrt(sqrt(5) + 5) - 1) - 1/50*sqrt(5*sqrt(
10*sqrt(5) + 50) - 25)*log(40380434179095140551907282511641644514738176000
0*(3*sqrt(5)*sqrt(10*sqrt(5) + 50)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) + 20
*sqrt(5)*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) - 5*sqrt(10*sqrt(5) + 50)*sqrt
(5*sqrt(10*sqrt(5) + 50) - 25) - 40*sqrt(5*sqrt(10*sqrt(5) + 50) - 25) + 1
00*tan(1/2*x))^2 + 403804341790951405519072825116416445147381760000000*ta
n(1/2*x)^2) + 1/50*sqrt(5*sqrt(10*sqrt(5) + 50) - 25)*log(4038043417909...
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.40

$$\int \frac{1}{1+\cos^5(x)} \, dx = \text{Too large to display}$$

input
```
int(1/(cos(x)^5 + 1),x)
```

output
```
tan(x/2)/5 + 2*atanh((603979776*tan(x/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50
 - 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))
/1220703125 - (134217728*5^(1/2))/1220703125 + (67108864*(- (2*5^(1/2))/5
- 1)^(1/2))/1220703125 - 301989888/1220703125)) + (268435456*5^(1/2)*tan(x
/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*((33554432
*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 - (134217728*5^(1/2))/122
0703125 + (67108864*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 - 301989888/12
20703125)))*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((603
979776*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*
((33554432*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 + (134217728*5^
(1/2))/1220703125 + (67108864*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 + 30
1989888/1220703125)) + (268435456*5^(1/2)*tan(x/2)*((- (2*5^(1/2))/5 - 1)^
(1/2)/50 - 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*(- (2*5^(1/2))/5 - 1
)^(1/2))/1220703125 + (134217728*5^(1/2))/1220703125 + (67108864*(- (2*5^(
1/2))/5 - 1)^(1/2))/1220703125 + 301989888/1220703125)))*((- (2*5^(1/2))/5
 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((603979776*tan(x/2)*(- ((2*5^(1/2))
/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*((2*5^(1/2))
/5 - 1)^(1/2))/1220703125 - (134217728*5^(1/2))/1220703125 - (67108864*((2
*5^(1/2))/5 - 1)^(1/2))/1220703125 + 301989888/1220703125)) - (268435456*5
^(1/2)*tan(x/2)*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(2441406...
```

Reduce [F]

$$\int \frac{1}{1+\cos^5(x)}\,dx = 4\left(\int \frac{\tan\left(\frac{x}{2}\right)^6}{5\tan\left(\frac{x}{2}\right)^8+10\tan\left(\frac{x}{2}\right)^4+1}dx\right) + \frac{12\left(\int \frac{\tan\left(\frac{x}{2}\right)^2}{5\tan\left(\frac{x}{2}\right)^8+10\tan\left(\frac{x}{2}\right)^4+1}dx\right)}{5} + \frac{\tan\left(\frac{x}{2}\right)}{5} + \frac{2x}{5}$$

input
```
int(1/(1+cos(x)^5),x)
```

output
```
(20*int(tan(x/2)**6/(5*tan(x/2)**8 + 10*tan(x/2)**4 + 1),x) + 12*int(tan(x
/2)**2/(5*tan(x/2)**8 + 10*tan(x/2)**4 + 1),x) + tan(x/2) + 2*x)/5
```

3.15 $\int \frac{1}{a-b\cos^2(x)}\,dx$

Optimal result . . . 160
Mathematica [A] (verified) . . . 160
Rubi [A] (verified) . . . 161
Maple [A] (verified) . . . 162
Fricas [A] (verification not implemented) . . . 162
Sympy [B] (verification not implemented) . . . 163
Maxima [F(-2)] . . . 164
Giac [A] (verification not implemented) . . . 164
Mupad [B] (verification not implemented) . . . 164
Reduce [B] (verification not implemented) . . . 165

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{1}{a-b\cos^2(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}}$$

output
```
-arctan((a-b)^(1/2)*cot(x)/a^(1/2))/a^(1/2)/(a-b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{1}{a-b\cos^2(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a-b}}\right)}{\sqrt{a}\sqrt{a-b}}$$

input
```
Integrate[(a - b*Cos[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a - b]]/(Sqrt[a]*Sqrt[a - b])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cos^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a-b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3660}$$

$$-\int \frac{1}{(a-b)\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{\arctan\left(\frac{\sqrt{a-b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}}$$

input
```
Int[(a - b*Cos[x]^2)^(-1),x]
```

output
```
-(ArcTan[(Sqrt[a - b]*Cot[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a-b)a}}\right)}{\sqrt{(a-b)a}}$	25
risch	$-\frac{\ln\left(e^{2ix}-\frac{2ia^2-2iab+2a\sqrt{-a^2+ba}-b\sqrt{-a^2+ba}}{b\sqrt{-a^2+ba}}\right)}{2\sqrt{-a^2+ba}}+\frac{\ln\left(e^{2ix}+\frac{2ia^2-2iab-2a\sqrt{-a^2+ba}+b\sqrt{-a^2+ba}}{b\sqrt{-a^2+ba}}\right)}{2\sqrt{-a^2+ba}}$	152

input
```
int(1/(a-b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/((a-b)*a)^(1/2)*arctan(a*tan(x)/((a-b)*a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 4.94

$$\int \frac{1}{a-b\cos^2(x)}\,dx = \left[-\frac{\sqrt{-a^2+ab}\log\left(\frac{(8a^2-8ab+b^2)\cos(x)^4-2(4a^2-3ab)\cos(x)^2+4\left((2a-b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2+ab}\sin(x)+a^2}{b^2\cos(x)^4-2ab\cos(x)^2+a^2}\right)}{4(a^2-ab)}, -\frac{\arctan\left(\frac{(2a-b)\cos(x)^2-a}{2\sqrt{a^2-ab}\cos(x)\sin(x)}\right)}{2\sqrt{a^2-ab}}\right]$$

input
```
integrate(1/(a-b*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 + a*b)*log(((8*a^2 - 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 - 3*
a*b)*cos(x)^2 + 4*((2*a - b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 + a*b)*sin(x)
+ a^2)/(b^2*cos(x)^4 - 2*a*b*cos(x)^2 + a^2))/(a^2 - a*b), -1/2*arctan(1/2
*((2*a - b)*cos(x)^2 - a)/(sqrt(a^2 - a*b)*cos(x)*sin(x)))/sqrt(a^2 - a*b)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10533 vs. $2(29) = 58$.

Time = 15.93 (sec) , antiderivative size = 10533, normalized size of antiderivative = 309.79

$$\int \frac{1}{a - b\cos^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*cos(x)**2),x)
```

output
```
Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (tan(x/2)
/(2*b) - 1/(2*b*tan(x/2)), Eq(a, b)), (2*tan(x/2)/(b*(tan(x/2)**2 - 1)), E
q(a, 0)), (a**3*sqrt(-a/(a - b) - b/(a - b) - 2*sqrt(a*b)/(a - b))*log(-sq
rt(-a/(a - b) - b/(a - b) + 2*sqrt(a*b)/(a - b)) + tan(x/2))/(2*a**4*sqrt(
-a/(a - b) - b/(a - b) - 2*sqrt(a*b)/(a - b))*sqrt(-a/(a - b) - b/(a - b)
+ 2*sqrt(a*b)/(a - b)) + 10*a**3*b*sqrt(-a/(a - b) - b/(a - b) - 2*sqrt(a*
b)/(a - b))*sqrt(-a/(a - b) - b/(a - b) + 2*sqrt(a*b)/(a - b)) - 8*a**3*sq
rt(a*b)*sqrt(-a/(a - b) - b/(a - b) - 2*sqrt(a*b)/(a - b))*sqrt(-a/(a - b)
 - b/(a - b) + 2*sqrt(a*b)/(a - b)) - 10*a**2*b**2*sqrt(-a/(a - b) - b/(a
- b) - 2*sqrt(a*b)/(a - b))*sqrt(-a/(a - b) - b/(a - b) + 2*sqrt(a*b)/(a -
 b)) - 2*a*b**3*sqrt(-a/(a - b) - b/(a - b) - 2*sqrt(a*b)/(a - b))*sqrt(-a
/(a - b) - b/(a - b) + 2*sqrt(a*b)/(a - b)) + 8*a*b**2*sqrt(a*b)*sqrt(-a/(
a - b) - b/(a - b) - 2*sqrt(a*b)/(a - b))*sqrt(-a/(a - b) - b/(a - b) + 2*
sqrt(a*b)/(a - b))) - a**3*sqrt(-a/(a - b) - b/(a - b) - 2*sqrt(a*b)/(a -
b))*log(sqrt(-a/(a - b) - b/(a - b) + 2*sqrt(a*b)/(a - b)) + tan(x/2))/(2*
a**4*sqrt(-a/(a - b) - b/(a - b) - 2*sqrt(a*b)/(a - b))*sqrt(-a/(a - b) -
b/(a - b) + 2*sqrt(a*b)/(a - b)) + 10*a**3*b*sqrt(-a/(a - b) - b/(a - b) -
 2*sqrt(a*b)/(a - b))*sqrt(-a/(a - b) - b/(a - b) + 2*sqrt(a*b)/(a - b)) -
 8*a**3*sqrt(a*b)*sqrt(-a/(a - b) - b/(a - b) - 2*sqrt(a*b)/(a - b))*sqrt(
-a/(a - b) - b/(a - b) + 2*sqrt(a*b)/(a - b)) - 10*a**2*b**2*sqrt(-a/(a...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a-b\cos^2(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a-b*cos(x)^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{1}{a-b\cos^2(x)}\,dx = \frac{\pi\left\lfloor \frac{x}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2-ab}}\right)}{\sqrt{a^2-ab}}$$

input
```
integrate(1/(a-b*cos(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 - a*b)))/sqrt(a^2
- a*b)
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{a-b\cos^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{a\tan(x)}{\sqrt{a^2-a\,b}}\right)}{\sqrt{a^2-a\,b}}$$

input
```
int(1/(a - b*cos(x)^2),x)
```

output
```
atan((a*tan(x))/(a^2 - a*b)^(1/2))/(a^2 - a*b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 310, normalized size of antiderivative = 9.12

$$\int \frac{1}{a-b\cos^2(x)}\,dx$$
$$= \frac{\sqrt{a-b}\left(-2\sqrt{b}\,\sqrt{a}\,\sqrt{2\sqrt{b}\,\sqrt{a}+a+b}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a-\tan\left(\frac{x}{2}\right)b}{\sqrt{a-b}\,\sqrt{2\sqrt{b}\,\sqrt{a}+a+b}}\right)+2\sqrt{2\sqrt{b}\,\sqrt{a}+a+b}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a-\tan}{\sqrt{a-b}\,\sqrt{2\sqrt{b}\,\sqrt{}}}\right.\right.}{}$$

input
```
int(1/(a-b*cos(x)^2),x)
```

output
```
(sqrt(a - b)*( - 2*sqrt(b)*sqrt(a)*sqrt(2*sqrt(b)*sqrt(a) + a + b)*atan((t
an(x/2)*a - tan(x/2)*b)/(sqrt(a - b)*sqrt(2*sqrt(b)*sqrt(a) + a + b))) + 2
*sqrt(2*sqrt(b)*sqrt(a) + a + b)*atan((tan(x/2)*a - tan(x/2)*b)/(sqrt(a -
b)*sqrt(2*sqrt(b)*sqrt(a) + a + b)))*a - sqrt(b)*sqrt(a)*sqrt(2*sqrt(b)*sq
rt(a) - a - b)*log( - sqrt(2*sqrt(b)*sqrt(a) - a - b) + sqrt(a - b)*tan(x/
2)) + sqrt(b)*sqrt(a)*sqrt(2*sqrt(b)*sqrt(a) - a - b)*log(sqrt(2*sqrt(b)*s
qrt(a) - a - b) + sqrt(a - b)*tan(x/2)) - sqrt(2*sqrt(b)*sqrt(a) - a - b)*
log( - sqrt(2*sqrt(b)*sqrt(a) - a - b) + sqrt(a - b)*tan(x/2))*a + sqrt(2*
sqrt(b)*sqrt(a) - a - b)*log(sqrt(2*sqrt(b)*sqrt(a) - a - b) + sqrt(a - b)
*tan(x/2))*a))/(2*a*(a**2 - 2*a*b + b**2))
```

3.16 $\int \frac{1}{a-b\cos^4(x)}\,dx$

Optimal result 166
Mathematica [A] (verified) 166
Rubi [A] (verified) 167
Maple [A] (verified) 169
Fricas [B] (verification not implemented) 169
Sympy [F(-1)] 170
Maxima [F] 171
Giac [B] (verification not implemented) 171
Mupad [B] (verification not implemented) 172
Reduce [F] 173

Optimal result

Integrand size = 11, antiderivative size = 101

$$\int \frac{1}{a-b\cos^4(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}$$

output
```
-1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*cot(x)/a^(1/4))/a^(3/4)/(a^(1/2)-b^(1/
2))^(1/2)-1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*cot(x)/a^(1/4))/a^(3/4)/(a^(1
/2)+b^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{1}{a-b\cos^4(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}$$

input
```
Integrate[(a - b*Cos[x]^4)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a]*Sqrt[a + Sqr
t[a]*Sqrt[b]]) - ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/(2*S
qrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.54, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3688, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cos^4(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin\left(x+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3688}$$

$$-\int \frac{\cot^2(x)+1}{(a-b)\cot^4(x)+2a\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \text{1480}$$

$$-\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{(a-b)\cot^2(x)+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\cot(x)-\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\int \frac{1}{(a-b)\cot^2(x)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\cot(x)$$

$$\Big\downarrow \text{218}$$

$$-\frac{\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\arctan\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}-\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\arctan\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}$$

input
```
Int[(a - b*Cos[x]^4)^(-1),x]
```

output
```
-1/2*((1 + Sqrt[b]/Sqrt[a])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Cot[x])/a^(1/4
)])/(a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a] + Sqrt[b])) - ((1 - Sqrt[b]/
Sqrt[a])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Cot[x])/a^(1/4)])/(2*a^(1/4)*(Sqr
t[a] - Sqrt[b])*Sqrt[Sqrt[a] + Sqrt[b]])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

method	result
default	$a\left(\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{\left(\sqrt{ba}+a\right)a}}\right)}{2a\sqrt{\left(\sqrt{ba}+a\right)a}}-\frac{\operatorname{arctanh}\left(\frac{a\tan(x)}{\sqrt{\left(\sqrt{ba}-a\right)a}}\right)}{2a\sqrt{\left(\sqrt{ba}-a\right)a}}\right)$
risch	$\sum_{\_R=\operatorname{RootOf}\left(1+(256a^4-256a^3b)\_Z^4+32a^2\_Z^2\right)}\_R\ln\left(e^{2ix}+\left(-\frac{128ia^4}{b}+128ia^3\right)\_R^3+\left(\frac{32a^3}{b}-32a^2\right)\_\right.$

input
```
int(1/(a-b*cos(x)^4),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2/a/(((b*a)^(1/2)+a)*a)^(1/2)*arctan(a*tan(x)/(((b*a)^(1/2)+a)*a)^(1/
2))-1/2/a/(((b*a)^(1/2)-a)*a)^(1/2)*arctanh(a*tan(x)/(((b*a)^(1/2)-a)*a)^(
1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. 2(65) = 130.

Time = 0.23 (sec) , antiderivative size = 817, normalized size of antiderivative = 8.09

$$\int\frac{1}{a-b\cos^4(x)}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a-b*cos(x)^4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b)
)*log(b*cos(x)^2 + 2*(a*b*cos(x)*sin(x) - (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^
4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b +
a^3*b^2)) + 1)/(a^2 - a*b)) + (a^3 - a^2*b - 2*(a^3 - a^2*b)*cos(x)^2)*sqr
t(b/(a^5 - 2*a^4*b + a^3*b^2))) + 1/8*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a
^4*b + a^3*b^2)) + 1)/(a^2 - a*b))*log(b*cos(x)^2 - 2*(a*b*cos(x)*sin(x) -
 (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^
2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b)) + (a^3 - a^2*
b - 2*(a^3 - a^2*b)*cos(x)^2)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))) + 1/8*sqr
t(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b))*log(-b*
cos(x)^2 + 2*(a*b*cos(x)*sin(x) + (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^
3*b^2))*cos(x)*sin(x))*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))
 - 1)/(a^2 - a*b)) + (a^3 - a^2*b - 2*(a^3 - a^2*b)*cos(x)^2)*sqrt(b/(a^5
- 2*a^4*b + a^3*b^2))) - 1/8*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3
*b^2)) - 1)/(a^2 - a*b))*log(-b*cos(x)^2 - 2*(a*b*cos(x)*sin(x) + (a^4 - a
^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 - a*b)*s
qrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b)) + (a^3 - a^2*b - 2*(a^3
 - a^2*b)*cos(x)^2)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a - b\cos^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-b*cos(x)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a-b\cos^4(x)}\,dx = \int -\frac{1}{b\cos(x)^4-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^4),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cos(x)^4 - a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(65) = 130.

Time = 0.14 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.96

$$\int \frac{1}{a-b\cos^4(x)}\,dx$$
$$= \frac{\left(3\sqrt{a^2+\sqrt{ab}a}a^2-4\sqrt{a^2+\sqrt{ab}a}ab-3\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}a+4\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}b\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\mathrm{arct}\right.}{2\left(3a^5-7a^4b+4a^3b^2\right)}$$
$$+\frac{\left(3\sqrt{a^2-\sqrt{ab}a}a^2-4\sqrt{a^2-\sqrt{ab}a}ab+3\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}a-4\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}b\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\mathrm{a}\right.}{2\left(3a^5-7a^4b+4a^3b^2\right)}$$

input
```
integrate(1/(a-b*cos(x)^4),x, algorithm="giac")
```

output
```
1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*a^2 - 4*sqrt(a^2 + sqrt(a*b)*a)*a*b - 3*sqr
t(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a + 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*b)*
(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a - b)*a + 1
6*a^2))/a)))*abs(a)/(3*a^5 - 7*a^4*b + 4*a^3*b^2) + 1/2*(3*sqrt(a^2 - sqrt
(a*b)*a)*a^2 - 4*sqrt(a^2 - sqrt(a*b)*a)*a*b + 3*sqrt(a^2 - sqrt(a*b)*a)*s
qrt(a*b)*a - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*b)*(pi*floor(x/pi + 1/2)
+ arctan(2*tan(x)/sqrt((4*a - sqrt(-16*(a - b)*a + 16*a^2))/a)))*abs(a)/(3
*a^5 - 7*a^4*b + 4*a^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 938, normalized size of antiderivative = 9.29

$$\int \frac{1}{a-b\cos^4(x)}\,dx$$
$$= 2\operatorname{atanh}\left(\frac{8a^6b\tan(x)\sqrt{\frac{a^2}{16(a^3b-a^4)}+\frac{\sqrt{a^3b}}{16(a^3b-a^4)}}}{2a^5b-2a^4b^2-\frac{2a^8b^2}{a^3b-a^4}+\frac{2a^9b}{a^3b-a^4}-\frac{2a^6b^2\sqrt{a^3b}}{a^3b-a^4}+\frac{2a^7b\sqrt{a^3b}}{a^3b-a^4}} - \frac{8a^2b\tan(x)\sqrt{\frac{a^2}{16(a^3b-a^4)}+\frac{\sqrt{a^3b}}{16(a^3b-a^4)}}}{2ab+\frac{2a^5b}{a^3b-a^4}+\frac{2a^3b\sqrt{a^3b}}{a^3b-a^4}} + \frac{8a^4b\tan(x)\sqrt{\frac{a^2}{16(a^3b-a^4)}+\frac{\sqrt{a^3b}}{16(a^3b-a^4)}}\sqrt{a^3b}}{2a^5b-2a^4b^2-\frac{2a^8b^2}{a^3b-a^4}+\frac{2a^9b}{a^3b-a^4}-\frac{2a^6b^2\sqrt{a^3b}}{a^3b-a^4}+\frac{2a^7b\sqrt{a^3b}}{a^3b-a^4}}\right)\sqrt{\frac{a^2+\sqrt{a^3b}}{16\,(a^3b-a^4)}}$$
$$- 2\operatorname{atanh}\left(\frac{8a^2b\tan(x)\sqrt{\frac{a^2}{16(a^3b-a^4)}-\frac{\sqrt{a^3b}}{16(a^3b-a^4)}}}{2ab+\frac{2a^5b}{a^3b-a^4}-\frac{2a^3b\sqrt{a^3b}}{a^3b-a^4}} - \frac{8a^6b\tan(x)\sqrt{\frac{a^2}{16(a^3b-a^4)}-\frac{\sqrt{a^3b}}{16(a^3b-a^4)}}}{2a^5b-2a^4b^2-\frac{2a^8b^2}{a^3b-a^4}+\frac{2a^9b}{a^3b-a^4}+\frac{2a^6b^2\sqrt{a^3b}}{a^3b-a^4}-\frac{2a^7b\sqrt{a^3b}}{a^3b-a^4}} + \frac{8a^4b\tan(x)\sqrt{\frac{a^2}{16(a^3b-a^4)}-\frac{\sqrt{a^3b}}{16(a^3b-a^4)}}\sqrt{a^3b}}{2a^5b-2a^4b^2-\frac{2a^8b^2}{a^3b-a^4}+\frac{2a^9b}{a^3b-a^4}+\frac{2a^6b^2\sqrt{a^3b}}{a^3b-a^4}-\frac{2a^7b\sqrt{a^3b}}{a^3b-a^4}}\right)\sqrt{\frac{a^2-\sqrt{a^3b}}{16\,(a^3b-a^4)}}$$

input
```
int(1/(a - b*cos(x)^4),x)
```

output
```
2*atanh((8*a^6*b*tan(x)*(a^2/(16*(a^3*b - a^4)) + (a^3*b)^(1/2)/(16*(a^3*b
 - a^4)))^(1/2))/(2*a^5*b - 2*a^4*b^2 - (2*a^8*b^2)/(a^3*b - a^4) + (2*a^9
*b)/(a^3*b - a^4) - (2*a^6*b^2*(a^3*b)^(1/2))/(a^3*b - a^4) + (2*a^7*b*(a^
3*b)^(1/2))/(a^3*b - a^4)) - (8*a^2*b*tan(x)*(a^2/(16*(a^3*b - a^4)) + (a^
3*b)^(1/2)/(16*(a^3*b - a^4)))^(1/2))/(2*a*b + (2*a^5*b)/(a^3*b - a^4) + (
2*a^3*b*(a^3*b)^(1/2))/(a^3*b - a^4)) + (8*a^4*b*tan(x)*(a^2/(16*(a^3*b -
a^4)) + (a^3*b)^(1/2)/(16*(a^3*b - a^4)))^(1/2)*(a^3*b)^(1/2))/(2*a^5*b -
2*a^4*b^2 - (2*a^8*b^2)/(a^3*b - a^4) + (2*a^9*b)/(a^3*b - a^4) - (2*a^6*b
^2*(a^3*b)^(1/2))/(a^3*b - a^4) + (2*a^7*b*(a^3*b)^(1/2))/(a^3*b - a^4)))*
((a^2 + (a^3*b)^(1/2))/(16*(a^3*b - a^4)))^(1/2) - 2*atanh((8*a^2*b*tan(x)
*(a^2/(16*(a^3*b - a^4)) - (a^3*b)^(1/2)/(16*(a^3*b - a^4)))^(1/2))/(2*a*b
 + (2*a^5*b)/(a^3*b - a^4) - (2*a^3*b*(a^3*b)^(1/2))/(a^3*b - a^4)) - (8*a
^6*b*tan(x)*(a^2/(16*(a^3*b - a^4)) - (a^3*b)^(1/2)/(16*(a^3*b - a^4)))^(1
/2))/(2*a^5*b - 2*a^4*b^2 - (2*a^8*b^2)/(a^3*b - a^4) + (2*a^9*b)/(a^3*b -
 a^4) + (2*a^6*b^2*(a^3*b)^(1/2))/(a^3*b - a^4) - (2*a^7*b*(a^3*b)^(1/2))/
(a^3*b - a^4)) + (8*a^4*b*tan(x)*(a^2/(16*(a^3*b - a^4)) - (a^3*b)^(1/2)/(
16*(a^3*b - a^4)))^(1/2)*(a^3*b)^(1/2))/(2*a^5*b - 2*a^4*b^2 - (2*a^8*b^2)
/(a^3*b - a^4) + (2*a^9*b)/(a^3*b - a^4) + (2*a^6*b^2*(a^3*b)^(1/2))/(a^3*
b - a^4) - (2*a^7*b*(a^3*b)^(1/2))/(a^3*b - a^4)))*((a^2 - (a^3*b)^(1/2))/
(16*(a^3*b - a^4)))^(1/2)
```

Reduce [F]

$$\int \frac{1}{a-b\cos^4(x)}\,dx = -\left(\int \frac{1}{\cos(x)^4 b-a}dx\right)$$

input
```
int(1/(a-b*cos(x)^4),x)
```

output
```
 - int(1/(cos(x)**4*b - a),x)
```

3.17 $\int \frac{1}{a-b\cos^6(x)}\,dx$

Optimal result 174
Mathematica [C] (verified) 175
Rubi [A] (verified) 175
Maple [C] (verified) 177
Fricas [C] (verification not implemented) 178
Sympy [F] 178
Maxima [F] 178
Giac [F(-1)] 179
Mupad [B] (verification not implemented) 179
Reduce [F] 180

Optimal result

Integrand size = 11, antiderivative size = 175

$$\int \frac{1}{a-b\cos^6(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
-1/3*arctan((a^(1/3)-b^(1/3))^(1/2)*cot(x)/a^(1/6))/a^(5/6)/(a^(1/3)-b^(1/
3))^(1/2)-1/3*arctan((a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)*cot(x)/a^(1/6))/a^
(5/6)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)-1/3*arctan((a^(1/3)-(-1)^(2/3)*b^
(1/3))^(1/2)*cot(x)/a^(1/6))/a^(5/6)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.83

$$\int \frac{1}{a-b\cos^6(x)}\,dx = -\frac{8}{3}\text{RootSum}\Big[b+6b\#1+15b\#1^2-64a\#1^3+20b\#1^3+15b\#1^4+6b\#1^5 + b\#1^6\&, \frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^2-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^2}{b+5b\#1-32a\#1^2+10b\#1^2+10b\#1^3+5b\#1^4+b\#1^5}\&\Big]$$

input
```
Integrate[(a - b*Cos[x]^6)^(-1),x]
```

output
```
(-8*RootSum[b + 6*b*#1 + 15*b*#1^2 - 64*a*#1^3 + 20*b*#1^3 + 15*b*#1^4 + 6
*b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 -
2*Cos[2*x]*#1 + #1^2]*#1^2)/(b + 5*b*#1 - 32*a*#1^2 + 10*b*#1^2 + 10*b*#1^
3 + 5*b*#1^4 + b*#1^5) & ])/3
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cos^6(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a-b\sin\left(x+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{\sqrt[3]{-1}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}}dx}{3a}$$

$$\Big\downarrow \text{3660}$$

$$-\frac{\int \frac{1}{\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)\cot^2(x)+1}d\cot(x)}{3a}-\frac{\int \frac{1}{\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\cot^2(x)+1}d\cot(x)}{3a}-\frac{\int \frac{1}{\left(1-\frac{(-1)^{2/3}\sqrt[3]{b}}{\sqrt[3]{a}}\right)\cot^2(x)+1}d\cot(x)}{3a}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a - b*Cos[x]^6)^(-1),x]
```

output
```
-1/3*ArcTan[(Sqrt[a^(1/3) - b^(1/3)]*Cot[x])/a^(1/6)]/(a^(5/6)*Sqrt[a^(1/3
) - b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Cot[x])/a^(1/6)
]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) -
(-1)^(2/3)*b^(1/3)]*Cot[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*
b^(1/3)])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.35

method	result
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}a+3\_Z^{4}a+3\_Z^{2}a+a-b\right)}\frac{\left(\_R^{4}+2\_R^{2}+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^{5}+2\_R^{3}+\_R}}{6a}$
risch	$\sum_{\_R=\operatorname{RootOf}\left(1+\left(46656a^{6}-46656a^{5}b\right)\_Z^{6}+3888a^{4}\_Z^{4}+108a^{2}\_Z^{2}\right)}\_R\ln\left(e^{2ix}+\left(\frac{15552ia^{6}}{b}-15552ia^{5}\right)\_R^{5}+\right.$

input
```
int(1/(a-b*cos(x)^6),x,method=_RETURNVERBOSE)
```

output
```
1/6/a*sum((_R^4+2*_R^2+1)/(_R^5+2*_R^3+_R)*ln(tan(x)-_R),_R=RootOf(_Z^6*a+
3*_Z^4*a+3*_Z^2*a+a-b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 16679, normalized size of antiderivative = 95.31

$$\int \frac{1}{a-b\cos^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*cos(x)^6),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{a-b\cos^6(x)}\,dx = \int \frac{1}{a-b\cos^6(x)}\,dx$$

input
```
integrate(1/(a-b*cos(x)**6),x)
```

output
```
Integral(1/(a - b*cos(x)**6), x)
```

Maxima [F]

$$\int \frac{1}{a-b\cos^6(x)}\,dx = \int -\frac{1}{b\cos(x)^6-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^6),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cos(x)^6 - a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{a-b\cos^6(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-b*cos(x)^6),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

$$\int \frac{1}{a-b\cos^6(x)}\,dx$$
$$=\sum_{k=1}^{6}\ln\Big(-\mathrm{root}\big(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k\big)^2\,a^3\,b^3\,\Big(\mathrm{root}\big(46656\,a^5\,b\,d^6-$$
$$+1\Big)\,\big(\mathrm{root}\big(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k\big)\;a\tan(x)\;6$$
$$-1\big)\;36\Big)\;\mathrm{root}\big(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k\big)$$

input
```
int(1/(a - b*cos(x)^6),x)
```

output
```
symsum(log(-36*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a
^2*d^2 - 1, d, k)^2*a^3*b^3*(36*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 388
8*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^2*a^2 + 1)*(6*root(46656*a^5*b*d^6 - 46
656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)*a*tan(x) - 1))*root(46
656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k), k,
1, 6)
```

Reduce [F]

$$\int \frac{1}{a-b\cos^6(x)}\,dx = -\left(\int \frac{1}{\cos(x)^6 b-a}dx\right)$$

input
```
int(1/(a-b*cos(x)^6),x)
```

output
```
 - int(1/(cos(x)**6*b - a),x)
```

3.18 $\int \frac{1}{a-b\cos^8(x)}\,dx$

Optimal result 181
Mathematica [C] (warning: unable to verify) 182
Rubi [A] (verified) 182
Maple [C] (verified) 184
Fricas [B] (verification not implemented) 185
Sympy [F] 185
Maxima [F] 185
Giac [F] 186
Mupad [B] (verification not implemented) 186
Reduce [F] 187

Optimal result

Integrand size = 11, antiderivative size = 213

$$\int \frac{1}{a-b\cos^8(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

output
```
-1/4*arctan((a^(1/4)-b^(1/4))^(1/2)*cot(x)/a^(1/8))/a^(7/8)/(a^(1/4)-b^(1/
4))^(1/2)-1/4*arctan((a^(1/4)-I*b^(1/4))^(1/2)*cot(x)/a^(1/8))/a^(7/8)/(a^
(1/4)-I*b^(1/4))^(1/2)-1/4*arctan((a^(1/4)+I*b^(1/4))^(1/2)*cot(x)/a^(1/8)
)/a^(7/8)/(a^(1/4)+I*b^(1/4))^(1/2)-1/4*arctan((a^(1/4)+b^(1/4))^(1/2)*cot
(x)/a^(1/8))/a^(7/8)/(a^(1/4)+b^(1/4))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.81

$$\int \frac{1}{a-b\cos^8(x)}\,dx = -8\text{RootSum}\Bigg[b+8b\#1+28b\#1^2+56b\#1^3-256a\#1^4+70b\#1^4+56b\#1^5+28b\#1^6+8b\#1^7+b\#1^8\&,\frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{b+7b\#1+21b\#1^2-128a\#1^3+35b\#1^3+35b\#1^4+21b\#1^5+7b\#1^6+b\#1^7}\&\Bigg]$$

input
```
Integrate[(a - b*Cos[x]^8)^(-1),x]
```

output
```
-8*RootSum[b + 8*b*#1 + 28*b*#1^2 + 56*b*#1^3 - 256*a*#1^4 + 70*b*#1^4 + 5
6*b*#1^5 + 28*b*#1^6 + 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x]
- #1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(b + 7*b*#1 + 21*b*#1^
2 - 128*a*#1^3 + 35*b*#1^3 + 35*b*#1^4 + 21*b*#1^5 + 7*b*#1^6 + b*#1^7) &
]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cos^8(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a-b\sin\left(x+\frac{\pi}{2}\right)^8}dx$$

$$\Big\downarrow \ 3690$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}+1}dx}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{a}}+1}dx}{4a}$$

$$\Big\downarrow \ 3660$$

$$-\frac{\int \frac{1}{\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right)\cot^2(x)+1}d\cot(x)}{4a}-\frac{\int \frac{1}{\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}\right)\cot^2(x)+1}d\cot(x)}{4a}-\frac{\int \frac{1}{\left(\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}+1\right)\cot^2(x)+1}d\cot(x)}{4a}-\frac{\int \frac{1}{\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}+1\right)\cot^2(x)+1}d\cot(x)}{4a}$$

$$\Big\downarrow \ 216$$

$$-\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

input
```
Int[(a - b*Cos[x]^8)^(-1),x]
```

output
```
-1/4*ArcTan[(Sqrt[a^(1/4) - b^(1/4)]*Cot[x])/a^(1/8)]/(a^(7/8)*Sqrt[a^(1/4
) - b^(1/4)]) - ArcTan[(Sqrt[a^(1/4) - I*b^(1/4)]*Cot[x])/a^(1/8)]/(4*a^(7
/8)*Sqrt[a^(1/4) - I*b^(1/4)]) - ArcTan[(Sqrt[a^(1/4) + I*b^(1/4)]*Cot[x])
/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + I*b^(1/4)]) - ArcTan[(Sqrt[a^(1/4) + b
^(1/4)]*Cot[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + b^(1/4)])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.37

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(\_Z^{8}a+4\_Z^{6}a+6\_Z^{4}a+4\_Z^{2}a+a-b\right)}\frac{\left(\_R^{6}+3\_R^{4}+3\_R^{2}+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^{7}+3\_R^{5}+3\_R^{3}+\_R}}{8a}$
risch	$\sum_{\_R=\text{RootOf}\left(1+\left(16777216a^{8}-16777216a^{7}b\right)\_Z^{8}+1048576a^{6}\_Z^{6}+24576a^{4}\_Z^{4}+256a^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{4194304ia^{8}}{b}\right.\right.$

input
```
int(1/(a-b*cos(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=
RootOf(_Z^8*a+4*_Z^6*a+6*_Z^4*a+4*_Z^2*a+a-b))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 643291 vs. 2(133) = 266.

Time = 6.51 (sec) , antiderivative size = 643291, normalized size of antiderivative = 3020.15

$$\int \frac{1}{a-b\cos^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*cos(x)^8),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{a-b\cos^8(x)}\,dx = \int \frac{1}{a-b\cos^8{(x)}}\,dx$$

input
```
integrate(1/(a-b*cos(x)**8),x)
```

output
```
Integral(1/(a - b*cos(x)**8), x)
```

Maxima [F]

$$\int \frac{1}{a-b\cos^8(x)}\,dx = \int -\frac{1}{b\cos{(x)}^8-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^8),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cos(x)^8 - a), x)
```

Giac [F]

$$\int \frac{1}{a-b\cos^8(x)}\,dx=\int -\frac{1}{b\cos{(x)}^8-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^8),x, algorithm="giac")
```

output
```
integrate(-1/(b*cos(x)^8 - a), x)
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.01

$$\int \frac{1}{a-b\cos^8(x)}\,dx$$
$$=\sum_{k=1}^{8}\ln\Big(-\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)^4\,a^5$$
$$+1\Big)\,\big(\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\big)\,a\tan(x)$$
$$-1\big)\,4096\Big)\,\mathrm{root}\big(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4$$
$$-256\,a^2\,d^2-1,d,k\big)$$

input
```
int(1/(a - b*cos(x)^8),x)
```

output
```
symsum(log(-4096*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*
d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^4*a^5*b^5*(64*root(16777216*a
^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^
2 - 1, d, k)^2*a^2 + 1)*(8*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 10
48576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)*a*tan(x) - 1))*root
(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 -
 256*a^2*d^2 - 1, d, k), k, 1, 8)
```

Reduce [F]

$$\int \frac{1}{a-b\cos^8(x)}\,dx=-\left(\int \frac{1}{\cos(x)^8 b-a}dx\right)$$

input
```
int(1/(a-b*cos(x)^8),x)
```

output
```
- int(1/(cos(x)**8*b - a),x)
```

3.19 $\int \frac{1}{a-b\cos(x)}\,dx$

Optimal result 188
Mathematica [A] (verified) 188
Rubi [A] (verified) 189
Maple [A] (verified) 190
Fricas [A] (verification not implemented) 190
Sympy [B] (verification not implemented) 191
Maxima [F(-2)] 192
Giac [A] (verification not implemented) 192
Mupad [B] (verification not implemented) 192
Reduce [B] (verification not implemented) 193

Optimal result

Integrand size = 9, antiderivative size = 42

$$\int \frac{1}{a-b\cos(x)}\,dx = \frac{2\arctan\left(\frac{\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

output
```
2*arctan((a+b)^(1/2)*tan(1/2*x)/(a-b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{1}{a-b\cos(x)}\,dx = -\frac{2\text{arctanh}\left(\frac{(a+b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
Integrate[(a - b*Cos[x])^(-1),x]
```

output
```
(-2*ArcTanh[((a + b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cos(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a-b\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3138}$$

$$2\int \frac{1}{(a+b)\tan^2\left(\frac{x}{2}\right)+a-b}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2\arctan\left(\frac{\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a - b*Cos[x])^(-1),x]
```

output
```
(2*ArcTan[(Sqrt[a + b]*Tan[x/2])/Sqrt[a - b]])/(Sqrt[a - b]*Sqrt[a + b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{2 \arctan\left(\frac{(a+b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}$	34
risch	$-\frac{\ln\left(e^{ix}-\frac{i a^2-i b^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{\ln\left(e^{ix}-\frac{-i a^2+i b^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	125

input
```
int(1/(a-b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
2/((a-b)*(a+b))^(1/2)*arctan((a+b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.36

$$\int \frac{1}{a-b\cos(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(-\frac{2\,ab\cos(x)-(2\,a^2-b^2)\cos(x)^2-2\,\sqrt{-a^2+b^2}(a\cos(x)-b)\sin(x)+a^2-2\,b^2}{b^2\cos(x)^2-2\,ab\cos(x)+a^2}\right)}{2\,(a^2-b^2)}, \frac{\arctan\left(-\frac{a\cos(x)-b}{\sqrt{a^2-b^2}\sin(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

input
```
integrate(1/(a-b*cos(x)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 + b^2)*log(-(2*a*b*cos(x) - (2*a^2 - b^2)*cos(x)^2 - 2*sqr
t(-a^2 + b^2)*(a*cos(x) - b)*sin(x) + a^2 - 2*b^2)/(b^2*cos(x)^2 - 2*a*b*c
os(x) + a^2))/(a^2 - b^2), arctan(-(a*cos(x) - b)/(sqrt(a^2 - b^2)*sin(x))
)/sqrt(a^2 - b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(34) = 68$.

Time = 1.54 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.29

$$\int \frac{1}{a - b\cos(x)}\,dx = \begin{cases} \tilde{\infty}\left(-\log\left(\tan\left(\frac{x}{2}\right) - 1\right) + \log\left(\tan\left(\frac{x}{2}\right) + 1\right)\right) & \text{for } a = 0 \wedge b = 0 \\ -\frac{\tan\left(\frac{x}{2}\right)}{b} & \text{for } a = -b \\ -\frac{1}{b\tan\left(\frac{x}{2}\right)} & \text{for } a = b \\ \frac{\log\left(-\sqrt{-\frac{a}{a+b} + \frac{b}{a+b}} + \tan\left(\frac{x}{2}\right)\right)}{a\sqrt{-\frac{a}{a+b} + \frac{b}{a+b}} + b\sqrt{-\frac{a}{a+b} + \frac{b}{a+b}}} - \frac{\log\left(\sqrt{-\frac{a}{a+b} + \frac{b}{a+b}} + \tan\left(\frac{x}{2}\right)\right)}{a\sqrt{-\frac{a}{a+b} + \frac{b}{a+b}} + b\sqrt{-\frac{a}{a+b} + \frac{b}{a+b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a-b*cos(x)),x)
```

output
```
Piecewise((zoo*(-log(tan(x/2) - 1) + log(tan(x/2) + 1)), Eq(a, 0) & Eq(b,
0)), (-tan(x/2)/b, Eq(a, -b)), (-1/(b*tan(x/2)), Eq(a, b)), (log(-sqrt(-a/
(a + b) + b/(a + b)) + tan(x/2))/(a*sqrt(-a/(a + b) + b/(a + b)) + b*sqrt(
-a/(a + b) + b/(a + b))) - log(sqrt(-a/(a + b) + b/(a + b)) + tan(x/2))/(a
*sqrt(-a/(a + b) + b/(a + b)) + b*sqrt(-a/(a + b) + b/(a + b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a-b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a-b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int \frac{1}{a-b\cos(x)}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

input
```
integrate(1/(a-b*cos(x)),x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(1/2*x) + b*tan(
1/2*x))/sqrt(a^2 - b^2)))/sqrt(a^2 - b^2)
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{a-b\cos(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)\,(2a+2b)}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
int(1/(a - b*cos(x)),x)
```

output
```
(2*atan((tan(x/2)*(2*a + 2*b))/(2*(a^2 - b^2)^(1/2))))/(a^2 - b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \frac{1}{a - b\cos(x)}\,dx = \frac{2\sqrt{a^2 - b^2}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a + \tan\left(\frac{x}{2}\right)b}{\sqrt{a^2 - b^2}}\right)}{a^2 - b^2}$$

input
```
int(1/(a-b*cos(x)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan(x/2)*a + tan(x/2)*b)/sqrt(a**2 - b**2)))/(a
**2 - b**2)
```

3.20 $\int \frac{1}{a-b\cos^3(x)}\,dx$

Optimal result 194
Mathematica [C] (verified) 195
Rubi [A] (verified) 196
Maple [C] (verified) 197
Fricas [C] (verification not implemented) 198
Sympy [F(-1)] 198
Maxima [F] 199
Giac [F] 199
Mupad [B] (verification not implemented) 199
Reduce [F] 200

Optimal result

Integrand size = 11, antiderivative size = 288

$$\int \frac{1}{a-b\cos^3(x)}\,dx = \frac{2\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} + \frac{2\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{2\arctan\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
2/3*arctan((a^(1/3)+b^(1/3))^(1/2)*tan(1/2*x)/(a^(1/3)-b^(1/3))^(1/2))/a^(
2/3)/(a^(1/3)-b^(1/3))^(1/2)/(a^(1/3)+b^(1/3))^(1/2)+2/3*arctan((a^(1/3)-(
-1)^(1/3)*b^(1/3))^(1/2)*tan(1/2*x)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2))/a^
(2/3)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2
)+2/3*arctan((a^(1/3)+(-1)^(2/3)*b^(1/3))^(1/2)*tan(1/2*x)/(a^(1/3)-(-1)^(
2/3)*b^(1/3))^(1/2))/a^(2/3)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)/(a^(1/3)+(
-1)^(2/3)*b^(1/3))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.33

$$\int \frac{1}{a-b\cos^3(x)}\,dx$$
$$= -\frac{2}{3}\text{RootSum}\left[b+3b\#1^2-8a\#1^3+3b\#1^4+b\#1^6\&,\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1}{b-4a\#1+2b\#1^2+b\#1^4}\&\right]$$

input
```
Integrate[(a - b*Cos[x]^3)^(-1),x]
```

output
```
(-2*RootSum[b + 3*b*#1^2 - 8*a*#1^3 + 3*b*#1^4 + b*#1^6 & , (2*ArcTan[Sin[
x]/(Cos[x] - #1)]*#1 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1)/(b - 4*a*#1 + 2*b
*#1^2 + b*#1^4) & ])/3
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cos^3(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin\left(x+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(\frac{1}{3a^{2/3}\left(\sqrt[3]{a}-\sqrt[3]{b}\cos(x)\right)}+\frac{1}{3a^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\cos(x)\right)}+\frac{1}{3a^{2/3}\left(\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\cos(x)\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a - b*Cos[x]^3)^(-1),x]
```

output
```
(2*ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*Tan[x/2])/Sqrt[a^(1/3) - b^(1/3)]])/(3*
a^(2/3)*Sqrt[a^(1/3) - b^(1/3)]*Sqrt[a^(1/3) + b^(1/3)]) + (2*ArcTan[(Sqrt
[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Tan[x/2])/Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)
]])/(3*a^(2/3)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Sqrt[a^(1/3) + (-1)^(1/3
)*b^(1/3)]) + (2*ArcTan[(Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)]*Tan[x/2])/Sqrt
[a^(1/3) - (-1)^(2/3)*b^(1/3)]])/(3*a^(2/3)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(1
/3)]*Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.32

method	result
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left((a+b)\_Z^{6}+(3a-3b)\_Z^{4}+(3a+3b)\_Z^{2}+a-b\right)}\frac{\left(\_R^{4}+2\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{5}a+\_R^{5}b+2\_R^{3}a-2\_R^{3}b+\_Ra+\_Rb}\right)}{3}$
risch	$\sum_{\_R=\operatorname{RootOf}\left(1+(729a^{6}-729a^{4}b^{2})\_Z^{6}+243a^{4}\_Z^{4}+27a^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{ix}+\left(-\frac{486ia^{6}}{b}+486ib\,a^{4}\right)\_R^{5}+\left(\frac{81a^{5}}{b}\right.\right.$

input
```
int(1/(a-b*cos(x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*a-2*_R^3*b+_R*a+_R*b)*ln(tan
(1/2*x)-_R),_R=RootOf((a+b)*_Z^6+(3*a-3*b)*_Z^4+(3*a+3*b)*_Z^2+a-b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 18595, normalized size of antiderivative = 64.57

$$\int \frac{1}{a-b\cos^3(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*cos(x)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a-b\cos^3(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-b*cos(x)**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a-b\cos^3(x)}\,dx = \int -\frac{1}{b\cos{(x)}^3-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^3),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cos(x)^3 - a), x)
```

Giac [F]

$$\int \frac{1}{a-b\cos^3(x)}\,dx = \int -\frac{1}{b\cos{(x)}^3-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^3),x, algorithm="giac")
```

output
```
integrate(-1/(b*cos(x)^3 - a), x)
```

Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.98

$$\int \frac{1}{a-b\cos^3(x)}\,dx = \sum_{k=1}^{6} \ln\left(-\frac{24576\,b^3\,(a+b)\,\left(324\tan\left(\frac{x}{2}\right)\,a^5+648\tan\left(\frac{x}{2}\right)\,a^4\,b-81\,a^4\,\mathrm{root}(d^6+27\,a^2\,d^4+243\,a^4\,d^2+72}{}\right.$$

$$-729\,a^6\,d^6-243\,a^4\,d^4-27\,a^2\,d^2-1,d,k)$$

input
```
int(1/(a - b*cos(x)^3),x)
```

output
```
symsum(log(-(24576*b^3*(a + b)*(324*a^5*tan(x/2) - 81*a^4*root(d^6 + 27*a^
2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) - root(d^6 + 27*a^2*d^4 +
 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5 - 18*a^2*root(d^6 + 27*a^2*d^4
 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3 + 324*a^3*b^2*tan(x/2) + 72*
a^3*tan(x/2)*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d,
 k)^2 - 27*a^2*b^2*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^
2), d, k) + 648*a^4*b*tan(x/2) + 4*a*tan(x/2)*root(d^6 + 27*a^2*d^4 + 243*
a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 + b*tan(x/2)*root(d^6 + 27*a^2*d^4
+ 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 9*a*b*root(d^6 + 27*a^2*d^4
 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3 - 108*b*a^3*root(d^6 + 27*a^
2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 72*a^2*b*tan(x/2)*root(
d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2))/root(d^6 +
 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5)*root(729*a^4*b^2
*d^6 - 729*a^6*d^6 - 243*a^4*d^4 - 27*a^2*d^2 - 1, d, k), k, 1, 6)
```

Reduce [F]

$$\int \frac{1}{a-b\cos^3(x)}\,dx = -\left(\int \frac{1}{\cos(x)^3 b-a}dx\right)$$

input
```
int(1/(a-b*cos(x)^3),x)
```

output
```
- int(1/(cos(x)**3*b - a),x)
```

3.21 $\int \frac{1}{a-b\cos^5(x)}\,dx$

Optimal result 202
Mathematica [C] (warning: unable to verify) 203
Rubi [A] (verified) 203
Maple [C] (verified) 205
Fricas [F(-2)] 206
Sympy [F] 206
Maxima [F] 206
Giac [F] 207
Mupad [B] (verification not implemented) 207
Reduce [F] 208

Optimal result

Integrand size = 11, antiderivative size = 494

$$\int \frac{1}{a-b\cos^5(x)}\,dx = \frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}} + \frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}} + \frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}} + \frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}} + \frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}$$

output
```
2/5*arctan((a^(1/5)+b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-b^(1/5))^(1/2))/a^(
4/5)/(a^(1/5)-b^(1/5))^(1/2)/(a^(1/5)+b^(1/5))^(1/2)+2/5*arctan((a^(1/5)-(
-1)^(1/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2))/a^
(4/5)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2
)+2/5*arctan((a^(1/5)+(-1)^(2/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-(-1)^(
2/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)/(a^(1/5)+(
-1)^(2/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)-(-1)^(3/5)*b^(1/5))^(1/2)*tan
(1/2*x)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(3/5)*b^
(1/5))^(1/2)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)+(-1)^(
4/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2))/a^(4/5)
/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.26

$$\int \frac{1}{a-b\cos^5(x)}\,dx = -\frac{8}{5}\text{RootSum}\left[b+5b\#1^2+10b\#1^4-32a\#1^5+10b\#1^6+5b\#1^8 + b\#1^{10}\&, \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3}{b+4b\#1^2-16a\#1^3+6b\#1^4+4b\#1^6+b\#1^8}\&\right]$$

input
```
Integrate[(a - b*Cos[x]^5)^(-1),x]
```

output
```
(-8*RootSum[b + 5*b*#1^2 + 10*b*#1^4 - 32*a*#1^5 + 10*b*#1^6 + 5*b*#1^8 +
b*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - I*Log[1 - 2*Cos[x]*#1 +
 #1^2]*#1^3)/(b + 4*b*#1^2 - 16*a*#1^3 + 6*b*#1^4 + 4*b*#1^6 + b*#1^8) & ]
)/5
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cos^5(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin\left(x+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-\sqrt[5]{b}\cos(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cos(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cos(x)\right)}+\frac{}{5a^{4/5}\left(\sqrt[5]{a}\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}$$

input
```
Int[(a - b*Cos[x]^5)^(-1),x]
```

output
```
(2*ArcTan[(Sqrt[a^(1/5) + b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - b^(1/5)]])/(5*
a^(4/5)*Sqrt[a^(1/5) - b^(1/5)]*Sqrt[a^(1/5) + b^(1/5)]) + (2*ArcTan[(Sqrt
[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + (-1)^(1/5)*b^(1/5)
]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(1/5
)*b^(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]*Tan[x/2])/Sqrt
[a^(1/5) - (-1)^(2/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(2/5)*b^(1
/5)]*Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) - (-1)^
(3/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]])/(5*a^(4/5)*S
qrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]) + (2
*ArcTan[(Sqrt[a^(1/5) + (-1)^(4/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - (-1)^
(4/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Sqrt[a^(1/5
) + (-1)^(4/5)*b^(1/5)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.94 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.30

method	result
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left((a+b)\_Z^{10}+(5a-5b)\_Z^{8}+(10a+10b)\_Z^{6}+(10a-10b)\_Z^{4}+(5a+5b)\_Z^{2}+a-b\right)} \frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}\right.}{\_R^{9}a+\_R^{9}b+4\_R^{7}a-4\_R^{7}b+6\_}\right.}{5}$
risch	$\sum_{\_R=\text{RootOf}\left(1+(9765625a^{10}-9765625a^{8}b^{2})\_Z^{10}+1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}+6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)} \_R\ln\left(e^{ix}+\right.$

input
```
int(1/(a-b*cos(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+_R^9*b+4*_R^7*a-4*_R^7*b+6*_
R^5*a+6*_R^5*b+4*_R^3*a-4*_R^3*b+_R*a+_R*b)*ln(tan(1/2*x)-_R),_R=RootOf((a
+b)*_Z^10+(5*a-5*b)*_Z^8+(10*a+10*b)*_Z^6+(10*a-10*b)*_Z^4+(5*a+5*b)*_Z^2+
a-b))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{a-b\cos^5(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-b*cos(x)^5),x, algorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F]

$$\int \frac{1}{a-b\cos^5(x)}\,dx = \int \frac{1}{a-b\cos^5{(x)}}\,dx$$

input
```
integrate(1/(a-b*cos(x)**5),x)
```

output
```
Integral(1/(a - b*cos(x)**5), x)
```

Maxima [F]

$$\int \frac{1}{a-b\cos^5(x)}\,dx = \int -\frac{1}{b\cos{(x)}^5-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^5),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cos(x)^5 - a), x)
```

Giac [F]

$$\int \frac{1}{a-b\cos^5(x)}\,dx=\int -\frac{1}{b\cos{(x)}^5-a}\,dx$$

input
```
integrate(1/(a-b*cos(x)^5),x, algorithm="giac")
```

output
```
integrate(-1/(b*cos(x)^5 - a), x)
```

Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 1518, normalized size of antiderivative = 3.07

$$\int \frac{1}{a-b\cos^5(x)}\,dx=\text{Too large to display}$$

input
```
int(1/(a - b*cos(x)^5),x)
```

output
```
symsum(log(-(10995116277760*b^7*(a + b)*(56*root(9765625*a^8*b^2*d^10 - 97
65625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^
2*d^2 - 1, d, k)*a - 7*cot(x/2) + root(9765625*a^8*b^2*d^10 - 9765625*a^10
*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
 d, k)*b + 5800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^
8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^3 + 225
000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1562
50*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 + 3875000*root(97
65625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6
- 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 + 25000000*root(9765625*a^8*
b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4
*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^9 - 735*root(9765625*a^8*b^2*d^10 - 9765
625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*
d^2 - 1, d, k)^2*a^2*cot(x/2) - 28875*root(9765625*a^8*b^2*d^10 - 9765625*
a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2
- 1, d, k)^4*a^4*cot(x/2) - 503125*root(9765625*a^8*b^2*d^10 - 9765625*a^1
0*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1
, d, k)^6*a^6*cot(x/2) - 3281250*root(9765625*a^8*b^2*d^10 - 9765625*a^10*
d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
d, k)^8*a^8*cot(x/2) + 800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^1...
```

Reduce [F]

$$\int \frac{1}{a-b\cos^5(x)}\,dx = -\left(\int \frac{1}{\cos(x)^5 b-a}dx\right)$$

input
```
int(1/(a-b*cos(x)^5),x)
```

output
```
 - int(1/(cos(x)**5*b - a),x)
```

3.22 $\int \frac{1}{a+b\cos^2(x)}\,dx$

Optimal result 209
Mathematica [A] (verified) 209
Rubi [A] (verified) 210
Maple [A] (verified) 211
Fricas [B] (verification not implemented) 211
Sympy [B] (verification not implemented) 212
Maxima [A] (verification not implemented) 213
Giac [A] (verification not implemented) 213
Mupad [B] (verification not implemented) 213
Reduce [B] (verification not implemented) 214

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{1}{a+b\cos^2(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

output
```
-arctan((a+b)^(1/2)*cot(x)/a^(1/2))/a^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \; 3660$$

$$-\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \; 218$$

$$-\frac{\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Int[(a + b*Cos[x]^2)^(-1),x]
```

output
```
-(ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$	21
risch	$-\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ba}+b\sqrt{-a^2-ba}}{b\sqrt{-a^2-ba}}\right)}{2\sqrt{-a^2-ba}}+\frac{\ln\left(e^{2ix}-\frac{2ia^2+2iab-2a\sqrt{-a^2-ba}-b\sqrt{-a^2-ba}}{b\sqrt{-a^2-ba}}\right)}{2\sqrt{-a^2-ba}}$	160

input
```
int(1/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 5.43

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2+4\,\left((2\,a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right)}{4\,(a^2+ab)}, -\frac{\arctan\left(\frac{(2\,a+b)\cos(x)^2-a}{2\,\sqrt{a^2+ab}\cos(x)\sin(x)}\right)}{2\,\sqrt{a^2+ab}}\right]$$

input
```
integrate(1/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*
a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x)
+ a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))/(a^2 + a*b), -1/2*arctan(1/2
*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))/sqrt(a^2 + a*b)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10924 vs. $2(29) = 58$.

Time = 14.66 (sec) , antiderivative size = 10924, normalized size of antiderivative = 364.13

$$\int \frac{1}{a + b\cos^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)**2),x)
```

output
```
Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (-tan(x/2
)/(2*b) + 1/(2*b*tan(x/2)), Eq(a, -b)), (-2*tan(x/2)/(b*(tan(x/2)**2 - 1))
, Eq(a, 0)), (a**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*log
(-sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**4*
sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a
 + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a/(a + b) + b/(a + b) - 2*
sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) -
8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt
(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(-a/(a
+ b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*s
qrt(-a*b)/(a + b)) + 2*a*b**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(
a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 8*a*b**2*sqr
t(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b
) + b/(a + b) + 2*sqrt(-a*b)/(a + b))) - a**3*sqrt(-a/(a + b) + b/(a + b)
- 2*sqrt(-a*b)/(a + b))*log(sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a
+ b)) + tan(x/2))/(2*a**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a +
b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-
a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b)
+ 2*sqrt(-a*b)/(a + b)) - 8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) -
2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$$

input
```
integrate(1/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
arctan(a*tan(x)/sqrt((a + b)*a))/sqrt((a + b)*a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}}$$

input
```
integrate(1/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))/sqrt(a^2
+ a*b)
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\mathrm{atan}\left(\frac{a\tan(x)}{\sqrt{a^2+b\,a}}\right)}{\sqrt{a^2+b\,a}}$$

input
```
int(1/(a + b*cos(x)^2),x)
```

output
```
atan((a*tan(x))/(a*b + a^2)^(1/2))/(a*b + a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.80

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\sqrt{a}\,\sqrt{a+b}\,\left(atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{x}{2}\right)-\sqrt{b}}{\sqrt{a}}\right)+atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{x}{2}\right)+\sqrt{b}}{\sqrt{a}}\right)\right)}{a\,(a+b)}$$

input

```
int(1/(a+b*cos(x)^2),x)
```

output

```
(sqrt(a)*sqrt(a + b)*(atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a)) + ata
n((sqrt(a + b)*tan(x/2) + sqrt(b))/sqrt(a))))/(a*(a + b))
```

3.23 $\int \frac{1}{a+b\cos^4(x)}\,dx$

Optimal result . . . 215
Mathematica [C] (verified) . . . 216
Rubi [A] (verified) . . . 216
Maple [C] (verified) . . . 221
Fricas [B] (verification not implemented) . . . 222
Sympy [F(-1)] . . . 222
Maxima [F] . . . 223
Giac [A] (verification not implemented) . . . 223
Mupad [B] (verification not implemented) . . . 224
Reduce [F] . . . 225

Optimal result

Integrand size = 10, antiderivative size = 350

$$\int \frac{1}{a+b\cos^4(x)}\,dx = \frac{\left(\sqrt{a}+\sqrt{a+b}\right)\arctan\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}-\sqrt{2}(a+b)^{3/4}\cot(x)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}} - \frac{\left(\sqrt{a}+\sqrt{a+b}\right)\arctan\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}+\sqrt{2}(a+b)^{3/4}\cot(x)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}} - \frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\cot(x)}{\sqrt{a+b}\left(\frac{\sqrt{a}}{\sqrt{a+b}}+\cot^2(x)\right)}\right)}{2\sqrt{2}a^{3/4}\sqrt{a+b}}$$

output
```
1/4*(a^(1/2)+(a+b)^(1/2))*arctan((a^(1/4)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)-
2^(1/2)*(a+b)^(3/4)*cot(x))/a^(1/4)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2))*2^(1/
2)/a^(3/4)/(a+b)^(1/4)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2)-1/4*(a^(1/2)+(a+b)^
(1/2))*arctan((a^(1/4)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)+2^(1/2)*(a+b)^(3/4)
*cot(x))/a^(1/4)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2))*2^(1/2)/a^(3/4)/(a+b)^(1
/4)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2)-1/4*(-a^(1/2)+(a+b)^(1/2))^(1/2)*arcta
nh(2^(1/2)*a^(1/4)*(-a^(1/2)+(a+b)^(1/2))^(1/2)*cot(x)/(a+b)^(1/2)/(a^(1/2
)/(a+b)^(1/2)+cot(x)^2))*2^(1/2)/a^(3/4)/(a+b)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.35

$$\int \frac{1}{a+b\cos^4(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{a+i\sqrt{a}\sqrt{b}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{-a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{-a+i\sqrt{a}\sqrt{b}}}$$

input
```
Integrate[(a + b*Cos[x]^4)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + I*Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a]*Sqrt[a + I
*Sqrt[a]*Sqrt[b]]) - ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[-a + I*Sqrt[a]*Sqrt[b]]
]/(2*Sqrt[a]*Sqrt[-a + I*Sqrt[a]*Sqrt[b]])
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.55, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3688, 1483, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^4(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3688}$$

$$-\int \frac{\cot^2(x)+1}{(a+b)\cot^4(x)+2a\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \text{1483}$$

$$-\frac{\sqrt[4]{a+b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\cot(x)}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}-\frac{\sqrt[4]{a+b}\int \frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\cot(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$\big\downarrow$ 1142

$$-\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}(a+b)^{5/4}}-\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int -\frac{\sqrt{2}\left(\cdots\right.}{\cot^2(x)-\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\left(\sqrt{2}\cot\cdots\right.}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$\big\downarrow$ 25

$$-\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\left(\frac{\sqrt[4]{\cdots}}{\cdots}\right.}{\cot^2(x)-\frac{\sqrt{2}\cdots}{\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\left(\sqrt{2}\cot\cdots\right.}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$\big\downarrow$ 27

$$-\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\frac{\sqrt[4]{a}\sqrt{a-\sqrt{\cdots}}}{(a+b)\cdots}}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{\cdots}}{\cdots}}\cdots}{\cdots}\cdots\right)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}(a+b)^{5/4}}+\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\cot(x)+\frac{\sqrt[4]{\cdots}}{\cdots}}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\cdots}}{(a\cdots}}\cdots}{\sqrt{2}\cdots}\cdots\right)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$\downarrow$ 1083

$$-\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\sqrt{2}\cot(x)}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}}-\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{-\left(2\cot(x)-\frac{\sqrt{2}\cdots}{\cdots}\right.\cdots}}{\cdots}\cdots\right)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\cot(x)+\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}}-\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{-\left(2\cot(x)+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\right.\cdots}}{\cdots}\cdots\right)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$\downarrow$ 217

$$-\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\sqrt{2}\cot(x)}{\cot^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}}+\frac{(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(2\cot(x\ldots\right.}{\sqrt{2}\sqrt[4]{\ldots}}\right.}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\int \frac{\sqrt{2}\cot(x)+\frac{\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\cot^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a-\sqrt{a+b}\sqrt{a}+b}\cot(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\cot(x)}{\sqrt{2}}+\frac{(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt{2}\sqrt[4]{a}\ldots}{\ldots}\right.}{\sqrt{2}\sqrt[4]{a}\ldots}\right.}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$\big\downarrow$ 1103

$$-\frac{\sqrt[4]{a+b}\left(\frac{(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(2\cot(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}{(a+b)^{3/4}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}-\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left((a+b)^{3/4}\ldots\right.\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{(\sqrt{a+b}+\sqrt{a})\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}{(a+b)^{3/4}}+2\cot(x)\right)}{\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{\sqrt{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left((a+b)^{3/4}c\ldots\right.\right.}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

input
```
Int[(a + b*Cos[x]^4)^(-1),x]
```

output
```
-1/2*((a + b)^(1/4)*(((Sqrt[a] + Sqrt[a + b])*Sqrt[a + b - Sqrt[a]*Sqrt[a
+ b]]*ArcTan[((a + b)^(3/4)*(-((Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[
a + b]])/(a + b)^(3/4)) + 2*Cot[x]))/(Sqrt[2]*a^(1/4)*Sqrt[a + b + Sqrt[a]
*Sqrt[a + b]])])/(Sqrt[a + b]*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) - ((1 - S
qrt[a]/Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/4) - Sqrt[2]*a^(1/4)*Sqrt[a + b
 - Sqrt[a]*Sqrt[a + b]]*Cot[x] + (a + b)^(3/4)*Cot[x]^2])/2))/(Sqrt[2]*a^(
3/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]) - ((a + b)^(1/4)*(((Sqrt[a] + Sqrt
[a + b])*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]*ArcTan[((a + b)^(3/4)*((Sqrt[2]
*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]])/(a + b)^(3/4) + 2*Cot[x]))/(Sq
rt[2]*a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])])/(Sqrt[a + b]*Sqrt[a + b
 + Sqrt[a]*Sqrt[a + b]]) + ((1 - Sqrt[a]/Sqrt[a + b])*Log[Sqrt[a]*(a + b)^
(1/4) + Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]*Cot[x] + (a + b)
^(3/4)*Cot[x]^2])/2))/(2*Sqrt[2]*a^(3/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.29

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+\left(256a^4+256a^3b\right)\_Z^4+32a^2\_Z^2\right)} \_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{128ia^4}{b}+128ia^3\right)\_R^3+\left(-\frac{32a^3}{b}-32a^2\right)\_$
default	Expression too large to display

input
```
int(1/(a+b*cos(x)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*I*x)+(128*I/b*a^4+128*I*a^3)*_R^3+(-32/b*a^3-32*a^2)*_R^2+
(8*I/b*a^2-8*I*a)*_R-2/b*a+1),_R=RootOf(1+(256*a^4+256*a^3*b)*_Z^4+32*a^2*
_Z^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 809 vs. $2(245) = 490$.

Time = 0.19 (sec) , antiderivative size = 809, normalized size of antiderivative = 2.31

$$\int \frac{1}{a + b\cos^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)^4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b
))*log(b*cos(x)^2 + 2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*
a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b
 + a^3*b^2)) + 1)/(a^2 + a*b)) - (a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*
sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))) + 1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5
+ 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b))*log(b*cos(x)^2 - 2*(a*b*cos(x)*sin
(x) + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt
(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)) - (a^3
 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))) +
 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)
)*log(-b*cos(x)^2 + 2*(a*b*cos(x)*sin(x) - (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*
a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b
+ a^3*b^2)) - 1)/(a^2 + a*b)) - (a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*s
qrt(-b/(a^5 + 2*a^4*b + a^3*b^2))) - 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 +
2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b))*log(-b*cos(x)^2 - 2*(a*b*cos(x)*sin(
x) - (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(
((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - (a^3 +
 a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a + b\cos^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(x)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a+b\cos^4(x)}\,dx=\int \frac{1}{b\cos{(x)}^4+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^4),x, algorithm="maxima")
```

output
```
integrate(1/(b*cos(x)^4 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+b\cos^4(x)}\,dx = \frac{\left(3\sqrt{a^2+\sqrt{-ab}a}a^2+4\sqrt{a^2+\sqrt{-ab}a}ab-3\sqrt{a^2+\sqrt{-ab}a}\sqrt{-ab}a-4\sqrt{a^2+\sqrt{-ab}a}\sqrt{-ab}b\right)\left(\pi\left\lfloor\frac{x}{\pi}\right.\right.}{2\left(3a^5+7a^4b+4a^3b^2\right)} + \frac{\left(3\sqrt{a^2-\sqrt{-ab}a}a^2+4\sqrt{a^2-\sqrt{-ab}a}ab-3\sqrt{a^2-\sqrt{-ab}a}\sqrt{-ab}a-4\sqrt{a^2-\sqrt{-ab}a}\sqrt{-ab}b\right)\left(\pi\left\lfloor\right.\right.}{2\left(3a^5+7a^4b+4a^3b^2\right)}$$

input
```
integrate(1/(a+b*cos(x)^4),x, algorithm="giac")
```

output
```
1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*a^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b - 3*s
qrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a - 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*
b)*b)*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a + b)
*a + 16*a^2))/a)))*abs(a)/(3*a^5 + 7*a^4*b + 4*a^3*b^2) + 1/2*(3*sqrt(a^2
- sqrt(-a*b)*a)*a^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b - 3*sqrt(a^2 - sqrt(-
a*b)*a)*sqrt(-a*b)*a - 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*b)*(pi*floor(
x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a - sqrt(-16*(a + b)*a + 16*a^2))/a)
))*abs(a)/(3*a^5 + 7*a^4*b + 4*a^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 926, normalized size of antiderivative = 2.65

$$\int \frac{1}{a+b\cos^4(x)}\,dx$$
$$= -2\,\mathrm{atanh}\left(\frac{8\,a^6\,b\tan(x)\,\sqrt{-\frac{a^2}{16\,(a^4+b\,a^3)}-\frac{\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}}}{\frac{2\,a^9\,b}{a^4+b\,a^3}-2\,a^4\,b^2-2\,a^5\,b+\frac{2\,a^8\,b^2}{a^4+b\,a^3}+\frac{2\,a^7\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}+\frac{2\,a^6\,b^2\,\sqrt{-a^3\,b}}{a^4+b\,a^3}} - \frac{8\,a^2\,b\tan(x)\,\sqrt{-\frac{a^2}{16\,(a^4+b\,a^3)}-\frac{\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}}}{\frac{2\,a^5\,b}{a^4+b\,a^3}-2\,a\,b+\frac{2\,a^3\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}} + \frac{8\,a^4\,b\tan(x)\,\sqrt{-a^3\,b}\,\sqrt{-\frac{a^2}{16\,(a^4+b\,a^3)}-\frac{\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}}}{\frac{2\,a^9\,b}{a^4+b\,a^3}-2\,a^4\,b^2-2\,a^5\,b+\frac{2\,a^8\,b^2}{a^4+b\,a^3}+\frac{2\,a^7\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}+\frac{2\,a^6\,b^2\,\sqrt{-a^3\,b}}{a^4+b\,a^3}}\right)\sqrt{-\frac{a^2+\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}}$$
$$- 2\,\mathrm{atanh}\left(\frac{8\,a^2\,b\tan(x)\,\sqrt{\frac{\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}-\frac{a^2}{16\,(a^4+b\,a^3)}}}{2\,a\,b-\frac{2\,a^5\,b}{a^4+b\,a^3}+\frac{2\,a^3\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}} - \frac{8\,a^6\,b\tan(x)\,\sqrt{\frac{\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}-\frac{a^2}{16\,(a^4+b\,a^3)}}}{2\,a^5\,b+2\,a^4\,b^2-\frac{2\,a^9\,b}{a^4+b\,a^3}-\frac{2\,a^8\,b^2}{a^4+b\,a^3}+\frac{2\,a^7\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}+\frac{2\,a^6\,b^2\,\sqrt{-a^3\,b}}{a^4+b\,a^3}} + \frac{8\,a^4\,b\tan(x)\,\sqrt{-a^3\,b}\,\sqrt{\frac{\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}-\frac{a^2}{16\,(a^4+b\,a^3)}}}{2\,a^5\,b+2\,a^4\,b^2-\frac{2\,a^9\,b}{a^4+b\,a^3}-\frac{2\,a^8\,b^2}{a^4+b\,a^3}+\frac{2\,a^7\,b\,\sqrt{-a^3\,b}}{a^4+b\,a^3}+\frac{2\,a^6\,b^2\,\sqrt{-a^3\,b}}{a^4+b\,a^3}}\right)\sqrt{-\frac{a^2-\sqrt{-a^3\,b}}{16\,(a^4+b\,a^3)}}$$

input
```
int(1/(a + b*cos(x)^4),x)
```

output
```
- 2*atanh((8*a^6*b*tan(x)*(- a^2/(16*(a^3*b + a^4)) - (-a^3*b)^(1/2)/(16*(
a^3*b + a^4)))^(1/2))/((2*a^9*b)/(a^3*b + a^4) - 2*a^4*b^2 - 2*a^5*b + (2*
a^8*b^2)/(a^3*b + a^4) + (2*a^7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*a^6*b
^2*(-a^3*b)^(1/2))/(a^3*b + a^4)) - (8*a^2*b*tan(x)*(- a^2/(16*(a^3*b + a^
4)) - (-a^3*b)^(1/2)/(16*(a^3*b + a^4)))^(1/2))/((2*a^5*b)/(a^3*b + a^4) -
 2*a*b + (2*a^3*b*(-a^3*b)^(1/2))/(a^3*b + a^4)) + (8*a^4*b*tan(x)*(-a^3*b
)^(1/2)*(- a^2/(16*(a^3*b + a^4)) - (-a^3*b)^(1/2)/(16*(a^3*b + a^4)))^(1/
2))/((2*a^9*b)/(a^3*b + a^4) - 2*a^4*b^2 - 2*a^5*b + (2*a^8*b^2)/(a^3*b +
a^4) + (2*a^7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*a^6*b^2*(-a^3*b)^(1/2))
/(a^3*b + a^4)))*(-(a^2 + (-a^3*b)^(1/2))/(16*(a^3*b + a^4)))^(1/2) - 2*at
anh((8*a^2*b*tan(x)*((-a^3*b)^(1/2)/(16*(a^3*b + a^4)) - a^2/(16*(a^3*b +
a^4)))^(1/2))/(2*a*b - (2*a^5*b)/(a^3*b + a^4) + (2*a^3*b*(-a^3*b)^(1/2))/
(a^3*b + a^4)) - (8*a^6*b*tan(x)*((-a^3*b)^(1/2)/(16*(a^3*b + a^4)) - a^2/
(16*(a^3*b + a^4)))^(1/2))/(2*a^5*b + 2*a^4*b^2 - (2*a^9*b)/(a^3*b + a^4)
- (2*a^8*b^2)/(a^3*b + a^4) + (2*a^7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*
a^6*b^2*(-a^3*b)^(1/2))/(a^3*b + a^4)) + (8*a^4*b*tan(x)*(-a^3*b)^(1/2)*((
-a^3*b)^(1/2)/(16*(a^3*b + a^4)) - a^2/(16*(a^3*b + a^4)))^(1/2))/(2*a^5*b
 + 2*a^4*b^2 - (2*a^9*b)/(a^3*b + a^4) - (2*a^8*b^2)/(a^3*b + a^4) + (2*a^
7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*a^6*b^2*(-a^3*b)^(1/2))/(a^3*b + a^
4)))*(-(a^2 - (-a^3*b)^(1/2))/(16*(a^3*b + a^4)))^(1/2)
```

Reduce [F]

$$\int \frac{1}{a+b\cos^4(x)}\,dx=\int \frac{1}{\cos{(x)}^4\,b+a}dx$$

input
```
int(1/(a+b*cos(x)^4),x)
```

output
```
int(1/(cos(x)**4*b + a),x)
```

3.24 $\int \frac{1}{a+b\cos^6(x)}\,dx$

Optimal result . . . 226
Mathematica [C] (verified) . . . 227
Rubi [A] (verified) . . . 227
Maple [C] (verified) . . . 229
Fricas [C] (verification not implemented) . . . 230
Sympy [F] . . . 230
Maxima [F] . . . 230
Giac [F(-1)] . . . 231
Mupad [B] (verification not implemented) . . . 231
Reduce [F] . . . 232

Optimal result

Integrand size = 10, antiderivative size = 171

$$\int \frac{1}{a+b\cos^6(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}} - \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
-1/3*arctan((a^(1/3)+b^(1/3))^(1/2)*cot(x)/a^(1/6))/a^(5/6)/(a^(1/3)+b^(1/
3))^(1/2)-1/3*arctan((a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)*cot(x)/a^(1/6))/a^
(5/6)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)-1/3*arctan((a^(1/3)+(-1)^(2/3)*b^
(1/3))^(1/2)*cot(x)/a^(1/6))/a^(5/6)/(a^(1/3)+(-1)^(2/3)*b^(1/3))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+b\cos^6(x)}\,dx$$
$$=\frac{8}{3}\text{RootSum}\left[b+6b\#1+15b\#1^2+64a\#1^3+20b\#1^3+15b\#1^4+6b\#1^5+b\#1^6\&,\frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^2-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^2}{b+5b\#1+32a\#1^2+10b\#1^2+10b\#1^3+5b\#1^4+b\#1^5}\&\right]$$

input
```
Integrate[(a + b*Cos[x]^6)^(-1),x]
```

output
```
(8*RootSum[b + 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 + 20*b*#1^3 + 15*b*#1^4 + 6*
b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2
*Cos[2*x]*#1 + #1^2]*#1^2)/(b + 5*b*#1 + 32*a*#1^2 + 10*b*#1^2 + 10*b*#1^3
 + 5*b*#1^4 + b*#1^5) & ])/3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^6(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{\frac{\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{(-1)^{2/3}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}+1}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\frac{\sqrt[3]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}+1}dx}{3a}$$

$$\Big\downarrow \text{3660}$$

$$-\frac{\int \frac{1}{\left(\frac{\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\cot^2(x)+1}d\cot(x)}{3a}-\frac{\int \frac{1}{\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)\cot^2(x)+1}d\cot(x)}{3a}-\frac{\int \frac{1}{\left(\frac{(-1)^{2/3}\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\cot^2(x)+1}d\cot(x)}{3a}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

input

```
Int[(a + b*Cos[x]^6)^(-1),x]
```

output

```
-1/3*ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*Cot[x])/a^(1/6)]/(a^(5/6)*Sqrt[a^(1/3
) + b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Cot[x])/a^(1/6)
]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) +
(-1)^(2/3)*b^(1/3)]*Cot[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(2/3)*
b^(1/3)])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.35

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6 a+3\_Z^4 a+3\_Z^2 a+a+b\right)} \frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^5+2\_R^3+\_R}}{6a}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(46656a^6+46656a^5 b)\_Z^6+3888a^4\_Z^4+108a^2\_Z^2\right)} \_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{15552ia^6}{b}-15552ia^5\right)\_R^5\right.$

input
```
int(1/(a+b*cos(x)^6),x,method=_RETURNVERBOSE)
```

output
```
1/6/a*sum((_R^4+2*_R^2+1)/(_R^5+2*_R^3+_R)*ln(tan(x)-_R),_R=RootOf(_Z^6*a+
3*_Z^4*a+3*_Z^2*a+a+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.32 (sec) , antiderivative size = 15483, normalized size of antiderivative = 90.54

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)^6),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \int \frac{1}{a+b\cos^6{(x)}}\,dx$$

input
```
integrate(1/(a+b*cos(x)**6),x)
```

output
```
Integral(1/(a + b*cos(x)**6), x)
```

Maxima [F]

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \int \frac{1}{b\cos{(x)}^6+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^6),x, algorithm="maxima")
```

output
```
integrate(1/(b*cos(x)^6 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(x)^6),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \sum_{k=1}^{6} \ln\left(\mathrm{root}\left(46656\,a^5\,b\,d^6+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k\right)^2\,a^3\,b^3\,\left(\mathrm{root}\left(46656\,a^5\,b\,d^6+4\right.\right.\right.$$
$$+1\Big)\,\left(\mathrm{root}\left(46656\,a^5\,b\,d^6+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k\right)\,a\tan(x)\,6\right.$$
$$\left.-1\right)\,36\Big)\,\mathrm{root}\left(46656\,a^5\,b\,d^6+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k\right)$$

input
```
int(1/(a + b*cos(x)^6),x)
```

output
```
symsum(log(36*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^
2*d^2 + 1, d, k)^2*a^3*b^3*(36*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888
*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^2*a^2 + 1)*(6*root(46656*a^5*b*d^6 + 466
56*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)*a*tan(x) - 1))*root(466
56*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k), k, 1
, 6)
```

Reduce [F]

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \int \frac{1}{\cos(x)^6 b + a}dx$$

input
```
int(1/(a+b*cos(x)^6),x)
```

output
```
int(1/(cos(x)**6*b + a),x)
```

3.25 $\int \frac{1}{a+b\cos^8(x)}\,dx$

Optimal result . . . 233
Mathematica [C] (warning: unable to verify) . . . 234
Rubi [A] (verified) . . . 234
Maple [C] (verified) . . . 236
Fricas [B] (verification not implemented) . . . 237
Sympy [F] . . . 237
Maxima [F] . . . 237
Giac [F] . . . 238
Mupad [B] (verification not implemented) . . . 238
Reduce [F] . . . 239

Optimal result

Integrand size = 10, antiderivative size = 245

$$\int \frac{1}{a+b\cos^8(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}+\frac{\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

output

```
1/4*arctan(((-a)^(1/4)-b^(1/4))^(1/2)*cot(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^
(1/4)-b^(1/4))^(1/2)+1/4*arctan(((-a)^(1/4)-I*b^(1/4))^(1/2)*cot(x)/(-a)^(
1/8))/(-a)^(7/8)/((-a)^(1/4)-I*b^(1/4))^(1/2)+1/4*arctan(((-a)^(1/4)+I*b^(
1/4))^(1/2)*cot(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+I*b^(1/4))^(1/2)+1/4
*arctan(((-a)^(1/4)+b^(1/4))^(1/2)*cot(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/
4)+b^(1/4))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.70

$$\int \frac{1}{a+b\cos^8(x)}\,dx = 8\text{RootSum}\left[b+8b\#1+28b\#1^2+56b\#1^3+256a\#1^4+70b\#1^4+56b\#1^5+28b\#1^6+8b\#1^7+b\#1^8\&,\frac{2\arctan\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{b+7b\#1+21b\#1^2+128a\#1^3+35b\#1^3+35b\#1^4+21b\#1^5+7b\#1^6+b\#1^7}\&\right]$$

input
```
Integrate[(a + b*Cos[x]^8)^(-1),x]
```

output
```
8*RootSum[b + 8*b*#1 + 28*b*#1^2 + 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 + 56
*b*#1^5 + 28*b*#1^6 + 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] -
 #1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(b + 7*b*#1 + 21*b*#1^2
 + 128*a*#1^3 + 35*b*#1^3 + 35*b*#1^4 + 21*b*#1^5 + 7*b*#1^6 + b*#1^7) & ]
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3690, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^8(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^8}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}+1}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{-a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin\left(x+\frac{\pi}{2}\right)^2}{\sqrt[4]{-a}}+1}dx}{4a}$$

$$\Big\downarrow \text{3660}$$

$$-\frac{\int \frac{1}{\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)\cot^2(x)+1}d\cot(x)}{4a}-\frac{\int \frac{1}{\left(\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}+1\right)\cot^2(x)+1}d\cot(x)}{4a}-\frac{\int \frac{1}{\left(\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}+1\right)\cot^2(x)+1}d\cot(x)}{4a}-\frac{\int \frac{1}{\left(\frac{\sqrt[4]{b}a}{(-a)^{5/4}}+1\right)\cot^2(x)+1}d\cot(x)}{4a}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}-\frac{(-a)^{5/8}\arctan\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\cot(x)}{(-a)^{5/8}}\right)}{4a\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

input
```
Int[(a + b*Cos[x]^8)^(-1),x]
```

output
```
-1/4*((-a)^(1/8)*ArcTan[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Cot[x])/(-a)^(1/8)])
/(a*Sqrt[(-a)^(1/4) - I*b^(1/4)]) - ((-a)^(1/8)*ArcTan[(Sqrt[(-a)^(1/4) +
I*b^(1/4)]*Cot[x])/(-a)^(1/8)])/(4*a*Sqrt[(-a)^(1/4) + I*b^(1/4)]) - ((-a)
^(1/8)*ArcTan[(Sqrt[(-a)^(1/4) + b^(1/4)]*Cot[x])/(-a)^(1/8)])/(4*a*Sqrt[(
-a)^(1/4) + b^(1/4)]) - ((-a)^(5/8)*ArcTan[(Sqrt[(-a)^(5/4) + a*b^(1/4)]*C
ot[x])/(-a)^(5/8)])/(4*a*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.31

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8 a+4\_Z^6 a+6\_Z^4 a+4\_Z^2 a+a+b\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}}{8a}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(16777216a^8+16777216a^7b)\_Z^8+1048576a^6\_Z^6+24576a^4\_Z^4+256a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{4194304ia^8}{b}\right.\right.$

input
```
int(1/(a+b*cos(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=
RootOf(_Z^8*a+4*_Z^6*a+6*_Z^4*a+4*_Z^2*a+a+b))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 665467 vs. 2(165) = 330.

Time = 6.80 (sec) , antiderivative size = 665467, normalized size of antiderivative = 2716.19

$$\int \frac{1}{a+b\cos^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)^8),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{a+b\cos^8(x)}\,dx = \int \frac{1}{a+b\cos^8{(x)}}\,dx$$

input
```
integrate(1/(a+b*cos(x)**8),x)
```

output
```
Integral(1/(a + b*cos(x)**8), x)
```

Maxima [F]

$$\int \frac{1}{a+b\cos^8(x)}\,dx = \int \frac{1}{b\cos{(x)}^8+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^8),x, algorithm="maxima")
```

output
```
integrate(1/(b*cos(x)^8 + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\cos^8(x)}\,dx=\int \frac{1}{b\cos{(x)}^8+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^8),x, algorithm="giac")
```

output
```
integrate(1/(b*cos(x)^8 + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+b\cos^8(x)}\,dx$$
$$=\sum_{k=1}^{8}\ln\Big(\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)^4\,a^5\,b^5$$
$$+1\Big)\ \big(\mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\big)\ a\tan(x)\ 8$$
$$-1\big)\ 4096\Big)\ \mathrm{root}\big(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4$$
$$+256\,a^2\,d^2+1,d,k\big)$$

input
```
int(1/(a + b*cos(x)^8),x)
```

output
```
symsum(log(4096*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d
^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^4*a^5*b^5*(64*root(16777216*a^
7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2
 + 1, d, k)^2*a^2 + 1)*(8*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 104
8576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)*a*tan(x) - 1))*root(
16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 +
256*a^2*d^2 + 1, d, k), k, 1, 8)
```

Reduce [F]

$$\int \frac{1}{a+b\cos^8(x)}\,dx=\int \frac{1}{\cos(x)^8 b+a}dx$$

input
```
int(1/(a+b*cos(x)^8),x)
```

output
```
int(1/(cos(x)**8*b + a),x)
```

3.26 $\int \frac{1}{a+b\cos(x)}\,dx$

Optimal result . . . 240
Mathematica [A] (verified) . . . 240
Rubi [A] (verified) . . . 241
Maple [A] (verified) . . . 242
Fricas [A] (verification not implemented) . . . 242
Sympy [B] (verification not implemented) . . . 243
Maxima [F(-2)] . . . 244
Giac [A] (verification not implemented) . . . 244
Mupad [B] (verification not implemented) . . . 244
Reduce [B] (verification not implemented) . . . 245

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \frac{1}{a+b\cos(x)}\,dx=\frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

output
```
2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\cos(x)}\,dx=-\frac{2\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
Integrate[(a + b*Cos[x])^(-1),x]
```

output
```
(-2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3138$$

$$2\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a + b*Cos[x])^(-1),x]
```

output
```
(2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}$	36
risch	$-\frac{\ln\left(\mathrm{e}^{ix}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{\ln\left(\mathrm{e}^{ix}+\frac{-ia^2+ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	123

input
```
int(1/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.26

$$\int \frac{1}{a+b\cos(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2+2\,\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)}{2\,(a^2-b^2)}, \frac{\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\sin(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

input
```
integrate(1/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*co
s(x) + a^2))/(a^2 - b^2), arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x)))
/sqrt(a^2 - b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(34) = 68$.

Time = 1.56 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.43

$$\int \frac{1}{a+b\cos(x)}\,dx = \begin{cases} \tilde{\infty}\left(-\log\left(\tan\left(\frac{x}{2}\right)-1\right)+\log\left(\tan\left(\frac{x}{2}\right)+1\right)\right) & \text{for } a=0 \wedge b=0 \\ \frac{\tan\left(\frac{x}{2}\right)}{b} & \text{for } a=b \\ \frac{1}{b\tan\left(\frac{x}{2}\right)} & \text{for } a=-b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*cos(x)),x)
```

output
```
Piecewise((zoo*(-log(tan(x/2) - 1) + log(tan(x/2) + 1)), Eq(a, 0) & Eq(b,
0)), (tan(x/2)/b, Eq(a, b)), (1/(b*tan(x/2)), Eq(a, -b)), (log(-sqrt(-a/(a
 - b) - b/(a - b)) + tan(x/2))/(a*sqrt(-a/(a - b) - b/(a - b)) - b*sqrt(-a
/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(a*s
qrt(-a/(a - b) - b/(a - b)) - b*sqrt(-a/(a - b) - b/(a - b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{1}{a+b\cos(x)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

input
```
integrate(1/(a+b*cos(x)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*t
an(1/2*x))/sqrt(a^2 - b^2)))/sqrt(a^2 - b^2)
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{a+b\cos(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)(2a-2b)}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
int(1/(a + b*cos(x)),x)
```

output
```
(2*atan((tan(x/2)*(2*a - 2*b))/(2*(a^2 - b^2)^(1/2))))/(a^2 - b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{1}{a + b\cos(x)}\,dx = \frac{2\sqrt{a^2 - b^2}\,atan\left(\frac{\tan\left(\frac{x}{2}\right)a - \tan\left(\frac{x}{2}\right)b}{\sqrt{a^2 - b^2}}\right)}{a^2 - b^2}$$

input
```
int(1/(a+b*cos(x)),x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan(x/2)*a - tan(x/2)*b)/sqrt(a**2 - b**2)))/(a
**2 - b**2)
```

3.27 $\int \frac{1}{a+b\cos^3(x)}\,dx$

Optimal result 246
Mathematica [C] (verified) 247
Rubi [A] (verified) 248
Maple [C] (verified) 249
Fricas [C] (verification not implemented) 250
Sympy [F(-1)] 250
Maxima [F] 251
Giac [F] 251
Mupad [B] (verification not implemented) 251
Reduce [F] 252

Optimal result

Integrand size = 10, antiderivative size = 288

$$\int \frac{1}{a+b\cos^3(x)}\,dx = \frac{2\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} + \frac{2\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{2\arctan\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
2/3*arctan((a^(1/3)-b^(1/3))^(1/2)*tan(1/2*x)/(a^(1/3)+b^(1/3))^(1/2))/a^(
2/3)/(a^(1/3)-b^(1/3))^(1/2)/(a^(1/3)+b^(1/3))^(1/2)+2/3*arctan((a^(1/3)+(
-1)^(1/3)*b^(1/3))^(1/2)*tan(1/2*x)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2))/a^
(2/3)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2
)+2/3*arctan((a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)*tan(1/2*x)/(a^(1/3)+(-1)^(
2/3)*b^(1/3))^(1/2))/a^(2/3)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)/(a^(1/3)+(
-1)^(2/3)*b^(1/3))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.33

$$\int \frac{1}{a+b\cos^3(x)}\,dx$$
$$= \frac{2}{3}\text{RootSum}\left[b+3b\#1^2+8a\#1^3+3b\#1^4 + b\#1^6\&, \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1}{b+4a\#1+2b\#1^2+b\#1^4}\&\right]$$

input
```
Integrate[(a + b*Cos[x]^3)^(-1),x]
```

output
```
(2*RootSum[b + 3*b*#1^2 + 8*a*#1^3 + 3*b*#1^4 + b*#1^6 & , (2*ArcTan[Sin[x
]/(Cos[x] - #1)]*#1 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1)/(b + 4*a*#1 + 2*b*
#1^2 + b*#1^4) & ])/3
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^3(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\cos(x)\right)}-\frac{1}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}\cos(x)-\sqrt[3]{a}\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\cos(x)\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a + b*Cos[x]^3)^(-1),x]
```

output
```
(2*ArcTan[(Sqrt[a^(1/3) - b^(1/3)]*Tan[x/2])/Sqrt[a^(1/3) + b^(1/3)]])/(3*
a^(2/3)*Sqrt[a^(1/3) - b^(1/3)]*Sqrt[a^(1/3) + b^(1/3)]) + (2*ArcTan[(Sqrt
[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tan[x/2])/Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)
]])/(3*a^(2/3)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Sqrt[a^(1/3) + (-1)^(1/3
)*b^(1/3)]) + (2*ArcTan[(Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)]*Tan[x/2])/Sqrt
[a^(1/3) + (-1)^(2/3)*b^(1/3)]])/(3*a^(2/3)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(1
/3)]*Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.63 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.33

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^6+(3a+3b)\_Z^4+(3a-3b)\_Z^2+a+b\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^5a-\_R^5b+2\_R^3a+2\_R^3b+\_Ra-\_Rb}\right)}{3}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(729a^6-729a^4b^2)\_Z^6+243a^4\_Z^4+27a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{ix}+\left(\frac{486ia^6}{b}-486ib\,a^4\right)\_R^5+\left(-\frac{81a^5}{b}\right.\right.$

input
```
int(1/(a+b*cos(x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((_R^4+2*_R^2+1)/(_R^5*a-_R^5*b+2*_R^3*a+2*_R^3*b+_R*a-_R*b)*ln(tan
(1/2*x)-_R),_R=RootOf((a-b)*_Z^6+(3*a+3*b)*_Z^4+(3*a-3*b)*_Z^2+a+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 18599, normalized size of antiderivative = 64.58

$$\int \frac{1}{a+b\cos^3(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+b\cos^3(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(x)**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a+b\cos^3(x)}\,dx=\int \frac{1}{b\cos{(x)}^3+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^3),x, algorithm="maxima")
```

output
```
integrate(1/(b*cos(x)^3 + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\cos^3(x)}\,dx=\int \frac{1}{b\cos{(x)}^3+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^3),x, algorithm="giac")
```

output
```
integrate(1/(b*cos(x)^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.98

$$\int \frac{1}{a+b\cos^3(x)}\,dx$$
$$=\sum_{k=1}^{6}\ln\left(-\frac{24576\,b^3\,(a-b)\,\left(-324\tan\left(\frac{x}{2}\right)\,a^5+648\tan\left(\frac{x}{2}\right)\,a^4\,b+81\,a^4\,\mathrm{root}(d^6+27\,a^2\,d^4+243\,a^4\,d^2+7\right.}{}\right.$$
$$-729\,a^6\,d^6-243\,a^4\,d^4-27\,a^2\,d^2-1,d,k\big)$$

input
```
int(1/(a + b*cos(x)^3),x)
```

output
```
symsum(log(-(24576*b^3*(a - b)*(81*a^4*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2
 + 729*a^4*(a^2 - b^2), d, k) - 324*a^5*tan(x/2) + root(d^6 + 27*a^2*d^4 +
 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5 + 18*a^2*root(d^6 + 27*a^2*d^4
 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3 - 324*a^3*b^2*tan(x/2) - 72*
a^3*tan(x/2)*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d,
 k)^2 + 27*a^2*b^2*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^
2), d, k) + 648*a^4*b*tan(x/2) - 4*a*tan(x/2)*root(d^6 + 27*a^2*d^4 + 243*
a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 + b*tan(x/2)*root(d^6 + 27*a^2*d^4
+ 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 9*a*b*root(d^6 + 27*a^2*d^4
 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3 - 108*b*a^3*root(d^6 + 27*a^
2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 72*a^2*b*tan(x/2)*root(
d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2))/root(d^6 +
 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5)*root(729*a^4*b^2
*d^6 - 729*a^6*d^6 - 243*a^4*d^4 - 27*a^2*d^2 - 1, d, k), k, 1, 6)
```

Reduce [F]

$$\int \frac{1}{a + b\cos^3(x)}\,dx = \int \frac{1}{\cos(x)^3 b + a}dx$$

input
```
int(1/(a+b*cos(x)^3),x)
```

output
```
int(1/(cos(x)**3*b + a),x)
```

3.28 $\int \frac{1}{a+b\cos^5(x)}\,dx$

Optimal result 254
Mathematica [C] (warning: unable to verify) 255
Rubi [A] (verified) 255
Maple [C] (verified) 257
Fricas [F(-2)] 258
Sympy [F] 258
Maxima [F] 258
Giac [F] 259
Mupad [B] (verification not implemented) 259
Reduce [F] 260

Optimal result

Integrand size = 10, antiderivative size = 494

$$\int \frac{1}{a+b\cos^5(x)}\,dx=\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}$$

output
```
2/5*arctan((a^(1/5)-b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+b^(1/5))^(1/2))/a^(
4/5)/(a^(1/5)-b^(1/5))^(1/2)/(a^(1/5)+b^(1/5))^(1/2)+2/5*arctan((a^(1/5)+(
-1)^(1/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2))/a^
(4/5)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2
)+2/5*arctan((a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+(-1)^(
2/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)/(a^(1/5)+(
-1)^(2/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)*tan
(1/2*x)/(a^(1/5)-(-1)^(3/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(3/5)*b^
(1/5))^(1/2)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)-(-1)^(
4/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2))/a^(4/5)
/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.26

$$\int \frac{1}{a+b\cos^5(x)}\,dx$$
$$=\frac{8}{5}\text{RootSum}\left[b+5b\#1^2+10b\#1^4+32a\#1^5+10b\#1^6+5b\#1^8+b\#1^{10}\&,\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3}{b+4b\#1^2+16a\#1^3+6b\#1^4+4b\#1^6+b\#1^8}\&\right]$$

input
```
Integrate[(a + b*Cos[x]^5)^(-1),x]
```

output
```
(8*RootSum[b + 5*b*#1^2 + 10*b*#1^4 + 32*a*#1^5 + 10*b*#1^6 + 5*b*#1^8 + b
*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - I*Log[1 - 2*Cos[x]*#1 +
#1^2]*#1^3)/(b + 4*b*#1^2 + 16*a*#1^3 + 6*b*#1^4 + 4*b*#1^6 + b*#1^8) & ])
/5
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^5(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-\sqrt[5]{b}\cos(x)\right)}-\frac{1}{5a^{4/5}\left(\sqrt[5]{-1}\sqrt[5]{b}\cos(x)-\sqrt[5]{a}\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cos(x)\right)}-\frac{}{5a^{4/5}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}+\frac{2\arctan\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}$$

input
```
Int[(a + b*Cos[x]^5)^(-1),x]
```

output
```
(2*ArcTan[(Sqrt[a^(1/5) - b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + b^(1/5)]])/(5*
a^(4/5)*Sqrt[a^(1/5) - b^(1/5)]*Sqrt[a^(1/5) + b^(1/5)]) + (2*ArcTan[(Sqrt
[a^(1/5) + (-1)^(1/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)
]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(1/5
)*b^(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) - (-1)^(2/5)*b^(1/5)]*Tan[x/2])/Sqrt
[a^(1/5) + (-1)^(2/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(2/5)*b^(1
/5)]*Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) + (-1)^
(3/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]])/(5*a^(4/5)*S
qrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]) + (2
*ArcTan[(Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + (-1)^
(4/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Sqrt[a^(1/5
) + (-1)^(4/5)*b^(1/5)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.93 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.30

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^{10}+(5a+5b)\_Z^{8}+(10a-10b)\_Z^{6}+(10a+10b)\_Z^{4}+(5a-5b)\_Z^{2}+a+b\right)}\frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}\right.}{\_R^{9}a-\_R^{9}b+4\_R^{7}a+4\_R^{7}b+6\_}\right.}{5}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(9765625a^{10}-9765625a^{8}b^{2})\_Z^{10}+1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}+6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{ix}+\right.$

input
```
int(1/(a+b*cos(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a-_R^9*b+4*_R^7*a+4*_R^7*b+6*_
R^5*a-6*_R^5*b+4*_R^3*a+4*_R^3*b+_R*a-_R*b)*ln(tan(1/2*x)-_R),_R=RootOf((a
-b)*_Z^10+(5*a+5*b)*_Z^8+(10*a-10*b)*_Z^6+(10*a+10*b)*_Z^4+(5*a-5*b)*_Z^2+
a+b))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{a + b\cos^5(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*cos(x)^5),x, algorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F]

$$\int \frac{1}{a + b\cos^5(x)}\,dx = \int \frac{1}{a + b\cos^5{(x)}}\,dx$$

input
```
integrate(1/(a+b*cos(x)**5),x)
```

output
```
Integral(1/(a + b*cos(x)**5), x)
```

Maxima [F]

$$\int \frac{1}{a + b\cos^5(x)}\,dx = \int \frac{1}{b\cos{(x)}^5 + a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^5),x, algorithm="maxima")
```

output
```
integrate(1/(b*cos(x)^5 + a), x)
```

Giac [F]

$$\int \frac{1}{a+b\cos^5(x)}\,dx=\int \frac{1}{b\cos{(x)}^5+a}\,dx$$

input
```
integrate(1/(a+b*cos(x)^5),x, algorithm="giac")
```

output
```
integrate(1/(b*cos(x)^5 + a), x)
```

Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 1520, normalized size of antiderivative = 3.08

$$\int \frac{1}{a+b\cos^5(x)}\,dx=\text{Too large to display}$$

input
```
int(1/(a + b*cos(x)^5),x)
```

output
```
symsum(log(-(10995116277760*b^7*(a - b)*(7*cot(x/2) - 56*root(9765625*a^8*
b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4
*d^4 - 125*a^2*d^2 - 1, d, k)*a + root(9765625*a^8*b^2*d^10 - 9765625*a^10
*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
 d, k)*b - 5800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^
8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^3 - 225
000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1562
50*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 - 3875000*root(97
65625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6
- 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 - 25000000*root(9765625*a^8*
b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4
*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^9 + 735*root(9765625*a^8*b^2*d^10 - 9765
625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*
d^2 - 1, d, k)^2*a^2*cot(x/2) + 28875*root(9765625*a^8*b^2*d^10 - 9765625*
a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2
- 1, d, k)^4*a^4*cot(x/2) + 503125*root(9765625*a^8*b^2*d^10 - 9765625*a^1
0*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1
, d, k)^6*a^6*cot(x/2) + 3281250*root(9765625*a^8*b^2*d^10 - 9765625*a^10*
d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
d, k)^8*a^8*cot(x/2) + 800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^1...
```

Reduce [F]

$$\int \frac{1}{a+b\cos^5(x)}\,dx=\int \frac{1}{\cos \left(x\right)^{5} b+a}dx$$

input
```
int(1/(a+b*cos(x)^5),x)
```

output
```
int(1/(cos(x)**5*b + a),x)
```

3.29 $\int \frac{1}{a-a\cos^2(x)}\,dx$

Optimal result 261
Mathematica [A] (verified) 261
Rubi [A] (verified) 262
Maple [A] (verified) 263
Fricas [A] (verification not implemented) 264
Sympy [B] (verification not implemented) 264
Maxima [A] (verification not implemented) 264
Giac [A] (verification not implemented) 265
Mupad [B] (verification not implemented) 265
Reduce [B] (verification not implemented) 265

Optimal result

Integrand size = 11, antiderivative size = 7

$$\int \frac{1}{a-a\cos^2(x)}\,dx=-\frac{\cot(x)}{a}$$

output
```
-cot(x)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-a\cos^2(x)}\,dx=-\frac{\cot(x)}{a}$$

input
```
Integrate[(a - a*Cos[x]^2)^(-1),x]
```

output
```
-(Cot[x]/a)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-a\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \csc^2(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(x)^2dx}{a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d\cot(x)}{a}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\cot(x)}{a}$$

input
```
Int[(a - a*Cos[x]^2)^(-1),x]
```

output
```
-(Cot[x]/a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

method	result	size
default	$-\frac{1}{a\tan(x)}$	10
risch	$-\frac{2i}{(e^{2ix}-1)a}$	16
parallelrisch	$\frac{-\cot(\frac{x}{2})+\tan(\frac{x}{2})}{2a}$	17
norman	$\frac{-\frac{1}{2a}+\frac{\tan(\frac{x}{2})^2}{2a}}{\tan(\frac{x}{2})}$	25

input
```
int(1/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/a/tan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{1}{a-a\cos^2(x)}\,dx = -\frac{\cos(x)}{a\sin(x)}$$

input
```
integrate(1/(a-a*cos(x)^2),x, algorithm="fricas")
```

output
```
-cos(x)/(a*sin(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \frac{1}{a-a\cos^2(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{2a} - \frac{1}{2a\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(a-a*cos(x)**2),x)
```

output
```
tan(x/2)/(2*a) - 1/(2*a*tan(x/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{1}{a-a\cos^2(x)}\,dx = -\frac{1}{a\tan(x)}$$

input
```
integrate(1/(a-a*cos(x)^2),x, algorithm="maxima")
```

output
```
-1/(a*tan(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{1}{a-a\cos^2(x)}\,dx=-\frac{1}{a\tan(x)}$$

input
```
integrate(1/(a-a*cos(x)^2),x, algorithm="giac")
```

output
```
-1/(a*tan(x))
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-a\cos^2(x)}\,dx=-\frac{\cot(x)}{a}$$

input
```
int(1/(a - a*cos(x)^2),x)
```

output
```
-cot(x)/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{1}{a-a\cos^2(x)}\,dx=-\frac{\cos(x)}{\sin(x)\,a}$$

input
```
int(1/(a-a*cos(x)^2),x)
```

output
```
( - cos(x))/(sin(x)*a)
```

3.30 $\int \frac{1}{(a-a\cos^2(x))^2}\,dx$

Optimal result 266
Mathematica [A] (verified) 266
Rubi [A] (verified) 267
Maple [A] (verified) 268
Fricas [A] (verification not implemented) 269
Sympy [B] (verification not implemented) 269
Maxima [A] (verification not implemented) 269
Giac [A] (verification not implemented) 270
Mupad [B] (verification not implemented) 270
Reduce [B] (verification not implemented) 270

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1}{(a-a\cos^2(x))^2}\,dx=-\frac{\cot(x)}{a^2}-\frac{\cot^3(x)}{3a^2}$$

output
```
-cot(x)/a^2-1/3*cot(x)^3/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a-a\cos^2(x))^2}\,dx=\frac{-\frac{2\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)}{a^2}$$

input
```
Integrate[(a - a*Cos[x]^2)^(-2),x]
```

output
```
((-2*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3)/a^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\cos^2(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\sin\left(x+\frac{\pi}{2}\right)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \csc^4(x)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(x)^4dx}{a^2}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int \left(\cot^2(x)+1\right)d\cot(x)}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\cot^3(x)}{3}+\cot(x)}{a^2}$$

input
```
Int[(a - a*Cos[x]^2)^(-2),x]
```

output
```
-((Cot[x] + Cot[x]^3/3)/a^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-\frac{1}{\tan(x)}-\frac{1}{3\tan(x)^{3}}}{a^{2}}$	18
risch	$\frac{4i\left(3\,\mathrm{e}^{2ix}-1\right)}{3\left(\mathrm{e}^{2ix}-1\right)^{3}a^{2}}$	25
parallelrisch	$\frac{\tan\left(\frac{x}{2}\right)^{3}-\cot\left(\frac{x}{2}\right)^{3}+9\tan\left(\frac{x}{2}\right)-9\cot\left(\frac{x}{2}\right)}{24a^{2}}$	33
norman	$\frac{-\frac{1}{24a}-\frac{3\tan\left(\frac{x}{2}\right)^{2}}{8a}+\frac{3\tan\left(\frac{x}{2}\right)^{4}}{8a}+\frac{\tan\left(\frac{x}{2}\right)^{6}}{24a}}{a\tan\left(\frac{x}{2}\right)^{3}}$	50

input
```
int(1/(a-a*cos(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/tan(x)-1/3/tan(x)^3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{1}{(a-a\cos^2(x))^2}\,dx = -\frac{2\cos(x)^3-3\cos(x)}{3\left(a^2\cos(x)^2-a^2\right)\sin(x)}$$

input
```
integrate(1/(a-a*cos(x)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(2*cos(x)^3 - 3*cos(x))/((a^2*cos(x)^2 - a^2)*sin(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.46 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \frac{1}{(a-a\cos^2(x))^2}\,dx = \frac{\tan^3\left(\frac{x}{2}\right)}{24a^2}+\frac{3\tan\left(\frac{x}{2}\right)}{8a^2}-\frac{3}{8a^2\tan\left(\frac{x}{2}\right)}-\frac{1}{24a^2\tan^3\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(a-a*cos(x)**2)**2,x)
```

output
```
tan(x/2)**3/(24*a**2) + 3*tan(x/2)/(8*a**2) - 3/(8*a**2*tan(x/2)) - 1/(24*
a**2*tan(x/2)**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a-a\cos^2(x))^2}\,dx = -\frac{3\tan(x)^2+1}{3\,a^2\tan(x)^3}$$

input
```
integrate(1/(a-a*cos(x)^2)^2,x, algorithm="maxima")
```

output
```
-1/3*(3*tan(x)^2 + 1)/(a^2*tan(x)^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a - a\cos^2(x))^2}\,dx = -\frac{3\,\tan(x)^2 + 1}{3\,a^2\tan(x)^3}$$

input
```
integrate(1/(a-a*cos(x)^2)^2,x, algorithm="giac")
```

output
```
-1/3*(3*tan(x)^2 + 1)/(a^2*tan(x)^3)
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a - a\cos^2(x))^2}\,dx = -\frac{\cot(x)\,\left(\cot(x)^2 + 3\right)}{3\,a^2}$$

input
```
int(1/(a - a*cos(x)^2)^2,x)
```

output
```
-(cot(x)*(cot(x)^2 + 3))/(3*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a - a\cos^2(x))^2}\,dx = \frac{\cos(x)\left(-2\sin(x)^2 - 1\right)}{3\sin(x)^3\,a^2}$$

input
```
int(1/(a-a*cos(x)^2)^2,x)
```

output
```
(cos(x)*( - 2*sin(x)**2 - 1))/(3*sin(x)**3*a**2)
```

3.31 $\int \frac{1}{\left(a - a\cos^2(x)\right)^3}\,dx$

Optimal result . . . 271
Mathematica [A] (verified) . . . 271
Rubi [A] (verified) . . . 272
Maple [A] (verified) . . . 273
Fricas [A] (verification not implemented) . . . 274
Sympy [B] (verification not implemented) . . . 274
Maxima [A] (verification not implemented) . . . 275
Giac [A] (verification not implemented) . . . 275
Mupad [B] (verification not implemented) . . . 275
Reduce [B] (verification not implemented) . . . 276

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{1}{\left(a - a\cos^2(x)\right)^3}\,dx = -\frac{\cot(x)}{a^3} - \frac{2\cot^3(x)}{3a^3} - \frac{\cot^5(x)}{5a^3}$$

output
```
-cot(x)/a^3-2/3*cot(x)^3/a^3-1/5*cot(x)^5/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a - a\cos^2(x)\right)^3}\,dx = \frac{-\frac{8\cot(x)}{15} - \frac{4}{15}\cot(x)\csc^2(x) - \frac{1}{5}\cot(x)\csc^4(x)}{a^3}$$

input
```
Integrate[(a - a*Cos[x]^2)^(-3),x]
```

output
```
((-8*Cot[x])/15 - (4*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5)/a^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\cos^2(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\sin\left(x+\frac{\pi}{2}\right)^2\right)^3}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \csc^6(x)dx}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(x)^6dx}{a^3}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int \left(\cot^4(x)+2\cot^2(x)+1\right)d\cot(x)}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\cot^5(x)}{5}+\frac{2\cot^3(x)}{3}+\cot(x)}{a^3}$$

input
```
Int[(a - a*Cos[x]^2)^(-3),x]
```

output
```
-((Cot[x] + (2*Cot[x]^3)/3 + Cot[x]^5/5)/a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{-\frac{1}{\tan(x)}-\frac{1}{5\tan(x)^{5}}-\frac{2}{3\tan(x)^{3}}}{a^{3}}$	24
risch	$-\frac{16i\left(10\,\mathrm{e}^{4ix}-5\,\mathrm{e}^{2ix}+1\right)}{15\left(\mathrm{e}^{2ix}-1\right)^{5}a^{3}}$	32
parallelrisch	$\frac{-3\cot\left(\frac{x}{2}\right)^{5}+3\tan\left(\frac{x}{2}\right)^{5}-25\cot\left(\frac{x}{2}\right)^{3}+25\tan\left(\frac{x}{2}\right)^{3}-150\cot\left(\frac{x}{2}\right)+150\tan\left(\frac{x}{2}\right)}{480a^{3}}$	51
norman	$\frac{-\frac{1}{160a}-\frac{5\tan\left(\frac{x}{2}\right)^{2}}{96a}-\frac{5\tan\left(\frac{x}{2}\right)^{4}}{16a}+\frac{5\tan\left(\frac{x}{2}\right)^{6}}{16a}+\frac{5\tan\left(\frac{x}{2}\right)^{8}}{96a}+\frac{\tan\left(\frac{x}{2}\right)^{10}}{160a}}{a^{2}\tan\left(\frac{x}{2}\right)^{5}}$	72

input
```
int(1/(a-a*cos(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/tan(x)-1/5/tan(x)^5-2/3/tan(x)^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{1}{(a - a\cos^2(x))^3}\,dx = -\frac{8\,\cos(x)^5 - 20\,\cos(x)^3 + 15\,\cos(x)}{15\,(a^3\cos(x)^4 - 2\,a^3\cos(x)^2 + a^3)\sin(x)}$$

input
```
integrate(1/(a-a*cos(x)^2)^3,x, algorithm="fricas")
```

output
```
-1/15*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos(x))/((a^3*cos(x)^4 - 2*a^3*cos(x)
^2 + a^3)*sin(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(29) = 58.

Time = 1.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a - a\cos^2(x))^3}\,dx = \frac{\tan^5\left(\frac{x}{2}\right)}{160a^3} + \frac{5\tan^3\left(\frac{x}{2}\right)}{96a^3} + \frac{5\tan\left(\frac{x}{2}\right)}{16a^3} - \frac{5}{16a^3\tan\left(\frac{x}{2}\right)} - \frac{5}{96a^3\tan^3\left(\frac{x}{2}\right)} - \frac{1}{160a^3\tan^5\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(a-a*cos(x)**2)**3,x)
```

output
```
tan(x/2)**5/(160*a**3) + 5*tan(x/2)**3/(96*a**3) + 5*tan(x/2)/(16*a**3) -
5/(16*a**3*tan(x/2)) - 5/(96*a**3*tan(x/2)**3) - 1/(160*a**3*tan(x/2)**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a-a\cos^2(x))^3}\,dx = -\frac{15\,\tan(x)^4+10\,\tan(x)^2+3}{15\,a^3\tan(x)^5}$$

input
```
integrate(1/(a-a*cos(x)^2)^3,x, algorithm="maxima")
```

output
```
-1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/(a^3*tan(x)^5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a-a\cos^2(x))^3}\,dx = -\frac{15\,\tan(x)^4+10\,\tan(x)^2+3}{15\,a^3\tan(x)^5}$$

input
```
integrate(1/(a-a*cos(x)^2)^3,x, algorithm="giac")
```

output
```
-1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/(a^3*tan(x)^5)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a-a\cos^2(x))^3}\,dx = -\frac{\tan(x)^4+\frac{2\tan(x)^2}{3}+\frac{1}{5}}{a^3\tan(x)^5}$$

input
```
int(1/(a - a*cos(x)^2)^3,x)
```

output
```
-((2*tan(x)^2)/3 + tan(x)^4 + 1/5)/(a^3*tan(x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a-a\cos^2(x))^3}\,dx=\frac{\cos(x)\left(-8\sin(x)^4-4\sin(x)^2-3\right)}{15\sin(x)^5\,a^3}$$

input
```
int(1/(a-a*cos(x)^2)^3,x)
```

output
```
(cos(x)*( - 8*sin(x)**4 - 4*sin(x)**2 - 3))/(15*sin(x)**5*a**3)
```

3.32 $\int \frac{1}{a+a\cos^2(x)}\,dx$

Optimal result 277
Mathematica [A] (verified) 277
Rubi [A] (verified) 278
Maple [A] (verified) 279
Fricas [A] (verification not implemented) 279
Sympy [A] (verification not implemented) 280
Maxima [A] (verification not implemented) 280
Giac [A] (verification not implemented) 280
Mupad [B] (verification not implemented) 281
Reduce [B] (verification not implemented) 281

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \frac{1}{a+a\cos^2(x)}\,dx = \frac{x}{\sqrt{2}a} - \frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}a}$$

output
```
1/2*x*2^(1/2)/a-1/2*arctan(cos(x)*sin(x)/(1+2^(1/2)+cos(x)^2))*2^(1/2)/a
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.45

$$\int \frac{1}{a+a\cos^2(x)}\,dx = \frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}a}$$

input
```
Integrate[(a + a*Cos[x]^2)^(-1),x]
```

output
```
ArcTan[Tan[x]/Sqrt[2]]/(Sqrt[2]*a)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cos^2(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\sin\left(x+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \text{3660}$$

$$-\int \frac{1}{2a\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \text{218}$$

$$-\frac{\arctan\left(\sqrt{2}\cot(x)\right)}{\sqrt{2}a}$$

input
```
Int[(a + a*Cos[x]^2)^(-1),x]
```

output
```
-(ArcTan[Sqrt[2]*Cot[x]]/(Sqrt[2]*a))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.42

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{2a}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}+3\right)}{4a}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}+3\right)}{4a}$	46

input
```
int(1/(a+a*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*2^(1/2)*arctan(1/2*tan(x)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+a\cos^2(x)}\,dx=-\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)}{4\,a}$$

input
```
integrate(1/(a+a*cos(x)^2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))/a
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{1}{a + a\cos^2(x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right) - 1\right) + \pi\left\lfloor\frac{\frac{x}{2} - \frac{\pi}{2}}{\pi}\right\rfloor\right)}{2a} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right) + 1\right) + \pi\left\lfloor\frac{\frac{x}{2} - \frac{\pi}{2}}{\pi}\right\rfloor\right)}{2a}$$

input
```
integrate(1/(a+a*cos(x)**2),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/(2*a) + s
qrt(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.40

$$\int \frac{1}{a + a\cos^2(x)}\,dx = \frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)}{2\,a}$$

input
```
integrate(1/(a+a*cos(x)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*tan(x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \frac{1}{a + a\cos^2(x)}\,dx = \frac{\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2\,x) - \sin(2\,x)}{\sqrt{2}\cos(2\,x) + \sqrt{2} - \cos(2\,x) + 1}\right)\right)}{2\,a}$$

input
```
integrate(1/(a+a*cos(x)^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1)))/a
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.40

$$\int \frac{1}{a + a\cos^2(x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{2\,a}$$

input
```
int(1/(a + a*cos(x)^2),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*tan(x))/2))/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{a + a\cos^2(x)}\,dx = \frac{\sqrt{2}\left(-atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + atan\left(\frac{\sqrt{2}+2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\right)}{2a}$$

input
```
int(1/(a+a*cos(x)^2),x)
```

output
```
(sqrt(2)*( - atan((sqrt(2) - 2*tan(x/2))/sqrt(2)) + atan((sqrt(2) + 2*tan(
x/2))/sqrt(2))))/(2*a)
```

3.33 $\int \frac{1}{(a+a\cos^2(x))^2}\,dx$

Optimal result 282
Mathematica [A] (verified) 282
Rubi [A] (verified) 283
Maple [A] (verified) 285
Fricas [A] (verification not implemented) 285
Sympy [B] (verification not implemented) 286
Maxima [A] (verification not implemented) 286
Giac [A] (verification not implemented) 287
Mupad [B] (verification not implemented) 287
Reduce [B] (verification not implemented) 288

Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{1}{(a+a\cos^2(x))^2}\,dx = \frac{3x}{4\sqrt{2}a^2} - \frac{3\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}a^2} - \frac{\cos(x)\sin(x)}{4\,(a^2+a^2\cos^2(x))}$$

output
```
3/8*x*2^(1/2)/a^2-3/8*arctan(cos(x)*sin(x)/(1+2^(1/2)+cos(x)^2))*2^(1/2)/a
^2-cos(x)*sin(x)/(4*a^2+4*a^2*cos(x)^2)
```

Mathematica [A] (verified)

Time = 5.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a+a\cos^2(x))^2}\,dx = \frac{3\sqrt{2}\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right) - \frac{2\sin(2x)}{3+\cos(2x)}}{8a^2}$$

input
```
Integrate[(a + a*Cos[x]^2)^(-2),x]
```

output
```
(3*Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]] - (2*Sin[2*x])/(3 + Cos[2*x]))/(8*a^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3663, 27, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos^2(x)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^2}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\frac{3}{\cos^2(x)+1}dx}{4a^2}-\frac{\sin(x)\cos(x)}{4\left(a^2\cos^2(x)+a^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{1}{\cos^2(x)+1}dx}{4a^2}-\frac{\sin(x)\cos(x)}{4\left(a^2\cos^2(x)+a^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx}{4a^2}-\frac{\sin(x)\cos(x)}{4\left(a^2\cos^2(x)+a^2\right)}$$

$$\Big\downarrow \text{3660}$$

$$-\frac{3\int \frac{1}{2\cot^2(x)+1}d\cot(x)}{4a^2}-\frac{\sin(x)\cos(x)}{4\left(a^2\cos^2(x)+a^2\right)}$$

$$\Big\downarrow \text{216}$$

$$-\frac{3\arctan\left(\sqrt{2}\cot(x)\right)}{4\sqrt{2}a^2}-\frac{\sin(x)\cos(x)}{4\left(a^2\cos^2(x)+a^2\right)}$$

input
```
Int[(a + a*Cos[x]^2)^(-2),x]
```

output
```
(-3*ArcTan[Sqrt[2]*Cot[x]])/(4*Sqrt[2]*a^2) - (Cos[x]*Sin[x])/(4*(a^2 + a^
2*Cos[x]^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

method	result	size
default	$\frac{-\frac{\tan(x)}{4\left(\tan(x)^2+2\right)}+\frac{3\sqrt{2}\,\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{8}}{a^2}$	31
risch	$-\frac{i(3\,\mathrm{e}^{2ix}+1)}{2(\mathrm{e}^{4ix}+6\,\mathrm{e}^{2ix}+1)a^2}+\frac{3i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)}{16a^2}-\frac{3i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)}{16a^2}$	77

input
```
int(1/(a+a*cos(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/4*tan(x)/(tan(x)^2+2)+3/8*2^(1/2)*arctan(1/2*tan(x)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+a\cos^2(x))^2}\,dx$$
$$= -\frac{3\left(\sqrt{2}\cos(x)^2+\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)+4\cos(x)\sin(x)}{16\left(a^2\cos(x)^2+a^2\right)}$$

input
```
integrate(1/(a+a*cos(x)^2)^2,x, algorithm="fricas")
```

output
```
-1/16*(3*(sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqr
t(2))/(cos(x)*sin(x))) + 4*cos(x)*sin(x))/(a^2*cos(x)^2 + a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(66) = 132$.

Time = 3.83 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.85

$$\int \frac{1}{(a + a\cos^2(x))^2}\,dx = \frac{3\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right) - 1\right) + \pi\left\lfloor\frac{\frac{x}{2} - \frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{x}{2}\right)}{8a^2\tan^4\left(\frac{x}{2}\right) + 8a^2} + \frac{3\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right) - 1\right) + \pi\left\lfloor\frac{\frac{x}{2} - \frac{\pi}{2}}{\pi}\right\rfloor\right)}{8a^2\tan^4\left(\frac{x}{2}\right) + 8a^2} + \frac{3\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right) + 1\right) + \pi\left\lfloor\frac{\frac{x}{2} - \frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^4\left(\frac{x}{2}\right)}{8a^2\tan^4\left(\frac{x}{2}\right) + 8a^2} + \frac{3\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right) + 1\right) + \pi\left\lfloor\frac{\frac{x}{2} - \frac{\pi}{2}}{\pi}\right\rfloor\right)}{8a^2\tan^4\left(\frac{x}{2}\right) + 8a^2} + \frac{2\tan^3\left(\frac{x}{2}\right)}{8a^2\tan^4\left(\frac{x}{2}\right) + 8a^2} - \frac{2\tan\left(\frac{x}{2}\right)}{8a^2\tan^4\left(\frac{x}{2}\right) + 8a^2}$$

input
```
integrate(1/(a+a*cos(x)**2)**2,x)
```

output
```
3*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2
)**4/(8*a**2*tan(x/2)**4 + 8*a**2) + 3*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1)
 + pi*floor((x/2 - pi/2)/pi))/(8*a**2*tan(x/2)**4 + 8*a**2) + 3*sqrt(2)*(a
tan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**4/(8*a**2
*tan(x/2)**4 + 8*a**2) + 3*sqrt(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor(
(x/2 - pi/2)/pi))/(8*a**2*tan(x/2)**4 + 8*a**2) + 2*tan(x/2)**3/(8*a**2*ta
n(x/2)**4 + 8*a**2) - 2*tan(x/2)/(8*a**2*tan(x/2)**4 + 8*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a + a\cos^2(x))^2}\,dx = -\frac{\tan(x)}{4\left(a^2\tan(x)^2 + 2a^2\right)} + \frac{3\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)}{8a^2}$$

input
```
integrate(1/(a+a*cos(x)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*tan(x)/(a^2*tan(x)^2 + 2*a^2) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)
)/a^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+a\cos^2(x))^2}\,dx = \frac{3\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)}{8a^2} - \frac{\tan(x)}{4\left(\tan(x)^2+2\right)a^2}$$

input
```
integrate(1/(a+a*cos(x)^2)^2,x, algorithm="giac")
```

output
```
3/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1)))/a^2 - 1/4*tan(x)/((tan(x)^2 + 2)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a+a\cos^2(x))^2}\,dx = \frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{8a^2} - \frac{\tan(x)}{4a^2\left(\tan(x)^2+2\right)}$$

input
```
int(1/(a + a*cos(x)^2)^2,x)
```

output
```
(3*2^(1/2)*atan((2^(1/2)*tan(x))/2))/(8*a^2) - tan(x)/(4*a^2*(tan(x)^2 + 2
))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{1}{\left(a+a\cos^2(x)\right)^2}\,dx$$
$$= \frac{-3\sqrt{2}\,atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\sin\left(x\right)^2+6\sqrt{2}\,atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+3\sqrt{2}\,atan\left(\frac{\sqrt{2}+2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\sin\left(x\right)^2-6\sqrt{2}\,ata}{8a^2\left(\sin\left(x\right)^2-2\right)}$$

input
```
int(1/(a+a*cos(x)^2)^2,x)
```

output
```
( - 3*sqrt(2)*atan((sqrt(2) - 2*tan(x/2))/sqrt(2))*sin(x)**2 + 6*sqrt(2)*a
tan((sqrt(2) - 2*tan(x/2))/sqrt(2)) + 3*sqrt(2)*atan((sqrt(2) + 2*tan(x/2)
)/sqrt(2))*sin(x)**2 - 6*sqrt(2)*atan((sqrt(2) + 2*tan(x/2))/sqrt(2)) + 2*
cos(x)*sin(x))/(8*a**2*(sin(x)**2 - 2))
```

3.34 $\int \frac{1}{(a+a\cos^2(x))^3}\,dx$

Optimal result 289
Mathematica [A] (verified) 289
Rubi [A] (verified) 290
Maple [A] (verified) 292
Fricas [A] (verification not implemented) 293
Sympy [B] (verification not implemented) 293
Maxima [A] (verification not implemented) 294
Giac [A] (verification not implemented) 295
Mupad [B] (verification not implemented) 295
Reduce [B] (verification not implemented) 296

Optimal result

Integrand size = 10, antiderivative size = 88

$$\int \frac{1}{(a+a\cos^2(x))^3}\,dx = \frac{19x}{32\sqrt{2}a^3} - \frac{19\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{32\sqrt{2}a^3} - \frac{\cos(x)\sin(x)}{8a\left(a+a\cos^2(x)\right)^2} - \frac{9\cos(x)\sin(x)}{32\left(a^3+a^3\cos^2(x)\right)}$$

output
```
19/64*x*2^(1/2)/a^3-19/64*arctan(cos(x)*sin(x)/(1+2^(1/2)+cos(x)^2))*2^(1/
2)/a^3-1/8*cos(x)*sin(x)/a/(a+a*cos(x)^2)^2-9*cos(x)*sin(x)/(32*a^3+32*a^3
*cos(x)^2)
```

Mathematica [A] (verified)

Time = 5.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a+a\cos^2(x))^3}\,dx = \frac{19\sqrt{2}\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)(3+\cos(2x))^2 - 70\sin(2x) - 9\sin(4x)}{64a^3(3+\cos(2x))^2}$$

input
```
Integrate[(a + a*Cos[x]^2)^(-3),x]
```

output
```
(19*Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]]*(3 + Cos[2*x])^2 - 70*Sin[2*x] - 9*Sin[
4*x])/(64*a^3*(3 + Cos[2*x])^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos^2(x)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^3}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\frac{7a-2a\cos^2(x)}{(a\cos^2(x)+a)^2}dx}{8a^2}-\frac{\sin(x)\cos(x)}{8a\left(a\cos^2(x)+a\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7a-2a\cos^2(x)}{(a\cos^2(x)+a)^2}dx}{8a^2}-\frac{\sin(x)\cos(x)}{8a\left(a\cos^2(x)+a\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-2a\sin\left(x+\frac{\pi}{2}\right)^2}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^2}dx}{8a^2}-\frac{\sin(x)\cos(x)}{8a\left(a\cos^2(x)+a\right)^2}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\int \frac{19a^2}{a\cos^2(x)+a}dx}{4a^2}-\frac{9\sin(x)\cos(x)}{4(a\cos^2(x)+a)}}{8a^2}-\frac{\sin(x)\cos(x)}{8a\left(a\cos^2(x)+a\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{19}{4}\int \frac{1}{a\cos^2(x)+a}dx-\frac{9\sin(x)\cos(x)}{4(a\cos^2(x)+a)}}{8a^2}-\frac{\sin(x)\cos(x)}{8a\left(a\cos^2(x)+a\right)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{19}{4}\int \frac{1}{a\sin\left(x+\frac{\pi}{2}\right)^2+a}dx-\frac{9\sin(x)\cos(x)}{4(a\cos^2(x)+a)}}{8a^2}-\frac{\sin(x)\cos(x)}{8a\left(a\cos^2(x)+a\right)^2}$$

$$\Big\downarrow \ 3660$$

$$\frac{-\frac{19}{4}\int \frac{1}{2a\cot^2(x)+a}d\cot(x)-\frac{9\sin(x)\cos(x)}{4(a\cos^2(x)+a)}}{8a^2}-\frac{\sin(x)\cos(x)}{8a\left(a\cos^2(x)+a\right)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{-\frac{19\arctan\left(\sqrt{2}\cot(x)\right)}{4\sqrt{2}a}-\frac{9\sin(x)\cos(x)}{4(a\cos^2(x)+a)}}{8a^2}-\frac{\sin(x)\cos(x)}{8a\left(a\cos^2(x)+a\right)^2}$$

input
```
Int[(a + a*Cos[x]^2)^(-3),x]
```

output
```
-1/8*(Cos[x]*Sin[x])/(a*(a + a*Cos[x]^2)^2) + ((-19*ArcTan[Sqrt[2]*Cot[x]]
)/(4*Sqrt[2]*a) - (9*Cos[x]*Sin[x])/(4*(a + a*Cos[x]^2)))/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{\frac{-\frac{13 \tan\left(x\right)^{3}}{32}-\frac{11 \tan\left(x\right)}{16}}{\left(\tan\left(x\right)^{2}+2\right)^{2}}+\frac{19 \sqrt{2}\, \arctan\left(\frac{\tan\left(x\right) \sqrt{2}}{2}\right)}{64}}{a^{3}}$	39
risch	$-\frac{i\left(19\,{\mathrm e}^{6 i x}+171\,{\mathrm e}^{4 i x}+89\,{\mathrm e}^{2 i x}+9\right)}{16\left({\mathrm e}^{4 i x}+6\,{\mathrm e}^{2 i x}+1\right)^{2} a^{3}}+\frac{19 i \sqrt{2}\, \ln\left({\mathrm e}^{2 i x}+2 \sqrt{2}+3\right)}{128 a^{3}}-\frac{19 i \sqrt{2}\, \ln\left({\mathrm e}^{2 i x}-2 \sqrt{2}+3\right)}{128 a^{3}}$	91

input
```
int(1/(a+a*cos(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*((-13/32*tan(x)^3-11/16*tan(x))/(tan(x)^2+2)^2+19/64*2^(1/2)*arctan(
1/2*tan(x)*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+a\cos^2(x))^3}\,dx = -\frac{19\left(\sqrt{2}\cos(x)^4+2\sqrt{2}\cos(x)^2+\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)+4\left(9\cos(x)^3+13\cos(x)\right)\sin(x)}{128\left(a^3\cos(x)^4+2a^3\cos(x)^2+a^3\right)}$$

input
```
integrate(1/(a+a*cos(x)^2)^3,x, algorithm="fricas")
```

output
```
-1/128*(19*(sqrt(2)*cos(x)^4 + 2*sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3
*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) + 4*(9*cos(x)^3 + 13*cos(x))
*sin(x))/(a^3*cos(x)^4 + 2*a^3*cos(x)^2 + a^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(88) = 176.

Time = 24.19 (sec) , antiderivative size = 541, normalized size of antiderivative = 6.15

$$\int \frac{1}{(a+a\cos^2(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*cos(x)**2)**3,x)
```

output
```
19*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/
2)**8/(64*a**3*tan(x/2)**8 + 128*a**3*tan(x/2)**4 + 64*a**3) + 38*sqrt(2)*
(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**4/(64*a
**3*tan(x/2)**8 + 128*a**3*tan(x/2)**4 + 64*a**3) + 19*sqrt(2)*(atan(sqrt(
2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/(64*a**3*tan(x/2)**8 + 128*a
**3*tan(x/2)**4 + 64*a**3) + 19*sqrt(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*f
loor((x/2 - pi/2)/pi))*tan(x/2)**8/(64*a**3*tan(x/2)**8 + 128*a**3*tan(x/2
)**4 + 64*a**3) + 38*sqrt(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 -
 pi/2)/pi))*tan(x/2)**4/(64*a**3*tan(x/2)**8 + 128*a**3*tan(x/2)**4 + 64*a
**3) + 19*sqrt(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))
/(64*a**3*tan(x/2)**8 + 128*a**3*tan(x/2)**4 + 64*a**3) + 22*tan(x/2)**7/(
64*a**3*tan(x/2)**8 + 128*a**3*tan(x/2)**4 + 64*a**3) - 14*tan(x/2)**5/(64
*a**3*tan(x/2)**8 + 128*a**3*tan(x/2)**4 + 64*a**3) + 14*tan(x/2)**3/(64*a
**3*tan(x/2)**8 + 128*a**3*tan(x/2)**4 + 64*a**3) - 22*tan(x/2)/(64*a**3*t
an(x/2)**8 + 128*a**3*tan(x/2)**4 + 64*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(a+a\cos^2(x)\right)^3}\,dx = -\frac{13\,\tan\left(x\right)^3+22\,\tan\left(x\right)}{32\,\left(a^3\tan\left(x\right)^4+4\,a^3\tan\left(x\right)^2+4\,a^3\right)} + \frac{19\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\tan\left(x\right)\right)}{64\,a^3}$$

input
```
integrate(1/(a+a*cos(x)^2)^3,x, algorithm="maxima")
```

output
```
-1/32*(13*tan(x)^3 + 22*tan(x))/(a^3*tan(x)^4 + 4*a^3*tan(x)^2 + 4*a^3) +
19/64*sqrt(2)*arctan(1/2*sqrt(2)*tan(x))/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+a\cos^2(x))^3}\,dx = \frac{19\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)}{64a^3} - \frac{13\tan(x)^3+22\tan(x)}{32\left(\tan(x)^2+2\right)^2a^3}$$

input
```
integrate(1/(a+a*cos(x)^2)^3,x, algorithm="giac")
```

output
```
19/64*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - cos(2*x) + 1)))/a^3 - 1/32*(13*tan(x)^3 + 22*tan(x))/((tan(x)
^2 + 2)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

$$\int \frac{1}{(a+a\cos^2(x))^3}\,dx = \frac{19\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{64a^3} - \frac{\frac{13\tan(x)^3}{32}+\frac{11\tan(x)}{16}}{a^3\left(\tan(x)^2+2\right)^2}$$

input
```
int(1/(a + a*cos(x)^2)^3,x)
```

output
```
(19*2^(1/2)*atan((2^(1/2)*tan(x))/2))/(64*a^3) - ((11*tan(x))/16 + (13*tan
(x)^3)/32)/(a^3*(tan(x)^2 + 2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.86

$$\int \frac{1}{\left(a+a\cos^2(x)\right)^3}\,dx$$
$$= \frac{-19\sqrt{2}\,atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\sin\left(x\right)^4+76\sqrt{2}\,atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\sin\left(x\right)^2-76\sqrt{2}\,atan\left(\frac{\sqrt{2}-2\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+19\sqrt{2}}{64a}$$

input
```
int(1/(a+a*cos(x)^2)^3,x)
```

output
```
( - 19*sqrt(2)*atan((sqrt(2) - 2*tan(x/2))/sqrt(2))*sin(x)**4 + 76*sqrt(2)
*atan((sqrt(2) - 2*tan(x/2))/sqrt(2))*sin(x)**2 - 76*sqrt(2)*atan((sqrt(2)
 - 2*tan(x/2))/sqrt(2)) + 19*sqrt(2)*atan((sqrt(2) + 2*tan(x/2))/sqrt(2))*
sin(x)**4 - 76*sqrt(2)*atan((sqrt(2) + 2*tan(x/2))/sqrt(2))*sin(x)**2 + 76
*sqrt(2)*atan((sqrt(2) + 2*tan(x/2))/sqrt(2)) + 18*cos(x)*sin(x)**3 - 44*c
os(x)*sin(x))/(64*a**3*(sin(x)**4 - 4*sin(x)**2 + 4))
```

3.35 $\int (1-\cos^2(x))^{5/2}\,dx$

Optimal result . . . 297
Mathematica [A] (verified) . . . 297
Rubi [A] (verified) . . . 298
Maple [A] (verified) . . . 300
Fricas [A] (verification not implemented) . . . 300
Sympy [F(-1)] . . . 301
Maxima [A] (verification not implemented) . . . 301
Giac [B] (verification not implemented) . . . 301
Mupad [F(-1)] . . . 302
Reduce [F] . . . 302

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int (1-\cos^2(x))^{5/2}\,dx = -\frac{8}{15}\cot(x)\sqrt{\sin^2(x)} - \frac{4}{15}\cot(x)\sin^2(x)^{3/2} - \frac{1}{5}\cot(x)\sin^2(x)^{5/2}$$

output
```
-8/15*cot(x)*(sin(x)^2)^(1/2)-4/15*cot(x)*(sin(x)^2)^(3/2)-1/5*cot(x)*(sin
(x)^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int (1-\cos^2(x))^{5/2}\,dx = -\frac{1}{240}(150\cos(x) - 25\cos(3x) + 3\cos(5x))\csc(x)\sqrt{\sin^2(x)}$$

input
```
Integrate[(1 - Cos[x]^2)^(5/2),x]
```

output
```
-1/240*((150*Cos[x] - 25*Cos[3*x] + 3*Cos[5*x])*Csc[x]*Sqrt[Sin[x]^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3042, 3655, 3042, 3682, 3042, 3682, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\cos^2(x)\right)^{5/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2} \, dx$$

$$\Big\downarrow \text{3655}$$

$$\int \sin^2(x)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sin(x)^2\right)^{5/2} \, dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}\int \sin^2(x)^{3/2}dx-\frac{1}{5}\sin^2(x)^{5/2}\cot(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \left(\sin(x)^2\right)^{3/2} \, dx-\frac{1}{5}\sin^2(x)^{5/2}\cot(x)$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \sqrt{\sin^2(x)}dx-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)\right)-\frac{1}{5}\sin^2(x)^{5/2}\cot(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \sqrt{\sin(x)^2}dx-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)\right)-\frac{1}{5}\sin^2(x)^{5/2}\cot(x)$$

$$\Big\downarrow \text{3686}$$

$$\frac{4}{5}\left(\frac{2}{3}\sqrt{\sin^2(x)}\csc(x)\int\sin(x)dx-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)\right)-\frac{1}{5}\sin^2(x)^{5/2}\cot(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\sqrt{\sin^2(x)}\csc(x)\int\sin(x)dx-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)\right)-\frac{1}{5}\sin^2(x)^{5/2}\cot(x)$$

$$\Big\downarrow \text{3118}$$

$$\frac{4}{5}\left(-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)-\frac{2}{3}\sqrt{\sin^2(x)}\cot(x)\right)-\frac{1}{5}\sin^2(x)^{5/2}\cot(x)$$

input
```
Int[(1 - Cos[x]^2)^(5/2),x]
```

output
```
-1/5*(Cot[x]*(Sin[x]^2)^(5/2)) + (4*((-2*Cot[x]*Sqrt[Sin[x]^2])/3 - (Cot[x
]*(Sin[x]^2)^(3/2))/3))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] && !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

method	result
default	$-\frac{2\sin(x)\cos(x)\left(3\sin(x)^{4}+4\sin(x)^{2}+8\right)}{15\sqrt{2-2\cos(2x)}}$
risch	$-\frac{i\mathrm{e}^{6ix}\sqrt{-(\mathrm{e}^{2ix}-1)^{2}\mathrm{e}^{-2ix}}}{160(\mathrm{e}^{2ix}-1)}-\frac{5i\mathrm{e}^{2ix}\sqrt{-(\mathrm{e}^{2ix}-1)^{2}\mathrm{e}^{-2ix}}}{16(\mathrm{e}^{2ix}-1)}-\frac{5i\sqrt{-(\mathrm{e}^{2ix}-1)^{2}\mathrm{e}^{-2ix}}}{16(\mathrm{e}^{2ix}-1)}+\frac{5i\mathrm{e}^{-2ix}\sqrt{-(\mathrm{e}^{2ix}-1)^{2}\mathrm{e}^{-2ix}}}{96(\mathrm{e}^{2ix}-1)}+\frac{11i\sqrt{-(}{}$

input
```
int((1-cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*sin(x)*cos(x)*(3*sin(x)^4+4*sin(x)^2+8)/(sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \left(1-\cos^{2}(x)\right)^{5/2}\,dx=-\frac{1}{5}\cos\left(x\right)^{5}+\frac{2}{3}\cos\left(x\right)^{3}-\cos\left(x\right)$$

input
```
integrate((1-cos(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(1-\cos^2(x)\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((1-cos(x)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \left(1-\cos^2(x)\right)^{5/2}\,dx=\frac{1}{80}\cos(5x)-\frac{5}{48}\cos(3x)+\frac{5}{8}\cos(x)$$

input
```
integrate((1-cos(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/80*cos(5*x) - 5/48*cos(3*x) + 5/8*cos(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(31) = 62.

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \left(1-\cos^2(x)\right)^{5/2}\,dx=$$
$$-\frac{16\left(10\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^4+5\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^2+\mathrm{sgn}\left(\tan\left(\frac{1}{2}\right.\right.\right.}{15\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^5}$$

input
```
integrate((1-cos(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-16/15*(10*sgn(tan(1/2*x)^3 + tan(1/2*x))*tan(1/2*x)^4 + 5*sgn(tan(1/2*x)^
3 + tan(1/2*x))*tan(1/2*x)^2 + sgn(tan(1/2*x)^3 + tan(1/2*x)))/(tan(1/2*x)
^2 + 1)^5
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-\cos^2(x)\right)^{5/2} dx = \int \left(1-\cos(x)^2\right)^{5/2} dx$$

input
```
int((1 - cos(x)^2)^(5/2),x)
```

output
```
int((1 - cos(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \left(1-\cos^2(x)\right)^{5/2} dx = \int \sqrt{-\cos(x)^2+1}dx + \int \sqrt{-\cos(x)^2+1}\,\cos(x)^4 dx - 2\left(\int \sqrt{-\cos(x)^2+1}\,\cos(x)^2 dx\right)$$

input
```
int((1-cos(x)^2)^(5/2),x)
```

output
```
int(sqrt( - cos(x)**2 + 1),x) + int(sqrt( - cos(x)**2 + 1)*cos(x)**4,x) -
2*int(sqrt( - cos(x)**2 + 1)*cos(x)**2,x)
```

3.36 $\int \left(1-\cos^2(x)\right)^{3/2} dx$

Optimal result . . . 303
Mathematica [A] (verified) . . . 303
Rubi [A] (verified) . . . 304
Maple [A] (verified) . . . 306
Fricas [A] (verification not implemented) . . . 306
Sympy [F] . . . 306
Maxima [A] (verification not implemented) . . . 307
Giac [B] (verification not implemented) . . . 307
Mupad [F(-1)] . . . 308
Reduce [F] . . . 308

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \left(1-\cos^2(x)\right)^{3/2} dx = -\frac{2}{3}\cot(x)\sqrt{\sin^2(x)}-\frac{1}{3}\cot(x)\sin^2(x)^{3/2}$$

output
```
-2/3*cot(x)*(sin(x)^2)^(1/2)-1/3*cot(x)*(sin(x)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \left(1-\cos^2(x)\right)^{3/2} dx = \frac{1}{12}(-9\cos(x)+\cos(3x))\csc(x)\sqrt{\sin^2(x)}$$

input
```
Integrate[(1 - Cos[x]^2)^(3/2),x]
```

output
```
((-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[Sin[x]^2])/12
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3655, 3042, 3682, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\cos^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3655}$$

$$\int \sin^2(x)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sin(x)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{2}{3}\int \sqrt{\sin^2(x)}dx-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \sqrt{\sin(x)^2}dx-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)$$

$$\Big\downarrow \text{3686}$$

$$\frac{2}{3}\sqrt{\sin^2(x)}\csc(x)\int \sin(x)dx-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\sqrt{\sin^2(x)}\csc(x)\int \sin(x)dx-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)$$

$$\Big\downarrow \text{3118}$$

$$-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)-\frac{2}{3}\sqrt{\sin^2(x)}\cot(x)$$

input
```
Int[(1 - Cos[x]^2)^(3/2),x]
```

output
```
(-2*Cot[x]*Sqrt[Sin[x]^2])/3 - (Cot[x]*(Sin[x]^2)^(3/2))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{2\sin(x)\cos(x)\left(\cos(x)^{2}-3\right)}{3\sqrt{2-2\cos(2x)}}$	19
risch	$\frac{i e^{4ix}\sqrt{-\left(e^{2ix}-1\right)^{2} e^{-2ix}}}{24\,e^{2ix}-24}-\frac{3i e^{2ix}\sqrt{-\left(e^{2ix}-1\right)^{2} e^{-2ix}}}{8\left(e^{2ix}-1\right)}-\frac{3i\sqrt{-\left(e^{2ix}-1\right)^{2} e^{-2ix}}}{8\left(e^{2ix}-1\right)}+\frac{i e^{-2ix}\sqrt{-\left(e^{2ix}-1\right)^{2} e^{-2ix}}}{24\,e^{2ix}-24}$	137

input
```
int((1-cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sin(x)*cos(x)*(cos(x)^2-3)/(sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.38

$$\int \left(1-\cos^{2}(x)\right)^{3/2}\,dx=\frac{1}{3}\cos\left(x\right)^{3}-\cos\left(x\right)$$

input
```
integrate((1-cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*cos(x)^3 - cos(x)
```

Sympy [F]

$$\int \left(1-\cos^{2}(x)\right)^{3/2}\,dx=\int \left(1-\cos^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1-cos(x)**2)**(3/2),x)
```

output
```
Integral((1 - cos(x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.38

$$\int \left(1-\cos^2(x)\right)^{3/2} dx = -\frac{1}{12}\cos(3x) + \frac{3}{4}\cos(x)$$

input
```
integrate((1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/12*cos(3*x) + 3/4*cos(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(21) = 42.

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \left(1-\cos^2(x)\right)^{3/2} dx = -\frac{4\left(3\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^2+\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\right)}{3\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^3}$$

input
```
integrate((1-cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-4/3*(3*sgn(tan(1/2*x)^3 + tan(1/2*x))*tan(1/2*x)^2 + sgn(tan(1/2*x)^3 + t
an(1/2*x)))/(tan(1/2*x)^2 + 1)^3
```

Mupad [F(-1)]

Timed out.

$$\int \left(1-\cos^2(x)\right)^{3/2} dx = \int \left(1-\cos(x)^2\right)^{3/2} dx$$

input
```
int((1 - cos(x)^2)^(3/2),x)
```

output
```
int((1 - cos(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(1-\cos^2(x)\right)^{3/2} dx = \int \sqrt{-\cos(x)^2+1}dx - \left(\int \sqrt{-\cos(x)^2+1}\,\cos(x)^2\,dx\right)$$

input
```
int((1-cos(x)^2)^(3/2),x)
```

output
```
int(sqrt( - cos(x)**2 + 1),x) - int(sqrt( - cos(x)**2 + 1)*cos(x)**2,x)
```

3.37 $\int \sqrt{1-\cos^2(x)}\,dx$

Optimal result . . . 309
Mathematica [A] (verified) . . . 309
Rubi [A] (verified) . . . 310
Maple [A] (verified) . . . 311
Fricas [A] (verification not implemented) . . . 312
Sympy [F] . . . 312
Maxima [A] (verification not implemented) . . . 312
Giac [B] (verification not implemented) . . . 313
Mupad [B] (verification not implemented) . . . 313
Reduce [F] . . . 313

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \sqrt{1-\cos^2(x)}\,dx=-\cot(x)\sqrt{\sin^2(x)}$$

output
```
-cot(x)*(sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \sqrt{1-\cos^2(x)}\,dx=-\cot(x)\sqrt{\sin^2(x)}$$

input
```
Integrate[Sqrt[1 - Cos[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[Sin[x]^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3655, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\cos^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{1-\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \sqrt{\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(x)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\sqrt{\sin^2(x)}\csc(x)\int \sin(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sin^2(x)}\csc(x)\int \sin(x)dx$$

$$\Big\downarrow \text{3118}$$

$$\sqrt{\sin^2(x)}(-\cot(x))$$

input
```
Int[Sqrt[1 - Cos[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[Sin[x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{2\sin(x)\cos(x)}{\sqrt{2-2\cos(2x)}}$	13
risch	$-\frac{i e^{2ix}\sqrt{-\left(e^{2ix}-1\right)^{2} e^{-2ix}}}{2\left(e^{2ix}-1\right)}-\frac{i\sqrt{-\left(e^{2ix}-1\right)^{2} e^{-2ix}}}{2\left(e^{2ix}-1\right)}$	67

input
```
int((1-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-sin(x)*cos(x)/(sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.33

$$\int \sqrt{1-\cos^2(x)}\,dx = -\cos(x)$$

input
```
integrate((1-cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-cos(x)
```

Sympy [F]

$$\int \sqrt{1-\cos^2(x)}\,dx = \int \sqrt{1-\cos^2(x)}\,dx$$

input
```
integrate((1-cos(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(1 - cos(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sqrt{1-\cos^2(x)}\,dx = -\frac{1}{\sqrt{\tan(x)^2+1}}$$

input
```
integrate((1-cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/sqrt(tan(x)^2 + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \sqrt{1-\cos^2(x)}\,dx = -\frac{2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}$$

input
```
integrate((1-cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*sgn(tan(1/2*x)^3 + tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sqrt{1-\cos^2(x)}\,dx = -\cot(x)\,\sqrt{\sin(x)^2}$$

input
```
int((1 - cos(x)^2)^(1/2),x)
```

output
```
-cot(x)*(sin(x)^2)^(1/2)
```

Reduce [F]

$$\int \sqrt{1-\cos^2(x)}\,dx = \int \sqrt{-\cos(x)^2+1}dx$$

input
```
int((1-cos(x)^2)^(1/2),x)
```

output
```
int(sqrt( - cos(x)**2 + 1),x)
```

3.38 $\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx$

Optimal result . . . 314
Mathematica [A] (verified) . . . 314
Rubi [A] (verified) . . . 315
Maple [A] (verified) . . . 316
Fricas [A] (verification not implemented) . . . 317
Sympy [F] . . . 317
Maxima [B] (verification not implemented) . . . 317
Giac [A] (verification not implemented) . . . 318
Mupad [F(-1)] . . . 318
Reduce [F] . . . 319

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx=-\frac{\operatorname{arctanh}(\cos(x))\sin(x)}{\sqrt{\sin^2(x)}}$$

output
```
-arctanh(cos(x))*sin(x)/(sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx=-\frac{\operatorname{arctanh}(\cos(x))\sin(x)}{\sqrt{\sin^2(x)}}$$

input
```
Integrate[1/Sqrt[1 - Cos[x]^2],x]
```

output
```
-((ArcTanh[Cos[x]]*Sin[x])/Sqrt[Sin[x]^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3655, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{1-\sin\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{1}{\sqrt{\sin^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin(x)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin(x)\int \csc(x)dx}{\sqrt{\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(x)\int \csc(x)dx}{\sqrt{\sin^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sin(x)\text{arctanh}(\cos(x))}{\sqrt{\sin^2(x)}}$$

input
```
Int[1/Sqrt[1 - Cos[x]^2],x]
```

output
```
-((ArcTanh[Cos[x]]*Sin[x])/Sqrt[Sin[x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{2\,\operatorname{arctanh}(\cos(x))\sin(x)}{\sqrt{2-2\cos(2x)}}$	14
risch	$\frac{2\ln(\mathrm{e}^{ix}-1)\sin(x)}{\sqrt{-(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}-\frac{2\ln(\mathrm{e}^{ix}+1)\sin(x)}{\sqrt{-(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}$	62

input
```
int(1/(1-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(cos(x))*sin(x)/(sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = -\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

input
```
integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx$$

input
```
integrate(1/(1-cos(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(1 - cos(x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = \frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right) - \frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

input
```
integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/2*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = \frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)}$$

input
```
integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(tan(1/2*x)))/sgn(tan(1/2*x)^3 + tan(1/2*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{1-\cos\left(x\right)^2}}\,dx$$

input
```
int(1/(1 - cos(x)^2)^(1/2),x)
```

output
```
int(1/(1 - cos(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = -\left(\int \frac{\sqrt{-\cos(x)^2+1}}{\cos(x)^2-1}dx\right)$$

input
```
int(1/(1-cos(x)^2)^(1/2),x)
```

output
```
- int(sqrt( - cos(x)**2 + 1)/(cos(x)**2 - 1),x)
```

3.39 $\int \frac{1}{(1-\cos^2(x))^{3/2}} \, dx$

Optimal result 320
Mathematica [A] (verified) 320
Rubi [A] (verified) 321
Maple [A] (verified) 323
Fricas [A] (verification not implemented) 323
Sympy [F] 324
Maxima [B] (verification not implemented) 324
Giac [B] (verification not implemented) 325
Mupad [F(-1)] 325
Reduce [F] 326

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{1}{(1-\cos^2(x))^{3/2}} \, dx = -\frac{\cot(x)}{2\sqrt{\sin^2(x)}} - \frac{\operatorname{arctanh}(\cos(x))\sin(x)}{2\sqrt{\sin^2(x)}}$$

output
```
-1/2*cot(x)/(sin(x)^2)^(1/2)-1/2*arctanh(cos(x))*sin(x)/(sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

$$\int \frac{1}{(1-\cos^2(x))^{3/2}} \, dx = -\frac{\left(\csc^2\left(\frac{x}{2}\right)+4\log\left(\cos\left(\frac{x}{2}\right)\right)-4\log\left(\sin\left(\frac{x}{2}\right)\right)-\sec^2\left(\frac{x}{2}\right)\right)\sin(x)}{8\sqrt{\sin^2(x)}}$$

input
```
Integrate[(1 - Cos[x]^2)^(-3/2),x]
```

output
```
-1/8*((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x]
)/Sqrt[Sin[x]^2]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3655, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-\cos^2(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{1}{\sin^2(x)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin^2(x)}}dx-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sin(x)^2}}dx-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin(x)\int \csc(x)dx}{2\sqrt{\sin^2(x)}}-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(x)\int \csc(x)dx}{2\sqrt{\sin^2(x)}}-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sin(x)\text{arctanh}(\cos(x))}{2\sqrt{\sin^2(x)}}-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}$$

input
```
Int[(1 - Cos[x]^2)^(-3/2),x]
```

output
```
-1/2*Cot[x]/Sqrt[Sin[x]^2] - (ArcTanh[Cos[x]]*Sin[x])/(2*Sqrt[Sin[x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{2\left(\frac{\cos(x)}{2}+\frac{(\ln(1+\cos(x))-\ln(-1+\cos(x)))\sin(x)^{2}}{4}\right)}{\sin(x)\sqrt{2-2\cos(2x)}}$	37
risch	$-\frac{i(\mathrm{e}^{2ix}+1)}{(\mathrm{e}^{2ix}-1)\sqrt{-(\mathrm{e}^{2ix}-1)^{2}\mathrm{e}^{-2ix}}}-\frac{\ln(\mathrm{e}^{ix}+1)\sin(x)}{\sqrt{-(\mathrm{e}^{2ix}-1)^{2}\mathrm{e}^{-2ix}}}+\frac{\ln(\mathrm{e}^{ix}-1)\sin(x)}{\sqrt{-(\mathrm{e}^{2ix}-1)^{2}\mathrm{e}^{-2ix}}}$	98

input
```
int(1/(1-cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(1/2*cos(x)+1/4*(ln(1+cos(x))-ln(-1+cos(x)))*sin(x)^2)/sin(x)/(sin(x)^2)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{1}{(1-\cos^2(x))^{3/2}}\,dx = -\frac{(\cos(x)^2-1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)^2-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\cos(x)}{4\left(\cos(x)^2-1\right)}$$

input
```
integrate(1/(1-cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*((cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) - (cos(x)^2 - 1)*log(-1/2*cos(x
) + 1/2) - 2*cos(x))/(cos(x)^2 - 1)
```

Sympy [F]

$$\int \frac{1}{\left(1-\cos^2(x)\right)^{3/2}}\, dx = \int \frac{1}{\left(1-\cos^2\left(x\right)\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(1-cos(x)**2)**(3/2),x)
```

output
```
Integral((1 - cos(x)**2)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(24) = 48$.

Time = 0.16 (sec) , antiderivative size = 300, normalized size of antiderivative = 9.38

$$\int \frac{1}{\left(1-\cos^2(x)\right)^{3/2}}\, dx = \frac{4\left(\cos\left(3\,x\right)+\cos\left(x\right)\right)\cos\left(4\,x\right)-4\left(2\,\cos\left(2\,x\right)-1\right)\cos\left(3\,x\right)-8\,\cos\left(2\,x\right)\cos\left(x\right)}{}$$

input
```
integrate(1/(1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*(cos(3*x) + cos(x))*cos(4*x) - 4*(2*cos(2*x) - 1)*cos(3*x) - 8*cos(
2*x)*cos(x) + (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - s
in(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*log(cos(x
)^2 + sin(x)^2 + 2*cos(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2
 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(
2*x) - 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*(sin(3*x) + sin(x))*
sin(4*x) - 8*sin(3*x)*sin(2*x) - 8*sin(2*x)*sin(x) + 4*cos(x))/(2*(2*cos(2
*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*si
n(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(24) = 48$.

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.44

$$\int \frac{1}{(1-\cos^2(x))^{3/2}}\,dx = \frac{\tan\left(\frac{1}{2}\,x\right)^2}{8\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)} + \frac{\log\left(\tan\left(\frac{1}{2}\,x\right)^2\right)}{4\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)} - \frac{2\,\tan\left(\frac{1}{2}\,x\right)^2+1}{8\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^2}$$

input
```
integrate(1/(1-cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*tan(1/2*x)^2/sgn(tan(1/2*x)^3 + tan(1/2*x)) + 1/4*log(tan(1/2*x)^2)/sg
n(tan(1/2*x)^3 + tan(1/2*x)) - 1/8*(2*tan(1/2*x)^2 + 1)/(sgn(tan(1/2*x)^3
+ tan(1/2*x))*tan(1/2*x)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(1-\cos\left(x\right)^2\right)^{3/2}}\,dx$$

input
```
int(1/(1 - cos(x)^2)^(3/2),x)
```

output
```
int(1/(1 - cos(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(1-\cos^2(x)\right)^{3/2}}\,dx = \int \frac{\sqrt{-\cos\left(x\right)^{2}+1}}{\cos\left(x\right)^{4}-2\cos\left(x\right)^{2}+1}dx$$

input
```
int(1/(1-cos(x)^2)^(3/2),x)
```

output
```
int(sqrt( - cos(x)**2 + 1)/(cos(x)**4 - 2*cos(x)**2 + 1),x)
```

3.40 $\int \frac{1}{(1-\cos^2(x))^{5/2}}\,dx$

Optimal result 327
Mathematica [A] (verified) 327
Rubi [A] (verified) 328
Maple [A] (verified) 330
Fricas [B] (verification not implemented) 331
Sympy [F] 331
Maxima [B] (verification not implemented) 331
Giac [B] (verification not implemented) 332
Mupad [F(-1)] 333
Reduce [F] 333

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{1}{(1-\cos^2(x))^{5/2}}\,dx = -\frac{\cot(x)}{4\sin^2(x)^{3/2}} - \frac{3\cot(x)}{8\sqrt{\sin^2(x)}} - \frac{3\text{arctanh}(\cos(x))\sin(x)}{8\sqrt{\sin^2(x)}}$$

output
```
-1/4*cot(x)/(sin(x)^2)^(3/2)-3/8*cot(x)/(sin(x)^2)^(1/2)-3/8*arctanh(cos(x
))*sin(x)/(sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int \frac{1}{(1-\cos^2(x))^{5/2}}\,dx = -\frac{\left(6\csc^2\left(\frac{x}{2}\right)+\csc^4\left(\frac{x}{2}\right)+24\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-6\sec^2\left(\frac{x}{2}\right)-\sec^4\left(\frac{x}{2}\right)\right)\sin(x)}{64\sqrt{\sin^2(x)}}$$

input
```
Integrate[(1 - Cos[x]^2)^(-5/2),x]
```

output
```
-1/64*((6*Csc[x/2]^2 + Csc[x/2]^4 + 24*(Log[Cos[x/2]] - Log[Sin[x/2]]) - 6
*Sec[x/2]^2 - Sec[x/2]^4)*Sin[x])/Sqrt[Sin[x]^2]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 3655, 3042, 3683, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-\cos^2(x))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(1-\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 3655$$

$$\int \frac{1}{\sin^2(x)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(\sin(x)^2)^{5/2}}dx$$

$$\Big\downarrow \; 3683$$

$$\frac{3}{4}\int \frac{1}{\sin^2(x)^{3/2}}dx-\frac{\cot(x)}{4\sin^2(x)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{4}\int \frac{1}{(\sin(x)^2)^{3/2}}dx-\frac{\cot(x)}{4\sin^2(x)^{3/2}}$$

$$\Big\downarrow \; 3683$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\sin^2(x)}}dx-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}\right)-\frac{\cot(x)}{4\sin^2(x)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\sin(x)^2}}dx-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}\right)-\frac{\cot(x)}{4\sin^2(x)^{3/2}}$$

$$\Big\downarrow \; 3686$$

$$\frac{3}{4}\left(\frac{\sin(x)\int \csc(x)dx}{2\sqrt{\sin^2(x)}}-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}\right)-\frac{\cot(x)}{4\sin^2(x)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{4}\left(\frac{\sin(x)\int \csc(x)dx}{2\sqrt{\sin^2(x)}}-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}\right)-\frac{\cot(x)}{4\sin^2(x)^{3/2}}$$

$$\Big\downarrow \; 4257$$

$$\frac{3}{4}\left(-\frac{\sin(x)\text{arctanh}(\cos(x))}{2\sqrt{\sin^2(x)}}-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}\right)-\frac{\cot(x)}{4\sin^2(x)^{3/2}}$$

input
```
Int[(1 - Cos[x]^2)^(-5/2),x]
```

output
```
-1/4*Cot[x]/(Sin[x]^2)^(3/2) + (3*(-1/2*Cot[x]/Sqrt[Sin[x]^2] - (ArcTanh[C
os[x]]*Sin[x])/(2*Sqrt[Sin[x]^2])))/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{6 \sin(x)^2 \cos(x)+4 \cos(x)+(3 \ln(1+\cos(x))-3 \ln(-1+\cos(x))) \sin(x)^4}{8(1+\cos(x))(-1+\cos(x)) \sin(x) \sqrt{2-2 \cos(2x)}}$	58
risch	$-\frac{i(3 \,\mathrm{e}^{6ix}-11 \,\mathrm{e}^{4ix}-11 \,\mathrm{e}^{2ix}+3)}{4(\mathrm{e}^{2ix}-1)^3 \sqrt{-(\mathrm{e}^{2ix}-1)^2 \mathrm{e}^{-2ix}}}-\frac{3 \ln(\mathrm{e}^{ix}+1) \sin(x)}{4\sqrt{-(\mathrm{e}^{2ix}-1)^2 \mathrm{e}^{-2ix}}}+\frac{3 \ln(\mathrm{e}^{ix}-1) \sin(x)}{4\sqrt{-(\mathrm{e}^{2ix}-1)^2 \mathrm{e}^{-2ix}}}$	115

input
```
int(1/(1-cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(6*sin(x)^2*cos(x)+4*cos(x)+(3*ln(1+cos(x))-3*ln(-1+cos(x)))*sin(x)^4
)/(1+cos(x))/(-1+cos(x))/sin(x)/(sin(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.50

$$\int \frac{1}{(1-\cos^2(x))^{5/2}}\,dx = \frac{6\,\cos(x)^3 - 3\left(\cos(x)^4 - 2\,\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\,\cos(x) + \frac{1}{2}\right) + 3\left(\cos(x)^4 - 2\,\cos\right.}{16\left(\cos(x)^4 - 2\,\cos(x)^2 + 1\right)}$$

input
```
integrate(1/(1-cos(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/16*(6*cos(x)^3 - 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + 3
*(cos(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) - 10*cos(x))/(cos(x)^4
 - 2*cos(x)^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(1-\cos^2(x))^{5/2}}\,dx = \int \frac{1}{(1-\cos^2(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(1-cos(x)**2)**(5/2),x)
```

output
```
Integral((1 - cos(x)**2)**(-5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 869 vs. $2(34) = 68$.

Time = 0.19 (sec) , antiderivative size = 869, normalized size of antiderivative = 18.89

$$\int \frac{1}{(1-\cos^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-cos(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/16*(4*(3*cos(7*x) - 11*cos(5*x) - 11*cos(3*x) + 3*cos(x))*cos(8*x) - 12*
(4*cos(6*x) - 6*cos(4*x) + 4*cos(2*x) - 1)*cos(7*x) + 16*(11*cos(5*x) + 11
*cos(3*x) - 3*cos(x))*cos(6*x) - 44*(6*cos(4*x) - 4*cos(2*x) + 1)*cos(5*x)
 - 24*(11*cos(3*x) - 3*cos(x))*cos(4*x) + 44*(4*cos(2*x) - 1)*cos(3*x) - 4
8*cos(2*x)*cos(x) + 3*(2*(4*cos(6*x) - 6*cos(4*x) + 4*cos(2*x) - 1)*cos(8*
x) - cos(8*x)^2 + 8*(6*cos(4*x) - 4*cos(2*x) + 1)*cos(6*x) - 16*cos(6*x)^2
 + 12*(4*cos(2*x) - 1)*cos(4*x) - 36*cos(4*x)^2 - 16*cos(2*x)^2 + 4*(2*sin
(6*x) - 3*sin(4*x) + 2*sin(2*x))*sin(8*x) - sin(8*x)^2 + 16*(3*sin(4*x) -
2*sin(2*x))*sin(6*x) - 16*sin(6*x)^2 - 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x
) - 16*sin(2*x)^2 + 8*cos(2*x) - 1)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1
) - 3*(2*(4*cos(6*x) - 6*cos(4*x) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^2
+ 8*(6*cos(4*x) - 4*cos(2*x) + 1)*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2*x
) - 1)*cos(4*x) - 36*cos(4*x)^2 - 16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(4*
x) + 2*sin(2*x))*sin(8*x) - sin(8*x)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*sin(
6*x) - 16*sin(6*x)^2 - 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x)^
2 + 8*cos(2*x) - 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*(3*sin(7*x
) - 11*sin(5*x) - 11*sin(3*x) + 3*sin(x))*sin(8*x) - 24*(2*sin(6*x) - 3*si
n(4*x) + 2*sin(2*x))*sin(7*x) + 16*(11*sin(5*x) + 11*sin(3*x) - 3*sin(x))*
sin(6*x) - 88*(3*sin(4*x) - 2*sin(2*x))*sin(5*x) - 24*(11*sin(3*x) - 3*sin
(x))*sin(4*x) + 176*sin(3*x)*sin(2*x) - 48*sin(2*x)*sin(x) + 12*cos(x))...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(34) = 68$.

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.26

$$\int \frac{1}{(1-\cos^2(x))^{5/2}}\,dx = \frac{1}{64}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^4 + \frac{1}{8}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^2 + \frac{3\log\left(\tan\left(\frac{1}{2}x\right)^2\right)}{16\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)} - \frac{18\tan\left(\frac{1}{2}x\right)^4+8\tan\left(\frac{1}{2}x\right)^2+1}{64\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^4}$$

input
```
integrate(1/(1-cos(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/64*sgn(tan(1/2*x)^3 + tan(1/2*x))*tan(1/2*x)^4 + 1/8*sgn(tan(1/2*x)^3 +
tan(1/2*x))*tan(1/2*x)^2 + 3/16*log(tan(1/2*x)^2)/sgn(tan(1/2*x)^3 + tan(1
/2*x)) - 1/64*(18*tan(1/2*x)^4 + 8*tan(1/2*x)^2 + 1)/(sgn(tan(1/2*x)^3 + t
an(1/2*x))*tan(1/2*x)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-\cos^2(x))^{5/2}}\,dx = \int \frac{1}{\left(1-\cos(x)^2\right)^{5/2}}\,dx$$

input
```
int(1/(1 - cos(x)^2)^(5/2),x)
```

output
```
int(1/(1 - cos(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(1-\cos^2(x))^{5/2}}\,dx = -\left(\int \frac{\sqrt{-\cos(x)^2+1}}{\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1}dx\right)$$

input
```
int(1/(1-cos(x)^2)^(5/2),x)
```

output
```
 - int(sqrt( - cos(x)**2 + 1)/(cos(x)**6 - 3*cos(x)**4 + 3*cos(x)**2 - 1),
x)
```

3.41 $\int (a-a\cos^2(x))^{5/2}\,dx$

Optimal result . . . 334
Mathematica [A] (verified) . . . 334
Rubi [A] (verified) . . . 335
Maple [A] (verified) . . . 337
Fricas [A] (verification not implemented) . . . 337
Sympy [F(-1)] . . . 338
Maxima [F] . . . 338
Giac [A] (verification not implemented) . . . 338
Mupad [F(-1)] . . . 339
Reduce [F] . . . 339

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int (a-a\cos^2(x))^{5/2}\,dx = -\frac{8}{15}a^2\cot(x)\sqrt{a\sin^2(x)}-\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

output
```
-8/15*a^2*cot(x)*(a*sin(x)^2)^(1/2)-4/15*a*cot(x)*(a*sin(x)^2)^(3/2)-1/5*c
ot(x)*(a*sin(x)^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int (a-a\cos^2(x))^{5/2}\,dx = -\frac{1}{240}a^2(150\cos(x)-25\cos(3x)+3\cos(5x))\csc(x)\sqrt{a\sin^2(x)}$$

input
```
Integrate[(a - a*Cos[x]^2)^(5/2),x]
```

output
```
-1/240*(a^2*(150*Cos[x] - 25*Cos[3*x] + 3*Cos[5*x])*Csc[x]*Sqrt[a*Sin[x]^2
])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 3655, 3042, 3682, 3042, 3682, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\cos^2(x)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \left(a\sin^2(x)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin(x)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\int \left(a\sin^2(x)\right)^{3/2}dx-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\int \left(a\sin(x)^2\right)^{3/2}dx-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\sin^2(x)}dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\right)-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\sin(x)^2}dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\right)-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\csc(x)\sqrt{a\sin^2(x)}\int\sin(x)dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\right)-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow\ 3042$$

$$\frac{4}{5}a\left(\frac{2}{3}a\csc(x)\sqrt{a\sin^2(x)}\int\sin(x)dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}\right)-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

$$\Big\downarrow\ 3118$$

$$\frac{4}{5}a\left(-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}\right)-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}$$

input
```
Int[(a - a*Cos[x]^2)^(5/2),x]
```

output
```
-1/5*(Cot[x]*(a*Sin[x]^2)^(5/2)) + (4*a*((-2*a*Cot[x]*Sqrt[a*Sin[x]^2])/3
- (Cot[x]*(a*Sin[x]^2)^(3/2))/3))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result
default	$-\frac{\sin(x) a^{3} \cos(x) \left(3 \sin(x)^{4}+4 \sin(x)^{2}+8\right)}{15 \sqrt{a \sin(x)^{2}}}$
risch	$-\frac{i a^{2} e^{6 i x} \sqrt{-a (e^{2 i x}-1)^{2} e^{-2 i x}}}{160 (e^{2 i x}-1)}-\frac{5 i a^{2} e^{2 i x} \sqrt{-a (e^{2 i x}-1)^{2} e^{-2 i x}}}{16 (e^{2 i x}-1)}-\frac{5 i \sqrt{-a (e^{2 i x}-1)^{2} e^{-2 i x}}\, a^{2}}{16 (e^{2 i x}-1)}+\frac{5 i a^{2} e^{-2 i x} \sqrt{-a (e^{2 i x}-1)^{2} e^{-2}}}{96 (e^{2 i x}-1)}$

input
```
int((a-a*cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*sin(x)*a^3*cos(x)*(3*sin(x)^4+4*sin(x)^2+8)/(a*sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \left(a-a\cos^{2}(x)\right)^{5/2} dx=-\frac{\left(3 a^{2} \cos (x)^{5}-10 a^{2} \cos (x)^{3}+15 a^{2} \cos (x)\right) \sqrt{-a \cos (x)^{2}+a}}{15 \sin (x)}$$

input
```
integrate((a-a*cos(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(3*a^2*cos(x)^5 - 10*a^2*cos(x)^3 + 15*a^2*cos(x))*sqrt(-a*cos(x)^2
+ a)/sin(x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a-a\cos^2(x)\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a-a*cos(x)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a-a\cos^2(x)\right)^{5/2}\,dx=\int \left(-a\cos\left(x\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a-a*cos(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-a*cos(x)^2 + a)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.42

$$\int \left(a-a\cos^2(x)\right)^{5/2}\,dx=$$
$$-\frac{16\left(10\,a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^4+5\,a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^2+a^{\frac{5}{2}}\mathrm{sgn}\left(\cdots\right)\right)}{15\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^5}$$

input
```
integrate((a-a*cos(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-16/15*(10*a^(5/2)*sgn(tan(1/2*x)^3 + tan(1/2*x))*tan(1/2*x)^4 + 5*a^(5/2)
*sgn(tan(1/2*x)^3 + tan(1/2*x))*tan(1/2*x)^2 + a^(5/2)*sgn(tan(1/2*x)^3 +
tan(1/2*x)))/(tan(1/2*x)^2 + 1)^5
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-a\cos^2(x)\right)^{5/2}\,dx=\int \left(a-a\cos\left(x\right)^2\right)^{5/2}\,dx$$

input
```
int((a - a*cos(x)^2)^(5/2),x)
```

output
```
int((a - a*cos(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \left(a-a\cos^2(x)\right)^{5/2}\,dx=\sqrt{a}\,a^2\left(\int \sqrt{-\cos\left(x\right)^2+1}dx+\int \sqrt{-\cos\left(x\right)^2+1}\,\cos\left(x\right)^4dx-2\left(\int \sqrt{-\cos\left(x\right)^2+1}\,\cos\left(x\right)^2dx\right)\right)$$

input
```
int((a-a*cos(x)^2)^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt( - cos(x)**2 + 1),x) + int(sqrt( - cos(x)**2 + 1)*c
os(x)**4,x) - 2*int(sqrt( - cos(x)**2 + 1)*cos(x)**2,x))
```

3.42 $\int \left(a-a\cos^2(x)\right)^{3/2}\,dx$

Optimal result . . . 340
Mathematica [A] (verified) . . . 340
Rubi [A] (verified) . . . 341
Maple [A] (verified) . . . 343
Fricas [A] (verification not implemented) . . . 343
Sympy [F(-1)] . . . 343
Maxima [F] . . . 344
Giac [A] (verification not implemented) . . . 344
Mupad [F(-1)] . . . 344
Reduce [F] . . . 345

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \left(a-a\cos^2(x)\right)^{3/2}\,dx=-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

output
```
-2/3*a*cot(x)*(a*sin(x)^2)^(1/2)-1/3*cot(x)*(a*sin(x)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a-a\cos^2(x)\right)^{3/2}\,dx=\frac{1}{12}a(-9\cos(x)+\cos(3x))\csc(x)\sqrt{a\sin^2(x)}$$

input
```
Integrate[(a - a*Cos[x]^2)^(3/2),x]
```

output
```
(a*(-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[a*Sin[x]^2])/12
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3042, 3655, 3042, 3682, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-a\cos^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3655}$$

$$\int \left(a\sin^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin(x)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{2}{3}a\int \sqrt{a\sin^2(x)}dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\int \sqrt{a\sin(x)^2}dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{2}{3}a\csc(x)\sqrt{a\sin^2(x)}\int \sin(x)dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\csc(x)\sqrt{a\sin^2(x)}\int \sin(x)dx-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}$$

input
```
Int[(a - a*Cos[x]^2)^(3/2),x]
```

output
```
(-2*a*Cot[x]*Sqrt[a*Sin[x]^2])/3 - (Cot[x]*(a*Sin[x]^2)^(3/2))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	s
default	$\frac{\sin(x) a^2 \cos(x) \left(\cos(x)^2-3\right)}{3\sqrt{a \sin(x)^2}}$	2
risch	$\frac{i a\, \mathrm{e}^{4 i x} \sqrt{-a (\mathrm{e}^{2 i x}-1)^2 \mathrm{e}^{-2 i x}}}{24\, \mathrm{e}^{2 i x}-24} - \frac{3 i a\, \mathrm{e}^{2 i x} \sqrt{-a (\mathrm{e}^{2 i x}-1)^2 \mathrm{e}^{-2 i x}}}{8(\mathrm{e}^{2 i x}-1)} - \frac{3 i \sqrt{-a (\mathrm{e}^{2 i x}-1)^2 \mathrm{e}^{-2 i x}}\, a}{8(\mathrm{e}^{2 i x}-1)} + \frac{i a\, \mathrm{e}^{-2 i x} \sqrt{-a (\mathrm{e}^{2 i x}-1)^2 \mathrm{e}^{-2 i x}}}{24\, \mathrm{e}^{2 i x}-24}$	1

input
```
int((a-a*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sin(x)*a^2*cos(x)*(cos(x)^2-3)/(a*sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \left(a-a\cos^2(x)\right)^{3/2}\,dx = \frac{\left(a\cos(x)^3 - 3\,a\cos(x)\right)\sqrt{-a\cos(x)^2+a}}{3\,\sin(x)}$$

input
```
integrate((a-a*cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(a*cos(x)^3 - 3*a*cos(x))*sqrt(-a*cos(x)^2 + a)/sin(x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a-a\cos^2(x)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a-a*cos(x)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a-a\cos^2(x)\right)^{3/2}\,dx=\int \left(-a\cos\left(x\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-a*cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*cos(x)^2 + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int \left(a-a\cos^2(x)\right)^{3/2}\,dx = -\frac{4\left(3\,a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^2+a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)\right)}{3\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3}$$

input
```
integrate((a-a*cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-4/3*(3*a^(3/2)*sgn(tan(1/2*x)^3 + tan(1/2*x))*tan(1/2*x)^2 + a^(3/2)*sgn(
tan(1/2*x)^3 + tan(1/2*x)))/(tan(1/2*x)^2 + 1)^3
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-a\cos^2(x)\right)^{3/2}\,dx=\int {\left(a-a\,{\cos\left(x\right)}^2\right)}^{3/2}\,dx$$

input
```
int((a - a*cos(x)^2)^(3/2),x)
```

output
```
int((a - a*cos(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a - a\cos^2(x)\right)^{3/2} dx = \sqrt{a}\, a\left(\int \sqrt{-\cos(x)^2 + 1}dx - \left(\int \sqrt{-\cos(x)^2 + 1}\cos(x)^2 dx\right)\right)$$

input
```
int((a-a*cos(x)^2)^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt( - cos(x)**2 + 1),x) - int(sqrt( - cos(x)**2 + 1)*cos(
x)**2,x))
```

3.43 $\int \sqrt{a-a\cos^2(x)}\,dx$

Optimal result . . . 346
Mathematica [A] (verified) . . . 346
Rubi [A] (verified) . . . 347
Maple [A] (verified) . . . 348
Fricas [A] (verification not implemented) . . . 349
Sympy [F] . . . 349
Maxima [A] (verification not implemented) . . . 349
Giac [B] (verification not implemented) . . . 350
Mupad [B] (verification not implemented) . . . 350
Reduce [F] . . . 350

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \sqrt{a-a\cos^2(x)}\,dx=-\cot(x)\sqrt{a\sin^2(x)}$$

output
```
-cot(x)*(a*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{a-a\cos^2(x)}\,dx=-\cot(x)\sqrt{a\sin^2(x)}$$

input
```
Integrate[Sqrt[a - a*Cos[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[a*Sin[x]^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3655, 3042, 3686, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\cos^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \sqrt{a\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin(x)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\csc(x)\sqrt{a\sin^2(x)}\int \sin(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(x)\sqrt{a\sin^2(x)}\int \sin(x)dx$$

$$\Big\downarrow \text{3118}$$

$$-\cot(x)\sqrt{a\sin^2(x)}$$

input
```
Int[Sqrt[a - a*Cos[x]^2],x]
```

output
```
-(Cot[x]*Sqrt[a*Sin[x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{a\sin(x)\cos(x)}{\sqrt{a\sin(x)^2}}$	16
risch	$-\frac{i\sqrt{-a(e^{2ix}-1)^2 e^{-2ix}}\, e^{2ix}}{2(e^{2ix}-1)}-\frac{i\sqrt{-a(e^{2ix}-1)^2 e^{-2ix}}}{2(e^{2ix}-1)}$	69

input
```
int((a-a*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a*sin(x)*cos(x)/(a*sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \sqrt{a-a\cos^2(x)}\,dx = -\frac{\sqrt{-a\cos(x)^2+a}\cos(x)}{\sin(x)}$$

input
```
integrate((a-a*cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-a*cos(x)^2 + a)*cos(x)/sin(x)
```

Sympy [F]

$$\int \sqrt{a-a\cos^2(x)}\,dx = \int \sqrt{-a\cos^2(x)+a}\,dx$$

input
```
integrate((a-a*cos(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-a*cos(x)**2 + a), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \sqrt{a-a\cos^2(x)}\,dx = -\frac{\sqrt{a}}{\sqrt{\tan(x)^2+1}}$$

input
```
integrate((a-a*cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(a)/sqrt(tan(x)^2 + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \sqrt{a - a\cos^2(x)}\, dx = -\frac{2\sqrt{a}\text{sgn}\left(\tan\left(\frac{1}{2}x\right)^3 + \tan\left(\frac{1}{2}x\right)\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}$$

input
```
integrate((a-a*cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(a)*sgn(tan(1/2*x)^3 + tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.86

$$\int \sqrt{a - a\cos^2(x)}\, dx = -\frac{\sqrt{2}\sqrt{a}\sqrt{2\sin(x)^2}\left(-\sin(x)^2 + \frac{\sin(2x)\,1\text{i}}{2} + 1\right)}{\sin(x)^2\,2\text{i} + \sin(2x)}$$

input
```
int((a - a*cos(x)^2)^(1/2),x)
```

output
```
-(2^(1/2)*a^(1/2)*(2*sin(x)^2)^(1/2)*((sin(2*x)*1i)/2 - sin(x)^2 + 1))/(si
n(2*x) + sin(x)^2*2i)
```

Reduce [F]

$$\int \sqrt{a - a\cos^2(x)}\, dx = \sqrt{a}\left(\int \sqrt{-\cos(x)^2 + 1}dx\right)$$

input
```
int((a-a*cos(x)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt( - cos(x)**2 + 1),x)
```

3.44 $\int \frac{1}{\sqrt{a-a\cos^2(x)}}\,dx$

Optimal result 351
Mathematica [A] (verified) 351
Rubi [A] (verified) 352
Maple [B] (verified) 353
Fricas [B] (verification not implemented) 354
Sympy [F] 354
Maxima [A] (verification not implemented) 355
Giac [A] (verification not implemented) 355
Mupad [F(-1)] 355
Reduce [F] 356

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{1}{\sqrt{a-a\cos^2(x)}}\,dx = -\frac{\operatorname{arctanh}(\cos(x))\sin(x)}{\sqrt{a\sin^2(x)}}$$

output
```
-arctanh(cos(x))*sin(x)/(a*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a-a\cos^2(x)}}\,dx = -\frac{\operatorname{arctanh}(\cos(x))\sin(x)}{\sqrt{a\sin^2(x)}}$$

input
```
Integrate[1/Sqrt[a - a*Cos[x]^2],x]
```

output
```
-((ArcTanh[Cos[x]]*Sin[x])/Sqrt[a*Sin[x]^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3655, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-a\cos^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{1}{\sqrt{a\sin^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin(x)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin(x)\int \csc(x)dx}{\sqrt{a\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(x)\int \csc(x)dx}{\sqrt{a\sin^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sin(x)\text{arctanh}(\cos(x))}{\sqrt{a\sin^2(x)}}$$

input
```
Int[1/Sqrt[a - a*Cos[x]^2],x]
```

output
```
-((ArcTanh[Cos[x]]*Sin[x])/Sqrt[a*Sin[x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.88

method	result	size
default	$-\frac{\sin(x)\sqrt{a\cos(x)^2}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\cos(x)^2}+2a}{\sin(x)}\right)}{\sqrt{a}\,\cos(x)\sqrt{a\sin(x)^2}}$	49
risch	$\frac{2\ln(\mathrm{e}^{ix}-1)\sin(x)}{\sqrt{-a(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}-\frac{2\ln(\mathrm{e}^{ix}+1)\sin(x)}{\sqrt{-a(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}$	64

input
```
int(1/(a-a*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-sin(x)*(a*cos(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2)+a)/sin
(x))/cos(x)/(a*sin(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.12

$$\int \frac{1}{\sqrt{a-a\cos^2(x)}}\,dx = \left[\frac{\sqrt{-a\cos(x)^2+a}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)}{2\,a\sin(x)}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a\cos(x)^2+a}\sqrt{-a}\cos(x)}{a\sin(x)}\right)}{a}\right]$$

input
```
integrate(1/(a-a*cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-a*cos(x)^2 + a)*log(-(cos(x) - 1)/(cos(x) + 1))/(a*sin(x)), sqr
t(-a)*arctan(sqrt(-a*cos(x)^2 + a)*sqrt(-a)*cos(x)/(a*sin(x)))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-a\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{-a\cos^2(x)+a}}\,dx$$

input
```
integrate(1/(a-a*cos(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(-a*cos(x)**2 + a), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{a - a\cos^2(x)}}\,dx = \frac{\sqrt{-a}(\arctan(\sin(x), \cos(x) + 1) - \arctan(\sin(x), \cos(x) - 1))}{a}$$

input
```
integrate(1/(a-a*cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-a)*(arctan2(sin(x), cos(x) + 1) - arctan2(sin(x), cos(x) - 1))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{a - a\cos^2(x)}}\,dx = \frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{\sqrt{a}\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3 + \tan\left(\frac{1}{2}x\right)\right)}$$

input
```
integrate(1/(a-a*cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(tan(1/2*x)))/(sqrt(a)*sgn(tan(1/2*x)^3 + tan(1/2*x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a - a\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{a - a\cos(x)^2}}\,dx$$

input
```
int(1/(a - a*cos(x)^2)^(1/2),x)
```

output
```
int(1/(a - a*cos(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a - a\cos^2(x)}}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{-\cos(x)^2+1}}{\cos(x)^2-1}dx\right)}{a}$$

input
```
int(1/(a-a*cos(x)^2)^(1/2),x)
```

output
```
( - sqrt(a)*int(sqrt( - cos(x)**2 + 1)/(cos(x)**2 - 1),x))/a
```

3.45 $\int \frac{1}{(a-a\cos^2(x))^{3/2}}\,dx$

Optimal result 357
Mathematica [A] (verified) 357
Rubi [A] (verified) 358
Maple [A] (verified) 360
Fricas [A] (verification not implemented) 360
Sympy [F] 361
Maxima [B] (verification not implemented) 361
Giac [B] (verification not implemented) 362
Mupad [F(-1)] 362
Reduce [F] 363

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{1}{(a-a\cos^2(x))^{3/2}}\,dx = -\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}} - \frac{\operatorname{arctanh}(\cos(x))\sin(x)}{2a\sqrt{a\sin^2(x)}}$$

output
```
-1/2*cot(x)/a/(a*sin(x)^2)^(1/2)-1/2*arctanh(cos(x))*sin(x)/a/(a*sin(x)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a-a\cos^2(x))^{3/2}}\,dx = -\frac{\left(\csc^2\left(\frac{x}{2}\right)+4\log\left(\cos\left(\frac{x}{2}\right)\right)-4\log\left(\sin\left(\frac{x}{2}\right)\right)-\sec^2\left(\frac{x}{2}\right)\right)\sin^3(x)}{8\left(a\sin^2(x)\right)^{3/2}}$$

input
```
Integrate[(a - a*Cos[x]^2)^(-3/2),x]
```

output
```
-1/8*((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x]
^3)/(a*Sin[x]^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 3655, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\cos^2(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(x)^2)^{3/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{\int \frac{1}{\sqrt{a\sin^2(x)}}dx}{2a}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{a\sin(x)^2}}dx}{2a}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin(x)\int \csc(x)dx}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(x)\int \csc(x)dx}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}$$

$$\downarrow 4257$$

$$-\frac{\sin(x)\text{arctanh}(\cos(x))}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}$$

input
```
Int[(a - a*Cos[x]^2)^(-3/2),x]
```

output
```
-1/2*Cot[x]/(a*Sqrt[a*Sin[x]^2]) - (ArcTanh[Cos[x]]*Sin[x])/(2*a*Sqrt[a*Si
n[x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{-2\cos(x)+(-\ln(1+\cos(x))+\ln(-1+\cos(x)))\sin(x)^2}{4a\sin(x)\sqrt{a\sin(x)^2}}$	41
risch	$-\frac{i(e^{2ix}+1)}{a(e^{2ix}-1)\sqrt{-a(e^{2ix}-1)^2 e^{-2ix}}}+\frac{\ln(e^{ix}-1)\sin(x)}{a\sqrt{-a(e^{2ix}-1)^2 e^{-2ix}}}-\frac{\ln(e^{ix}+1)\sin(x)}{a\sqrt{-a(e^{2ix}-1)^2 e^{-2ix}}}$	110

input
```
int(1/(a-a*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*(-2*cos(x)+(-ln(1+cos(x))+ln(-1+cos(x)))*sin(x)^2)/sin(x)/(a*sin(x)^
2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{1}{(a-a\cos^2(x))^{3/2}}\,dx=\frac{\sqrt{-a\cos(x)^2+a}\Big((\cos(x)^2-1)\log\Big(-\frac{\cos(x)-1}{\cos(x)+1}\Big)+2\cos(x)\Big)}{4\,(a^2\cos(x)^2-a^2)\sin(x)}$$

input
```
integrate(1/(a-a*cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(-a*cos(x)^2 + a)*((cos(x)^2 - 1)*log(-(cos(x) - 1)/(cos(x) + 1))
+ 2*cos(x))/((a^2*cos(x)^2 - a^2)*sin(x))
```

Sympy [F]

$$\int \frac{1}{\left(a-a\cos^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(-a\cos^2{(x)}+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a-a*cos(x)**2)**(3/2),x)
```

output
```
Integral((-a*cos(x)**2 + a)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(34) = 68$.

Time = 0.16 (sec) , antiderivative size = 314, normalized size of antiderivative = 7.48

$$\int \frac{1}{\left(a-a\cos^2(x)\right)^{3/2}}\,dx =$$
$$-\frac{\left(\left(2\left(2\cos\left(2x\right)-1\right)\cos\left(4x\right)-\cos\left(4x\right)^2-4\cos\left(2x\right)^2-\sin\left(4x\right)^2+4\sin\left(4x\right)\sin\left(2x\right)-4\sin\left(2x\right)^2 \cdots\right.\right.}{\cdots}$$

input
```
integrate(1/(a-a*cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*((2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^
2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x), c
os(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - si
n(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(si
n(x), cos(x) - 1) + 2*(sin(3*x) + sin(x))*cos(4*x) - 2*(cos(3*x) + cos(x))
*sin(4*x) - 2*(2*cos(2*x) - 1)*sin(3*x) + 4*cos(3*x)*sin(2*x) + 4*cos(x)*s
in(2*x) - 4*cos(2*x)*sin(x) + 2*sin(x))*sqrt(-a)/(a^2*cos(4*x)^2 + 4*a^2*c
os(2*x)^2 + a^2*sin(4*x)^2 - 4*a^2*sin(4*x)*sin(2*x) + 4*a^2*sin(2*x)^2 -
4*a^2*cos(2*x) + a^2 - 2*(2*a^2*cos(2*x) - a^2)*cos(4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.07

$$\int \frac{1}{(a-a\cos^2(x))^{3/2}}\,dx = \frac{\tan\left(\frac{1}{2}\,x\right)^2}{8\,a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)} + \frac{\log\left(\tan\left(\frac{1}{2}\,x\right)^2\right)}{4\,a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)} - \frac{2\,\tan\left(\frac{1}{2}\,x\right)^2+1}{8\,a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^2}$$

input
```
integrate(1/(a-a*cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*tan(1/2*x)^2/(a^(3/2)*sgn(tan(1/2*x)^3 + tan(1/2*x))) + 1/4*log(tan(1/
2*x)^2)/(a^(3/2)*sgn(tan(1/2*x)^3 + tan(1/2*x))) - 1/8*(2*tan(1/2*x)^2 + 1
)/(a^(3/2)*sgn(tan(1/2*x)^3 + tan(1/2*x))*tan(1/2*x)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(a-a\cos\left(x\right)^2\right)^{3/2}}\,dx$$

input
```
int(1/(a - a*cos(x)^2)^(3/2),x)
```

output
```
int(1/(a - a*cos(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a-a\cos^2(x)\right)^{3/2}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{-\cos(x)^2+1}}{\cos(x)^4-2\cos(x)^2+1}dx\right)}{a^2}$$

input
```
int(1/(a-a*cos(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt( - cos(x)**2 + 1)/(cos(x)**4 - 2*cos(x)**2 + 1),x))/a**2
```

3.46 $\int \frac{1}{\left(a-a\cos^2(x)\right)^{5/2}}\,dx$

Optimal result 364
Mathematica [A] (verified) 364
Rubi [A] (verified) 365
Maple [A] (verified) 367
Fricas [A] (verification not implemented) 368
Sympy [F] 368
Maxima [B] (verification not implemented) 368
Giac [B] (verification not implemented) 369
Mupad [F(-1)] 370
Reduce [F] 370

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{1}{\left(a-a\cos^2(x)\right)^{5/2}}\,dx=-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}-\frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}}-\frac{3\text{arctanh}(\cos(x))\sin(x)}{8a^2\sqrt{a\sin^2(x)}}$$

output
```
-1/4*cot(x)/a/(a*sin(x)^2)^(3/2)-3/8*cot(x)/a^2/(a*sin(x)^2)^(1/2)-3/8*arc
tanh(cos(x))*sin(x)/a^2/(a*sin(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a-a\cos^2(x)\right)^{5/2}}\,dx=$$
$$-\frac{\csc(x)\left(6\csc^2\left(\frac{x}{2}\right)+\csc^4\left(\frac{x}{2}\right)+24\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-6\sec^2\left(\frac{x}{2}\right)-\sec^4\left(\frac{x}{2}\right)\right)\sqrt{a\sin^2(x)}}{64a^3}$$

input
```
Integrate[(a - a*Cos[x]^2)^(-5/2),x]
```

output
```
-1/64*(Csc[x]*(6*Csc[x/2]^2 + Csc[x/2]^4 + 24*(Log[Cos[x/2]] - Log[Sin[x/2
]]) - 6*Sec[x/2]^2 - Sec[x/2]^4)*Sqrt[a*Sin[x]^2])/a^3
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 3655, 3042, 3683, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\cos^2(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{1}{(a\sin^2(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(x)^2)^{5/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\int \frac{1}{(a\sin^2(x))^{3/2}}dx}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{(a\sin(x)^2)^{3/2}}dx}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a\sin^2(x)}}dx}{2a}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a\sin(x)^2}}dx}{2a}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \; 3686$$

$$\frac{3\left(\frac{\sin(x)\int \csc(x)dx}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{\sin(x)\int \csc(x)dx}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

$$\Big\downarrow \; 4257$$

$$\frac{3\left(-\frac{\sin(x)\text{arctanh}(\cos(x))}{2a\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}\right)}{4a}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

input
```
Int[(a - a*Cos[x]^2)^(-5/2),x]
```

output
```
-1/4*Cot[x]/(a*(a*Sin[x]^2)^(3/2)) + (3*(-1/2*Cot[x]/(a*Sqrt[a*Sin[x]^2])
- (ArcTanh[Cos[x]]*Sin[x])/(2*a*Sqrt[a*Sin[x]^2])))/(4*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{6 \sin \left(x \right)^{2} \cos \left(x \right)+4 \cos \left(x \right)+\left(3 \ln \left(1+\cos \left(x \right)\right)-3 \ln \left(-1+\cos \left(x \right)\right)\right) \sin \left(x \right)^{4}}{16 a^{2} \left(1+\cos \left(x \right)\right) \left(-1+\cos \left(x \right)\right) \sin \left(x \right) \sqrt{a \sin \left(x \right)^{2}}}$	63
risch	$-\frac{i \left(3 \,{\mathrm e}^{6 i x}-11 \,{\mathrm e}^{4 i x}-11 \,{\mathrm e}^{2 i x}+3\right)}{4 a^{2} \left({\mathrm e}^{2 i x}-1\right)^{3} \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}-\frac{3 \ln \left({\mathrm e}^{i x}+1\right) \sin \left(x \right)}{4 a^{2} \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}+\frac{3 \ln \left({\mathrm e}^{i x}-1\right) \sin \left(x \right)}{4 a^{2} \sqrt{-a \left({\mathrm e}^{2 i x}-1\right)^{2} {\mathrm e}^{-2 i x}}}$	127

input
```
int(1/(a-a*cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/a^2*(6*sin(x)^2*cos(x)+4*cos(x)+(3*ln(1+cos(x))-3*ln(-1+cos(x)))*sin(
x)^4)/(1+cos(x))/(-1+cos(x))/sin(x)/(a*sin(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(a-a\cos^2(x)\right)^{5/2}}\,dx=\frac{\sqrt{-a\cos\left(x\right)^2+a}\left(6\cos\left(x\right)^3+3\left(\cos\left(x\right)^4-2\cos\left(x\right)^2+1\right)\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)-1\right.}{16\left(a^3\cos\left(x\right)^4-2a^3\cos\left(x\right)^2+a^3\right)\sin\left(x\right)}$$

input
```
integrate(1/(a-a*cos(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/16*sqrt(-a*cos(x)^2 + a)*(6*cos(x)^3 + 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log
(-(cos(x) - 1)/(cos(x) + 1)) - 10*cos(x))/((a^3*cos(x)^4 - 2*a^3*cos(x)^2
+ a^3)*sin(x))
```

Sympy [F]

$$\int \frac{1}{\left(a-a\cos^2(x)\right)^{5/2}}\,dx=\int \frac{1}{\left(-a\cos^2\left(x\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a-a*cos(x)**2)**(5/2),x)
```

output
```
Integral((-a*cos(x)**2 + a)**(-5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. 2(49) = 98.

Time = 0.31 (sec) , antiderivative size = 931, normalized size of antiderivative = 15.26

$$\int \frac{1}{\left(a-a\cos^2(x)\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a-a*cos(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/8*(3*(2*(4*cos(6*x) - 6*cos(4*x) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^
2 + 8*(6*cos(4*x) - 4*cos(2*x) + 1)*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2
*x) - 1)*cos(4*x) - 36*cos(4*x)^2 - 16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(
4*x) + 2*sin(2*x))*sin(8*x) - sin(8*x)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*si
n(6*x) - 16*sin(6*x)^2 - 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x
)^2 + 8*cos(2*x) - 1)*arctan2(sin(x), cos(x) + 1) - 3*(2*(4*cos(6*x) - 6*c
os(4*x) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 8*(6*cos(4*x) - 4*cos(2*
x) + 1)*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2*x) - 1)*cos(4*x) - 36*cos(4
*x)^2 - 16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(4*x) + 2*sin(2*x))*sin(8*x)
- sin(8*x)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*sin(6*x) - 16*sin(6*x)^2 - 36*
sin(4*x)^2 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x)^2 + 8*cos(2*x) - 1)*arctan
2(sin(x), cos(x) - 1) + 2*(3*sin(7*x) - 11*sin(5*x) - 11*sin(3*x) + 3*sin(
x))*cos(8*x) + 12*(2*sin(6*x) - 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 8*(11*
sin(5*x) + 11*sin(3*x) - 3*sin(x))*cos(6*x) + 44*(3*sin(4*x) - 2*sin(2*x))
*cos(5*x) - 12*(11*sin(3*x) - 3*sin(x))*cos(4*x) - 2*(3*cos(7*x) - 11*cos(
5*x) - 11*cos(3*x) + 3*cos(x))*sin(8*x) - 6*(4*cos(6*x) - 6*cos(4*x) + 4*c
os(2*x) - 1)*sin(7*x) - 8*(11*cos(5*x) + 11*cos(3*x) - 3*cos(x))*sin(6*x)
- 22*(6*cos(4*x) - 4*cos(2*x) + 1)*sin(5*x) + 12*(11*cos(3*x) - 3*cos(x))*
sin(4*x) + 22*(4*cos(2*x) - 1)*sin(3*x) - 88*cos(3*x)*sin(2*x) + 24*cos(x)
*sin(2*x) - 24*cos(2*x)*sin(x) + 6*sin(x))*sqrt(-a)/(a^3*cos(8*x)^2 + 1...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(49) = 98$.

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.98

$$\int \frac{1}{(a-a\cos^2(x))^{5/2}}\,dx = \frac{3\,\log\left(\tan\left(\frac{1}{2}\,x\right)^2\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)}$$
$$+\frac{a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^4+8\,a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^2}{64\,a^5}$$
$$-\frac{18\,\tan\left(\frac{1}{2}\,x\right)^4+8\,\tan\left(\frac{1}{2}\,x\right)^2+1}{64\,a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^4}$$

input
```
integrate(1/(a-a*cos(x)^2)^(5/2),x, algorithm="giac")
```

output
```
3/16*log(tan(1/2*x)^2)/(a^(5/2)*sgn(tan(1/2*x)^3 + tan(1/2*x))) + 1/64*(a^
(5/2)*sgn(tan(1/2*x)^3 + tan(1/2*x))*tan(1/2*x)^4 + 8*a^(5/2)*sgn(tan(1/2*
x)^3 + tan(1/2*x))*tan(1/2*x)^2)/a^5 - 1/64*(18*tan(1/2*x)^4 + 8*tan(1/2*x
)^2 + 1)/(a^(5/2)*sgn(tan(1/2*x)^3 + tan(1/2*x))*tan(1/2*x)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\cos^2(x))^{5/2}}\,dx = \int \frac{1}{\left(a-a\cos\left(x\right)^2\right)^{5/2}}\,dx$$

input
```
int(1/(a - a*cos(x)^2)^(5/2),x)
```

output
```
int(1/(a - a*cos(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a-a\cos^2(x))^{5/2}}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{-\cos(x)^2+1}}{\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1}dx\right)}{a^3}$$

input
```
int(1/(a-a*cos(x)^2)^(5/2),x)
```

output
```
( - sqrt(a)*int(sqrt( - cos(x)**2 + 1)/(cos(x)**6 - 3*cos(x)**4 + 3*cos(x)
**2 - 1),x))/a**3
```

3.47 $\int (1+\cos^2(x))^{5/2}\,dx$

Optimal result . . . 371
Mathematica [A] (verified) . . . 371
Rubi [A] (verified) . . . 372
Maple [B] (verified) . . . 375
Fricas [F] . . . 375
Sympy [F(-1)] . . . 376
Maxima [F] . . . 376
Giac [F] . . . 376
Mupad [F(-1)] . . . 377
Reduce [F] . . . 377

Optimal result

Integrand size = 10, antiderivative size = 63

$$\int \left(1+\cos^2(x)\right)^{5/2}\,dx=\frac{18}{5}E\left(\frac{\pi}{2}+x\middle|-1\right)-\frac{8}{5}\text{EllipticF}\left(\frac{\pi}{2}+x,-1\right)+\frac{4}{5}\cos(x)\sqrt{1+\cos^2(x)}\sin(x)+\frac{1}{5}\cos(x)\left(1+\cos^2(x)\right)^{3/2}\sin(x)$$

output
```
18/5*EllipticE(cos(x),I)-8/5*InverseJacobiAM(1/2*Pi+x,I)+4/5*cos(x)*(1+cos
(x)^2)^(1/2)*sin(x)+1/5*cos(x)*(1+cos(x)^2)^(3/2)*sin(x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \left(1+\cos^2(x)\right)^{5/2}\,dx=\frac{288E\left(x\middle|\frac{1}{2}\right)-64\,\text{EllipticF}\left(x,\frac{1}{2}\right)+\sqrt{3+\cos(2x)}(22\sin(2x)+\sin(4x))}{40\sqrt{2}}$$

input
```
Integrate[(1 + Cos[x]^2)^(5/2),x]
```

output
```
(288*EllipticE[x, 1/2] - 64*EllipticF[x, 1/2] + Sqrt[3 + Cos[2*x]]*(22*Sin
[2*x] + Sin[4*x]))/(40*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3659, 27, 3042, 3649, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\cos^2(x)+1\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^{5/2} dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{5}\int 6\sqrt{\cos^2(x)+1}\left(2\cos^2(x)+1\right)dx+\frac{1}{5}\sin(x)\cos(x)\left(\cos^2(x)+1\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{5}\int \sqrt{\cos^2(x)+1}\left(2\cos^2(x)+1\right)dx+\frac{1}{5}\sin(x)\cos(x)\left(\cos^2(x)+1\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}\left(2\sin\left(x+\frac{\pi}{2}\right)^2+1\right)dx+\frac{1}{5}\sin(x)\cos(x)\left(\cos^2(x)+1\right)^{3/2}$$

$$\Big\downarrow \text{3649}$$

$$\frac{6}{5}\left(\frac{1}{3}\int \frac{9\cos^2(x)+5}{\sqrt{\cos^2(x)+1}}dx+\frac{2}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}\right)+\frac{1}{5}\sin(x)\cos(x)\left(\cos^2(x)+1\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}\left(\frac{1}{3}\int \frac{9\sin\left(x+\frac{\pi}{2}\right)^2+5}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx+\frac{2}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}\right)+\frac{1}{5}\sin(x)\cos(x)\left(\cos^2(x)+1\right)^{3/2}$$

$$\Big\downarrow \text{3651}$$

$$\frac{6}{5}\left(\frac{1}{3}\left(9\int \sqrt{\cos^2(x)+1}dx-4\int \frac{1}{\sqrt{\cos^2(x)+1}}dx\right)+\frac{2}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}\right)+\frac{1}{5}\sin(x)\cos(x)\left(\cos^2(x)+1\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}\left(\frac{1}{3}\left(9\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx-4\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx\right)+\frac{2}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}\right)+\frac{1}{5}\sin(x)\cos(x)\left(\cos^2(x)+1\right)^{3/2}$$

$$\Big\downarrow \text{3656}$$

$$\frac{6}{5}\left(\frac{1}{3}\left(9E\left(x+\frac{\pi}{2}\middle|-1\right)-4\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx\right)+\frac{2}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}\right)+\frac{1}{5}\sin(x)\cos(x)\left(\cos^2(x)+1\right)^{3/2}$$

$$\Big\downarrow \text{3661}$$

$$\frac{1}{5}\sin(x)\cos(x)\left(\cos^2(x)+1\right)^{3/2}+\frac{6}{5}\left(\frac{2}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}+\frac{1}{3}\left(9E\left(x+\frac{\pi}{2}\middle|-1\right)-4\,\text{EllipticF}\left(x+\frac{\pi}{2},-1\right)\right)\right)$$

input
```
Int[(1 + Cos[x]^2)^(5/2),x]
```

output
```
(Cos[x]*(1 + Cos[x]^2)^(3/2)*Sin[x])/5 + (6*((9*EllipticE[Pi/2 + x, -1] -
4*EllipticF[Pi/2 + x, -1])/3 + (2*Cos[x]*Sqrt[1 + Cos[x]^2]*Sin[x])/3))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(46) = 92$.

Time = 1.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.71

method	result
default	$\frac{\sqrt{\left(1+\cos(x)^2\right)\sin(x)^2}\left(\sin(x)^6\cos(x)-8\sin(x)^4\cos(x)+8\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{-\sin(x)^2+2}\,\mathrm{EllipticF}(\cos(x),i)-18\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{-\mathrm{s}}\right.}{5\sqrt{1-\cos(x)^4}\,\sin(x)\sqrt{1+\cos(x)^2}}$

input
```
int((1+cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*((1+cos(x)^2)*sin(x)^2)^(1/2)*(sin(x)^6*cos(x)-8*sin(x)^4*cos(x)+8*(si
n(x)^2)^(1/2)*(-sin(x)^2+2)^(1/2)*EllipticF(cos(x),I)-18*(sin(x)^2)^(1/2)*
(-sin(x)^2+2)^(1/2)*EllipticE(cos(x),I)+12*sin(x)^2*cos(x))/(1-cos(x)^4)^(
1/2)/sin(x)/(1+cos(x)^2)^(1/2)
```

Fricas [F]

$$\int \left(1+\cos^2(x)\right)^{5/2}\,dx=\int \left(\cos\left(x\right)^2+1\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((1+cos(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral((cos(x)^4 + 2*cos(x)^2 + 1)*sqrt(cos(x)^2 + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(1+\cos^2(x)\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((1+cos(x)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(1+\cos^2(x)\right)^{5/2}\,dx=\int \left(\cos\left(x\right)^2+1\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((1+cos(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((cos(x)^2 + 1)^(5/2), x)
```

Giac [F]

$$\int \left(1+\cos^2(x)\right)^{5/2}\,dx=\int \left(\cos\left(x\right)^2+1\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((1+cos(x)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((cos(x)^2 + 1)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+\cos^2(x)\right)^{5/2}\,dx=\int \left(\cos(x)^2+1\right)^{5/2}\,dx$$

input
```
int((cos(x)^2 + 1)^(5/2),x)
```

output
```
int((cos(x)^2 + 1)^(5/2), x)
```

Reduce [F]

$$\int \left(1+\cos^2(x)\right)^{5/2}\,dx=\int \sqrt{\cos(x)^2+1}dx + \int \sqrt{\cos(x)^2+1}\,\cos(x)^4\,dx+2\left(\int \sqrt{\cos(x)^2+1}\,\cos(x)^2\,dx\right)$$

input
```
int((1+cos(x)^2)^(5/2),x)
```

output
```
int(sqrt(cos(x)**2 + 1),x) + int(sqrt(cos(x)**2 + 1)*cos(x)**4,x) + 2*int(
sqrt(cos(x)**2 + 1)*cos(x)**2,x)
```

3.48 $\int \left(1+\cos^2(x)\right)^{3/2} \, dx$

Optimal result 378
Mathematica [A] (verified) 378
Rubi [A] (verified) 379
Maple [B] (verified) 381
Fricas [F] 381
Sympy [F] 382
Maxima [F] 382
Giac [F] 382
Mupad [F(-1)] 383
Reduce [F] 383

Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \left(1+\cos^2(x)\right)^{3/2} \, dx = 2E\left(\frac{\pi}{2}+x\middle|-1\right) - \frac{2}{3}\operatorname{EllipticF}\left(\frac{\pi}{2}+x,-1\right)+\frac{1}{3}\cos(x)\sqrt{1+\cos^2(x)}\sin(x)$$

output
```
2*EllipticE(cos(x),I)-2/3*InverseJacobiAM(1/2*Pi+x,I)+1/3*cos(x)*(1+cos(x)
^2)^(1/2)*sin(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \left(1+\cos^2(x)\right)^{3/2} \, dx = \frac{24E\left(x\middle|\frac{1}{2}\right)-4\operatorname{EllipticF}\left(x,\frac{1}{2}\right)+\sqrt{3+\cos(2x)}\sin(2x)}{6\sqrt{2}}$$

input
```
Integrate[(1 + Cos[x]^2)^(3/2),x]
```

output
```
(24*EllipticE[x, 1/2] - 4*EllipticF[x, 1/2] + Sqrt[3 + Cos[2*x]]*Sin[2*x])
/(6*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3659, 27, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\cos^2(x)+1\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3659}$$

$$\frac{1}{3}\int \frac{2\left(3\cos^2(x)+2\right)}{\sqrt{\cos^2(x)+1}}dx+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}\int \frac{3\cos^2(x)+2}{\sqrt{\cos^2(x)+1}}dx+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2}{3}\int \frac{3\sin\left(x+\frac{\pi}{2}\right)^2+2}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

$$\Big\downarrow \text{ 3651}$$

$$\frac{2}{3}\left(3\int \sqrt{\cos^2(x)+1}dx-\int \frac{1}{\sqrt{\cos^2(x)+1}}dx\right)+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2}{3}\left(3\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx-\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx\right)+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

$$\Big\downarrow \text{ 3656}$$

$$\frac{2}{3}\left(3E\left(x+\frac{\pi}{2}\middle|-1\right)-\int\frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx\right)+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

$$\Big\downarrow \text{3661}$$

$$\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}+\frac{2}{3}\left(3E\left(x+\frac{\pi}{2}\middle|-1\right)-\text{EllipticF}\left(x+\frac{\pi}{2},-1\right)\right)$$

input
```
Int[(1 + Cos[x]^2)^(3/2),x]
```

output
```
(2*(3*EllipticE[Pi/2 + x, -1] - EllipticF[Pi/2 + x, -1]))/3 + (Cos[x]*Sqrt
[1 + Cos[x]^2]*Sin[x])/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(32) = 64$.

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.35

method	result
default	$\frac{\sqrt{\left(1+\cos(x)^{2}\right)\sin(x)^{2}}\,\left(-\sin(x)^{4}\cos(x)+2\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\sin(x)^{2}+2}\;\mathrm{EllipticF}(\cos(x),i)-6\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\sin(x)^{2}+2}\;\mathrm{Ellipt}\right.}{3\sqrt{1-\cos(x)^{4}}\,\sin(x)\sqrt{1+\cos(x)^{2}}}$

input
```
int((1+cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((1+cos(x)^2)*sin(x)^2)^(1/2)*(-sin(x)^4*cos(x)+2*(sin(x)^2)^(1/2)*(-s
in(x)^2+2)^(1/2)*EllipticF(cos(x),I)-6*(sin(x)^2)^(1/2)*(-sin(x)^2+2)^(1/2
)*EllipticE(cos(x),I)+2*sin(x)^2*cos(x))/(1-cos(x)^4)^(1/2)/sin(x)/(1+cos(
x)^2)^(1/2)
```

Fricas [F]

$$\int \left(1+\cos^2(x)\right)^{3/2}\,dx = \int \left(\cos\left(x\right)^{2}+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((cos(x)^2 + 1)^(3/2), x)
```

Sympy [F]

$$\int \left(1+\cos^2(x)\right)^{3/2}\,dx = \int \left(\cos^2\left(x\right)+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+cos(x)**2)**(3/2),x)
```

output
```
Integral((cos(x)**2 + 1)**(3/2), x)
```

Maxima [F]

$$\int \left(1+\cos^2(x)\right)^{3/2}\,dx = \int \left(\cos\left(x\right)^2+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((cos(x)^2 + 1)^(3/2), x)
```

Giac [F]

$$\int \left(1+\cos^2(x)\right)^{3/2}\,dx = \int \left(\cos\left(x\right)^2+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((cos(x)^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+\cos^2(x)\right)^{3/2} dx = \int \left(\cos(x)^2+1\right)^{3/2} dx$$

input
```
int((cos(x)^2 + 1)^(3/2),x)
```

output
```
int((cos(x)^2 + 1)^(3/2), x)
```

Reduce [F]

$$\int \left(1+\cos^2(x)\right)^{3/2} dx = \int \sqrt{\cos(x)^2+1}\,dx + \int \sqrt{\cos(x)^2+1}\,\cos(x)^2 dx$$

input
```
int((1+cos(x)^2)^(3/2),x)
```

output
```
int(sqrt(cos(x)**2 + 1),x) + int(sqrt(cos(x)**2 + 1)*cos(x)**2,x)
```

3.49 $\int \sqrt{1+\cos^2(x)}\,dx$

Optimal result 384
Mathematica [A] (verified) 384
Rubi [A] (verified) 385
Maple [B] (verified) 386
Fricas [F] 386
Sympy [F] 386
Maxima [F] 387
Giac [F] 387
Mupad [B] (verification not implemented) 387
Reduce [F] 388

Optimal result

Integrand size = 10, antiderivative size = 9

$$\int \sqrt{1+\cos^2(x)}\,dx = E\left(\frac{\pi}{2}+x\middle|-1\right)$$

output
```
EllipticE(cos(x),I)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \sqrt{1+\cos^2(x)}\,dx = \sqrt{2}E\left(x\middle|\frac{1}{2}\right)$$

input
```
Integrate[Sqrt[1 + Cos[x]^2],x]
```

output
```
Sqrt[2]*EllipticE[x, 1/2]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos^2(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \text{3656}$$

$$E\left(x+\frac{\pi}{2}\middle|-1\right)$$

input
```
Int[Sqrt[1 + Cos[x]^2],x]
```

output
```
EllipticE[Pi/2 + x, -1]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(5) = 10$.

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 4.56

method	result	size
default	$-\frac{\sqrt{\left(1+\cos \left(x \right)^{2}\right) \sin \left(x \right)^{2}}\, \sqrt{\frac{1}{2}-\frac{\cos \left(2 x \right)}{2}}\, \operatorname{EllipticE}\left(\cos \left(x \right), i\right)}{\sqrt{1-\cos \left(x \right)^{4}}\, \sin \left(x \right)}$	41

input
```
int((1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((1+cos(x)^2)*sin(x)^2)^(1/2)*(sin(x)^2)^(1/2)*EllipticE(cos(x),I)/(1-cos
(x)^4)^(1/2)/sin(x)
```

Fricas [F]

$$\int \sqrt{1+\cos^2(x)}\,dx = \int \sqrt{\cos\left(x\right)^{2}+1}\,dx$$

input
```
integrate((1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(cos(x)^2 + 1), x)
```

Sympy [F]

$$\int \sqrt{1+\cos^2(x)}\,dx = \int \sqrt{\cos^{2}\left(x\right)+1}\,dx$$

input
```
integrate((1+cos(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(cos(x)**2 + 1), x)
```

Maxima [F]

$$\int \sqrt{1+\cos^2(x)}\,dx = \int \sqrt{\cos\left(x\right)^2+1}\,dx$$

input
```
integrate((1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(x)^2 + 1), x)
```

Giac [F]

$$\int \sqrt{1+\cos^2(x)}\,dx = \int \sqrt{\cos\left(x\right)^2+1}\,dx$$

input
```
integrate((1+cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(x)^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \sqrt{1+\cos^2(x)}\,dx = \sqrt{2}\,\mathrm{E}\left(x\middle|\frac{1}{2}\right)$$

input
```
int((cos(x)^2 + 1)^(1/2),x)
```

output
```
2^(1/2)*ellipticE(x, 1/2)
```

Reduce [F]

$$\int \sqrt{1+\cos^2(x)}\,dx = \int \sqrt{\cos(x)^2+1}dx$$

input
```
int((1+cos(x)^2)^(1/2),x)
```

output
```
int(sqrt(cos(x)**2 + 1),x)
```

3.50 $\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal result 389
Mathematica [A] (verified) 389
Rubi [A] (verified) 390
Maple [B] (verified) 391
Fricas [B] (verification not implemented) 391
Sympy [F] 392
Maxima [F] 392
Giac [F] 392
Mupad [F(-1)] 393
Reduce [F] 393

Optimal result

Integrand size = 10, antiderivative size = 9

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \text{EllipticF}\left(\frac{\pi}{2}+x,-1\right)$$

output
```
InverseJacobiAM(1/2*Pi+x,I)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \frac{\text{EllipticF}\left(x,\frac{1}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/Sqrt[1 + Cos[x]^2],x]
```

output
```
EllipticF[x, 1/2]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos^2(x)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx$$

$$\Big\downarrow \text{3661}$$

$$\text{EllipticF}\left(x+\frac{\pi}{2},-1\right)$$

input
```
Int[1/Sqrt[1 + Cos[x]^2],x]
```

output
```
EllipticF[Pi/2 + x, -1]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(8) = 16.

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 4.56

method	result	size
default	$-\frac{\sqrt{\left(1+\cos\left(x\right)^{2}\right)\sin\left(x\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos\left(2x\right)}{2}}\,\operatorname{EllipticF}\left(\cos\left(x\right),i\right)}{\sqrt{1-\cos\left(x\right)^{4}}\,\sin\left(x\right)}$	41

input
```
int(1/(1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((1+cos(x)^2)*sin(x)^2)^(1/2)*(sin(x)^2)^(1/2)/(1-cos(x)^4)^(1/2)*Ellipti
cF(cos(x),I)/sin(x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(7) = 14.

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 9.67

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx$$
$$= \sqrt{2\sqrt{2}-3}\left(2i\sqrt{2}+3i\right)F(\arcsin\left(\sqrt{2\sqrt{2}-3}(\cos\left(x\right)+i\,\sin\left(x\right))\right) \mid 12\sqrt{2}+17)$$
$$+\sqrt{2\sqrt{2}-3}\left(-2i\sqrt{2}-3i\right)F(\arcsin\left(\sqrt{2\sqrt{2}-3}(\cos\left(x\right)-i\,\sin\left(x\right))\right) \mid 12\sqrt{2}+17)$$

input
```
integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2*sqrt(2) - 3)*(2*I*sqrt(2) + 3*I)*elliptic_f(arcsin(sqrt(2*sqrt(2) -
 3)*(cos(x) + I*sin(x))), 12*sqrt(2) + 17) + sqrt(2*sqrt(2) - 3)*(-2*I*sqr
t(2) - 3*I)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin(x))), 12
*sqrt(2) + 17)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{\cos^2{(x)}+1}}\,dx$$

input
```
integrate(1/(1+cos(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(cos(x)**2 + 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{\cos{(x)}^2+1}}\,dx$$

input
```
integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(cos(x)^2 + 1), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{\cos{(x)}^2+1}}\,dx$$

input
```
integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(cos(x)^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{\cos(x)^2+1}}\,dx$$

input
```
int(1/(cos(x)^2 + 1)^(1/2),x)
```

output
```
int(1/(cos(x)^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{\sqrt{\cos(x)^2+1}}{\cos(x)^2+1}dx$$

input
```
int(1/(1+cos(x)^2)^(1/2),x)
```

output
```
int(sqrt(cos(x)**2 + 1)/(cos(x)**2 + 1),x)
```

3.51 $\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx$

Optimal result . . . 394
Mathematica [A] (verified) . . . 394
Rubi [A] (verified) . . . 395
Maple [B] (verified) . . . 396
Fricas [B] (verification not implemented) . . . 397
Sympy [F] . . . 397
Maxima [F] . . . 398
Giac [F] . . . 398
Mupad [F(-1)] . . . 398
Reduce [F] . . . 399

Optimal result

Integrand size = 10, antiderivative size = 32

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx=\frac{1}{2}E\left(\frac{\pi}{2}+x\middle|-1\right)-\frac{\cos(x)\sin(x)}{2\sqrt{1+\cos^2(x)}}$$

output
```
1/2*EllipticE(cos(x),I)-1/2*cos(x)*sin(x)/(1+cos(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx=\frac{E\left(x\middle|\frac{1}{2}\right)}{\sqrt{2}}-\frac{\sin(2x)}{2\sqrt{2}\sqrt{3+\cos(2x)}}$$

input
```
Integrate[(1 + Cos[x]^2)^(-3/2),x]
```

output
```
EllipticE[x, 1/2]/Sqrt[2] - Sin[2*x]/(2*Sqrt[2]*Sqrt[3 + Cos[2*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3663, 25, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos^2(x)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{1}{2}\int -\sqrt{\cos^2(x)+1}dx-\frac{\sin(x)\cos(x)}{2\sqrt{\cos^2(x)+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \sqrt{\cos^2(x)+1}dx-\frac{\sin(x)\cos(x)}{2\sqrt{\cos^2(x)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx-\frac{\sin(x)\cos(x)}{2\sqrt{\cos^2(x)+1}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{1}{2}E\left(x+\frac{\pi}{2}\middle|-1\right)-\frac{\sin(x)\cos(x)}{2\sqrt{\cos^2(x)+1}}$$

input
```
Int[(1 + Cos[x]^2)^(-3/2),x]
```

output
```
EllipticE[Pi/2 + x, -1]/2 - (Cos[x]*Sin[x])/(2*Sqrt[1 + Cos[x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(22) = 44$.

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

method	result	size
default	$-\frac{\sqrt{-\sin(x)^4+2\sin(x)^2}\left(\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{-\sin(x)^2+2}\,\text{EllipticE}(\cos(x),i)+\sin(x)^2\cos(x)\right)}{2\sqrt{1-\cos(x)^4}\,\sin(x)\sqrt{1+\cos(x)^2}}$	70

input
```
int(1/(1+cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-sin(x)^4+2*sin(x)^2)^(1/2)*((sin(x)^2)^(1/2)*(-sin(x)^2+2)^(1/2)*El
lipticE(cos(x),I)+sin(x)^2*cos(x))/(1-cos(x)^4)^(1/2)/sin(x)/(1+cos(x)^2)^
(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(21) = 42$.

Time = 0.11 (sec) , antiderivative size = 247, normalized size of antiderivative = 7.72

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx = \frac{\left((2i\sqrt{2}-3i)\cos(x)^2 + 2i\sqrt{2}-3i\right)\sqrt{2\sqrt{2}-3}E\left(\arcsin\left(\sqrt{2\sqrt{2}-3}(\cos(x)+i\right.\right.}{}$$

input
```
integrate(1/(1+cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(((2*I*sqrt(2) - 3*I)*cos(x)^2 + 2*I*sqrt(2) - 3*I)*sqrt(2*sqrt(2) - 3
)*elliptic_e(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) + I*sin(x))), 12*sqrt(2) +
 17) + ((-2*I*sqrt(2) + 3*I)*cos(x)^2 - 2*I*sqrt(2) + 3*I)*sqrt(2*sqrt(2)
- 3)*elliptic_e(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin(x))), 12*sqrt(2
) + 17) - 4*((-I*sqrt(2) - 3*I)*cos(x)^2 - I*sqrt(2) - 3*I)*sqrt(2*sqrt(2)
 - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) + I*sin(x))), 12*sqrt(
2) + 17) - 4*((I*sqrt(2) + 3*I)*cos(x)^2 + I*sqrt(2) + 3*I)*sqrt(2*sqrt(2)
 - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin(x))), 12*sqrt(
2) + 17) - 2*sqrt(cos(x)^2 + 1)*cos(x)*sin(x))/(cos(x)^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx = \int \frac{1}{(\cos^2(x)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+cos(x)**2)**(3/2),x)
```

output
```
Integral((cos(x)**2 + 1)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(\cos\left(x\right)^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((cos(x)^2 + 1)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(\cos\left(x\right)^2+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((cos(x)^2 + 1)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(\cos\left(x\right)^2+1\right)^{3/2}}\,dx$$

input
```
int(1/(cos(x)^2 + 1)^(3/2),x)
```

output
```
int(1/(cos(x)^2 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx = \int \frac{\sqrt{\cos(x)^2+1}}{\cos(x)^4+2\cos(x)^2+1}dx$$

input
```
int(1/(1+cos(x)^2)^(3/2),x)
```

output
```
int(sqrt(cos(x)**2 + 1)/(cos(x)**4 + 2*cos(x)**2 + 1),x)
```

3.52 $\int \frac{1}{(1+\cos^2(x))^{5/2}}\,dx$

Optimal result 400
Mathematica [A] (verified) 400
Rubi [A] (verified) 401
Maple [B] (verified) 404
Fricas [B] (verification not implemented) 404
Sympy [F] 405
Maxima [F] 405
Giac [F] 406
Mupad [F(-1)] 406
Reduce [F] 406

Optimal result

Integrand size = 10, antiderivative size = 63

$$\int \frac{1}{(1+\cos^2(x))^{5/2}}\,dx = \frac{1}{2}E\left(\frac{\pi}{2}+x\middle|-1\right) - \frac{1}{6}\operatorname{EllipticF}\left(\frac{\pi}{2}+x,-1\right) - \frac{\cos(x)\sin(x)}{6\,(1+\cos^2(x))^{3/2}} - \frac{\cos(x)\sin(x)}{2\sqrt{1+\cos^2(x)}}$$

output
```
1/2*EllipticE(cos(x),I)-1/6*InverseJacobiAM(1/2*Pi+x,I)-1/6*cos(x)*sin(x)/
(1+cos(x)^2)^(3/2)-1/2*cos(x)*sin(x)/(1+cos(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{1}{(1+\cos^2(x))^{5/2}}\,dx = \frac{12E\left(x\middle|\frac{1}{2}\right) - 2\operatorname{EllipticF}\left(x,\frac{1}{2}\right) - \frac{22\sin(2x)+3\sin(4x)}{(3+\cos(2x))^{3/2}}}{12\sqrt{2}}$$

input
```
Integrate[(1 + Cos[x]^2)^(-5/2),x]
```

output
```
(12*EllipticE[x, 1/2] - 2*EllipticF[x, 1/2] - (22*Sin[2*x] + 3*Sin[4*x])/(
3 + Cos[2*x])^(3/2))/(12*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos^2(x)+1)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^{5/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{1}{6}\int -\frac{5-\cos^2(x)}{(\cos^2(x)+1)^{3/2}}dx-\frac{\sin(x)\cos(x)}{6(\cos^2(x)+1)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}\int \frac{5-\cos^2(x)}{(\cos^2(x)+1)^{3/2}}dx-\frac{\sin(x)\cos(x)}{6(\cos^2(x)+1)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{5-\sin\left(x+\frac{\pi}{2}\right)^2}{\left(\sin\left(x+\frac{\pi}{2}\right)^2+1\right)^{3/2}}dx-\frac{\sin(x)\cos(x)}{6(\cos^2(x)+1)^{3/2}}$$

$$\Big\downarrow \text{3652}$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{2(3\cos^2(x)+2)}{\sqrt{\cos^2(x)+1}}dx-\frac{3\sin(x)\cos(x)}{\sqrt{\cos^2(x)+1}}\right)-\frac{\sin(x)\cos(x)}{6(\cos^2(x)+1)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\int \frac{3\cos^2(x)+2}{\sqrt{\cos^2(x)+1}}dx-\frac{3\sin(x)\cos(x)}{\sqrt{\cos^2(x)+1}}\right)-\frac{\sin(x)\cos(x)}{6\left(\cos^2(x)+1\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{1}{6}\left(\int \frac{3\sin\left(x+\frac{\pi}{2}\right)^2+2}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx-\frac{3\sin(x)\cos(x)}{\sqrt{\cos^2(x)+1}}\right)-\frac{\sin(x)\cos(x)}{6\left(\cos^2(x)+1\right)^{3/2}}$$

$\big\downarrow$ 3651

$$\frac{1}{6}\left(-\int \frac{1}{\sqrt{\cos^2(x)+1}}dx+3\int \sqrt{\cos^2(x)+1}dx-\frac{3\sin(x)\cos(x)}{\sqrt{\cos^2(x)+1}}\right)-\frac{\sin(x)\cos(x)}{6\left(\cos^2(x)+1\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{1}{6}\left(-\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx+3\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx-\frac{3\sin(x)\cos(x)}{\sqrt{\cos^2(x)+1}}\right)-\frac{\sin(x)\cos(x)}{6\left(\cos^2(x)+1\right)^{3/2}}$$

$\big\downarrow$ 3656

$$\frac{1}{6}\left(-\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx+3E\left(x+\frac{\pi}{2}\middle|-1\right)-\frac{3\sin(x)\cos(x)}{\sqrt{\cos^2(x)+1}}\right)-\frac{\sin(x)\cos(x)}{6\left(\cos^2(x)+1\right)^{3/2}}$$

$\big\downarrow$ 3661

$$\frac{1}{6}\left(-\text{EllipticF}\left(x+\frac{\pi}{2},-1\right)+3E\left(x+\frac{\pi}{2}\middle|-1\right)-\frac{3\sin(x)\cos(x)}{\sqrt{\cos^2(x)+1}}\right)-\frac{\sin(x)\cos(x)}{6\left(\cos^2(x)+1\right)^{3/2}}$$

input
```
Int[(1 + Cos[x]^2)^(-5/2),x]
```

output
```
-1/6*(Cos[x]*Sin[x])/(1 + Cos[x]^2)^(3/2) + (3*EllipticE[Pi/2 + x, -1] - E
llipticF[Pi/2 + x, -1] - (3*Cos[x]*Sin[x])/Sqrt[1 + Cos[x]^2])/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(46) = 92$.

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.35

method	result
default	$-\frac{\sqrt{-\left(-1-\cos(x)^2\right)\sin(x)^2}\left(\frac{\cos(x)\sqrt{1-\cos(x)^4}}{6\left(1+\cos(x)^2\right)^2}+\frac{\sin(x)^2\cos(x)}{2\sqrt{-\left(-1-\cos(x)^2\right)\sin(x)^2}}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{1+\cos(x)^2}\,\mathrm{EllipticF}(\cos(x),i)}{3\sqrt{1-\cos(x)^4}}-\frac{\sqrt{\frac{1}{2}-\cos}}{}\right.}{\sin(x)\sqrt{1+\cos(x)^2}}$

input
```
int(1/(1+cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-1-cos(x)^2)*sin(x)^2)^(1/2)*(1/6*cos(x)*(1-cos(x)^4)^(1/2)/(1+cos(x)^
2)^2+1/2*sin(x)^2*cos(x)/(-(-1-cos(x)^2)*sin(x)^2)^(1/2)+1/3*(sin(x)^2)^(1
/2)*(1+cos(x)^2)^(1/2)/(1-cos(x)^4)^(1/2)*EllipticF(cos(x),I)-1/2*(sin(x)^
2)^(1/2)*(1+cos(x)^2)^(1/2)/(1-cos(x)^4)^(1/2)*(EllipticF(cos(x),I)-Ellipt
icE(cos(x),I)))/sin(x)/(1+cos(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(44) = 88$.

Time = 0.15 (sec) , antiderivative size = 316, normalized size of antiderivative = 5.02

$$\int \frac{1}{\left(1+\cos^2(x)\right)^{5/2}}\,dx =$$

$$-\frac{3\left(\left(-2i\sqrt{2}+3i\right)\cos(x)^4+2\left(-2i\sqrt{2}+3i\right)\cos(x)^2-2i\sqrt{2}+3i\right)\sqrt{2\sqrt{2}-3}E\left(\arcsin\left(\sqrt{2\sqrt{2}-3}(\cos\right.\right.}{}$$

input
```
integrate(1/(1+cos(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*((-2*I*sqrt(2) + 3*I)*cos(x)^4 + 2*(-2*I*sqrt(2) + 3*I)*cos(x)^2
- 2*I*sqrt(2) + 3*I)*sqrt(2*sqrt(2) - 3)*elliptic_e(arcsin(sqrt(2*sqrt(2)
- 3)*(cos(x) + I*sin(x))), 12*sqrt(2) + 17) + 3*((2*I*sqrt(2) - 3*I)*cos(x
)^4 + 2*(2*I*sqrt(2) - 3*I)*cos(x)^2 + 2*I*sqrt(2) - 3*I)*sqrt(2*sqrt(2) -
 3)*elliptic_e(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin(x))), 12*sqrt(2)
 + 17) + 2*((-4*I*sqrt(2) - 15*I)*cos(x)^4 + 2*(-4*I*sqrt(2) - 15*I)*cos(x
)^2 - 4*I*sqrt(2) - 15*I)*sqrt(2*sqrt(2) - 3)*elliptic_f(arcsin(sqrt(2*sqr
t(2) - 3)*(cos(x) + I*sin(x))), 12*sqrt(2) + 17) + 2*((4*I*sqrt(2) + 15*I)
*cos(x)^4 + 2*(4*I*sqrt(2) + 15*I)*cos(x)^2 + 4*I*sqrt(2) + 15*I)*sqrt(2*s
qrt(2) - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin(x))), 12
*sqrt(2) + 17) + 2*(3*cos(x)^3 + 4*cos(x))*sqrt(cos(x)^2 + 1)*sin(x))/(cos
(x)^4 + 2*cos(x)^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+\cos^2(x))^{5/2}}\,dx = \int \frac{1}{\left(\cos^2\left(x\right)+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(1+cos(x)**2)**(5/2),x)
```

output
```
Integral((cos(x)**2 + 1)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(1+\cos^2(x))^{5/2}}\,dx = \int \frac{1}{\left(\cos\left(x\right)^2+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(1+cos(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((cos(x)^2 + 1)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(1+\cos^2(x))^{5/2}}\,dx = \int \frac{1}{\left(\cos\left(x\right)^2+1\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(1+cos(x)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((cos(x)^2 + 1)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+\cos^2(x))^{5/2}}\,dx = \int \frac{1}{\left(\cos\left(x\right)^2+1\right)^{5/2}}\,dx$$

input
```
int(1/(cos(x)^2 + 1)^(5/2),x)
```

output
```
int(1/(cos(x)^2 + 1)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(1+\cos^2(x))^{5/2}}\,dx = \int \frac{\sqrt{\cos\left(x\right)^2+1}}{\cos\left(x\right)^6+3\cos\left(x\right)^4+3\cos\left(x\right)^2+1}dx$$

input
```
int(1/(1+cos(x)^2)^(5/2),x)
```

output
```
int(sqrt(cos(x)**2 + 1)/(cos(x)**6 + 3*cos(x)**4 + 3*cos(x)**2 + 1),x)
```

3.53 $\int (a+a\cos^2(x))^{5/2}\,dx$

Optimal result 407
Mathematica [A] (verified) 408
Rubi [A] (verified) 408
Maple [A] (verified) 412
Fricas [F] 413
Sympy [F(-1)] 413
Maxima [F] 413
Giac [F] 414
Mupad [F(-1)] 414
Reduce [F] 414

Optimal result

Integrand size = 12, antiderivative size = 121

$$\int \left(a+a\cos^2(x)\right)^{5/2}\,dx=\frac{18a^2\sqrt{a+a\cos^2(x)}E\left(\frac{\pi}{2}+x\middle|-1\right)}{5\sqrt{1+\cos^2(x)}}-\frac{8a^3\sqrt{1+\cos^2(x)}\,\text{EllipticF}\left(\frac{\pi}{2}+x,-1\right)}{5\sqrt{a+a\cos^2(x)}}+\frac{4}{5}a^2\cos(x)\sqrt{a+a\cos^2(x)}\sin(x)+\frac{1}{5}a\cos(x)\left(a+a\cos^2(x)\right)^{3/2}\sin(x)$$

output
```
18/5*a^2*(a+a*cos(x)^2)^(1/2)*EllipticE(cos(x),I)/(1+cos(x)^2)^(1/2)-8/5*a
^3*(1+cos(x)^2)^(1/2)*InverseJacobiAM(1/2*Pi+x,I)/(a+a*cos(x)^2)^(1/2)+4/5
*a^2*cos(x)*(a+a*cos(x)^2)^(1/2)*sin(x)+1/5*a*cos(x)*(a+a*cos(x)^2)^(3/2)*
sin(x)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.55

$$\int (a + a\cos^2(x))^{5/2}\,dx = \frac{a^2\sqrt{a(3+\cos(2x))}\left(288E\left(x\middle|\frac{1}{2}\right) - 64\,\text{EllipticF}\left(x,\frac{1}{2}\right) + \sqrt{3+\cos(2x)}(22\sin(2x) + \text{si}\right.}{80\sqrt{1+\cos^2(x)}}$$

input
```
Integrate[(a + a*Cos[x]^2)^(5/2),x]
```

output
```
(a^2*Sqrt[a*(3 + Cos[2*x])]*(288*EllipticE[x, 1/2] - 64*EllipticF[x, 1/2]
+ Sqrt[3 + Cos[2*x]]*(22*Sin[2*x] + Sin[4*x])))/(80*Sqrt[1 + Cos[x]^2])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {3042, 3659, 27, 3042, 3649, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos^2(x)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^{5/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{5}\int 6\sqrt{a\cos^2(x)+a}\,(2\cos^2(x)a^2+a^2)\,dx+\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{5}\int \sqrt{a\cos^2(x)+a}\,(2\cos^2(x)a^2+a^2)\,dx+\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}\left(2\sin\left(x+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

$$\Big\downarrow \text{3649}$$

$$\frac{6}{5}\left(\frac{1}{3}\int \frac{9\cos^2(x)a^3+5a^3}{\sqrt{a\cos^2(x)+a}}dx+\frac{2}{3}a^2\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}\right)+\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}\left(\frac{1}{3}\int \frac{9\sin\left(x+\frac{\pi}{2}\right)^2a^3+5a^3}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx+\frac{2}{3}a^2\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}\right)+\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

$$\Big\downarrow \text{3651}$$

$$\frac{6}{5}\left(\frac{1}{3}\left(9a^2\int \sqrt{a\cos^2(x)+a}dx-4a^3\int \frac{1}{\sqrt{a\cos^2(x)+a}}dx\right)+\frac{2}{3}a^2\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}\right)+\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}\left(\frac{1}{3}\left(9a^2\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}dx-4a^3\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+\frac{2}{3}a^2\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}\right)+\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

$$\Big\downarrow \text{3657}$$

$$\frac{6}{5}\left(\frac{1}{3}\left(\frac{9a^2\sqrt{a\cos^2(x)+a}\int \sqrt{\cos^2(x)+1}dx}{\sqrt{\cos^2(x)+1}}-4a^3\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+\frac{2}{3}a^2\sin(x)\cos(x)\sqrt{a\cos^2(x)\right.$$

$$\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}\left(\frac{1}{3}\left(\frac{9a^2\sqrt{a\cos^2(x)+a}\int\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx}{\sqrt{\cos^2(x)+1}}-4a^3\int\frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+\frac{2}{3}a^2\sin(x)\cos(x)\sqrt{a\cos\right.$$
$$\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

$\downarrow$ 3656

$$\frac{6}{5}\left(\frac{1}{3}\left(\frac{9a^2E\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}-4a^3\int\frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+\frac{2}{3}a^2\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}\right.$$
$$\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

$\downarrow$ 3662

$$\frac{6}{5}\left(\frac{1}{3}\left(\frac{9a^2E\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}-\frac{4a^3\sqrt{\cos^2(x)+1}\int\frac{1}{\sqrt{\cos^2(x)+1}}dx}{\sqrt{a\cos^2(x)+a}}\right)+\frac{2}{3}a^2\sin(x)\cos(x)\sqrt{a\cos^2(x)}\right.$$
$$\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

$\downarrow$ 3042

$$\frac{6}{5}\left(\frac{1}{3}\left(\frac{9a^2E\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}-\frac{4a^3\sqrt{\cos^2(x)+1}\int\frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx}{\sqrt{a\cos^2(x)+a}}\right)+\frac{2}{3}a^2\sin(x)\cos(x)\sqrt{a\cos^2(}\right.$$
$$\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

$\downarrow$ 3661

$$\frac{6}{5}\left(\frac{2}{3}a^2\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}+\frac{1}{3}\left(\frac{9a^2E\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}-\frac{4a^3\sqrt{\cos^2(x)+1}\,\text{EllipticF}\left(x+\right.}{\sqrt{a\cos^2(x)+a}}\right.\right.$$
$$\frac{1}{5}a\sin(x)\cos(x)\left(a\cos^2(x)+a\right)^{3/2}$$

input
```
Int[(a + a*Cos[x]^2)^(5/2),x]
```

output
```
(a*Cos[x]*(a + a*Cos[x]^2)^(3/2)*Sin[x])/5 + (6*(((9*a^2*Sqrt[a + a*Cos[x]
^2]*EllipticE[Pi/2 + x, -1])/Sqrt[1 + Cos[x]^2] - (4*a^3*Sqrt[1 + Cos[x]^2
]*EllipticF[Pi/2 + x, -1])/Sqrt[a + a*Cos[x]^2])/3 + (2*a^2*Cos[x]*Sqrt[a
+ a*Cos[x]^2]*Sin[x])/3))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

method	result
default	$\frac{\sqrt{a\left(1+\cos(x)^2\right)\sin(x)^2}\, a^3 \left(\sin(x)^6 \cos(x)-8\sin(x)^4\cos(x)+8\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\sin(x)^2+2}\ \text{EllipticF}(\cos(x),i)-18\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\right.}{5\sqrt{-a\left(\cos(x)^4-1\right)}\,\sin(x)\sqrt{a\left(1+\cos(x)^2\right)}}$

input
```
int((a+a*cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(a*(1+cos(x)^2)*sin(x)^2)^(1/2)*a^3*(sin(x)^6*cos(x)-8*sin(x)^4*cos(x)
+8*(sin(x)^2)^(1/2)*(-sin(x)^2+2)^(1/2)*EllipticF(cos(x),I)-18*(sin(x)^2)^
(1/2)*(-sin(x)^2+2)^(1/2)*EllipticE(cos(x),I)+12*sin(x)^2*cos(x))/(-a*(cos
(x)^4-1))^(1/2)/sin(x)/(a*(1+cos(x)^2))^(1/2)
```

Fricas [F]

$$\int \left(a+a\cos^2(x)\right)^{5/2}\,dx=\int \left(a\cos\left(x\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*cos(x)^4 + 2*a^2*cos(x)^2 + a^2)*sqrt(a*cos(x)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+a\cos^2(x)\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(x)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+a\cos^2(x)\right)^{5/2}\,dx=\int \left(a\cos\left(x\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(x)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \left(a+a\cos^2(x)\right)^{5/2}\,dx=\int \left(a\cos\left(x\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*cos(x)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(x)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+a\cos^2(x)\right)^{5/2}\,dx=\int \left(a\cos\left(x\right)^2+a\right)^{5/2}\,dx$$

input
```
int((a + a*cos(x)^2)^(5/2),x)
```

output
```
int((a + a*cos(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \left(a+a\cos^2(x)\right)^{5/2}\,dx=\sqrt{a}\,a^2\left(\int \sqrt{\cos\left(x\right)^2+1}dx+\int \sqrt{\cos\left(x\right)^2+1}\,\cos\left(x\right)^4dx+2\left(\int \sqrt{\cos\left(x\right)^2+1}\,\cos\left(x\right)^2dx\right)\right)$$

input
```
int((a+a*cos(x)^2)^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(cos(x)**2 + 1),x) + int(sqrt(cos(x)**2 + 1)*cos(x)*
*4,x) + 2*int(sqrt(cos(x)**2 + 1)*cos(x)**2,x))
```

3.54 $\int \left(a+a\cos^2(x)\right)^{3/2}\,dx$

Optimal result 415
Mathematica [A] (verified) 415
Rubi [A] (verified) 416
Maple [A] (verified) 419
Fricas [F] 420
Sympy [F(-1)] 420
Maxima [F] 420
Giac [F] 421
Mupad [F(-1)] 421
Reduce [F] 421

Optimal result

Integrand size = 12, antiderivative size = 94

$$\int \left(a+a\cos^2(x)\right)^{3/2}\,dx=\frac{2a\sqrt{a+a\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{1+\cos^2(x)}}-\frac{2a^2\sqrt{1+\cos^2(x)}\,\text{EllipticF}\left(\frac{\pi}{2}+x,-1\right)}{3\sqrt{a+a\cos^2(x)}}+\frac{1}{3}a\cos(x)\sqrt{a+a\cos^2(x)}\sin(x)$$

output
```
2*a*(a+a*cos(x)^2)^(1/2)*EllipticE(cos(x),I)/(1+cos(x)^2)^(1/2)-2/3*a^2*(1
+cos(x)^2)^(1/2)*InverseJacobiAM(1/2*Pi+x,I)/(a+a*cos(x)^2)^(1/2)+1/3*a*co
s(x)*(a+a*cos(x)^2)^(1/2)*sin(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \left(a+a\cos^2(x)\right)^{3/2}\,dx=\frac{a\sqrt{a\left(1+\cos^2(x)\right)}\left(24E\left(x\middle|\frac{1}{2}\right)-4\,\text{EllipticF}\left(x,\frac{1}{2}\right)+\sqrt{3+\cos(2x)}\sin(2x)\right)}{6\sqrt{3+\cos(2x)}}$$

input
```
Integrate[(a + a*Cos[x]^2)^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[x]^2)]*(24*EllipticE[x, 1/2] - 4*EllipticF[x, 1/2] + Sq
rt[3 + Cos[2*x]]*Sin[2*x]))/(6*Sqrt[3 + Cos[2*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3659, 27, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cos^2(x)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{2\left(3\cos^2(x)a^2+2a^2\right)}{\sqrt{a\cos^2(x)+a}}dx+\frac{1}{3}a\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{3\cos^2(x)a^2+2a^2}{\sqrt{a\cos^2(x)+a}}dx+\frac{1}{3}a\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{3\sin\left(x+\frac{\pi}{2}\right)^2a^2+2a^2}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx+\frac{1}{3}a\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}$$

$$\Big\downarrow \text{3651}$$

$$\frac{2}{3}\left(3a\int \sqrt{a\cos^2(x)+a}dx-a^2\int \frac{1}{\sqrt{a\cos^2(x)+a}}dx\right)+\frac{1}{3}a\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(3a\int\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}dx-a^2\int\frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+$$
$$\frac{1}{3}a\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}$$

$\downarrow$ 3657

$$\frac{2}{3}\left(\frac{3a\sqrt{a\cos^2(x)+a}\int\sqrt{\cos^2(x)+1}dx}{\sqrt{\cos^2(x)+1}}-a^2\int\frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+$$
$$\frac{1}{3}a\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}$$

$\downarrow$ 3042

$$\frac{2}{3}\left(\frac{3a\sqrt{a\cos^2(x)+a}\int\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx}{\sqrt{\cos^2(x)+1}}-a^2\int\frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+$$
$$\frac{1}{3}a\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}$$

$\downarrow$ 3656

$$\frac{2}{3}\left(\frac{3aE\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}-a^2\int\frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+$$
$$\frac{1}{3}a\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}$$

$\downarrow$ 3662

$$\frac{2}{3}\left(\frac{3aE\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}-\frac{a^2\sqrt{\cos^2(x)+1}\int\frac{1}{\sqrt{\cos^2(x)+1}}dx}{\sqrt{a\cos^2(x)+a}}\right)+$$
$$\frac{1}{3}a\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}$$

$\downarrow$ 3042

$$\frac{2}{3}\left(\frac{3aE\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}-\frac{a^2\sqrt{\cos^2(x)+1}\int\frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx}{\sqrt{a\cos^2(x)+a}}\right)+$$
$$\frac{1}{3}a\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}$$

$\downarrow$ 3661

$$\frac{2}{3}\left(\frac{3aE\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}-\frac{a^2\sqrt{\cos^2(x)+1}\,\text{EllipticF}\left(x+\frac{\pi}{2},-1\right)}{\sqrt{a\cos^2(x)+a}}\right)+\frac{1}{3}a\sin(x)\cos(x)\sqrt{a\cos^2(x)+a}$$

input
```
Int[(a + a*Cos[x]^2)^(3/2),x]
```

output
```
(2*((3*a*Sqrt[a + a*Cos[x]^2]*EllipticE[Pi/2 + x, -1])/Sqrt[1 + Cos[x]^2]
- (a^2*Sqrt[1 + Cos[x]^2]*EllipticF[Pi/2 + x, -1])/Sqrt[a + a*Cos[x]^2]))/
3 + (a*Cos[x]*Sqrt[a + a*Cos[x]^2]*Sin[x])/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

method	result
default	$\frac{\sqrt{a\left(1+\cos(x)^2\right)\sin(x)^2}\, a^2 \left(-\sin(x)^4\cos(x)+2\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\sin(x)^2+2}\,\text{EllipticF}(\cos(x),i)-6\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\sin(x)^2+2}\,\text{El}\right.}{3\sqrt{-a\left(\cos(x)^4-1\right)}\,\sin(x)\sqrt{a\left(1+\cos(x)^2\right)}}$

input
```
int((a+a*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*(1+cos(x)^2)*sin(x)^2)^(1/2)*a^2*(-sin(x)^4*cos(x)+2*(sin(x)^2)^(1/
2)*(-sin(x)^2+2)^(1/2)*EllipticF(cos(x),I)-6*(sin(x)^2)^(1/2)*(-sin(x)^2+2
)^(1/2)*EllipticE(cos(x),I)+2*sin(x)^2*cos(x))/(-a*(cos(x)^4-1))^(1/2)/sin
(x)/(a*(1+cos(x)^2))^(1/2)
```

Fricas [F]

$$\int \left(a+a\cos^2(x)\right)^{3/2}\,dx=\int \left(a\cos\left(x\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*cos(x)^2 + a)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+a\cos^2(x)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a+a*cos(x)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+a\cos^2(x)\right)^{3/2}\,dx=\int \left(a\cos\left(x\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(x)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+a\cos^2(x)\right)^{3/2}\,dx=\int \left(a\cos{(x)}^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(x)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+a\cos^2(x)\right)^{3/2}\,dx=\int \left(a\cos{(x)}^2+a\right)^{3/2}\,dx$$

input
```
int((a + a*cos(x)^2)^(3/2),x)
```

output
```
int((a + a*cos(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+a\cos^2(x)\right)^{3/2}\,dx=\sqrt{a}\,a\left(\int \sqrt{\cos{(x)}^2+1}dx+\int \sqrt{\cos{(x)}^2+1}\,\cos{(x)}^2\,dx\right)$$

input
```
int((a+a*cos(x)^2)^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(cos(x)**2 + 1),x) + int(sqrt(cos(x)**2 + 1)*cos(x)**2,
x))
```

3.55 $\int \sqrt{a+a\cos^2(x)}\,dx$

Optimal result . . . 422
Mathematica [A] (verified) . . . 422
Rubi [A] (verified) . . . 423
Maple [B] (verified) . . . 424
Fricas [F] . . . 425
Sympy [F] . . . 425
Maxima [F] . . . 425
Giac [F] . . . 426
Mupad [F(-1)] . . . 426
Reduce [F] . . . 426

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \sqrt{a+a\cos^2(x)}\,dx=\frac{\sqrt{a+a\cos^2(x)}E\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{1+\cos^2(x)}}$$

output
```
(a+a*cos(x)^2)^(1/2)*EllipticE(cos(x),I)/(1+cos(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{a+a\cos^2(x)}\,dx=\frac{\sqrt{a(3+\cos(2x))}E\left(x\middle|\frac{1}{2}\right)}{\sqrt{1+\cos^2(x)}}$$

input
```
Integrate[Sqrt[a + a*Cos[x]^2],x]
```

output
```
(Sqrt[a*(3 + Cos[2*x])]*EllipticE[x, 1/2])/Sqrt[1 + Cos[x]^2]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cos^2(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a\cos^2(x)+a}\int \sqrt{\cos^2(x)+1}dx}{\sqrt{\cos^2(x)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\cos^2(x)+a}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx}{\sqrt{\cos^2(x)+1}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{E\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}$$

input
```
Int[Sqrt[a + a*Cos[x]^2],x]
```

output
```
(Sqrt[a + a*Cos[x]^2]*EllipticE[Pi/2 + x, -1])/Sqrt[1 + Cos[x]^2]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(24) = 48$.

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.94

method	result	size
default	$-\frac{\sqrt{a\left(1+\cos(x)^{2}\right)\sin(x)^{2}}\, a\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{1+\cos(x)^{2}}\,\operatorname{EllipticE}(\cos(x), i)}{\sqrt{-a\left(\cos(x)^{4}-1\right)}\,\sin(x)\sqrt{a\left(1+\cos(x)^{2}\right)}}$	62

input
```
int((a+a*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*(1+cos(x)^2)*sin(x)^2)^(1/2)*a*(sin(x)^2)^(1/2)*(1+cos(x)^2)^(1/2)*Ell
ipticE(cos(x),I)/(-a*(cos(x)^4-1))^(1/2)/sin(x)/(a*(1+cos(x)^2))^(1/2)
```

Fricas [F]

$$\int \sqrt{a+a\cos^2(x)}\,dx = \int \sqrt{a\cos{(x)}^2 + a}\,dx$$

input
```
integrate((a+a*cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cos(x)^2 + a), x)
```

Sympy [F]

$$\int \sqrt{a+a\cos^2(x)}\,dx = \int \sqrt{a\cos^2{(x)} + a}\,dx$$

input
```
integrate((a+a*cos(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a*cos(x)**2 + a), x)
```

Maxima [F]

$$\int \sqrt{a+a\cos^2(x)}\,dx = \int \sqrt{a\cos{(x)}^2 + a}\,dx$$

input
```
integrate((a+a*cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cos(x)^2 + a), x)
```

Giac [F]

$$\int \sqrt{a+a\cos^2(x)}\,dx = \int \sqrt{a\cos(x)^2+a}\,dx$$

input
```
integrate((a+a*cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cos(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos^2(x)}\,dx = \int \sqrt{a\cos(x)^2+a}\,dx$$

input
```
int((a + a*cos(x)^2)^(1/2),x)
```

output
```
int((a + a*cos(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+a\cos^2(x)}\,dx = \sqrt{a}\left(\int \sqrt{\cos(x)^2+1}dx\right)$$

input
```
int((a+a*cos(x)^2)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(cos(x)**2 + 1),x)
```

3.56 $\int \frac{1}{\sqrt{a+a\cos^2(x)}}\,dx$

Optimal result 427
Mathematica [A] (verified) 427
Rubi [A] (verified) 428
Maple [B] (verified) 429
Fricas [B] (verification not implemented) 430
Sympy [F] 430
Maxima [F] 431
Giac [F] 431
Mupad [F(-1)] 431
Reduce [F] 432

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{1}{\sqrt{a+a\cos^2(x)}}\,dx = \frac{\sqrt{1+\cos^2(x)}\,\text{EllipticF}\left(\frac{\pi}{2}+x,-1\right)}{\sqrt{a+a\cos^2(x)}}$$

output
```
(1+cos(x)^2)^(1/2)*InverseJacobiAM(1/2*Pi+x,I)/(a+a*cos(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a+a\cos^2(x)}}\,dx = \frac{\sqrt{1+\cos^2(x)}\,\text{EllipticF}\left(x,\frac{1}{2}\right)}{\sqrt{a(3+\cos(2x))}}$$

input
```
Integrate[1/Sqrt[a + a*Cos[x]^2],x]
```

output
```
(Sqrt[1 + Cos[x]^2]*EllipticF[x, 1/2])/Sqrt[a*(3 + Cos[2*x])]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cos^2(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sqrt{\cos^2(x)+1}\int \frac{1}{\sqrt{\cos^2(x)+1}}dx}{\sqrt{a\cos^2(x)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos^2(x)+1}\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx}{\sqrt{a\cos^2(x)+a}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\sqrt{\cos^2(x)+1}\,\text{EllipticF}\left(x+\frac{\pi}{2},-1\right)}{\sqrt{a\cos^2(x)+a}}$$

input
```
Int[1/Sqrt[a + a*Cos[x]^2],x]
```

output
```
(Sqrt[1 + Cos[x]^2]*EllipticF[Pi/2 + x, -1])/Sqrt[a + a*Cos[x]^2]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

method	result	size
default	$-\frac{\sqrt{a\left(1+\cos(x)^2\right)\sin(x)^2}\,\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{1+\cos(x)^2}\,\operatorname{EllipticF}(\cos(x),i)}{\sqrt{-a\left(\cos(x)^4-1\right)}\,\sin(x)\sqrt{a\left(1+\cos(x)^2\right)}}$	61

input
```
int(1/(a+a*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*(1+cos(x)^2)*sin(x)^2)^(1/2)*(sin(x)^2)^(1/2)*(1+cos(x)^2)^(1/2)/(-a*(
cos(x)^4-1))^(1/2)*EllipticF(cos(x),I)/sin(x)/(a*(1+cos(x)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(26) = 52.

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.03

$$\int \frac{1}{\sqrt{a+a\cos^2(x)}}\,dx = \frac{\sqrt{a}\sqrt{2\sqrt{2}-3}(2i\sqrt{2}+3i)F\left(\arcsin\left(\sqrt{2\sqrt{2}-3}(\cos(x)+i\sin(x))\right) \mid 12\sqrt{2}+17\right)+\sqrt{a}\sqrt{2\sqrt{2}-3}(-}{a}$$

input
```
integrate(1/(a+a*cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(a)*sqrt(2*sqrt(2) - 3)*(2*I*sqrt(2) + 3*I)*elliptic_f(arcsin(sqrt(2*
sqrt(2) - 3)*(cos(x) + I*sin(x))), 12*sqrt(2) + 17) + sqrt(a)*sqrt(2*sqrt(
2) - 3)*(-2*I*sqrt(2) - 3*I)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x)
 - I*sin(x))), 12*sqrt(2) + 17))/a
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{a\cos^2(x)+a}}\,dx$$

input
```
integrate(1/(a+a*cos(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*cos(x)**2 + a), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + a\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{a\cos{(x)}^2 + a}}\,dx$$

input
```
integrate(1/(a+a*cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*cos(x)^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + a\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{a\cos{(x)}^2 + a}}\,dx$$

input
```
integrate(1/(a+a*cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*cos(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + a\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{a\cos{(x)}^2 + a}}\,dx$$

input
```
int(1/(a + a*cos(x)^2)^(1/2),x)
```

output
```
int(1/(a + a*cos(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+a\cos^2(x)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cos(x)^2+1}}{\cos(x)^2+1}dx\right)}{a}$$

input
```
int(1/(a+a*cos(x)^2)^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(cos(x)**2 + 1)/(cos(x)**2 + 1),x))/a
```

3.57 $\int \frac{1}{\left(a+a\cos^2(x)\right)^{3/2}}\,dx$

Optimal result 433
Mathematica [A] (verified) 433
Rubi [A] (verified) 434
Maple [A] (verified) 436
Fricas [B] (verification not implemented) 436
Sympy [F] 437
Maxima [F] 437
Giac [F] 438
Mupad [F(-1)] 438
Reduce [F] 438

Optimal result

Integrand size = 12, antiderivative size = 62

$$\int \frac{1}{\left(a+a\cos^2(x)\right)^{3/2}}\,dx=\frac{\sqrt{a+a\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{2a^2\sqrt{1+\cos^2(x)}}-\frac{\cos(x)\sin(x)}{2a\sqrt{a+a\cos^2(x)}}$$

output
```
1/2*(a+a*cos(x)^2)^(1/2)*EllipticE(cos(x),I)/a^2/(1+cos(x)^2)^(1/2)-1/2*co
s(x)*sin(x)/a/(a+a*cos(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+a\cos^2(x)\right)^{3/2}}\,dx=\frac{2\sqrt{3+\cos(2x)}\,E\left(x\middle|\frac{1}{2}\right)-\sin(2x)}{2\sqrt{2}a\sqrt{a(3+\cos(2x))}}$$

input
```
Integrate[(a + a*Cos[x]^2)^(-3/2),x]
```

output
```
(2*Sqrt[3 + Cos[2*x]]*EllipticE[x, 1/2] - Sin[2*x])/(2*Sqrt[2]*a*Sqrt[a*(3
 + Cos[2*x])])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 3663, 25, 3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos^2(x)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\sqrt{a\cos^2(x)+a}dx}{2a^2}-\frac{\sin(x)\cos(x)}{2a\sqrt{a\cos^2(x)+a}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sqrt{a\cos^2(x)+a}dx}{2a^2}-\frac{\sin(x)\cos(x)}{2a\sqrt{a\cos^2(x)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}dx}{2a^2}-\frac{\sin(x)\cos(x)}{2a\sqrt{a\cos^2(x)+a}}$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a\cos^2(x)+a}\int \sqrt{\cos^2(x)+1}dx}{2a^2\sqrt{\cos^2(x)+1}}-\frac{\sin(x)\cos(x)}{2a\sqrt{a\cos^2(x)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\cos^2(x)+a}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx}{2a^2\sqrt{\cos^2(x)+1}}-\frac{\sin(x)\cos(x)}{2a\sqrt{a\cos^2(x)+a}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{E\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{2a^2\sqrt{\cos^2(x)+1}}-\frac{\sin(x)\cos(x)}{2a\sqrt{a\cos^2(x)+a}}$$

input
```
Int[(a + a*Cos[x]^2)^(-3/2),x]
```

output
```
(Sqrt[a + a*Cos[x]^2]*EllipticE[Pi/2 + x, -1])/(2*a^2*Sqrt[1 + Cos[x]^2])
- (Cos[x]*Sin[x])/(2*a*Sqrt[a + a*Cos[x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{\sqrt{-a\sin(x)^4+2a\sin(x)^2}\left(\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\sin(x)^2+2}\,\mathrm{EllipticE}(\cos(x),i)+\sin(x)^2\cos(x)\right)}{2a\sqrt{-a\left(\cos(x)^4-1\right)}\,\sin(x)\sqrt{a\left(1+\cos(x)^2\right)}}$	78

input
```
int(1/(a+a*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*(-a*sin(x)^4+2*a*sin(x)^2)^(1/2)*((sin(x)^2)^(1/2)*(-sin(x)^2+2)^(1
/2)*EllipticE(cos(x),I)+sin(x)^2*cos(x))/(-a*(cos(x)^4-1))^(1/2)/sin(x)/(a
*(1+cos(x)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(47) = 94.

Time = 0.11 (sec) , antiderivative size = 267, normalized size of antiderivative = 4.31

$$\int \frac{1}{\left(a+a\cos^2(x)\right)^{3/2}}\,dx = \frac{\left(\left(2i\sqrt{2}-3i\right)\cos(x)^2+2i\sqrt{2}-3i\right)\sqrt{a}\sqrt{2\sqrt{2}-3}E\left(\arcsin\left(\sqrt{2\sqrt{2}-3}(\cos(x\right.\right.}{}$$

input
```
integrate(1/(a+a*cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(((2*I*sqrt(2) - 3*I)*cos(x)^2 + 2*I*sqrt(2) - 3*I)*sqrt(a)*sqrt(2*sqr
t(2) - 3)*elliptic_e(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) + I*sin(x))), 12*s
qrt(2) + 17) + ((-2*I*sqrt(2) + 3*I)*cos(x)^2 - 2*I*sqrt(2) + 3*I)*sqrt(a)
*sqrt(2*sqrt(2) - 3)*elliptic_e(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin
(x))), 12*sqrt(2) + 17) - 4*((-I*sqrt(2) - 3*I)*cos(x)^2 - I*sqrt(2) - 3*I
)*sqrt(a)*sqrt(2*sqrt(2) - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x
) + I*sin(x))), 12*sqrt(2) + 17) - 4*((I*sqrt(2) + 3*I)*cos(x)^2 + I*sqrt(
2) + 3*I)*sqrt(a)*sqrt(2*sqrt(2) - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3
)*(cos(x) - I*sin(x))), 12*sqrt(2) + 17) - 2*sqrt(a*cos(x)^2 + a)*cos(x)*s
in(x))/(a^2*cos(x)^2 + a^2)
```

Sympy [F]

$$\int \frac{1}{(a+a\cos^2(x))^{3/2}}\,dx = \int \frac{1}{(a\cos^2(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(x)**2)**(3/2),x)
```

output
```
Integral((a*cos(x)**2 + a)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\cos^2(x))^{3/2}}\,dx = \int \frac{1}{(a\cos(x)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(x)^2 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+a\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(a\cos(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(x)^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(a\cos(x)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(a + a*cos(x)^2)^(3/2),x)
```

output
```
int(1/(a + a*cos(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\cos^2(x))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cos(x)^2+1}}{\cos(x)^4+2\cos(x)^2+1}dx\right)}{a^2}$$

input
```
int(1/(a+a*cos(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(cos(x)**2 + 1)/(cos(x)**4 + 2*cos(x)**2 + 1),x))/a**2
```

3.58 $\int \frac{1}{\left(a+a\cos^2(x)\right)^{5/2}}\,dx$

Optimal result 439
Mathematica [A] (verified) 439
Rubi [A] (verified) 440
Maple [A] (verified) 444
Fricas [B] (verification not implemented) 444
Sympy [F] 445
Maxima [F] 445
Giac [F] 446
Mupad [F(-1)] 446
Reduce [F] 446

Optimal result

Integrand size = 12, antiderivative size = 123

$$\int \frac{1}{\left(a+a\cos^2(x)\right)^{5/2}}\,dx = \frac{\sqrt{a+a\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{2a^3\sqrt{1+\cos^2(x)}} - \frac{\sqrt{1+\cos^2(x)}\,\text{EllipticF}\left(\frac{\pi}{2}+x,-1\right)}{6a^2\sqrt{a+a\cos^2(x)}} - \frac{\cos(x)\sin(x)}{6a\left(a+a\cos^2(x)\right)^{3/2}} - \frac{\cos(x)\sin(x)}{2a^2\sqrt{a+a\cos^2(x)}}$$

output
```
1/2*(a+a*cos(x)^2)^(1/2)*EllipticE(cos(x),I)/a^3/(1+cos(x)^2)^(1/2)-1/6*(1
+cos(x)^2)^(1/2)*InverseJacobiAM(1/2*Pi+x,I)/a^2/(a+a*cos(x)^2)^(1/2)-1/6*
cos(x)*sin(x)/a/(a+a*cos(x)^2)^(3/2)-1/2*cos(x)*sin(x)/a^2/(a+a*cos(x)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(a+a\cos^2(x)\right)^{5/2}}\,dx = \frac{12(3+\cos(2x))^{3/2}E\left(x\middle|\frac{1}{2}\right) - 2(3+\cos(2x))^{3/2}\,\text{EllipticF}\left(x,\frac{1}{2}\right) - 22\sin(2x) - 3}{12\sqrt{2}a(a(3+\cos(2x)))^{3/2}}$$

input
```
Integrate[(a + a*Cos[x]^2)^(-5/2),x]
```

output
```
(12*(3 + Cos[2*x])^(3/2)*EllipticE[x, 1/2] - 2*(3 + Cos[2*x])^(3/2)*Ellipt
icF[x, 1/2] - 22*Sin[2*x] - 3*Sin[4*x])/(12*Sqrt[2]*a*(a*(3 + Cos[2*x]))^(
3/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a \cos^2(x)+a\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a \sin \left(x+\frac{\pi}{2}\right)^2+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3663}$$

$$-\frac{\int -\frac{5a-a\cos^2(x)}{\left(a\cos^2(x)+a\right)^{3/2}}dx}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{5a-a\cos^2(x)}{\left(a\cos^2(x)+a\right)^{3/2}}dx}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{5a-a\sin\left(x+\frac{\pi}{2}\right)^2}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^{3/2}}dx}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \text{ 3652}$$

$$\frac{\frac{\int \frac{2\left(3\cos^2(x)a^2+2a^2\right)}{\sqrt{a\cos^2(x)+a}}dx}{2a^2}-\frac{3\sin(x)\cos(x)}{\sqrt{a\cos^2(x)+a}}}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3\cos^2(x)a^2+2a^2}{\sqrt{a\cos^2(x)+a}}dx}{a^2}-\frac{3\sin(x)\cos(x)}{\sqrt{a\cos^2(x)+a}}}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{3\sin\left(x+\frac{\pi}{2}\right)^2a^2+2a^2}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx}{a^2}-\frac{3\sin(x)\cos(x)}{\sqrt{a\cos^2(x)+a}}}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \; 3651$$

$$\frac{\frac{3a\int \sqrt{a\cos^2(x)+a}dx-a^2\int \frac{1}{\sqrt{a\cos^2(x)+a}}dx}{a^2}-\frac{3\sin(x)\cos(x)}{\sqrt{a\cos^2(x)+a}}}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3a\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}dx-a^2\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx}{a^2}-\frac{3\sin(x)\cos(x)}{\sqrt{a\cos^2(x)+a}}}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \; 3657$$

$$\frac{\frac{\frac{3a\sqrt{a\cos^2(x)+a}\int \sqrt{\cos^2(x)+1}dx}{\sqrt{\cos^2(x)+1}}-a^2\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx}{a^2}-\frac{3\sin(x)\cos(x)}{\sqrt{a\cos^2(x)+a}}}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3a\sqrt{a\cos^2(x)+a}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx}{\sqrt{\cos^2(x)+1}}-a^2\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx}{a^2}-\frac{3\sin(x)\cos(x)}{\sqrt{a\cos^2(x)+a}}}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \; 3656$$

$$\frac{\frac{\frac{3aE\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}-a^2\int\frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx}{a^2}-\frac{3\sin(x)\cos(x)}{\sqrt{a\cos^2(x)+a}}}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \text{3662}$$

$$\frac{\frac{\frac{3aE\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}-\frac{a^2\sqrt{\cos^2(x)+1}\int\frac{1}{\sqrt{\cos^2(x)+1}}dx}{\sqrt{a\cos^2(x)+a}}}{a^2}-\frac{3\sin(x)\cos(x)}{\sqrt{a\cos^2(x)+a}}}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3aE\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}-\frac{a^2\sqrt{\cos^2(x)+1}\int\frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx}{\sqrt{a\cos^2(x)+a}}}{a^2}-\frac{3\sin(x)\cos(x)}{\sqrt{a\cos^2(x)+a}}}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\frac{\frac{3aE\left(x+\frac{\pi}{2}\middle|-1\right)\sqrt{a\cos^2(x)+a}}{\sqrt{\cos^2(x)+1}}-\frac{a^2\sqrt{\cos^2(x)+1}\,\text{EllipticF}\left(x+\frac{\pi}{2},-1\right)}{\sqrt{a\cos^2(x)+a}}}{a^2}-\frac{3\sin(x)\cos(x)}{\sqrt{a\cos^2(x)+a}}}{6a^2}-\frac{\sin(x)\cos(x)}{6a\left(a\cos^2(x)+a\right)^{3/2}}$$

input
```
Int[(a + a*Cos[x]^2)^(-5/2),x]
```

output
```
-1/6*(Cos[x]*Sin[x])/(a*(a + a*Cos[x]^2)^(3/2)) + (((3*a*Sqrt[a + a*Cos[x]
^2]*EllipticE[Pi/2 + x, -1])/Sqrt[1 + Cos[x]^2] - (a^2*Sqrt[1 + Cos[x]^2]*
EllipticF[Pi/2 + x, -1])/Sqrt[a + a*Cos[x]^2])/a^2 - (3*Cos[x]*Sin[x])/Sqr
t[a + a*Cos[x]^2])/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.47

method	result
default	$-\frac{\sqrt{a\left(1+\cos(x)^2\right)\sin(x)^2}\left(\sqrt{-\sin(x)^2+2}\ \mathrm{EllipticF}(\cos(x),i)\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\ \sin(x)^2-3\sqrt{-\sin(x)^2+2}\ \mathrm{EllipticE}(\cos(x),i)\sqrt{\frac{1}{2}-\frac{\mathrm{co}}{}}\right.}{6a}$

input
```
int(1/(a+a*cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*(1+cos(x)^2)*sin(x)^2)^(1/2)/a^3/sin(x)^3/(sin(x)^4-4*sin(x)^2+4)*
((-sin(x)^2+2)^(1/2)*EllipticF(cos(x),I)*(sin(x)^2)^(1/2)*sin(x)^2-3*(-sin
(x)^2+2)^(1/2)*EllipticE(cos(x),I)*(sin(x)^2)^(1/2)*sin(x)^2-3*sin(x)^4*co
s(x)-2*(sin(x)^2)^(1/2)*(-sin(x)^2+2)^(1/2)*EllipticF(cos(x),I)+6*(sin(x)^
2)^(1/2)*(-sin(x)^2+2)^(1/2)*EllipticE(cos(x),I)+7*sin(x)^2*cos(x))*(-a*si
n(x)^4+2*a*sin(x)^2)^(1/2)/(a*(1+cos(x)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(96) = 192.

Time = 0.12 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.76

$$\int \frac{1}{\left(a+a\cos^2(x)\right)^{5/2}}\,dx =$$

$$-\frac{3\left(\left(-2i\sqrt{2}+3i\right)\cos(x)^4+2\left(-2i\sqrt{2}+3i\right)\cos(x)^2-2i\sqrt{2}+3i\right)\sqrt{a}\sqrt{2\sqrt{2}-3}E\left(\arcsin\left(\sqrt{2\sqrt{2}-3}\right.\right.}{}$$

input
```
integrate(1/(a+a*cos(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*((-2*I*sqrt(2) + 3*I)*cos(x)^4 + 2*(-2*I*sqrt(2) + 3*I)*cos(x)^2
- 2*I*sqrt(2) + 3*I)*sqrt(a)*sqrt(2*sqrt(2) - 3)*elliptic_e(arcsin(sqrt(2*
sqrt(2) - 3)*(cos(x) + I*sin(x))), 12*sqrt(2) + 17) + 3*((2*I*sqrt(2) - 3*
I)*cos(x)^4 + 2*(2*I*sqrt(2) - 3*I)*cos(x)^2 + 2*I*sqrt(2) - 3*I)*sqrt(a)*
sqrt(2*sqrt(2) - 3)*elliptic_e(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin(
x))), 12*sqrt(2) + 17) + 2*((-4*I*sqrt(2) - 15*I)*cos(x)^4 + 2*(-4*I*sqrt(
2) - 15*I)*cos(x)^2 - 4*I*sqrt(2) - 15*I)*sqrt(a)*sqrt(2*sqrt(2) - 3)*elli
ptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) + I*sin(x))), 12*sqrt(2) + 17) +
 2*((4*I*sqrt(2) + 15*I)*cos(x)^4 + 2*(4*I*sqrt(2) + 15*I)*cos(x)^2 + 4*I*
sqrt(2) + 15*I)*sqrt(a)*sqrt(2*sqrt(2) - 3)*elliptic_f(arcsin(sqrt(2*sqrt(
2) - 3)*(cos(x) - I*sin(x))), 12*sqrt(2) + 17) + 2*sqrt(a*cos(x)^2 + a)*(3
*cos(x)^3 + 4*cos(x))*sin(x))/(a^3*cos(x)^4 + 2*a^3*cos(x)^2 + a^3)
```

Sympy [F]

$$\int \frac{1}{(a + a\cos^2(x))^{5/2}}\,dx = \int \frac{1}{(a\cos^2{(x)} + a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(x)**2)**(5/2),x)
```

output
```
Integral((a*cos(x)**2 + a)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a + a\cos^2(x))^{5/2}}\,dx = \int \frac{1}{(a\cos{(x)}^2 + a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(x)^2 + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a + a\cos^2(x))^{5/2}}\,dx = \int \frac{1}{\left(a\cos\left(x\right)^2 + a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cos(x)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(x)^2 + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a\cos^2(x))^{5/2}}\,dx = \int \frac{1}{\left(a\cos\left(x\right)^2 + a\right)^{5/2}}\,dx$$

input
```
int(1/(a + a*cos(x)^2)^(5/2),x)
```

output
```
int(1/(a + a*cos(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a + a\cos^2(x))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cos(x)^2+1}}{\cos(x)^6+3\cos(x)^4+3\cos(x)^2+1}dx\right)}{a^3}$$

input
```
int(1/(a+a*cos(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*int(sqrt(cos(x)**2 + 1)/(cos(x)**6 + 3*cos(x)**4 + 3*cos(x)**2 +
1),x))/a**3
```

3.59 $\int \left(a+b\cos^2(x)\right)^4\,dx$

Optimal result 447
Mathematica [A] (verified) 448
Rubi [A] (verified) 448
Maple [A] (verified) 450
Fricas [A] (verification not implemented) 451
Sympy [B] (verification not implemented) 452
Maxima [A] (verification not implemented) 453
Giac [A] (verification not implemented) 453
Mupad [B] (verification not implemented) 454
Reduce [B] (verification not implemented) 455

Optimal result

Integrand size = 10, antiderivative size = 140

$$\begin{aligned}\int \left(a+b\cos^2(x)\right)^4\,dx =& \frac{1}{128}\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)x\\ &+\frac{1}{384}b\left(608a^3+808a^2b+480ab^2+105b^3\right)\cos(x)\sin(x)\\ &+\frac{1}{192}b^2\left(104a^2+104ab+35b^2\right)\cos^3(x)\sin(x)\\ &+\frac{7}{48}b(2a+b)\cos(x)\left(a+b\cos^2(x)\right)^2\sin(x)\\ &+\frac{1}{8}b\cos(x)\left(a+b\cos^2(x)\right)^3\sin(x)\end{aligned}$$

output
```
1/128*(128*a^4+256*a^3*b+288*a^2*b^2+160*a*b^3+35*b^4)*x+1/384*b*(608*a^3+
808*a^2*b+480*a*b^2+105*b^3)*cos(x)*sin(x)+1/192*b^2*(104*a^2+104*a*b+35*b
^2)*cos(x)^3*sin(x)+7/48*b*(2*a+b)*cos(x)*(a+b*cos(x)^2)^2*sin(x)+1/8*b*co
s(x)*(a+b*cos(x)^2)^3*sin(x)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

$$\int \left(a+b\cos^2(x)\right)^4\,dx$$
$$=\frac{24(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4)\,x+96b(2a+b)\left(16a^2+16ab+7b^2\right)\sin(2x)+24b^2(24a^2}{3072}$$

input
```
Integrate[(a + b*Cos[x]^2)^4,x]
```

output
```
(24*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x + 96*b*(2*a
 + b)*(16*a^2 + 16*a*b + 7*b^2)*Sin[2*x] + 24*b^2*(24*a^2 + 24*a*b + 7*b^2
)*Sin[4*x] + 32*b^3*(2*a + b)*Sin[6*x] + 3*b^4*Sin[8*x])/3072
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3659, 3042, 3649, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cos^2(x)\right)^4\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^4 dx$$

$$\Big\downarrow \ 3659$$

$$\frac{1}{8}\int \left(b\cos^2(x)+a\right)^2\left(7b(2a+b)\cos^2(x)+a(8a+b)\right)dx+\frac{1}{8}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^3$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{8}\int \left(b\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^2\left(7b(2a+b)\sin\left(x+\frac{\pi}{2}\right)^2+a(8a+b)\right)dx+\frac{1}{8}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^3$$

$\big\downarrow$ 3649

$$\frac{1}{8}\left(\frac{1}{6}\int \left(b\cos^2(x)+a\right)\left(b\left(104a^2+104ba+35b^2\right)\cos^2(x)+a\left(48a^2+20ba+7b^2\right)\right)dx+\frac{7}{6}b(2a+b)\sin(x)\cos(x)\right.$$
$$\frac{1}{8}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^3$$

$\big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\int \left(b\sin\left(x+\frac{\pi}{2}\right)^2+a\right)\left(b\left(104a^2+104ba+35b^2\right)\sin\left(x+\frac{\pi}{2}\right)^2+a\left(48a^2+20ba+7b^2\right)\right)dx+\frac{7}{6}b(2a+\right.$$
$$\frac{1}{8}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^3$$

$\big\downarrow$ 3648

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{1}{4}b^2\left(104a^2+104ab+35b^2\right)\sin(x)\cos^3(x)+\frac{1}{8}b\left(608a^3+808a^2b+480ab^2+105b^3\right)\sin(x)\cos(x)+\frac{3}{8}x(12\right.\right.$$
$$\frac{1}{8}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^3$$

input
```
Int[(a + b*Cos[x]^2)^4,x]
```

output
```
(b*Cos[x]*(a + b*Cos[x]^2)^3*Sin[x])/8 + ((7*b*(2*a + b)*Cos[x]*(a + b*Cos
[x]^2)^2*Sin[x])/6 + ((3*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 +
35*b^4)*x)/8 + (b*(608*a^3 + 808*a^2*b + 480*a*b^2 + 105*b^3)*Cos[x]*Sin[x
])/8 + (b^2*(104*a^2 + 104*a*b + 35*b^2)*Cos[x]^3*Sin[x])/4)/6)/8
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

Maple [A] (verified)

Time = 11.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

method	result
default	$b^4\left(\frac{\left(\cos(x)^7+\frac{7\cos(x)^5}{6}+\frac{35\cos(x)^3}{24}+\frac{35\cos(x)}{16}\right)\sin(x)}{8}+\frac{35x}{128}\right)+4a\,b^3\left(\frac{\left(\cos(x)^5+\frac{5\cos(x)^3}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+\right.$
parts	$b^4\left(\frac{\left(\cos(x)^7+\frac{7\cos(x)^5}{6}+\frac{35\cos(x)^3}{24}+\frac{35\cos(x)}{16}\right)\sin(x)}{8}+\frac{35x}{128}\right)+4a\,b^3\left(\frac{\left(\cos(x)^5+\frac{5\cos(x)^3}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+\right.$
parallelrisch	$\frac{(32a^3b+48b^2a^2+30a\,b^3+7b^4)\sin(2x)}{32}+\frac{(24b^2a^2+24a\,b^3+7b^4)\sin(4x)}{128}+\frac{(2a\,b^3+b^4)\sin(6x)}{96}+\frac{b^4\sin(8x)}{1024}+(a^4+$
risch	$a^4x+2a^3bx+\frac{9a^2b^2x}{4}+\frac{5a\,b^3x}{4}+\frac{35b^4x}{128}+\frac{b^4\sin(8x)}{1024}+\frac{\sin(6x)a\,b^3}{48}+\frac{\sin(6x)b^4}{96}+\frac{3\sin(4x)a^2b^2}{16}+$
norman	$\left(-36a^3b-\frac{63}{2}b^2a^2-\frac{113}{6}a\,b^3-\frac{1799}{192}b^4\right)\tan\left(\frac{x}{2}\right)^{11}+\left(-20a^3b-\frac{51}{2}b^2a^2-\frac{61}{6}a\,b^3-\frac{91}{192}b^4\right)\tan\left(\frac{x}{2}\right)^{13}+\left(-20a^3b-\frac{27}{2}b^2a^2-\frac{85}{6}a\,b^3+\right.$

input
```
int((a+b*cos(x)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
b^4*(1/8*(cos(x)^7+7/6*cos(x)^5+35/24*cos(x)^3+35/16*cos(x))*sin(x)+35/128
*x)+4*a*b^3*(1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+5/16*x)+6*b^2*
a^2*(1/4*(cos(x)^3+3/2*cos(x))*sin(x)+3/8*x)+4*a^3*b*(1/2*cos(x)*sin(x)+1/
2*x)+a^4*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int \left(a+b\cos^2(x)\right)^4 \, dx = \frac{1}{128} \left(128 \, a^4 + 256 \, a^3 b + 288 \, a^2 b^2 + 160 \, a b^3 + 35 \, b^4\right) x + \frac{1}{384} \left(48 \, b^4 \cos\left(x\right)^7 + 8 \left(32 \, a b^3 + 7 \, b^4\right) \cos\left(x\right)^5 + 2 \left(288 \, a^2 b^2 + 160 \, a b^3 + 35 \, b^4\right) \cos\left(x\right)^3 + 3 \left(256 \, a^3 b + \right.\right.$$

input
```
integrate((a+b*cos(x)^2)^4,x, algorithm="fricas")
```

output
```
1/128*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x + 1/384*(
48*b^4*cos(x)^7 + 8*(32*a*b^3 + 7*b^4)*cos(x)^5 + 2*(288*a^2*b^2 + 160*a*b
^3 + 35*b^4)*cos(x)^3 + 3*(256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*c
os(x))*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(146) = 292.

Time = 0.49 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.93

$$\int \left(a+b\cos^2(x)\right)^4 dx = a^4x + 2a^3bx\sin^2(x) + 2a^3bx\cos^2(x) + 2a^3b\sin(x)\cos(x) + \frac{9a^2b^2x\sin^4(x)}{4} + \frac{9a^2b^2x\sin^2(x)\cos^2(x)}{2} + \frac{9a^2b^2x\cos^4(x)}{4} + \frac{9a^2b^2\sin^3(x)\cos(x)}{4} + \frac{15a^2b^2\sin(x)\cos^3(x)}{4} + \frac{5ab^3x\sin^6(x)}{4} + \frac{15ab^3x\sin^4(x)\cos^2(x)}{4} + \frac{15ab^3x\sin^2(x)\cos^4(x)}{4} + \frac{5ab^3x\cos^6(x)}{4} + \frac{5ab^3\sin^5(x)\cos(x)}{4} + \frac{10ab^3\sin^3(x)\cos^3(x)}{3} + \frac{11ab^3\sin(x)\cos^5(x)}{4} + \frac{35b^4x\sin^8(x)}{128} + \frac{35b^4x\sin^6(x)\cos^2(x)}{32} + \frac{105b^4x\sin^4(x)\cos^4(x)}{64} + \frac{35b^4x\sin^2(x)\cos^6(x)}{32} + \frac{35b^4x\cos^8(x)}{128} + \frac{35b^4\sin^7(x)\cos(x)}{128} + \frac{385b^4\sin^5(x)\cos^3(x)}{384} + \frac{511b^4\sin^3(x)\cos^5(x)}{384} + \frac{93b^4\sin(x)\cos^7(x)}{128}$$

input
```
integrate((a+b*cos(x)**2)**4,x)
```

output
```
a**4*x + 2*a**3*b*x*sin(x)**2 + 2*a**3*b*x*cos(x)**2 + 2*a**3*b*sin(x)*cos
(x) + 9*a**2*b**2*x*sin(x)**4/4 + 9*a**2*b**2*x*sin(x)**2*cos(x)**2/2 + 9*
a**2*b**2*x*cos(x)**4/4 + 9*a**2*b**2*sin(x)**3*cos(x)/4 + 15*a**2*b**2*si
n(x)*cos(x)**3/4 + 5*a*b**3*x*sin(x)**6/4 + 15*a*b**3*x*sin(x)**4*cos(x)**
2/4 + 15*a*b**3*x*sin(x)**2*cos(x)**4/4 + 5*a*b**3*x*cos(x)**6/4 + 5*a*b**
3*sin(x)**5*cos(x)/4 + 10*a*b**3*sin(x)**3*cos(x)**3/3 + 11*a*b**3*sin(x)*
cos(x)**5/4 + 35*b**4*x*sin(x)**8/128 + 35*b**4*x*sin(x)**6*cos(x)**2/32 +
 105*b**4*x*sin(x)**4*cos(x)**4/64 + 35*b**4*x*sin(x)**2*cos(x)**6/32 + 35
*b**4*x*cos(x)**8/128 + 35*b**4*sin(x)**7*cos(x)/128 + 385*b**4*sin(x)**5*
cos(x)**3/384 + 511*b**4*sin(x)**3*cos(x)**5/384 + 93*b**4*sin(x)*cos(x)**
7/128
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76

$$\int \left(a+b\cos^2(x)\right)^4 \,dx$$
$$= -\frac{1}{48}\left(4\,\sin\left(2\,x\right)^3-60\,x-9\,\sin\left(4\,x\right)-48\,\sin\left(2\,x\right)\right)ab^3$$
$$-\frac{1}{3072}\left(128\,\sin\left(2\,x\right)^3-840\,x-3\,\sin\left(8\,x\right)-168\,\sin\left(4\,x\right)-768\,\sin\left(2\,x\right)\right)b^4$$
$$+\frac{3}{16}\,a^2b^2(12\,x+\sin\left(4\,x\right)+8\,\sin\left(2\,x\right))+a^3b(2\,x+\sin\left(2\,x\right))+a^4x$$

input
```
integrate((a+b*cos(x)^2)^4,x, algorithm="maxima")
```

output
```
-1/48*(4*sin(2*x)^3 - 60*x - 9*sin(4*x) - 48*sin(2*x))*a*b^3 - 1/3072*(128
*sin(2*x)^3 - 840*x - 3*sin(8*x) - 168*sin(4*x) - 768*sin(2*x))*b^4 + 3/16
*a^2*b^2*(12*x + sin(4*x) + 8*sin(2*x)) + a^3*b*(2*x + sin(2*x)) + a^4*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int \left(a+b\cos^2(x)\right)^4 \,dx = \frac{1}{1024}\,b^4\sin\left(8\,x\right)$$
$$+\frac{1}{128}\left(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4\right)x$$
$$+\frac{1}{96}\left(2\,ab^3+b^4\right)\sin\left(6\,x\right)$$
$$+\frac{1}{128}\left(24\,a^2b^2+24\,ab^3+7\,b^4\right)\sin\left(4\,x\right)$$
$$+\frac{1}{32}\left(32\,a^3b+48\,a^2b^2+30\,ab^3+7\,b^4\right)\sin\left(2\,x\right)$$

input
```
integrate((a+b*cos(x)^2)^4,x, algorithm="giac")
```

output
```
1/1024*b^4*sin(8*x) + 1/128*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3
 + 35*b^4)*x + 1/96*(2*a*b^3 + b^4)*sin(6*x) + 1/128*(24*a^2*b^2 + 24*a*b^
3 + 7*b^4)*sin(4*x) + 1/32*(32*a^3*b + 48*a^2*b^2 + 30*a*b^3 + 7*b^4)*sin(
2*x)
```

Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

$$\int \left(a+b\cos^2(x)\right)^4\,dx = x\,a^4+2\,\sin(x)\,a^3\,b\,\cos(x)+2\,x\,a^3\,b+\frac{3\,\sin(x)\,a^2\,b^2\,\cos(x)^3}{2}+\frac{9\,\sin(x)\,a^2\,b^2\,\cos(x)}{4}+\frac{9\,x\,a^2\,b^2}{4}+\frac{2\,\sin(x)\,a\,b^3\,\cos(x)^5}{3}+\frac{5\,\sin(x)\,a\,b^3\,\cos(x)^3}{6}+\frac{5\,\sin(x)\,a\,b^3\,\cos(x)}{4}+\frac{5\,x\,a\,b^3}{4}+\frac{\sin(x)\,b^4\,\cos(x)^7}{8}+\frac{7\,\sin(x)\,b^4\,\cos(x)^5}{48}+\frac{35\,\sin(x)\,b^4\,\cos(x)^3}{192}+\frac{35\,\sin(x)\,b^4\,\cos(x)}{128}+\frac{35\,x\,b^4}{128}$$

input
```
int((a + b*cos(x)^2)^4,x)
```

output
```
a^4*x + (35*b^4*x)/128 + (35*b^4*cos(x)^3*sin(x))/192 + (7*b^4*cos(x)^5*si
n(x))/48 + (b^4*cos(x)^7*sin(x))/8 + (9*a^2*b^2*x)/4 + (35*b^4*cos(x)*sin(
x))/128 + (5*a*b^3*x)/4 + 2*a^3*b*x + (3*a^2*b^2*cos(x)^3*sin(x))/2 + (5*a
*b^3*cos(x)*sin(x))/4 + 2*a^3*b*cos(x)*sin(x) + (9*a^2*b^2*cos(x)*sin(x))/
4 + (5*a*b^3*cos(x)^3*sin(x))/6 + (2*a*b^3*cos(x)^5*sin(x))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

$$\int \left(a+b\cos^2(x)\right)^4\,dx=-\frac{\cos(x)\sin(x)^7 b^4}{8}+\frac{2\cos(x)\sin(x)^5 a\,b^3}{3}+\frac{25\cos(x)\sin(x)^5 b^4}{48}-\frac{3\cos(x)\sin(x)^3 a^2b^2}{2}-\frac{13\cos(x)\sin(x)^3 a\,b^3}{6}-\frac{163\cos(x)\sin(x)^3 b^4}{192}+2\cos(x)\sin(x)\,a^3b+\frac{15\cos(x)\sin(x)\,a^2b^2}{4}+\frac{11\cos(x)\sin(x)\,a\,b^3}{4}+\frac{93\cos(x)\sin(x)\,b^4}{128}+a^4x+2a^3bx+\frac{9a^2b^2x}{4}+\frac{5a\,b^3x}{4}+\frac{35b^4x}{128}$$

input
```
int((a+b*cos(x)^2)^4,x)
```

output
```
( - 48*cos(x)*sin(x)**7*b**4 + 256*cos(x)*sin(x)**5*a*b**3 + 200*cos(x)*si
n(x)**5*b**4 - 576*cos(x)*sin(x)**3*a**2*b**2 - 832*cos(x)*sin(x)**3*a*b**
3 - 326*cos(x)*sin(x)**3*b**4 + 768*cos(x)*sin(x)*a**3*b + 1440*cos(x)*sin
(x)*a**2*b**2 + 1056*cos(x)*sin(x)*a*b**3 + 279*cos(x)*sin(x)*b**4 + 384*a
**4*x + 768*a**3*b*x + 864*a**2*b**2*x + 480*a*b**3*x + 105*b**4*x)/384
```

3.60 $\int (a+b\cos^2(x))^3\,dx$

Optimal result 456
Mathematica [C] (verified) 456
Rubi [A] (verified) 457
Maple [A] (verified) 458
Fricas [A] (verification not implemented) 459
Sympy [B] (verification not implemented) 460
Maxima [A] (verification not implemented) 460
Giac [A] (verification not implemented) 461
Mupad [B] (verification not implemented) 461
Reduce [B] (verification not implemented) 462

Optimal result

Integrand size = 10, antiderivative size = 87

$$\int (a+b\cos^2(x))^3\,dx = \frac{1}{16}(2a+b)\left(8a^2+8ab+5b^2\right)x + \frac{1}{48}b\left(64a^2+54ab+15b^2\right)\cos(x)\sin(x) + \frac{5}{24}b^2(2a+b)\cos^3(x)\sin(x) + \frac{1}{6}b\cos(x)\left(a+b\cos^2(x)\right)^2\sin(x)$$

output
```
1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*x+1/48*b*(64*a^2+54*a*b+15*b^2)*cos(x)*si
n(x)+5/24*b^2*(2*a+b)*cos(x)^3*sin(x)+1/6*b*cos(x)*(a+b*cos(x)^2)^2*sin(x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int (a+b\cos^2(x))^3\,dx = \frac{1}{192}\left(12(2a+b)\left(8a^2+8ab+5b^2\right)x + 9b(4a+(2-i)b)(4a+(2+i)b)\sin(2x) + 9b^2(2a+b)\sin(4x)+b^3\sin(6x)\right)$$

input
```
Integrate[(a + b*Cos[x]^2)^3,x]
```

output
```
(12*(2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*x + 9*b*(4*a + (2 - I)*b)*(4*a + (2
+ I)*b)*Sin[2*x] + 9*b^2*(2*a + b)*Sin[4*x] + b^3*Sin[6*x])/192
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3659, 3042, 3648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cos^2(x)\right)^3 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^3 dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{6}\int \left(b\cos^2(x)+a\right)\left(5b(2a+b)\cos^2(x)+a(6a+b)\right)dx+\frac{1}{6}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \left(b\sin\left(x+\frac{\pi}{2}\right)^2+a\right)\left(5b(2a+b)\sin\left(x+\frac{\pi}{2}\right)^2+a(6a+b)\right)dx+\frac{1}{6}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^2$$

$$\Big\downarrow \text{3648}$$

$$\frac{1}{6}\left(\frac{3}{8}x(2a+b)\left(8a^2+8ab+5b^2\right)+\frac{1}{8}b\left(64a^2+54ab+15b^2\right)\sin(x)\cos(x)+\frac{5}{4}b^2(2a+b)\sin(x)\cos^3(x)\right)+\frac{1}{6}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^2$$

input
```
Int[(a + b*Cos[x]^2)^3,x]
```

output
```
(b*Cos[x]*(a + b*Cos[x]^2)^2*Sin[x])/6 + ((3*(2*a + b)*(8*a^2 + 8*a*b + 5*
b^2)*x)/8 + (b*(64*a^2 + 54*a*b + 15*b^2)*Cos[x]*Sin[x])/8 + (5*b^2*(2*a +
 b)*Cos[x]^3*Sin[x])/4)/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3648
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(4*A*(2*a + b) + B*(4*a + 3*b))*(x/8), x] +
(-Simp[b*B*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)), x] - Simp[(4*A*b + B*(4*a +
 3*b))*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{3(16a^2b+16a\,b^2+5b^3)\sin(2x)}{64}+\frac{3(2a\,b^2+b^3)\sin(4x)}{64}+\frac{b^3\sin(6x)}{192}+x\left(a+\frac{b}{2}\right)\left(a^2+ba+\frac{5}{8}b^2\right)$
default	$b^3\left(\frac{\left(\cos(x)^5+\frac{5\cos(x)^3}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+\frac{5x}{16}\right)+3a\,b^2\left(\frac{\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{3x}{8}\right)+3a^2b\left(\frac{\cos(x)\,\mathrm{si}}{2}\right.$
parts	$b^3\left(\frac{\left(\cos(x)^5+\frac{5\cos(x)^3}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+\frac{5x}{16}\right)+3a\,b^2\left(\frac{\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{3x}{8}\right)+3a^2b\left(\frac{\cos(x)\,\mathrm{si}}{2}\right.$
risch	$a^3x+\frac{3a^2bx}{2}+\frac{9a\,b^2x}{8}+\frac{5b^3x}{16}+\frac{b^3\sin(6x)}{192}+\frac{3\sin(4x)a\,b^2}{32}+\frac{3\sin(4x)b^3}{64}+\frac{3\sin(2x)a^2b}{4}+\frac{3\sin(2x)a\,b^2}{4}+15$
norman	$\frac{\left(-9a^2b-\frac{21}{4}a\,b^2+\frac{5}{24}b^3\right)\tan\left(\frac{x}{2}\right)^9+\left(-6a^2b-\frac{3}{2}a\,b^2-\frac{15}{4}b^3\right)\tan\left(\frac{x}{2}\right)^7+\left(-3a^2b-\frac{15}{4}a\,b^2-\frac{11}{8}b^3\right)\tan\left(\frac{x}{2}\right)^{11}+\left(3a^2b+\frac{15}{4}a\,b^2+\frac{11}{8}b^3\right.}{}$
orering	Expression too large to display

input
```
int((a+b*cos(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
3/64*(16*a^2*b+16*a*b^2+5*b^3)*sin(2*x)+3/64*(2*a*b^2+b^3)*sin(4*x)+1/192*
b^3*sin(6*x)+x*(a+1/2*b)*(a^2+b*a+5/8*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \left(a+b\cos^2(x)\right)^3\,dx=\frac{1}{16}\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)x+\frac{1}{48}\left(8\,b^3\cos(x)^5+2\left(18\,ab^2+5\,b^3\right)\cos(x)^3+3\left(24\,a^2b+18\,ab^2+5\,b^3\right)\cos(x)\right)\sin(x)$$

input
```
integrate((a+b*cos(x)^2)^3,x, algorithm="fricas")
```

output
```
1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*x + 1/48*(8*b^3*cos(x)^5 + 2*(
18*a*b^2 + 5*b^3)*cos(x)^3 + 3*(24*a^2*b + 18*a*b^2 + 5*b^3)*cos(x))*sin(x
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(88) = 176$.

Time = 0.25 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.83

$$\int \left(a + b\cos^2(x)\right)^3 \, dx = a^3x + \frac{3a^2bx\sin^2(x)}{2} + \frac{3a^2bx\cos^2(x)}{2} + \frac{3a^2b\sin(x)\cos(x)}{2} + \frac{9ab^2x\sin^4(x)}{8} + \frac{9ab^2x\sin^2(x)\cos^2(x)}{4} + \frac{9ab^2x\cos^4(x)}{8} + \frac{9ab^2\sin^3(x)\cos(x)}{8} + \frac{15ab^2\sin(x)\cos^3(x)}{8} + \frac{5b^3x\sin^6(x)}{16} + \frac{15b^3x\sin^4(x)\cos^2(x)}{16} + \frac{15b^3x\sin^2(x)\cos^4(x)}{16} + \frac{5b^3x\cos^6(x)}{16} + \frac{5b^3\sin^5(x)\cos(x)}{16} + \frac{5b^3\sin^3(x)\cos^3(x)}{6} + \frac{11b^3\sin(x)\cos^5(x)}{16}$$

input
```
integrate((a+b*cos(x)**2)**3,x)
```

output
```
a**3*x + 3*a**2*b*x*sin(x)**2/2 + 3*a**2*b*x*cos(x)**2/2 + 3*a**2*b*sin(x)
*cos(x)/2 + 9*a*b**2*x*sin(x)**4/8 + 9*a*b**2*x*sin(x)**2*cos(x)**2/4 + 9*
a*b**2*x*cos(x)**4/8 + 9*a*b**2*sin(x)**3*cos(x)/8 + 15*a*b**2*sin(x)*cos(
x)**3/8 + 5*b**3*x*sin(x)**6/16 + 15*b**3*x*sin(x)**4*cos(x)**2/16 + 15*b*
*3*x*sin(x)**2*cos(x)**4/16 + 5*b**3*x*cos(x)**6/16 + 5*b**3*sin(x)**5*cos
(x)/16 + 5*b**3*sin(x)**3*cos(x)**3/6 + 11*b**3*sin(x)*cos(x)**5/16
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int \left(a + b\cos^2(x)\right)^3 \, dx = -\frac{1}{192} \left(4 \sin(2x)^3 - 60x - 9 \sin(4x) - 48 \sin(2x)\right)b^3 + \frac{3}{32}ab^2(12x + \sin(4x) + 8 \sin(2x)) + \frac{3}{4}a^2b(2x + \sin(2x)) + a^3x$$

input
```
integrate((a+b*cos(x)^2)^3,x, algorithm="maxima")
```

output
```
-1/192*(4*sin(2*x)^3 - 60*x - 9*sin(4*x) - 48*sin(2*x))*b^3 + 3/32*a*b^2*(
12*x + sin(4*x) + 8*sin(2*x)) + 3/4*a^2*b*(2*x + sin(2*x)) + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \left(a+b\cos^2(x)\right)^3 \,dx = \frac{1}{192}\,b^3\sin\left(6\,x\right)+\frac{1}{16}\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)x + \frac{3}{64}\left(2\,ab^2+b^3\right)\sin\left(4\,x\right)+\frac{3}{64}\left(16\,a^2b+16\,ab^2+5\,b^3\right)\sin\left(2\,x\right)$$

input
```
integrate((a+b*cos(x)^2)^3,x, algorithm="giac")
```

output
```
1/192*b^3*sin(6*x) + 1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*x + 3/64*
(2*a*b^2 + b^3)*sin(4*x) + 3/64*(16*a^2*b + 16*a*b^2 + 5*b^3)*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.34

$$\int \left(a+b\cos^2(x)\right)^3 \,dx = a^3\,x+\frac{5\,b^3\,x}{16} + \frac{\left(72\,a^2\,b+54\,a\,b^2+15\,b^3\right)\,\tan(x)^5+\left(144\,a^2\,b+144\,a\,b^2+40\,b^3\right)\,\tan(x)^3+\left(72\,a^2\,b+90\,a\,b^2+33\,b^3\right)\,\mathrm{t}}{48\tan\left(x\right)^6+144\tan\left(x\right)^4+144\tan\left(x\right)^2+48} + \frac{9\,a\,b^2\,x}{8}+\frac{3\,a^2\,b\,x}{2}$$

input
```
int((a + b*cos(x)^2)^3,x)
```

output
```
a^3*x + (5*b^3*x)/16 + (tan(x)^5*(54*a*b^2 + 72*a^2*b + 15*b^3) + tan(x)^3
*(144*a*b^2 + 144*a^2*b + 40*b^3) + tan(x)*(90*a*b^2 + 72*a^2*b + 33*b^3))
/(144*tan(x)^2 + 144*tan(x)^4 + 48*tan(x)^6 + 48) + (9*a*b^2*x)/8 + (3*a^2
*b*x)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \left(a+b\cos^2(x)\right)^3\,dx = \frac{\cos(x)\sin(x)^5 b^3}{6} - \frac{3\cos(x)\sin(x)^3 a\,b^2}{4} - \frac{13\cos(x)\sin(x)^3 b^3}{24} + \frac{3\cos(x)\sin(x)\,a^2 b}{2} + \frac{15\cos(x)\sin(x)\,a\,b^2}{8} + \frac{11\cos(x)\sin(x)\,b^3}{16} + a^3x + \frac{3a^2bx}{2} + \frac{9a\,b^2x}{8} + \frac{5b^3x}{16}$$

input
```
int((a+b*cos(x)^2)^3,x)
```

output
```
(8*cos(x)*sin(x)**5*b**3 - 36*cos(x)*sin(x)**3*a*b**2 - 26*cos(x)*sin(x)**
3*b**3 + 72*cos(x)*sin(x)*a**2*b + 90*cos(x)*sin(x)*a*b**2 + 33*cos(x)*sin
(x)*b**3 + 48*a**3*x + 72*a**2*b*x + 54*a*b**2*x + 15*b**3*x)/48
```

3.61 $\int (a+b\cos^2(x))^2\, dx$

Optimal result 463
Mathematica [A] (verified) 463
Rubi [A] (verified) 464
Maple [A] (verified) 465
Fricas [A] (verification not implemented) 465
Sympy [B] (verification not implemented) 466
Maxima [A] (verification not implemented) 466
Giac [A] (verification not implemented) 467
Mupad [B] (verification not implemented) 467
Reduce [B] (verification not implemented) 467

Optimal result

Integrand size = 10, antiderivative size = 50

$$\int (a+b\cos^2(x))^2\, dx = \frac{1}{8}(8a^2+8ab+3b^2)\,x + \frac{1}{8}b(8a+3b)\cos(x)\sin(x) + \frac{1}{4}b^2\cos^3(x)\sin(x)$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*x+1/8*b*(8*a+3*b)*cos(x)*sin(x)+1/4*b^2*cos(x)^3*s
in(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int (a+b\cos^2(x))^2\, dx = \frac{1}{32}(4(8a^2+8ab+3b^2)\,x+8b(2a+b)\sin(2x)+b^2\sin(4x))$$

input
```
Integrate[(a + b*Cos[x]^2)^2,x]
```

output
```
(4*(8*a^2 + 8*a*b + 3*b^2)*x + 8*b*(2*a + b)*Sin[2*x] + b^2*Sin[4*x])/32
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3658}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cos^2(x)\right)^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^2dx$$

$$\Big\downarrow \text{ 3658}$$

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)+\frac{1}{8}b(8a+3b)\sin(x)\cos(x)+\frac{1}{4}b^2\sin(x)\cos^3(x)$$

input
```
Int[(a + b*Cos[x]^2)^2,x]
```

output
```
((8*a^2 + 8*a*b + 3*b^2)*x)/8 + (b*(8*a + 3*b)*Cos[x]*Sin[x])/8 + (b^2*Cos
[x]^3*Sin[x])/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3658
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[(8*a^2 +
8*a*b + 3*b^2)*(x/8), x] + (-Simp[b^2*Cos[e + f*x]*(Sin[e + f*x]^3/(4*f)),
x] - Simp[b*(8*a + 3*b)*Cos[e + f*x]*(Sin[e + f*x]/(8*f)), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{(2ba+b^2)\sin(2x)}{4}+\frac{b^2\sin(4x)}{32}+\left(a^2+ba+\frac{3}{8}b^2\right)x$
default	$b^2\left(\frac{\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{3x}{8}\right)+2ba\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+a^2x$
parts	$b^2\left(\frac{\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{3x}{8}\right)+2ba\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+a^2x$
risch	$a^2x+abx+\frac{3b^2x}{8}+\frac{b^2\sin(4x)}{32}+\frac{ab\sin(2x)}{2}+\frac{b^2\sin(2x)}{4}$
orering	$x\left(a+b\cos(x)^2\right)^2+\left(a+b\cos(x)^2\right)b\cos(x)\sin(x)+\frac{5x(8b^2\cos(x)^2\sin(x)^2+4(a+b\cos(x)^2)b\sin(x)^2}{16}$
norman	$\frac{(-2ba-\frac{5}{4}b^2)\tan(\frac{x}{2})^7+(-2ba+\frac{3}{4}b^2)\tan(\frac{x}{2})^5+(2ba-\frac{3}{4}b^2)\tan(\frac{x}{2})^3+(2ba+\frac{5}{4}b^2)\tan(\frac{x}{2})+(a^2+ba+\frac{3}{8}b^2)x+(a^2+ba+\frac{3}{8}b^2)x}{(1+\tan(\frac{x}{2})^2)^4}$

input
```
int((a+b*cos(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*a*b+b^2)*sin(2*x)+1/32*b^2*sin(4*x)+(a^2+b*a+3/8*b^2)*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \left(a+b\cos^2(x)\right)^2\,dx=\frac{1}{8}\left(8a^2+8ab+3b^2\right)x+\frac{1}{8}\left(2b^2\cos(x)^3+\left(8ab+3b^2\right)\cos(x)\right)\sin(x)$$

input
```
integrate((a+b*cos(x)^2)^2,x, algorithm="fricas")
```

output
```
1/8*(8*a^2 + 8*a*b + 3*b^2)*x + 1/8*(2*b^2*cos(x)^3 + (8*a*b + 3*b^2)*cos(
x))*sin(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(44) = 88$.

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.20

$$\int \left(a+b\cos^2(x)\right)^2\,dx = a^2x + abx\sin^2(x) + abx\cos^2(x) + ab\sin(x)\cos(x) + \frac{3b^2x\sin^4(x)}{8} + \frac{3b^2x\sin^2(x)\cos^2(x)}{4} + \frac{3b^2x\cos^4(x)}{8} + \frac{3b^2\sin^3(x)\cos(x)}{8} + \frac{5b^2\sin(x)\cos^3(x)}{8}$$

input
```
integrate((a+b*cos(x)**2)**2,x)
```

output
```
a**2*x + a*b*x*sin(x)**2 + a*b*x*cos(x)**2 + a*b*sin(x)*cos(x) + 3*b**2*x*
sin(x)**4/8 + 3*b**2*x*sin(x)**2*cos(x)**2/4 + 3*b**2*x*cos(x)**4/8 + 3*b*
*2*sin(x)**3*cos(x)/8 + 5*b**2*sin(x)*cos(x)**3/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \left(a+b\cos^2(x)\right)^2\,dx = \frac{1}{32}\,b^2(12\,x + \sin(4\,x) + 8\,\sin(2\,x)) + \frac{1}{2}\,ab(2\,x + \sin(2\,x)) + a^2x$$

input
```
integrate((a+b*cos(x)^2)^2,x, algorithm="maxima")
```

output
```
1/32*b^2*(12*x + sin(4*x) + 8*sin(2*x)) + 1/2*a*b*(2*x + sin(2*x)) + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \left(a+b\cos^2(x)\right)^2 dx = \frac{1}{32}b^2\sin(4x) + \frac{1}{8}\left(8a^2+8ab+3b^2\right)x + \frac{1}{4}\left(2ab+b^2\right)\sin(2x)$$

input
```
integrate((a+b*cos(x)^2)^2,x, algorithm="giac")
```

output
```
1/32*b^2*sin(4*x) + 1/8*(8*a^2 + 8*a*b + 3*b^2)*x + 1/4*(2*a*b + b^2)*sin(
2*x)
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \left(a+b\cos^2(x)\right)^2 dx = x\,a^2 + \sin(x)\;a\,b\;\cos(x) + x\,a\,b + \frac{\sin(x)\;b^2\cos(x)^3}{4} + \frac{3\;\sin(x)\;b^2\;\cos(x)}{8} + \frac{3\,x\,b^2}{8}$$

input
```
int((a + b*cos(x)^2)^2,x)
```

output
```
a^2*x + (3*b^2*x)/8 + (b^2*cos(x)^3*sin(x))/4 + a*b*x + (3*b^2*cos(x)*sin(
x))/8 + a*b*cos(x)*sin(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \left(a+b\cos^2(x)\right)^2 dx = -\frac{\cos(x)\sin(x)^3 b^2}{4} + \cos(x)\sin(x)\,ab + \frac{5\cos(x)\sin(x)\,b^2}{8} + a^2x + abx + \frac{3b^2x}{8}$$

input
```
int((a+b*cos(x)^2)^2,x)
```

output
```
( - 2*cos(x)*sin(x)**3*b**2 + 8*cos(x)*sin(x)*a*b + 5*cos(x)*sin(x)*b**2 +
 8*a**2*x + 8*a*b*x + 3*b**2*x)/8
```

3.62 $\int (a+b\cos^2(x))\,dx$

Optimal result 469
Mathematica [A] (verified) 469
Rubi [A] (verified) 470
Maple [A] (verified) 471
Fricas [A] (verification not implemented) 471
Sympy [A] (verification not implemented) 472
Maxima [A] (verification not implemented) 472
Giac [A] (verification not implemented) 472
Mupad [B] (verification not implemented) 473
Reduce [B] (verification not implemented) 473

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int (a+b\cos^2(x))\,dx = ax+\frac{bx}{2}+\frac{1}{2}b\cos(x)\sin(x)$$

output
```
a*x+1/2*b*x+1/2*b*cos(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (a+b\cos^2(x))\,dx = ax+\frac{bx}{2}+\frac{1}{4}b\sin(2x)$$

input
```
Integrate[a + b*Cos[x]^2,x]
```

output
```
a*x + (b*x)/2 + (b*Sin[2*x])/4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cos^2(x)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{bx}{2}+\frac{1}{2}b\sin(x)\cos(x)$$

input
```
Int[a + b*Cos[x]^2,x]
```

output
```
a*x + (b*x)/2 + (b*Cos[x]*Sin[x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
risch	$ax+\frac{bx}{2}+\frac{b\sin(2x)}{4}$	16
default	$ax+b\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)$	17
parallelrisch	$b\left(\frac{\sin(2x)}{4}+\frac{x}{2}\right)+ax$	17
parts	$ax+b\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)$	17
orering	$x\left(a+b\cos\left(x\right)^{2}\right)+\frac{b\cos(x)\sin(x)}{2}+\frac{x\left(2b\sin(x)^{2}-2b\cos(x)^{2}\right)}{4}$	37
norman	$\frac{b\tan\left(\frac{x}{2}\right)+\left(a+\frac{b}{2}\right)x+\left(a+\frac{b}{2}\right)x\tan\left(\frac{x}{2}\right)^{4}+(2a+b)x\tan\left(\frac{x}{2}\right)^{2}-b\tan\left(\frac{x}{2}\right)^{3}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$	61

input
```
int(a+b*cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*x+1/4*b*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \left(a+b\cos^2(x)\right)\,dx=\frac{1}{2}\,b\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,(2\,a+b)x$$

input
```
integrate(a+b*cos(x)^2,x, algorithm="fricas")
```

output
```
1/2*b*cos(x)*sin(x) + 1/2*(2*a + b)*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\cos^2(x)\right) \, dx = ax + b\left(\frac{x}{2} + \frac{\sin(x)\cos(x)}{2}\right)$$

input
```
integrate(a+b*cos(x)**2,x)
```

output
```
a*x + b*(x/2 + sin(x)*cos(x)/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\cos^2(x)\right) \, dx = \frac{1}{4}b(2x + \sin(2x)) + ax$$

input
```
integrate(a+b*cos(x)^2,x, algorithm="maxima")
```

output
```
1/4*b*(2*x + sin(2*x)) + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\cos^2(x)\right) \, dx = \frac{1}{4}b(2x + \sin(2x)) + ax$$

input
```
integrate(a+b*cos(x)^2,x, algorithm="giac")
```

output
```
1/4*b*(2*x + sin(2*x)) + a*x
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int (a + b\cos^2(x))\ dx = \frac{b\ \sin(2\,x)}{4} + x\left(a + \frac{b}{2}\right)$$

input
```
int(a + b*cos(x)^2,x)
```

output
```
(b*sin(2*x))/4 + x*(a + b/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int (a + b\cos^2(x))\ dx = \frac{\cos(x)\sin(x)\,b}{2} + ax + \frac{bx}{2}$$

input
```
int(a+b*cos(x)^2,x)
```

output
```
(cos(x)*sin(x)*b + 2*a*x + b*x)/2
```

3.63 $\int \frac{1}{a+b\cos^2(x)}\,dx$

Optimal result 474
Mathematica [A] (verified) 474
Rubi [A] (verified) 475
Maple [A] (verified) 476
Fricas [B] (verification not implemented) 476
Sympy [B] (verification not implemented) 477
Maxima [A] (verification not implemented) 478
Giac [A] (verification not implemented) 478
Mupad [B] (verification not implemented) 478
Reduce [B] (verification not implemented) 479

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{1}{a+b\cos^2(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

output
```
-arctan((a+b)^(1/2)*cot(x)/a^(1/2))/a^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3660}$$

$$-\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)$$

$$\Big\downarrow \text{218}$$

$$-\frac{\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Int[(a + b*Cos[x]^2)^(-1),x]
```

output
```
-(ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$	21
risch	$-\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ba}+b\sqrt{-a^2-ba}}{b\sqrt{-a^2-ba}}\right)}{2\sqrt{-a^2-ba}}+\frac{\ln\left(e^{2ix}-\frac{2ia^2+2iab-2a\sqrt{-a^2-ba}-b\sqrt{-a^2-ba}}{b\sqrt{-a^2-ba}}\right)}{2\sqrt{-a^2-ba}}$	160

input
```
int(1/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 5.43

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2+4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)}{4(a^2+ab)}, -\frac{\arctan\left(\frac{(2a+b)\cos(x)^2-a}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)}{2\sqrt{a^2+ab}}\right]$$

input
```
integrate(1/(a+b*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*
a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x)
+ a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))/(a^2 + a*b), -1/2*arctan(1/2
*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))/sqrt(a^2 + a*b)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10924 vs. $2(29) = 58$.

Time = 14.64 (sec) , antiderivative size = 10924, normalized size of antiderivative = 364.13

$$\int \frac{1}{a + b\cos^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)**2),x)
```

output
```
Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (-tan(x/2
)/(2*b) + 1/(2*b*tan(x/2)), Eq(a, -b)), (-2*tan(x/2)/(b*(tan(x/2)**2 - 1))
, Eq(a, 0)), (a**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*log
(-sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**4*
sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a
 + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a/(a + b) + b/(a + b) - 2*
sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) -
8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt
(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(-a/(a
+ b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*s
qrt(-a*b)/(a + b)) + 2*a*b**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(
a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 8*a*b**2*sqr
t(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b
) + b/(a + b) + 2*sqrt(-a*b)/(a + b))) - a**3*sqrt(-a/(a + b) + b/(a + b)
- 2*sqrt(-a*b)/(a + b))*log(sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a
+ b)) + tan(x/2))/(2*a**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a +
b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-
a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b)
+ 2*sqrt(-a*b)/(a + b)) - 8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) -
2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$$

input
```
integrate(1/(a+b*cos(x)^2),x, algorithm="maxima")
```

output
```
arctan(a*tan(x)/sqrt((a + b)*a))/sqrt((a + b)*a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}}$$

input
```
integrate(1/(a+b*cos(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))/sqrt(a^2
+ a*b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{a\tan(x)}{\sqrt{a^2+b\,a}}\right)}{\sqrt{a^2+b\,a}}$$

input
```
int(1/(a + b*cos(x)^2),x)
```

output
```
atan((a*tan(x))/(a*b + a^2)^(1/2))/(a*b + a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.80

$$\int \frac{1}{a+b\cos^2(x)}\,dx = \frac{\sqrt{a}\,\sqrt{a+b}\left(atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{x}{2}\right)-\sqrt{b}}{\sqrt{a}}\right)+atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{x}{2}\right)+\sqrt{b}}{\sqrt{a}}\right)\right)}{a\,(a+b)}$$

input
```
int(1/(a+b*cos(x)^2),x)
```

output
```
(sqrt(a)*sqrt(a + b)*(atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a)) + ata
n((sqrt(a + b)*tan(x/2) + sqrt(b))/sqrt(a))))/(a*(a + b))
```

3.64 $\int \frac{1}{\left(a+b\cos^2(x)\right)^2}\,dx$

Optimal result 480
Mathematica [A] (verified) 480
Rubi [A] (verified) 481
Maple [A] (verified) 483
Fricas [B] (verification not implemented) 483
Sympy [F(-1)] 484
Maxima [A] (verification not implemented) 484
Giac [A] (verification not implemented) 485
Mupad [B] (verification not implemented) 485
Reduce [B] (verification not implemented) 486

Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^2}\,dx=-\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\cos(x)\sin(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

output
```
-1/2*(2*a+b)*arctan((a+b)^(1/2)*cot(x)/a^(1/2))/a^(3/2)/(a+b)^(3/2)-1/2*b*
cos(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)
```

Mathematica [A] (verified)

Time = 5.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^2}\,dx=-\frac{(-2a-b)\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sin(2x)}{2a(a+b)(2a+b+b\cos(2x))}$$

input
```
Integrate[(a + b*Cos[x]^2)^(-2),x]
```

output
```
-1/2*((-2*a - b)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(3/2)*(a + b)^(3
/2)) - (b*Sin[2*x])/(2*a*(a + b)*(2*a + b + b*Cos[2*x]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 3663, 25, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos^2(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^2}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\frac{2a+b}{b\cos^2(x)+a}dx}{2a(a+b)}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2a+b}{b\cos^2(x)+a}dx}{2a(a+b)}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(2a+b)\int \frac{1}{b\cos^2(x)+a}dx}{2a(a+b)}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2a+b)\int \frac{1}{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx}{2a(a+b)}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

$$\Big\downarrow \text{3660}$$

$$-\frac{(2a+b)\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{2a(a+b)}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

input
```
Int[(a + b*Cos[x]^2)^(-2),x]
```

output
```
-1/2*((2*a + b)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(3/
2)) - (b*Cos[x]*Sin[x])/(2*a*(a + b)*(a + b*Cos[x]^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

method	result
default	$-\frac{b\tan(x)}{2(a+b)a\left(a\tan(x)^2+a+b\right)}+\frac{(2a+b)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{2(a+b)a\sqrt{(a+b)a}}$
risch	$-\frac{i(2a\,e^{2ix}+e^{2ix}b+b)}{(a+b)a(e^{4ix}b+4a\,e^{2ix}+2\,e^{2ix}b+b)}-\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ba}+b\sqrt{-a^2-ba}}{b\sqrt{-a^2-ba}}\right)}{2\sqrt{-a^2-ba}\,(a+b)}-\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ba}+b\sqrt{-}}{b\sqrt{-a^2-ba}}\right.}{4\sqrt{-a^2-ba}\,(a+b)a}$

input
```
int(1/(a+b*cos(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/(a+b)/a*tan(x)/(a*tan(x)^2+a+b)+1/2*(2*a+b)/(a+b)/a/((a+b)*a)^(1/2)
*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(53) = 106.

Time = 0.12 (sec) , antiderivative size = 326, normalized size of antiderivative = 5.02

$$\int\frac{1}{\left(a+b\cos^2(x)\right)^2}\,dx$$

$$=\left[-\frac{4\left(a^2b+ab^2\right)\cos(x)\sin(x)+\left(\left(2\,ab+b^2\right)\cos(x)^2+2\,a^2+ab\right)\sqrt{-a^2-ab}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\,(\cdots}{\cdots}\right.}{8\left(a^5+2\,a^4b+a^3b^2+\left(a^4b+2\,a^3b^2+a^2b^3\right)\cos(x\cdots\right.}\right.$$

$$\left.-\frac{2\left(a^2b+ab^2\right)\cos(x)\sin(x)+\left(\left(2\,ab+b^2\right)\cos(x)^2+2\,a^2+ab\right)\sqrt{a^2+ab}\arctan\left(\frac{(2\,a+b)\cos(x)^2-a}{2\,\sqrt{a^2+ab}\cos(x)\sin(x)}\right)}{4\left(a^5+2\,a^4b+a^3b^2+\left(a^4b+2\,a^3b^2+a^2b^3\right)\cos(x)^2\right)}\right]$$

input
```
integrate(1/(a+b*cos(x)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(a^2*b + a*b^2)*cos(x)*sin(x) + ((2*a*b + b^2)*cos(x)^2 + 2*a^2 +
 a*b)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*
a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x)
+ a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)))/(a^5 + 2*a^4*b + a^3*b^2 +
(a^4*b + 2*a^3*b^2 + a^2*b^3)*cos(x)^2), -1/4*(2*(a^2*b + a*b^2)*cos(x)*si
n(x) + ((2*a*b + b^2)*cos(x)^2 + 2*a^2 + a*b)*sqrt(a^2 + a*b)*arctan(1/2*(
(2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x))))/(a^5 + 2*a^4*b +
 a^3*b^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*cos(x)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*cos(x)**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^2}\,dx=-\frac{b\tan\left(x\right)}{2\left(a^3+2\,a^2b+ab^2+\left(a^3+a^2b\right)\tan\left(x\right)^2\right)}+\frac{\left(2\,a+b\right)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{2\,\sqrt{(a+b)a}\left(a^2+ab\right)}$$

input
```
integrate(1/(a+b*cos(x)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*b*tan(x)/(a^3 + 2*a^2*b + a*b^2 + (a^3 + a^2*b)*tan(x)^2) + 1/2*(2*a
+ b)*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + a*b))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^2}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(2\,a+b)}{2\,(a^2+ab)^{\frac{3}{2}}} - \frac{b\tan\left(x\right)}{2\,\left(a\tan\left(x\right)^2+a+b\right)(a^2+ab)}$$

input
```
integrate(1/(a+b*cos(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*(2*a
+ b)/(a^2 + a*b)^(3/2) - 1/2*b*tan(x)/((a*tan(x)^2 + a + b)*(a^2 + a*b))
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)\,(2\,a+b)}{2\,a^{3/2}\,(a+b)^{3/2}} - \frac{b\tan(x)}{2\,a\,\,(a+b)\,\,\left(a\tan\left(x\right)^2+a+b\right)}$$

input
```
int(1/(a + b*cos(x)^2)^2,x)
```

output
```
(atan((a^(1/2)*tan(x))/(a + b)^(1/2))*(2*a + b))/(2*a^(3/2)*(a + b)^(3/2))
 - (b*tan(x))/(2*a*(a + b)*(a + b + a*tan(x)^2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 386, normalized size of antiderivative = 5.94

$$\int \frac{1}{(a+b\cos^2(x))^2}\,dx$$

$$= \frac{2\sqrt{a}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{x}{2}\right)-\sqrt{b}}{\sqrt{a}}\right)\sin(x)^2\,ab+\sqrt{a}\,\sqrt{a+b}\,atan\left(\frac{\sqrt{a+b}\,\tan\left(\frac{x}{2}\right)-\sqrt{b}}{\sqrt{a}}\right)\sin(x)^2\,b^2-2\sqrt{a}\,\sqrt{a+}}{}$$

input
```
int(1/(a+b*cos(x)^2)^2,x)
```

output
```
(2*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*sin(
x)**2*a*b + sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt
(a))*sin(x)**2*b**2 - 2*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - s
qrt(b))/sqrt(a))*a**2 - 3*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) -
 sqrt(b))/sqrt(a))*a*b - sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) -
sqrt(b))/sqrt(a))*b**2 + 2*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2)
+ sqrt(b))/sqrt(a))*sin(x)**2*a*b + sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*
tan(x/2) + sqrt(b))/sqrt(a))*sin(x)**2*b**2 - 2*sqrt(a)*sqrt(a + b)*atan((
sqrt(a + b)*tan(x/2) + sqrt(b))/sqrt(a))*a**2 - 3*sqrt(a)*sqrt(a + b)*atan
((sqrt(a + b)*tan(x/2) + sqrt(b))/sqrt(a))*a*b - sqrt(a)*sqrt(a + b)*atan(
(sqrt(a + b)*tan(x/2) + sqrt(b))/sqrt(a))*b**2 + cos(x)*sin(x)*a**2*b + co
s(x)*sin(x)*a*b**2)/(2*a**2*(sin(x)**2*a**2*b + 2*sin(x)**2*a*b**2 + sin(x
)**2*b**3 - a**3 - 3*a**2*b - 3*a*b**2 - b**3))
```

3.65 $\int \frac{1}{\left(a+b\cos^2(x)\right)^3}\,dx$

Optimal result 487
Mathematica [A] (verified) 487
Rubi [A] (verified) 488
Maple [A] (verified) 490
Fricas [B] (verification not implemented) 491
Sympy [F(-1)] 492
Maxima [A] (verification not implemented) 492
Giac [A] (verification not implemented) 493
Mupad [B] (verification not implemented) 493
Reduce [B] (verification not implemented) 494

Optimal result

Integrand size = 10, antiderivative size = 107

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^3}\,dx = -\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}} - \frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2} - \frac{3b(2a+b)\cos(x)\sin(x)}{8a^2(a+b)^2\left(a+b\cos^2(x)\right)}$$

output
```
-1/8*(8*a^2+8*a*b+3*b^2)*arctan((a+b)^(1/2)*cot(x)/a^(1/2))/a^(5/2)/(a+b)^
(5/2)-1/4*b*cos(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)^2-3/8*b*(2*a+b)*cos(x)*si
n(x)/a^2/(a+b)^2/(a+b*cos(x)^2)
```

Mathematica [A] (verified)

Time = 5.64 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^3}\,dx = \frac{\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{\sqrt{a}b(16a^2+16ab+3b^2+3b(2a+b)\cos(2x))\sin(2x)}{(a+b)^2(2a+b+b\cos(2x))^2}}{8a^{5/2}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(-3),x]
```

output
```
(((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a + b)^(5
/2) - (Sqrt[a]*b*(16*a^2 + 16*a*b + 3*b^2 + 3*b*(2*a + b)*Cos[2*x])*Sin[2*
x])/((a + b)^2*(2*a + b + b*Cos[2*x])^2))/(8*a^(5/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos^2(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^3}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\frac{-2b\cos^2(x)+4a+3b}{(b\cos^2(x)+a)^2}dx}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-2b\cos^2(x)+4a+3b}{(b\cos^2(x)+a)^2}dx}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-2b\sin\left(x+\frac{\pi}{2}\right)^2+4a+3b}{\left(b\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^2}dx}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\int \frac{8a^2+8ba+3b^2}{b\cos^2(x)+a}dx}{2a(a+b)}-\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)(a+b\cos^2(x))}}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{b\cos^2(x)+a}dx}{2a(a+b)}-\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)(a+b\cos^2(x))}}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx}{2a(a+b)}-\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)(a+b\cos^2(x))}}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{3660}$$

$$\frac{-\frac{(8a^2+8ab+3b^2)\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{2a(a+b)}-\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)(a+b\cos^2(x))}}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{3b(2a+b)\sin(x)\cos(x)}{2a(a+b)(a+b\cos^2(x))}}{4a(a+b)}-\frac{b\sin(x)\cos(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2}$$

input

```
Int[(a + b*Cos[x]^2)^(-3),x]
```

output

```
-1/4*(b*Cos[x]*Sin[x])/(a*(a + b)*(a + b*Cos[x]^2)^2) + (-1/2*((8*a^2 + 8*
a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(3/2))
 - (3*b*(2*a + b)*Cos[x]*Sin[x])/(2*a*(a + b)*(a + b*Cos[x]^2)))/(4*a*(a +
 b))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

method	result
default	$\frac{-\frac{b(8a+5b)\tan(x)^3}{8a\left(a^2+2ba+b^2\right)}-\frac{(8a+3b)b\tan(x)}{8a^2(a+b)}}{\left(a\tan(x)^2+a+b\right)^2}+\frac{(8a^2+8ba+3b^2)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{8(a^2+2ba+b^2)a^2\sqrt{(a+b)a}}$
risch	$-\frac{i(8a^2b\,e^{6ix}+8a\,b^2e^{6ix}+3b^3e^{6ix}+48a^3e^{4ix}+72a^2b\,e^{4ix}+42a\,b^2e^{4ix}+9b^3e^{4ix}+40a^2b\,e^{2ix}+40a\,b^2e^{2ix}+9b^3e^{2ix}+6a\,b^2+3b^3)}{4(a+b)^2a^2(e^{4ix}b+4a\,e^{2ix}+2\,e^{2ix}b+b)^2}-$

input
```
int(1/(a+b*cos(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*tan(x)^3-1/8*(8*a+3*b)*b/a^2/(a+b)*tan
(x))/(a*tan(x)^2+a+b)^2+1/8*(8*a^2+8*a*b+3*b^2)/(a^2+2*a*b+b^2)/a^2/((a+b)
*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(93) = 186$.

Time = 0.13 (sec) , antiderivative size = 616, normalized size of antiderivative = 5.76

$$\int \frac{1}{(a+b\cos^2(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(x)^4 + 8*a^4 + 8*a^3*b + 3*a^2*
b^2 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(x)^2)*sqrt(-a^2 - a*b)*log(((8
*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 + 4*((2*a + b)*c
os(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*c
os(x)^2 + a^2)) + 4*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^3 + (8*a^4*b
 + 13*a^3*b^2 + 5*a^2*b^3)*cos(x))*sin(x))/(a^8 + 3*a^7*b + 3*a^6*b^2 + a^
5*b^3 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cos(x)^4 + 2*(a^7*b +
3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*cos(x)^2), -1/16*(((8*a^2*b^2 + 8*a*b^3 +
 3*b^4)*cos(x)^4 + 8*a^4 + 8*a^3*b + 3*a^2*b^2 + 2*(8*a^3*b + 8*a^2*b^2 +
3*a*b^3)*cos(x)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sq
rt(a^2 + a*b)*cos(x)*sin(x))) + 2*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x
)^3 + (8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos(x))*sin(x))/(a^8 + 3*a^7*b +
3*a^6*b^2 + a^5*b^3 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cos(x)^4
 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*cos(x)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(x)**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.74

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^3}\,dx = \frac{(8\,a^2+8\,ab+3\,b^2)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{8\,(a^4+2\,a^3b+a^2b^2)\sqrt{(a+b)a}} - \frac{(8\,a^2b+5\,ab^2)\tan(x)^3+(8\,a^2b+11\,ab^2+3\,b^3)\tan(x)}{8\,\left(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+(a^6+2\,a^5b+a^4b^2)\tan(x)^4+2\,(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\,\text{ta}\right.}$$

input
```
integrate(1/(a+b*cos(x)^2)^3,x, algorithm="maxima")
```

output
```
1/8*(8*a^2 + 8*a*b + 3*b^2)*arctan(a*tan(x)/sqrt((a + b)*a))/((a^4 + 2*a^3
*b + a^2*b^2)*sqrt((a + b)*a)) - 1/8*((8*a^2*b + 5*a*b^2)*tan(x)^3 + (8*a^
2*b + 11*a*b^2 + 3*b^3)*tan(x))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a
^2*b^4 + (a^6 + 2*a^5*b + a^4*b^2)*tan(x)^4 + 2*(a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*tan(x)^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.39

$$\int \frac{1}{(a+b\cos^2(x))^3}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(8\,a^2+8\,ab+3\,b^2)}{8\,(a^4+2\,a^3b+a^2b^2)\sqrt{a^2+ab}} - \frac{8\,a^2b\tan(x)^3+5\,ab^2\tan(x)^3+8\,a^2b\tan(x)+11\,ab^2\tan(x)+3\,b^3\tan(x)}{8\,(a^4+2\,a^3b+a^2b^2)\left(a\tan(x)^2+a+b\right)^2}$$

input
```
integrate(1/(a+b*cos(x)^2)^3,x, algorithm="giac")
```

output
```
1/8*(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*(8*a^
2 + 8*a*b + 3*b^2)/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a^2 + a*b)) - 1/8*(8*a^
2*b*tan(x)^3 + 5*a*b^2*tan(x)^3 + 8*a^2*b*tan(x) + 11*a*b^2*tan(x) + 3*b^3
*tan(x))/((a^4 + 2*a^3*b + a^2*b^2)*(a*tan(x)^2 + a + b)^2)
```

Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+b\cos^2(x))^3}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)\,(8\,a^2+8\,a\,b+3\,b^2)}{8\,a^{5/2}\,(a+b)^{5/2}} - \frac{\frac{\tan(x)\,(3\,b^2+8\,a\,b)}{8\,a^2\,(a+b)}+\frac{\tan(x)^3\,(5\,b^2+8\,a\,b)}{8\,a\,(a+b)^2}}{2\,a\,b+\tan(x)^2\,(2\,a^2+2\,b\,a)+a^2\tan(x)^4+a^2+b^2}$$

input
```
int(1/(a + b*cos(x)^2)^3,x)
```

output
```
(atan((a^(1/2)*tan(x))/(a + b)^(1/2))*(8*a*b + 8*a^2 + 3*b^2))/(8*a^(5/2)*
(a + b)^(5/2)) - ((tan(x)*(8*a*b + 3*b^2))/(8*a^2*(a + b)) + (tan(x)^3*(8*
a*b + 5*b^2))/(8*a*(a + b)^2))/(2*a*b + tan(x)^2*(2*a*b + 2*a^2) + a^2*tan
(x)^4 + a^2 + b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1025, normalized size of antiderivative = 9.58

$$\int \frac{1}{(a+b\cos^2(x))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*cos(x)^2)^3,x)
```

output
```
(8*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*sin(
x)**4*a**2*b**2 + 8*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(
b))/sqrt(a))*sin(x)**4*a*b**3 + 3*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*ta
n(x/2) - sqrt(b))/sqrt(a))*sin(x)**4*b**4 - 16*sqrt(a)*sqrt(a + b)*atan((s
qrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*sin(x)**2*a**3*b - 32*sqrt(a)*sqrt
(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*sin(x)**2*a**2*b**2
 - 22*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*s
in(x)**2*a*b**3 - 6*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(
b))/sqrt(a))*sin(x)**2*b**4 + 8*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(
x/2) - sqrt(b))/sqrt(a))*a**4 + 24*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*t
an(x/2) - sqrt(b))/sqrt(a))*a**3*b + 27*sqrt(a)*sqrt(a + b)*atan((sqrt(a +
 b)*tan(x/2) - sqrt(b))/sqrt(a))*a**2*b**2 + 14*sqrt(a)*sqrt(a + b)*atan((
sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*a*b**3 + 3*sqrt(a)*sqrt(a + b)*at
an((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*b**4 + 8*sqrt(a)*sqrt(a + b)*
atan((sqrt(a + b)*tan(x/2) + sqrt(b))/sqrt(a))*sin(x)**4*a**2*b**2 + 8*sqr
t(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) + sqrt(b))/sqrt(a))*sin(x)**4*
a*b**3 + 3*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) + sqrt(b))/sqrt(
a))*sin(x)**4*b**4 - 16*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) + s
qrt(b))/sqrt(a))*sin(x)**2*a**3*b - 32*sqrt(a)*sqrt(a + b)*atan((sqrt(a +
b)*tan(x/2) + sqrt(b))/sqrt(a))*sin(x)**2*a**2*b**2 - 22*sqrt(a)*sqrt(a...
```

3.66 $\int \frac{1}{(a+b\cos^2(x))^4}\,dx$

Optimal result 495
Mathematica [A] (verified) 496
Rubi [A] (verified) 496
Maple [A] (verified) 499
Fricas [B] (verification not implemented) 500
Sympy [F(-1)] 501
Maxima [B] (verification not implemented) 502
Giac [A] (verification not implemented) 502
Mupad [B] (verification not implemented) 503
Reduce [B] (verification not implemented) 503

Optimal result

Integrand size = 10, antiderivative size = 154

$$\int \frac{1}{(a+b\cos^2(x))^4}\,dx = -\frac{(2a+b)\,(8a^2+8ab+5b^2)\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^{7/2}} - \frac{b\cos(x)\sin(x)}{6a(a+b)\,(a+b\cos^2(x))^3} - \frac{5b(2a+b)\cos(x)\sin(x)}{24a^2(a+b)^2\,(a+b\cos^2(x))^2} - \frac{b(44a^2+44ab+15b^2)\cos(x)\sin(x)}{48a^3(a+b)^3\,(a+b\cos^2(x))}$$

output

```
-1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan((a+b)^(1/2)*cot(x)/a^(1/2))/a^(7/
2)/(a+b)^(7/2)-1/6*b*cos(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)^3-5/24*b*(2*a+b)
*cos(x)*sin(x)/a^2/(a+b)^2/(a+b*cos(x)^2)^2-1/48*b*(44*a^2+44*a*b+15*b^2)*
cos(x)*sin(x)/a^3/(a+b)^3/(a+b*cos(x)^2)
```

Mathematica [A] (verified)

Time = 6.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.09

$$\int \frac{1}{(a+b\cos^2(x))^4}\,dx = \frac{\frac{3(16a^3+24a^2b+18ab^2+5b^3)\arctan\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{(a+b)^{7/2}} - \frac{32a^{5/2}b\sin(2x)}{(a+b)(2a+b+b\cos(2x))^3} - \frac{20a^{3/2}b(2a+b)\sin(2x)}{(a+b)^2(2a+b+b\cos(2x))^2} - \frac{\sqrt{a}b(44a^2+44ab+15b^2)\sin(2}{(a+b)^3(2a+b+b\cos(2x))}}{48a^{7/2}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(-4),x]
```

output
```
((3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a
+ b]])/(a + b)^(7/2) - (32*a^(5/2)*b*Sin[2*x])/((a + b)*(2*a + b + b*Cos[2
*x])^3) - (20*a^(3/2)*b*(2*a + b)*Sin[2*x])/((a + b)^2*(2*a + b + b*Cos[2*
x])^2) - (Sqrt[a]*b*(44*a^2 + 44*a*b + 15*b^2)*Sin[2*x])/((a + b)^3*(2*a +
 b + b*Cos[2*x])))/(48*a^(7/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {3042, 3663, 25, 3042, 3652, 3042, 3652, 27, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos^2(x))^4}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^4}dx$$

$$\Big\downarrow \; 3663$$

$$-\frac{\int -\frac{-4b\cos^2(x)+6a+5b}{(b\cos^2(x)+a)^3}dx}{6a(a+b)} - \frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\cos^2(x)\right)^3}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{-4b\cos^2(x)+6a+5b}{(b\cos^2(x)+a)^3}dx}{6a(a+b)}-\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\cos^2(x)\right)^3}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{-4b\sin\left(x+\frac{\pi}{2}\right)^2+6a+5b}{\left(b\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^3}dx}{6a(a+b)}-\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\cos^2(x)\right)^3}$$

$$\Big\downarrow 3652$$

$$\frac{\frac{\int \frac{24a^2+34ba+15b^2-10b(2a+b)\cos^2(x)}{\left(b\cos^2(x)+a\right)^2}dx}{4a(a+b)}-\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)(a+b\cos^2(x))^2}}{6a(a+b)}-\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\cos^2(x)\right)^3}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\int \frac{24a^2+34ba+15b^2-10b(2a+b)\sin\left(x+\frac{\pi}{2}\right)^2}{\left(b\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^2}dx}{4a(a+b)}-\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)(a+b\cos^2(x))^2}}{6a(a+b)}-\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\cos^2(x)\right)^3}$$

$$\Big\downarrow 3652$$

$$\frac{\frac{\frac{\int \frac{3(2a+b)\left(8a^2+8ba+5b^2\right)}{b\cos^2(x)+a}dx}{2a(a+b)}-\frac{b\left(44a^2+44ab+15b^2\right)\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}}{4a(a+b)}-\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)(a+b\cos^2(x))^2}}{6a(a+b)}-\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\cos^2(x)\right)^3}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\frac{3(2a+b)\left(8a^2+8ab+5b^2\right)\int \frac{1}{b\cos^2(x)+a}dx}{2a(a+b)}-\frac{b\left(44a^2+44ab+15b^2\right)\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}}{4a(a+b)}-\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)(a+b\cos^2(x))^2}}{6a(a+b)}-\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\cos^2(x)\right)^3}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\frac{3(2a+b)\left(8a^2+8ab+5b^2\right)\int \frac{1}{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx}{2a(a+b)}-\frac{b\left(44a^2+44ab+15b^2\right)\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}}{4a(a+b)}-\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)(a+b\cos^2(x))^2}}{6a(a+b)}-\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\cos^2(x)\right)^3}$$

$$\Big\downarrow \ 3660$$

$$\frac{\frac{-\frac{3(2a+b)\left(8a^2+8ab+5b^2\right)\int \frac{1}{(a+b)\cot^2(x)+a}d\cot(x)}{2a(a+b)}-\frac{b\left(44a^2+44ab+15b^2\right)\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}}{4a(a+b)}-\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)(a+b\cos^2(x))^2}}{6a(a+b)}-\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\cos^2(x)\right)^3}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{-\frac{b\left(44a^2+44ab+15b^2\right)\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}-\frac{3(2a+b)\left(8a^2+8ab+5b^2\right)\arctan\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}}{4a(a+b)}-\frac{5b(2a+b)\sin(x)\cos(x)}{4a(a+b)(a+b\cos^2(x))^2}}{6a(a+b)}-\frac{b\sin(x)\cos(x)}{6a(a+b)\left(a+b\cos^2(x)\right)^3}$$

input
```
Int[(a + b*Cos[x]^2)^(-4),x]
```

output
```
-1/6*(b*Cos[x]*Sin[x])/(a*(a + b)*(a + b*Cos[x]^2)^3) + ((-5*b*(2*a + b)*C
os[x]*Sin[x])/(4*a*(a + b)*(a + b*Cos[x]^2)^2) + ((-3*(2*a + b)*(8*a^2 + 8
*a*b + 5*b^2)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/
2)) - (b*(44*a^2 + 44*a*b + 15*b^2)*Cos[x]*Sin[x])/(2*a*(a + b)*(a + b*Cos
[x]^2)))/(4*a*(a + b)))/(6*a*(a + b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.27

method	result
default	$\frac{-\frac{b\left(24a^2+30ba+11b^2\right)\tan(x)^5}{16a\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{\left(18a^2+18ba+5b^2\right)b\tan(x)^3}{6a^2\left(a^2+2ba+b^2\right)}-\frac{\left(24a^2+18ba+5b^2\right)b\tan(x)}{16a^3(a+b)}}{\left(a\tan(x)^2+a+b\right)^3}+\frac{(16a^3+24a^2b+18a\,b^2+5b^3)\arctan\left(\frac{a\tan(x}{\sqrt{(a+b}}\right.}{16a^3(a^3+3a^2b+3a\,b^2+b^3)\sqrt{(a+b)a}}$
risch	$-\frac{i(2592a^2b^3e^{4ix}+960a\,b^4e^{4ix}+480a^3b^2e^{2ix}+720a^2b^3e^{2ix}+390a\,b^4e^{2ix}+48a^3b^2e^{10ix}+420a\,b^4e^{8ix}+3520a^4b\,e^{6ix}+15b^5+150b^5e^{6}}{}$

input
```
int(1/(a+b*cos(x)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/16*b*(24*a^2+30*a*b+11*b^2)/a/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(x)^5-1/6*(
18*a^2+18*a*b+5*b^2)*b/a^2/(a^2+2*a*b+b^2)*tan(x)^3-1/16*(24*a^2+18*a*b+5*
b^2)*b/a^3/(a+b)*tan(x))/(a*tan(x)^2+a+b)^3+1/16*(16*a^3+24*a^2*b+18*a*b^2
+5*b^3)/a^3/(a^3+3*a^2*b+3*a*b^2+b^3)/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+
b)*a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(138) = 276$.

Time = 0.18 (sec) , antiderivative size = 994, normalized size of antiderivative = 6.45

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)^2)^4,x, algorithm="fricas")
```

output
```
[-1/192*(3*((16*a^3*b^3 + 24*a^2*b^4 + 18*a*b^5 + 5*b^6)*cos(x)^6 + 16*a^6
 + 24*a^5*b + 18*a^4*b^2 + 5*a^3*b^3 + 3*(16*a^4*b^2 + 24*a^3*b^3 + 18*a^2
*b^4 + 5*a*b^5)*cos(x)^4 + 3*(16*a^5*b + 24*a^4*b^2 + 18*a^3*b^3 + 5*a^2*b
^4)*cos(x)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*
a^2 + 3*a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)
*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) + 4*((44*a^4*b^3 + 8
8*a^3*b^4 + 59*a^2*b^5 + 15*a*b^6)*cos(x)^5 + 2*(54*a^5*b^2 + 113*a^4*b^3
+ 79*a^3*b^4 + 20*a^2*b^5)*cos(x)^3 + 3*(24*a^6*b + 54*a^5*b^2 + 41*a^4*b^
3 + 11*a^3*b^4)*cos(x))*sin(x))/(a^11 + 4*a^10*b + 6*a^9*b^2 + 4*a^8*b^3 +
 a^7*b^4 + (a^8*b^3 + 4*a^7*b^4 + 6*a^6*b^5 + 4*a^5*b^6 + a^4*b^7)*cos(x)^
6 + 3*(a^9*b^2 + 4*a^8*b^3 + 6*a^7*b^4 + 4*a^6*b^5 + a^5*b^6)*cos(x)^4 + 3
*(a^10*b + 4*a^9*b^2 + 6*a^8*b^3 + 4*a^7*b^4 + a^6*b^5)*cos(x)^2), -1/96*(
3*((16*a^3*b^3 + 24*a^2*b^4 + 18*a*b^5 + 5*b^6)*cos(x)^6 + 16*a^6 + 24*a^5
*b + 18*a^4*b^2 + 5*a^3*b^3 + 3*(16*a^4*b^2 + 24*a^3*b^3 + 18*a^2*b^4 + 5*
a*b^5)*cos(x)^4 + 3*(16*a^5*b + 24*a^4*b^2 + 18*a^3*b^3 + 5*a^2*b^4)*cos(x
)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*
cos(x)*sin(x))) + 2*((44*a^4*b^3 + 88*a^3*b^4 + 59*a^2*b^5 + 15*a*b^6)*cos
(x)^5 + 2*(54*a^5*b^2 + 113*a^4*b^3 + 79*a^3*b^4 + 20*a^2*b^5)*cos(x)^3 +
3*(24*a^6*b + 54*a^5*b^2 + 41*a^4*b^3 + 11*a^3*b^4)*cos(x))*sin(x))/(a^11
+ 4*a^10*b + 6*a^9*b^2 + 4*a^8*b^3 + a^7*b^4 + (a^8*b^3 + 4*a^7*b^4 + 6...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos^2(x))^4}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*cos(x)**2)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(138) = 276$.

Time = 0.13 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.23

$$\int \frac{1}{(a+b\cos^2(x))^4}\,dx = \frac{(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{16\,(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sqrt{(a+b)a}} - \frac{3\,(24\,a^4b+30\,a^3b^2+11\,a^2b^3)\tan(x)^5+8\,(18\,a^4b+36\,a^3b^2+23\,a^2b}{48\,(a^9+6\,a^8b+15\,a^7b^2+20\,a^6b^3+15\,a^5b^4+6\,a^4b^5+a^3b^6+(a^9+3\,a^8b+3\,a^7b^2+a^6b^3)\tan(x)^6+3}$$

input
```
integrate(1/(a+b*cos(x)^2)^4,x, algorithm="maxima")
```

output
```
1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*arctan(a*tan(x)/sqrt((a + b)*a
))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt((a + b)*a)) - 1/48*(3*(24*a
^4*b + 30*a^3*b^2 + 11*a^2*b^3)*tan(x)^5 + 8*(18*a^4*b + 36*a^3*b^2 + 23*a
^2*b^3 + 5*a*b^4)*tan(x)^3 + 3*(24*a^4*b + 66*a^3*b^2 + 65*a^2*b^3 + 28*a*
b^4 + 5*b^5)*tan(x))/(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4
 + 6*a^4*b^5 + a^3*b^6 + (a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*tan(x)^6 +
3*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*tan(x)^4 + 3*(a^9 + 5*
a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*tan(x)^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.63

$$\int \frac{1}{(a+b\cos^2(x))^4}\,dx$$
$$= \frac{(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{16\,(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sqrt{a^2+ab}} - \frac{72\,a^4b\tan(x)^5+90\,a^3b^2\tan(x)^5+33\,a^2b^3\tan(x)^5+144\,a^4b\tan(x)^3+288\,a^3b^2\tan(x)^3+184\,a^2b^3\,\mathrm{ta}}{48\,(a^6+3\,a^5b+3\,a^4b^2}$$

input
```
integrate(1/(a+b*cos(x)^2)^4,x, algorithm="giac")
```

output
```
1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*(pi*floor(x/pi + 1/2)*sgn(a) +
 arctan(a*tan(x)/sqrt(a^2 + a*b)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*
sqrt(a^2 + a*b)) - 1/48*(72*a^4*b*tan(x)^5 + 90*a^3*b^2*tan(x)^5 + 33*a^2*
b^3*tan(x)^5 + 144*a^4*b*tan(x)^3 + 288*a^3*b^2*tan(x)^3 + 184*a^2*b^3*tan
(x)^3 + 40*a*b^4*tan(x)^3 + 72*a^4*b*tan(x) + 198*a^3*b^2*tan(x) + 195*a^2
*b^3*tan(x) + 84*a*b^4*tan(x) + 15*b^5*tan(x))/((a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*(a*tan(x)^2 + a + b)^3)
```

Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.65

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^4}\,dx=\frac{\mathrm{atan}\left(\frac{\sqrt{a}\tan(x)\,(2\,a+b)\,(8\,a^2+8\,a\,b+5\,b^2)}{\sqrt{a+b}\,(16\,a^3+24\,a^2\,b+18\,a\,b^2+5\,b^3)}\right)\,(2\,a+b)\,\left(8\,a^2+8\,a\,b+5\,b^2\right)}{16\,a^{7/2}\,{(a+b)}^{7/2}}-\frac{\frac{\tan(x)\,(24\,a^2\,b+18\,a\,b^2+5\,b^3)}{16\,a^3\,(a+b)}+\frac{\tan(x)^3\,(18\,a^2\,b+18\,a\,b^2+5\,b^3)}{6\,a^2\,{(a+b)}^2}+\frac{\tan(x)^5\,(24\,a^2\,b+30\,a\,b^2+11\,b^3)}{16\,a\,{(a+b)}^3}}{{\tan\left(x\right)}^2\,\left(3\,a^3+6\,a^2\,b+3\,a\,b^2\right)+a^3\,{\tan\left(x\right)}^6+3\,a\,b^2+3\,a^2\,b+{\tan\left(x\right)}^4\,\left(3\,a^3+3\,b\,a^2\right)+a^3+b^3}$$

input
```
int(1/(a + b*cos(x)^2)^4,x)
```

output
```
(atan((a^(1/2)*tan(x)*(2*a + b)*(8*a*b + 8*a^2 + 5*b^2))/((a + b)^(1/2)*(1
8*a*b^2 + 24*a^2*b + 16*a^3 + 5*b^3)))*(2*a + b)*(8*a*b + 8*a^2 + 5*b^2))/
(16*a^(7/2)*(a + b)^(7/2)) - ((tan(x)*(18*a*b^2 + 24*a^2*b + 5*b^3))/(16*a
^3*(a + b)) + (tan(x)^3*(18*a*b^2 + 18*a^2*b + 5*b^3))/(6*a^2*(a + b)^2) +
 (tan(x)^5*(30*a*b^2 + 24*a^2*b + 11*b^3))/(16*a*(a + b)^3))/(tan(x)^2*(3*
a*b^2 + 6*a^2*b + 3*a^3) + a^3*tan(x)^6 + 3*a*b^2 + 3*a^2*b + tan(x)^4*(3*
a^2*b + 3*a^3) + a^3 + b^3)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1968, normalized size of antiderivative = 12.78

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^4}\,dx=\text{Too large to display}$$

input
```
int(1/(a+b*cos(x)^2)^4,x)
```

output
```
(48*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*sin
(x)**6*a**3*b**3 + 72*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqr
t(b))/sqrt(a))*sin(x)**6*a**2*b**4 + 54*sqrt(a)*sqrt(a + b)*atan((sqrt(a +
 b)*tan(x/2) - sqrt(b))/sqrt(a))*sin(x)**6*a*b**5 + 15*sqrt(a)*sqrt(a + b)
*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*sin(x)**6*b**6 - 144*sqrt(
a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*sin(x)**4*a*
*4*b**2 - 360*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sq
rt(a))*sin(x)**4*a**3*b**3 - 378*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan
(x/2) - sqrt(b))/sqrt(a))*sin(x)**4*a**2*b**4 - 207*sqrt(a)*sqrt(a + b)*at
an((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*sin(x)**4*a*b**5 - 45*sqrt(a)
*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*sin(x)**4*b**6
 + 144*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*
sin(x)**2*a**5*b + 504*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sq
rt(b))/sqrt(a))*sin(x)**2*a**4*b**2 + 738*sqrt(a)*sqrt(a + b)*atan((sqrt(a
 + b)*tan(x/2) - sqrt(b))/sqrt(a))*sin(x)**2*a**3*b**3 + 585*sqrt(a)*sqrt(
a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*sin(x)**2*a**2*b**4
+ 252*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt(b))/sqrt(a))*s
in(x)**2*a*b**5 + 45*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*tan(x/2) - sqrt
(b))/sqrt(a))*sin(x)**2*b**6 - 48*sqrt(a)*sqrt(a + b)*atan((sqrt(a + b)*ta
n(x/2) - sqrt(b))/sqrt(a))*a**6 - 216*sqrt(a)*sqrt(a + b)*atan((sqrt(a ...
```

3.67 $\int (a+b\cos^2(x))^{5/2}\,dx$

Optimal result 505
Mathematica [A] (verified) 506
Rubi [A] (verified) 506
Maple [B] (verified) 510
Fricas [F] 511
Sympy [F(-1)] 511
Maxima [F] 512
Giac [F] 512
Mupad [F(-1)] 512
Reduce [F] 513

Optimal result

Integrand size = 12, antiderivative size = 164

$$\int (a+b\cos^2(x))^{5/2}\,dx=\frac{(23a^2+23ab+8b^2)\sqrt{a+b\cos^2(x)}E\left(\frac{\pi}{2}+x|-\frac{b}{a}\right)}{15\sqrt{\frac{a+b\cos^2(x)}{a}}}-\frac{4a(a+b)(2a+b)\sqrt{\frac{a+b\cos^2(x)}{a}}\,\text{EllipticF}\left(\frac{\pi}{2}+x,-\frac{b}{a}\right)}{15\sqrt{a+b\cos^2(x)}}+\frac{4}{15}b(2a+b)\cos(x)\sqrt{a+b\cos^2(x)}\sin(x)+\frac{1}{5}b\cos(x)\left(a+b\cos^2(x)\right)^{3/2}\sin(x)$$

output

```
1/15*(23*a^2+23*a*b+8*b^2)*(a+b*cos(x)^2)^(1/2)*EllipticE(cos(x),(-b/a)^(1
/2))/((a+b*cos(x)^2)/a)^(1/2)-4/15*a*(a+b)*(2*a+b)*((a+b*cos(x)^2)/a)^(1/2
)*InverseJacobiAM(1/2*Pi+x,(-b/a)^(1/2))/(a+b*cos(x)^2)^(1/2)+4/15*b*(2*a+
b)*cos(x)*(a+b*cos(x)^2)^(1/2)*sin(x)+1/5*b*cos(x)*(a+b*cos(x)^2)^(3/2)*si
n(x)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02

$$\int (a+b\cos^2(x))^{5/2}\,dx = \frac{16(23a^3+46a^2b+31ab^2+8b^3)\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}E\left(x\middle|\frac{b}{a+b}\right)-64a(2a^2+3ab+b^2)\sqrt{\frac{2a+}{}}}{240\sqrt{}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(5/2),x]
```

output
```
(16*(23*a^3 + 46*a^2*b + 31*a*b^2 + 8*b^3)*Sqrt[(2*a + b + b*Cos[2*x])/(a
+ b)]*EllipticE[x, b/(a + b)] - 64*a*(2*a^2 + 3*a*b + b^2)*Sqrt[(2*a + b +
 b*Cos[2*x])/(a + b)]*EllipticF[x, b/(a + b)] + Sqrt[2]*b*(88*a^2 + 88*a*b
 + 25*b^2 + 28*b*(2*a + b)*Cos[2*x] + 3*b^2*Cos[4*x])*Sin[2*x])/(240*Sqrt[
2*a + b + b*Cos[2*x]])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {3042, 3659, 3042, 3649, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos^2(x))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{5}\int \sqrt{b\cos^2(x)+a}\,(4b(2a+b)\cos^2(x)+a(5a+b))\,dx+\frac{1}{5}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}\left(4b(2a+b)\sin\left(x+\frac{\pi}{2}\right)^2+a(5a+b)\right)dx+$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^{3/2}$$

$\Big\downarrow$ 3649

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{b\left(23a^2+23ba+8b^2\right)\cos^2(x)+a\left(15a^2+11ba+4b^2\right)}{\sqrt{b\cos^2(x)+a}}dx+\frac{4}{3}b(2a+b)\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}\right)+$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^{3/2}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{b\left(23a^2+23ba+8b^2\right)\sin\left(x+\frac{\pi}{2}\right)^2+a\left(15a^2+11ba+4b^2\right)}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx+\frac{4}{3}b(2a+b)\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^{3/2}$$

$\Big\downarrow$ 3651

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2+23ab+8b^2\right)\int \sqrt{b\cos^2(x)+a}dx-4a(a+b)(2a+b)\int \frac{1}{\sqrt{b\cos^2(x)+a}}dx\right)+\frac{4}{3}b(2a+b)\sin(x)\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^{3/2}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2+23ab+8b^2\right)\int \sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx-4a(a+b)(2a+b)\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+\frac{4}{3}b(2a\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^{3/2}$$

$\Big\downarrow$ 3657

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\cos^2(x)}{a}+1}dx}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-4a(a+b)(2a+b)\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^{3/2}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{(23a^2+23ab+8b^2)\sqrt{a+b\cos^2(x)}\int\sqrt{\frac{b\sin(x+\frac{\pi}{2})^2}{a}+1}dx}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-4a(a+b)(2a+b)\int\frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right.\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3656}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{(23a^2+23ab+8b^2)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-4a(a+b)(2a+b)\int\frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+\frac{4}{3}b\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3662}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{(23a^2+23ab+8b^2)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{4a(a+b)(2a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\int\frac{1}{\sqrt{\frac{b\cos^2(x)}{a}+1}}dx}{\sqrt{a+b\cos^2(x)}}\right)-\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{(23a^2+23ab+8b^2)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{4a(a+b)(2a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}}dx}{\sqrt{a+b\cos^2(x)}}\right.\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3661}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{(23a^2+23ab+8b^2)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{4a(a+b)(2a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\,\text{EllipticF}\left(x+\frac{\pi}{2},-\right.}{\sqrt{a+b\cos^2(x)}}\right.\right.$$
$$\frac{1}{5}b\sin(x)\cos(x)\left(a+b\cos^2(x)\right)^{3/2}$$

input
```
Int[(a + b*Cos[x]^2)^(5/2),x]
```

output
```
(b*Cos[x]*(a + b*Cos[x]^2)^(3/2)*Sin[x])/5 + ((((23*a^2 + 23*a*b + 8*b^2)*
Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/Sqrt[1 + (b*Cos[x]^2)/a]
 - (4*a*(a + b)*(2*a + b)*Sqrt[1 + (b*Cos[x]^2)/a]*EllipticF[Pi/2 + x, -(b
/a)])/Sqrt[a + b*Cos[x]^2])/3 + (4*b*(2*a + b)*Cos[x]*Sqrt[a + b*Cos[x]^2]
*Sin[x])/3)/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3649
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*Sin[e + f*x]*((a + b*
Sin[e + f*x]^2)^p/(2*f*(p + 1))), x] + Simp[1/(2*(p + 1))   Int[(a + b*Sin[
e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*
p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && G
tQ[p, 0]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(141) = 282$.

Time = 5.38 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.02

method	result
default	$-\frac{-\frac{b^{3}\cos(x)\sin(x)^{6}}{5}+\frac{\left(14a\,b^{2}+10b^{3}\right)\sin(x)^{4}\cos(x)}{15}+\frac{\left(-11a^{2}b-18a\,b^{2}-7b^{3}\right)\sin(x)^{2}\cos(x)}{15}-\frac{8a^{3}\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\frac{b\sin(x)^{2}}{a}+\frac{a+b}{a}}\ \mathrm{EllipticF}\left(\right.}{15}$

input
```
int((a+b*cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-1/5*b^3*cos(x)*sin(x)^6+1/15*(14*a*b^2+10*b^3)*sin(x)^4*cos(x)+1/15*(-1
1*a^2*b-18*a*b^2-7*b^3)*sin(x)^2*cos(x)-8/15*a^3*(sin(x)^2)^(1/2)*(-b/a*si
n(x)^2+(a+b)/a)^(1/2)*EllipticF(cos(x),(-b/a)^(1/2))-4/5*a^2*(sin(x)^2)^(1
/2)*(-b/a*sin(x)^2+(a+b)/a)^(1/2)*EllipticF(cos(x),(-b/a)^(1/2))*b-4/15*a*
(sin(x)^2)^(1/2)*(-b/a*sin(x)^2+(a+b)/a)^(1/2)*EllipticF(cos(x),(-b/a)^(1/
2))*b^2+23/15*(sin(x)^2)^(1/2)*(-b/a*sin(x)^2+(a+b)/a)^(1/2)*EllipticE(cos
(x),(-b/a)^(1/2))*a^3+23/15*(sin(x)^2)^(1/2)*(-b/a*sin(x)^2+(a+b)/a)^(1/2)
*EllipticE(cos(x),(-b/a)^(1/2))*a^2*b+8/15*(sin(x)^2)^(1/2)*(-b/a*sin(x)^2
+(a+b)/a)^(1/2)*EllipticE(cos(x),(-b/a)^(1/2))*a*b^2)/sin(x)/(a+b*cos(x)^2
)^(1/2)
```

Fricas [F]

$$\int \left(a+b\cos^2(x)\right)^{5/2}\,dx=\int \left(b\cos\left(x\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)*sqrt(b*cos(x)^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\cos^2(x)\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*cos(x)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\cos^2(x)\right)^{5/2}\,dx=\int \left(b\cos\left(x\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(x)^2 + a)^(5/2), x)
```

Giac [F]

$$\int \left(a+b\cos^2(x)\right)^{5/2}\,dx=\int \left(b\cos\left(x\right)^2+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(x)^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\cos^2(x)\right)^{5/2}\,dx=\int \left(b\cos\left(x\right)^2+a\right)^{5/2}\,dx$$

input
```
int((a + b*cos(x)^2)^(5/2),x)
```

output
```
int((a + b*cos(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \left(a+b\cos^2(x)\right)^{5/2}\,dx = \left(\int \sqrt{\cos\left(x\right)^2 b+a}dx\right)a^2 + \left(\int \sqrt{\cos\left(x\right)^2 b+a}\,\cos\left(x\right)^4 dx\right)b^2 + 2\left(\int \sqrt{\cos\left(x\right)^2 b+a}\,\cos\left(x\right)^2 dx\right)ab$$

input
```
int((a+b*cos(x)^2)^(5/2),x)
```

output
```
int(sqrt(cos(x)**2*b + a),x)*a**2 + int(sqrt(cos(x)**2*b + a)*cos(x)**4,x)
*b**2 + 2*int(sqrt(cos(x)**2*b + a)*cos(x)**2,x)*a*b
```

3.68 $\int (a+b\cos^2(x))^{3/2}\,dx$

Optimal result 514
Mathematica [A] (verified) 514
Rubi [A] (verified) 515
Maple [A] (verified) 518
Fricas [F] 519
Sympy [F(-1)] 519
Maxima [F] 519
Giac [F] 520
Mupad [F(-1)] 520
Reduce [F] 520

Optimal result

Integrand size = 12, antiderivative size = 123

$$\int (a+b\cos^2(x))^{3/2}\,dx = \frac{2(2a+b)\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{3\sqrt{\frac{a+b\cos^2(x)}{a}}} - \frac{a(a+b)\sqrt{\frac{a+b\cos^2(x)}{a}}\,\operatorname{EllipticF}\left(\frac{\pi}{2}+x,-\frac{b}{a}\right)}{3\sqrt{a+b\cos^2(x)}} + \frac{1}{3}b\cos(x)\sqrt{a+b\cos^2(x)}\sin(x)$$

output
```
2/3*(2*a+b)*(a+b*cos(x)^2)^(1/2)*EllipticE(cos(x),(-b/a)^(1/2))/((a+b*cos(
x)^2)/a)^(1/2)-1/3*a*(a+b)*((a+b*cos(x)^2)/a)^(1/2)*InverseJacobiAM(1/2*Pi
+x,(-b/a)^(1/2))/(a+b*cos(x)^2)^(1/2)+1/3*b*cos(x)*(a+b*cos(x)^2)^(1/2)*si
n(x)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00

$$\int (a+b\cos^2(x))^{3/2}\,dx = \frac{8(2a^2+3ab+b^2)\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}\,E\left(x\middle|\frac{b}{a+b}\right) - 4a(a+b)\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}\,\operatorname{EllipticF}\left(x,\frac{b}{a+b}\right.}{12\sqrt{2a+b+b\cos(2x)}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(3/2),x]
```

output
```
(8*(2*a^2 + 3*a*b + b^2)*Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticE[x,
 b/(a + b)] - 4*a*(a + b)*Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticF[x
, b/(a + b)] + Sqrt[2]*b*(2*a + b + b*Cos[2*x])*Sin[2*x])/(12*Sqrt[2*a + b
 + b*Cos[2*x]])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3659, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cos^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{2b(2a+b)\cos^2(x)+a(3a+b)}{\sqrt{b\cos^2(x)+a}}dx+\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{2b(2a+b)\sin\left(x+\frac{\pi}{2}\right)^2+a(3a+b)}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx+\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\cos^2(x)+a}dx-a(a+b)\int \frac{1}{\sqrt{b\cos^2(x)+a}}dx\right)+\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3657}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\cos^2(x)}{a}+1}dx}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}dx}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3656}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx\right)+\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3662}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\cos^2(x)}{a}+1}}dx}{\sqrt{a+b\cos^2(x)}}\right)+\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}}dx}{\sqrt{a+b\cos^2(x)}}\right)+\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}$$

$$\Big\downarrow \text{3661}$$

$$\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}+$$
$$\frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\,\text{EllipticF}\left(x+\frac{\pi}{2},-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}\right)$$

input
```
Int[(a + b*Cos[x]^2)^(3/2),x]
```

output
```
((2*(2*a + b)*Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/Sqrt[1 + (
b*Cos[x]^2)/a] - (a*(a + b)*Sqrt[1 + (b*Cos[x]^2)/a]*EllipticF[Pi/2 + x, -
(b/a)])/Sqrt[a + b*Cos[x]^2])/3 + (b*Cos[x]*Sqrt[a + b*Cos[x]^2]*Sin[x])/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.56

method	result
default	$-\frac{-\frac{a^{2}\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\cos(x)^{2}}{a}}\,\operatorname{EllipticF}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)}{3}-\frac{a\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\cos(x)^{2}}{a}}\,\operatorname{EllipticF}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)b}{3}+\frac{4\sqrt{\frac{a+b\cos(x)^{2}}{a}}\,\operatorname{Ellip}}{}}{\sin(x)\sqrt{}}$

input
```
int((a+b*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-1/3*a^2*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x),(-b/
a)^(1/2))-1/3*a*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x)
,(-b/a)^(1/2))*b+4/3*((a+b*cos(x)^2)/a)^(1/2)*EllipticE(cos(x),(-b/a)^(1/2
))*(sin(x)^2)^(1/2)*a^2+2/3*((a+b*cos(x)^2)/a)^(1/2)*EllipticE(cos(x),(-b/
a)^(1/2))*(sin(x)^2)^(1/2)*a*b+1/3*b^2*cos(x)^5+1/3*a*b*cos(x)^3-1/3*b^2*c
os(x)^3-1/3*cos(x)*a*b)/sin(x)/(a+b*cos(x)^2)^(1/2)
```

Fricas [F]

$$\int \left(a+b\cos^2(x)\right)^{3/2} dx = \int \left(b\cos{(x)}^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(x)^2 + a)^(3/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\cos^2(x)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a+b*cos(x)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\cos^2(x)\right)^{3/2} dx = \int \left(b\cos{(x)}^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(x)^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+b\cos^2(x)\right)^{3/2}\,dx=\int \left(b\cos\left(x\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(x)^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\cos^2(x)\right)^{3/2}\,dx=\int \left(b\cos\left(x\right)^2+a\right)^{3/2}\,dx$$

input
```
int((a + b*cos(x)^2)^(3/2),x)
```

output
```
int((a + b*cos(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\cos^2(x)\right)^{3/2}\,dx=\left(\int \sqrt{\cos\left(x\right)^2 b+a}dx\right)a+\left(\int \sqrt{\cos\left(x\right)^2 b+a}\,\cos\left(x\right)^2dx\right)b$$

input
```
int((a+b*cos(x)^2)^(3/2),x)
```

output
```
int(sqrt(cos(x)**2*b + a),x)*a + int(sqrt(cos(x)**2*b + a)*cos(x)**2,x)*b
```

3.69 $\int \sqrt{a+b\cos^2(x)}\,dx$

Optimal result 521
Mathematica [A] (verified) 521
Rubi [A] (verified) 522
Maple [A] (verified) 523
Fricas [F] 523
Sympy [F] 524
Maxima [F] 524
Giac [F] 524
Mupad [F(-1)] 525
Reduce [F] 525

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \sqrt{a+b\cos^2(x)}\,dx=\frac{\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{\sqrt{\frac{a+b\cos^2(x)}{a}}}$$

output
```
(a+b*cos(x)^2)^(1/2)*EllipticE(cos(x),(-b/a)^(1/2))/((a+b*cos(x)^2)/a)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \sqrt{a+b\cos^2(x)}\,dx=\frac{\sqrt{2a+b+b\cos(2x)}\,E\left(x\middle|\frac{b}{a+b}\right)}{\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}}$$

input
```
Integrate[Sqrt[a + b*Cos[x]^2],x]
```

output
```
(Sqrt[2*a + b + b*Cos[2*x]]*EllipticE[x, b/(a + b)])/Sqrt[(2*a + b + b*Cos
[2*x])/(a + b)]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cos^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\cos^2(x)}{a}+1}dx}{\sqrt{\frac{b\cos^2(x)}{a}+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}dx}{\sqrt{\frac{b\cos^2(x)}{a}+1}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}$$

input
```
Int[Sqrt[a + b*Cos[x]^2],x]
```

output
```
(Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/Sqrt[1 + (b*Cos[x]^2)/a
]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{a\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\cos(x)^{2}}{a}}\,\operatorname{EllipticE}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)}{\sin(x)\sqrt{a+b\cos(x)^{2}}}$	49

input
```
int((a+b*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticE(cos(x),(-b/a)^(1/2)
)/sin(x)/(a+b*cos(x)^2)^(1/2)
```

Fricas [F]

$$\int \sqrt{a+b\cos^2(x)}\,dx=\int \sqrt{b\cos{(x)}^{2}+a}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(x)^2 + a), x)
```

Sympy [F]

$$\int \sqrt{a+b\cos^2(x)}\,dx = \int \sqrt{a+b\cos^2{(x)}}\,dx$$

input
```
integrate((a+b*cos(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(x)**2), x)
```

Maxima [F]

$$\int \sqrt{a+b\cos^2(x)}\,dx = \int \sqrt{b\cos{(x)}^2+a}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(x)^2 + a), x)
```

Giac [F]

$$\int \sqrt{a+b\cos^2(x)}\,dx = \int \sqrt{b\cos{(x)}^2+a}\,dx$$

input
```
integrate((a+b*cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos^2(x)}\,dx=\int \sqrt{b\cos{(x)}^2+a}\,dx$$

input
```
int((a + b*cos(x)^2)^(1/2),x)
```

output
```
int((a + b*cos(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\cos^2(x)}\,dx=\int \sqrt{\cos{(x)}^2\,b+a}dx$$

input
```
int((a+b*cos(x)^2)^(1/2),x)
```

output
```
int(sqrt(cos(x)**2*b + a),x)
```

3.70 $\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx$

Optimal result 526
Mathematica [A] (verified) 526
Rubi [A] (verified) 527
Maple [A] (verified) 528
Fricas [C] (verification not implemented) 529
Sympy [F] 529
Maxima [F] 530
Giac [F] 530
Mupad [F(-1)] 530
Reduce [F] 531

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx = \frac{\sqrt{\frac{a+b\cos^2(x)}{a}}\,\text{EllipticF}\left(\frac{\pi}{2}+x,-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}$$

output
```
((a+b*cos(x)^2)/a)^(1/2)*InverseJacobiAM(1/2*Pi+x,(-b/a)^(1/2))/(a+b*cos(x
)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx = \frac{\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}\,\text{EllipticF}\left(x,\frac{b}{a+b}\right)}{\sqrt{2a+b+b\cos(2x)}}$$

input
```
Integrate[1/Sqrt[a + b*Cos[x]^2],x]
```

output
```
(Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticF[x, b/(a + b)])/Sqrt[2*a +
b + b*Cos[2*x]]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sin\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sqrt{\frac{b\cos^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\cos^2(x)}{a}+1}}dx}{\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{b\cos^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}}dx}{\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\sqrt{\frac{b\cos^2(x)}{a}+1}\,\text{EllipticF}\left(x+\frac{\pi}{2},-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}$$

input
```
Int[1/Sqrt[a + b*Cos[x]^2],x]
```

output
```
(Sqrt[1 + (b*Cos[x]^2)/a]*EllipticF[Pi/2 + x, -(b/a)])/Sqrt[a + b*Cos[x]^2
]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\cos(x)^2}{a}}\,\operatorname{EllipticF}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)}{\sin(x)\sqrt{a+b\cos(x)^2}}$	48

input
```
int(1/(a+b*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x),(-b/a)^(1/2))/
sin(x)/(a+b*cos(x)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 276, normalized size of antiderivative = 6.42

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx = -\frac{\left(-2i\,b\sqrt{\frac{a^2+ab}{b^2}}-2i\,a-i\,b\right)\sqrt{b}\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}-2\,a-b}{b}}F\left(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}-2\,a-b}{b}}(\cos(x)+i\,\sin(x))\right)\mid \frac{8\,a}{}\right.}{}$$

input
```
integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-((-2*I*b*sqrt((a^2 + a*b)/b^2) - 2*I*a - I*b)*sqrt(b)*sqrt((2*b*sqrt((a^2
 + a*b)/b^2) - 2*a - b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^
2) - 2*a - b)/b)*(cos(x) + I*sin(x))), (8*a^2 + 8*a*b + b^2 + 4*(2*a*b + b
^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (2*I*b*sqrt((a^2 + a*b)/b^2) + 2*I*a + I
*b)*sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*elliptic_f(arcsi
n(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*(cos(x) - I*sin(x))), (8*a
^2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/b^2
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{a+b\cos^2{(x)}}}\,dx$$

input
```
integrate(1/(a+b*cos(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*cos(x)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + b\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{b\cos(x)^2 + a}}\,dx$$

input
```
integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*cos(x)^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + b\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{b\cos(x)^2 + a}}\,dx$$

input
```
integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*cos(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{b\cos(x)^2 + a}}\,dx$$

input
```
int(1/(a + b*cos(x)^2)^(1/2),x)
```

output
```
int(1/(a + b*cos(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx = \int \frac{\sqrt{\cos{(x)}^2 b+a}}{\cos{(x)}^2 b+a}dx$$

input
```
int(1/(a+b*cos(x)^2)^(1/2),x)
```

output
```
int(sqrt(cos(x)**2*b + a)/(cos(x)**2*b + a),x)
```

3.71 $\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx$

Optimal result 532
Mathematica [A] (verified) 532
Rubi [A] (verified) 533
Maple [A] (verified) 535
Fricas [C] (verification not implemented) 535
Sympy [F] 536
Maxima [F] 537
Giac [F] 537
Mupad [F(-1)] 537
Reduce [F] 538

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{1}{(a+b\cos^2(x))^{3/2}}\,dx=\frac{\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{a(a+b)\sqrt{\frac{a+b\cos^2(x)}{a}}}-\frac{b\cos(x)\sin(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

output
```
(a+b*cos(x)^2)^(1/2)*EllipticE(cos(x),(-b/a)^(1/2))/a/(a+b)/((a+b*cos(x)^2
)/a)^(1/2)-b*cos(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+b\cos^2(x))^{3/2}}\,dx=\frac{2(a+b)\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}\,E\left(x\middle|\frac{b}{a+b}\right)-\sqrt{2}b\sin(2x)}{2a(a+b)\sqrt{2a+b+b\cos(2x)}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(-3/2),x]
```

output
```
(2*(a + b)*Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticE[x, b/(a + b)] -
Sqrt[2]*b*Sin[2*x])/(2*a*(a + b)*Sqrt[2*a + b + b*Cos[2*x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 3663, 25, 3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos^2(x))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3663}$$

$$-\frac{\int -\sqrt{b\cos^2(x)+a}dx}{a(a+b)}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \sqrt{b\cos^2(x)+a}dx}{a(a+b)}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx}{a(a+b)}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{ 3657}$$

$$\frac{\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\cos^2(x)}{a}+1}dx}{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}dx}{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

$$\Big\downarrow \text{ 3656}$$

$$\frac{\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

input
```
Int[(a + b*Cos[x]^2)^(-3/2),x]
```

output
```
(Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/(a*(a + b)*Sqrt[1 + (b*
Cos[x]^2)/a]) - (b*Cos[x]*Sin[x])/(a*(a + b)*Sqrt[a + b*Cos[x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\frac{b\sin(x)^2}{a}+\frac{a+b}{a}}\,a\,\mathrm{EllipticE}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)+b\cos(x)\sin(x)^2}{a(a+b)\sin(x)\sqrt{a+b\cos(x)^2}}$	73

input
```
int(1/(a+b*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-((sin(x)^2)^(1/2)*(-b/a*sin(x)^2+(a+b)/a)^(1/2)*a*EllipticE(cos(x),(-b/a)
^(1/2))+b*cos(x)*sin(x)^2)/a/(a+b)/sin(x)/(a+b*cos(x)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 775, normalized size of antiderivative = 9.81

$$\int \frac{1}{(a+b\cos^2(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(b*cos(x)^2 + a)*b^3*cos(x)*sin(x) + (2*I*a^2*b + I*a*b^2 + (2
*I*a*b^2 + I*b^3)*cos(x)^2 - 2*(I*b^3*cos(x)^2 + I*a*b^2)*sqrt((a^2 + a*b)
/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*elliptic_e(ar
csin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*(cos(x) + I*sin(x))), (
8*a^2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*
a^2*b - I*a*b^2 + (-2*I*a*b^2 - I*b^3)*cos(x)^2 - 2*(-I*b^3*cos(x)^2 - I*a
*b^2)*sqrt((a^2 + a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a
 - b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*(
cos(x) - I*sin(x))), (8*a^2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*
b)/b^2))/b^2) + 2*(-2*I*a^3 - 3*I*a^2*b - I*a*b^2 + (-2*I*a^2*b - 3*I*a*b^
2 - I*b^3)*cos(x)^2 + 2*(-I*a*b^2*cos(x)^2 - I*a^2*b)*sqrt((a^2 + a*b)/b^2
))*sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*elliptic_f(arcsin
(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*(cos(x) + I*sin(x))), (8*a^
2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*(2*I*a^3
 + 3*I*a^2*b + I*a*b^2 + (2*I*a^2*b + 3*I*a*b^2 + I*b^3)*cos(x)^2 + 2*(I*a
*b^2*cos(x)^2 + I*a^2*b)*sqrt((a^2 + a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^
2 + a*b)/b^2) - 2*a - b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b
^2) - 2*a - b)/b)*(cos(x) - I*sin(x))), (8*a^2 + 8*a*b + b^2 + 4*(2*a*b +
b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/(a^3*b^2 + a^2*b^3 + (a^2*b^3 + a*b^4)*c
os(x)^2)
```

Sympy [F]

$$\int \frac{1}{(a+b\cos^2(x))^{3/2}}\,dx = \int \frac{1}{(a+b\cos^2(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(x)**2)**(3/2),x)
```

output
```
Integral((a + b*cos(x)**2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\cos\left(x\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(x)^2 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\cos\left(x\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(x)^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\cos\left(x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*cos(x)^2)^(3/2),x)
```

output
```
int(1/(a + b*cos(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx=\int \frac{\sqrt{\cos\left(x\right)^{2}b+a}}{\cos\left(x\right)^{4}b^{2}+2\cos\left(x\right)^{2}ab+a^{2}}dx$$

input
```
int(1/(a+b*cos(x)^2)^(3/2),x)
```

output
```
int(sqrt(cos(x)**2*b + a)/(cos(x)**4*b**2 + 2*cos(x)**2*a*b + a**2),x)
```

3.72 $\int \frac{1}{\left(a+b\cos^2(x)\right)^{5/2}}\,dx$

Optimal result 539
Mathematica [A] (verified) 540
Rubi [A] (verified) 540
Maple [B] (verified) 544
Fricas [C] (verification not implemented) 545
Sympy [F] 546
Maxima [F] 547
Giac [F] 547
Mupad [F(-1)] 547
Reduce [F] 548

Optimal result

Integrand size = 12, antiderivative size = 177

$$\int \frac{1}{(a+b\cos^2(x))^{5/2}}\,dx = \frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{3a^2(a+b)^2\sqrt{\frac{a+b\cos^2(x)}{a}}} - \frac{\sqrt{\frac{a+b\cos^2(x)}{a}}\operatorname{EllipticF}\left(\frac{\pi}{2}+x,-\frac{b}{a}\right)}{3a(a+b)\sqrt{a+b\cos^2(x)}} - \frac{b\cos(x)\sin(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}} - \frac{2b(2a+b)\cos(x)\sin(x)}{3a^2(a+b)^2\sqrt{a+b\cos^2(x)}}$$

output
```
2/3*(2*a+b)*(a+b*cos(x)^2)^(1/2)*EllipticE(cos(x),(-b/a)^(1/2))/a^2/(a+b)^
2/((a+b*cos(x)^2)/a)^(1/2)-1/3*((a+b*cos(x)^2)/a)^(1/2)*InverseJacobiAM(1/
2*Pi+x,(-b/a)^(1/2))/a/(a+b)/(a+b*cos(x)^2)^(1/2)-1/3*b*cos(x)*sin(x)/a/(a
+b)/(a+b*cos(x)^2)^(3/2)-2/3*b*(2*a+b)*cos(x)*sin(x)/a^2/(a+b)^2/(a+b*cos(
x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^{5/2}}\,dx = \frac{2(a+b)^2(2a+b)\left(\frac{2a+b+b\cos(2x)}{a+b}\right)^{3/2} E\left(x\middle|\frac{b}{a+b}\right) - a(a+b)^2\left(\frac{2a+b+b\cos(2x)}{a+b}\right)^{3/2} \text{Elli}}{3a^2(a+b)^2(2a+b+b\,\text{c}}$$

input
```
Integrate[(a + b*Cos[x]^2)^(-5/2),x]
```

output
```
(2*(a + b)^2*(2*a + b)*((2*a + b + b*Cos[2*x])/(a + b))^(3/2)*EllipticE[x,
 b/(a + b)] - a*(a + b)^2*((2*a + b + b*Cos[2*x])/(a + b))^(3/2)*EllipticF
[x, b/(a + b)] - Sqrt[2]*b*(5*a^2 + 5*a*b + b^2 + b*(2*a + b)*Cos[2*x])*Si
n[2*x])/(3*a^2*(a + b)^2*(2*a + b + b*Cos[2*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {3042, 3663, 25, 3042, 3652, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\cos^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a+b\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 3663$$

$$-\frac{\int -\frac{-b\cos^2(x)+3a+2b}{\left(b\cos^2(x)+a\right)^{3/2}}dx}{3a(a+b)} - \frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-b\cos^2(x)+3a+2b}{(b\cos^2(x)+a)^{3/2}}dx}{3a(a+b)} - \frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{-b\sin\left(x+\frac{\pi}{2}\right)^2+3a+2b}{\left(b\sin\left(x+\frac{\pi}{2}\right)^2+a\right)^{3/2}}dx}{3a(a+b)} - \frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3652

$$\frac{\frac{\int \frac{2b(2a+b)\cos^2(x)+a(3a+b)}{\sqrt{b\cos^2(x)+a}}dx}{a(a+b)} - \frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}}{3a(a+b)} - \frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\int \frac{2b(2a+b)\sin\left(x+\frac{\pi}{2}\right)^2+a(3a+b)}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx}{a(a+b)} - \frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}}{3a(a+b)} - \frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3651

$$\frac{\frac{2(2a+b)\int \sqrt{b\cos^2(x)+a}dx-a(a+b)\int \frac{1}{\sqrt{b\cos^2(x)+a}}dx}{a(a+b)} - \frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}}{3a(a+b)} - \frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{2(2a+b)\int \sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}dx-a(a+b)\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx}{a(a+b)} - \frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}}{3a(a+b)} - \frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3657

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\cos^2(x)}\int \sqrt{\frac{b\cos^2(x)}{a}+1}dx}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-a(a+b)\int \frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx}{a(a+b)} - \frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}}{3a(a+b)} - \frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\cos^2(x)}\int\sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}dx}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-a(a+b)\int\frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx}{a(a+b)}-\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}}{3a(a+b)}-\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3656}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-a(a+b)\int\frac{1}{\sqrt{b\sin\left(x+\frac{\pi}{2}\right)^2+a}}dx}{a(a+b)}-\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}}{3a(a+b)}-\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3662}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\int\frac{1}{\sqrt{\frac{b\cos^2(x)}{a}+1}}dx}{\sqrt{a+b\cos^2(x)}}}{a(a+b)}-\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}}{3a(a+b)}-\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin\left(x+\frac{\pi}{2}\right)^2}{a}+1}}dx}{\sqrt{a+b\cos^2(x)}}}{a(a+b)}-\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}}{3a(a+b)}-\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3661}$$

$$\frac{\frac{\frac{2(2a+b)\sqrt{a+b\cos^2(x)}E\left(x+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\,\text{EllipticF}\left(x+\frac{\pi}{2},-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}}{a(a+b)}-\frac{2b(2a+b)\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}}{3a(a+b)}-\frac{b\sin(x)\cos(x)}{3a(a+b)\left(a+b\cos^2(x)\right)^{3/2}}$$

input
```
Int[(a + b*Cos[x]^2)^(-5/2),x]
```

output
```
-1/3*(b*Cos[x]*Sin[x])/(a*(a + b)*(a + b*Cos[x]^2)^(3/2)) + (((2*(2*a + b)
*Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/Sqrt[1 + (b*Cos[x]^2)/a
] - (a*(a + b)*Sqrt[1 + (b*Cos[x]^2)/a]*EllipticF[Pi/2 + x, -(b/a)])/Sqrt[
a + b*Cos[x]^2])/(a*(a + b)) - (2*b*(2*a + b)*Cos[x]*Sin[x])/(a*(a + b)*Sq
rt[a + b*Cos[x]^2]))/(3*a*(a + b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(154) = 308$.

Time = 0.86 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.24

method	result
default	$\frac{\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\cos(x)^2}{a}}\,\text{EllipticF}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)a^2b\cos(x)^2+\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b\cos(x)^2}{a}}\,\text{EllipticF}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)a\,b^2\cos(x)}{}$

input
```
int(1/(a+b*cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x),(-b/a)^(1/
2))*a^2*b*cos(x)^2+(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos
(x),(-b/a)^(1/2))*a*b^2*cos(x)^2-4*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/
2)*EllipticE(cos(x),(-b/a)^(1/2))*a^2*b*cos(x)^2-2*(sin(x)^2)^(1/2)*((a+b*
cos(x)^2)/a)^(1/2)*EllipticE(cos(x),(-b/a)^(1/2))*a*b^2*cos(x)^2+4*a*b^2*c
os(x)^5+2*b^3*cos(x)^5+(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF
(cos(x),(-b/a)^(1/2))*a^3+(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*Ellipt
icF(cos(x),(-b/a)^(1/2))*a^2*b-4*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)
*EllipticE(cos(x),(-b/a)^(1/2))*a^3-2*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^
(1/2)*EllipticE(cos(x),(-b/a)^(1/2))*a^2*b+5*a^2*b*cos(x)^3-a*b^2*cos(x)^3
-2*b^3*cos(x)^3-5*a^2*b*cos(x)-3*a*b^2*cos(x))/(a+b*cos(x)^2)^(3/2)/a^2/(a
+b)^2/sin(x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 1213, normalized size of antiderivative = 6.85

$$\int \frac{1}{(a+b\cos^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*((4*I*a^4*b + 4*I*a^3*b^2 + I*a^2*b^3 + (4*I*a^2*b^3 + 4*I*a*b^4 + I*
b^5)*cos(x)^4 + 2*(4*I*a^3*b^2 + 4*I*a^2*b^3 + I*a*b^4)*cos(x)^2 - 2*(2*I*
a^3*b^2 + I*a^2*b^3 + (2*I*a*b^4 + I*b^5)*cos(x)^4 - 2*(-2*I*a^2*b^3 - I*a
*b^4)*cos(x)^2)*sqrt((a^2 + a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/
b^2) - 2*a - b)/b)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a
 - b)/b)*(cos(x) + I*sin(x))), (8*a^2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt
((a^2 + a*b)/b^2))/b^2) + (-4*I*a^4*b - 4*I*a^3*b^2 - I*a^2*b^3 + (-4*I*a^
2*b^3 - 4*I*a*b^4 - I*b^5)*cos(x)^4 + 2*(-4*I*a^3*b^2 - 4*I*a^2*b^3 - I*a*
b^4)*cos(x)^2 - 2*(-2*I*a^3*b^2 - I*a^2*b^3 + (-2*I*a*b^4 - I*b^5)*cos(x)^
4 - 2*(2*I*a^2*b^3 + I*a*b^4)*cos(x)^2)*sqrt((a^2 + a*b)/b^2))*sqrt(b)*sqr
t((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*elliptic_e(arcsin(sqrt((2*b*sqr
t((a^2 + a*b)/b^2) - 2*a - b)/b)*(cos(x) - I*sin(x))), (8*a^2 + 8*a*b + b^
2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-6*I*a^5 - 13*I*a^4*b -
 9*I*a^3*b^2 - 2*I*a^2*b^3 + (-6*I*a^3*b^2 - 13*I*a^2*b^3 - 9*I*a*b^4 - 2*
I*b^5)*cos(x)^4 + 2*(-6*I*a^4*b - 13*I*a^3*b^2 - 9*I*a^2*b^3 - 2*I*a*b^4)*
cos(x)^2 - 2*(3*I*a^4*b + I*a^3*b^2 + (3*I*a^2*b^3 + I*a*b^4)*cos(x)^4 - 2
*(-3*I*a^3*b^2 - I*a^2*b^3)*cos(x)^2)*sqrt((a^2 + a*b)/b^2))*sqrt(b)*sqrt(
(2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) - 2*a - b)/b)*(cos(x) + I*sin(x))), (8*a^2 + 8*a*b + b^2
+ 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (6*I*a^5 + 13*I*a^4*b +...
```

Sympy [F]

$$\int \frac{1}{(a+b\cos^2(x))^{5/2}}\,dx = \int \frac{1}{\left(a+b\cos^2{(x)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(x)**2)**(5/2),x)
```

output
```
Integral((a + b*cos(x)**2)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\cos^2(x))^{5/2}}\,dx = \int \frac{1}{(b\cos(x)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(x)^2 + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cos^2(x))^{5/2}}\,dx = \int \frac{1}{(b\cos(x)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(x)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(x)^2 + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos^2(x))^{5/2}}\,dx = \int \frac{1}{(b\cos(x)^2+a)^{5/2}}\,dx$$

input
```
int(1/(a + b*cos(x)^2)^(5/2),x)
```

output
```
int(1/(a + b*cos(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a + b\cos^2(x))^{5/2}}\,dx = \int \frac{\sqrt{\cos(x)^2 b + a}}{\cos(x)^6 b^3 + 3\cos(x)^4 a b^2 + 3\cos(x)^2 a^2 b + a^3}dx$$

input
```
int(1/(a+b*cos(x)^2)^(5/2),x)
```

output
```
int(sqrt(cos(x)**2*b + a)/(cos(x)**6*b**3 + 3*cos(x)**4*a*b**2 + 3*cos(x)*
*2*a**2*b + a**3),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 549
4.2 Links to plain text integration problems used in this report for each CAS . 567

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file