# Computer Algebra Independent Integration Tests

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4-Trig-functions/4.3-Tangent/214-4.3.0

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# Contents

**1 Introduction** **14**
1.1 Listing of CAS systems tested . . . . . . . . . . . . . . . . . . . . . . . . . . 15
1.2 Results . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 16
1.3 Time and leaf size Performance . . . . . . . . . . . . . . . . . . . . . . . . 20
1.4 Performance based on number of rules Rubi used . . . . . . . . . . . . . 22
1.5 Performance based on number of steps Rubi used . . . . . . . . . . . . . 23
1.6 Solved integrals histogram based on leaf size of result . . . . . . . . . . 24
1.7 Solved integrals histogram based on CPU time used . . . . . . . . . . . 25
1.8 Leaf size vs. CPU time used . . . . . . . . . . . . . . . . . . . . . . . . . 26
1.9 list of integrals with no known antiderivative . . . . . . . . . . . . . . . 27
1.10 List of integrals solved by CAS but has no known antiderivative . . . . 27
1.11 list of integrals solved by CAS but failed verification . . . . . . . . . . 27
1.12 Timing . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 28
1.13 Verification . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 29
1.14 Important notes about some of the results . . . . . . . . . . . . . . . . . 29
1.15 Current tree layout of integration tests . . . . . . . . . . . . . . . . . . 32
1.16 Design of the test system . . . . . . . . . . . . . . . . . . . . . . . . . . 33

**2 detailed summary tables of results** **34**
2.1 List of integrals sorted by grade for each CAS . . . . . . . . . . . . . . . 35
2.2 Detailed conclusion table per each integral for all CAS systems . . . . . 43
2.3 Detailed conclusion table specific for Rubi results . . . . . . . . . . . . . 140

**3 Listing of integrals** **153**
3.1 $\int \tan(c+dx)\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 165
3.2 $\int \tan^2(c+dx)\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . 170
3.3 $\int \tan^3(c+dx)\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . 175
3.4 $\int \tan^4(c+dx)\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . 180
3.5 $\int \tan^5(c+dx)\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . 185
3.6 $\int \tan^6(c+dx)\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . 190
3.7 $\int \tan^7(c+dx)\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . 195

3.8 $\int \tan^8(c+dx)\,dx$ . . . 201
3.9 $\int (b\tan(c+dx))^{7/2}\,dx$ . . . 207
3.10 $\int (b\tan(c+dx))^{5/2}\,dx$ . . . 217
3.11 $\int (b\tan(c+dx))^{3/2}\,dx$ . . . 227
3.12 $\int \sqrt{b\tan(c+dx)}\,dx$ . . . 237
3.13 $\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$ . . . 245
3.14 $\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx$ . . . 254
3.15 $\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx$ . . . 264
3.16 $\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx$ . . . 273
3.17 $\int (b\tan(c+dx))^{4/3}\,dx$ . . . 283
3.18 $\int (b\tan(c+dx))^{2/3}\,dx$ . . . 294
3.19 $\int \sqrt[3]{b\tan(c+dx)}\,dx$ . . . 305
3.20 $\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$ . . . 314
3.21 $\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$ . . . 323
3.22 $\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx$ . . . 333
3.23 $\int (b\tan(c+dx))^n\,dx$ . . . 344
3.24 $\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx$ . . . 349
3.25 $\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx$ . . . 355
3.26 $\int \sqrt{b\tan^2(c+dx)}\,dx$ . . . 361
3.27 $\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx$ . . . 366
3.28 $\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx$ . . . 372
3.29 $\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx$ . . . 379
3.30 $\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx$ . . . 386
3.31 $\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx$ . . . 397
3.32 $\int \sqrt{b\tan^3(c+dx)}\,dx$ . . . 407
3.33 $\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$ . . . 417
3.34 $\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx$ . . . 427
3.35 $\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx$ . . . 437
3.36 $\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx$ . . . 448
3.37 $\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx$ . . . 455
3.38 $\int \sqrt{b\tan^4(c+dx)}\,dx$ . . . 462
3.39 $\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx$ . . . 467
3.40 $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx$ . . . 472
3.41 $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx$ . . . 478
3.42 $\int \left(b\tan^p(c+dx)\right)^n\,dx$ . . . 485

3.43 $\int (b\tan^2(c+dx))^n \, dx$ . . . 490
3.44 $\int (b\tan^3(c+dx))^n \, dx$ . . . 495
3.45 $\int (b\tan^4(c+dx))^n \, dx$ . . . 500
3.46 $\int (b\tan^p(c+dx))^{5/2} \, dx$ . . . 505
3.47 $\int (b\tan^p(c+dx))^{3/2} \, dx$ . . . 510
3.48 $\int \sqrt{b\tan^p(c+dx)}\, dx$ . . . 515
3.49 $\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\, dx$ . . . 520
3.50 $\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\, dx$ . . . 525
3.51 $\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\, dx$ . . . 530
3.52 $\int (b\tan^p(c+dx))^{\frac{1}{p}} \, dx$ . . . 535
3.53 $\int (a(b\tan(c+dx))^p)^n \, dx$ . . . 540
3.54 $\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\, dx$ . . . 545
3.55 $\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\, dx$ . . . 555
3.56 $\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\, dx$ . . . 565
3.57 $\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\, dx$ . . . 570
3.58 $\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\, dx$ . . . 576
3.59 $\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\, dx$ . . . 582
3.60 $\int \sin(a+bx)\sqrt{d\tan(a+bx)}\, dx$ . . . 589
3.61 $\int \csc(a+bx)\sqrt{d\tan(a+bx)}\, dx$ . . . 595
3.62 $\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\, dx$ . . . 601
3.63 $\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\, dx$ . . . 607
3.64 $\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\, dx$ . . . 614
3.65 $\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\, dx$ . . . 624
3.66 $\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\, dx$ . . . 634
3.67 $\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\, dx$ . . . 639
3.68 $\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\, dx$ . . . 645
3.69 $\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\, dx$ . . . 651
3.70 $\int \sin(a+bx)(d\tan(a+bx))^{3/2}\, dx$ . . . 658
3.71 $\int \csc(a+bx)(d\tan(a+bx))^{3/2}\, dx$ . . . 664
3.72 $\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\, dx$ . . . 671
3.73 $\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\, dx$ . . . 678
3.74 $\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\, dx$ . . . 688
3.75 $\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\, dx$ . . . 698
3.76 $\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\, dx$ . . . 703
3.77 $\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\, dx$ . . . 709
3.78 $\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\, dx$ . . . 715
3.79 $\int \sin(a+bx)(d\tan(a+bx))^{5/2}\, dx$ . . . 723
3.80 $\int \csc(a+bx)(d\tan(a+bx))^{5/2}\, dx$ . . . 730

3.81 $\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 737
3.82 $\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 744
3.83 $\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 751
3.84 $\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 759
3.85 $\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 769
3.86 $\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 779
3.87 $\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 784
3.88 $\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 790
3.89 $\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 796
3.90 $\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 803
3.91 $\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 809
3.92 $\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 815
3.93 $\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 822
3.94 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . . . 829
3.95 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . . . 839
3.96 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . . . 849
3.97 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . . . 854
3.98 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . . . 860
3.99 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . . . 866
3.100 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . . . 873
3.101 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . . . 879
3.102 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . . . 885
3.103 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . . . 892
3.104 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . . . 902
3.105 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . . . 912
3.106 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . . . 917
3.107 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . . . 923
3.108 $\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . . . 929
3.109 $\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . . . 936
3.110 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . . . 943
3.111 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . . . 949

3.112 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 955
3.113 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 962
3.114 $\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$ . . . 970
3.115 $\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$ . . . 976
3.116 $\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx$ . . . 982
3.117 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$ . . . 987
3.118 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$ . . . 993
3.119 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$ . . . 1000
3.120 $\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$ . . . 1006
3.121 $\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$ . . . 1013
3.122 $\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx$ . . . 1019
3.123 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx$ . . . 1025
3.124 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx$ . . . 1030
3.125 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx$ . . . 1036
3.126 $\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 1044
3.127 $\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 1051
3.128 $\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 1057
3.129 $\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 1063
3.130 $\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 1068
3.131 $\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$ . . . 1074
3.132 $\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$ . . . 1081
3.133 $\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$ . . . 1087
3.134 $\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1095
3.135 $\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1102
3.136 $\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1108
3.137 $\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1113
3.138 $\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$ . . . 1121
3.139 $\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1130
3.140 $\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1137
3.141 $\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1143
3.142 $\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$ . . . 1149
3.143 $\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$ . . . 1155

3.144 $\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx$ . . . 1162
3.145 $\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$ . . . 1170
3.146 $\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx$ . . . 1175
3.147 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx$ . . . 1180
3.148 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx$ . . . 1185
3.149 $\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$ . . . 1190
3.150 $\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx$ . . . 1195
3.151 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx$ . . . 1200
3.152 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx$ . . . 1205
3.153 $\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx$ . . . 1210
3.154 $\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$ . . . 1215
3.155 $\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$ . . . 1220
3.156 $\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$ . . . 1225
3.157 $\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$ . . . 1230
3.158 $\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$ . . . 1235
3.159 $\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$ . . . 1240
3.160 $\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$ . . . 1245
3.161 $\int (a\sin(e+fx))^m\tan^3(e+fx)\,dx$ . . . 1250
3.162 $\int (a\sin(e+fx))^m\tan(e+fx)\,dx$ . . . 1255
3.163 $\int \cot(e+fx)(a\sin(e+fx))^m\,dx$ . . . 1260
3.164 $\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx$ . . . 1265
3.165 $\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx$ . . . 1271
3.166 $\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx$ . . . 1277
3.167 $\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx$ . . . 1282
3.168 $\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx$ . . . 1287
3.169 $\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx$ . . . 1292
3.170 $\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx$ . . . 1297
3.171 $\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx$ . . . 1302
3.172 $\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 1307
3.173 $\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1312
3.174 $\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$ . . . 1317
3.175 $\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1322
3.176 $\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1327
3.177 $\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1332

3.178 $\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1337
3.179 $\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1343
3.180 $\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1348
3.181 $\int \sin(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1353
3.182 $\int \csc(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1358
3.183 $\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1363
3.184 $\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1369
3.185 $\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx$ . . . 1375
3.186 $\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx$ . . . 1380
3.187 $\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx$ . . . 1385
3.188 $\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx$ . . . 1390
3.189 $\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx$ . . . 1395
3.190 $\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx$ . . . 1400
3.191 $\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx$ . . . 1405
3.192 $\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$ . . . 1416
3.193 $\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$ . . . 1427
3.194 $\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx$ . . . 1437
3.195 $\int \sqrt{d\cot(e+fx)}\,dx$ . . . 1447
3.196 $\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$ . . . 1456
3.197 $\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$ . . . 1466
3.198 $\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$ . . . 1476
3.199 $\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx$ . . . 1487
3.200 $\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx$ . . . 1498
3.201 $\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx$ . . . 1508
3.202 $\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx$ . . . 1518
3.203 $\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx$ . . . 1527
3.204 $\int (d\cot(e+fx))^{3/2}\,dx$ . . . 1536
3.205 $\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx$ . . . 1546
3.206 $\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$ . . . 1556
3.207 $\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1567
3.208 $\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1578
3.209 $\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1588
3.210 $\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1598
3.211 $\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1607
3.212 $\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1616
3.213 $\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1626

3.214 $\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1635
3.215 $\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1646
3.216 $\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1656
3.217 $\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1666
3.218 $\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1675
3.219 $\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1684
3.220 $\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1694
3.221 $\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1703
3.222 $\int \cot^m(e+fx)\tan^n(e+fx)\,dx$ . . . 1714
3.223 $\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1719
3.224 $\int (a\cot(e+fx))^m\tan^n(e+fx)\,dx$ . . . 1724
3.225 $\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx$ . . . 1729
3.226 $\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1734
3.227 $\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1740
3.228 $\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1746
3.229 $\int \sqrt{d\tan(e+fx)}\,dx$ . . . 1751
3.230 $\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1760
3.231 $\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1770
3.232 $\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1777
3.233 $\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1784
3.234 $\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1790
3.235 $\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1796
3.236 $\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1803
3.237 $\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1809
3.238 $\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1815
3.239 $\int (d\tan(a+bx))^{3/2}\,dx$ . . . 1820
3.240 $\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1830
3.241 $\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1840
3.242 $\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1848
3.243 $\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1855
3.244 $\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1861
3.245 $\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1867
3.246 $\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1874
3.247 $\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx$ . . . 1882
3.248 $\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$ . . . 1888
3.249 $\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$ . . . 1894
3.250 $\int (d\tan(e+fx))^{5/2}\,dx$ . . . 1899

3.251 $\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$ . . . 1909
3.252 $\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$ . . . 1919
3.253 $\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$ . . . 1929
3.254 $\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$ . . . 1936
3.255 $\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$ . . . 1942
3.256 $\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$ . . . 1948
3.257 $\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$ . . . 1954
3.258 $\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 1961
3.259 $\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 1967
3.260 $\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 1972
3.261 $\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 1977
3.262 $\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 1987
3.263 $\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 1997
3.264 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 2005
3.265 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 2012
3.266 $\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 2019
3.267 $\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 2025
3.268 $\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 2032
3.269 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 2040
3.270 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx$ . . . 2046
3.271 $\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx$ . . . 2053
3.272 $\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx$ . . . 2058
3.273 $\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx$ . . . 2063
3.274 $\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx$ . . . 2068
3.275 $\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx$ . . . 2073
3.276 $\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx$ . . . 2078
3.277 $\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx$ . . . 2083
3.278 $\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx$ . . . 2088
3.279 $\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx$ . . . 2093
3.280 $\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx$ . . . 2098
3.281 $\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx$ . . . 2103
3.282 $\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx$ . . . 2108
3.283 $\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx$ . . . 2113
3.284 $\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$ . . . 2118

3.285 $\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$ . . . 2123
3.286 $\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx$ . . . 2128
3.287 $\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx$ . . . 2133
3.288 $\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx$ . . . 2138
3.289 $\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$ . . . 2143
3.290 $\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$ . . . 2148
3.291 $\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$ . . . 2153
3.292 $\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$ . . . 2162
3.293 $\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx$ . . . 2168
3.294 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx$ . . . 2175
3.295 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx$ . . . 2181
3.296 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx$ . . . 2186
3.297 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx$ . . . 2192
3.298 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx$ . . . 2197
3.299 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$ . . . 2204
3.300 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$ . . . 2212
3.301 $\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx$ . . . 2221
3.302 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx$ . . . 2227
3.303 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx$ . . . 2235
3.304 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx$ . . . 2241
3.305 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx$ . . . 2246
3.306 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx$ . . . 2253
3.307 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx$ . . . 2259
3.308 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx$ . . . 2268
3.309 $\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx$ . . . 2276
3.310 $\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx$ . . . 2285
3.311 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx$ . . . 2291
3.312 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx$ . . . 2299
3.313 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx$ . . . 2305
3.314 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx$ . . . 2310
3.315 $\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 2317
3.316 $\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 2326

3.317 $\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 2333
3.318 $\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 2341
3.319 $\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$ . . . 2347
3.320 $\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$ . . . 2352
3.321 $\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$ . . . 2358
3.322 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 2364
3.323 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 2372
3.324 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 2379
3.325 $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$ . . . 2384
3.326 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$ . . . 2391
3.327 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$ . . . 2397
3.328 $\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx$ . . . 2404
3.329 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx$ . . . 2413
3.330 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx$ . . . 2420
3.331 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx$ . . . 2425
3.332 $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx$ . . . 2432
3.333 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx$ . . . 2438
3.334 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx$ . . . 2445
3.335 $\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$ . . . 2451
3.336 $\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx$ . . . 2456
3.337 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx$ . . . 2461
3.338 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx$ . . . 2466
3.339 $\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$ . . . 2471
3.340 $\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx$ . . . 2476
3.341 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx$ . . . 2481
3.342 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx$ . . . 2486
3.343 $\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx$ . . . 2491
3.344 $\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$ . . . 2496
3.345 $\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$ . . . 2501
3.346 $\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$ . . . 2506
3.347 $\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$ . . . 2511
3.348 $\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$ . . . 2516

3.349 $\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$ . . . 2521
3.350 $\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$ . . . 2526
3.351 $\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx$ . . . 2531
3.352 $\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx$ . . . 2538
3.353 $\int (b\sec(e+fx))^m \tan(e+fx)\,dx$ . . . 2545
3.354 $\int \cot(e+fx)(b\sec(e+fx))^m\,dx$ . . . 2550
3.355 $\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx$ . . . 2555
3.356 $\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx$ . . . 2560
3.357 $\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx$ . . . 2565
3.358 $\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx$ . . . 2570
3.359 $\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx$ . . . 2575
3.360 $\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx$ . . . 2580
3.361 $\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx$ . . . 2585
3.362 $\int (a\sec(e+fx))^m (b\tan(e+fx))^n\,dx$ . . . 2590
3.363 $\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2595
3.364 $\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2601
3.365 $\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2607
3.366 $\int (d\tan(a+bx))^n\,dx$ . . . 2612
3.367 $\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2617
3.368 $\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2622
3.369 $\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2627
3.370 $\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2632
3.371 $\int \sec(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2637
3.372 $\int \cos(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2642
3.373 $\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2647
3.374 $\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx$ . . . 2652
3.375 $\int (b\csc(e+fx))^m \tan(e+fx)\,dx$ . . . 2657
3.376 $\int \cot(e+fx)(b\csc(e+fx))^m\,dx$ . . . 2662
3.377 $\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$ . . . 2668
3.378 $\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx$ . . . 2675
3.379 $\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx$ . . . 2681
3.380 $\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx$ . . . 2686
3.381 $\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx$ . . . 2691
3.382 $\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx$ . . . 2696
3.383 $\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx$ . . . 2701
3.384 $\int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx$ . . . 2707
3.385 $\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx$ . . . 2713
3.386 $\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx$ . . . 2719

3.387 $\int (a\csc(e+fx))^m (b\tan(e+fx))^n \, dx$ . . . . . . . . . . . . . . . . . . . 2725

**4 Appendix** **2731**

4.1 Listing of Grading functions . . . . . . . . . . . . . . . . . . . . . . . . . 2731

4.2 Links to plain text integration problems used in this report for each CAS 2749

# CHAPTER 1

## Introduction

1.1 Listing of CAS systems tested . . . . . . . . . . . . . . . . . . . . . . . . . 15
1.2 Results . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 16
1.3 Time and leaf size Performance . . . . . . . . . . . . . . . . . . . . . . . . 20
1.4 Performance based on number of rules Rubi used . . . . . . . . . . . . . . . 22
1.5 Performance based on number of steps Rubi used . . . . . . . . . . . . . . . 23
1.6 Solved integrals histogram based on leaf size of result . . . . . . . . . . . . 24
1.7 Solved integrals histogram based on CPU time used . . . . . . . . . . . . . 25
1.8 Leaf size vs. CPU time used . . . . . . . . . . . . . . . . . . . . . . . . . . 26
1.9 list of integrals with no known antiderivative . . . . . . . . . . . . . . . . . 27
1.10 List of integrals solved by CAS but has no known antiderivative . . . . . . 27
1.11 list of integrals solved by CAS but failed verification . . . . . . . . . . . . 27
1.12 Timing . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 28
1.13 Verification . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 29
1.14 Important notes about some of the results . . . . . . . . . . . . . . . . . . 29
1.15 Current tree layout of integration tests . . . . . . . . . . . . . . . . . . . . 32
1.16 Design of the test system . . . . . . . . . . . . . . . . . . . . . . . . . . . 33

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ **387** ]. This is test number [ 214 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 100.00 ( 387 ) | 0.00 ( 0 ) |
| Mathematica | 100.00 ( 387 ) | 0.00 ( 0 ) |
| Maple | 68.99 ( 267 ) | 31.01 ( 120 ) |
| Fricas | 62.27 ( 241 ) | 37.73 ( 146 ) |
| Maxima | 35.40 ( 137 ) | 64.60 ( 250 ) |
| Mupad | 31.52 ( 122 ) | 68.48 ( 265 ) |
| Giac | 23.77 ( 92 ) | 76.23 ( 295 ) |
| Reduce | 7.49 ( 29 ) | 92.51 ( 358 ) |
| Sympy | 4.65 ( 18 ) | 95.35 ( 369 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

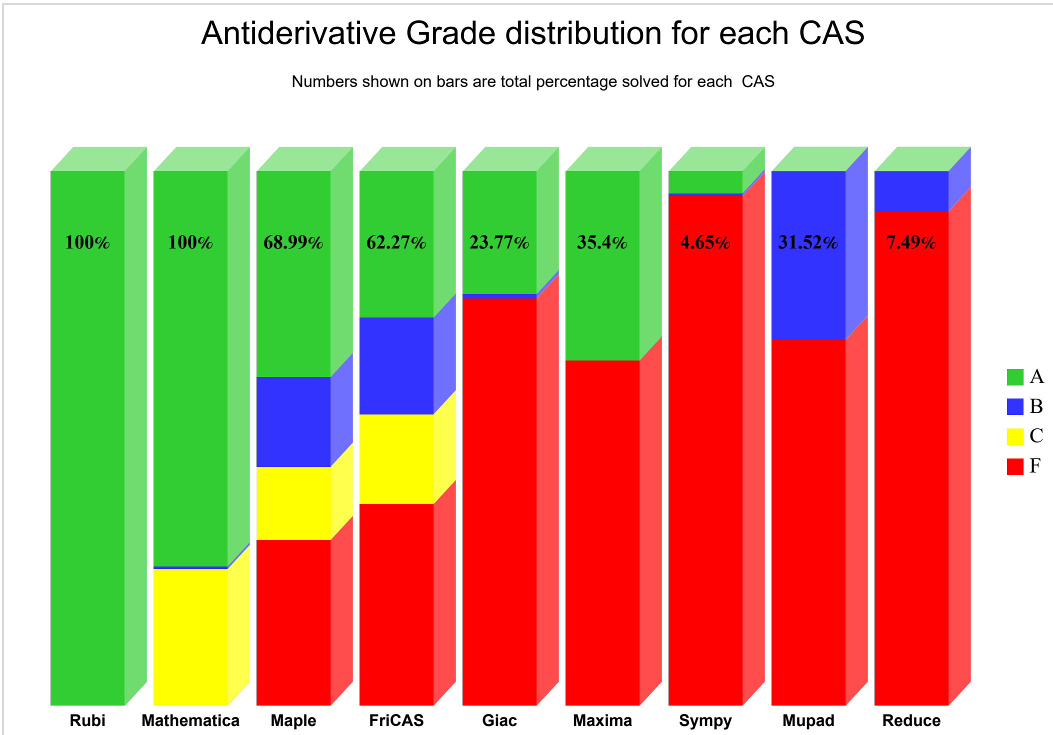
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

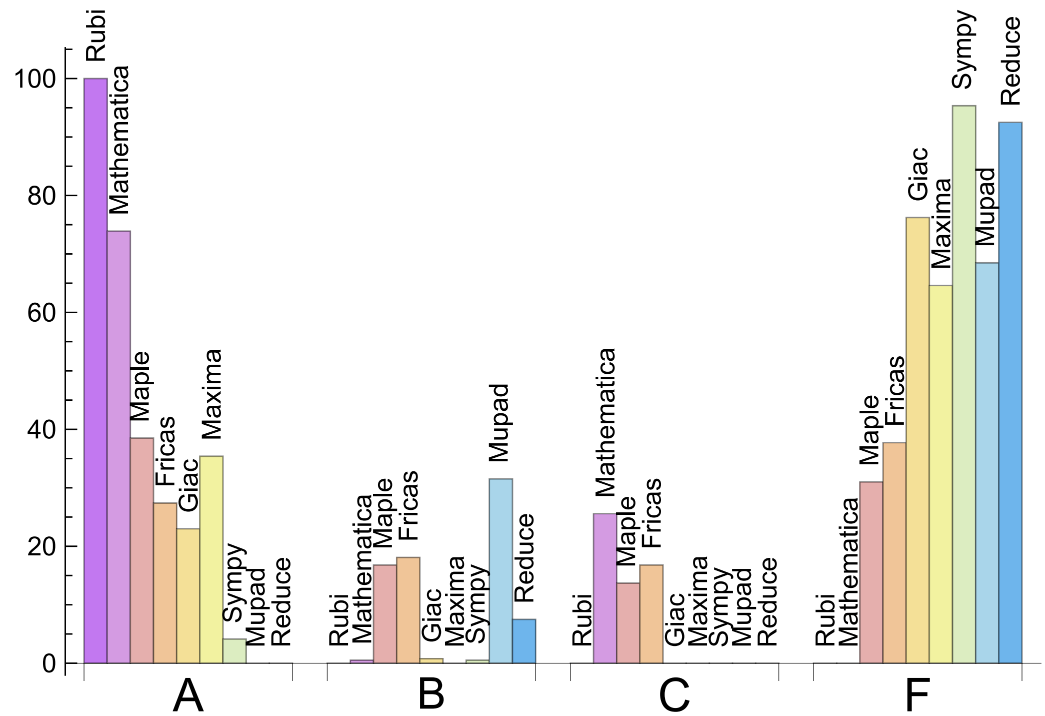
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 100.000 | 0.000 | 0.000 | 0.000 |
| Mathematica | 73.902 | 0.517 | 25.581 | 0.000 |
| Maple | 38.501 | 16.796 | 13.695 | 31.008 |
| Maxima | 35.401 | 0.000 | 0.000 | 64.599 |
| Fricas | 27.390 | 18.088 | 16.796 | 37.726 |
| Giac | 22.997 | 0.775 | 0.000 | 76.227 |
| Sympy | 4.134 | 0.517 | 0.000 | 95.349 |
| Mupad | 0.000 | 31.525 | 0.000 | 68.475 |
| Reduce | 0.000 | 7.494 | 0.000 | 92.506 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.




The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 | 0.00 | 0.00 |
| Mathematica | 0 | 0.00 | 0.00 | 0.00 |
| Maple | 120 | 100.00 | 0.00 | 0.00 |
| Fricas | 146 | 95.89 | 0.00 | 4.11 |
| Maxima | 250 | 99.60 | 0.40 | 0.00 |
| Mupad | 265 | 0.00 | 100.00 | 0.00 |
| Giac | 295 | 83.73 | 7.46 | 8.81 |
| Reduce | 358 | 100.00 | 0.00 | 0.00 |
| Sympy | 369 | 66.67 | 33.33 | 0.00 |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) |
|---|---|
| Maxima | 0.09 |
| Fricas | 0.13 |
| Reduce | 0.19 |
| Giac | 0.27 |
| Rubi | 0.37 |
| Mathematica | 1.33 |
| Mupad | 1.74 |
| Maple | 5.46 |
| Sympy | 13.12 |

Table 1.5: Time performance for each CAS

| System | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|
| Reduce | 38.31 | 0.79 | 36.00 | 0.86 |
| Sympy | 50.72 | 1.43 | 52.00 | 1.29 |
| Mathematica | 92.91 | 1.10 | 71.00 | 0.94 |
| Rubi | 98.91 | 1.03 | 77.00 | 1.00 |
| Giac | 112.90 | 1.12 | 64.00 | 1.14 |
| Maxima | 115.38 | 0.99 | 133.00 | 1.07 |
| Mupad | 146.28 | 2.69 | 77.50 | 1.10 |
| Fricas | 194.91 | 1.66 | 129.00 | 1.36 |
| Maple | 407.20 | 5.37 | 156.00 | 1.23 |

Table 1.6: Leaf size performance for each CAS

## 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the $y$ axis is the percentage solved which Rubi itself needed the number of rules given the $x$ axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

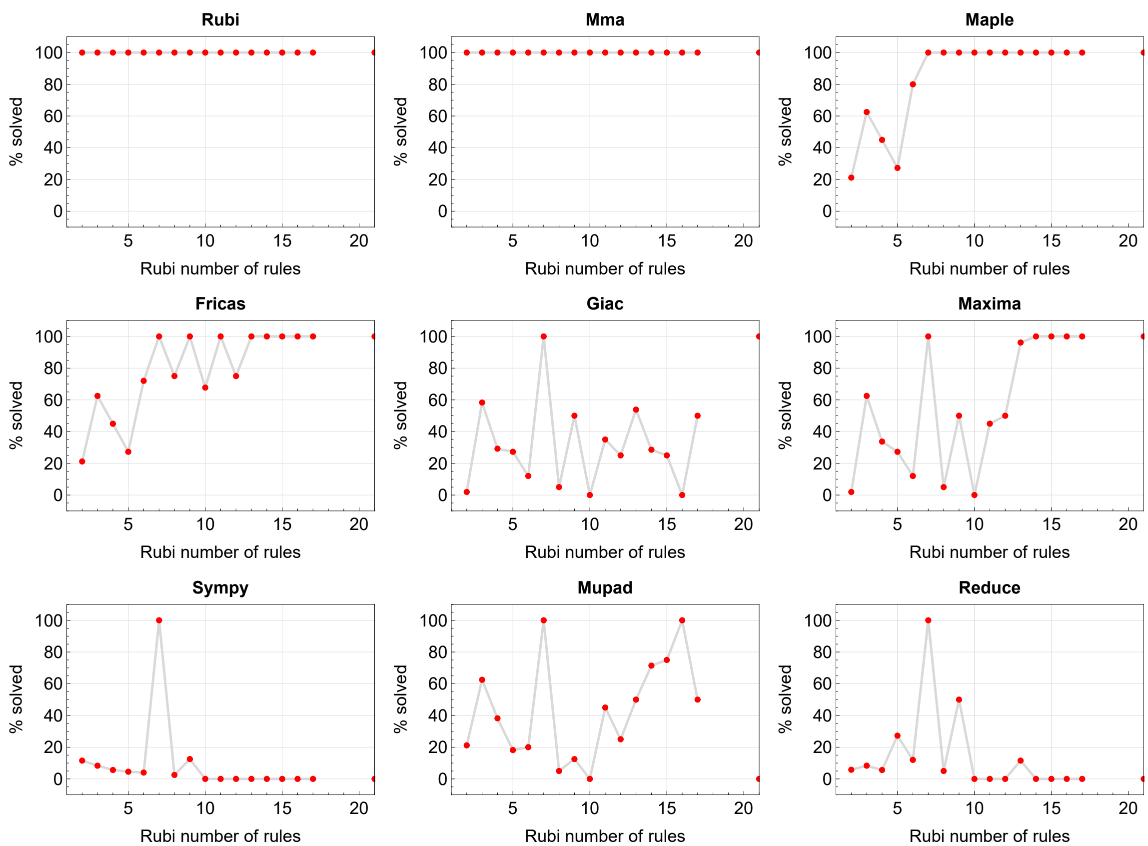



Figure 1.1: Solving statistics per number of Rubi rules used

## 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

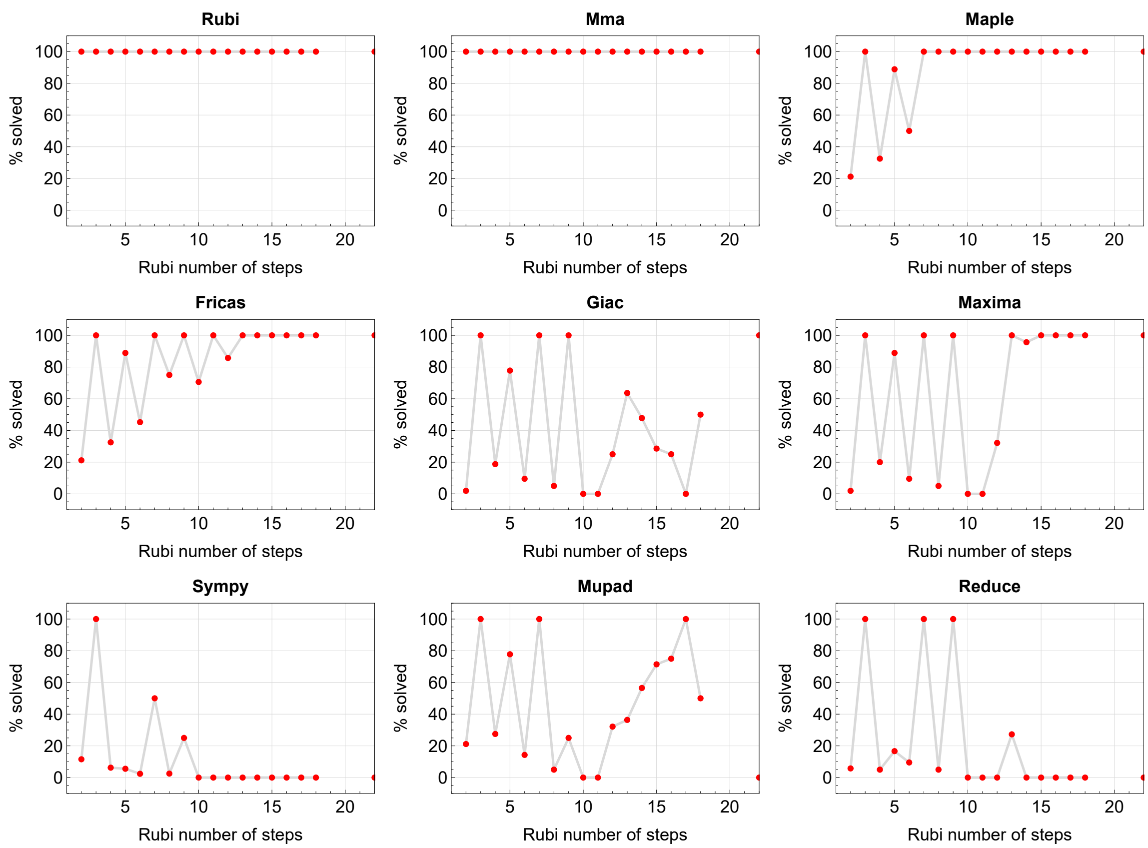



Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

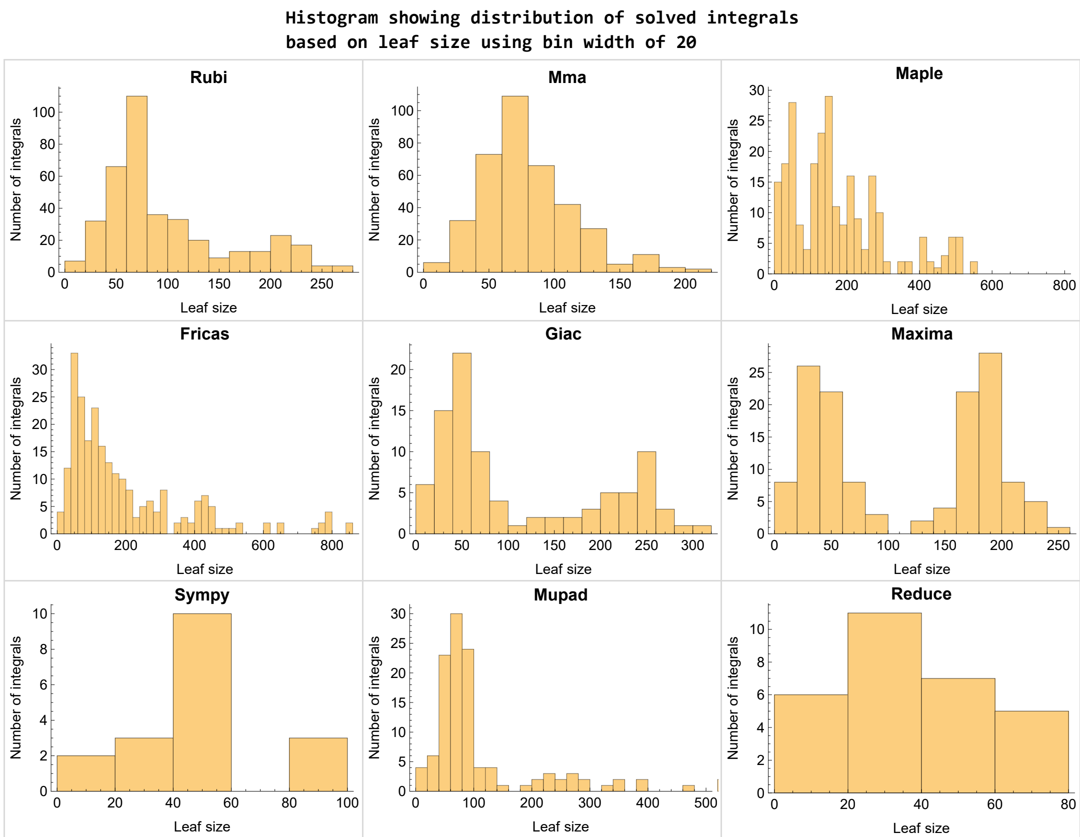



Figure 1.3: Solved integrals based on leaf size distribution

## 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

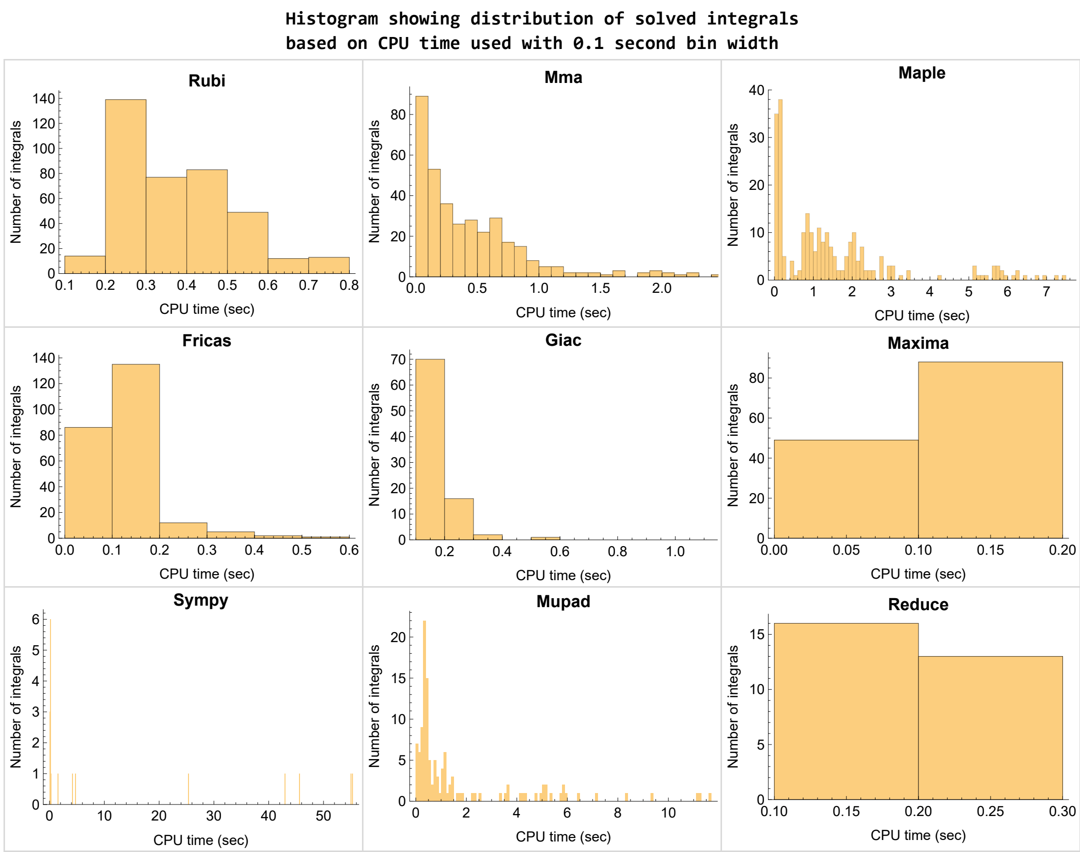



Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

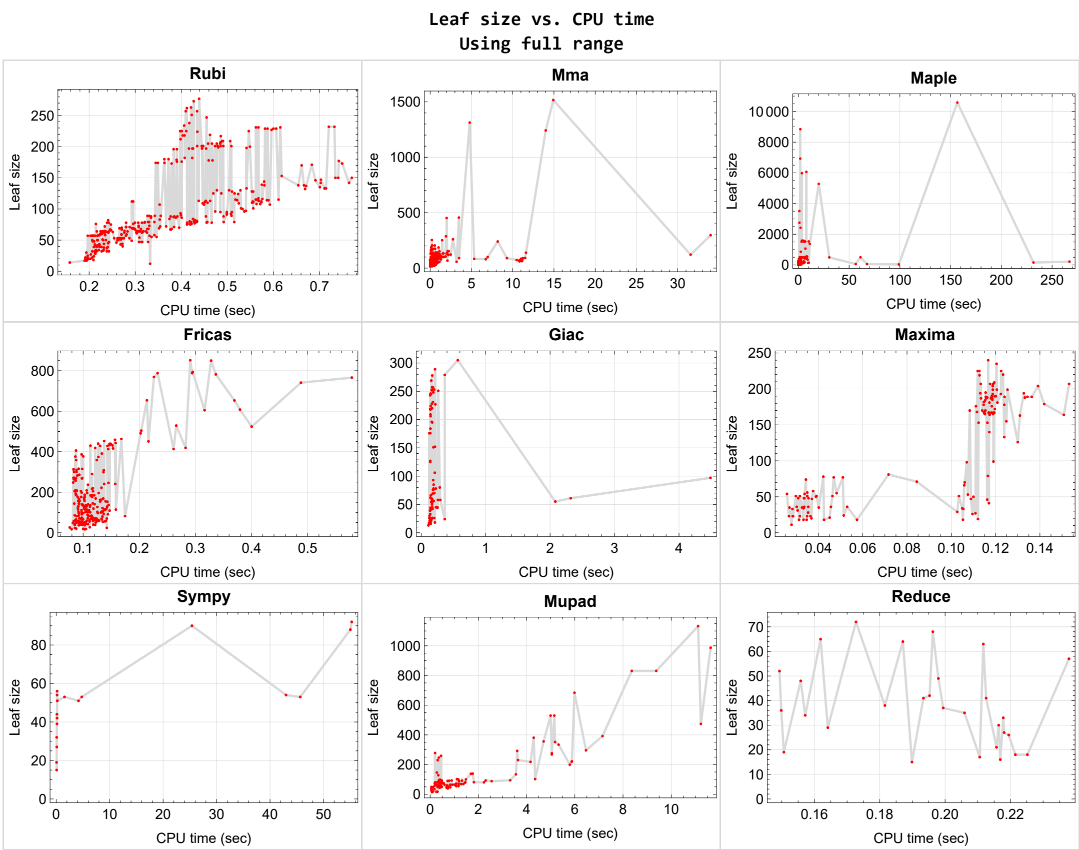



Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{}

## 1.10 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Reduce** {}

**Mupad** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

**Rubi** {9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 54, 55, 64, 65, 73, 74, 84, 85, 94, 95, 103, 104, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 229, 239, 250, 261, 291, 293, 300, 302, 307, 309, 311, 315, 317, 322, 328}

**Mathematica** $\{35, 59, 62, 63, 78, 83, 100, 101, 102, 119, 139, 144, 174, 180, 181, 183, 184, 185, 246, 256, 269, 372, 373, 379, 381, 387\}$

**Maple** $\{52, 59, 64, 65, 73, 74, 78, 99, 164, 165, 178, 179, 191, 192, 199, 200, 201, 214, 215, 245, 246, 351, 352, 377, 378\}$

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Reduce** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.14 Important notes about some of the results

### Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

## Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

## Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

## Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.15 Current tree layout of integration tests

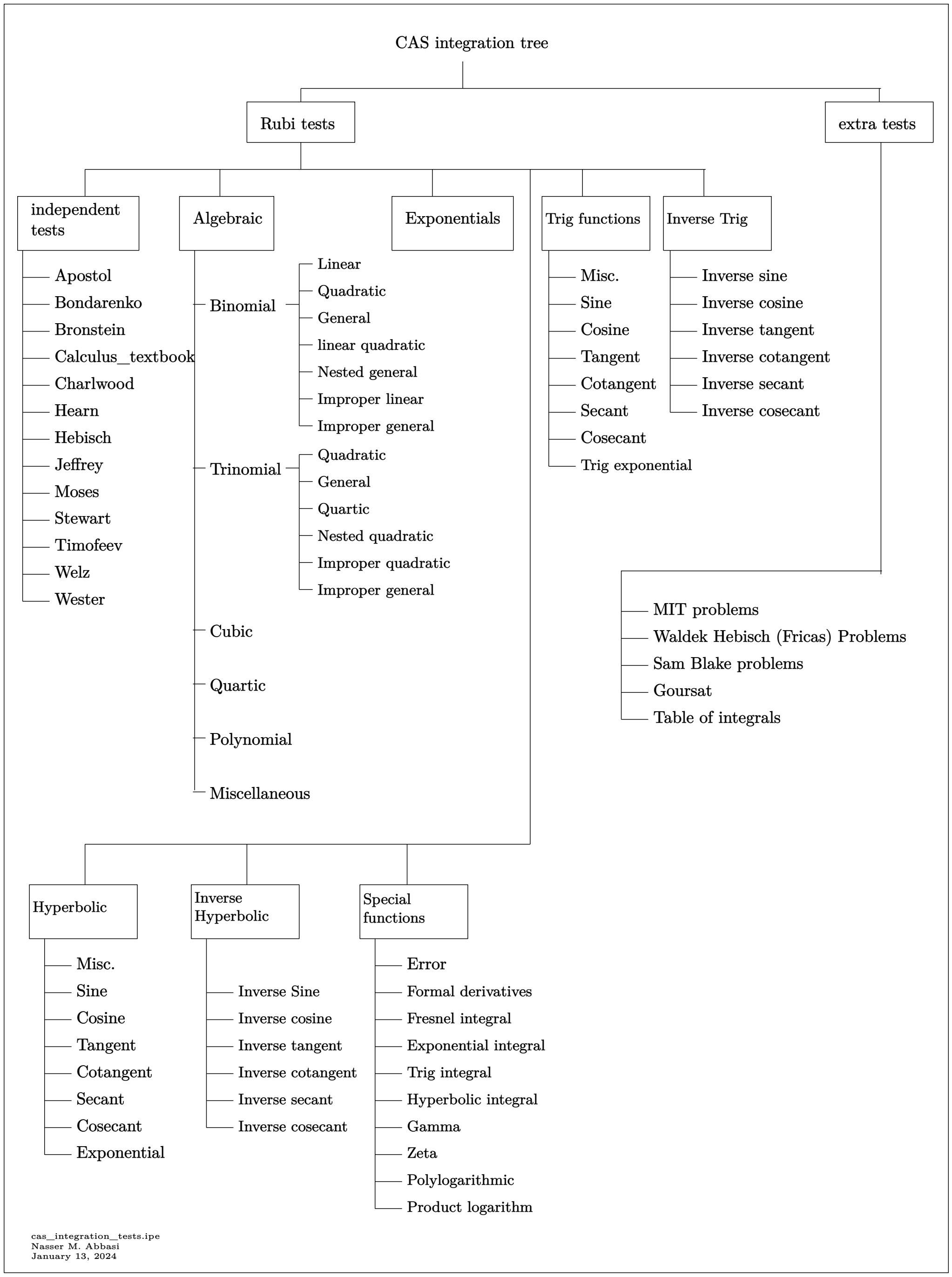
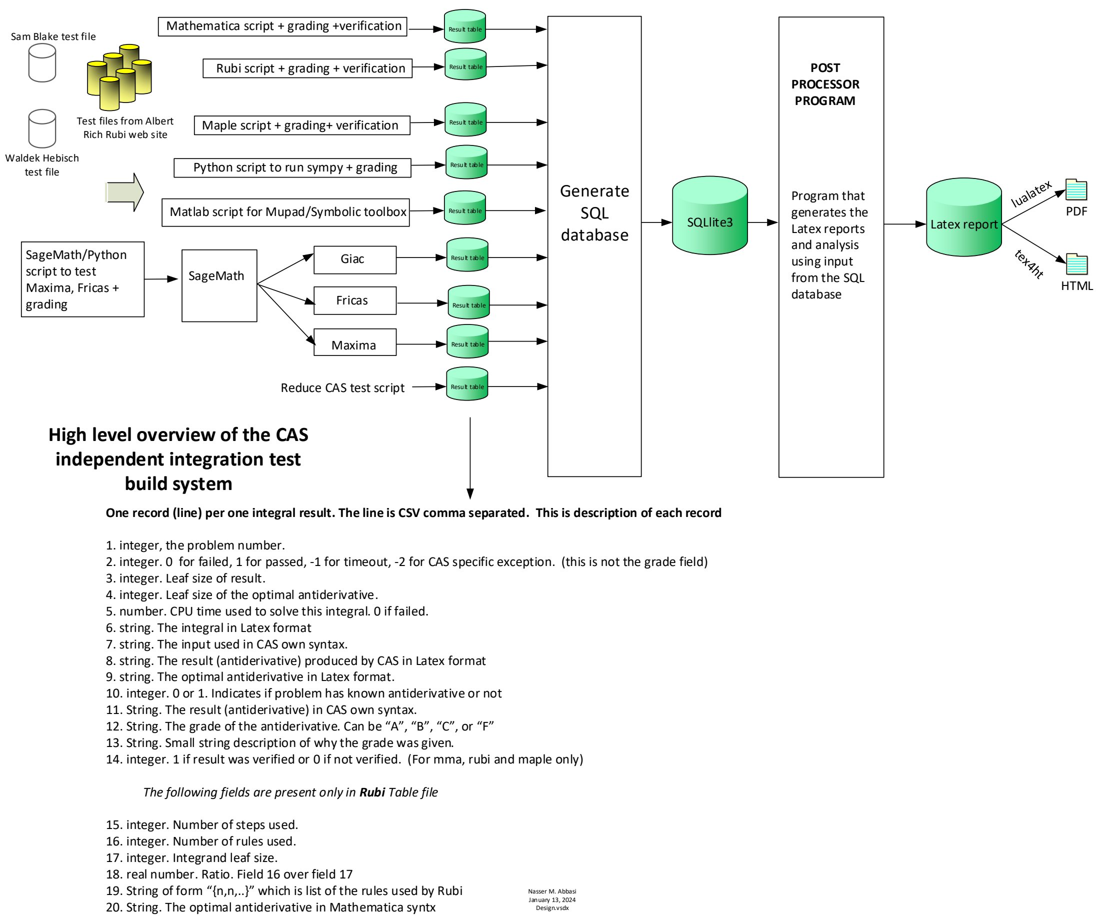



Figure 1.6: CAS integration tests tree

## 1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS . . . . . . . . . . . . . . . . . 35
2.2 Detailed conclusion table per each integral for all CAS systems . . . . . . 43
2.3 Detailed conclusion table specific for Rubi results . . . . . . . . . . . . . . . 140

## 2.1 List of integrals sorted by grade for each CAS

Rubi . . . . . 35
Mma . . . . . 36
Maple . . . . . 37
Fricas . . . . . 37
Maxima . . . . . 38
Giac . . . . . 39
Mupad . . . . . 40
Sympy . . . . . 40
Reduce . . . . . 41

### Rubi

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

**B grade** { }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Mma

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 114, 115, 116, 117, 118, 119, 121, 123, 125, 127, 129, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 182, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 258, 259, 260, 261, 262, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 295, 297, 300, 304, 306, 307, 309, 311, 313, 315, 317, 319, 321, 322, 324, 326, 328, 330, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 376, 377, 378, 380, 382, 383, 384, 385, 386 }

**B grade** { 379, 381 }

**C grade** { 17, 18, 21, 22, 39, 40, 41, 59, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 120, 122, 124, 126, 128, 130, 132, 174, 180, 181, 183, 184, 185, 231, 232, 233, 234, 235, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 263, 264, 265, 266, 267, 268, 269, 270, 292, 294, 296, 298, 299, 301, 302, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333, 372, 373, 387 }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Maple

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 56, 57, 58, 60, 62, 63, 66, 67, 68, 75, 76, 77, 79, 80, 81, 82, 83, 86, 88, 96, 97, 98, 100, 101, 102, 105, 108, 118, 125, 127, 129, 131, 134, 135, 136, 137, 163, 177, 195, 196, 197, 198, 203, 204, 205, 206, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 236, 237, 238, 239, 241, 242, 243, 244, 247, 248, 249, 250, 253, 254, 256, 258, 259, 260, 261, 263, 269, 291, 293, 295, 297, 300, 302, 304, 306, 307, 309, 311, 313, 315, 317, 319, 321, 322, 324, 326, 328, 330, 332, 334, 353, 363, 364, 365, 376 }

**B grade** { 54, 55, 61, 64, 65, 69, 70, 71, 72, 73, 74, 84, 85, 87, 89, 90, 91, 92, 93, 94, 95, 103, 104, 106, 107, 109, 110, 111, 112, 113, 114, 121, 123, 133, 138, 191, 192, 193, 194, 199, 200, 201, 202, 207, 208, 209, 214, 215, 230, 231, 232, 233, 234, 235, 240, 251, 252, 255, 262, 264, 265, 266, 267, 268, 270 }

**C grade** { 52, 59, 78, 99, 115, 116, 117, 119, 120, 122, 124, 126, 128, 130, 132, 139, 140, 141, 142, 143, 144, 164, 165, 178, 179, 245, 246, 257, 292, 294, 296, 298, 299, 301, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333, 351, 352, 377, 378 }

**F normal fail** { 23, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Fricas

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 52, 57, 58, 66, 67, 68, 76, 77, 87, 88, 114, 116, 121, 123, 127, 134, 135, 163, 164, 165, 177, 178, 179, 191, 192, 193, 194, 199, 200, 201, 202, 203, 207, 208, 209, 214, 215, 226, 227, 229, 236, 237, 239, 247, 248, 250, 258, 259, 261, 295, 297, 306, 319, 321, 324, 326, 332, 334, 351, 352, 353, 363, 364, 365, 376, 377, 378 }

**B grade** { 22, 54, 55, 56, 64, 65, 73, 74, 75, 84, 85, 86, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 118, 125, 129, 131, 133, 136, 137, 138, 195, 196, 197, 198, 204, 205, 206, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 228, 230, 238, 240, 249, 251, 252, 260, 262, 291, 293, 300, 302, 304, 307, 309, 311, 313, 315, 317, 322, 328, 330 }

**C grade** { 61, 62, 63, 71, 72, 80, 81, 82, 83, 92, 93, 101, 102, 112, 113, 115, 117, 119, 120, 122, 124, 126, 128, 130, 132, 139, 140, 141, 142, 143, 144, 231, 232, 241, 242, 243, 253, 254, 255, 263, 264, 265, 269, 270, 292, 294, 296, 298, 299, 301, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333 }

**F normal fail** { 23, 42, 43, 44, 45, 53, 59, 60, 69, 70, 78, 79, 89, 90, 91, 99, 100, 108, 109, 110, 111, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 233, 234, 235, 244, 245, 246, 256, 257, 266, 267, 268, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 46, 47, 48, 49, 50, 51 }

## Maxima

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 163, 164, 165, 177, 178, 179, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 230, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 258, 259, 260, 261, 262, 351, 352, 353, 363, 364, 365, 376, 377, 378 }

**B grade** { }

**C grade** { }

**F normal fail** { 23, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 59, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 231, 232, 233, 234, 235, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

**F(-1) timedout fail** { 286 }

**F(-2) exception fail** { }

## Giac

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 12, 13, 17, 18, 19, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 163, 226, 227, 228, 229, 230, 236, 237, 238, 240, 247, 248, 249, 251, 252, 258, 259, 260, 262, 352, 353, 363, 364, 365, 376, 377 }

**B grade** { 27, 351, 378 }

**C grade** { }

**F normal fail** { 23, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 60, 61, 62, 63, 71, 72, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 122, 123, 126, 127, 128, 129, 130, 131, 132, 133, 137, 142, 146, 147, 148, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 231, 232, 233, 235, 241, 242, 243, 253, 254, 255, 256, 257, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 327, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

**F(-1) timedout fail** { 14, 15, 16, 134, 135, 136, 139, 140, 141, 143, 144, 145, 150, 158, 261, 325, 326, 328, 329, 330, 331, 333 }

**F(-2) exception fail** { 9, 10, 11, 59, 69, 70, 78, 79, 120, 121, 124, 125, 138, 149, 234, 239, 244, 245, 246, 250, 285, 290, 307, 308, 309, 311 }

## Mupad

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 27, 56, 57, 58, 66, 67, 68, 75, 76, 77, 86, 87, 88, 96, 97, 98, 105, 106, 107, 114, 116, 121, 123, 127, 129, 134, 135, 136, 163, 164, 165, 177, 178, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 236, 237, 238, 239, 247, 248, 249, 250, 258, 259, 260, 261, 295, 297, 304, 306, 313, 319, 321, 324, 326, 330, 332, 334, 351, 352, 353, 364, 365, 376, 377, 378 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 122, 124, 125, 126, 128, 130, 131, 132, 133, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 246, 251, 252, 253, 254, 255, 256, 257, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 298, 299, 300, 301, 302, 303, 305, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 320, 322, 323, 325, 327, 328, 329, 331, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

**F(-2) exception fail** { }

## Sympy

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 295, 304, 319, 321, 324, 326, 330, 332 }

**B grade** { 353, 376 }

**C grade** { }

**F normal fail** { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 70, 71, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 111, 112, 113, 116, 117, 118, 130, 131, 137, 146, 147, 154, 155, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 237, 238, 239, 241, 242, 243, 244, 250, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 269, 270, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 296, 301, 302, 303, 317, 318, 320, 323, 325, 331, 336, 337, 338, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 374, 375, 377, 379, 380, 381, 382, 384, 385, 386, 387 }

**F(-1) timedout fail** { 58, 59, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 89, 90, 99, 107, 108, 109, 110, 114, 115, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 148, 149, 150, 151, 152, 153, 156, 157, 158, 159, 160, 170, 179, 180, 185, 199, 234, 235, 236, 240, 245, 246, 247, 248, 249, 251, 252, 257, 267, 268, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 291, 297, 298, 299, 300, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 322, 327, 328, 329, 333, 334, 335, 339, 343, 347, 373, 378, 383 }

**F(-2) exception fail** { }

## Reduce

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 24, 25, 26, 27, 28, 29, 36, 37, 38, 39, 40, 41, 52, 324, 330, 351, 352, 353, 376, 377, 378 }

**C grade** { }

**F normal fail** { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246,

247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387
}

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 17 | 11 | 18 | 19 | 13 | 16 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 1.42 | 0.92 | 1.50 | 1.58 | 1.08 | 1.33 | 1.33 |
| time (sec) | N/A | 0.332 | 0.035 | 0.022 | 0.028 | 0.105 | 0.067 | 0.111 | 0.217 | 0.257 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 23 | 15 | 18 | 17 | 15 | 21 | 15 | 14 |
| N.S. | 1 | 1.00 | 1.64 | 1.07 | 1.29 | 1.21 | 1.07 | 1.50 | 1.07 | 1.00 |
| time (sec) | N/A | 0.158 | 0.006 | 0.019 | 0.105 | 0.103 | 0.080 | 0.134 | 0.190 | 0.079 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 25 | 28 | 31 | 27 | 32 | 30 | 27 | 30 |
| N.S. | 1 | 1.00 | 0.93 | 1.04 | 1.15 | 1.00 | 1.19 | 1.11 | 1.00 | 1.11 |
| time (sec) | N/A | 0.204 | 0.020 | 0.025 | 0.031 | 0.112 | 0.084 | 0.130 | 0.218 | 0.055 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 38 | 27 | 29 | 26 | 27 | 39 | 26 | 24 |
| N.S. | 1 | 1.00 | 1.36 | 0.96 | 1.04 | 0.93 | 0.96 | 1.39 | 0.93 | 0.86 |
| time (sec) | N/A | 0.214 | 0.008 | 0.025 | 0.103 | 0.076 | 0.106 | 0.136 | 0.219 | 0.053 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 41 | 38 | 54 | 39 | 44 | 44 | 37 | 38 |
| N.S. | 1 | 1.00 | 0.95 | 0.88 | 1.26 | 0.91 | 1.02 | 1.02 | 0.86 | 0.88 |
| time (sec) | N/A | 0.268 | 0.069 | 0.046 | 0.026 | 0.092 | 0.133 | 0.143 | 0.199 | 0.054 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 53 | 39 | 41 | 38 | 39 | 54 | 38 | 35 |
| N.S. | 1 | 1.00 | 1.20 | 0.89 | 0.93 | 0.86 | 0.89 | 1.23 | 0.86 | 0.80 |
| time (sec) | N/A | 0.272 | 0.014 | 0.035 | 0.117 | 0.085 | 0.146 | 0.155 | 0.182 | 0.060 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 47 | 49 | 74 | 51 | 56 | 62 | 49 | 49 |
| N.S. | 1 | 1.00 | 0.82 | 0.86 | 1.30 | 0.89 | 0.98 | 1.09 | 0.86 | 0.86 |
| time (sec) | N/A | 0.331 | 0.044 | 0.059 | 0.034 | 0.091 | 0.168 | 0.143 | 0.198 | 0.051 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 68 | 49 | 51 | 48 | 51 | 66 | 48 | 44 |
| N.S. | 1 | 1.00 | 1.17 | 0.84 | 0.88 | 0.83 | 0.88 | 1.14 | 0.83 | 0.76 |
| time (sec) | N/A | 0.342 | 0.009 | 0.048 | 0.103 | 0.082 | 0.211 | 0.149 | 0.156 | 0.061 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F(-2) | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 227 | 174 | 169 | 186 | 175 | 0 | 0 | 58 | 93 |
| N.S. | 1 | 1.29 | 0.99 | 0.96 | 1.06 | 0.99 | 0.00 | 0.00 | 0.33 | 0.53 |
| time (sec) | N/A | 0.596 | 0.604 | 0.103 | 0.118 | 0.090 | 0.000 | 0.000 | 0.159 | 0.482 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F(-2) | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 197 | 101 | 154 | 176 | 166 | 0 | 0 | 24 | 74 |
| N.S. | 1 | 1.26 | 0.65 | 0.99 | 1.13 | 1.06 | 0.00 | 0.00 | 0.15 | 0.47 |
| time (sec) | N/A | 0.417 | 0.173 | 0.043 | 0.111 | 0.102 | 0.000 | 0.000 | 0.173 | 0.224 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F(-2) | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 201 | 160 | 149 | 170 | 157 | 0 | 0 | 38 | 73 |
| N.S. | 1 | 1.30 | 1.03 | 0.96 | 1.10 | 1.01 | 0.00 | 0.00 | 0.25 | 0.47 |
| time (sec) | N/A | 0.444 | 0.099 | 0.042 | 0.108 | 0.094 | 0.000 | 0.000 | 0.190 | 0.193 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 174 | 71 | 136 | 153 | 144 | 0 | 176 | 12 | 49 |
| N.S. | 1 | 1.27 | 0.52 | 0.99 | 1.12 | 1.05 | 0.00 | 1.28 | 0.09 | 0.36 |
| time (sec) | N/A | 0.345 | 0.039 | 0.098 | 0.112 | 0.092 | 0.000 | 0.125 | 0.210 | 0.124 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 180 | 131 | 138 | 155 | 130 | 0 | 184 | 24 | 59 |
| N.S. | 1 | 1.32 | 0.96 | 1.01 | 1.14 | 0.96 | 0.00 | 1.35 | 0.18 | 0.43 |
| time (sec) | N/A | 0.372 | 0.067 | 0.066 | 0.125 | 0.101 | 0.000 | 0.143 | 0.208 | 0.192 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F(-1) | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 197 | 82 | 157 | 167 | 177 | 0 | 0 | 24 | 76 |
| N.S. | 1 | 1.26 | 0.53 | 1.01 | 1.07 | 1.13 | 0.00 | 0.00 | 0.15 | 0.49 |
| time (sec) | N/A | 0.445 | 0.082 | 0.041 | 0.118 | 0.096 | 0.000 | 0.000 | 0.225 | 0.191 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F(-1) | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 205 | 86 | 157 | 168 | 186 | 0 | 0 | 24 | 75 |
| N.S. | 1 | 1.29 | 0.54 | 0.99 | 1.06 | 1.17 | 0.00 | 0.00 | 0.15 | 0.47 |
| time (sec) | N/A | 0.458 | 0.126 | 0.043 | 0.115 | 0.087 | 0.000 | 0.000 | 0.199 | 0.352 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F(-1) | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 225 | 96 | 171 | 195 | 198 | 0 | 0 | 24 | 92 |
| N.S. | 1 | 1.26 | 0.54 | 0.96 | 1.09 | 1.11 | 0.00 | 0.00 | 0.13 | 0.51 |
| time (sec) | N/A | 0.547 | 0.194 | 0.046 | 0.115 | 0.097 | 0.000 | 0.000 | 0.223 | 0.382 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | F | A | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 243 | 217 | 205 | 215 | 185 | 312 | 0 | 209 | 31 | 247 |
| N.S. | 1 | 0.89 | 0.84 | 0.88 | 0.76 | 1.28 | 0.00 | 0.86 | 0.13 | 1.02 |
| time (sec) | N/A | 0.486 | 0.235 | 0.111 | 0.119 | 0.083 | 0.000 | 0.144 | 0.223 | 0.358 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | F | A | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 196 | 185 | 191 | 168 | 314 | 0 | 206 | 14 | 259 |
| N.S. | 1 | 0.88 | 0.83 | 0.85 | 0.75 | 1.40 | 0.00 | 0.92 | 0.06 | 1.16 |
| time (sec) | N/A | 0.391 | 0.137 | 0.100 | 0.112 | 0.083 | 0.000 | 0.146 | 0.241 | 0.452 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 112 | 106 | 108 | 98 | 124 | 0 | 127 | 14 | 146 |
| N.S. | 1 | 0.85 | 0.81 | 0.82 | 0.75 | 0.95 | 0.00 | 0.97 | 0.11 | 1.11 |
| time (sec) | N/A | 0.296 | 0.198 | 0.067 | 0.107 | 0.091 | 0.000 | 0.141 | 0.206 | 0.278 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 112 | 100 | 108 | 99 | 299 | 0 | 125 | 14 | 128 |
| N.S. | 1 | 0.85 | 0.76 | 0.82 | 0.76 | 2.28 | 0.00 | 0.95 | 0.11 | 0.98 |
| time (sec) | N/A | 0.293 | 0.079 | 0.059 | 0.119 | 0.136 | 0.000 | 0.141 | 0.188 | 0.338 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | F | A | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 196 | 189 | 203 | 170 | 282 | 0 | 196 | 14 | 230 |
| N.S. | 1 | 0.88 | 0.84 | 0.91 | 0.76 | 1.26 | 0.00 | 0.88 | 0.06 | 1.03 |
| time (sec) | N/A | 0.383 | 0.135 | 0.085 | 0.114 | 0.090 | 0.000 | 0.163 | 0.170 | 0.313 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | B | F | A | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 219 | 254 | 212 | 182 | 430 | 0 | 227 | 14 | 278 |
| N.S. | 1 | 0.89 | 1.04 | 0.87 | 0.74 | 1.76 | 0.00 | 0.93 | 0.06 | 1.13 |
| time (sec) | N/A | 0.462 | 0.201 | 0.064 | 0.115 | 0.113 | 0.000 | 0.220 | 0.180 | 0.204 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 51 | 0 | 0 | 0 | 0 | 0 | 14 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 0.207 | 0.189 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.176 | 0.000 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 67 | 58 | 58 | 47 | 74 | 0 | 55 | 42 | 0 |
| N.S. | 1 | 0.68 | 0.59 | 0.59 | 0.48 | 0.76 | 0.00 | 0.56 | 0.43 | 0.00 |
| time (sec) | N/A | 0.377 | 0.121 | 0.100 | 0.109 | 0.098 | 0.000 | 0.161 | 0.195 | 0.000 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 49 | 47 | 48 | 34 | 52 | 0 | 41 | 30 | 0 |
| N.S. | 1 | 0.80 | 0.77 | 0.79 | 0.56 | 0.85 | 0.00 | 0.67 | 0.49 | 0.00 |
| time (sec) | N/A | 0.292 | 0.051 | 0.049 | 0.105 | 0.097 | 0.000 | 0.151 | 0.216 | 0.000 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 32 | 37 | 19 | 38 | 0 | 26 | 18 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.16 | 0.59 | 1.19 | 0.00 | 0.81 | 0.56 | 0.00 |
| time (sec) | N/A | 0.238 | 0.024 | 0.048 | 0.112 | 0.121 | 0.000 | 0.127 | 0.222 | 0.000 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 33 | 31 | 47 | 33 | 50 | 0 | 59 | 33 | 34 |
| N.S. | 1 | 1.06 | 1.00 | 1.52 | 1.06 | 1.61 | 0.00 | 1.90 | 1.06 | 1.10 |
| time (sec) | N/A | 0.230 | 0.039 | 0.053 | 0.105 | 0.122 | 0.000 | 0.167 | 0.218 | 0.175 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 54 | 47 | 64 | 46 | 69 | 0 | 93 | 57 | 0 |
| N.S. | 1 | 0.82 | 0.71 | 0.97 | 0.70 | 1.05 | 0.00 | 1.41 | 0.86 | 0.00 |
| time (sec) | N/A | 0.309 | 0.089 | 0.052 | 0.116 | 0.114 | 0.000 | 0.186 | 0.238 | 0.000 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 70 | 58 | 74 | 66 | 82 | 0 | 106 | 68 | 0 |
| N.S. | 1 | 0.72 | 0.60 | 0.76 | 0.68 | 0.85 | 0.00 | 1.09 | 0.70 | 0.00 |
| time (sec) | N/A | 0.396 | 0.093 | 0.061 | 0.106 | 0.114 | 0.000 | 0.211 | 0.196 | 0.000 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 232 | 204 | 263 | 178 | 310 | 0 | 289 | 92 | 0 |
| N.S. | 1 | 0.78 | 0.68 | 0.88 | 0.60 | 1.04 | 0.00 | 0.97 | 0.31 | 0.00 |
| time (sec) | N/A | 0.733 | 0.560 | 0.101 | 0.124 | 0.088 | 0.000 | 0.215 | 0.169 | 0.000 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 198 | 115 | 236 | 140 | 248 | 0 | 253 | 22 | 0 |
| N.S. | 1 | 0.89 | 0.52 | 1.06 | 0.63 | 1.11 | 0.00 | 1.13 | 0.10 | 0.00 |
| time (sec) | N/A | 0.542 | 0.412 | 0.047 | 0.117 | 0.082 | 0.000 | 0.201 | 0.163 | 0.000 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 178 | 162 | 205 | 133 | 264 | 0 | 195 | 37 | 0 |
| N.S. | 1 | 0.92 | 0.84 | 1.06 | 0.69 | 1.37 | 0.00 | 1.01 | 0.19 | 0.00 |
| time (sec) | N/A | 0.460 | 0.127 | 0.054 | 0.124 | 0.083 | 0.000 | 0.171 | 0.167 | 0.000 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 178 | 87 | 211 | 126 | 257 | 0 | 251 | 24 | 0 |
| N.S. | 1 | 0.93 | 0.45 | 1.10 | 0.66 | 1.34 | 0.00 | 1.31 | 0.12 | 0.00 |
| time (sec) | N/A | 0.468 | 0.202 | 0.055 | 0.130 | 0.086 | 0.000 | 0.265 | 0.195 | 0.000 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 232 | 200 | 98 | 233 | 163 | 270 | 0 | 279 | 24 | 0 |
| N.S. | 1 | 0.86 | 0.42 | 1.00 | 0.70 | 1.16 | 0.00 | 1.20 | 0.10 | 0.00 |
| time (sec) | N/A | 0.549 | 0.189 | 0.051 | 0.131 | 0.098 | 0.000 | 0.365 | 0.220 | 0.000 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 232 | 139 | 272 | 172 | 290 | 0 | 305 | 24 | 0 |
| N.S. | 1 | 0.78 | 0.46 | 0.91 | 0.58 | 0.97 | 0.00 | 1.02 | 0.08 | 0.00 |
| time (sec) | N/A | 0.720 | 0.351 | 0.054 | 0.120 | 0.116 | 0.000 | 0.568 | 0.216 | 0.000 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 182 | 100 | 86 | 84 | 79 | 96 | 0 | 85 | 63 | 0 |
| N.S. | 1 | 0.55 | 0.47 | 0.46 | 0.43 | 0.53 | 0.00 | 0.47 | 0.35 | 0.00 |
| time (sec) | N/A | 0.533 | 0.475 | 0.116 | 0.117 | 0.092 | 0.000 | 0.193 | 0.212 | 0.000 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 68 | 66 | 64 | 53 | 62 | 0 | 59 | 41 | 0 |
| N.S. | 1 | 0.62 | 0.60 | 0.58 | 0.48 | 0.56 | 0.00 | 0.54 | 0.37 | 0.00 |
| time (sec) | N/A | 0.379 | 0.499 | 0.051 | 0.107 | 0.119 | 0.000 | 0.165 | 0.193 | 0.000 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 37 | 41 | 42 | 26 | 37 | 0 | 26 | 17 | 0 |
| N.S. | 1 | 0.74 | 0.82 | 0.84 | 0.52 | 0.74 | 0.00 | 0.52 | 0.34 | 0.00 |
| time (sec) | N/A | 0.239 | 0.064 | 0.050 | 0.110 | 0.103 | 0.000 | 0.130 | 0.211 | 0.000 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 38 | 43 | 40 | 27 | 39 | 0 | 33 | 29 | 0 |
| N.S. | 1 | 0.75 | 0.84 | 0.78 | 0.53 | 0.76 | 0.00 | 0.65 | 0.57 | 0.00 |
| time (sec) | N/A | 0.232 | 0.054 | 0.051 | 0.111 | 0.097 | 0.000 | 0.140 | 0.164 | 0.000 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 71 | 45 | 63 | 50 | 62 | 0 | 56 | 52 | 0 |
| N.S. | 1 | 0.60 | 0.38 | 0.53 | 0.42 | 0.52 | 0.00 | 0.47 | 0.44 | 0.00 |
| time (sec) | N/A | 0.384 | 0.033 | 0.056 | 0.110 | 0.086 | 0.000 | 0.194 | 0.149 | 0.000 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 101 | 45 | 83 | 70 | 82 | 0 | 76 | 72 | 0 |
| N.S. | 1 | 0.55 | 0.25 | 0.45 | 0.38 | 0.45 | 0.00 | 0.42 | 0.39 | 0.00 |
| time (sec) | N/A | 0.556 | 0.023 | 0.065 | 0.106 | 0.088 | 0.000 | 0.189 | 0.173 | 0.000 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 59 | 0 | 0 | 0 | 0 | 0 | 16 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 0.270 | 0.072 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.185 | 0.000 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 51 | 0 | 0 | 0 | 0 | 0 | 16 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 0.270 | 0.062 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.166 | 0.000 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 57 | 0 | 0 | 0 | 0 | 0 | 16 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 0.280 | 0.058 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.153 | 0.000 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 55 | 0 | 0 | 0 | 0 | 0 | 16 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 0.281 | 0.057 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.166 | 0.000 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 63 | 0 | 0 | 0 | 0 | 0 | 18 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.25 | 0.00 |
| time (sec) | N/A | 0.288 | 0.086 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.189 | 0.000 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 61 | 0 | 0 | 0 | 0 | 0 | 16 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.25 | 0.00 |
| time (sec) | N/A | 0.277 | 0.073 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.165 | 0.000 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 70 | 56 | 0 | 0 | 0 | 0 | 0 | 15 | 0 |
| N.S. | 1 | 1.25 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 0.271 | 0.062 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.163 | 0.000 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 60 | 0 | 0 | 0 | 0 | 0 | 20 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.32 | 0.00 |
| time (sec) | N/A | 0.276 | 0.070 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.166 | 0.000 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 61 | 0 | 0 | 0 | 0 | 0 | 20 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 0.280 | 0.084 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.187 | 0.000 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 63 | 0 | 0 | 0 | 0 | 0 | 20 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 0.280 | 0.082 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.218 | 0.000 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 32 | 5979 | 0 | 23 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 186.84 | 0.00 | 0.72 | 0.00 | 0.00 | 0.66 | 0.00 |
| time (sec) | N/A | 0.243 | 0.015 | 3.476 | 0.000 | 0.085 | 0.000 | 0.000 | 0.216 | 0.000 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 61 | 0 | 0 | 0 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.303 | 0.062 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.222 | 0.000 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | A | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 251 | 122 | 497 | 225 | 453 | 0 | 245 | 21 | 0 |
| N.S. | 1 | 1.23 | 0.60 | 2.44 | 1.10 | 2.22 | 0.00 | 1.20 | 0.10 | 0.00 |
| time (sec) | N/A | 0.423 | 0.499 | 4.260 | 0.112 | 0.137 | 0.000 | 0.150 | 0.194 | 0.000 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | A | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 212 | 104 | 404 | 194 | 440 | 0 | 219 | 21 | 0 |
| N.S. | 1 | 1.22 | 0.60 | 2.32 | 1.11 | 2.53 | 0.00 | 1.26 | 0.12 | 0.00 |
| time (sec) | N/A | 0.398 | 0.268 | 0.543 | 0.113 | 0.127 | 0.000 | 0.140 | 0.164 | 0.000 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 17 | 23 | 37 | 0 | 16 | 21 | 48 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.28 | 2.06 | 0.00 | 0.89 | 1.17 | 2.67 |
| time (sec) | N/A | 0.196 | 0.271 | 0.107 | 0.036 | 0.092 | 0.000 | 0.148 | 0.164 | 0.680 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 39 | 30 | 43 | 33 | 63 | 0 | 43 | 21 | 102 |
| N.S. | 1 | 0.95 | 0.73 | 1.05 | 0.80 | 1.54 | 0.00 | 1.05 | 0.51 | 2.49 |
| time (sec) | N/A | 0.227 | 0.239 | 0.410 | 0.028 | 0.127 | 0.000 | 0.150 | 0.163 | 4.356 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 58 | 50 | 52 | 48 | 82 | 0 | 58 | 21 | 356 |
| N.S. | 1 | 0.92 | 0.79 | 0.83 | 0.76 | 1.30 | 0.00 | 0.92 | 0.33 | 5.65 |
| time (sec) | N/A | 0.222 | 0.285 | 0.446 | 0.031 | 0.110 | 0.000 | 0.169 | 0.188 | 4.706 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 108 | 139 | 1585 | 0 | 0 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.03 | 1.32 | 15.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.00 |
| time (sec) | N/A | 0.563 | 11.611 | 5.189 | 0.000 | 0.000 | 0.000 | 0.000 | 0.226 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 57 | 122 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 1.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.25 | 0.00 |
| time (sec) | N/A | 0.413 | 3.202 | 2.709 | 0.000 | 0.000 | 0.000 | 0.000 | 0.215 | 0.000 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 73 | 107 | 0 | 54 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.55 | 2.28 | 0.00 | 1.15 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.315 | 0.181 | 0.707 | 0.000 | 0.135 | 0.000 | 0.000 | 0.213 | 0.000 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 115 | 124 | 0 | 113 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.00 | 1.49 | 1.61 | 0.00 | 1.47 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 0.429 | 0.534 | 0.778 | 0.000 | 0.137 | 0.000 | 0.000 | 0.168 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 110 | 124 | 141 | 0 | 157 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.05 | 1.18 | 1.34 | 0.00 | 1.50 | 0.00 | 0.00 | 0.20 | 0.00 |
| time (sec) | N/A | 0.554 | 1.163 | 0.897 | 0.000 | 0.102 | 0.000 | 0.000 | 0.172 | 0.000 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F(-1) | A | F | F(-1) |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 277 | 123 | 1375 | 235 | 451 | 0 | 269 | 28 | 0 |
| N.S. | 1 | 1.24 | 0.55 | 6.14 | 1.05 | 2.01 | 0.00 | 1.20 | 0.12 | 0.00 |
| time (sec) | N/A | 0.439 | 0.594 | 11.257 | 0.120 | 0.216 | 0.000 | 0.156 | 0.185 | 0.000 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F(-1) | A | F | F(-1) |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 238 | 113 | 1065 | 204 | 439 | 0 | 243 | 28 | 0 |
| N.S. | 1 | 1.23 | 0.58 | 5.49 | 1.05 | 2.26 | 0.00 | 1.25 | 0.14 | 0.00 |
| time (sec) | N/A | 0.418 | 0.351 | 6.926 | 0.119 | 0.149 | 0.000 | 0.140 | 0.202 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 17 | 23 | 24 | 0 | 16 | 28 | 43 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.28 | 1.33 | 0.00 | 0.89 | 1.56 | 2.39 |
| time (sec) | N/A | 0.195 | 0.267 | 0.122 | 0.027 | 0.142 | 0.000 | 0.146 | 0.230 | 0.395 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 39 | 30 | 38 | 34 | 51 | 0 | 50 | 28 | 100 |
| N.S. | 1 | 0.95 | 0.73 | 0.93 | 0.83 | 1.24 | 0.00 | 1.22 | 0.68 | 2.44 |
| time (sec) | N/A | 0.216 | 0.228 | 0.818 | 0.033 | 0.104 | 0.000 | 0.168 | 0.224 | 1.166 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 58 | 42 | 47 | 58 | 71 | 0 | 69 | 28 | 292 |
| N.S. | 1 | 0.92 | 0.67 | 0.75 | 0.92 | 1.13 | 0.00 | 1.10 | 0.44 | 4.63 |
| time (sec) | N/A | 0.223 | 0.329 | 0.896 | 0.038 | 0.108 | 0.000 | 0.177 | 0.210 | 3.611 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 111 | 90 | 288 | 0 | 0 | 0 | 0 | 28 | 0 |
| N.S. | 1 | 1.01 | 0.82 | 2.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.25 | 0.00 |
| time (sec) | N/A | 0.574 | 3.498 | 1.399 | 0.000 | 0.000 | 0.000 | 0.000 | 0.170 | 0.000 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 58 | 278 | 0 | 0 | 0 | 0 | 26 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 3.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.414 | 0.271 | 1.048 | 0.000 | 0.000 | 0.000 | 0.000 | 0.171 | 0.000 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 61 | 267 | 0 | 136 | 0 | 0 | 26 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 3.51 | 0.00 | 1.79 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.420 | 0.284 | 0.972 | 0.000 | 0.105 | 0.000 | 0.000 | 0.205 | 0.000 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 138 | 71 | 212 | 0 | 176 | 0 | 0 | 28 | 0 |
| N.S. | 1 | 1.35 | 0.70 | 2.08 | 0.00 | 1.73 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 0.580 | 0.537 | 0.968 | 0.000 | 0.116 | 0.000 | 0.000 | 0.221 | 0.000 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F(-1) | A | F | F(-1) |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 273 | 142 | 493 | 240 | 504 | 0 | 278 | 32 | 0 |
| N.S. | 1 | 1.22 | 0.63 | 2.20 | 1.07 | 2.25 | 0.00 | 1.24 | 0.14 | 0.00 |
| time (sec) | N/A | 0.427 | 0.677 | 30.458 | 0.117 | 0.204 | 0.000 | 0.171 | 0.238 | 0.000 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F(-1) | A | F | F(-1) |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 234 | 126 | 488 | 209 | 491 | 0 | 252 | 32 | 0 |
| N.S. | 1 | 1.21 | 0.65 | 2.52 | 1.08 | 2.53 | 0.00 | 1.30 | 0.16 | 0.00 |
| time (sec) | N/A | 0.406 | 0.381 | 5.279 | 0.119 | 0.202 | 0.000 | 0.166 | 0.218 | 0.000 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 17 | 23 | 40 | 0 | 24 | 32 | 56 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 1.15 | 2.00 | 0.00 | 1.20 | 1.60 | 2.80 |
| time (sec) | N/A | 0.199 | 0.307 | 0.182 | 0.035 | 0.118 | 0.000 | 0.161 | 0.180 | 0.340 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 39 | 32 | 42 | 36 | 58 | 0 | 42 | 32 | 64 |
| N.S. | 1 | 0.95 | 0.78 | 1.02 | 0.88 | 1.41 | 0.00 | 1.02 | 0.78 | 1.56 |
| time (sec) | N/A | 0.210 | 0.269 | 3.011 | 0.034 | 0.140 | 0.000 | 0.176 | 0.190 | 0.598 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 58 | 42 | 55 | 56 | 82 | 0 | 70 | 32 | 134 |
| N.S. | 1 | 0.92 | 0.67 | 0.87 | 0.89 | 1.30 | 0.00 | 1.11 | 0.51 | 2.13 |
| time (sec) | N/A | 0.220 | 0.415 | 67.622 | 0.035 | 0.110 | 0.000 | 0.204 | 0.180 | 3.558 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 142 | 153 | 1532 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.04 | 1.12 | 11.18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 0.703 | 2.240 | 10.211 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 109 | 133 | 131 | 0 | 0 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.01 | 1.23 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 0.525 | 1.603 | 3.094 | 0.000 | 0.000 | 0.000 | 0.000 | 0.228 | 0.000 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 71 | 125 | 0 | 103 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.56 | 0.00 | 1.29 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.427 | 0.405 | 0.936 | 0.000 | 0.107 | 0.000 | 0.000 | 0.174 | 0.000 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 71 | 119 | 0 | 103 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.49 | 0.00 | 1.29 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.434 | 0.478 | 1.517 | 0.000 | 0.125 | 0.000 | 0.000 | 0.181 | 0.000 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 111 | 110 | 146 | 0 | 162 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.01 | 1.00 | 1.33 | 0.00 | 1.47 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.557 | 0.594 | 10.316 | 0.000 | 0.130 | 0.000 | 0.000 | 0.207 | 0.000 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 146 | 130 | 159 | 0 | 209 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.04 | 0.93 | 1.14 | 0.00 | 1.49 | 0.00 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 0.691 | 1.520 | 231.624 | 0.000 | 0.106 | 0.000 | 0.000 | 0.249 | 0.000 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | A | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 257 | 122 | 548 | 220 | 429 | 0 | 246 | 32 | 0 |
| N.S. | 1 | 1.26 | 0.60 | 2.69 | 1.08 | 2.10 | 0.00 | 1.21 | 0.16 | 0.00 |
| time (sec) | N/A | 0.410 | 0.495 | 7.861 | 0.123 | 0.143 | 0.000 | 0.144 | 0.164 | 0.000 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | A | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 218 | 109 | 403 | 188 | 413 | 0 | 218 | 32 | 0 |
| N.S. | 1 | 1.25 | 0.63 | 2.32 | 1.08 | 2.37 | 0.00 | 1.25 | 0.18 | 0.00 |
| time (sec) | N/A | 0.403 | 0.313 | 1.822 | 0.116 | 0.145 | 0.000 | 0.141 | 0.163 | 0.000 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 17 | 23 | 46 | 0 | 23 | 32 | 102 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 1.15 | 2.30 | 0.00 | 1.15 | 1.60 | 5.10 |
| time (sec) | N/A | 0.192 | 0.227 | 0.131 | 0.028 | 0.094 | 0.000 | 0.141 | 0.166 | 1.114 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 41 | 40 | 98 | 35 | 70 | 0 | 45 | 32 | 530 |
| N.S. | 1 | 0.95 | 0.93 | 2.28 | 0.81 | 1.63 | 0.00 | 1.05 | 0.74 | 12.33 |
| time (sec) | N/A | 0.217 | 0.221 | 0.676 | 0.040 | 0.142 | 0.000 | 0.170 | 0.200 | 4.996 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 60 | 50 | 107 | 48 | 93 | 0 | 58 | 32 | 831 |
| N.S. | 1 | 0.92 | 0.77 | 1.65 | 0.74 | 1.43 | 0.00 | 0.89 | 0.49 | 12.78 |
| time (sec) | N/A | 0.227 | 0.242 | 0.784 | 0.036 | 0.137 | 0.000 | 0.173 | 0.221 | 8.363 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 112 | 86 | 227 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.05 | 0.80 | 2.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.30 | 0.00 |
| time (sec) | N/A | 0.574 | 0.772 | 1.263 | 0.000 | 0.000 | 0.000 | 0.000 | 0.163 | 0.000 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 98 | 216 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.00 | 1.24 | 2.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.41 | 0.00 |
| time (sec) | N/A | 0.421 | 0.642 | 0.903 | 0.000 | 0.000 | 0.000 | 0.000 | 0.171 | 0.000 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 60 | 207 | 0 | 0 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.00 | 1.28 | 4.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.64 | 0.00 |
| time (sec) | N/A | 0.308 | 0.189 | 0.749 | 0.000 | 0.000 | 0.000 | 0.000 | 0.185 | 0.000 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 109 | 69 | 281 | 0 | 169 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.51 | 0.96 | 3.90 | 0.00 | 2.35 | 0.00 | 0.00 | 0.42 | 0.00 |
| time (sec) | N/A | 0.446 | 0.272 | 0.725 | 0.000 | 0.136 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 139 | 104 | 264 | 0 | 237 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.36 | 1.02 | 2.59 | 0.00 | 2.32 | 0.00 | 0.00 | 0.31 | 0.00 |
| time (sec) | N/A | 0.575 | 0.592 | 0.871 | 0.000 | 0.109 | 0.000 | 0.000 | 0.219 | 0.000 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | A | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 257 | 123 | 519 | 225 | 433 | 0 | 257 | 32 | 0 |
| N.S. | 1 | 1.26 | 0.60 | 2.54 | 1.10 | 2.12 | 0.00 | 1.26 | 0.16 | 0.00 |
| time (sec) | N/A | 0.434 | 0.436 | 6.418 | 0.122 | 0.147 | 0.000 | 0.168 | 0.168 | 0.000 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | A | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 218 | 105 | 427 | 193 | 419 | 0 | 228 | 32 | 0 |
| N.S. | 1 | 1.25 | 0.60 | 2.45 | 1.11 | 2.41 | 0.00 | 1.31 | 0.18 | 0.00 |
| time (sec) | N/A | 0.407 | 0.299 | 6.243 | 0.122 | 0.121 | 0.000 | 0.166 | 0.160 | 0.000 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 17 | 23 | 58 | 0 | 26 | 32 | 381 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 1.15 | 2.90 | 0.00 | 1.30 | 1.60 | 19.05 |
| time (sec) | N/A | 0.203 | 0.250 | 0.129 | 0.032 | 0.092 | 0.000 | 0.178 | 0.162 | 4.294 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 41 | 42 | 48 | 35 | 84 | 0 | 45 | 32 | 684 |
| N.S. | 1 | 0.95 | 0.98 | 1.12 | 0.81 | 1.95 | 0.00 | 1.05 | 0.74 | 15.91 |
| time (sec) | N/A | 0.215 | 0.203 | 0.678 | 0.027 | 0.115 | 0.000 | 0.229 | 0.194 | 5.986 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 60 | 54 | 57 | 48 | 109 | 0 | 58 | 32 | 987 |
| N.S. | 1 | 0.92 | 0.83 | 0.88 | 0.74 | 1.68 | 0.00 | 0.89 | 0.49 | 15.18 |
| time (sec) | N/A | 0.224 | 0.249 | 0.817 | 0.033 | 0.147 | 0.000 | 0.260 | 0.216 | 11.635 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 113 | 102 | 1510 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.01 | 0.91 | 13.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.568 | 0.401 | 3.213 | 0.000 | 0.000 | 0.000 | 0.000 | 0.225 | 0.000 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 113 | 126 | 114 | 0 | 0 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.43 | 1.59 | 1.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.463 | 0.616 | 2.741 | 0.000 | 0.000 | 0.000 | 0.000 | 0.227 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 110 | 116 | 0 | 119 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.00 | 1.34 | 1.41 | 0.00 | 1.45 | 0.00 | 0.00 | 0.37 | 0.00 |
| time (sec) | N/A | 0.429 | 0.609 | 0.743 | 0.000 | 0.118 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 115 | 136 | 132 | 0 | 163 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.03 | 1.21 | 1.18 | 0.00 | 1.46 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.562 | 2.046 | 0.890 | 0.000 | 0.132 | 0.000 | 0.000 | 0.224 | 0.000 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | A | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 263 | 123 | 550 | 219 | 429 | 0 | 248 | 32 | 0 |
| N.S. | 1 | 1.29 | 0.60 | 2.70 | 1.07 | 2.10 | 0.00 | 1.22 | 0.16 | 0.00 |
| time (sec) | N/A | 0.421 | 1.084 | 8.614 | 0.113 | 0.130 | 0.000 | 0.188 | 0.160 | 0.000 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | A | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 224 | 113 | 407 | 189 | 416 | 0 | 220 | 32 | 0 |
| N.S. | 1 | 1.29 | 0.65 | 2.34 | 1.09 | 2.39 | 0.00 | 1.26 | 0.18 | 0.00 |
| time (sec) | N/A | 0.435 | 0.684 | 7.982 | 0.116 | 0.153 | 0.000 | 0.169 | 0.163 | 0.000 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 17 | 23 | 63 | 0 | 26 | 32 | 530 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 1.15 | 3.15 | 0.00 | 1.30 | 1.60 | 26.50 |
| time (sec) | N/A | 0.198 | 0.497 | 0.130 | 0.029 | 0.124 | 0.000 | 0.202 | 0.165 | 5.151 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 41 | 50 | 103 | 35 | 91 | 0 | 45 | 32 | 831 |
| N.S. | 1 | 0.95 | 1.16 | 2.40 | 0.81 | 2.12 | 0.00 | 1.05 | 0.74 | 19.33 |
| time (sec) | N/A | 0.219 | 0.464 | 0.775 | 0.033 | 0.144 | 0.000 | 0.252 | 0.195 | 9.378 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 60 | 60 | 112 | 48 | 114 | 0 | 58 | 32 | 1132 |
| N.S. | 1 | 0.92 | 0.92 | 1.72 | 0.74 | 1.75 | 0.00 | 0.89 | 0.49 | 17.42 |
| time (sec) | N/A | 0.228 | 0.597 | 0.921 | 0.034 | 0.158 | 0.000 | 0.296 | 0.224 | 11.115 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 150 | 122 | 240 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.04 | 0.85 | 1.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.00 |
| time (sec) | N/A | 0.741 | 2.488 | 2.219 | 0.000 | 0.000 | 0.000 | 0.000 | 0.216 | 0.000 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 117 | 100 | 230 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.03 | 0.88 | 2.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 0.568 | 1.672 | 1.413 | 0.000 | 0.000 | 0.000 | 0.000 | 0.212 | 0.000 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 97 | 219 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.00 | 1.15 | 2.61 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.425 | 0.972 | 0.994 | 0.000 | 0.000 | 0.000 | 0.000 | 0.181 | 0.000 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 69 | 264 | 0 | 0 | 0 | 0 | 145 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 3.38 | 0.00 | 0.00 | 0.00 | 0.00 | 1.86 | 0.00 |
| time (sec) | N/A | 0.413 | 0.673 | 0.850 | 0.000 | 0.000 | 0.000 | 0.000 | 0.230 | 0.000 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 144 | 105 | 270 | 0 | 246 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.31 | 0.95 | 2.45 | 0.00 | 2.24 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 0.580 | 1.863 | 0.877 | 0.000 | 0.145 | 0.000 | 0.000 | 0.176 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 177 | 116 | 286 | 0 | 309 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.26 | 0.83 | 2.04 | 0.00 | 2.21 | 0.00 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 0.742 | 1.057 | 0.998 | 0.000 | 0.137 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | A | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 51 | 340 | 0 | 65 | 0 | 0 | 33 | 80 |
| N.S. | 1 | 1.00 | 0.75 | 5.00 | 0.00 | 0.96 | 0.00 | 0.00 | 0.49 | 1.18 |
| time (sec) | N/A | 0.312 | 0.992 | 1.765 | 0.000 | 0.097 | 0.000 | 0.000 | 0.179 | 2.233 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 80 | 108 | 0 | 102 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 1.23 | 0.00 | 1.16 | 0.00 | 0.00 | 0.33 | 0.00 |
| time (sec) | N/A | 0.392 | 6.739 | 2.961 | 0.000 | 0.097 | 0.000 | 0.000 | 0.178 | 0.000 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 71 | 0 | 47 | 0 | 0 | 22 | 60 |
| N.S. | 1 | 1.00 | 1.00 | 2.37 | 0.00 | 1.57 | 0.00 | 0.00 | 0.73 | 2.00 |
| time (sec) | N/A | 0.194 | 0.703 | 1.620 | 0.000 | 0.087 | 0.000 | 0.000 | 0.200 | 0.704 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 60 | 79 | 0 | 65 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 1.00 | 1.20 | 1.58 | 0.00 | 1.30 | 0.00 | 0.00 | 0.66 | 0.00 |
| time (sec) | N/A | 0.261 | 0.512 | 1.542 | 0.000 | 0.087 | 0.000 | 0.000 | 0.208 | 0.000 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 73 | 72 | 164 | 0 | 413 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 0.68 | 0.67 | 1.53 | 0.00 | 3.86 | 0.00 | 0.00 | 0.31 | 0.00 |
| time (sec) | N/A | 0.331 | 0.867 | 1.263 | 0.000 | 0.261 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 79 | 100 | 0 | 135 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.16 | 0.00 | 1.57 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.401 | 0.781 | 1.539 | 0.000 | 0.106 | 0.000 | 0.000 | 0.215 | 0.000 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 124 | 99 | 228 | 0 | 127 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 0.98 | 0.79 | 1.81 | 0.00 | 1.01 | 0.00 | 0.00 | 0.32 | 0.00 |
| time (sec) | N/A | 0.523 | 6.970 | 5.112 | 0.000 | 0.125 | 0.000 | 0.000 | 0.168 | 0.000 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | A | F(-1) | F(-2) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 45 | 285 | 0 | 57 | 0 | 0 | 36 | 69 |
| N.S. | 1 | 1.00 | 0.66 | 4.19 | 0.00 | 0.84 | 0.00 | 0.00 | 0.53 | 1.01 |
| time (sec) | N/A | 0.326 | 0.663 | 2.203 | 0.000 | 0.097 | 0.000 | 0.000 | 0.173 | 1.379 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 83 | 208 | 0 | 102 | 0 | 0 | 88 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 2.48 | 0.00 | 1.21 | 0.00 | 0.00 | 1.05 | 0.00 |
| time (sec) | N/A | 0.374 | 0.672 | 2.710 | 0.000 | 0.110 | 0.000 | 0.000 | 0.171 | 0.000 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | A | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 253 | 0 | 45 | 0 | 0 | 40 | 39 |
| N.S. | 1 | 1.00 | 1.00 | 8.43 | 0.00 | 1.50 | 0.00 | 0.00 | 1.33 | 1.30 |
| time (sec) | N/A | 0.213 | 0.488 | 2.183 | 0.000 | 0.094 | 0.000 | 0.000 | 0.211 | 0.735 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 92 | 197 | 0 | 103 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 2.19 | 0.00 | 1.14 | 0.00 | 0.00 | 0.44 | 0.00 |
| time (sec) | N/A | 0.399 | 0.700 | 2.075 | 0.000 | 0.104 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 110 | 104 | 209 | 0 | 524 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 0.76 | 0.72 | 1.44 | 0.00 | 3.61 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 0.457 | 0.805 | 2.227 | 0.000 | 0.400 | 0.000 | 0.000 | 0.173 | 0.000 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 128 | 100 | 225 | 0 | 136 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.04 | 0.81 | 1.83 | 0.00 | 1.11 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.553 | 1.614 | 5.433 | 0.000 | 0.100 | 0.000 | 0.000 | 0.154 | 0.000 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 52 | 48 | 0 | 71 | 0 | 0 | 44 | 88 |
| N.S. | 1 | 1.00 | 0.76 | 0.71 | 0.00 | 1.04 | 0.00 | 0.00 | 0.65 | 1.29 |
| time (sec) | N/A | 0.329 | 0.843 | 0.804 | 0.000 | 0.121 | 0.000 | 0.000 | 0.156 | 2.550 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 87 | 201 | 0 | 119 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 2.28 | 0.00 | 1.35 | 0.00 | 0.00 | 0.50 | 0.00 |
| time (sec) | N/A | 0.385 | 0.653 | 3.032 | 0.000 | 0.141 | 0.000 | 0.000 | 0.159 | 0.000 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 32 | 33 | 0 | 53 | 0 | 0 | 40 | 69 |
| N.S. | 1 | 1.00 | 1.00 | 1.03 | 0.00 | 1.66 | 0.00 | 0.00 | 1.25 | 2.16 |
| time (sec) | N/A | 0.207 | 0.534 | 0.725 | 0.000 | 0.098 | 0.000 | 0.000 | 0.200 | 1.286 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 69 | 187 | 0 | 71 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 1.00 | 1.38 | 3.74 | 0.00 | 1.42 | 0.00 | 0.00 | 0.66 | 0.00 |
| time (sec) | N/A | 0.263 | 0.522 | 1.355 | 0.000 | 0.093 | 0.000 | 0.000 | 0.215 | 0.000 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 73 | 80 | 157 | 0 | 419 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 0.69 | 0.75 | 1.48 | 0.00 | 3.95 | 0.00 | 0.00 | 0.42 | 0.00 |
| time (sec) | N/A | 0.308 | 0.487 | 0.765 | 0.000 | 0.282 | 0.000 | 0.000 | 0.164 | 0.000 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 89 | 171 | 0 | 145 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 1.97 | 0.00 | 1.67 | 0.00 | 0.00 | 0.51 | 0.00 |
| time (sec) | N/A | 0.383 | 0.737 | 1.108 | 0.000 | 0.117 | 0.000 | 0.000 | 0.171 | 0.000 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 111 | 112 | 294 | 0 | 605 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 0.76 | 0.77 | 2.01 | 0.00 | 4.14 | 0.00 | 0.00 | 0.30 | 0.00 |
| time (sec) | N/A | 0.451 | 1.031 | 0.872 | 0.000 | 0.316 | 0.000 | 0.000 | 0.162 | 0.000 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 153 | 67 | 63 | 0 | 84 | 0 | 0 | 44 | 296 |
| N.S. | 1 | 1.05 | 0.46 | 0.43 | 0.00 | 0.58 | 0.00 | 0.00 | 0.30 | 2.03 |
| time (sec) | N/A | 0.618 | 2.093 | 0.872 | 0.000 | 0.110 | 0.000 | 0.000 | 0.195 | 6.465 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 113 | 57 | 53 | 0 | 71 | 0 | 0 | 44 | 94 |
| N.S. | 1 | 1.04 | 0.52 | 0.49 | 0.00 | 0.65 | 0.00 | 0.00 | 0.40 | 0.86 |
| time (sec) | N/A | 0.438 | 1.132 | 0.874 | 0.000 | 0.095 | 0.000 | 0.000 | 0.217 | 3.319 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F(-1) | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 45 | 40 | 0 | 55 | 0 | 0 | 44 | 81 |
| N.S. | 1 | 1.00 | 1.41 | 1.25 | 0.00 | 1.72 | 0.00 | 0.00 | 1.38 | 2.53 |
| time (sec) | N/A | 0.213 | 0.639 | 0.793 | 0.000 | 0.093 | 0.000 | 0.000 | 0.217 | 1.814 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 107 | 88 | 200 | 0 | 529 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 0.76 | 0.62 | 1.42 | 0.00 | 3.75 | 0.00 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 0.449 | 0.771 | 1.322 | 0.000 | 0.266 | 0.000 | 0.000 | 0.207 | 0.000 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 113 | 103 | 288 | 0 | 608 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 0.75 | 0.68 | 1.91 | 0.00 | 4.03 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.453 | 0.714 | 0.884 | 0.000 | 0.379 | 0.000 | 0.000 | 0.187 | 0.000 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | C | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 173 | 118 | 127 | 0 | 140 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.04 | 0.71 | 0.76 | 0.00 | 0.84 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.749 | 2.201 | 9.553 | 0.000 | 0.109 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | C | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 133 | 97 | 117 | 0 | 128 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.02 | 0.75 | 0.90 | 0.00 | 0.98 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.542 | 1.175 | 5.117 | 0.000 | 0.112 | 0.000 | 0.000 | 0.165 | 0.000 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | C | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 80 | 102 | 0 | 103 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.10 | 0.00 | 1.11 | 0.00 | 0.00 | 0.43 | 0.00 |
| time (sec) | N/A | 0.399 | 0.780 | 2.250 | 0.000 | 0.092 | 0.000 | 0.000 | 0.168 | 0.000 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 79 | 102 | 0 | 139 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.19 | 0.00 | 1.62 | 0.00 | 0.00 | 0.51 | 0.00 |
| time (sec) | N/A | 0.388 | 0.646 | 1.501 | 0.000 | 0.098 | 0.000 | 0.000 | 0.189 | 0.000 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | C | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 128 | 96 | 121 | 0 | 187 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 0.98 | 0.74 | 0.93 | 0.00 | 1.44 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.556 | 0.983 | 1.469 | 0.000 | 0.103 | 0.000 | 0.000 | 0.225 | 0.000 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | C | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 171 | 106 | 132 | 0 | 241 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.02 | 0.63 | 0.79 | 0.00 | 1.44 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.684 | 1.323 | 2.469 | 0.000 | 0.138 | 0.000 | 0.000 | 0.170 | 0.000 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 69 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.305 | 11.082 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.182 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 69 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.306 | 10.818 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.166 | 0.000 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 64 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.305 | 10.783 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 67 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 |
| time (sec) | N/A | 0.315 | 10.875 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 63 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.323 | 11.028 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.189 | 0.000 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 63 | 0 | 0 | 0 | 0 | 0 | 85 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.33 | 0.00 |
| time (sec) | N/A | 0.311 | 10.809 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.183 | 0.000 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 63 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.317 | 10.818 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.164 | 0.000 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 69 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.307 | 10.860 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.221 | 0.000 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 65 | 0 | 0 | 0 | 0 | 0 | 84 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.31 | 0.00 |
| time (sec) | N/A | 0.318 | 11.143 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.224 | 0.000 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 66 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.305 | 10.839 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.180 | 0.000 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 66 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.303 | 10.873 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.176 | 0.000 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 121 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 |
| time (sec) | N/A | 0.326 | 31.560 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 85 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.309 | 11.031 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.246 | 0.000 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 72 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.306 | 10.971 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 67 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.312 | 11.148 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.166 | 0.000 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 70 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.308 | 10.945 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.167 | 0.000 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 53 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.211 | 0.066 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.205 | 0.000 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 53 | 0 | 0 | 0 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.44 | 0.00 |
| time (sec) | N/A | 0.208 | 0.037 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.207 | 0.000 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 18 | 18 | 17 | 0 | 17 | 21 | 17 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.06 | 1.00 | 0.00 | 1.00 | 1.24 | 1.00 |
| time (sec) | N/A | 0.190 | 0.011 | 0.482 | 0.057 | 0.079 | 0.000 | 0.121 | 0.182 | 0.305 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F | F | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 44 | 37 | 2751 | 47 | 57 | 0 | 0 | 23 | 91 |
| N.S. | 1 | 0.96 | 0.80 | 59.80 | 1.02 | 1.24 | 0.00 | 0.00 | 0.50 | 1.98 |
| time (sec) | N/A | 0.232 | 0.070 | 1.035 | 0.037 | 0.083 | 0.000 | 0.000 | 0.217 | 1.198 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F | F | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 67 | 62 | 6931 | 71 | 112 | 0 | 0 | 23 | 219 |
| N.S. | 1 | 0.93 | 0.86 | 96.26 | 0.99 | 1.56 | 0.00 | 0.00 | 0.32 | 3.04 |
| time (sec) | N/A | 0.245 | 0.391 | 1.850 | 0.084 | 0.110 | 0.000 | 0.000 | 0.219 | 5.868 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 71 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.299 | 0.143 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.213 | 0.000 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 71 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.296 | 0.097 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.208 | 0.000 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 66 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.33 | 0.00 |
| time (sec) | N/A | 0.296 | 0.085 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.171 | 0.000 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 71 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.32 | 0.00 |
| time (sec) | N/A | 0.307 | 0.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.158 | 0.000 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 87 | 0 | 0 | 0 | 0 | 0 | 93 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.18 | 0.00 |
| time (sec) | N/A | 0.330 | 1.224 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.167 | 0.000 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 87 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.30 | 0.00 |
| time (sec) | N/A | 0.328 | 0.833 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.163 | 0.000 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 87 | 0 | 0 | 0 | 0 | 0 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.44 | 0.00 |
| time (sec) | N/A | 0.321 | 0.730 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 87 | 0 | 0 | 0 | 0 | 0 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.44 | 0.00 |
| time (sec) | N/A | 0.342 | 1.237 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.222 | 0.000 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 260 | 0 | 0 | 0 | 0 | 0 | 26 | 0 |
| N.S. | 1 | 1.00 | 3.13 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.31 | 0.00 |
| time (sec) | N/A | 0.353 | 2.763 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.159 | 0.000 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 53 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.46 | 0.00 |
| time (sec) | N/A | 0.218 | 0.532 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.171 | 0.000 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 53 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.46 | 0.00 |
| time (sec) | N/A | 0.219 | 0.357 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.175 | 0.000 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 22 | 30 | 28 | 42 | 0 | 0 | 23 | 53 |
| N.S. | 1 | 1.00 | 0.88 | 1.20 | 1.12 | 1.68 | 0.00 | 0.00 | 0.92 | 2.12 |
| time (sec) | N/A | 0.197 | 0.350 | 1.845 | 0.035 | 0.108 | 0.000 | 0.000 | 0.186 | 0.443 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F | F | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 51 | 46 | 5281 | 55 | 86 | 0 | 0 | 23 | 138 |
| N.S. | 1 | 0.96 | 0.87 | 99.64 | 1.04 | 1.62 | 0.00 | 0.00 | 0.43 | 2.60 |
| time (sec) | N/A | 0.229 | 0.377 | 20.091 | 0.048 | 0.085 | 0.000 | 0.000 | 0.200 | 1.691 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 75 | 69 | 10580 | 81 | 144 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 0.94 | 0.86 | 132.25 | 1.01 | 1.80 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.247 | 0.521 | 156.640 | 0.072 | 0.093 | 0.000 | 0.000 | 0.225 | 0.000 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 456 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 5.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.320 | 3.485 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.166 | 0.000 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 252 | 0 | 0 | 0 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.00 | 3.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 0.296 | 1.455 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.164 | 0.000 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 64 | 0 | 0 | 0 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 0.315 | 0.367 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.168 | 0.000 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 1242 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 15.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.307 | 14.005 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.164 | 0.000 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 1516 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 19.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.299 | 14.922 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 297 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 3.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.35 | 0.00 |
| time (sec) | N/A | 0.355 | 33.988 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.232 | 0.000 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 91 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 0.327 | 11.510 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.233 | 0.000 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 89 | 0 | 0 | 0 | 0 | 0 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 |
| time (sec) | N/A | 0.330 | 11.246 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.224 | 0.000 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 90 | 0 | 0 | 0 | 0 | 0 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 |
| time (sec) | N/A | 0.343 | 9.301 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.186 | 0.000 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 81 | 0 | 0 | 0 | 0 | 0 | 26 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.30 | 0.00 |
| time (sec) | N/A | 0.334 | 0.483 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 64 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.30 | 0.00 |
| time (sec) | N/A | 0.251 | 0.087 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.151 | 0.000 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F | B |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 229 | 109 | 489 | 207 | 217 | 0 | 0 | 21 | 97 |
| N.S. | 1 | 1.30 | 0.62 | 2.78 | 1.18 | 1.23 | 0.00 | 0.00 | 0.12 | 0.55 |
| time (sec) | N/A | 0.587 | 0.540 | 7.464 | 0.113 | 0.116 | 0.000 | 0.000 | 0.160 | 0.425 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F | B |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 209 | 91 | 473 | 190 | 208 | 0 | 0 | 21 | 83 |
| N.S. | 1 | 1.32 | 0.58 | 2.99 | 1.20 | 1.32 | 0.00 | 0.00 | 0.13 | 0.53 |
| time (sec) | N/A | 0.506 | 0.157 | 8.305 | 0.120 | 0.089 | 0.000 | 0.000 | 0.148 | 0.320 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 201 | 80 | 456 | 189 | 205 | 0 | 0 | 21 | 80 |
| N.S. | 1 | 1.30 | 0.52 | 2.94 | 1.22 | 1.32 | 0.00 | 0.00 | 0.14 | 0.52 |
| time (sec) | N/A | 0.491 | 0.143 | 7.261 | 0.136 | 0.118 | 0.000 | 0.000 | 0.158 | 0.318 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 182 | 132 | 345 | 167 | 185 | 0 | 0 | 19 | 61 |
| N.S. | 1 | 1.33 | 0.96 | 2.52 | 1.22 | 1.35 | 0.00 | 0.00 | 0.14 | 0.45 |
| time (sec) | N/A | 0.407 | 0.135 | 9.194 | 0.115 | 0.088 | 0.000 | 0.000 | 0.173 | 0.234 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 174 | 71 | 136 | 165 | 286 | 0 | 0 | 12 | 50 |
| N.S. | 1 | 1.28 | 0.52 | 1.00 | 1.21 | 2.10 | 0.00 | 0.00 | 0.09 | 0.37 |
| time (sec) | N/A | 0.364 | 0.065 | 0.181 | 0.115 | 0.093 | 0.000 | 0.000 | 0.210 | 0.376 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 205 | 164 | 149 | 178 | 315 | 0 | 0 | 37 | 74 |
| N.S. | 1 | 1.34 | 1.07 | 0.97 | 1.16 | 2.06 | 0.00 | 0.00 | 0.24 | 0.48 |
| time (sec) | N/A | 0.477 | 0.141 | 0.164 | 0.115 | 0.098 | 0.000 | 0.000 | 0.198 | 0.396 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 201 | 100 | 156 | 187 | 374 | 0 | 0 | 21 | 76 |
| N.S. | 1 | 1.26 | 0.63 | 0.98 | 1.18 | 2.35 | 0.00 | 0.00 | 0.13 | 0.48 |
| time (sec) | N/A | 0.485 | 0.197 | 0.181 | 0.118 | 0.127 | 0.000 | 0.000 | 0.163 | 0.423 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 231 | 172 | 171 | 199 | 387 | 0 | 0 | 55 | 90 |
| N.S. | 1 | 1.31 | 0.98 | 0.97 | 1.13 | 2.20 | 0.00 | 0.00 | 0.31 | 0.51 |
| time (sec) | N/A | 0.562 | 0.336 | 0.185 | 0.121 | 0.097 | 0.000 | 0.000 | 0.162 | 0.722 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F(-1) | F | F | B |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 229 | 109 | 489 | 207 | 220 | 0 | 0 | 28 | 97 |
| N.S. | 1 | 1.29 | 0.61 | 2.75 | 1.16 | 1.24 | 0.00 | 0.00 | 0.16 | 0.54 |
| time (sec) | N/A | 0.600 | 0.682 | 2.571 | 0.118 | 0.117 | 0.000 | 0.000 | 0.179 | 0.387 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F | B |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 209 | 91 | 473 | 190 | 209 | 0 | 0 | 28 | 83 |
| N.S. | 1 | 1.32 | 0.58 | 2.99 | 1.20 | 1.32 | 0.00 | 0.00 | 0.18 | 0.53 |
| time (sec) | N/A | 0.489 | 0.177 | 1.479 | 0.120 | 0.086 | 0.000 | 0.000 | 0.169 | 0.323 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F | B |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 201 | 82 | 477 | 189 | 206 | 0 | 0 | 28 | 82 |
| N.S. | 1 | 1.28 | 0.52 | 3.04 | 1.20 | 1.31 | 0.00 | 0.00 | 0.18 | 0.52 |
| time (sec) | N/A | 0.509 | 0.156 | 2.319 | 0.134 | 0.088 | 0.000 | 0.000 | 0.202 | 0.317 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 182 | 134 | 360 | 167 | 185 | 0 | 0 | 28 | 61 |
| N.S. | 1 | 1.33 | 0.98 | 2.63 | 1.22 | 1.35 | 0.00 | 0.00 | 0.20 | 0.45 |
| time (sec) | N/A | 0.424 | 0.083 | 1.110 | 0.119 | 0.116 | 0.000 | 0.000 | 0.191 | 0.298 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 176 | 72 | 138 | 167 | 185 | 0 | 0 | 26 | 54 |
| N.S. | 1 | 1.29 | 0.53 | 1.01 | 1.23 | 1.36 | 0.00 | 0.00 | 0.19 | 0.40 |
| time (sec) | N/A | 0.401 | 0.004 | 0.190 | 0.117 | 0.120 | 0.000 | 0.000 | 0.166 | 0.359 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 201 | 161 | 149 | 179 | 316 | 0 | 0 | 38 | 75 |
| N.S. | 1 | 1.31 | 1.05 | 0.97 | 1.16 | 2.05 | 0.00 | 0.00 | 0.25 | 0.49 |
| time (sec) | N/A | 0.447 | 0.119 | 0.146 | 0.142 | 0.092 | 0.000 | 0.000 | 0.159 | 0.498 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 201 | 100 | 152 | 184 | 376 | 0 | 0 | 22 | 73 |
| N.S. | 1 | 1.29 | 0.64 | 0.97 | 1.18 | 2.41 | 0.00 | 0.00 | 0.14 | 0.47 |
| time (sec) | N/A | 0.448 | 0.182 | 0.161 | 0.115 | 0.086 | 0.000 | 0.000 | 0.161 | 0.455 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 231 | 172 | 171 | 199 | 406 | 0 | 0 | 56 | 91 |
| N.S. | 1 | 1.31 | 0.97 | 0.97 | 1.12 | 2.29 | 0.00 | 0.00 | 0.32 | 0.51 |
| time (sec) | N/A | 0.568 | 0.168 | 0.166 | 0.125 | 0.087 | 0.000 | 0.000 | 0.174 | 0.785 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 229 | 94 | 503 | 207 | 205 | 0 | 0 | 32 | 97 |
| N.S. | 1 | 1.31 | 0.54 | 2.87 | 1.18 | 1.17 | 0.00 | 0.00 | 0.18 | 0.55 |
| time (sec) | N/A | 0.582 | 0.226 | 2.499 | 0.119 | 0.085 | 0.000 | 0.000 | 0.185 | 0.380 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 209 | 84 | 498 | 190 | 196 | 0 | 0 | 32 | 81 |
| N.S. | 1 | 1.34 | 0.54 | 3.19 | 1.22 | 1.26 | 0.00 | 0.00 | 0.21 | 0.52 |
| time (sec) | N/A | 0.491 | 0.183 | 1.383 | 0.118 | 0.107 | 0.000 | 0.000 | 0.183 | 0.319 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 199 | 79 | 859 | 189 | 193 | 0 | 0 | 30 | 79 |
| N.S. | 1 | 1.29 | 0.51 | 5.58 | 1.23 | 1.25 | 0.00 | 0.00 | 0.19 | 0.51 |
| time (sec) | N/A | 0.480 | 0.071 | 2.376 | 0.124 | 0.087 | 0.000 | 0.000 | 0.156 | 0.208 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 180 | 131 | 138 | 165 | 268 | 0 | 0 | 24 | 57 |
| N.S. | 1 | 1.31 | 0.96 | 1.01 | 1.20 | 1.96 | 0.00 | 0.00 | 0.18 | 0.42 |
| time (sec) | N/A | 0.363 | 0.052 | 0.194 | 0.119 | 0.087 | 0.000 | 0.000 | 0.155 | 0.443 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 173 | 74 | 135 | 164 | 268 | 0 | 0 | 15 | 58 |
| N.S. | 1 | 1.27 | 0.54 | 0.99 | 1.21 | 1.97 | 0.00 | 0.00 | 0.11 | 0.43 |
| time (sec) | N/A | 0.379 | 0.004 | 0.193 | 0.151 | 0.098 | 0.000 | 0.000 | 0.155 | 0.297 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 205 | 161 | 151 | 181 | 300 | 0 | 0 | 40 | 77 |
| N.S. | 1 | 1.31 | 1.03 | 0.97 | 1.16 | 1.92 | 0.00 | 0.00 | 0.26 | 0.49 |
| time (sec) | N/A | 0.467 | 0.102 | 0.179 | 0.117 | 0.088 | 0.000 | 0.000 | 0.162 | 0.447 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 201 | 100 | 156 | 187 | 359 | 0 | 0 | 24 | 76 |
| N.S. | 1 | 1.26 | 0.63 | 0.98 | 1.18 | 2.26 | 0.00 | 0.00 | 0.15 | 0.48 |
| time (sec) | N/A | 0.474 | 0.123 | 0.186 | 0.118 | 0.124 | 0.000 | 0.000 | 0.187 | 0.478 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F | B |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 229 | 97 | 506 | 207 | 205 | 0 | 0 | 32 | 93 |
| N.S. | 1 | 1.30 | 0.55 | 2.88 | 1.18 | 1.16 | 0.00 | 0.00 | 0.18 | 0.53 |
| time (sec) | N/A | 0.606 | 0.260 | 2.556 | 0.153 | 0.096 | 0.000 | 0.000 | 0.220 | 0.374 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | F | F | B |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 207 | 83 | 501 | 190 | 196 | 0 | 0 | 30 | 80 |
| N.S. | 1 | 1.34 | 0.54 | 3.23 | 1.23 | 1.26 | 0.00 | 0.00 | 0.19 | 0.52 |
| time (sec) | N/A | 0.485 | 0.097 | 1.411 | 0.120 | 0.128 | 0.000 | 0.000 | 0.159 | 0.218 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 197 | 82 | 157 | 187 | 377 | 0 | 0 | 24 | 76 |
| N.S. | 1 | 1.25 | 0.52 | 1.00 | 1.19 | 2.40 | 0.00 | 0.00 | 0.15 | 0.48 |
| time (sec) | N/A | 0.454 | 0.060 | 0.159 | 0.116 | 0.100 | 0.000 | 0.000 | 0.157 | 0.470 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 179 | 134 | 138 | 164 | 271 | 0 | 0 | 24 | 57 |
| N.S. | 1 | 1.31 | 0.98 | 1.01 | 1.20 | 1.98 | 0.00 | 0.00 | 0.18 | 0.42 |
| time (sec) | N/A | 0.390 | 0.101 | 0.200 | 0.117 | 0.084 | 0.000 | 0.000 | 0.152 | 0.388 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 176 | 74 | 138 | 167 | 271 | 0 | 0 | 15 | 58 |
| N.S. | 1 | 1.29 | 0.54 | 1.01 | 1.23 | 1.99 | 0.00 | 0.00 | 0.11 | 0.43 |
| time (sec) | N/A | 0.391 | 0.004 | 0.196 | 0.117 | 0.083 | 0.000 | 0.000 | 0.155 | 0.319 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 205 | 161 | 151 | 181 | 300 | 0 | 0 | 40 | 77 |
| N.S. | 1 | 1.31 | 1.03 | 0.97 | 1.16 | 1.92 | 0.00 | 0.00 | 0.26 | 0.49 |
| time (sec) | N/A | 0.485 | 0.147 | 0.174 | 0.121 | 0.084 | 0.000 | 0.000 | 0.194 | 0.477 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 201 | 100 | 156 | 187 | 359 | 0 | 0 | 24 | 76 |
| N.S. | 1 | 1.26 | 0.63 | 0.98 | 1.18 | 2.26 | 0.00 | 0.00 | 0.15 | 0.48 |
| time (sec) | N/A | 0.479 | 0.160 | 0.174 | 0.113 | 0.088 | 0.000 | 0.000 | 0.202 | 0.512 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 231 | 172 | 171 | 199 | 391 | 0 | 0 | 58 | 93 |
| N.S. | 1 | 1.29 | 0.96 | 0.96 | 1.11 | 2.18 | 0.00 | 0.00 | 0.32 | 0.52 |
| time (sec) | N/A | 0.615 | 0.311 | 0.179 | 0.119 | 0.133 | 0.000 | 0.000 | 0.173 | 0.892 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 62 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.31 | 0.00 |
| time (sec) | N/A | 0.303 | 0.054 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.158 | 0.000 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 64 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.300 | 0.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.167 | 0.000 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 64 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.283 | 0.045 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.157 | 0.000 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 67 | 0 | 0 | 0 | 0 | 0 | 26 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.293 | 0.072 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.154 | 0.000 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 62 | 52 | 52 | 51 | 59 | 0 | 76 | 21 | 334 |
| N.S. | 1 | 0.93 | 0.78 | 0.78 | 0.76 | 0.88 | 0.00 | 1.13 | 0.31 | 4.99 |
| time (sec) | N/A | 0.227 | 0.349 | 0.202 | 0.035 | 0.106 | 0.000 | 0.183 | 0.185 | 5.328 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 43 | 34 | 37 | 36 | 49 | 0 | 53 | 21 | 218 |
| N.S. | 1 | 0.96 | 0.76 | 0.82 | 0.80 | 1.09 | 0.00 | 1.18 | 0.47 | 4.84 |
| time (sec) | N/A | 0.212 | 0.235 | 0.183 | 0.053 | 0.099 | 0.000 | 0.170 | 0.232 | 4.165 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 22 | 19 | 18 | 37 | 0 | 21 | 21 | 53 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.82 | 1.68 | 0.00 | 0.95 | 0.95 | 2.41 |
| time (sec) | N/A | 0.192 | 0.023 | 0.129 | 0.033 | 0.092 | 0.000 | 0.155 | 0.215 | 0.419 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 174 | 71 | 136 | 153 | 144 | 0 | 176 | 12 | 49 |
| N.S. | 1 | 1.27 | 0.52 | 0.99 | 1.12 | 1.05 | 0.00 | 1.28 | 0.09 | 0.36 |
| time (sec) | N/A | 0.349 | 0.051 | 0.195 | 0.116 | 0.084 | 0.000 | 0.141 | 0.206 | 0.311 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 225 | 102 | 403 | 193 | 439 | 0 | 218 | 21 | 0 |
| N.S. | 1 | 1.33 | 0.60 | 2.38 | 1.14 | 2.60 | 0.00 | 1.29 | 0.12 | 0.00 |
| time (sec) | N/A | 0.411 | 0.299 | 1.193 | 0.119 | 0.147 | 0.000 | 0.146 | 0.159 | 0.000 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 108 | 102 | 295 | 0 | 184 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.01 | 0.95 | 2.76 | 0.00 | 1.72 | 0.00 | 0.00 | 0.20 | 0.00 |
| time (sec) | N/A | 0.571 | 0.456 | 1.339 | 0.000 | 0.113 | 0.000 | 0.000 | 0.159 | 0.000 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 61 | 266 | 0 | 131 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 3.55 | 0.00 | 1.75 | 0.00 | 0.00 | 0.25 | 0.00 |
| time (sec) | N/A | 0.420 | 0.274 | 1.141 | 0.000 | 0.091 | 0.000 | 0.000 | 0.164 | 0.000 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 57 | 229 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.21 | 4.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.328 | 0.155 | 0.854 | 0.000 | 0.000 | 0.000 | 0.000 | 0.171 | 0.000 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 94 | 238 | 0 | 0 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.00 | 1.16 | 2.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.434 | 0.475 | 1.095 | 0.000 | 0.000 | 0.000 | 0.000 | 0.189 | 0.000 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 116 | 86 | 250 | 0 | 0 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.05 | 0.77 | 2.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.19 | 0.00 |
| time (sec) | N/A | 0.576 | 0.773 | 1.478 | 0.000 | 0.000 | 0.000 | 0.000 | 0.152 | 0.000 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 62 | 52 | 52 | 51 | 68 | 0 | 78 | 55 | 392 |
| N.S. | 1 | 0.93 | 0.78 | 0.78 | 0.76 | 1.01 | 0.00 | 1.16 | 0.82 | 5.85 |
| time (sec) | N/A | 0.242 | 0.276 | 0.200 | 0.030 | 0.123 | 0.000 | 0.190 | 0.168 | 7.150 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 43 | 42 | 37 | 36 | 56 | 0 | 55 | 55 | 276 |
| N.S. | 1 | 0.96 | 0.93 | 0.82 | 0.80 | 1.24 | 0.00 | 1.22 | 1.22 | 6.13 |
| time (sec) | N/A | 0.223 | 0.217 | 0.203 | 0.030 | 0.139 | 0.000 | 0.175 | 0.167 | 5.048 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 22 | 19 | 18 | 45 | 0 | 24 | 55 | 100 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.82 | 2.05 | 0.00 | 1.09 | 2.50 | 4.55 |
| time (sec) | N/A | 0.196 | 0.035 | 0.128 | 0.043 | 0.105 | 0.000 | 0.148 | 0.168 | 1.481 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F(-2) | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 201 | 160 | 149 | 170 | 157 | 0 | 0 | 38 | 73 |
| N.S. | 1 | 1.30 | 1.03 | 0.96 | 1.10 | 1.01 | 0.00 | 0.00 | 0.25 | 0.47 |
| time (sec) | N/A | 0.435 | 0.167 | 0.157 | 0.115 | 0.091 | 0.000 | 0.000 | 0.157 | 0.509 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 225 | 110 | 404 | 188 | 431 | 0 | 224 | 28 | 0 |
| N.S. | 1 | 1.35 | 0.66 | 2.42 | 1.13 | 2.58 | 0.00 | 1.34 | 0.17 | 0.00 |
| time (sec) | N/A | 0.401 | 0.353 | 2.028 | 0.133 | 0.152 | 0.000 | 0.147 | 0.201 | 0.000 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 150 | 90 | 141 | 0 | 127 | 0 | 0 | 55 | 0 |
| N.S. | 1 | 1.10 | 0.66 | 1.04 | 0.00 | 0.93 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.735 | 0.727 | 1.675 | 0.000 | 0.104 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 115 | 80 | 127 | 0 | 114 | 0 | 0 | 55 | 0 |
| N.S. | 1 | 1.06 | 0.74 | 1.18 | 0.00 | 1.06 | 0.00 | 0.00 | 0.51 | 0.00 |
| time (sec) | N/A | 0.563 | 0.458 | 1.372 | 0.000 | 0.140 | 0.000 | 0.000 | 0.170 | 0.000 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 69 | 123 | 0 | 97 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.54 | 0.00 | 1.21 | 0.00 | 0.00 | 0.64 | 0.00 |
| time (sec) | N/A | 0.418 | 0.306 | 1.115 | 0.000 | 0.126 | 0.000 | 0.000 | 0.161 | 0.000 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 58 | 123 | 0 | 0 | 0 | 0 | 26 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 1.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.33 | 0.00 |
| time (sec) | N/A | 0.430 | 0.195 | 2.940 | 0.000 | 0.000 | 0.000 | 0.000 | 0.169 | 0.000 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 112 | 96 | 1588 | 0 | 0 | 0 | 0 | 28 | 0 |
| N.S. | 1 | 1.04 | 0.89 | 14.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.555 | 0.924 | 3.493 | 0.000 | 0.000 | 0.000 | 0.000 | 0.205 | 0.000 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 147 | 131 | 1526 | 0 | 0 | 0 | 0 | 28 | 0 |
| N.S. | 1 | 1.08 | 0.96 | 11.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 0.703 | 1.986 | 6.794 | 0.000 | 0.000 | 0.000 | 0.000 | 0.173 | 0.000 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 62 | 52 | 52 | 51 | 82 | 0 | 78 | 32 | 474 |
| N.S. | 1 | 0.93 | 0.78 | 0.78 | 0.76 | 1.22 | 0.00 | 1.16 | 0.48 | 7.07 |
| time (sec) | N/A | 0.230 | 0.581 | 0.090 | 0.032 | 0.175 | 0.000 | 0.226 | 0.187 | 11.230 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 43 | 42 | 37 | 36 | 69 | 0 | 55 | 32 | 352 |
| N.S. | 1 | 0.96 | 0.93 | 0.82 | 0.80 | 1.53 | 0.00 | 1.22 | 0.71 | 7.82 |
| time (sec) | N/A | 0.221 | 0.339 | 98.997 | 0.046 | 0.133 | 0.000 | 2.081 | 0.194 | 5.181 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F(-1) | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 22 | 19 | 18 | 55 | 0 | 26 | 32 | 230 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.82 | 2.50 | 0.00 | 1.18 | 1.45 | 10.45 |
| time (sec) | N/A | 0.207 | 0.037 | 1.828 | 0.031 | 0.111 | 0.000 | 0.172 | 0.195 | 3.644 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F(-2) | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 197 | 101 | 154 | 176 | 166 | 0 | 0 | 24 | 74 |
| N.S. | 1 | 1.26 | 0.65 | 0.99 | 1.13 | 1.06 | 0.00 | 0.00 | 0.15 | 0.47 |
| time (sec) | N/A | 0.424 | 0.158 | 0.177 | 0.118 | 0.093 | 0.000 | 0.000 | 0.163 | 0.564 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 225 | 107 | 408 | 194 | 443 | 0 | 231 | 32 | 0 |
| N.S. | 1 | 1.35 | 0.64 | 2.44 | 1.16 | 2.65 | 0.00 | 1.38 | 0.19 | 0.00 |
| time (sec) | N/A | 0.398 | 0.360 | 2.793 | 0.133 | 0.158 | 0.000 | 0.182 | 0.174 | 0.000 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F(-1) | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 262 | 125 | 501 | 225 | 459 | 0 | 257 | 32 | 0 |
| N.S. | 1 | 1.34 | 0.64 | 2.57 | 1.15 | 2.35 | 0.00 | 1.32 | 0.16 | 0.00 |
| time (sec) | N/A | 0.412 | 0.302 | 61.210 | 0.113 | 0.157 | 0.000 | 0.183 | 0.183 | 0.000 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 114 | 79 | 134 | 0 | 115 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.05 | 0.72 | 1.23 | 0.00 | 1.06 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.589 | 0.621 | 1.574 | 0.000 | 0.103 | 0.000 | 0.000 | 0.165 | 0.000 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 68 | 118 | 0 | 97 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.49 | 0.00 | 1.23 | 0.00 | 0.00 | 0.41 | 0.00 |
| time (sec) | N/A | 0.434 | 0.357 | 1.253 | 0.000 | 0.100 | 0.000 | 0.000 | 0.157 | 0.000 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 77 | 102 | 0 | 57 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.00 | 1.64 | 2.17 | 0.00 | 1.21 | 0.00 | 0.00 | 0.64 | 0.00 |
| time (sec) | N/A | 0.313 | 0.178 | 0.910 | 0.000 | 0.125 | 0.000 | 0.000 | 0.181 | 0.000 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 126 | 111 | 0 | 0 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.00 | 1.66 | 1.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 |
| time (sec) | N/A | 0.422 | 0.483 | 2.914 | 0.000 | 0.000 | 0.000 | 0.000 | 0.156 | 0.000 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 111 | 94 | 1507 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.02 | 0.86 | 13.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.556 | 0.948 | 5.767 | 0.000 | 0.000 | 0.000 | 0.000 | 0.154 | 0.000 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 60 | 45 | 41 | 54 | 64 | 0 | 80 | 32 | 268 |
| N.S. | 1 | 0.92 | 0.69 | 0.63 | 0.83 | 0.98 | 0.00 | 1.23 | 0.49 | 4.12 |
| time (sec) | N/A | 0.225 | 0.478 | 1.382 | 0.034 | 0.119 | 0.000 | 0.289 | 0.168 | 5.046 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 41 | 32 | 31 | 36 | 54 | 0 | 44 | 32 | 64 |
| N.S. | 1 | 0.95 | 0.74 | 0.72 | 0.84 | 1.26 | 0.00 | 1.02 | 0.74 | 1.49 |
| time (sec) | N/A | 0.221 | 0.253 | 1.066 | 0.035 | 0.105 | 0.000 | 0.251 | 0.159 | 0.829 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 19 | 18 | 40 | 0 | 18 | 32 | 51 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.90 | 2.00 | 0.00 | 0.90 | 1.60 | 2.55 |
| time (sec) | N/A | 0.197 | 0.041 | 0.145 | 0.035 | 0.100 | 0.000 | 0.213 | 0.185 | 0.415 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F(-1) | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 197 | 82 | 157 | 167 | 177 | 0 | 0 | 24 | 76 |
| N.S. | 1 | 1.26 | 0.53 | 1.01 | 1.07 | 1.13 | 0.00 | 0.00 | 0.15 | 0.49 |
| time (sec) | N/A | 0.425 | 0.082 | 0.167 | 0.119 | 0.103 | 0.000 | 0.000 | 0.165 | 0.533 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 247 | 115 | 505 | 204 | 463 | 0 | 252 | 32 | 0 |
| N.S. | 1 | 1.29 | 0.60 | 2.64 | 1.07 | 2.42 | 0.00 | 1.32 | 0.17 | 0.00 |
| time (sec) | N/A | 0.455 | 0.394 | 6.286 | 0.139 | 0.168 | 0.000 | 0.162 | 0.157 | 0.000 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 142 | 104 | 217 | 0 | 222 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.03 | 0.75 | 1.57 | 0.00 | 1.61 | 0.00 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 0.764 | 0.808 | 1.473 | 0.000 | 0.133 | 0.000 | 0.000 | 0.164 | 0.000 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 107 | 93 | 363 | 0 | 172 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.03 | 0.89 | 3.49 | 0.00 | 1.65 | 0.00 | 0.00 | 0.31 | 0.00 |
| time (sec) | N/A | 0.592 | 0.478 | 1.212 | 0.000 | 0.102 | 0.000 | 0.000 | 0.159 | 0.000 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 69 | 306 | 0 | 169 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 3.92 | 0.00 | 2.17 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.455 | 0.352 | 0.957 | 0.000 | 0.122 | 0.000 | 0.000 | 0.179 | 0.000 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 66 | 264 | 0 | 0 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 3.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.466 | 0.380 | 1.048 | 0.000 | 0.000 | 0.000 | 0.000 | 0.167 | 0.000 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 115 | 77 | 274 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.03 | 0.69 | 2.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.611 | 0.580 | 1.292 | 0.000 | 0.000 | 0.000 | 0.000 | 0.165 | 0.000 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 150 | 89 | 284 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.06 | 0.63 | 2.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 0.770 | 0.831 | 1.788 | 0.000 | 0.000 | 0.000 | 0.000 | 0.163 | 0.000 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 113 | 116 | 0 | 119 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.00 | 1.38 | 1.41 | 0.00 | 1.45 | 0.00 | 0.00 | 0.37 | 0.00 |
| time (sec) | N/A | 0.429 | 0.660 | 0.959 | 0.000 | 0.102 | 0.000 | 0.000 | 0.170 | 0.000 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 114 | 103 | 267 | 0 | 241 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.04 | 0.94 | 2.43 | 0.00 | 2.19 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.586 | 1.859 | 1.100 | 0.000 | 0.157 | 0.000 | 0.000 | 0.165 | 0.000 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 77 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.287 | 0.268 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.170 | 0.000 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 56 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.276 | 0.152 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.174 | 0.000 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 56 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.265 | 0.129 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 56 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.272 | 0.127 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.178 | 0.000 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 54 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.37 | 0.00 |
| time (sec) | N/A | 0.269 | 0.118 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.171 | 0.000 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 89 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.263 | 0.906 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.177 | 0.000 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 78 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.47 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.269 | 0.712 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.175 | 0.000 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 77 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.264 | 0.635 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.169 | 0.000 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 78 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.47 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.262 | 0.242 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.181 | 0.000 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 77 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.254 | 0.260 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.172 | 0.000 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 57 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.210 | 0.117 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.239 | 0.000 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 57 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.206 | 0.071 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.177 | 0.000 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 55 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.206 | 0.074 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.173 | 0.000 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 55 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.207 | 0.104 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.180 | 0.000 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 57 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.218 | 0.110 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.166 | 0.000 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-1) | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 58 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.209 | 0.109 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.241 | 0.000 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 57 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.213 | 0.091 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.181 | 0.000 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 55 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.197 | 0.087 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.174 | 0.000 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 57 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.203 | 0.239 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.173 | 0.000 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 59 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.210 | 0.258 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.184 | 0.000 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 130 | 131 | 186 | 0 | 788 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 0.73 | 0.74 | 1.04 | 0.00 | 4.43 | 0.00 | 0.00 | 0.19 | 0.00 |
| time (sec) | N/A | 0.477 | 0.767 | 5.688 | 0.000 | 0.294 | 0.000 | 0.000 | 0.194 | 0.000 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 73 | 269 | 0 | 105 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 2.89 | 0.00 | 1.13 | 0.00 | 0.00 | 0.31 | 0.00 |
| time (sec) | N/A | 0.499 | 10.485 | 1.960 | 0.000 | 0.135 | 0.000 | 0.000 | 0.179 | 0.000 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 88 | 101 | 136 | 0 | 654 | 0 | 0 | 22 | 0 |
| N.S. | 1 | 0.67 | 0.77 | 1.03 | 0.00 | 4.95 | 0.00 | 0.00 | 0.17 | 0.00 |
| time (sec) | N/A | 0.334 | 0.508 | 5.342 | 0.000 | 0.214 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 66 | 253 | 0 | 66 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 1.00 | 1.20 | 4.60 | 0.00 | 1.20 | 0.00 | 0.00 | 0.60 | 0.00 |
| time (sec) | N/A | 0.346 | 0.449 | 1.960 | 0.000 | 0.133 | 0.000 | 0.000 | 0.169 | 0.000 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 34 | 35 | 0 | 50 | 53 | 0 | 33 | 55 |
| N.S. | 1 | 1.00 | 1.00 | 1.03 | 0.00 | 1.47 | 1.56 | 0.00 | 0.97 | 1.62 |
| time (sec) | N/A | 0.221 | 0.391 | 1.107 | 0.000 | 0.093 | 4.777 | 0.000 | 0.177 | 1.178 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 70 | 265 | 0 | 112 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 2.79 | 0.00 | 1.18 | 0.00 | 0.00 | 0.35 | 0.00 |
| time (sec) | N/A | 0.485 | 0.807 | 2.073 | 0.000 | 0.097 | 0.000 | 0.000 | 0.163 | 0.000 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 53 | 47 | 0 | 63 | 0 | 0 | 33 | 69 |
| N.S. | 1 | 1.00 | 0.74 | 0.65 | 0.00 | 0.88 | 0.00 | 0.00 | 0.46 | 0.96 |
| time (sec) | N/A | 0.367 | 0.630 | 1.315 | 0.000 | 0.120 | 0.000 | 0.000 | 0.174 | 1.187 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 137 | 86 | 277 | 0 | 125 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 1.04 | 0.65 | 2.10 | 0.00 | 0.95 | 0.00 | 0.00 | 0.25 | 0.00 |
| time (sec) | N/A | 0.671 | 1.069 | 2.184 | 0.000 | 0.102 | 0.000 | 0.000 | 0.180 | 0.000 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 132 | 83 | 159 | 0 | 140 | 0 | 0 | 74 | 0 |
| N.S. | 1 | 1.01 | 0.63 | 1.21 | 0.00 | 1.07 | 0.00 | 0.00 | 0.56 | 0.00 |
| time (sec) | N/A | 0.669 | 5.332 | 1.957 | 0.000 | 0.098 | 0.000 | 0.000 | 0.176 | 0.000 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 125 | 130 | 177 | 0 | 769 | 0 | 0 | 68 | 0 |
| N.S. | 1 | 0.74 | 0.77 | 1.05 | 0.00 | 4.55 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.516 | 0.760 | 5.816 | 0.000 | 0.226 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 65 | 143 | 0 | 91 | 0 | 0 | 55 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 1.62 | 0.00 | 1.03 | 0.00 | 0.00 | 0.62 | 0.00 |
| time (sec) | N/A | 0.453 | 0.488 | 2.026 | 0.000 | 0.097 | 0.000 | 0.000 | 0.174 | 0.000 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | B | F | F | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 107 | 66 | 208 | 0 | 741 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 0.64 | 0.40 | 1.25 | 0.00 | 4.44 | 0.00 | 0.00 | 0.24 | 0.00 |
| time (sec) | N/A | 0.353 | 0.600 | 5.821 | 0.000 | 0.488 | 0.000 | 0.000 | 0.170 | 0.000 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 67 | 156 | 0 | 104 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 1.00 | 0.70 | 1.62 | 0.00 | 1.08 | 0.00 | 0.00 | 0.42 | 0.00 |
| time (sec) | N/A | 0.474 | 0.539 | 1.895 | 0.000 | 0.133 | 0.000 | 0.000 | 0.164 | 0.000 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | A | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 43 | 38 | 0 | 58 | 53 | 0 | 40 | 65 |
| N.S. | 1 | 1.00 | 1.26 | 1.12 | 0.00 | 1.71 | 1.56 | 0.00 | 1.18 | 1.91 |
| time (sec) | N/A | 0.214 | 0.810 | 1.129 | 0.000 | 0.088 | 45.649 | 0.000 | 0.164 | 1.000 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 138 | 81 | 169 | 0 | 116 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 1.05 | 0.62 | 1.29 | 0.00 | 0.89 | 0.00 | 0.00 | 0.31 | 0.00 |
| time (sec) | N/A | 0.666 | 0.808 | 2.234 | 0.000 | 0.137 | 0.000 | 0.000 | 0.159 | 0.000 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 115 | 55 | 50 | 0 | 70 | 0 | 0 | 40 | 78 |
| N.S. | 1 | 1.12 | 0.53 | 0.49 | 0.00 | 0.68 | 0.00 | 0.00 | 0.39 | 0.76 |
| time (sec) | N/A | 0.518 | 0.748 | 1.392 | 0.000 | 0.124 | 0.000 | 0.000 | 0.184 | 1.444 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 170 | 153 | 216 | 0 | 852 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 0.82 | 0.74 | 1.04 | 0.00 | 4.10 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 0.661 | 2.126 | 267.098 | 0.000 | 0.291 | 0.000 | 0.000 | 0.203 | 0.000 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 133 | 80 | 302 | 0 | 155 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 1.02 | 0.61 | 2.31 | 0.00 | 1.18 | 0.00 | 0.00 | 0.31 | 0.00 |
| time (sec) | N/A | 0.711 | 0.634 | 1.946 | 0.000 | 0.140 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 125 | 130 | 186 | 0 | 788 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 0.74 | 0.77 | 1.10 | 0.00 | 4.66 | 0.00 | 0.00 | 0.20 | 0.00 |
| time (sec) | N/A | 0.470 | 0.675 | 5.645 | 0.000 | 0.233 | 0.000 | 0.000 | 0.241 | 0.000 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 65 | 293 | 0 | 114 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 3.33 | 0.00 | 1.30 | 0.00 | 0.00 | 0.50 | 0.00 |
| time (sec) | N/A | 0.492 | 0.530 | 2.074 | 0.000 | 0.107 | 0.000 | 0.000 | 0.223 | 0.000 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 125 | 140 | 190 | 0 | 766 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 0.74 | 0.83 | 1.13 | 0.00 | 4.56 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.487 | 0.880 | 5.661 | 0.000 | 0.579 | 0.000 | 0.000 | 0.227 | 0.000 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 79 | 272 | 0 | 122 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 2.83 | 0.00 | 1.27 | 0.00 | 0.00 | 0.46 | 0.00 |
| time (sec) | N/A | 0.537 | 0.705 | 2.074 | 0.000 | 0.134 | 0.000 | 0.000 | 0.223 | 0.000 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 45 | 40 | 0 | 68 | 0 | 0 | 44 | 72 |
| N.S. | 1 | 1.00 | 1.32 | 1.18 | 0.00 | 2.00 | 0.00 | 0.00 | 1.29 | 2.12 |
| time (sec) | N/A | 0.231 | 0.726 | 1.282 | 0.000 | 0.115 | 0.000 | 0.000 | 0.217 | 1.081 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 138 | 91 | 282 | 0 | 137 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.05 | 0.69 | 2.15 | 0.00 | 1.05 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.654 | 1.109 | 2.052 | 0.000 | 0.145 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 130 | 133 | 192 | 0 | 782 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 0.73 | 0.75 | 1.08 | 0.00 | 4.39 | 0.00 | 0.00 | 0.25 | 0.00 |
| time (sec) | N/A | 0.504 | 0.909 | 5.993 | 0.000 | 0.336 | 0.000 | 0.000 | 0.226 | 0.000 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 83 | 146 | 0 | 100 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 1.59 | 0.00 | 1.09 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.508 | 0.572 | 2.080 | 0.000 | 0.132 | 0.000 | 0.000 | 0.162 | 0.000 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 88 | 99 | 139 | 0 | 653 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 0.67 | 0.76 | 1.06 | 0.00 | 4.98 | 0.00 | 0.00 | 0.31 | 0.00 |
| time (sec) | N/A | 0.377 | 0.544 | 5.713 | 0.000 | 0.369 | 0.000 | 0.000 | 0.157 | 0.000 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 68 | 147 | 0 | 58 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 1.00 | 1.24 | 2.67 | 0.00 | 1.05 | 0.00 | 0.00 | 0.60 | 0.00 |
| time (sec) | N/A | 0.335 | 0.462 | 1.906 | 0.000 | 0.128 | 0.000 | 0.000 | 0.165 | 0.000 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 32 | 32 | 0 | 47 | 51 | 0 | 44 | 52 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.00 | 1.47 | 1.59 | 0.00 | 1.38 | 1.62 |
| time (sec) | N/A | 0.217 | 0.518 | 1.124 | 0.000 | 0.095 | 4.172 | 0.000 | 0.170 | 0.748 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 70 | 148 | 0 | 101 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 1.56 | 0.00 | 1.06 | 0.00 | 0.00 | 0.46 | 0.00 |
| time (sec) | N/A | 0.499 | 0.619 | 2.055 | 0.000 | 0.113 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 50 | 45 | 0 | 58 | 88 | 0 | 44 | 64 |
| N.S. | 1 | 1.00 | 0.69 | 0.62 | 0.00 | 0.81 | 1.22 | 0.00 | 0.61 | 0.89 |
| time (sec) | N/A | 0.354 | 0.611 | 1.323 | 0.000 | 0.117 | 55.030 | 0.000 | 0.167 | 0.912 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 128 | 118 | 198 | 0 | 794 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 0.75 | 0.69 | 1.16 | 0.00 | 4.64 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.510 | 0.974 | 5.704 | 0.000 | 0.295 | 0.000 | 0.000 | 0.174 | 0.000 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 80 | 236 | 0 | 130 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 2.43 | 0.00 | 1.34 | 0.00 | 0.00 | 0.41 | 0.00 |
| time (sec) | N/A | 0.540 | 0.662 | 1.977 | 0.000 | 0.106 | 0.000 | 0.000 | 0.174 | 0.000 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 32 | 29 | 0 | 52 | 53 | 0 | 34 | 46 |
| N.S. | 1 | 1.00 | 1.00 | 0.91 | 0.00 | 1.62 | 1.66 | 0.00 | 1.06 | 1.44 |
| time (sec) | N/A | 0.227 | 0.349 | 1.118 | 0.000 | 0.102 | 1.557 | 0.000 | 0.157 | 0.836 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 91 | 78 | 434 | 0 | 129 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 4.77 | 0.00 | 1.42 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.499 | 0.519 | 1.946 | 0.000 | 0.143 | 0.000 | 0.000 | 0.173 | 0.000 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 67 | 52 | 48 | 0 | 66 | 90 | 0 | 44 | 60 |
| N.S. | 1 | 0.93 | 0.72 | 0.67 | 0.00 | 0.92 | 1.25 | 0.00 | 0.61 | 0.83 |
| time (sec) | N/A | 0.377 | 0.467 | 1.238 | 0.000 | 0.100 | 25.397 | 0.000 | 0.230 | 1.055 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 133 | 89 | 270 | 0 | 141 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.02 | 0.68 | 2.08 | 0.00 | 1.08 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.714 | 0.691 | 2.261 | 0.000 | 0.120 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 130 | 151 | 175 | 0 | 850 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 0.76 | 0.88 | 1.02 | 0.00 | 4.94 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.528 | 1.137 | 6.113 | 0.000 | 0.328 | 0.000 | 0.000 | 0.216 | 0.000 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 81 | 150 | 0 | 154 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 1.49 | 0.00 | 1.52 | 0.00 | 0.00 | 0.44 | 0.00 |
| time (sec) | N/A | 0.530 | 0.822 | 2.021 | 0.000 | 0.130 | 0.000 | 0.000 | 0.198 | 0.000 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 34 | 36 | 0 | 61 | 54 | 0 | 41 | 55 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 0.00 | 1.79 | 1.59 | 0.00 | 1.21 | 1.62 |
| time (sec) | N/A | 0.231 | 0.462 | 1.134 | 0.000 | 0.111 | 42.991 | 0.000 | 0.213 | 1.078 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 80 | 148 | 0 | 136 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.56 | 0.00 | 1.43 | 0.00 | 0.00 | 0.35 | 0.00 |
| time (sec) | N/A | 0.510 | 0.697 | 2.124 | 0.000 | 0.113 | 0.000 | 0.000 | 0.209 | 0.000 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | A | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 44 | 52 | 0 | 75 | 92 | 0 | 44 | 81 |
| N.S. | 1 | 1.00 | 0.64 | 0.75 | 0.00 | 1.09 | 1.33 | 0.00 | 0.64 | 1.17 |
| time (sec) | N/A | 0.352 | 0.628 | 1.252 | 0.000 | 0.117 | 55.274 | 0.000 | 0.224 | 1.460 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | C | F(-1) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 135 | 95 | 163 | 0 | 153 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.02 | 0.72 | 1.23 | 0.00 | 1.16 | 0.00 | 0.00 | 0.33 | 0.00 |
| time (sec) | N/A | 0.701 | 0.897 | 2.227 | 0.000 | 0.143 | 0.000 | 0.000 | 0.217 | 0.000 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 114 | 65 | 65 | 0 | 89 | 0 | 0 | 44 | 93 |
| N.S. | 1 | 1.08 | 0.61 | 0.61 | 0.00 | 0.84 | 0.00 | 0.00 | 0.42 | 0.88 |
| time (sec) | N/A | 0.521 | 0.834 | 1.437 | 0.000 | 0.108 | 0.000 | 0.000 | 0.192 | 2.302 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 64 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.225 | 0.073 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.254 | 0.000 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 62 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.234 | 0.057 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.180 | 0.000 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 62 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.230 | 0.068 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.180 | 0.000 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 64 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.232 | 0.083 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.189 | 0.000 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 64 | 0 | 0 | 0 | 0 | 0 | 58 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.91 | 0.00 |
| time (sec) | N/A | 0.243 | 0.100 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.272 | 0.000 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 62 | 0 | 0 | 0 | 0 | 0 | 58 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.91 | 0.00 |
| time (sec) | N/A | 0.229 | 0.091 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.249 | 0.000 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 69 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.232 | 0.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.228 | 0.000 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 71 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.234 | 0.052 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.207 | 0.000 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 62 | 0 | 0 | 0 | 0 | 0 | 49 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.77 | 0.00 |
| time (sec) | N/A | 0.234 | 0.095 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.255 | 0.000 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 62 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.217 | 0.057 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.229 | 0.000 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 62 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.229 | 0.080 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.193 | 0.000 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 62 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 |
| time (sec) | N/A | 0.227 | 0.155 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 64 | 0 | 0 | 0 | 0 | 0 | 62 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.97 | 0.00 |
| time (sec) | N/A | 0.232 | 0.103 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.189 | 0.000 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 64 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.230 | 0.055 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.184 | 0.000 |

| Problem 349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 64 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.236 | 0.068 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.182 | 0.000 |

| Problem 350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 64 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.00 |
| time (sec) | N/A | 0.234 | 0.138 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | B | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 66 | 62 | 6067 | 77 | 80 | 0 | 151 | 64 | 199 |
| N.S. | 1 | 0.99 | 0.93 | 90.55 | 1.15 | 1.19 | 0.00 | 2.25 | 0.96 | 2.97 |
| time (sec) | N/A | 0.248 | 0.312 | 7.916 | 0.051 | 0.134 | 0.000 | 0.189 | 0.187 | 5.793 |

| Problem 352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 45 | 37 | 2423 | 51 | 50 | 0 | 60 | 35 | 87 |
| N.S. | 1 | 1.05 | 0.86 | 56.35 | 1.19 | 1.16 | 0.00 | 1.40 | 0.81 | 2.02 |
| time (sec) | N/A | 0.241 | 0.135 | 2.154 | 0.046 | 0.104 | 0.000 | 0.168 | 0.206 | 1.199 |

| Problem 353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 18 | 20 | 19 | 42 | 19 | 18 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.18 | 1.12 | 2.47 | 1.12 | 1.06 | 1.12 |
| time (sec) | N/A | 0.194 | 0.048 | 0.420 | 0.037 | 0.100 | 0.163 | 0.144 | 0.225 | 0.123 |

| Problem 354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 40 | 0 | 0 | 0 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.52 | 0.00 |
| time (sec) | N/A | 0.216 | 0.045 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 39 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.59 | 0.00 |
| time (sec) | N/A | 0.222 | 0.116 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.208 | 0.000 |

| Problem 356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 40 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.58 | 0.00 |
| time (sec) | N/A | 0.222 | 0.173 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.221 | 0.000 |

| Problem 357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 61 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.37 | 0.00 |
| time (sec) | N/A | 0.232 | 0.155 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.151 | 0.000 |

| Problem 358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 61 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.37 | 0.00 |
| time (sec) | N/A | 0.228 | 0.129 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.158 | 0.000 |

| Problem 359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 62 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 |
| time (sec) | N/A | 0.225 | 0.158 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.161 | 0.000 |

| Problem 360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 61 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.37 | 0.00 |
| time (sec) | N/A | 0.228 | 0.146 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.157 | 0.000 |

| Problem 361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 62 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.37 | 0.00 |
| time (sec) | N/A | 0.228 | 0.415 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 80 | 0 | 0 | 0 | 0 | 0 | 26 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.32 | 0.00 |
| time (sec) | N/A | 0.241 | 0.098 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.215 | 0.000 |

| Problem 363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 69 | 101 | 88 | 77 | 85 | 0 | 97 | 23 | 0 |
| N.S. | 1 | 0.93 | 1.36 | 1.19 | 1.04 | 1.15 | 0.00 | 1.31 | 0.31 | 0.00 |
| time (sec) | N/A | 0.266 | 1.442 | 0.105 | 0.047 | 0.092 | 0.000 | 4.488 | 0.221 | 0.000 |

| Problem 364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 47 | 78 | 58 | 51 | 61 | 0 | 61 | 23 | 139 |
| N.S. | 1 | 0.96 | 1.59 | 1.18 | 1.04 | 1.24 | 0.00 | 1.24 | 0.47 | 2.84 |
| time (sec) | N/A | 0.243 | 0.812 | 56.519 | 0.039 | 0.087 | 0.000 | 2.320 | 0.214 | 1.764 |

| Problem 365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 25 | 25 | 24 | 40 | 0 | 24 | 23 | 49 |
| N.S. | 1 | 1.00 | 1.04 | 1.04 | 1.00 | 1.67 | 0.00 | 1.00 | 0.96 | 2.04 |
| time (sec) | N/A | 0.210 | 0.015 | 2.740 | 0.051 | 0.085 | 0.000 | 0.365 | 0.190 | 0.484 |

| Problem 366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 51 | 0 | 0 | 0 | 0 | 0 | 14 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 0.218 | 0.066 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.208 | 0.000 |

| Problem 367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 51 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.46 | 0.00 |
| time (sec) | N/A | 0.236 | 0.295 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.185 | 0.000 |

| Problem 368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 51 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.46 | 0.00 |
| time (sec) | N/A | 0.230 | 0.299 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.177 | 0.000 |

| Problem 369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 72 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.243 | 0.062 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.160 | 0.000 |

| Problem 370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 72 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.239 | 0.065 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.157 | 0.000 |

| Problem 371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 64 | 0 | 0 | 0 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 0.215 | 0.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.160 | 0.000 |

| Problem 372 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 452 | 0 | 0 | 0 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.00 | 6.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.234 | 1.995 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 373 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 1313 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 16.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.280 | 4.790 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 374 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 52 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.58 | 0.00 |
| time (sec) | N/A | 0.273 | 0.058 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.224 | 0.000 |

| Problem 375 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 52 | 0 | 0 | 0 | 0 | 0 | 21 | 0 |
| N.S. | 1 | 1.00 | 1.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.54 | 0.00 |
| time (sec) | N/A | 0.240 | 0.038 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.162 | 0.000 |

| Problem 376 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 19 | 21 | 20 | 54 | 20 | 19 | 43 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.17 | 1.11 | 3.00 | 1.11 | 1.06 | 2.39 |
| time (sec) | N/A | 0.209 | 0.014 | 0.266 | 0.045 | 0.088 | 0.164 | 0.135 | 0.151 | 0.684 |

| Problem 377 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 46 | 36 | 3514 | 50 | 60 | 0 | 59 | 36 | 92 |
| N.S. | 1 | 1.07 | 0.84 | 81.72 | 1.16 | 1.40 | 0.00 | 1.37 | 0.84 | 2.14 |
| time (sec) | N/A | 0.259 | 0.060 | 1.004 | 0.039 | 0.086 | 0.000 | 0.165 | 0.150 | 1.325 |

| Problem 378 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 67 | 63 | 8846 | 78 | 115 | 0 | 152 | 65 | 222 |
| N.S. | 1 | 0.97 | 0.91 | 128.20 | 1.13 | 1.67 | 0.00 | 2.20 | 0.94 | 3.22 |
| time (sec) | N/A | 0.271 | 0.208 | 1.947 | 0.042 | 0.091 | 0.000 | 0.213 | 0.162 | 5.881 |

| Problem 379 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 240 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 3.81 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.37 | 0.00 |
| time (sec) | N/A | 0.240 | 8.187 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.156 | 0.000 |

| Problem 380 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 79 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.36 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.235 | 1.309 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.199 | 0.000 |

| Problem 381 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 186 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 2.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.37 | 0.00 |
| time (sec) | N/A | 0.233 | 1.017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.157 | 0.000 |

| Problem 382 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 106 | 0 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.37 | 0.00 |
| time (sec) | N/A | 0.236 | 0.476 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.170 | 0.000 |

| Problem 383 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 87 | 0 | 0 | 0 | 0 | 0 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 |
| time (sec) | N/A | 0.515 | 0.874 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.172 | 0.000 |

| Problem 384 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 87 | 0 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.30 | 0.00 |
| time (sec) | N/A | 0.456 | 0.614 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.184 | 0.000 |

| Problem 385 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 87 | 0 | 0 | 0 | 0 | 0 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.44 | 0.00 |
| time (sec) | N/A | 0.457 | 0.665 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.164 | 0.000 |

| Problem 386 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 87 | 0 | 0 | 0 | 0 | 0 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.44 | 0.00 |
| time (sec) | N/A | 0.492 | 0.676 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.161 | 0.000 |

| Problem 387 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 287 | 0 | 0 | 0 | 0 | 0 | 26 | 0 |
| N.S. | 1 | 1.00 | 3.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.509 | 1.915 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.160 | 0.000 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [30] had the largest ratio of [1.50000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 2 | 2 | 1.00 | 6 | 0.333 |
| 2 | A | 3 | 3 | 1.00 | 8 | 0.375 |
| 3 | A | 4 | 4 | 1.00 | 8 | 0.500 |
| 4 | A | 5 | 5 | 1.00 | 8 | 0.625 |
| 5 | A | 6 | 6 | 1.00 | 8 | 0.750 |
| 6 | A | 7 | 7 | 1.00 | 8 | 0.875 |
| 7 | A | 8 | 8 | 1.00 | 8 | 1.000 |
| 8 | A | 9 | 9 | 1.00 | 8 | 1.125 |
| 9 | A | 16 | 15 | 1.29 | 12 | 1.250 |
| 10 | A | 14 | 13 | 1.26 | 12 | 1.083 |
| 11 | A | 14 | 13 | 1.30 | 12 | 1.083 |
| 12 | A | 12 | 11 | 1.27 | 12 | 0.917 |
| 13 | A | 12 | 11 | 1.32 | 12 | 0.917 |
| 14 | A | 14 | 13 | 1.26 | 12 | 1.083 |
| 15 | A | 14 | 13 | 1.29 | 12 | 1.083 |
| 16 | A | 16 | 15 | 1.26 | 12 | 1.250 |
| 17 | A | 14 | 13 | 0.89 | 12 | 1.083 |
| 18 | A | 12 | 11 | 0.88 | 12 | 0.917 |
| 19 | A | 12 | 11 | 0.85 | 12 | 0.917 |
| 20 | A | 12 | 11 | 0.85 | 12 | 0.917 |
| 21 | A | 12 | 11 | 0.88 | 12 | 0.917 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 22 | A | 14 | 13 | 0.89 | 12 | 1.083 |
| 23 | A | 4 | 3 | 1.00 | 10 | 0.300 |
| 24 | A | 8 | 8 | 0.68 | 14 | 0.571 |
| 25 | A | 6 | 6 | 0.80 | 14 | 0.429 |
| 26 | A | 4 | 4 | 1.00 | 14 | 0.286 |
| 27 | A | 5 | 5 | 1.06 | 14 | 0.357 |
| 28 | A | 9 | 9 | 0.82 | 14 | 0.643 |
| 29 | A | 13 | 13 | 0.72 | 14 | 0.929 |
| 30 | A | 22 | 21 | 0.78 | 14 | 1.500 |
| 31 | A | 18 | 17 | 0.89 | 14 | 1.214 |
| 32 | A | 16 | 15 | 0.92 | 14 | 1.071 |
| 33 | A | 16 | 15 | 0.93 | 14 | 1.071 |
| 34 | A | 18 | 17 | 0.86 | 14 | 1.214 |
| 35 | A | 22 | 21 | 0.78 | 14 | 1.500 |
| 36 | A | 13 | 13 | 0.55 | 14 | 0.929 |
| 37 | A | 9 | 9 | 0.62 | 14 | 0.643 |
| 38 | A | 5 | 5 | 0.74 | 14 | 0.357 |
| 39 | A | 5 | 5 | 0.75 | 14 | 0.357 |
| 40 | A | 9 | 9 | 0.60 | 14 | 0.643 |
| 41 | A | 13 | 13 | 0.55 | 14 | 0.929 |
| 42 | A | 6 | 5 | 1.00 | 12 | 0.417 |
| 43 | A | 6 | 5 | 1.00 | 12 | 0.417 |
| 44 | A | 6 | 5 | 1.00 | 12 | 0.417 |
| 45 | A | 6 | 5 | 1.00 | 12 | 0.417 |
| 46 | A | 6 | 5 | 1.00 | 14 | 0.357 |
| 47 | A | 6 | 5 | 1.00 | 14 | 0.357 |
| 48 | A | 6 | 5 | 1.25 | 14 | 0.357 |
| 49 | A | 6 | 5 | 1.00 | 14 | 0.357 |
| 50 | A | 6 | 5 | 1.00 | 14 | 0.357 |
| 51 | A | 6 | 5 | 1.00 | 14 | 0.357 |
| 52 | A | 4 | 4 | 1.00 | 14 | 0.286 |
| 53 | A | 6 | 5 | 1.00 | 14 | 0.357 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 54 | A | 14 | 13 | 1.23 | 21 | 0.619 |
| 55 | A | 13 | 12 | 1.22 | 21 | 0.571 |
| 56 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 57 | A | 5 | 4 | 0.95 | 21 | 0.190 |
| 58 | A | 5 | 4 | 0.92 | 21 | 0.190 |
| 59 | A | 10 | 10 | 1.03 | 21 | 0.476 |
| 60 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 61 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 62 | A | 8 | 8 | 1.00 | 21 | 0.381 |
| 63 | A | 10 | 10 | 1.05 | 21 | 0.476 |
| 64 | A | 15 | 14 | 1.24 | 21 | 0.667 |
| 65 | A | 14 | 13 | 1.23 | 21 | 0.619 |
| 66 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 67 | A | 5 | 4 | 0.95 | 21 | 0.190 |
| 68 | A | 5 | 4 | 0.92 | 21 | 0.190 |
| 69 | A | 10 | 10 | 1.01 | 21 | 0.476 |
| 70 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 71 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 72 | A | 10 | 10 | 1.35 | 21 | 0.476 |
| 73 | A | 15 | 14 | 1.22 | 21 | 0.667 |
| 74 | A | 14 | 13 | 1.21 | 21 | 0.619 |
| 75 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 76 | A | 5 | 4 | 0.95 | 21 | 0.190 |
| 77 | A | 5 | 4 | 0.92 | 21 | 0.190 |
| 78 | A | 12 | 12 | 1.04 | 21 | 0.571 |
| 79 | A | 10 | 10 | 1.01 | 19 | 0.526 |
| 80 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 81 | A | 8 | 8 | 1.00 | 21 | 0.381 |
| 82 | A | 10 | 10 | 1.01 | 21 | 0.476 |
| 83 | A | 12 | 12 | 1.04 | 21 | 0.571 |
| 84 | A | 14 | 13 | 1.26 | 21 | 0.619 |
| 85 | A | 13 | 12 | 1.25 | 21 | 0.571 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 86 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 87 | A | 5 | 4 | 0.95 | 21 | 0.190 |
| 88 | A | 5 | 4 | 0.92 | 21 | 0.190 |
| 89 | A | 10 | 10 | 1.05 | 21 | 0.476 |
| 90 | A | 8 | 8 | 1.00 | 21 | 0.381 |
| 91 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 92 | A | 8 | 8 | 1.51 | 19 | 0.421 |
| 93 | A | 10 | 10 | 1.36 | 21 | 0.476 |
| 94 | A | 14 | 13 | 1.26 | 21 | 0.619 |
| 95 | A | 13 | 12 | 1.25 | 21 | 0.571 |
| 96 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 97 | A | 5 | 4 | 0.95 | 21 | 0.190 |
| 98 | A | 5 | 4 | 0.92 | 21 | 0.190 |
| 99 | A | 10 | 10 | 1.01 | 21 | 0.476 |
| 100 | A | 8 | 8 | 1.43 | 19 | 0.421 |
| 101 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 102 | A | 10 | 10 | 1.03 | 21 | 0.476 |
| 103 | A | 14 | 13 | 1.29 | 21 | 0.619 |
| 104 | A | 13 | 12 | 1.29 | 21 | 0.571 |
| 105 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 106 | A | 5 | 4 | 0.95 | 21 | 0.190 |
| 107 | A | 5 | 4 | 0.92 | 21 | 0.190 |
| 108 | A | 12 | 12 | 1.04 | 21 | 0.571 |
| 109 | A | 10 | 10 | 1.03 | 21 | 0.476 |
| 110 | A | 8 | 8 | 1.00 | 21 | 0.381 |
| 111 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 112 | A | 10 | 10 | 1.31 | 19 | 0.526 |
| 113 | A | 12 | 12 | 1.26 | 21 | 0.571 |
| 114 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 115 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 116 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 117 | A | 4 | 4 | 1.00 | 25 | 0.160 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 118 | A | 10 | 9 | 0.68 | 25 | 0.360 |
| 119 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 120 | A | 8 | 8 | 0.98 | 25 | 0.320 |
| 121 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 122 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 123 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 124 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 125 | A | 12 | 11 | 0.76 | 25 | 0.440 |
| 126 | A | 8 | 8 | 1.04 | 25 | 0.320 |
| 127 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 128 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 129 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 130 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 131 | A | 10 | 9 | 0.69 | 25 | 0.360 |
| 132 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 133 | A | 12 | 11 | 0.76 | 25 | 0.440 |
| 134 | A | 8 | 8 | 1.05 | 25 | 0.320 |
| 135 | A | 6 | 6 | 1.04 | 25 | 0.240 |
| 136 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 137 | A | 12 | 11 | 0.76 | 25 | 0.440 |
| 138 | A | 12 | 11 | 0.75 | 25 | 0.440 |
| 139 | A | 10 | 10 | 1.04 | 25 | 0.400 |
| 140 | A | 8 | 8 | 1.02 | 25 | 0.320 |
| 141 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 142 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 143 | A | 8 | 8 | 0.98 | 25 | 0.320 |
| 144 | A | 10 | 10 | 1.02 | 25 | 0.400 |
| 145 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 146 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 147 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 148 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 149 | A | 4 | 4 | 1.00 | 25 | 0.160 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 150 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 151 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 152 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 153 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 154 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 155 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 156 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 157 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 158 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 159 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 160 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 161 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 162 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 163 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 164 | A | 5 | 4 | 0.96 | 19 | 0.211 |
| 165 | A | 5 | 4 | 0.93 | 19 | 0.211 |
| 166 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 167 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 168 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 169 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 170 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 171 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 172 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 173 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 174 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 175 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 176 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 177 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 178 | A | 5 | 4 | 0.96 | 19 | 0.211 |
| 179 | A | 5 | 4 | 0.94 | 19 | 0.211 |
| 180 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 181 | A | 4 | 4 | 1.00 | 17 | 0.235 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 182 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 183 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 184 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 185 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 186 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 187 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 188 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 189 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 190 | A | 5 | 4 | 1.00 | 21 | 0.190 |
| 191 | A | 17 | 16 | 1.30 | 21 | 0.762 |
| 192 | A | 16 | 15 | 1.32 | 21 | 0.714 |
| 193 | A | 15 | 14 | 1.30 | 21 | 0.667 |
| 194 | A | 14 | 13 | 1.33 | 19 | 0.684 |
| 195 | A | 12 | 11 | 1.28 | 12 | 0.917 |
| 196 | A | 15 | 14 | 1.34 | 19 | 0.737 |
| 197 | A | 15 | 14 | 1.26 | 21 | 0.667 |
| 198 | A | 17 | 16 | 1.31 | 21 | 0.762 |
| 199 | A | 18 | 17 | 1.29 | 21 | 0.810 |
| 200 | A | 15 | 14 | 1.32 | 21 | 0.667 |
| 201 | A | 16 | 15 | 1.28 | 21 | 0.714 |
| 202 | A | 13 | 12 | 1.33 | 21 | 0.571 |
| 203 | A | 14 | 13 | 1.29 | 19 | 0.684 |
| 204 | A | 14 | 13 | 1.31 | 12 | 1.083 |
| 205 | A | 15 | 14 | 1.29 | 19 | 0.737 |
| 206 | A | 17 | 16 | 1.31 | 21 | 0.762 |
| 207 | A | 18 | 17 | 1.31 | 21 | 0.810 |
| 208 | A | 15 | 14 | 1.34 | 21 | 0.667 |
| 209 | A | 16 | 15 | 1.29 | 19 | 0.789 |
| 210 | A | 12 | 11 | 1.31 | 12 | 0.917 |
| 211 | A | 13 | 12 | 1.27 | 19 | 0.632 |
| 212 | A | 15 | 14 | 1.31 | 21 | 0.667 |
| 213 | A | 15 | 14 | 1.26 | 21 | 0.667 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 214 | A | 17 | 16 | 1.30 | 21 | 0.762 |
| 215 | A | 16 | 15 | 1.34 | 19 | 0.789 |
| 216 | A | 14 | 13 | 1.25 | 12 | 1.083 |
| 217 | A | 13 | 12 | 1.31 | 19 | 0.632 |
| 218 | A | 13 | 12 | 1.29 | 21 | 0.571 |
| 219 | A | 15 | 14 | 1.31 | 21 | 0.667 |
| 220 | A | 15 | 14 | 1.26 | 21 | 0.667 |
| 221 | A | 17 | 16 | 1.29 | 21 | 0.762 |
| 222 | A | 6 | 5 | 1.00 | 17 | 0.294 |
| 223 | A | 6 | 5 | 1.00 | 19 | 0.263 |
| 224 | A | 6 | 5 | 1.00 | 19 | 0.263 |
| 225 | A | 6 | 5 | 1.00 | 21 | 0.238 |
| 226 | A | 5 | 4 | 0.93 | 21 | 0.190 |
| 227 | A | 5 | 4 | 0.96 | 21 | 0.190 |
| 228 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 229 | A | 12 | 11 | 1.27 | 12 | 0.917 |
| 230 | A | 14 | 13 | 1.33 | 21 | 0.619 |
| 231 | A | 10 | 10 | 1.01 | 21 | 0.476 |
| 232 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 233 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 234 | A | 8 | 8 | 1.00 | 21 | 0.381 |
| 235 | A | 10 | 10 | 1.05 | 21 | 0.476 |
| 236 | A | 5 | 4 | 0.93 | 21 | 0.190 |
| 237 | A | 5 | 4 | 0.96 | 21 | 0.190 |
| 238 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 239 | A | 14 | 13 | 1.30 | 12 | 1.083 |
| 240 | A | 14 | 13 | 1.35 | 21 | 0.619 |
| 241 | A | 12 | 12 | 1.10 | 21 | 0.571 |
| 242 | A | 10 | 10 | 1.06 | 21 | 0.476 |
| 243 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 244 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 245 | A | 10 | 10 | 1.04 | 21 | 0.476 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 246 | A | 12 | 12 | 1.08 | 21 | 0.571 |
| 247 | A | 5 | 4 | 0.93 | 21 | 0.190 |
| 248 | A | 5 | 4 | 0.96 | 21 | 0.190 |
| 249 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 250 | A | 14 | 13 | 1.26 | 12 | 1.083 |
| 251 | A | 14 | 13 | 1.35 | 21 | 0.619 |
| 252 | A | 15 | 14 | 1.34 | 21 | 0.667 |
| 253 | A | 10 | 10 | 1.05 | 21 | 0.476 |
| 254 | A | 8 | 8 | 1.00 | 21 | 0.381 |
| 255 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 256 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 257 | A | 10 | 10 | 1.02 | 21 | 0.476 |
| 258 | A | 5 | 4 | 0.92 | 21 | 0.190 |
| 259 | A | 5 | 4 | 0.95 | 21 | 0.190 |
| 260 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 261 | A | 14 | 13 | 1.26 | 12 | 1.083 |
| 262 | A | 15 | 14 | 1.29 | 21 | 0.667 |
| 263 | A | 12 | 12 | 1.03 | 21 | 0.571 |
| 264 | A | 10 | 10 | 1.03 | 21 | 0.476 |
| 265 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 266 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 267 | A | 10 | 10 | 1.03 | 21 | 0.476 |
| 268 | A | 12 | 12 | 1.06 | 21 | 0.571 |
| 269 | A | 8 | 8 | 1.00 | 19 | 0.421 |
| 270 | A | 10 | 10 | 1.04 | 21 | 0.476 |
| 271 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 272 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 273 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 274 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 275 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 276 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 277 | A | 4 | 4 | 1.00 | 19 | 0.211 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 278 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 279 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 280 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 281 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 282 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 283 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 284 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 285 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 286 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 287 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 288 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 289 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 290 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 291 | A | 12 | 11 | 0.73 | 25 | 0.440 |
| 292 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 293 | A | 10 | 9 | 0.67 | 25 | 0.360 |
| 294 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 295 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 296 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 297 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 298 | A | 10 | 10 | 1.04 | 25 | 0.400 |
| 299 | A | 10 | 10 | 1.01 | 25 | 0.400 |
| 300 | A | 12 | 11 | 0.74 | 25 | 0.440 |
| 301 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 302 | A | 11 | 10 | 0.64 | 25 | 0.400 |
| 303 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 304 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 305 | A | 10 | 10 | 1.05 | 25 | 0.400 |
| 306 | A | 6 | 6 | 1.12 | 25 | 0.240 |
| 307 | A | 14 | 13 | 0.82 | 25 | 0.520 |
| 308 | A | 10 | 10 | 1.02 | 25 | 0.400 |
| 309 | A | 12 | 11 | 0.74 | 25 | 0.440 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 310 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 311 | A | 12 | 11 | 0.74 | 25 | 0.440 |
| 312 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 313 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 314 | A | 10 | 10 | 1.05 | 25 | 0.400 |
| 315 | A | 12 | 11 | 0.73 | 25 | 0.440 |
| 316 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 317 | A | 10 | 9 | 0.67 | 25 | 0.360 |
| 318 | A | 6 | 6 | 1.00 | 25 | 0.240 |
| 319 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 320 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 321 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 322 | A | 12 | 11 | 0.75 | 25 | 0.440 |
| 323 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 324 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 325 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 326 | A | 4 | 4 | 0.93 | 25 | 0.160 |
| 327 | A | 10 | 10 | 1.02 | 25 | 0.400 |
| 328 | A | 12 | 11 | 0.76 | 25 | 0.440 |
| 329 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 330 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 331 | A | 8 | 8 | 1.00 | 25 | 0.320 |
| 332 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 333 | A | 10 | 10 | 1.02 | 25 | 0.400 |
| 334 | A | 6 | 6 | 1.08 | 25 | 0.240 |
| 335 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 336 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 337 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 338 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 339 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 340 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 341 | A | 2 | 2 | 1.00 | 25 | 0.080 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 342 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 343 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 344 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 345 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 346 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 347 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 348 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 349 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 350 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 351 | A | 5 | 4 | 0.99 | 19 | 0.211 |
| 352 | A | 6 | 5 | 1.05 | 19 | 0.263 |
| 353 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 354 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 355 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 356 | A | 5 | 4 | 1.00 | 19 | 0.211 |
| 357 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 358 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 359 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 360 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 361 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 362 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 363 | A | 5 | 4 | 0.93 | 19 | 0.211 |
| 364 | A | 5 | 4 | 0.96 | 19 | 0.211 |
| 365 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 366 | A | 4 | 3 | 1.00 | 10 | 0.300 |
| 367 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 368 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 369 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 370 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 371 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 372 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 373 | A | 2 | 2 | 1.00 | 19 | 0.105 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 374 | A | 5 | 4 | 1.00 | 19 | 0.211 |
| 375 | A | 6 | 5 | 1.00 | 17 | 0.294 |
| 376 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 377 | A | 7 | 6 | 1.07 | 19 | 0.316 |
| 378 | A | 6 | 5 | 0.97 | 19 | 0.263 |
| 379 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 380 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 381 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 382 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 383 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 384 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 385 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 386 | A | 6 | 6 | 1.00 | 23 | 0.261 |
| 387 | A | 6 | 6 | 1.00 | 21 | 0.286 |

# CHAPTER 3

## Listing of integrals


3.1 $\int \tan(c+dx)\,dx$ . . . 165
3.2 $\int \tan^2(c+dx)\,dx$ . . . 170
3.3 $\int \tan^3(c+dx)\,dx$ . . . 175
3.4 $\int \tan^4(c+dx)\,dx$ . . . 180
3.5 $\int \tan^5(c+dx)\,dx$ . . . 185
3.6 $\int \tan^6(c+dx)\,dx$ . . . 190
3.7 $\int \tan^7(c+dx)\,dx$ . . . 195
3.8 $\int \tan^8(c+dx)\,dx$ . . . 201
3.9 $\int (b\tan(c+dx))^{7/2}\,dx$ . . . 207
3.10 $\int (b\tan(c+dx))^{5/2}\,dx$ . . . 217
3.11 $\int (b\tan(c+dx))^{3/2}\,dx$ . . . 227
3.12 $\int \sqrt{b\tan(c+dx)}\,dx$ . . . 237
3.13 $\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$ . . . 245
3.14 $\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx$ . . . 254
3.15 $\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx$ . . . 264
3.16 $\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx$ . . . 273
3.17 $\int (b\tan(c+dx))^{4/3}\,dx$ . . . 283
3.18 $\int (b\tan(c+dx))^{2/3}\,dx$ . . . 294
3.19 $\int \sqrt[3]{b\tan(c+dx)}\,dx$ . . . 305
3.20 $\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$ . . . 314
3.21 $\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$ . . . 323
3.22 $\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx$ . . . 333
3.23 $\int (b\tan(c+dx))^n\,dx$ . . . 344
3.24 $\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx$ . . . 349
3.25 $\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx$ . . . 355

3.26 $\int \sqrt{b\tan^2(c+dx)}\,dx$ . . . 361
3.27 $\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx$ . . . 366
3.28 $\int \frac{1}{(b\tan^2(c+dx))^{3/2}}\,dx$ . . . 372
3.29 $\int \frac{1}{(b\tan^2(c+dx))^{5/2}}\,dx$ . . . 379
3.30 $\int (b\tan^3(c+dx))^{5/2}\,dx$ . . . 386
3.31 $\int (b\tan^3(c+dx))^{3/2}\,dx$ . . . 397
3.32 $\int \sqrt{b\tan^3(c+dx)}\,dx$ . . . 407
3.33 $\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$ . . . 417
3.34 $\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx$ . . . 427
3.35 $\int \frac{1}{(b\tan^3(c+dx))^{5/2}}\,dx$ . . . 437
3.36 $\int (b\tan^4(c+dx))^{5/2}\,dx$ . . . 448
3.37 $\int (b\tan^4(c+dx))^{3/2}\,dx$ . . . 455
3.38 $\int \sqrt{b\tan^4(c+dx)}\,dx$ . . . 462
3.39 $\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx$ . . . 467
3.40 $\int \frac{1}{(b\tan^4(c+dx))^{3/2}}\,dx$ . . . 472
3.41 $\int \frac{1}{(b\tan^4(c+dx))^{5/2}}\,dx$ . . . 478
3.42 $\int (b\tan^p(c+dx))^n\,dx$ . . . 485
3.43 $\int (b\tan^2(c+dx))^n\,dx$ . . . 490
3.44 $\int (b\tan^3(c+dx))^n\,dx$ . . . 495
3.45 $\int (b\tan^4(c+dx))^n\,dx$ . . . 500
3.46 $\int (b\tan^p(c+dx))^{5/2}\,dx$ . . . 505
3.47 $\int (b\tan^p(c+dx))^{3/2}\,dx$ . . . 510
3.48 $\int \sqrt{b\tan^p(c+dx)}\,dx$ . . . 515
3.49 $\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx$ . . . 520
3.50 $\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx$ . . . 525
3.51 $\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx$ . . . 530
3.52 $\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx$ . . . 535
3.53 $\int (a(b\tan(c+dx))^p)^n\,dx$ . . . 540
3.54 $\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$ . . . 545
3.55 $\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$ . . . 555
3.56 $\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$ . . . 565
3.57 $\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$ . . . 570
3.58 $\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx$ . . . 576
3.59 $\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$ . . . 582
3.60 $\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx$ . . . 589
3.61 $\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$ . . . 595

3.62 $\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$ . . . . . 601
3.63 $\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx$ . . . . . 607
3.64 $\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . . . 614
3.65 $\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . . . 624
3.66 $\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . . . 634
3.67 $\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . . . 639
3.68 $\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . . . 645
3.69 $\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . . . 651
3.70 $\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . . . 658
3.71 $\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . . . 664
3.72 $\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . . . 671
3.73 $\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 678
3.74 $\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 688
3.75 $\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 698
3.76 $\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 703
3.77 $\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 709
3.78 $\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 715
3.79 $\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 723
3.80 $\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 730
3.81 $\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 737
3.82 $\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 744
3.83 $\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx$ . . . . . 751
3.84 $\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 759
3.85 $\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 769
3.86 $\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 779
3.87 $\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 784
3.88 $\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 790
3.89 $\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 796
3.90 $\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 803
3.91 $\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 809
3.92 $\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 815
3.93 $\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$ . . . . . 822
3.94 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . . . 829
3.95 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . . . 839
3.96 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . . . 849

3.97 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 854
3.98 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 860
3.99 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 866
3.100 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 873
3.101 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 879
3.102 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 885
3.103 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 892
3.104 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 902
3.105 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 912
3.106 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 917
3.107 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 923
3.108 $\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 929
3.109 $\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 936
3.110 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 943
3.111 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 949
3.112 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 955
3.113 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 962
3.114 $\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$ . . . 970
3.115 $\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$ . . . 976
3.116 $\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx$ . . . 982
3.117 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$ . . . 987
3.118 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$ . . . 993
3.119 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$ . . . 1000
3.120 $\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$ . . . 1006
3.121 $\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$ . . . 1013
3.122 $\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx$ . . . 1019
3.123 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx$ . . . 1025
3.124 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx$ . . . 1030
3.125 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx$ . . . 1036
3.126 $\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 1044
3.127 $\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 1051
3.128 $\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 1057

3.129 $\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 1063
3.130 $\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 1068
3.131 $\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$ . . . 1074
3.132 $\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$ . . . 1081
3.133 $\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$ . . . 1087
3.134 $\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1095
3.135 $\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1102
3.136 $\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1108
3.137 $\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1113
3.138 $\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$ . . . 1121
3.139 $\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1130
3.140 $\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1137
3.141 $\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1143
3.142 $\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$ . . . 1149
3.143 $\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$ . . . 1155
3.144 $\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx$ . . . 1162
3.145 $\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$ . . . 1170
3.146 $\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx$ . . . 1175
3.147 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx$ . . . 1180
3.148 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx$ . . . 1185
3.149 $\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$ . . . 1190
3.150 $\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx$ . . . 1195
3.151 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx$ . . . 1200
3.152 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx$ . . . 1205
3.153 $\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx$ . . . 1210
3.154 $\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$ . . . 1215
3.155 $\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$ . . . 1220
3.156 $\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$ . . . 1225
3.157 $\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$ . . . 1230
3.158 $\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$ . . . 1235
3.159 $\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$ . . . 1240

3.160 $\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$ . . . 1245
3.161 $\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx$ . . . 1250
3.162 $\int (a\sin(e+fx))^m \tan(e+fx)\,dx$ . . . 1255
3.163 $\int \cot(e+fx)(a\sin(e+fx))^m\,dx$ . . . 1260
3.164 $\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx$ . . . 1265
3.165 $\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx$ . . . 1271
3.166 $\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx$ . . . 1277
3.167 $\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx$ . . . 1282
3.168 $\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx$ . . . 1287
3.169 $\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx$ . . . 1292
3.170 $\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx$ . . . 1297
3.171 $\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx$ . . . 1302
3.172 $\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 1307
3.173 $\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 1312
3.174 $\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$ . . . 1317
3.175 $\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1322
3.176 $\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1327
3.177 $\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1332
3.178 $\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1337
3.179 $\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1343
3.180 $\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1348
3.181 $\int \sin(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1353
3.182 $\int \csc(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1358
3.183 $\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1363
3.184 $\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1369
3.185 $\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx$ . . . 1375
3.186 $\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx$ . . . 1380
3.187 $\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx$ . . . 1385
3.188 $\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx$ . . . 1390
3.189 $\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx$ . . . 1395
3.190 $\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx$ . . . 1400
3.191 $\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx$ . . . 1405
3.192 $\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$ . . . 1416
3.193 $\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$ . . . 1427
3.194 $\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx$ . . . 1437
3.195 $\int \sqrt{d\cot(e+fx)}\,dx$ . . . 1447
3.196 $\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$ . . . 1456
3.197 $\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$ . . . 1466

3.198 $\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$ . . . 1476
3.199 $\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx$ . . . 1487
3.200 $\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx$ . . . 1498
3.201 $\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx$ . . . 1508
3.202 $\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx$ . . . 1518
3.203 $\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx$ . . . 1527
3.204 $\int (d\cot(e+fx))^{3/2}\,dx$ . . . 1536
3.205 $\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx$ . . . 1546
3.206 $\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$ . . . 1556
3.207 $\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1567
3.208 $\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1578
3.209 $\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1588
3.210 $\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1598
3.211 $\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1607
3.212 $\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1616
3.213 $\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$ . . . 1626
3.214 $\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1635
3.215 $\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1646
3.216 $\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1656
3.217 $\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1666
3.218 $\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1675
3.219 $\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1684
3.220 $\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1694
3.221 $\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$ . . . 1703
3.222 $\int \cot^m(e+fx)\tan^n(e+fx)\,dx$ . . . 1714
3.223 $\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx$ . . . 1719
3.224 $\int (a\cot(e+fx))^m\tan^n(e+fx)\,dx$ . . . 1724
3.225 $\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx$ . . . 1729
3.226 $\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1734
3.227 $\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1740
3.228 $\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1746
3.229 $\int \sqrt{d\tan(e+fx)}\,dx$ . . . 1751
3.230 $\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1760
3.231 $\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1770
3.232 $\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1777

3.233 $\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1784
3.234 $\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1790
3.235 $\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx$ . . . 1796
3.236 $\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1803
3.237 $\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1809
3.238 $\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1815
3.239 $\int (d\tan(a+bx))^{3/2}\,dx$ . . . 1820
3.240 $\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1830
3.241 $\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1840
3.242 $\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1848
3.243 $\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1855
3.244 $\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1861
3.245 $\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1867
3.246 $\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$ . . . 1874
3.247 $\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx$ . . . 1882
3.248 $\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$ . . . 1888
3.249 $\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$ . . . 1894
3.250 $\int (d\tan(e+fx))^{5/2}\,dx$ . . . 1899
3.251 $\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$ . . . 1909
3.252 $\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$ . . . 1919
3.253 $\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$ . . . 1929
3.254 $\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$ . . . 1936
3.255 $\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$ . . . 1942
3.256 $\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$ . . . 1948
3.257 $\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$ . . . 1954
3.258 $\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 1961
3.259 $\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 1967
3.260 $\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 1972
3.261 $\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 1977
3.262 $\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 1987
3.263 $\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 1997
3.264 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 2005
3.265 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 2012
3.266 $\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 2019
3.267 $\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 2025

3.268 $\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$ . . . 2032
3.269 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$ . . . 2040
3.270 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx$ . . . 2046
3.271 $\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx$ . . . 2053
3.272 $\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx$ . . . 2058
3.273 $\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx$ . . . 2063
3.274 $\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx$ . . . 2068
3.275 $\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx$ . . . 2073
3.276 $\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx$ . . . 2078
3.277 $\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx$ . . . 2083
3.278 $\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx$ . . . 2088
3.279 $\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx$ . . . 2093
3.280 $\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx$ . . . 2098
3.281 $\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx$ . . . 2103
3.282 $\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx$ . . . 2108
3.283 $\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx$ . . . 2113
3.284 $\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$ . . . 2118
3.285 $\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$ . . . 2123
3.286 $\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx$ . . . 2128
3.287 $\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx$ . . . 2133
3.288 $\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx$ . . . 2138
3.289 $\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$ . . . 2143
3.290 $\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$ . . . 2148
3.291 $\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$ . . . 2153
3.292 $\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$ . . . 2162
3.293 $\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx$ . . . 2168
3.294 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx$ . . . 2175
3.295 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx$ . . . 2181
3.296 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx$ . . . 2186
3.297 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx$ . . . 2192
3.298 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx$ . . . 2197
3.299 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$ . . . 2204
3.300 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$ . . . 2212
3.301 $\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx$ . . . 2221

3.302 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx$ . . . 2227
3.303 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx$ . . . 2235
3.304 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx$ . . . 2241
3.305 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx$ . . . 2246
3.306 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx$ . . . 2253
3.307 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx$ . . . 2259
3.308 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx$ . . . 2268
3.309 $\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx$ . . . 2276
3.310 $\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx$ . . . 2285
3.311 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx$ . . . 2291
3.312 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx$ . . . 2299
3.313 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx$ . . . 2305
3.314 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx$ . . . 2310
3.315 $\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 2317
3.316 $\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 2326
3.317 $\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 2333
3.318 $\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$ . . . 2341
3.319 $\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$ . . . 2347
3.320 $\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$ . . . 2352
3.321 $\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$ . . . 2358
3.322 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 2364
3.323 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 2372
3.324 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$ . . . 2379
3.325 $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$ . . . 2384
3.326 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$ . . . 2391
3.327 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$ . . . 2397
3.328 $\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx$ . . . 2404
3.329 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx$ . . . 2413
3.330 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx$ . . . 2420
3.331 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx$ . . . 2425
3.332 $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx$ . . . 2432

3.333 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx$ . . . 2438
3.334 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx$ . . . 2445
3.335 $\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$ . . . 2451
3.336 $\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx$ . . . 2456
3.337 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx$ . . . 2461
3.338 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx$ . . . 2466
3.339 $\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$ . . . 2471
3.340 $\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx$ . . . 2476
3.341 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx$ . . . 2481
3.342 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx$ . . . 2486
3.343 $\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx$ . . . 2491
3.344 $\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$ . . . 2496
3.345 $\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$ . . . 2501
3.346 $\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$ . . . 2506
3.347 $\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$ . . . 2511
3.348 $\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$ . . . 2516
3.349 $\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$ . . . 2521
3.350 $\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$ . . . 2526
3.351 $\int (b\sec(e+fx))^m\tan^5(e+fx)\,dx$ . . . 2531
3.352 $\int (b\sec(e+fx))^m\tan^3(e+fx)\,dx$ . . . 2538
3.353 $\int (b\sec(e+fx))^m\tan(e+fx)\,dx$ . . . 2545
3.354 $\int \cot(e+fx)(b\sec(e+fx))^m\,dx$ . . . 2550
3.355 $\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx$ . . . 2555
3.356 $\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx$ . . . 2560
3.357 $\int (b\sec(e+fx))^m\tan^4(e+fx)\,dx$ . . . 2565
3.358 $\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx$ . . . 2570
3.359 $\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx$ . . . 2575
3.360 $\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx$ . . . 2580
3.361 $\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx$ . . . 2585
3.362 $\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx$ . . . 2590
3.363 $\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2595
3.364 $\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2601
3.365 $\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2607
3.366 $\int (d\tan(a+bx))^n\,dx$ . . . 2612
3.367 $\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx$ . . . 2617

3.368 $\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2622
3.369 $\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2627
3.370 $\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2632
3.371 $\int \sec(a+bx)(d\tan(a+bx))^n\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . 2637
3.372 $\int \cos(a+bx)(d\tan(a+bx))^n\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . 2642
3.373 $\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2647
3.374 $\int (b\csc(e+fx))^m\tan^3(e+fx)\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2652
3.375 $\int (b\csc(e+fx))^m\tan(e+fx)\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2657
3.376 $\int \cot(e+fx)(b\csc(e+fx))^m\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2662
3.377 $\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2668
3.378 $\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2675
3.379 $\int (b\csc(e+fx))^m\tan^4(e+fx)\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2681
3.380 $\int (b\csc(e+fx))^m\tan^2(e+fx)\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2686
3.381 $\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2691
3.382 $\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2696
3.383 $\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx$ . . . . . . . . . . . . . . . . . . . . . 2701
3.384 $\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx$ . . . . . . . . . . . . . . . . . . . . . 2707
3.385 $\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . . 2713
3.386 $\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx$ . . . . . . . . . . . . . . . . . . . . . . . . 2719
3.387 $\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx$ . . . . . . . . . . . . . . . . . . . . . . 2725

# 3.1 $\int \tan(c+dx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 165
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 165
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 166
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 167
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 167
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 168
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 168
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 168
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 169
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 169

## Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \tan(c+dx)\,dx = -\frac{\log(\cos(c+dx))}{d}$$

output
```
-ln(cos(d*x+c))/d
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan(c+dx)\,dx = -\frac{\log(\cos(c+dx))}{d}$$

input
```
Integrate[Tan[c + d*x],x]
```

output
```
-(Log[Cos[c + d*x]]/d)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\log(\cos(c+dx))}{d}$$

input
```
Int[Tan[c + d*x],x]
```

output
```
-(Log[Cos[c + d*x]]/d)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln\left(1+\tan(dx+c)^2\right)}{2d}$ | 17 |
| default | $\frac{\ln\left(1+\tan(dx+c)^2\right)}{2d}$ | 17 |
| norman | $\frac{\ln\left(1+\tan(dx+c)^2\right)}{2d}$ | 17 |
| parallelrisch | $\frac{\ln\left(1+\tan(dx+c)^2\right)}{2d}$ | 17 |
| risch | $ix + \frac{2ic}{d} - \frac{\ln\left(e^{2i(dx+c)}+1\right)}{d}$ | 30 |

input
```
int(tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*ln(1+tan(d*x+c)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \tan(c+dx)\,dx = -\frac{\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate(tan(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*log(1/(tan(d*x + c)^2 + 1))/d
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \tan(c+dx)\,dx = \begin{cases} \frac{\log(\tan^2(c+dx)+1)}{2d} & \text{for } d \neq 0 \\ x\tan(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c),x)
```

output
```
Piecewise((log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*tan(c), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \tan(c+dx)\,dx = \frac{\log(\sec(dx+c))}{d}$$

input
```
integrate(tan(d*x+c),x, algorithm="maxima")
```

output
```
log(sec(d*x + c))/d
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \tan(c+dx)\,dx = -\frac{\log(|\cos(dx+c)|)}{d}$$

input
```
integrate(tan(d*x+c),x, algorithm="giac")
```

output
```
-log(abs(cos(d*x + c)))/d
```

## Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan(c+dx)\,dx = \frac{\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}$$

input
```
int(tan(c + d*x),x)
```

output
```
log(tan(c + d*x)^2 + 1)/(2*d)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan(c+dx)\,dx = \frac{\log\left(\tan\left(dx+c\right)^2+1\right)}{2d}$$

input
```
int(tan(d*x+c),x)
```

output
```
log(tan(c + d*x)**2 + 1)/(2*d)
```

# 3.2 $\int \tan^2(c+dx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 170
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 170
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 171
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 172
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 172
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 173
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 173
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 173
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 174
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 174

## Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \tan^2(c+dx)\,dx = -x+\frac{\tan(c+dx)}{d}$$

output
```
-x+tan(d*x+c)/d
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \tan^2(c+dx)\,dx = -\frac{\arctan(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}$$

input
```
Integrate[Tan[c + d*x]^2,x]
```

output
```
-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d
```

## Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan(c+dx)}{d}-\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan(c+dx)}{d}-x$$

input
```
Int[Tan[c + d*x]^2,x]
```

output
```
-x + Tan[c + d*x]/d
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| norman | $-x+\frac{\tan(dx+c)}{d}$ | 15 |
| parallelrisch | $-\frac{dx-\tan(dx+c)}{d}$ | 18 |
| derivativedivides | $\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{d}$ | 21 |
| default | $\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{d}$ | 21 |
| risch | $-x+\frac{2i}{d\left(e^{2i(dx+c)}+1\right)}$ | 24 |

input
```
int(tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-x+tan(d*x+c)/d
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \tan^2(c+dx)\,dx=-\frac{dx-\tan(dx+c)}{d}$$

input
```
integrate(tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(d*x - tan(d*x + c))/d
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \tan^2(c+dx)\,dx = \begin{cases} -x + \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^2(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**2,x)
```

output
```
Piecewise((-x + tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**2, True))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \tan^2(c+dx)\,dx = -\frac{dx + c - \tan(dx+c)}{d}$$

input
```
integrate(tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(d*x + c - tan(d*x + c))/d
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \tan^2(c+dx)\,dx = -\frac{dx+c}{d} + \frac{\tan(dx+c)}{d}$$

input
```
integrate(tan(d*x+c)^2,x, algorithm="giac")
```

output
```
-(d*x + c)/d + tan(d*x + c)/d
```

## Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \tan^2(c+dx)\,dx = \frac{\tan(c+d\,x)}{d} - x$$

input
```
int(tan(c + d*x)^2,x)
```

output
```
tan(c + d*x)/d - x
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \tan^2(c+dx)\,dx = \frac{\tan(dx+c) - dx}{d}$$

input
```
int(tan(d*x+c)^2,x)
```

output
```
(tan(c + d*x) - d*x)/d
```

# 3.3 $\int \tan^3(c+dx)\,dx$

Optimal result . . . . . 175
Mathematica [A] (verified) . . . . . 175
Rubi [A] (verified) . . . . . 176
Maple [A] (verified) . . . . . 177
Fricas [A] (verification not implemented) . . . . . 178
Sympy [A] (verification not implemented) . . . . . 178
Maxima [A] (verification not implemented) . . . . . 178
Giac [A] (verification not implemented) . . . . . 179
Mupad [B] (verification not implemented) . . . . . 179
Reduce [B] (verification not implemented) . . . . . 179

## Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \tan^3(c+dx)\,dx = \frac{\log(\cos(c+dx))}{d} + \frac{\tan^2(c+dx)}{2d}$$

output
```
ln(cos(d*x+c))/d+1/2*tan(d*x+c)^2/d
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \tan^3(c+dx)\,dx = \frac{2\log(\cos(c+dx)) + \sec^2(c+dx)}{2d}$$

input
```
Integrate[Tan[c + d*x]^3,x]
```

output
```
(2*Log[Cos[c + d*x]] + Sec[c + d*x]^2)/(2*d)
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^3dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(c+dx)}{2d}-\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(c+dx)}{2d}-\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}$$

input
```
Int[Tan[c + d*x]^3,x]
```

output
```
Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{-\tan(dx+c)^2+\ln\left(1+\tan(dx+c)^2\right)}{2d}$ | 28 |
| derivativedivides | $\frac{\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}}{d}$ | 29 |
| default | $\frac{\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}}{d}$ | 29 |
| norman | $\frac{\tan(dx+c)^2}{2d}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2d}$ | 31 |
| risch | $-ix-\frac{2ic}{d}+\frac{2\,e^{2i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d}$ | 56 |

input
```
int(tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-tan(d*x+c)^2+ln(1+tan(d*x+c)^2))/d
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \tan^3(c+dx)\,dx = \frac{\tan(dx+c)^2 + \log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate(tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(tan(d*x + c)^2 + log(1/(tan(d*x + c)^2 + 1)))/d
```

## Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \tan^3(c+dx)\,dx = \begin{cases} -\frac{\log(\tan^2(c+dx)+1)}{2d} + \frac{\tan^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\tan^3(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**3,x)
```

output
```
Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**2/(2*d), Ne(d,
0)), (x*tan(c)**3, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \tan^3(c+dx)\,dx = -\frac{\frac{1}{\sin(dx+c)^2-1} - \log\left(\sin(dx+c)^2 - 1\right)}{2\,d}$$

input
```
integrate(tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \tan^3(c+dx)\,dx = \frac{\tan(dx+c)^2}{2\,d} - \frac{\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$

input
```
integrate(tan(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*tan(d*x + c)^2/d - 1/2*log(tan(d*x + c)^2 + 1)/d
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \tan^3(c+dx)\,dx = \frac{\tan(c+d\,x)^2}{2\,d} - \frac{\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}$$

input
```
int(tan(c + d*x)^3,x)
```

output
```
tan(c + d*x)^2/(2*d) - log(tan(c + d*x)^2 + 1)/(2*d)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \tan^3(c+dx)\,dx = \frac{-\log\left(\tan(dx+c)^2+1\right)+\tan(dx+c)^2}{2d}$$

input
```
int(tan(d*x+c)^3,x)
```

output
```
( - log(tan(c + d*x)**2 + 1) + tan(c + d*x)**2)/(2*d)
```

## 3.4 $\int \tan^4(c+dx)\,dx$

Optimal result . . . 180
Mathematica [A] (verified) . . . 180
Rubi [A] (verified) . . . 181
Maple [A] (verified) . . . 182
Fricas [A] (verification not implemented) . . . 183
Sympy [A] (verification not implemented) . . . 183
Maxima [A] (verification not implemented) . . . 183
Giac [A] (verification not implemented) . . . 184
Mupad [B] (verification not implemented) . . . 184
Reduce [B] (verification not implemented) . . . 184

### Optimal result

Integrand size = 8, antiderivative size = 28

$$\int \tan^4(c+dx)\,dx = x - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d}$$

output
```
x-tan(d*x+c)/d+1/3*tan(d*x+c)^3/d
```

### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \tan^4(c+dx)\,dx = \frac{\arctan(\tan(c+dx))}{d} - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d}$$

input
```
Integrate[Tan[c + d*x]^4,x]
```

output
```
ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^4dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^3(c+dx)}{3d}-\int \tan^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^3(c+dx)}{3d}-\int \tan(c+dx)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}+x$$

input
```
Int[Tan[c + d*x]^4,x]
```

output
```
x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d)
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

| method | result | size |
|---|---|---|
| norman | $x-\frac{\tan(dx+c)}{d}+\frac{\tan(dx+c)^{3}}{3d}$ | 27 |
| parallelrisch | $\frac{\tan(dx+c)^{3}+3dx-3\tan(dx+c)}{3d}$ | 27 |
| derivativedivides | $\frac{\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$ | 31 |
| default | $\frac{\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$ | 31 |
| risch | $x-\frac{4i\left(3\,e^{4i(dx+c)}+3\,e^{2i(dx+c)}+2\right)}{3d\left(e^{2i(dx+c)}+1\right)^{3}}$ | 46 |

input
```
int(tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
x-tan(d*x+c)/d+1/3*tan(d*x+c)^3/d
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \tan^4(c+dx)\,dx = \frac{\tan(dx+c)^3 + 3\,dx - 3\,\tan(dx+c)}{3\,d}$$

input
```
integrate(tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(tan(d*x + c)^3 + 3*d*x - 3*tan(d*x + c))/d
```

## Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \tan^4(c+dx)\,dx = \begin{cases} x + \frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^4(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**4,x)
```

output
```
Piecewise((x + tan(c + d*x)**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (x*tan(c
)**4, True))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \tan^4(c+dx)\,dx = \frac{\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)}{3\,d}$$

input
```
integrate(tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))/d
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \tan^4(c+dx)\,dx = \frac{dx+c}{d} + \frac{d^2\tan\left(dx+c\right)^3 - 3\,d^2\tan\left(dx+c\right)}{3\,d^3}$$

input
```
integrate(tan(d*x+c)^4,x, algorithm="giac")
```

output
```
(d*x + c)/d + 1/3*(d^2*tan(d*x + c)^3 - 3*d^2*tan(d*x + c))/d^3
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \tan^4(c+dx)\,dx = x - \frac{\tan(c+d\,x) - \frac{\tan(c+d\,x)^3}{3}}{d}$$

input
```
int(tan(c + d*x)^4,x)
```

output
```
x - (tan(c + d*x) - tan(c + d*x)^3/3)/d
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \tan^4(c+dx)\,dx = \frac{\tan\left(dx+c\right)^3 - 3\tan\left(dx+c\right) + 3dx}{3d}$$

input
```
int(tan(d*x+c)^4,x)
```

output
```
(tan(c + d*x)**3 - 3*tan(c + d*x) + 3*d*x)/(3*d)
```

## 3.5 $\int \tan^5(c+dx)\,dx$

Optimal result . . . 185
Mathematica [A] (verified) . . . 185
Rubi [A] (verified) . . . 186
Maple [A] (verified) . . . 187
Fricas [A] (verification not implemented) . . . 188
Sympy [A] (verification not implemented) . . . 188
Maxima [A] (verification not implemented) . . . 188
Giac [A] (verification not implemented) . . . 189
Mupad [B] (verification not implemented) . . . 189
Reduce [B] (verification not implemented) . . . 189

### Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \tan^5(c+dx)\,dx = -\frac{\log(\cos(c+dx))}{d} - \frac{\tan^2(c+dx)}{2d} + \frac{\tan^4(c+dx)}{4d}$$

output
```
-ln(cos(d*x+c))/d-1/2*tan(d*x+c)^2/d+1/4*tan(d*x+c)^4/d
```

### Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \tan^5(c+dx)\,dx = -\frac{\log(\cos(c+dx))}{d} - \frac{\sec^2(c+dx)}{d} + \frac{\sec^4(c+dx)}{4d}$$

input
```
Integrate[Tan[c + d*x]^5,x]
```

output
```
-(Log[Cos[c + d*x]]/d) - Sec[c + d*x]^2/d + Sec[c + d*x]^4/(4*d)
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^5dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^4(c+dx)}{4d}-\int \tan^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^4(c+dx)}{4d}-\int \tan(c+dx)^3dx$$

$$\Big\downarrow \text{3954}$$

$$\int \tan(c+dx)dx+\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)dx+\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}-\frac{\log(\cos(c+dx))}{d}$$

input
```
Int[Tan[c + d*x]^5,x]
```

output
```
-(Log[Cos[c + d*x]]/d) - Tan[c + d*x]^2/(2*d) + Tan[c + d*x]^4/(4*d)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{\tan(dx+c)^4-2\tan(dx+c)^2+2\ln\left(1+\tan(dx+c)^2\right)}{4d}$ | 38 |
| derivativedivides | $\frac{\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}}{d}$ | 39 |
| default | $\frac{\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}}{d}$ | 39 |
| norman | $-\frac{\tan(dx+c)^2}{2d}+\frac{\tan(dx+c)^4}{4d}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2d}$ | 44 |
| risch | $ix+\frac{2ic}{d}-\frac{4\left(e^{6i(dx+c)}+e^{4i(dx+c)}+e^{2i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^4}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d}$ | 76 |

input
```
int(tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*(tan(d*x+c)^4-2*tan(d*x+c)^2+2*ln(1+tan(d*x+c)^2))/d
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \tan^5(c+dx)\,dx = \frac{\tan\left(dx+c\right)^4 - 2\,\tan\left(dx+c\right)^2 - 2\,\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{4\,d}$$

input
```
integrate(tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/4*(tan(d*x + c)^4 - 2*tan(d*x + c)^2 - 2*log(1/(tan(d*x + c)^2 + 1)))/d
```

## Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \tan^5(c+dx)\,dx = \begin{cases} \frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{\tan^4\left(c+dx\right)}{4d} - \frac{\tan^2\left(c+dx\right)}{2d} & \text{for } d \neq 0 \\ x\tan^5\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**5,x)
```

output
```
Piecewise((log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**4/(4*d) - tan(c
+ d*x)**2/(2*d), Ne(d, 0)), (x*tan(c)**5, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \tan^5(c+dx)\,dx = \frac{\frac{4\,\sin(dx+c)^2-3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - 2\,\log\left(\sin\left(dx+c\right)^2-1\right)}{4\,d}$$

input
```
integrate(tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/4*((4*sin(d*x + c)^2 - 3)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 2*lo
g(sin(d*x + c)^2 - 1))/d
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \tan^5(c+dx)\,dx = \frac{\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d} + \frac{d\tan\left(dx+c\right)^4 - 2\,d\tan\left(dx+c\right)^2}{4\,d^2}$$

input
```
integrate(tan(d*x+c)^5,x, algorithm="giac")
```

output
```
1/2*log(tan(d*x + c)^2 + 1)/d + 1/4*(d*tan(d*x + c)^4 - 2*d*tan(d*x + c)^2
)/d^2
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \tan^5(c+dx)\,dx = \frac{\frac{\ln\left(\tan(c+d\,x)^2+1\right)}{2} - \frac{\tan(c+d\,x)^2}{2} + \frac{\tan(c+d\,x)^4}{4}}{d}$$

input
```
int(tan(c + d*x)^5,x)
```

output
```
(log(tan(c + d*x)^2 + 1)/2 - tan(c + d*x)^2/2 + tan(c + d*x)^4/4)/d
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \tan^5(c+dx)\,dx = \frac{2\log\left(\tan\left(dx+c\right)^2+1\right) + \tan\left(dx+c\right)^4 - 2\tan\left(dx+c\right)^2}{4d}$$

input
```
int(tan(d*x+c)^5,x)
```

output
```
(2*log(tan(c + d*x)**2 + 1) + tan(c + d*x)**4 - 2*tan(c + d*x)**2)/(4*d)
```

# 3.6 $\int \tan^6(c+dx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 190
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 190
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 191
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 192
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 193
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 193
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 193
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 194
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 194
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 194

## Optimal result

Integrand size = 8, antiderivative size = 44

$$\int \tan^6(c+dx)\,dx = -x+\frac{\tan(c+dx)}{d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan^5(c+dx)}{5d}$$

output
```
-x+tan(d*x+c)/d-1/3*tan(d*x+c)^3/d+1/5*tan(d*x+c)^5/d
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \tan^6(c+dx)\,dx = -\frac{\arctan(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan^5(c+dx)}{5d}$$

input
```
Integrate[Tan[c + d*x]^6,x]
```

output
```
-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c
+ d*x]^5/(5*d)
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^6dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^5(c+dx)}{5d}-\int \tan^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^5(c+dx)}{5d}-\int \tan(c+dx)^4dx$$

$$\Big\downarrow \text{3954}$$

$$\int \tan^2(c+dx)dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3954}$$

$$-\int 1dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}-x$$

input
```
Int[Tan[c + d*x]^6,x]
```

output
```
-x + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c + d*x]^5/(5*d)
```

**Defintions of rubi rules used**

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{-3\tan(dx+c)^{5}+5\tan(dx+c)^{3}+15dx-15\tan(dx+c)}{15d}$ | 39 |
| derivativedivides | $\frac{\frac{\tan(dx+c)^{5}}{5}-\frac{\tan(dx+c)^{3}}{3}+\tan(dx+c)-\arctan(\tan(dx+c))}{d}$ | 41 |
| default | $\frac{\frac{\tan(dx+c)^{5}}{5}-\frac{\tan(dx+c)^{3}}{3}+\tan(dx+c)-\arctan(\tan(dx+c))}{d}$ | 41 |
| norman | $-x+\frac{\tan(dx+c)}{d}-\frac{\tan(dx+c)^{3}}{3d}+\frac{\tan(dx+c)^{5}}{5d}$ | 41 |
| risch | $-x+\frac{2i\left(45\,\mathrm{e}^{8i(dx+c)}+90\,\mathrm{e}^{6i(dx+c)}+140\,\mathrm{e}^{4i(dx+c)}+70\,\mathrm{e}^{2i(dx+c)}+23\right)}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}$ | 70 |

input
```
int(tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-3*tan(d*x+c)^5+5*tan(d*x+c)^3+15*d*x-15*tan(d*x+c))/d
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \tan^6(c+dx)\,dx = \frac{3\,\tan\left(dx+c\right)^5 - 5\,\tan\left(dx+c\right)^3 - 15\,dx + 15\,\tan\left(dx+c\right)}{15\,d}$$

input
```
integrate(tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x + 15*tan(d*x + c))/d
```

## Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \tan^6(c+dx)\,dx = \begin{cases} -x + \frac{\tan^5(c+dx)}{5d} - \frac{\tan^3(c+dx)}{3d} + \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^6(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**6,x)
```

output
```
Piecewise((-x + tan(c + d*x)**5/(5*d) - tan(c + d*x)**3/(3*d) + tan(c + d*
x)/d, Ne(d, 0)), (x*tan(c)**6, True))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \tan^6(c+dx)\,dx = \frac{3\,\tan\left(dx+c\right)^5 - 5\,\tan\left(dx+c\right)^3 - 15\,dx - 15\,c + 15\,\tan\left(dx+c\right)}{15\,d}$$

input
```
integrate(tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c
))/d
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \tan^6(c+dx)\,dx = -\frac{dx+c}{d} + \frac{3\,d^4\tan\left(dx+c\right)^5 - 5\,d^4\tan\left(dx+c\right)^3 + 15\,d^4\tan\left(dx+c\right)}{15\,d^5}$$

input
```
integrate(tan(d*x+c)^6,x, algorithm="giac")
```

output
```
-(d*x + c)/d + 1/15*(3*d^4*tan(d*x + c)^5 - 5*d^4*tan(d*x + c)^3 + 15*d^4*
tan(d*x + c))/d^5
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \tan^6(c+dx)\,dx = \frac{\frac{\tan(c+d\,x)^5}{5} - \frac{\tan(c+d\,x)^3}{3} + \tan(c+d\,x)}{d} - x$$

input
```
int(tan(c + d*x)^6,x)
```

output
```
(tan(c + d*x) - tan(c + d*x)^3/3 + tan(c + d*x)^5/5)/d - x
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \tan^6(c+dx)\,dx = \frac{3\tan\left(dx+c\right)^5 - 5\tan\left(dx+c\right)^3 + 15\tan\left(dx+c\right) - 15dx}{15d}$$

input
```
int(tan(d*x+c)^6,x)
```

output
```
(3*tan(c + d*x)**5 - 5*tan(c + d*x)**3 + 15*tan(c + d*x) - 15*d*x)/(15*d)
```

# 3.7 $\int \tan^7(c+dx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 195
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 195
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 196
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 197
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 198
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 198
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 199
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 199
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 199
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 200

## Optimal result

Integrand size = 8, antiderivative size = 57

$$\int \tan^7(c+dx)\,dx = \frac{\log(\cos(c+dx))}{d} + \frac{\tan^2(c+dx)}{2d} - \frac{\tan^4(c+dx)}{4d} + \frac{\tan^6(c+dx)}{6d}$$

output
```
ln(cos(d*x+c))/d+1/2*tan(d*x+c)^2/d-1/4*tan(d*x+c)^4/d+1/6*tan(d*x+c)^6/d
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \tan^7(c+dx)\,dx = \frac{12\log(\cos(c+dx)) + 18\sec^2(c+dx) - 9\sec^4(c+dx) + 2\sec^6(c+dx)}{12d}$$

input
```
Integrate[Tan[c + d*x]^7,x]
```

output
```
(12*Log[Cos[c + d*x]] + 18*Sec[c + d*x]^2 - 9*Sec[c + d*x]^4 + 2*Sec[c + d
*x]^6)/(12*d)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3954, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^7(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^7dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^6(c+dx)}{6d}-\int \tan^5(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^6(c+dx)}{6d}-\int \tan(c+dx)^5dx$$

$$\Big\downarrow \text{3954}$$

$$\int \tan^3(c+dx)dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^3dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3954}$$

$$-\int \tan(c+dx)dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \tan(c+dx)dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}$$

input
```
Int[Tan[c + d*x]^7,x]
```

output
```
Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d) - Tan[c + d*x]^4/(4*d) + Tan[c
+ d*x]^6/(6*d)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{\tan(dx+c)^6}{6}-\frac{\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}}{d}$ | 49 |
| default | $\frac{\frac{\tan(dx+c)^6}{6}-\frac{\tan(dx+c)^4}{4}+\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}}{d}$ | 49 |
| parallelrisch | $-\frac{-2\tan(dx+c)^6+3\tan(dx+c)^4-6\tan(dx+c)^2+6\ln\left(1+\tan(dx+c)^2\right)}{12d}$ | 50 |
| norman | $\frac{\tan(dx+c)^2}{2d}-\frac{\tan(dx+c)^4}{4d}+\frac{\tan(dx+c)^6}{6d}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2d}$ | 57 |
| risch | $-ix-\frac{2ic}{d}+\frac{6\,e^{10i(dx+c)}+12\,e^{8i(dx+c)}+\frac{68\,e^{6i(dx+c)}}{3}+12\,e^{4i(dx+c)}+6\,e^{2i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^6}+\frac{\ln(e^{2i(dx+c)}+1)}{d}$ | 103 |

input
```
int(tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/6*tan(d*x+c)^6-1/4*tan(d*x+c)^4+1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c
)^2))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \tan^7(c+dx)\,dx = \frac{2\,\tan\left(dx+c\right)^6 - 3\,\tan\left(dx+c\right)^4 + 6\,\tan\left(dx+c\right)^2 + 6\,\log\left(\frac{1}{\tan\left(dx+c\right)^2+1}\right)}{12\,d}$$

input
```
integrate(tan(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/12*(2*tan(d*x + c)^6 - 3*tan(d*x + c)^4 + 6*tan(d*x + c)^2 + 6*log(1/(ta
n(d*x + c)^2 + 1)))/d
```

## Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \tan^7(c+dx)\,dx = \begin{cases} -\frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{\tan^6\left(c+dx\right)}{6d} - \frac{\tan^4\left(c+dx\right)}{4d} + \frac{\tan^2\left(c+dx\right)}{2d} & \text{for } d \neq 0 \\ x\tan^7\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**7,x)
```

output
```
Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**6/(6*d) - tan(c
 + d*x)**4/(4*d) + tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*tan(c)**7, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \tan^7(c+dx)\,dx = -\frac{\frac{18\,\sin(dx+c)^4-27\,\sin(dx+c)^2+11}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1} - 6\,\log\left(\sin\left(dx+c\right)^2-1\right)}{12\,d}$$

input
```
integrate(tan(d*x+c)^7,x, algorithm="maxima")
```

output
```
-1/12*((18*sin(d*x + c)^4 - 27*sin(d*x + c)^2 + 11)/(sin(d*x + c)^6 - 3*si
n(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 6*log(sin(d*x + c)^2 - 1))/d
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \tan^7(c+dx)\,dx = -\frac{\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d} + \frac{2\,d^2\tan\left(dx+c\right)^6-3\,d^2\tan\left(dx+c\right)^4+6\,d^2\tan\left(dx+c\right)^2}{12\,d^3}$$

input
```
integrate(tan(d*x+c)^7,x, algorithm="giac")
```

output
```
-1/2*log(tan(d*x + c)^2 + 1)/d + 1/12*(2*d^2*tan(d*x + c)^6 - 3*d^2*tan(d*
x + c)^4 + 6*d^2*tan(d*x + c)^2)/d^3
```

## Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \tan^7(c+dx)\,dx = -\frac{\frac{\ln\left(\tan(c+d\,x)^2+1\right)}{2}-\frac{\tan(c+d\,x)^2}{2}+\frac{\tan(c+d\,x)^4}{4}-\frac{\tan(c+d\,x)^6}{6}}{d}$$

input
```
int(tan(c + d*x)^7,x)
```

output
```
-(log(tan(c + d*x)^2 + 1)/2 - tan(c + d*x)^2/2 + tan(c + d*x)^4/4 - tan(c
+ d*x)^6/6)/d
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \tan^7(c+dx)\,dx$$
$$= \frac{-6\log\left(\tan\left(dx+c\right)^2+1\right)+2\tan\left(dx+c\right)^6-3\tan\left(dx+c\right)^4+6\tan\left(dx+c\right)^2}{12d}$$

input
```
int(tan(d*x+c)^7,x)
```

output
```
( - 6*log(tan(c + d*x)**2 + 1) + 2*tan(c + d*x)**6 - 3*tan(c + d*x)**4 + 6
*tan(c + d*x)**2)/(12*d)
```

# 3.8 $\int \tan^8(c+dx)\,dx$

Optimal result . . . 201
Mathematica [A] (verified) . . . 201
Rubi [A] (verified) . . . 202
Maple [A] (verified) . . . 204
Fricas [A] (verification not implemented) . . . 204
Sympy [A] (verification not implemented) . . . 205
Maxima [A] (verification not implemented) . . . 205
Giac [A] (verification not implemented) . . . 205
Mupad [B] (verification not implemented) . . . 206
Reduce [B] (verification not implemented) . . . 206

## Optimal result

Integrand size = 8, antiderivative size = 58

$$\int \tan^8(c+dx)\,dx = x - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d}$$

output
```
x-tan(d*x+c)/d+1/3*tan(d*x+c)^3/d-1/5*tan(d*x+c)^5/d+1/7*tan(d*x+c)^7/d
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \tan^8(c+dx)\,dx = \frac{\arctan(\tan(c+dx))}{d} - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d}$$

input
```
Integrate[Tan[c + d*x]^8,x]
```

output
```
ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d
*x]^5/(5*d) + Tan[c + d*x]^7/(7*d)
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^8(c+dx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(c+dx)^8dx$$
$$\Big\downarrow \text{3954}$$
$$\frac{\tan^7(c+dx)}{7d}-\int \tan^6(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{\tan^7(c+dx)}{7d}-\int \tan(c+dx)^6dx$$
$$\Big\downarrow \text{3954}$$
$$\int \tan^4(c+dx)dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(c+dx)^4dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}$$
$$\Big\downarrow \text{3954}$$
$$-\int \tan^2(c+dx)dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}$$
$$\Big\downarrow \text{3042}$$
$$-\int \tan(c+dx)^2dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}$$
$$\Big\downarrow \text{3954}$$

$$\int 1dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}+x$$

input
```
Int[Tan[c + d*x]^8,x]
```

output
```
x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d*x]^5/(5*d) + Tan[c +
 d*x]^7/(7*d)
```

## Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{15\tan(dx+c)^{7}-21\tan(dx+c)^{5}+35\tan(dx+c)^{3}+105dx-105\tan(dx+c)}{105d}$ | 49 |
| derivativedivides | $\frac{\frac{\tan(dx+c)^{7}}{7}-\frac{\tan(dx+c)^{5}}{5}+\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$ | 51 |
| default | $\frac{\frac{\tan(dx+c)^{7}}{7}-\frac{\tan(dx+c)^{5}}{5}+\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$ | 51 |
| norman | $x-\frac{\tan(dx+c)}{d}+\frac{\tan(dx+c)^{3}}{3d}-\frac{\tan(dx+c)^{5}}{5d}+\frac{\tan(dx+c)^{7}}{7d}$ | 53 |
| risch | $x-\frac{8i\left(105\,\mathrm{e}^{12i(dx+c)}+315\,\mathrm{e}^{10i(dx+c)}+770\,\mathrm{e}^{8i(dx+c)}+770\,\mathrm{e}^{6i(dx+c)}+609\,\mathrm{e}^{4i(dx+c)}+203\,\mathrm{e}^{2i(dx+c)}+44\right)}{105d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{7}}$ | 90 |

input
```
int(tan(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
1/105*(15*tan(d*x+c)^7-21*tan(d*x+c)^5+35*tan(d*x+c)^3+105*d*x-105*tan(d*x
+c))/d
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \tan^8(c+dx)\,dx = \frac{15\,\tan\left(dx+c\right)^{7}-21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+105\,dx-105\,\tan\left(dx+c\right)}{105\,d}$$

input
```
integrate(tan(d*x+c)^8,x, algorithm="fricas")
```

output
```
1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x
 - 105*tan(d*x + c))/d
```

## Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \tan^8(c+dx)\,dx = \begin{cases} x + \frac{\tan^7(c+dx)}{7d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^8(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**8,x)
```

output
```
Piecewise((x + tan(c + d*x)**7/(7*d) - tan(c + d*x)**5/(5*d) + tan(c + d*x
)**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**8, True))
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \tan^8(c+dx)\,dx = \frac{15\tan(dx+c)^7 - 21\tan(dx+c)^5 + 35\tan(dx+c)^3 + 105\,dx + 105\,c - 105\tan(dx+c)}{105\,d}$$

input
```
integrate(tan(d*x+c)^8,x, algorithm="maxima")
```

output
```
1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x
 + 105*c - 105*tan(d*x + c))/d
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \tan^8(c+dx)\,dx = \frac{dx+c}{d} + \frac{15\,d^6\tan(dx+c)^7 - 21\,d^6\tan(dx+c)^5 + 35\,d^6\tan(dx+c)^3 - 105\,d^6\tan(dx+c)}{105\,d^7}$$

input
```
integrate(tan(d*x+c)^8,x, algorithm="giac")
```

output
```
(d*x + c)/d + 1/105*(15*d^6*tan(d*x + c)^7 - 21*d^6*tan(d*x + c)^5 + 35*d^
6*tan(d*x + c)^3 - 105*d^6*tan(d*x + c))/d^7
```

## Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \tan^8(c+dx)\,dx = x - \frac{-\frac{\tan(c+d\,x)^7}{7} + \frac{\tan(c+d\,x)^5}{5} - \frac{\tan(c+d\,x)^3}{3} + \tan(c+d\,x)}{d}$$

input
```
int(tan(c + d*x)^8,x)
```

output
```
x - (tan(c + d*x) - tan(c + d*x)^3/3 + tan(c + d*x)^5/5 - tan(c + d*x)^7/7
)/d
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \tan^8(c+dx)\,dx = \frac{15\tan(dx+c)^7 - 21\tan(dx+c)^5 + 35\tan(dx+c)^3 - 105\tan(dx+c) + 105dx}{105d}$$

input
```
int(tan(d*x+c)^8,x)
```

output
```
(15*tan(c + d*x)**7 - 21*tan(c + d*x)**5 + 35*tan(c + d*x)**3 - 105*tan(c
+ d*x) + 105*d*x)/(105*d)
```

# 3.9 $\int (b\tan(c+dx))^{7/2}\,dx$

Optimal result . . . 207
Mathematica [A] (verified) . . . 208
Rubi [A] (warning: unable to verify) . . . 208
Maple [A] (verified) . . . 213
Fricas [A] (verification not implemented) . . . 214
Sympy [F] . . . 214
Maxima [A] (verification not implemented) . . . 215
Giac [F(-2)] . . . 215
Mupad [B] (verification not implemented) . . . 216
Reduce [F] . . . 216

## Optimal result

Integrand size = 12, antiderivative size = 176

$$\int (b\tan(c+dx))^{7/2}\,dx = -\frac{b^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{b^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}+\sqrt{b}\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{2b^3\sqrt{b\tan(c+dx)}}{d} + \frac{2b(b\tan(c+dx))^{5/2}}{5d}$$

output

```
-1/2*b^(7/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/d+1/2*
b^(7/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/d+1/2*b^(7/
2)*arctanh(2^(1/2)*(b*tan(d*x+c))^(1/2)/(b^(1/2)+b^(1/2)*tan(d*x+c)))*2^(1
/2)/d-2*b^3*(b*tan(d*x+c))^(1/2)/d+2/5*b*(b*tan(d*x+c))^(5/2)/d
```

## Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{(b\tan(c+dx))^{7/2}\left(-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(\right.}{2\sqrt{2}}\right.}{d\tan^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[(b*Tan[c + d*x])^(7/2),x]
```

output
```
((b*Tan[c + d*x])^(7/2)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])
 + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*
x]] + Tan[c + d*x]]/(2*Sqrt[2]) - 2*Sqrt[Tan[c + d*x]] + (2*Tan[c + d*x]^(
5/2))/5))/(d*Tan[c + d*x]^(7/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.29, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {3042, 3954, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d} - b^2\int (b\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\int (b\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \textbf{3954}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\tan(c+dx)}}dx\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\tan(c+dx)}}dx\right)$$

$$\Big\downarrow \textbf{3957}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{b^3\int \frac{1}{\sqrt{b\tan(c+dx)}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}\right)$$

$$\Big\downarrow \textbf{266}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\int \frac{1}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{d}\right)$$

$$\Big\downarrow \textbf{755}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{d}\right)$$

$$\Big\downarrow \textbf{1476}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+}}{2b}\right.}{d}\right.$$

$$\Big\downarrow \textbf{1082}$$

$$b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{\frac{2b(b\tan(c+dx))^{5/2}}{5d}-2b^3\left(\frac{\frac{\int\frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int\frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\int\frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+}}{2}\right)}{d}\right)$$

$\downarrow$ 217

$$b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{\frac{2b(b\tan(c+dx))^{5/2}}{5d}-2b^3\left(\frac{\int\frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}\right)}{d}\right)$$

$\downarrow$ 1479

$$b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{\frac{2b(b\tan(c+dx))^{5/2}}{5d}-2b^3\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}\right)}{d}\right)$$

$\downarrow$ 25

$$b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{\frac{2b(b\tan(c+dx))^{5/2}}{5d}-2b^3\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\right)}{d}\right)$$

$\downarrow$ 27

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-$$

$$b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int\frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}}{2b}+\right.}{d}\right.$$

$\Big\downarrow$ 1103

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-$$

$$b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\log\left(\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(-\right.}{}}{2b}\right.}{d}\right.$$

input
```
Int[(b*Tan[c + d*x])^(7/2),x]
```

output
```
(2*b*(b*Tan[c + d*x])^(5/2))/(5*d) - b^2*((-2*b^3*((-(ArcTan[1 - Sqrt[2]*S
qrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) + ArcTan[1 + Sqrt[2]*Sqrt[b]*Tan[c
 + d*x]]/(Sqrt[2]*Sqrt[b]))/(2*b) + (-1/2*Log[b - Sqrt[2]*b^(3/2)*Tan[c +
d*x] + b^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[b]) + Log[b + Sqrt[2]*b^(3/2)*Tan
[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]))/(2*b)))/d + (2*b*Sqrt
[b*Tan[c + d*x]])/d)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| derivativedivides | $\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{5}{2}}}{5}-b^2\sqrt{b\tan(dx+c)}+\frac{b^2(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan}}{(b^2)}\right)\right)}{8}\right)}{d}$ |
| default | $\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{5}{2}}}{5}-b^2\sqrt{b\tan(dx+c)}+\frac{b^2(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan}}{(b^2)}\right)\right)}{8}\right)}{d}$ |

input
```
int((b*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*(1/5*(b*tan(d*x+c))^(5/2)-b^2*(b*tan(d*x+c))^(1/2)+1/8*b^2*(b^2)^(1/
4)*2^(1/2)*(ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2
)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2
)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)
/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{10\,\sqrt{2}b^{\frac{7}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b}{b}\right)+10\,\sqrt{2}b^{\frac{7}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}-b}{b}\right)+5\,\sqrt{2}b^{\frac{7}{2}}\log\left(b\right.}{}$$

input
```
integrate((b*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/20*(10*sqrt(2)*b^(7/2)*arctan((sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)
/b) + 10*sqrt(2)*b^(7/2)*arctan((sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) - b)
/b) + 5*sqrt(2)*b^(7/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*
sqrt(b) + b) - 5*sqrt(2)*b^(7/2)*log(b*tan(d*x + c) - sqrt(2)*sqrt(b*tan(d
*x + c))*sqrt(b) + b) + 8*(b^3*tan(d*x + c)^2 - 5*b^3)*sqrt(b*tan(d*x + c)
))/d
```

## Sympy [F]

$$\int (b\tan(c+dx))^{7/2}\,dx = \int \left(b\tan\left(c+dx\right)\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c))**(7/2),x)
```

output
```
Integral((b*tan(c + d*x))**(7/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.06

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{10\sqrt{2}b^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right) + 10\sqrt{2}b^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right) + 5$$

input
```
integrate((b*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
1/20*(10*sqrt(2)*b^(9/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*ta
n(d*x + c)))/sqrt(b)) + 10*sqrt(2)*b^(9/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sq
rt(b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + 5*sqrt(2)*b^(9/2)*log(b*tan(d*x
 + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - 5*sqrt(2)*b^(9/2)*log(
b*tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) + 8*(b*tan(d*x
+ c))^(5/2)*b^2 - 40*sqrt(b*tan(d*x + c))*b^4)/(b*d)
```

## Giac [F(-2)]

Exception generated.

$$\int (b\tan(c+dx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[5,19]%%%}+%%%{8,[5,17]%%%}+%%%{28,[5,15]%%%}+%%%{56
,[5,13]%%
```

## Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.53

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{2\,b\,(b\tan(c+d\,x))^{5/2}}{5\,d} - \frac{2\,b^3\,\sqrt{b\tan(c+d\,x)}}{d} - \frac{(-1)^{1/4}\,b^{7/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{d} - \frac{(-1)^{1/4}\,b^{7/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}\,1\mathrm{i}}{\sqrt{b}}\right)}{d}$$

input
```
int((b*tan(c + d*x))^(7/2),x)
```

output
```
(2*b*(b*tan(c + d*x))^(5/2))/(5*d) - (2*b^3*(b*tan(c + d*x))^(1/2))/d - ((
-1)^(1/4)*b^(7/2)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/d
- ((-1)^(1/4)*b^(7/2)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2)*1i)/b^(1/2))
)/d
```

## Reduce [F]

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{b}\,b^3\left(2\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 - 10\sqrt{\tan(dx+c)} + 5\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)d\right)}{5d}$$

input
```
int((b*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(b)*b**3*(2*sqrt(tan(c + d*x))*tan(c + d*x)**2 - 10*sqrt(tan(c + d*x)
) + 5*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*d))/(5*d)
```

# 3.10 $\int (b\tan(c+dx))^{5/2}\,dx$


Optimal result . . . 217
Mathematica [A] (verified) . . . 218
Rubi [A] (warning: unable to verify) . . . 218
Maple [A] (verified) . . . 222
Fricas [A] (verification not implemented) . . . 223
Sympy [F] . . . 224
Maxima [A] (verification not implemented) . . . 224
Giac [F(-2)] . . . 225
Mupad [B] (verification not implemented) . . . 225
Reduce [F] . . . 226


## Optimal result

Integrand size = 12, antiderivative size = 156

$$\int (b\tan(c+dx))^{5/2}\,dx = \frac{b^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}+\sqrt{b}\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{2b(b\tan(c+dx))^{3/2}}{3d}$$

output
```
1/2*b^(5/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/d-1/2*b
^(5/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/d+1/2*b^(5/2
)*arctanh(2^(1/2)*(b*tan(d*x+c))^(1/2)/(b^(1/2)+b^(1/2)*tan(d*x+c)))*2^(1/
2)/d+2/3*b*(b*tan(d*x+c))^(3/2)/d
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.65

$$\int (b\tan(c+dx))^{5/2}\,dx = \frac{b(b\tan(c+dx))^{3/2}\left(-3\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan(c+dx)}+3\text{arctanh}\left(\sqrt[4]{-\tan^2\right.\right.}{3d\tan^{\frac{7}{4}}(c+dx)}$$

input
```
Integrate[(b*Tan[c + d*x])^(5/2),x]
```

output
```
(b*(b*Tan[c + d*x])^(3/2)*(-3*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*
x])^(1/4) + 3*ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(1/4) + 2*T
an[c + d*x]^(7/4)))/(3*d*Tan[c + d*x]^(7/4))
```

## Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-b^2\int \sqrt{b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-b^2\int \sqrt{b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3957}$$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{b^3\int \frac{\sqrt{b\tan(c+dx)}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\int \frac{b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\left(\frac{1}{2}\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\Big(\frac{1}{2}\Big(\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}\Big)}{d}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\left(\frac{1}{2}\left(\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan}\right.}{d}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{d}$$

$\downarrow$ 25

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{d}$$

$\downarrow$ 27

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{d}$$

$\downarrow$ 1103

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(\sqrt{2}b^{3}\right.}{}\right.\right.}{d}$$

input
```
Int[(b*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*b^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) +
ArcTan[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/2 + (Log[b - S
qrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]) - Lo
g[b + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b
]))/2))/d + (2*b*(b*Tan[c + d*x])^(3/2))/(3*d)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| derivativedivides | $\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{b^2\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.\right.}{8(b^2)^{\frac{1}{4}}}\right.}{d}$ |
| default | $\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{b^2\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.\right.}{8(b^2)^{\frac{1}{4}}}\right.}{d}$ |

input
```
int((b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*(1/3*(b*tan(d*x+c))^(3/2)-1/8*b^2/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x
+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b
^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2
)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c
))^(1/2)+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06

$$\int (b\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{6\sqrt{2}b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b}{b}\right)+6\sqrt{2}b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}-b}{b}\right)-3\sqrt{2}b^{\frac{5}{2}}\log\left(b\tan(dx+c)+\right.}{}$$

input
```
integrate((b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(6*sqrt(2)*b^(5/2)*arctan((sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)
/b) + 6*sqrt(2)*b^(5/2)*arctan((sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) - b)/
b) - 3*sqrt(2)*b^(5/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*s
qrt(b) + b) + 3*sqrt(2)*b^(5/2)*log(b*tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*
x + c))*sqrt(b) + b) - 8*sqrt(b*tan(d*x + c))*b^2*tan(d*x + c))/d
```

## Sympy [F]

$$\int (b\tan(c+dx))^{5/2}\,dx = \int (b\tan{(c+dx)})^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((b*tan(c + d*x))**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.13

$$\int (b\tan(c+dx))^{5/2}\,dx = -\frac{3\,b^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{\ldots}\right.}{\sqrt{b}}\right.}{12\,bd}$$

input
```
integrate((b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/12*(3*b^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan
(d*x + c)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c
) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(
d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b)) - 8*(b*tan(d
*x + c))^(3/2)*b^2)/(b*d)
```

## Giac [F(-2)]

Exception generated.

$$\int (b\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[4,14]%%%}+%%%{6,[4,12]%%%}+%%%{15,[4,10]%%%}+%%%{20
,[4,8]%%%
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.47

$$\int (b\tan(c+dx))^{5/2}\,dx = \frac{2\,b\,(b\tan(c+d\,x))^{3/2}}{3\,d} - \frac{(-1)^{1/4}\,b^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{d} + \frac{(-1)^{1/4}\,b^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{d}$$

input
```
int((b*tan(c + d*x))^(5/2),x)
```

output
```
(2*b*(b*tan(c + d*x))^(3/2))/(3*d) - ((-1)^(1/4)*b^(5/2)*atan(((-1)^(1/4)*
(b*tan(c + d*x))^(1/2))/b^(1/2)))/d + ((-1)^(1/4)*b^(5/2)*atanh(((-1)^(1/4
)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/d
```

## Reduce [F]

$$\int (b\tan(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\tan(dx+c)}\,\tan(dx+c)^2\,dx\right)b^2$$

input
```
int((b*tan(d*x+c))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*b**2
```

# 3.11 $\int (b\tan(c+dx))^{3/2}\,dx$

Optimal result . . . . . 227
Mathematica [A] (verified) . . . . . 228
Rubi [A] (warning: unable to verify) . . . . . 228
Maple [A] (verified) . . . . . 232
Fricas [A] (verification not implemented) . . . . . 233
Sympy [F] . . . . . 234
Maxima [A] (verification not implemented) . . . . . 234
Giac [F(-2)] . . . . . 235
Mupad [B] (verification not implemented) . . . . . 235
Reduce [F] . . . . . 236

## Optimal result

Integrand size = 12, antiderivative size = 155

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{b^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}+\sqrt{b}\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{2b\sqrt{b\tan(c+dx)}}{d}$$

output
```
1/2*b^(3/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/d-1/2*b
^(3/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/d-1/2*b^(3/2
)*arctanh(2^(1/2)*(b*tan(d*x+c))^(1/2)/(b^(1/2)+b^(1/2)*tan(d*x+c)))*2^(1/
2)/d+2*b*(b*tan(d*x+c))^(1/2)/d
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}} + 2\sqrt{\tan(c+dx)}\right)(b\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Tan[c + d*x])^(3/2),x]
```

output
```
((ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqr
t[2]) + 2*Sqrt[Tan[c + d*x]])*(b*Tan[c + d*x])^(3/2))/(d*Tan[c + d*x]^(3/2
))
```

## Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.30, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d} - b^2\int \frac{1}{\sqrt{b\tan(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-b^2\int \frac{1}{\sqrt{b\tan(c+dx)}}dx$$

$$\Big\downarrow \; 3957$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{b^3\int \frac{1}{\sqrt{b\tan(c+dx)}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 266$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\int \frac{1}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 755$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{d}$$

$$\Big\downarrow \; 1476$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\cdots}}{2b}\right.}{d}$$

$$\Big\downarrow \; 1082$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{d}$$

$$\Big\downarrow \; 217$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}\right)}{d}$$

$$\Big\downarrow \; 1479$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}\cdots}{2\cdots}\right)}{d}$$

$\big\downarrow$ 25

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\text{arct}\cdots}{\cdots}}{2b}\cdots\right)}{d}$$

$\big\downarrow$ 27

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\text{arct}\cdots}{\cdots}}{2b}\cdots\right)}{d}$$

$\big\downarrow$ 1103

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\log\left(\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+\cdots\right)}{2\sqrt{2}\sqrt{b}}}{2b}\cdots\right)}{d}$$

input
```
Int[(b*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*b^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) +
ArcTan[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/(2*b) + (-1/2*
Log[b - Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[b
]) + Log[b + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]
*Sqrt[b]))/(2*b)))/d + (2*b*Sqrt[b*Tan[c + d*x]])/d
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| derivativedivides | $\frac{2b\left(\sqrt{b\tan(dx+c)}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\right.}{8}\right.}{d}$ |
| default | $\frac{2b\left(\sqrt{b\tan(dx+c)}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\right.}{8}\right.}{d}$ |

input
```
int((b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*((b*tan(d*x+c))^(1/2)-1/8*(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)+(b^2
)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4
)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*
(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)
+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

$$\int (b\tan(c+dx))^{3/2}\,dx =$$
$$-\frac{2\sqrt{2}b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b}{b}\right)+2\sqrt{2}b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}-b}{b}\right)+\sqrt{2}b^{\frac{3}{2}}\log\left(b\tan\left(dx+c\right)+\sqrt{\right.}{4}$$

input
```
integrate((b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*b^(3/2)*arctan((sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/
b) + 2*sqrt(2)*b^(3/2)*arctan((sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) - b)/b
) + sqrt(2)*b^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt
(b) + b) - sqrt(2)*b^(3/2)*log(b*tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c
))*sqrt(b) + b) - 8*sqrt(b*tan(d*x + c))*b)/d
```

## Sympy [F]

$$\int (b\tan(c+dx))^{3/2}\,dx = \int (b\tan(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((b*tan(c + d*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10

$$\int (b\tan(c+dx))^{3/2}\,dx =$$
$$-\frac{2\sqrt{2}b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+2\sqrt{2}b^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+\sqrt{2}b^{\frac{5}{2}}\log\left(b\tan\right.}{}$$

input
```
integrate((b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*b^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan
(d*x + c)))/sqrt(b)) + 2*sqrt(2)*b^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + sqrt(2)*b^(5/2)*log(b*tan(d*x + c
) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - sqrt(2)*b^(5/2)*log(b*tan(
d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - 8*sqrt(b*tan(d*x +
c))*b^2)/(b*d)
```

## Giac [F(-2)]

Exception generated.

$$\int (b\tan(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[3,9]%%%}+%%%{4,[3,7]%%%}+%%%{6,[3,5]%%%}+%%%{4,[3,3
]%%%}+%%%
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{2\,b\,\sqrt{b\tan\left(c+d\,x\right)}}{d} + \frac{{\left(-1\right)}^{1/4}\,b^{3/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{b\tan\left(c+d\,x\right)}}{\sqrt{b}}\right)\,1{}\mathrm{i}}{d} + \frac{{\left(-1\right)}^{1/4}\,b^{3/2}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{b\tan\left(c+d\,x\right)}}{\sqrt{b}}\right)\,1{}\mathrm{i}}{d}$$

input
```
int((b*tan(c + d*x))^(3/2),x)
```

output
```
(2*b*(b*tan(c + d*x))^(1/2))/d + ((-1)^(1/4)*b^(3/2)*atan(((-1)^(1/4)*(b*t
an(c + d*x))^(1/2))/b^(1/2))*1i)/d + ((-1)^(1/4)*b^(3/2)*atanh(((-1)^(1/4)
*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/d
```

## Reduce [F]

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{\sqrt{b}\,b\left(2\sqrt{\tan(dx+c)}-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)d\right)}{d}$$

input
```
int((b*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*b*(2*sqrt(tan(c + d*x)) - int(sqrt(tan(c + d*x))/tan(c + d*x),x)*
d))/d
```

# 3.12 $\int \sqrt{b\tan(c+dx)}\,dx$

Optimal result . . . 237
Mathematica [A] (verified) . . . 237
Rubi [A] (warning: unable to verify) . . . 238
Maple [A] (verified) . . . 241
Fricas [A] (verification not implemented) . . . 242
Sympy [F] . . . 242
Maxima [A] (verification not implemented) . . . 243
Giac [A] (verification not implemented) . . . 243
Mupad [B] (verification not implemented) . . . 244
Reduce [F] . . . 244

## Optimal result

Integrand size = 12, antiderivative size = 137

$$\int \sqrt{b\tan(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{\sqrt{b}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}+\sqrt{b}\tan(c+dx)}\right)}{\sqrt{2}d}$$

output
```
-1/2*b^(1/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/d+1/2*
b^(1/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/d-1/2*b^(1/
2)*arctanh(2^(1/2)*(b*tan(d*x+c))^(1/2)/(b^(1/2)+b^(1/2)*tan(d*x+c)))*2^(1
/2)/d
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int \sqrt{b\tan(c+dx)}\,dx = \frac{\left(\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)-\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\right)\sqrt[4]{-\tan(c+dx)}\sqrt{b\tan(c+dx)}}{d\tan^{\frac{3}{4}}(c+dx)}$$

input
```
Integrate[Sqrt[b*Tan[c + d*x]],x]
```

output
```
((ArcTan[(-Tan[c + d*x]^2)^(1/4)] - ArcTanh[(-Tan[c + d*x]^2)^(1/4)])*(-Ta
n[c + d*x])^(1/4)*Sqrt[b*Tan[c + d*x]])/(d*Tan[c + d*x]^(3/4))
```

## Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{b\tan(c+dx)}dx$$

$$\Big\downarrow \; 3957$$

$$\frac{b\int \frac{\sqrt{b\tan(c+dx)}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 266$$

$$\frac{2b\int \frac{b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 826$$

$$\frac{2b\left(\frac{1}{2}\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{d}$$

$$\Big\downarrow \; 1476$$

$$\frac{2b\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}\right)-\right.}{d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2b\left(\frac{1}{2}\left(\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(}\right.}{d}$$

$$\Big\downarrow \text{217}$$

$$\frac{2b\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2b\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\right.}}{\sqrt{}}\right.\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b\right.}}{\sqrt{}}\right.\right.}{d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2b\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(\sqrt{2}b^{3/\right.}}{}\right.\right.}{d}$$

input
```
Int[Sqrt[b*Tan[c + d*x]],x]
```

output
```
(2*b*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) + Arc
Tan[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/2 + (Log[b - Sqrt
[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]) - Log[b
 + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]))
/2))/d
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| derivativedivides | $\frac{b\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+\right.\right.}{4d(b^2)^{\frac{1}{4}}}$ |
| default | $\frac{b\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+\right.\right.}{4d(b^2)^{\frac{1}{4}}}$ |

input
```
int((b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*b/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(
1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2
^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-
2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.05

$$\int \sqrt{b\tan(c+dx)}\,dx = \frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b}{b}\right)+2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}-b}{b}\right)-\sqrt{2}\sqrt{b}\log\left(b\tan\left(dx+c\right)+\right.}{4\,d}$$

input
```
integrate((b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*sqrt(b)*arctan((sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/b
) + 2*sqrt(2)*sqrt(b)*arctan((sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) - b)/b)
 - sqrt(2)*sqrt(b)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(
b) + b) + sqrt(2)*sqrt(b)*log(b*tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c)
)*sqrt(b) + b))/d
```

## Sympy [F]

$$\int \sqrt{b\tan(c+dx)}\,dx = \int \sqrt{b\tan\left(c+dx\right)}\,dx$$

input
```
integrate((b*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*tan(c + d*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int \sqrt{b\tan(c+dx)}\,dx$$
$$= \frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+\right.}{\sqrt{b}}\right.}{4\,d}$$

input
```
integrate((b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x +
c)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2
*sqrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) + sqr
t(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x + c
) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b))/d
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.28

$$\int \sqrt{b\tan(c+dx)}\,dx$$
$$= \frac{\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}-\frac{\sqrt{2}|b|^{\frac{3}{2}}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(\right.}{d}}{4\,b}$$

input
```
integrate((b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*s
qrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 2*sqrt(2)*abs(b)^(3/2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d -
sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqr
t(abs(b)) + abs(b))/d + sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) - sqrt(2)*
sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d)/b
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.36

$$\int \sqrt{b\tan(c+dx)}\,dx = \frac{(-1)^{1/4}\sqrt{b}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\right)}{d}$$

input
```
int((b*tan(c + d*x))^(1/2),x)
```

output
```
((-1)^(1/4)*b^(1/2)*(atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)) - a
tanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))))/d
```

## Reduce [F]

$$\int \sqrt{b\tan(c+dx)}\,dx = \sqrt{b}\left(\int \sqrt{\tan(dx+c)}dx\right)$$

input
```
int((b*tan(d*x+c))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(tan(c + d*x)),x)
```

# 3.13 $\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$


Optimal result . . . . . 245
Mathematica [A] (verified) . . . . . 245
Rubi [A] (warning: unable to verify) . . . . . 246
Maple [A] (verified) . . . . . 250
Fricas [A] (verification not implemented) . . . . . 250
Sympy [F] . . . . . 251
Maxima [A] (verification not implemented) . . . . . 251
Giac [A] (verification not implemented) . . . . . 252
Mupad [B] (verification not implemented) . . . . . 252
Reduce [F] . . . . . 253


## Optimal result

Integrand size = 12, antiderivative size = 136

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}+\sqrt{b}\tan(c+dx)}\right)}{\sqrt{2}\sqrt{b}d}$$

output
```
-1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/b^(1/2)/d+1/2*
arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/b^(1/2)/d+1/2*arcta
nh(2^(1/2)*(b*tan(d*x+c))^(1/2)/(b^(1/2)+b^(1/2)*tan(d*x+c)))*2^(1/2)/b^(1
/2)/d
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx =\frac{\left(-2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\right.\right.}{2\sqrt{2}d\sqrt{b\tan(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[c + d*x]],x]
```

output
```
((-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[Ta
n[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Log[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/(2*Sqrt[
2]*d*Sqrt[b*Tan[c + d*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.32, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\int \frac{1}{\sqrt{b\tan(c+dx)}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b\int \frac{1}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{755}$$

$$\frac{2b\left(\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{d}$$

$$\Big\downarrow \text{1476}$$

$$2b\left(\frac{\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan}}{2b}\right.$$
$$\overline{\qquad d\qquad}$$

$\downarrow$ 1082

$$\frac{2b\left(\frac{\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{d}$$

$\downarrow$ 217

$$\frac{2b\left(\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}\right)}{d}$$

$\downarrow$ 1479

$$2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-}{2b}\right.$$
$$\overline{\qquad d\qquad}$$

$\downarrow$ 25

$$2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\text{arcta}}{2b}\right.$$
$$\overline{\qquad d\qquad}$$

$\downarrow$ 27

$$2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\text{arcta}}{2b}\right.$$
$$\overline{\qquad d\qquad}$$

$\downarrow$ 1103

$$\frac{2b\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\log\left(\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}}{2b}\right)}{d}$$

input
```
Int[1/Sqrt[b*Tan[c + d*x]],x]
```

output
```
(2*b*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) + Arc
Tan[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/(2*b) + (-1/2*Log
[b - Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[b])
+ Log[b + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sq
rt[b]))/(2*b)))/d
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| derivativedivides | $\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{b\tan(dx}}{(b^2)^{\frac{1}{4}}}\right.\right.}{4db}$ |
| default | $\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{b\tan(dx}}{(b^2)^{\frac{1}{4}}}\right.\right.}{4db}$ |

input
```
int(1/(b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/b*(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(
1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2
^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-
2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{\sqrt{b}}+1\right)}{\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{\sqrt{b}}-1\right)}{\sqrt{b}}+\frac{\sqrt{2}\log\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{\sqrt{b}}+\tan(dx+c)+1\right)}{\sqrt{b}}-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{b\tan}}{\sqrt{b}}\right.}{}}{4\,d}$$

input
```
integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*arctan(sqrt(2)*sqrt(b*tan(d*x + c))/sqrt(b) + 1)/sqrt(b) +
2*sqrt(2)*arctan(sqrt(2)*sqrt(b*tan(d*x + c))/sqrt(b) - 1)/sqrt(b) + sqrt(
2)*log(sqrt(2)*sqrt(b*tan(d*x + c))/sqrt(b) + tan(d*x + c) + 1)/sqrt(b) -
sqrt(2)*log(-sqrt(2)*sqrt(b*tan(d*x + c))/sqrt(b) + tan(d*x + c) + 1)/sqrt
(b))/d
```

## Sympy [F]

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(c + d*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = \frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+\sqrt{2}\sqrt{b}\log\left(b\,\mathrm{ta}\right.}{4\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(
d*x + c)))/sqrt(b)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + sqrt(2)*sqrt(b)*log(b*tan(d*x + c)
 + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - sqrt(2)*sqrt(b)*log(b*tan(d
*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b))/(b*d)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{2\,bd} + \frac{\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{2\,bd} + \frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{|b|}+|b|\right)}{4\,bd} - \frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan(dx+c)-\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{|b|}+|b|\right)}{4\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*sqrt
(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 1/2*sqrt(2)*sqrt(abs(b))*arctan(-1
/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(
b*d) + 1/4*sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*
x + c))*sqrt(abs(b)) + abs(b))/(b*d) - 1/4*sqrt(2)*sqrt(abs(b))*log(b*tan(
d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b*d)
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = -\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{\sqrt{b}\,d} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{\sqrt{b}\,d}$$

input
```
int(1/(b*tan(c + d*x))^(1/2),x)
```

output
```
- ((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/(b^(1/
2)*d) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)
/(b^(1/2)*d)
```

## Reduce [F]

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)}{b}$$

input
```
int(1/(b*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(tan(c + d*x))/tan(c + d*x),x))/b
```

# 3.14 $\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 254
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 254
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . 255
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 259
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 260
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 261
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 261
Giac [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 262
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 262
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 262

## Optimal result

Integrand size = 12, antiderivative size = 156

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx=\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}+\sqrt{b}\tan(c+dx)}\right)}{\sqrt{2}b^{3/2}d}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

output

```
1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/b^(3/2)/d-1/2*a
rctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/b^(3/2)/d+1/2*arctan
h(2^(1/2)*(b*tan(d*x+c))^(1/2)/(b^(1/2)+b^(1/2)*tan(d*x+c)))*2^(1/2)/b^(3/
2)/d-2/b/d/(b*tan(d*x+c))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx=\frac{-2-\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}+\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(c+a\right.}\right.}{bd\sqrt{b\tan(c+dx)}}$$

input
```
Integrate[(b*Tan[c + d*x])^(-3/2),x]
```

output
```
(-2 - ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4) + ArcTanh[(-
Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4))/(b*d*Sqrt[b*Tan[c + d*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$-\frac{\int \sqrt{b\tan(c+dx)}dx}{b^2}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sqrt{b\tan(c+dx)}dx}{b^2}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\int \frac{\sqrt{b\tan(c+dx)}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2\int \frac{b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}\right)\right.}{bd}$$

$$\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan\right.}{bd}$$

$$\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{bd}$$

$$\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.}{bd}$$

$$\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.}{bd}$$

$$\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$\downarrow$ 1103

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(\sqrt{2}b^{3}\right.}{}\right.}{bd}$$

$$\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x])^(-3/2),x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) + ArcT
an[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/2 + (Log[b - Sqrt[
2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]) - Log[b
+ Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]))/
2))/(b*d) - 2/(b*d*Sqrt[b*Tan[c + d*x]])
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| derivativedivides | $\frac{2b\left(-\frac{1}{b^2\sqrt{b\tan(dx+c)}}-\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.\right.}{8b^2\left(b^2\right)^{\frac{1}{4}}}}{d}$ |
| default | $\frac{2b\left(-\frac{1}{b^2\sqrt{b\tan(dx+c)}}-\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.\right.}{8b^2\left(b^2\right)^{\frac{1}{4}}}}{d}$ |

input
```
int(1/(b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*(-1/b^2/(b*tan(d*x+c))^(1/2)-1/8/b^2/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(
d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)
+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(
b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*
x+c))^(1/2)+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{\sqrt{b}}+1\right)\tan(dx+c)+2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{\sqrt{b}}-1\right)\tan(dx+c)-\sqrt{}}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*sqrt(b*tan(d*x + c))/sqrt(b) + 1)*t
an(d*x + c) + 2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*sqrt(b*tan(d*x + c))/sqrt(b
) - 1)*tan(d*x + c) - sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(b*tan(d*x + c))/sqr
t(b) + tan(d*x + c) + 1)*tan(d*x + c) + sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(
b*tan(d*x + c))/sqrt(b) + tan(d*x + c) + 1)*tan(d*x + c) + 8*sqrt(b*tan(d*
x + c)))/(b^2*d*tan(d*x + c))
```

## Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((b*tan(c + d*x))**(-3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.07

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}} - \frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{\sqrt{b}} - \cdots}{4\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x + c
)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2*
sqrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) + sqrt
(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x + c)
 - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + 8/sqrt(b*tan(d*x +
c)))/(b*d)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.49

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \frac{(-1)^{1/4}\,\text{atanh}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d} - \frac{(-1)^{1/4}\,\text{atan}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d} - \frac{2}{b\,d\,\sqrt{b\tan\left(c+d\,x\right)}}$$

input
```
int(1/(b*tan(c + d*x))^(3/2),x)
```

output
```
((-1)^(1/4)*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(3/2)*d
) - ((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(3/2
)*d) - 2/(b*d*(b*tan(c + d*x))^(1/2))
```

## Reduce [F]

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)}{b^2}$$

input
```
int(1/(b*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x))/b**2
```

# 3.15 $\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 264
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 264
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . 265
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 269
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 270
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 271
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 271
Giac [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 272
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 272
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 272

## Optimal result

Integrand size = 12, antiderivative size = 159

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}+\sqrt{b}\tan(c+dx)}\right)}{\sqrt{2}b^{5/2}d} - \frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

output
```
1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/b^(5/2)/d-1/2*a
rctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/b^(5/2)/d-1/2*arctan
h(2^(1/2)*(b*tan(d*x+c))^(1/2)/(b^(1/2)+b^(1/2)*tan(d*x+c)))*2^(1/2)/b^(5/
2)/d-2/3/b/d/(b*tan(d*x+c))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.54

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \frac{-2+3\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\left(-\tan^2(c+dx)\right)^{3/4}+3\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(\right.}{3bd(b\tan(c+dx))^{3/2}}$$

input
```
Integrate[(b*Tan[c + d*x])^(-5/2),x]
```

output
```
(-2 + 3*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(3/4) + 3*ArcTan
h[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(3/4))/(3*b*d*(b*Tan[c + d*x]
)^(3/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.29, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {3042, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(b\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3955}$$
$$-\frac{\int \frac{1}{\sqrt{b\tan(c+dx)}}dx}{b^2}-\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\int \frac{1}{\sqrt{b\tan(c+dx)}}dx}{b^2}-\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$
$$\Big\downarrow \text{3957}$$
$$-\frac{\int \frac{1}{\sqrt{b\tan(c+dx)}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{bd}-\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$
$$\Big\downarrow \text{266}$$
$$-\frac{2\int \frac{1}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{bd}-\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$
$$\Big\downarrow \text{755}$$

$$-\frac{2\left(\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{bd}-\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 1476

$$-\frac{2\left(\frac{\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\cdots}}{2b}\cdots\right)}{bd}\cdots$$

$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 1082

$$-\frac{2\left(\frac{\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{bd}-$$

$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 217

$$-\frac{2\left(\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}\right)}{bd}-$$

$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 1479

$$-\frac{2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}\cdots}{2\cdots}\cdots\right)}{bd}\cdots$$

$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 25

$$-\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\text{arct}}{}}{2b}\right.}{bd}$$

$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\text{arct}}{}}{2b}\right.}{bd}$$

$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\log\left(\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+\right.}{2\sqrt{2}\sqrt{b}}}{2b}\right.}{bd}$$

$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

input
```
Int[(b*Tan[c + d*x])^(-5/2),x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) + ArcT
an[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/(2*b) + (-1/2*Log[
b - Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[b]) +
 Log[b + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqr
t[b]))/(2*b)))/(b*d) - 2/(3*b*d*(b*Tan[c + d*x])^(3/2))
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| derivativedivides | $\frac{2b\left(-\frac{1}{3b^2(b\tan(dx+c))^{\frac{3}{2}}}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{ar}}{8b^4}\right.}{d}$ |
| default | $\frac{2b\left(-\frac{1}{3b^2(b\tan(dx+c))^{\frac{3}{2}}}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{ar}}{8b^4}\right.}{d}$ |

input
```
int(1/(b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*(-1/3/b^2/(b*tan(d*x+c))^(3/2)-1/8/b^4*(b^2)^(1/4)*2^(1/2)*(ln((b*ta
n(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+
c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)
/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(
d*x+c))^(1/2)+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.17

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{6\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{\sqrt{b}}+1\right)\tan(dx+c)^2+6\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{\sqrt{b}}-1\right)\tan(dx+c)^2+3}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(6*sqrt(2)*sqrt(b)*arctan(sqrt(2)*sqrt(b*tan(d*x + c))/sqrt(b) + 1)*
tan(d*x + c)^2 + 6*sqrt(2)*sqrt(b)*arctan(sqrt(2)*sqrt(b*tan(d*x + c))/sqr
t(b) - 1)*tan(d*x + c)^2 + 3*sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(b*tan(d*x +
c))/sqrt(b) + tan(d*x + c) + 1)*tan(d*x + c)^2 - 3*sqrt(2)*sqrt(b)*log(-sq
rt(2)*sqrt(b*tan(d*x + c))/sqrt(b) + tan(d*x + c) + 1)*tan(d*x + c)^2 + 8*
sqrt(b*tan(d*x + c)))/(b^3*d*tan(d*x + c)^2)
```

## Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((b*tan(c + d*x))**(-5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = -\frac{\frac{6\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{b^{\frac{3}{2}}} + \frac{6\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{b^{\frac{3}{2}}} + \frac{3\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{b^{\frac{3}{2}}}}{12\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/12*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x +
c)))/sqrt(b))/b^(3/2) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2
*sqrt(b*tan(d*x + c)))/sqrt(b))/b^(3/2) + 3*sqrt(2)*log(b*tan(d*x + c) + s
qrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/b^(3/2) - 3*sqrt(2)*log(b*tan(d*x
 + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/b^(3/2) + 8/(b*tan(d*x +
 c))^(3/2))/(b*d)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.47

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = -\frac{2}{3\,b\,d\,(b\tan(c+d\,x))^{3/2}} + \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{b^{5/2}\,d} + \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{b^{5/2}\,d}$$

input
```
int(1/(b*tan(c + d*x))^(5/2),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/(b^(5/2)
*d) - 2/(3*b*d*(b*tan(c + d*x))^(3/2)) + ((-1)^(1/4)*atanh(((-1)^(1/4)*(b*
tan(c + d*x))^(1/2))/b^(1/2))*1i)/(b^(5/2)*d)
```

## Reduce [F]

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)}{b^3}$$

input
```
int(1/(b*tan(d*x+c))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x))/b**3
```

# 3.16 $\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx$


Optimal result . . . . . . . . . . 273
Mathematica [A] (verified) . . . . . . . . . . 274
Rubi [A] (warning: unable to verify) . . . . . . . . . . 274
Maple [A] (verified) . . . . . . . . . . 279
Fricas [A] (verification not implemented) . . . . . . . . . . 279
Sympy [F] . . . . . . . . . . 280
Maxima [A] (verification not implemented) . . . . . . . . . . 280
Giac [F(-1)] . . . . . . . . . . 281
Mupad [B] (verification not implemented) . . . . . . . . . . 281
Reduce [F] . . . . . . . . . . 281


## Optimal result

Integrand size = 12, antiderivative size = 179

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}+\sqrt{b}\tan(c+dx)}\right)}{\sqrt{2}b^{7/2}d} - \frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}}$$

output
```
-1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/b^(7/2)/d+1/2*
arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*2^(1/2)/b^(7/2)/d-1/2*arcta
nh(2^(1/2)*(b*tan(d*x+c))^(1/2)/(b^(1/2)+b^(1/2)*tan(d*x+c)))*2^(1/2)/b^(7
/2)/d-2/5/b/d/(b*tan(d*x+c))^(5/2)+2/b^3/d/(b*tan(d*x+c))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.54

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{10-2\cot^2(c+dx)+5\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}-5\mathrm{arcta}}{5b^3d\sqrt{b\tan(c+dx)}}$$

input
```
Integrate[(b*Tan[c + d*x])^(-7/2),x]
```

output
```
(10 - 2*Cot[c + d*x]^2 + 5*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^
2)^(1/4) - 5*ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4))/(5*
b^3*d*Sqrt[b*Tan[c + d*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.26, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {3042, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$-\frac{\int \frac{1}{(b\tan(c+dx))^{3/2}}dx}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{1}{(b\tan(c+dx))^{3/2}}dx}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3955}$$

$$-\frac{-\frac{\int \sqrt{b\tan(c+dx)}dx}{b^2}-\frac{2}{bd\sqrt{b\tan(c+dx)}}}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{-\frac{\int \sqrt{b\tan(c+dx)}dx}{b^2}-\frac{2}{bd\sqrt{b\tan(c+dx)}}}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3957}$$

$$-\frac{-\frac{\int \frac{\sqrt{b\tan(c+dx)}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{266}$$

$$-\frac{-\frac{2\int \frac{b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{826}$$

$$-\frac{-\frac{2\left(\frac{1}{2}\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{1476}$$

$$-\frac{-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}}{bd}}{b^2}$$

$$\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{1082}$$

$$-\frac{-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{bd}-\frac{}{bd}}{b^2}$$

$$\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$\downarrow$ 217

$$-\frac{-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}}{b^2}-$$

$$\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$\downarrow$ 1479

$$-\frac{-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+d\right.}{\sqrt{2}\sqrt{b}}\right.}{bd}}{b^2}$$

$$\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$\downarrow$ 25

$$-\frac{-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx\right.}{\sqrt{2}\sqrt{b}}\right.}{bd}}{b^2}$$

$$\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$\downarrow$ 27

$$-\frac{-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx\right.}{\sqrt{2}\sqrt{b}}\right.}{bd}}{b^2}$$

$$\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$\downarrow$ 1103

$$-\frac{-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(\right.}{2\sqrt{2}\sqrt{b}}\right.}{bd}}{b^2}$$

$$\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

input
```
Int[(b*Tan[c + d*x])^(-7/2),x]
```

output
```
-2/(5*b*d*(b*Tan[c + d*x])^(5/2)) - ((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Ta
n[c + d*x]]/(Sqrt[2]*Sqrt[b])) + ArcTan[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/
(Sqrt[2]*Sqrt[b]))/2 + (Log[b - Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c +
 d*x]^2]/(2*Sqrt[2]*Sqrt[b]) - Log[b + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*
Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]))/2))/(b*d) - 2/(b*d*Sqrt[b*Tan[c + d*x
]]))/b^2
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| derivativedivides | $\frac{2b\left(-\frac{1}{5b^2(b\tan(dx+c))^{\frac{5}{2}}}+\frac{1}{b^4\sqrt{b\tan(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+}}{(b^2)^{\frac{1}{4}}}\right.}{8b^4(b^2)^{\frac{1}{4}}}\right.}{d}$ |
| default | $\frac{2b\left(-\frac{1}{5b^2(b\tan(dx+c))^{\frac{5}{2}}}+\frac{1}{b^4\sqrt{b\tan(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+}}{(b^2)^{\frac{1}{4}}}\right.}{8b^4(b^2)^{\frac{1}{4}}}\right.}{d}$ |

input
```
int(1/(b*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*(-1/5/b^2/(b*tan(d*x+c))^(5/2)+1/b^4/(b*tan(d*x+c))^(1/2)+1/8/b^4/(b
^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/
2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^
2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-
2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.11

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{10\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{\sqrt{b}}+1\right)\tan(dx+c)^3+10\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(}}{\sqrt{b}}\right.}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/20*(10*sqrt(2)*sqrt(b)*arctan(sqrt(2)*sqrt(b*tan(d*x + c))/sqrt(b) + 1)*
tan(d*x + c)^3 + 10*sqrt(2)*sqrt(b)*arctan(sqrt(2)*sqrt(b*tan(d*x + c))/sq
rt(b) - 1)*tan(d*x + c)^3 - 5*sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(b*tan(d*x +
 c))/sqrt(b) + tan(d*x + c) + 1)*tan(d*x + c)^3 + 5*sqrt(2)*sqrt(b)*log(-s
qrt(2)*sqrt(b*tan(d*x + c))/sqrt(b) + tan(d*x + c) + 1)*tan(d*x + c)^3 + 8
*sqrt(b*tan(d*x + c))*(5*tan(d*x + c)^2 - 1))/(b^4*d*tan(d*x + c)^3)
```

## Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\tan{(c+dx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(7/2),x)
```

output
```
Integral((b*tan(c + d*x))**(-7/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.09

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{2}\log\left(b\tan(dx+c)+\cdots\right.}{\cdots}\cdots\right)}{b^2}\cdots}{20\,bd\cdots}$$

input
```
integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
1/20*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x
+ c)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) -
 2*sqrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) + s
qrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x +
 c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b))/b^2 + 8*(5*b^2*ta
n(d*x + c)^2 - b^2)/((b*tan(d*x + c))^(5/2)*b^2))/(b*d)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.51

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}\,d} - \frac{\frac{2}{5\,b}-\frac{2\tan(c+d\,x)^2}{b}}{d\,(b\tan(c+d\,x))^{5/2}} - \frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}\,d}$$

input
```
int(1/(b*tan(c + d*x))^(7/2),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(7/2)*d)
 - (2/(5*b) - (2*tan(c + d*x)^2)/b)/(d*(b*tan(c + d*x))^(5/2)) - ((-1)^(1/
4)*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(7/2)*d)
```

## Reduce [F]

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4}dx\right)}{b^4}$$

input
```
int(1/(b*tan(d*x+c))^(7/2),x)
```

output
```
(sqrt(b)*int(sqrt(tan(c + d*x))/tan(c + d*x)**4,x))/b**4
```

# 3.17 $\int (b\tan(c+dx))^{4/3}\,dx$

Optimal result . . . 283
Mathematica [C] (verified) . . . 284
Rubi [A] (warning: unable to verify) . . . 284
Maple [A] (verified) . . . 289
Fricas [A] (verification not implemented) . . . 289
Sympy [F] . . . 290
Maxima [A] (verification not implemented) . . . 290
Giac [A] (verification not implemented) . . . 291
Mupad [B] (verification not implemented) . . . 292
Reduce [F] . . . 293

## Optimal result

Integrand size = 12, antiderivative size = 243

$$\int (b\tan(c+dx))^{4/3}\,dx = -\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{b^{4/3}\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d} - \frac{b^{4/3}\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d} + \frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} - \frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} + \frac{3b\sqrt[3]{b\tan(c+dx)}}{d}$$

output
```
-b^(4/3)*arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/d-1/2*b^(4/3)*arctan(-3^(1/2
)+2*(b*tan(d*x+c))^(1/3)/b^(1/3))/d-1/2*b^(4/3)*arctan(3^(1/2)+2*(b*tan(d*
x+c))^(1/3)/b^(1/3))/d+1/4*3^(1/2)*b^(4/3)*ln(b^(2/3)-3^(1/2)*b^(1/3)*(b*t
an(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))/d-1/4*3^(1/2)*b^(4/3)*ln(b^(2/3)+3^
(1/2)*b^(1/3)*(b*tan(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))/d+3*b*(b*tan(d*x+
c))^(1/3)/d
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.84

$$\int (b\tan(c+dx))^{4/3}\,dx = \frac{b\sqrt[3]{b\tan(c+dx)}\left(-i\log\left(1-i\sqrt[6]{\tan^2(c+dx)}\right)+i\log\left(1+i\sqrt[6]{\tan^2(c+dx)}\right)-(-1)^{5/6}\log\ldots\right.}{\ldots}$$

input
```
Integrate[(b*Tan[c + d*x])^(4/3),x]
```

output
```
(b*(b*Tan[c + d*x])^(1/3)*((-I)*Log[1 - I*(Tan[c + d*x]^2)^(1/6)] + I*Log[
1 + I*(Tan[c + d*x]^2)^(1/6)] - (-1)^(5/6)*Log[1 - (-1)^(1/6)*(Tan[c + d*x
]^2)^(1/6)] + (-1)^(5/6)*Log[1 + (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)] - (-1)
^(1/6)*Log[1 - (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)] + (-1)^(1/6)*Log[1 + (-1
)^(5/6)*(Tan[c + d*x]^2)^(1/6)] + 6*(Tan[c + d*x]^2)^(1/6)))/(2*d*(Tan[c +
 d*x]^2)^(1/6))
```

## Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3954, 3042, 3957, 266, 753, 27, 216, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (b\tan(c+dx))^{4/3}dx$$

$$\Big\downarrow \ 3954$$

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-b^2\int\frac{1}{(b\tan(c+dx))^{2/3}}dx$$

$\downarrow$ 3042

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-b^2\int\frac{1}{(b\tan(c+dx))^{2/3}}dx$$

$\downarrow$ 3957

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-\frac{b^3\int\frac{1}{(b\tan(c+dx))^{2/3}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}$$

$\downarrow$ 266

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-\frac{3b^3\int\frac{1}{b^6\tan^6(c+dx)+b^2}d\sqrt[3]{b\tan(c+dx)}}{d}$$

$\downarrow$ 753

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-\frac{3b^3\left(\frac{\int\frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{3b^{4/3}}+\frac{\int\frac{2\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{2\left(b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}\right)}d\sqrt[3]{b\tan(c+dx)}}{3b^{5/3}}+\frac{\int\frac{2\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b}}{2\left(b^2\tan^2(c+dx)+\sqrt{3}\right.}\right.}{d}$$

$\downarrow$ 27

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-\frac{3b^3\left(\frac{\int\frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{3b^{4/3}}+\frac{\int\frac{2\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\int\frac{2\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan}\right.}{d}$$

$\downarrow$ 216

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-\frac{3b^3\left(\frac{\int\frac{2\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\int\frac{2\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\arctan\right.}{d}$$

$\downarrow$ 1142

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$

$$3b^3\left(\frac{\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}\right.$$

$\downarrow$ 25

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$

$$3b^3\left(\frac{\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\right.$$

$\downarrow$ 1082

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$

$$3b^3\left(\frac{\frac{\int \frac{1}{-b^2\tan^2(c+dx)-\frac{1}{3}}d\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}+2\sqrt[3]{\ldots}}{b^2\tan^2(c+dx)+\ldots}}{\ldots}\right.$$

$$d$$

$\downarrow$ 217

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$

$$3b^3\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)}{6b^{5/3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}+2\sqrt[3]{b\tan\ldots}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\ldots}}{\ldots}\right.$$

$$d$$

$\downarrow$ 1103

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$

$$3b^3\left(\frac{\arctan\left(b^{2/3}\tan(c+dx)\right)}{3b^{5/3}}+\frac{-\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)-\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}+b^2\tan^2(c+dx)\right)}{6b^{5/3}}+\frac{\arctan\left(\sqrt{3}\ldots\right.}{\ldots}\right.$$

$$d$$

input

```
Int[(b*Tan[c + d*x])^(4/3),x]
```

output
```
(-3*b^3*(ArcTan[b^(2/3)*Tan[c + d*x]]/(3*b^(5/3)) + (-ArcTan[Sqrt[3]*(1 -
(2*b^(2/3)*Tan[c + d*x])/Sqrt[3])] - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*b^(4/3
)*Tan[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(5/3)) + (ArcTan[Sqrt[3]*(1
+ (2*b^(2/3)*Tan[c + d*x])/Sqrt[3])] + (Sqrt[3]*Log[b^(2/3) + Sqrt[3]*b^(4
/3)*Tan[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(5/3))))/d + (3*b*(b*Tan[c
 + d*x])^(1/3))/d
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| derivativedivides | $\frac{3b(b\tan(dx+c))^{\frac{1}{3}}}{d}-\frac{b\sqrt{3}\,(b^2)^{\frac{1}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{4d}-\frac{b(b^2)^{\frac{1}{6}}\arctan\left(\frac{2(b\tan}{(}\right.}{2d}$ |
| default | $\frac{3b(b\tan(dx+c))^{\frac{1}{3}}}{d}-\frac{b\sqrt{3}\,(b^2)^{\frac{1}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{4d}-\frac{b(b^2)^{\frac{1}{6}}\arctan\left(\frac{2(b\tan}{(}\right.}{2d}$ |

input
```
int((b*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3*b*(b*tan(d*x+c))^(1/3)/d-1/4/d*b*3^(1/2)*(b^2)^(1/6)*ln((b*tan(d*x+c))^(
2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))-1/2/d*b*(b^2)^(
1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)+3^(1/2))+1/4/d*b*3^(1/2)*(b
^2)^(1/6)*ln((b*tan(d*x+c))^(2/3)-3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)
+(b^2)^(1/3))-1/2/d*b*(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6
)-3^(1/2))-1/d*b*(b^2)^(1/6)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.28

$$\int (b\tan(c+dx))^{4/3}\,dx =$$

$$-\frac{\left(-\frac{b^8}{d^6}\right)^{\frac{1}{6}}\left(\sqrt{-3}d+d\right)\log\left((b\tan(dx+c))^{\frac{1}{3}}b+\frac{1}{2}\left(-\frac{b^8}{d^6}\right)^{\frac{1}{6}}\left(\sqrt{-3}d+d\right)\right)-\left(-\frac{b^8}{d^6}\right)^{\frac{1}{6}}\left(\sqrt{-3}d+d\right)\log\left((\right.}{}$$

input
```
integrate((b*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
-1/4*((-b^8/d^6)^(1/6)*(sqrt(-3)*d + d)*log((b*tan(d*x + c))^(1/3)*b + 1/2
*(-b^8/d^6)^(1/6)*(sqrt(-3)*d + d)) - (-b^8/d^6)^(1/6)*(sqrt(-3)*d + d)*lo
g((b*tan(d*x + c))^(1/3)*b - 1/2*(-b^8/d^6)^(1/6)*(sqrt(-3)*d + d)) + (-b^
8/d^6)^(1/6)*(sqrt(-3)*d - d)*log((b*tan(d*x + c))^(1/3)*b + 1/2*(-b^8/d^6
)^(1/6)*(sqrt(-3)*d - d)) - (-b^8/d^6)^(1/6)*(sqrt(-3)*d - d)*log((b*tan(d
*x + c))^(1/3)*b - 1/2*(-b^8/d^6)^(1/6)*(sqrt(-3)*d - d)) + 2*(-b^8/d^6)^(
1/6)*d*log((b*tan(d*x + c))^(1/3)*b + (-b^8/d^6)^(1/6)*d) - 2*(-b^8/d^6)^(
1/6)*d*log((b*tan(d*x + c))^(1/3)*b - (-b^8/d^6)^(1/6)*d) - 12*(b*tan(d*x
+ c))^(1/3)*b)/d
```

## Sympy [F]

$$\int (b\tan(c+dx))^{4/3}\,dx = \int (b\tan{(c+dx)})^{\frac{4}{3}}\,dx$$

input
```
integrate((b*tan(d*x+c))**(4/3),x)
```

output
```
Integral((b*tan(c + d*x))**(4/3), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.76

$$\int (b\tan(c+dx))^{4/3}\,dx =$$

$$-\frac{\sqrt{3}b^{\frac{7}{3}}\log\left(\sqrt{3}(b\tan{(dx+c)})^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan{(dx+c)})^{\frac{2}{3}}+b^{\frac{2}{3}}\right)-\sqrt{3}b^{\frac{7}{3}}\log\left(-\sqrt{3}(b\tan{(dx+c)})^{\frac{1}{3}}b^{\frac{1}{3}}+(b\,t\right.}{}$$

input
```
integrate((b*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
-1/4*(sqrt(3)*b^(7/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(
d*x + c))^(2/3) + b^(2/3)) - sqrt(3)*b^(7/3)*log(-sqrt(3)*(b*tan(d*x + c))
^(1/3)*b^(1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3)) + 2*b^(7/3)*arctan((sqr
t(3)*b^(1/3) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 2*b^(7/3)*arctan(-(sqr
t(3)*b^(1/3) - 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 4*b^(7/3)*arctan((b*ta
n(d*x + c))^(1/3)/b^(1/3)) - 12*(b*tan(d*x + c))^(1/3)*b^2)/(b*d)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.86

$$\int (b\tan(c+dx))^{4/3}\,dx =$$

$$-\frac{1}{4}b\left(\frac{\sqrt{3}|b|^{\frac{1}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{d}-\frac{\sqrt{3}|b|^{\frac{1}{3}}\log\left(-\sqrt{3}(b\tan(dx+c)\right.}{}\right.$$

input
```
integrate((b*tan(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
-1/4*b*(sqrt(3)*abs(b)^(1/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/
3) + (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/d - sqrt(3)*abs(b)^(1/3)*log(-
sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) + abs
(b)^(2/3))/d + 2*abs(b)^(1/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*tan(d*x
+ c))^(1/3))/abs(b)^(1/3))/d + 2*abs(b)^(1/3)*arctan(-(sqrt(3)*abs(b)^(1/3
) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/d + 4*abs(b)^(1/3)*arctan((b*t
an(d*x + c))^(1/3)/abs(b)^(1/3))/d - 12*(b*tan(d*x + c))^(1/3)/d)
```

## Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.02

$$\int (b\tan(c+dx))^{4/3}\,dx = \frac{3\,b\,(b\tan(c+d\,x))^{1/3}}{d} - \frac{(-1)^{1/6}\,b^{4/3}\,\mathrm{atan}\left(\frac{(-1)^{5/6}\,(b\tan(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{d} - \frac{(-1)^{1/6}\,b^{4/3}\,\ln\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} - \frac{(-1)^{1/6}\,b^{4/3}\,\ln\left(2\,(b\tan(c+d\,x))^{1/3}-(-1)^{1/6}\,b^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} + \frac{(-1)^{1/6}\,b^{4/3}\,\ln\left((-1)^{1/6}\,b^{1/3}-2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d} + \frac{(-1)^{1/6}\,b^{4/3}\,\ln\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}-(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

input
```
int((b*tan(c + d*x))^(4/3),x)
```

output
```
(3*b*(b*tan(c + d*x))^(1/3))/d - ((-1)^(1/6)*b^(4/3)*atan(((-1)^(5/6)*(b*t
an(c + d*x))^(1/3)*1i)/b^(1/3))*1i)/d - ((-1)^(1/6)*b^(4/3)*log((-1)^(1/6)
*b^(1/3) + 2*(b*tan(c + d*x))^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2
)*1i)/2 + 1/2))/(2*d) - ((-1)^(1/6)*b^(4/3)*log(2*(b*tan(c + d*x))^(1/3) -
 (-1)^(1/6)*b^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/
(2*d) + ((-1)^(1/6)*b^(4/3)*log((-1)^(1/6)*b^(1/3) - 2*(b*tan(c + d*x))^(1
/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/4 + 1/4))/d + ((-1)^(1/6)*
b^(4/3)*log((-1)^(1/6)*b^(1/3) + 2*(b*tan(c + d*x))^(1/3) - (-1)^(2/3)*3^(
1/2)*b^(1/3))*((3^(1/2)*1i)/4 - 1/4))/d
```

## Reduce [F]

$$\int (b\tan(c+dx))^{4/3}\,dx = \frac{b^{\frac{4}{3}}\left(3\tan\left(dx+c\right)^{\frac{1}{3}}-\left(\int \frac{1}{\tan(dx+c)^{\frac{2}{3}}}dx\right)d\right)}{d}$$

input
```
int((b*tan(d*x+c))^(4/3),x)
```

output
```
(b**(1/3)*b*(3*tan(c + d*x)**(1/3) - int(tan(c + d*x)**(1/3)/tan(c + d*x),
x)*d))/d
```

# 3.18 $\int (b\tan(c+dx))^{2/3}\,dx$


Optimal result . . . 294
Mathematica [C] (verified) . . . 295
Rubi [A] (warning: unable to verify) . . . 295
Maple [A] (verified) . . . 299
Fricas [A] (verification not implemented) . . . 300
Sympy [F] . . . 301
Maxima [A] (verification not implemented) . . . 301
Giac [A] (verification not implemented) . . . 302
Mupad [B] (verification not implemented) . . . 303
Reduce [F] . . . 304


## Optimal result

Integrand size = 12, antiderivative size = 224

$$\int (b\tan(c+dx))^{2/3}\,dx = \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d} + \frac{b^{2/3}\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d} + \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} - \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d}$$

output
```
b^(2/3)*arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/d+1/2*b^(2/3)*arctan(-3^(1/2)
+2*(b*tan(d*x+c))^(1/3)/b^(1/3))/d+1/2*b^(2/3)*arctan(3^(1/2)+2*(b*tan(d*x
+c))^(1/3)/b^(1/3))/d+1/4*3^(1/2)*b^(2/3)*ln(b^(2/3)-3^(1/2)*b^(1/3)*(b*ta
n(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))/d-1/4*3^(1/2)*b^(2/3)*ln(b^(2/3)+3^(
1/2)*b^(1/3)*(b*tan(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))/d
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.83

$$\int (b\tan(c+dx))^{2/3}\,dx = \frac{\left(i\log\left(1-i\sqrt[6]{\tan^2(c+dx)}\right)-i\log\left(1+i\sqrt[6]{\tan^2(c+dx)}\right)+\sqrt[6]{-1}\left(\log\left(1-\sqrt[6]{-1}\sqrt[6]{\tan}\right.\right.\right.}{}$$

input
```
Integrate[(b*Tan[c + d*x])^(2/3),x]
```

output
```
((I*Log[1 - I*(Tan[c + d*x]^2)^(1/6)] - I*Log[1 + I*(Tan[c + d*x]^2)^(1/6)
] + (-1)^(1/6)*(Log[1 - (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)] - Log[1 + (-1)^
(1/6)*(Tan[c + d*x]^2)^(1/6)] + (-1)^(2/3)*(Log[1 - (-1)^(5/6)*(Tan[c + d*
x]^2)^(1/6)] - Log[1 + (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)])))*(b*Tan[c + d*
x])^(5/3))/(2*b*d*(Tan[c + d*x]^2)^(5/6))
```

## Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3957, 266, 824, 27, 216, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx))^{2/3}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\int \frac{(b\tan(c+dx))^{2/3}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{3b\int \frac{b^4\tan^4(c+dx)}{b^6\tan^6(c+dx)+b^2}d\sqrt[3]{b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{824}$$

$$\frac{3b\left(\frac{1}{3}\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}+\frac{\int -\frac{\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{2\left(b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}\right)}d\sqrt[3]{b\tan(c+dx)}}{3\sqrt[3]{b}}+\frac{\int -\frac{\sqrt[3]{b}+\sqrt{}}{2\left(b^2\tan^2(c+\right.}}{}\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{3b\left(\frac{1}{3}\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{\int \frac{\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/}}}{}\right.}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{3b\left(-\frac{\int \frac{\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}+\frac{\arctan}{}\right.}{d}$$

$$\Big\downarrow \text{1142}$$

$$3b\left(-\frac{-\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx}}{6\sqrt[3]{b}}\right.$$

$$\Big\downarrow \text{25}$$

$$3b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}\right.$$

$$\Big\downarrow \text{1082}$$

$$3b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-\frac{1}{3}}d\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}}}{6\sqrt[3]{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-\frac{1}{3}}}{\quad}\right.$$
$$\overline{\qquad\qquad\qquad d}$$

$\downarrow$ 217

$$3b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}+\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)}{6\sqrt[3]{b}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}+2\sqrt[3]{b\tan}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}}}{\quad}\right.$$
$$\overline{\qquad\qquad\qquad d}$$

$\downarrow$ 1103

$$3b\left(\frac{\arctan\left(b^{2/3}\tan(c+dx)\right)}{3\sqrt[3]{b}}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)-\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}+b^2\tan^2(c+dx)\right)}{6\sqrt[3]{b}}-\frac{\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}b^{4/3}\right.}{\quad}\right.$$
$$\overline{\qquad\qquad\qquad d}$$

input
```
Int[(b*Tan[c + d*x])^(2/3),x]
```

output
```
(3*b*(ArcTan[b^(2/3)*Tan[c + d*x]]/(3*b^(1/3)) - (ArcTan[Sqrt[3]*(1 - (2*b
^(2/3)*Tan[c + d*x])/Sqrt[3])] - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*b^(4/3)*Ta
n[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(1/3)) - (-ArcTan[Sqrt[3]*(1 + (
2*b^(2/3)*Tan[c + d*x])/Sqrt[3])] + (Sqrt[3]*Log[b^(2/3) + Sqrt[3]*b^(4/3)
*Tan[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(1/3))))/d
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.85

| method | result |
|---|---|
| derivativedivides | $\frac{3b\left(-\frac{\sqrt{3}\left(b^2\right)^{\frac{5}{6}}\ln\left(\left(b\tan(dx+c)\right)^{\frac{2}{3}}+\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\left(b\tan(dx+c)\right)^{\frac{1}{3}}+\left(b^2\right)^{\frac{1}{3}}\right)}{12b^2}+\frac{\arctan\left(\frac{2\left(b\tan(dx+c)\right)^{\frac{1}{3}}}{\left(b^2\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6\left(b^2\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(b^2\right)^{\frac{5}{6}}\ln\left((b\,\mathrm{ta}\right.}{}\ldots\right.}{d}$ |
| default | $\frac{3b\left(-\frac{\sqrt{3}\left(b^2\right)^{\frac{5}{6}}\ln\left(\left(b\tan(dx+c)\right)^{\frac{2}{3}}+\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\left(b\tan(dx+c)\right)^{\frac{1}{3}}+\left(b^2\right)^{\frac{1}{3}}\right)}{12b^2}+\frac{\arctan\left(\frac{2\left(b\tan(dx+c)\right)^{\frac{1}{3}}}{\left(b^2\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6\left(b^2\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(b^2\right)^{\frac{5}{6}}\ln\left((b\,\mathrm{ta}\right.}{}\ldots\right.}{d}$ |

input
```
int((b*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/d*b*(-1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)
^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/(b^2)^(1/6)*arctan(2*(b*tan(d
*x+c))^(1/3)/(b^2)^(1/6)+3^(1/2))+1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d
*x+c))^(2/3)-3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/(b^
2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))+1/3/(b^2)^(1/6
)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.40

$$\int (b\tan(c+dx))^{2/3}\,dx = -\frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b^3 + \frac{1}{2}\left(\sqrt{-3}d^5+d^5\right)\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}\right) + \frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b^3 - \frac{1}{2}\left(\sqrt{-3}d^5+d^5\right)\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}\right) - \frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b^3 + \frac{1}{2}\left(\sqrt{-3}d^5-d^5\right)\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}\right) + \frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b^3 - \frac{1}{2}\left(\sqrt{-3}d^5-d^5\right)\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}\right) + \frac{1}{2}\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left(d^5\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}} + (b\tan(dx+c))^{\frac{1}{3}}b^3\right) - \frac{1}{2}\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left(-d^5\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}} + (b\tan(dx+c))^{\frac{1}{3}}b^3\right)$$

input
```
integrate((b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(-3) - 1)*(-b^4/d^6)^(1/6)*log((b*tan(d*x + c))^(1/3)*b^3 + 1/2*
(sqrt(-3)*d^5 + d^5)*(-b^4/d^6)^(5/6)) + 1/4*(sqrt(-3) - 1)*(-b^4/d^6)^(1/
6)*log((b*tan(d*x + c))^(1/3)*b^3 - 1/2*(sqrt(-3)*d^5 + d^5)*(-b^4/d^6)^(5
/6)) - 1/4*(sqrt(-3) + 1)*(-b^4/d^6)^(1/6)*log((b*tan(d*x + c))^(1/3)*b^3
+ 1/2*(sqrt(-3)*d^5 - d^5)*(-b^4/d^6)^(5/6)) + 1/4*(sqrt(-3) + 1)*(-b^4/d^
6)^(1/6)*log((b*tan(d*x + c))^(1/3)*b^3 - 1/2*(sqrt(-3)*d^5 - d^5)*(-b^4/d
^6)^(5/6)) + 1/2*(-b^4/d^6)^(1/6)*log(d^5*(-b^4/d^6)^(5/6) + (b*tan(d*x +
c))^(1/3)*b^3) - 1/2*(-b^4/d^6)^(1/6)*log(-d^5*(-b^4/d^6)^(5/6) + (b*tan(d
*x + c))^(1/3)*b^3)
```

## Sympy [F]

$$\int (b\tan(c+dx))^{2/3}\,dx = \int (b\tan{(c+dx)})^{\frac{2}{3}}\,dx$$

input
```
integrate((b*tan(d*x+c))**(2/3),x)
```

output
```
Integral((b*tan(c + d*x))**(2/3), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.75

$$\int (b\tan(c+dx))^{2/3}\,dx = -\frac{\left(\frac{\sqrt{3}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}} - \frac{\sqrt{3}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}} - \frac{2\arctan\left(\frac{\sqrt{3}b^{\frac{1}{3}}+2(b\tan\cdots}{b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}\cdots}{4d}$$

input
```
integrate((b*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
-1/4*(sqrt(3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d*x + c)
)^(2/3) + b^(2/3))/b^(1/3) - sqrt(3)*log(-sqrt(3)*(b*tan(d*x + c))^(1/3)*b
^(1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3))/b^(1/3) - 2*arctan((sqrt(3)*b^(
1/3) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 2*arctan(-(sqrt(3)*b^(
1/3) - 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 4*arctan((b*tan(d*x +
c))^(1/3)/b^(1/3))/b^(1/3))*b/d
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.92

$$\int (b\tan(c+dx))^{2/3}\,dx = -\frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}\,|b|^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{4\,bd} + \frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}\,|b|^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{4\,bd} + \frac{|b|^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}|b|^{\frac{1}{3}}+2\,(b\tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{2\,bd} + \frac{|b|^{\frac{5}{3}}\arctan\left(-\frac{\sqrt{3}|b|^{\frac{1}{3}}-2\,(b\tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{2\,bd} + \frac{|b|^{\frac{5}{3}}\arctan\left(\frac{(b\tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{bd}$$

input
```
integrate((b*tan(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
-1/4*sqrt(3)*abs(b)^(5/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3)
+ (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b*d) + 1/4*sqrt(3)*abs(b)^(5/3)*
log(-sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3)
+ abs(b)^(2/3))/(b*d) + 1/2*abs(b)^(5/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*
(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b*d) + 1/2*abs(b)^(5/3)*arctan(-(sq
rt(3)*abs(b)^(1/3) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b*d) + abs(b
)^(5/3)*arctan((b*tan(d*x + c))^(1/3)/abs(b)^(1/3))/(b*d)
```

## Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.16

$$\int (b\tan(c+dx))^{2/3}\,dx = \frac{(-1)^{1/6}\,b^{2/3}\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,(b\tan(c+d\,x))^{1/3}}{b^{1/3}}\right)\,1\mathrm{i}}{d} - \frac{(-1)^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}+\frac{486\,(-1)^{1/6}\,b^{26/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,(b\tan(c+d\,x))^{1/3}}{d^3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} - \frac{(-1)^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}+\frac{486\,(-1)^{1/6}\,b^{26/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,(b\tan(c+d\,x))^{1/3}}{d^3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} + \frac{(-1)^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{486\,(-1)^{1/6}\,b^{26/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,(b\tan(c+d\,x))^{1/3}}{d^3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d} + \frac{(-1)^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{486\,(-1)^{1/6}\,b^{26/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,(b\tan(c+d\,x))^{1/3}}{d^3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

input
```
int((b*tan(c + d*x))^(2/3),x)
```

output
```
((-1)^(1/6)*b^(2/3)*atan(((-1)^(2/3)*(b*tan(c + d*x))^(1/3))/b^(1/3))*1i)/
d - ((-1)^(1/6)*b^(2/3)*log((972*b^9)/d^3 + (486*(-1)^(1/6)*b^(26/3)*(3^(1
/2)*1i - 1)*(b*tan(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/2 - 1/2))/(2*d) - (
(-1)^(1/6)*b^(2/3)*log((972*b^9)/d^3 + (486*(-1)^(1/6)*b^(26/3)*(3^(1/2)*1
i + 1)*(b*tan(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((-1)^
(1/6)*b^(2/3)*log((972*b^9)/d^3 - (486*(-1)^(1/6)*b^(26/3)*(3^(1/2)*1i - 1
)*(b*tan(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/4 - 1/4))/d + ((-1)^(1/6)*b^(
2/3)*log((972*b^9)/d^3 - (486*(-1)^(1/6)*b^(26/3)*(3^(1/2)*1i + 1)*(b*tan(
c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/4 + 1/4))/d
```

## Reduce [F]

$$\int (b\tan(c+dx))^{2/3}\,dx = b^{\frac{2}{3}}\left(\int \tan(dx+c)^{\frac{2}{3}}\,dx\right)$$

input
```
int((b*tan(d*x+c))^(2/3),x)
```

output
```
b**(2/3)*int(tan(c + d*x)**(2/3),x)
```

# 3.19 $\int \sqrt[3]{b \tan(c+dx)}\,dx$

Optimal result . . . 305
Mathematica [A] (verified) . . . 305
Rubi [A] (warning: unable to verify) . . . 306
Maple [A] (verified) . . . 310
Fricas [A] (verification not implemented) . . . 310
Sympy [F] . . . 311
Maxima [A] (verification not implemented) . . . 311
Giac [A] (verification not implemented) . . . 312
Mupad [B] (verification not implemented) . . . 312
Reduce [F] . . . 313

## Optimal result

Integrand size = 12, antiderivative size = 131

$$\int \sqrt[3]{b \tan(c+dx)}\,dx = -\frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2d} - \frac{\sqrt[3]{b}\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx))^{4/3}\right)}{4d}$$

output
```
-1/2*3^(1/2)*b^(1/3)*arctan(1/3*(b^(2/3)-2*(b*tan(d*x+c))^(2/3))*3^(1/2)/b
^(2/3))/d-1/2*b^(1/3)*ln(b^(2/3)+(b*tan(d*x+c))^(2/3))/d+1/4*b^(1/3)*ln(b^
(4/3)-b^(2/3)*(b*tan(d*x+c))^(2/3)+(b*tan(d*x+c))^(4/3))/d
```

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int \sqrt[3]{b \tan(c+dx)}\,dx = -\frac{\left(\log\left(1+\sqrt[3]{\tan^2(c+dx)}\right)-\sqrt[3]{-1}\log\left(1-\sqrt[3]{-1}\sqrt[3]{\tan^2(c+dx)}\right)+(-1)^{2/3}\log\left(1+(-1)^{2/3}\sqrt[3]{\tan\right.\right.}{2bd\tan^2(c+dx)^{2/3}}$$

input
```
Integrate[(b*Tan[c + d*x])^(1/3),x]
```

output
```
-1/2*((Log[1 + (Tan[c + d*x]^2)^(1/3)] - (-1)^(1/3)*Log[1 - (-1)^(1/3)*(Ta
n[c + d*x]^2)^(1/3)] + (-1)^(2/3)*Log[1 + (-1)^(2/3)*(Tan[c + d*x]^2)^(1/3
)])*(b*Tan[c + d*x])^(4/3))/(b*d*(Tan[c + d*x]^2)^(2/3))
```

## Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3957, 266, 807, 821, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\int \frac{\sqrt[3]{b\tan(c+dx)}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{3b\int \frac{b^3\tan^3(c+dx)}{b^6\tan^6(c+dx)+b^2}d\sqrt[3]{b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{807}$$

$$\frac{3b\int \frac{b^2\tan^2(c+dx)}{b^3\tan^3(c+dx)+b^2}d\left(b^2\tan^2(c+dx)\right)}{2d}$$

$$\Big\downarrow \text{821}$$

$$\frac{3b\left(\frac{\int \frac{b^2\tan^2(c+dx)+b^{2/3}}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)}{3b^{2/3}}-\frac{\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\left(b^2\tan^2(c+dx)\right)}{3b^{2/3}}\right)}{2d}$$

$\Big\downarrow$ 16

$$\frac{3b\left(\frac{\int \frac{b^2\tan^2(c+dx)+b^{2/3}}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)}{3b^{2/3}}-\frac{\log\left(b^{2/3}+b^2\tan^2(c+dx)\right)}{3b^{2/3}}\right)}{2d}$$

$\Big\downarrow$ 1142

$$\frac{3b\left(\frac{\frac{3}{2}b^{2/3}\int \frac{1}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)+\frac{1}{2}\int -\frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)}{3b^{2/3}}-\frac{\log\left(b^{2/3}+b^2\,t\ldots\right.}{3b^{2}\ldots}\right.}{2d}$$

$\Big\downarrow$ 25

$$\frac{3b\left(\frac{\frac{3}{2}b^{2/3}\int \frac{1}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)-\frac{1}{2}\int \frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)}{3b^{2/3}}-\frac{\log\left(b^{2/3}+b^2\tan\ldots\right.}{3b^{2/3}}\right.}{2d}$$

$\Big\downarrow$ 1082

$$\frac{3b\left(\frac{3\int \frac{1}{2\sqrt[3]{b}\tan(c+dx)-4}d\left(1-2\sqrt[3]{b}\tan(c+dx)\right)-\frac{1}{2}\int \frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)}{3b^{2/3}}-\frac{\log\left(b^{2/3}+b^2\tan^2(c+dx)\right)}{3b^{2/3}}\right)}{2d}$$

$\Big\downarrow$ 217

$$\frac{3b\left(\frac{-\frac{1}{2}\int \frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)-\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}}\right)}{3b^{2/3}}-\frac{\log\left(b^{2/3}+b^2\tan^2(c+dx)\right)}{3b^{2/3}}\right)}{2d}$$

$\Big\downarrow$ 1103

$$\frac{3b\left(\frac{\frac{1}{2}\log\left(-b^{5/3}\tan(c+dx)+b^{4/3}+b^2\tan^2(c+dx)\right)-\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}}\right)}{3b^{2/3}}-\frac{\log\left(b^{2/3}+b^2\tan^2(c+dx)\right)}{3b^{2/3}}\right)}{2d}$$

input
```
Int[(b*Tan[c + d*x])^(1/3),x]
```

output
```
(3*b*(-1/3*Log[b^(2/3) + b^2*Tan[c + d*x]^2]/b^(2/3) + (-(Sqrt[3]*ArcTan[(
1 - 2*b^(1/3)*Tan[c + d*x])/Sqrt[3]]) + Log[b^(4/3) - b^(5/3)*Tan[c + d*x]
 + b^2*Tan[c + d*x]^2]/2)/(3*b^(2/3))))/(2*d)
```

### Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| derivativedivides | $\frac{3b\left(-\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{1}{3}}}+\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}(b^2)^{\frac{1}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{(b^2)^{\frac{1}{3}}}\right.}{3}\right.}{6(b^2)^{\frac{1}{3}}}\right.}{d}$ |
| default | $\frac{3b\left(-\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{1}{3}}}+\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}(b^2)^{\frac{1}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{(b^2)^{\frac{1}{3}}}\right.}{3}\right.}{6(b^2)^{\frac{1}{3}}}\right.}{d}$ |

input
```
int((b*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/d*b*(-1/6/(b^2)^(1/3)*ln((b*tan(d*x+c))^(2/3)+(b^2)^(1/3))+1/12/(b^2)^(1
/3)*ln((b*tan(d*x+c))^(4/3)-(b*tan(d*x+c))^(2/3)*(b^2)^(1/3)+(b^2)^(2/3))+
1/6*3^(1/2)/(b^2)^(1/3)*arctan(1/3*3^(1/2)*(2*(b*tan(d*x+c))^(2/3)/(b^2)^(
1/3)-1)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int \sqrt[3]{b\tan(c+dx)}\,dx$$
$$= \frac{2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(b\tan(dx+c))^{\frac{2}{3}}(-b)^{\frac{1}{3}}+\sqrt{3}b}{3b}\right)-(-b)^{\frac{1}{3}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b\tan(dx+c)-(b\tan(dx\right.}{4d}$$

input
```
integrate((b*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*tan(d*x + c))^(2/3)*(-b
)^(1/3) + sqrt(3)*b)/b) - (-b)^(1/3)*log((b*tan(d*x + c))^(1/3)*b*tan(d*x
+ c) - (b*tan(d*x + c))^(2/3)*(-b)^(2/3) - (-b)^(1/3)*b) + 2*(-b)^(1/3)*lo
g((b*tan(d*x + c))^(2/3) + (-b)^(2/3)))/d
```

## Sympy [F]

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = \int \sqrt[3]{b\tan{(c+dx)}}\,dx$$

input
```
integrate((b*tan(d*x+c))**(1/3),x)
```

output
```
Integral((b*tan(c + d*x))**(1/3), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = \frac{2\sqrt{3}b^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-b^{\frac{2}{3}}\right)}{3\,b^{\frac{2}{3}}}\right)+b^{\frac{4}{3}}\log\left((b\tan{(dx+c)})^{\frac{4}{3}}-(b\tan{(dx+c)})^{\frac{2}{3}}\,b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)-2\,b^{\frac{4}{3}}\,\mathrm{lo}}{4\,bd}$$

input
```
integrate((b*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(3)*b^(4/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - b^(2
/3))/b^(2/3)) + b^(4/3)*log((b*tan(d*x + c))^(4/3) - (b*tan(d*x + c))^(2/3
)*b^(2/3) + b^(4/3)) - 2*b^(4/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b
*d)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97

$$\int \sqrt[3]{b\tan(c+dx)}\,dx$$
$$=\frac{1}{4}b\left(\frac{2\sqrt{3}|b|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{b^2d}+\frac{|b|^{\frac{4}{3}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b\tan(dx+c)-(b\tan(dx+\right.}{b^2d}\right.$$

input
```
integrate((b*tan(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
1/4*b*(2*sqrt(3)*abs(b)^(4/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3)
 - abs(b)^(2/3))/abs(b)^(2/3))/(b^2*d) + abs(b)^(4/3)*log((b*tan(d*x + c))
^(1/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*abs(b)^(2/3) + abs(b)^(4/3)
)/(b^2*d) - 2*abs(b)^(4/3)*log((b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^2
*d))
```

## Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int \sqrt[3]{b\tan(c+dx)}\,dx$$
$$=\frac{(-b)^{1/3}\ln\left(81\,(-b)^{16/3}\,(b\tan(c+d\,x))^{2/3}+81\,b^6\right)}{2\,d}$$
$$-\frac{(-b)^{1/3}\ln\left(\frac{81\,b^6}{d^4}-\frac{81\,(-b)^{16/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(b\tan(c+d\,x))^{2/3}}{d^4}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$+\frac{(-b)^{1/3}\ln\left(\frac{81\,b^6}{d^4}+\frac{162\,(-b)^{16/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)(b\tan(c+d\,x))^{2/3}}{d^4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

input
```
int((b*tan(c + d*x))^(1/3),x)
```

output
```
((-b)^(1/3)*log(81*(-b)^(16/3)*(b*tan(c + d*x))^(2/3) + 81*b^6))/(2*d) - (
(-b)^(1/3)*log((81*b^6)/d^4 - (81*(-b)^(16/3)*((3^(1/2)*1i)/2 + 1/2)*(b*ta
n(c + d*x))^(2/3))/d^4)*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((-b)^(1/3)*log((8
1*b^6)/d^4 + (162*(-b)^(16/3)*((3^(1/2)*1i)/4 - 1/4)*(b*tan(c + d*x))^(2/3
))/d^4)*((3^(1/2)*1i)/4 - 1/4))/d
```

## Reduce [F]

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = b^{\frac{1}{3}}\left(\int \tan\left(dx+c\right)^{\frac{1}{3}}dx\right)$$

input
```
int((b*tan(d*x+c))^(1/3),x)
```

output
```
b**(1/3)*int(tan(c + d*x)**(1/3),x)
```

# 3.20 $\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$

Optimal result . . . 314
Mathematica [A] (verified) . . . 315
Rubi [A] (warning: unable to verify) . . . 315
Maple [A] (verified) . . . 319
Fricas [A] (verification not implemented) . . . 320
Sympy [F] . . . 320
Maxima [A] (verification not implemented) . . . 321
Giac [A] (verification not implemented) . . . 321
Mupad [B] (verification not implemented) . . . 322
Reduce [F] . . . 322

## Optimal result

Integrand size = 12, antiderivative size = 131

$$\begin{aligned}\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx &= -\frac{\sqrt{3}\arctan\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2\sqrt[3]{b}d} \\ &\quad+\frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} \\ &\quad-\frac{\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}\end{aligned}$$

output

```
-1/2*3^(1/2)*arctan(1/3*(b^(2/3)-2*(b*tan(d*x+c))^(2/3))*3^(1/2)/b^(2/3))/
b^(1/3)/d+1/2*ln(b^(2/3)+(b*tan(d*x+c))^(2/3))/b^(1/3)/d-1/4*ln(b^(4/3)-b^
(2/3)*(b*tan(d*x+c))^(2/3)+(b*tan(d*x+c))^(4/3))/b^(1/3)/d
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$$
$$= \frac{\left(2\sqrt{3}\arctan\left(\frac{-1+2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)+2\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)-\log\left(1-\tan^{\frac{2}{3}}(c+dx)+\tan^{\frac{4}{3}}(c+dx)\right)\right)}{4d\sqrt[3]{b\tan(c+dx)}}$$

input
```
Integrate[(b*Tan[c + d*x])^(-1/3),x]
```

output
```
((2*Sqrt[3]*ArcTan[(-1 + 2*Tan[c + d*x]^(2/3))/Sqrt[3]] + 2*Log[1 + Tan[c
+ d*x]^(2/3)] - Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])*Tan[c +
d*x]^(1/3))/(4*d*(b*Tan[c + d*x])^(1/3))
```

## Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3957, 266, 807, 750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}dx$$
$$\Big\downarrow \; 3957$$
$$\frac{b\int \frac{1}{\sqrt[3]{b\tan(c+dx)}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}$$
$$\Big\downarrow \; 266$$

$$\frac{3b\int \frac{\sqrt[3]{b\tan(c+dx)}}{b^6\tan^6(c+dx)+b^2}d\sqrt[3]{b\tan(c+dx)}}{d}$$

$\downarrow$ 807

$$\frac{3b\int \frac{1}{b^3\tan^3(c+dx)+b^2}d\left(b^2\tan^2(c+dx)\right)}{2d}$$

$\downarrow$ 750

$$\frac{3b\left(\frac{\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\left(b^2\tan^2(c+dx)\right)}{3b^{4/3}}+\frac{\int \frac{2b^{2/3}-b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)}{3b^{4/3}}\right)}{2d}$$

$\downarrow$ 16

$$\frac{3b\left(\frac{\int \frac{2b^{2/3}-b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)}{3b^{4/3}}+\frac{\log\left(b^{2/3}+b^2\tan^2(c+dx)\right)}{3b^{4/3}}\right)}{2d}$$

$\downarrow$ 1142

$$\frac{3b\left(\frac{\frac{3}{2}b^{2/3}\int \frac{1}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)-\frac{1}{2}\int -\frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)}{3b^{4/3}}+\frac{\log\left(b^{2/3}+b^2\tan^2(c+dx)\right)}{3b^{4/3}}\right)}{2d}$$

$\downarrow$ 25

$$\frac{3b\left(\frac{\frac{3}{2}b^{2/3}\int \frac{1}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)+\frac{1}{2}\int \frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)}{3b^{4/3}}+\frac{\log\left(b^{2/3}+b^2\tan^2(c+dx)\right)}{3b^{4/3}}\right)}{2d}$$

$\downarrow$ 1082

$$\frac{3b\left(\frac{\frac{1}{2}\int \frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d\left(b^2\tan^2(c+dx)\right)+3\int \frac{1}{2\sqrt[3]{b}\tan(c+dx)-4}d\left(1-2\sqrt[3]{b}\tan(c+dx)\right)}{3b^{4/3}}+\frac{\log\left(b^{2/3}+b^2\tan^2(c+dx)\right)}{3b^{4/3}}\right)}{2d}$$

$\downarrow$ 217

$$\frac{3b\left(\frac{\frac{1}{2}\int \frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))-\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}}\right)}{3b^{4/3}}+\frac{\log(b^{2/3}+b^2\tan^2(c+dx))}{3b^{4/3}}\right)}{2d}$$

$$\Big\downarrow \ 1103$$

$$\frac{3b\left(\frac{-\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}}\right)-\frac{1}{2}\log(-b^{5/3}\tan(c+dx)+b^{4/3}+b^2\tan^2(c+dx))}{3b^{4/3}}+\frac{\log(b^{2/3}+b^2\tan^2(c+dx))}{3b^{4/3}}\right)}{2d}$$

input
```
Int[(b*Tan[c + d*x])^(-1/3),x]
```

output
```
(3*b*(Log[b^(2/3) + b^2*Tan[c + d*x]^2]/(3*b^(4/3)) + (-(Sqrt[3]*ArcTan[(1
 - 2*b^(1/3)*Tan[c + d*x])/Sqrt[3]]) - Log[b^(4/3) - b^(5/3)*Tan[c + d*x]
+ b^2*Tan[c + d*x]^2]/2)/(3*b^(4/3))))/(2*d)
```

## Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| derivativedivides | $\frac{3b\left(\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\left(b^{2}\right)^{\frac{1}{3}}\right)}{6\left(b^{2}\right)^{\frac{2}{3}}}-\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}\left(b^{2}\right)^{\frac{1}{3}}+\left(b^{2}\right)^{\frac{2}{3}}\right)}{12\left(b^{2}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{\left(b^{2}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{6\left(b^{2}\right)^{\frac{2}{3}}}\right.}{d}$ |
| default | $\frac{3b\left(\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\left(b^{2}\right)^{\frac{1}{3}}\right)}{6\left(b^{2}\right)^{\frac{2}{3}}}-\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}\left(b^{2}\right)^{\frac{1}{3}}+\left(b^{2}\right)^{\frac{2}{3}}\right)}{12\left(b^{2}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{\left(b^{2}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{6\left(b^{2}\right)^{\frac{2}{3}}}\right.}{d}$ |

input
```
int(1/(b*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/d*b*(1/6/(b^2)^(2/3)*ln((b*tan(d*x+c))^(2/3)+(b^2)^(1/3))-1/12/(b^2)^(2/
3)*ln((b*tan(d*x+c))^(4/3)-(b*tan(d*x+c))^(2/3)*(b^2)^(1/3)+(b^2)^(2/3))+1
/6/(b^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*tan(d*x+c))^(2/3)/(b^2)^(1
/3)-1)))
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.28

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{3}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\tan(dx+c)+2b\tan(dx+c)^2-\sqrt{3}b^{\frac{4}{3}}\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+(b\tan(dx+c))^{\frac{2}{3}}\left(\sqrt{3}b^{\frac{2}{3}}\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3b^{\frac{1}{3}}\right)-b}{\tan(dx+c)^2+1}\right.}{}\right.$$

input
```
integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((2*sqrt(3)*(b*tan(d*x + c))^(1/3)*b*s
qrt(-1/b^(2/3))*tan(d*x + c) + 2*b*tan(d*x + c)^2 - sqrt(3)*b^(4/3)*sqrt(-
1/b^(2/3)) + (b*tan(d*x + c))^(2/3)*(sqrt(3)*b^(2/3)*sqrt(-1/b^(2/3)) - 3*
b^(1/3)) - b)/(tan(d*x + c)^2 + 1)) - b^(2/3)*log((b*tan(d*x + c))^(1/3)*b
*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*b^(2/3) + b^(4/3)) + 2*b^(2/3)*log(
(b*tan(d*x + c))^(2/3) + b^(2/3)))/(b*d), 1/4*(2*sqrt(3)*b^(2/3)*arctan(1/
3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3)*b^(2/3) - b^(4/3))/b^(4/3)) - b^(2/3)*
log((b*tan(d*x + c))^(1/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*b^(2/3)
 + b^(4/3)) + 2*b^(2/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b*d)]
```

## Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\tan{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(1/3),x)
```

output
```
Integral((b*tan(c + d*x))**(-1/3), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$$
$$= \frac{2\sqrt{3}b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-b^{\frac{2}{3}}\right)}{3\,b^{\frac{2}{3}}}\right)-b^{\frac{2}{3}}\log\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)+2\,b^{\frac{2}{3}}\,\mathrm{lo}}{4\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(3)*b^(2/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - b^(2
/3))/b^(2/3)) - b^(2/3)*log((b*tan(d*x + c))^(4/3) - (b*tan(d*x + c))^(2/3
)*b^(2/3) + b^(4/3)) + 2*b^(2/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b
*d)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{3}|b|^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{2\,bd} - \frac{|b|^{\frac{2}{3}}\log\left((b\tan(dx+c))^{\frac{1}{3}}\,b\tan(dx+c)-(b\tan(dx+c))^{\frac{2}{3}}\,|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}\right)}{4\,bd} + \frac{|b|^{\frac{2}{3}}\log\left((b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{2\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*abs(b)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - ab
s(b)^(2/3))/abs(b)^(2/3))/(b*d) - 1/4*abs(b)^(2/3)*log((b*tan(d*x + c))^(1
/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*abs(b)^(2/3) + abs(b)^(4/3))/(
b*d) + 1/2*abs(b)^(2/3)*log((b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b*d)
```

## Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \frac{\ln\left((b\tan(c+d\,x))^{2/3}+b^{2/3}\right)}{2\,b^{1/3}\,d} + \frac{\ln\left(\frac{81\,b^{11/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{d^3}+\frac{162\,b^3\,(b\tan(c+d\,x))^{2/3}}{d^3}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,b^{1/3}\,d} - \frac{\ln\left(\frac{81\,b^{11/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{d^3}-\frac{162\,b^3\,(b\tan(c+d\,x))^{2/3}}{d^3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4\,b^{1/3}\,d}$$

input
```
int(1/(b*tan(c + d*x))^(1/3),x)
```

output
```
log((b*tan(c + d*x))^(2/3) + b^(2/3))/(2*b^(1/3)*d) + (log((81*b^(11/3)*(3
^(1/2)*1i - 1))/d^3 + (162*b^3*(b*tan(c + d*x))^(2/3))/d^3)*(3^(1/2)*1i -
1))/(4*b^(1/3)*d) - (log((81*b^(11/3)*(3^(1/2)*1i + 1))/d^3 - (162*b^3*(b*
tan(c + d*x))^(2/3))/d^3)*(3^(1/2)*1i + 1))/(4*b^(1/3)*d)
```

## Reduce [F]

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \frac{\int \frac{1}{\tan(dx+c)^{\frac{1}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(1/(b*tan(d*x+c))^(1/3),x)
```

output
```
int(1/tan(c + d*x)**(1/3),x)/b**(1/3)
```

# 3.21 $\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 323
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 324
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . . . 324
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 328
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 329
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 330
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 330
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 330
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 331
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 332

## Optimal result

Integrand size = 12, antiderivative size = 224

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d} + \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d} - \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d}$$

output
```
arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/b^(2/3)/d+1/2*arctan(-3^(1/2)+2*(b*ta
n(d*x+c))^(1/3)/b^(1/3))/b^(2/3)/d+1/2*arctan(3^(1/2)+2*(b*tan(d*x+c))^(1/
3)/b^(1/3))/b^(2/3)/d-1/4*3^(1/2)*ln(b^(2/3)-3^(1/2)*b^(1/3)*(b*tan(d*x+c)
)^(1/3)+(b*tan(d*x+c))^(2/3))/b^(2/3)/d+1/4*3^(1/2)*ln(b^(2/3)+3^(1/2)*b^(
1/3)*(b*tan(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))/b^(2/3)/d
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.84

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{\left(i\log\left(1-i\sqrt[6]{\tan^2(c+dx)}\right) - i\log\left(1+i\sqrt[6]{\tan^2(c+dx)}\right) + \sqrt[6]{-1}\left((-1)^{2/3}\right.\right.}{}$$

input
```
Integrate[(b*Tan[c + d*x])^(-2/3),x]
```

output
```
((I*Log[1 - I*(Tan[c + d*x]^2)^(1/6)] - I*Log[1 + I*(Tan[c + d*x]^2)^(1/6)
] + (-1)^(1/6)*((-1)^(2/3)*Log[1 - (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)] - (-
1)^(2/3)*Log[1 + (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)] + Log[1 - (-1)^(5/6)*(
Tan[c + d*x]^2)^(1/6)] - Log[1 + (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)]))*(b*T
an[c + d*x])^(1/3))/(2*b*d*(Tan[c + d*x]^2)^(1/6))
```

## Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3957, 266, 753, 27, 216, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\int \frac{1}{(b\tan(c+dx))^{2/3}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}$$

$\Big\downarrow$ 266

$$\frac{3b\int \frac{1}{b^6\tan^6(c+dx)+b^2}d\sqrt[3]{b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 753

$$\frac{3b\left(\frac{\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{3b^{4/3}}+\frac{\int \frac{2\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{2\left(b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}\right)}d\sqrt[3]{b\tan(c+dx)}}{3b^{5/3}}+\frac{\int \frac{2\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(\ldots}}{2\left(b^2\tan^2(c+dx)+\sqrt{3}b^{4\ldots}\right.}}{}\right.}{d}$$

$\Big\downarrow$ 27

$$\frac{3b\left(\frac{\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{3b^{4/3}}+\frac{\int \frac{2\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\int \frac{2\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(\ldots}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan\ldots}}{}\right.}{d}$$

$\Big\downarrow$ 216

$$\frac{3b\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\int \frac{2\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\arctan(b\ldots}{}\right.}{d}$$

$\Big\downarrow$ 1142

$$3b\left(\frac{\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}\ldots\right.$$

$\Big\downarrow$ 25

$$3b\left(\frac{\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\ldots\right.$$

$\Big\downarrow$ 1082

$$3b\left(\frac{\frac{\int \frac{1}{-b^2\tan^2(c+dx)-\frac{1}{3}}d\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}+2\sqrt[3]{\ldots}}{b^2\tan^2(c+dx)+\sqrt{\ldots}}}{\ldots}\right.$$
$$\overline{\qquad d\qquad}$$

$\downarrow$ 217

$$3b\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)}{6b^{5/3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}+2\sqrt[3]{b\tan(\ldots}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan\ldots}}{\ldots}\right.$$
$$\overline{\qquad d\qquad}$$

$\downarrow$ 1103

$$3b\left(\frac{\arctan\left(b^{2/3}\tan(c+dx)\right)}{3b^{5/3}}+\frac{-\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)-\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}+b^2\tan^2(c+dx)\right)}{6b^{5/3}}+\frac{\arctan\left(\sqrt{3}\left(\ldots\right.\right.}{\ldots}\right.$$
$$\overline{\qquad d\qquad}$$

input
```
Int[(b*Tan[c + d*x])^(-2/3),x]
```

output
```
(3*b*(ArcTan[b^(2/3)*Tan[c + d*x]]/(3*b^(5/3)) + (-ArcTan[Sqrt[3]*(1 - (2*
b^(2/3)*Tan[c + d*x])/Sqrt[3])] - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*b^(4/3)*T
an[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(5/3)) + (ArcTan[Sqrt[3]*(1 + (
2*b^(2/3)*Tan[c + d*x])/Sqrt[3])] + (Sqrt[3]*Log[b^(2/3) + Sqrt[3]*b^(4/3)
*Tan[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(5/3))))/d
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| derivativedivides | $\frac{3b\left(\frac{\sqrt{3}\,(b^2)^{\frac{1}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{12b^2}+\frac{(b^2)^{\frac{1}{6}}\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{(b^2)^{\frac{1}{6}}}+\sqrt{3}\right)}{6b^2}-\frac{\sqrt{3}\,(b^2)^{\frac{1}{6}}\ln \cdots}{\cdots}\right.}{\cdots}$ |
| default | $\frac{3b\left(\frac{\sqrt{3}\,(b^2)^{\frac{1}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{12b^2}+\frac{(b^2)^{\frac{1}{6}}\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{(b^2)^{\frac{1}{6}}}+\sqrt{3}\right)}{6b^2}-\frac{\sqrt{3}\,(b^2)^{\frac{1}{6}}\ln \cdots}{\cdots}\right.}{\cdots}$ |

input
```
int(1/(b*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/d*b*(1/12/b^2*3^(1/2)*(b^2)^(1/6)*ln((b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)^
(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/b^2*(b^2)^(1/6)*arctan(2*(b*ta
n(d*x+c))^(1/3)/(b^2)^(1/6)+3^(1/2))-1/12/b^2*3^(1/2)*(b^2)^(1/6)*ln(-(b*t
an(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)-(b^2)^(1/3))+1/6
/b^2*(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))+1/3/b^
2*(b^2)^(1/6)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.26

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(\frac{1}{2}\left(\sqrt{-3}bd+bd\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) - \frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(-\frac{1}{2}\left(\sqrt{-3}bd+bd\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) + \frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(\frac{1}{2}\left(\sqrt{-3}bd-bd\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) - \frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(-\frac{1}{2}\left(\sqrt{-3}bd-bd\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) + \frac{1}{2}\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(bd\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) - \frac{1}{2}\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(-bd\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right)$$

input
```
integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
1/4*(sqrt(-3) + 1)*(-1/(b^4*d^6))^(1/6)*log(1/2*(sqrt(-3)*b*d + b*d)*(-1/(
b^4*d^6))^(1/6) + (b*tan(d*x + c))^(1/3)) - 1/4*(sqrt(-3) + 1)*(-1/(b^4*d^
6))^(1/6)*log(-1/2*(sqrt(-3)*b*d + b*d)*(-1/(b^4*d^6))^(1/6) + (b*tan(d*x
+ c))^(1/3)) + 1/4*(sqrt(-3) - 1)*(-1/(b^4*d^6))^(1/6)*log(1/2*(sqrt(-3)*b
*d - b*d)*(-1/(b^4*d^6))^(1/6) + (b*tan(d*x + c))^(1/3)) - 1/4*(sqrt(-3) -
 1)*(-1/(b^4*d^6))^(1/6)*log(-1/2*(sqrt(-3)*b*d - b*d)*(-1/(b^4*d^6))^(1/6
) + (b*tan(d*x + c))^(1/3)) + 1/2*(-1/(b^4*d^6))^(1/6)*log(b*d*(-1/(b^4*d^
6))^(1/6) + (b*tan(d*x + c))^(1/3)) - 1/2*(-1/(b^4*d^6))^(1/6)*log(-b*d*(-
1/(b^4*d^6))^(1/6) + (b*tan(d*x + c))^(1/3))
```

## Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\tan(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(2/3),x)
```

output
```
Integral((b*tan(c + d*x))**(-2/3), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.76

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{\sqrt{3}b^{\frac{1}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}} + (b\tan(dx+c))^{\frac{2}{3}} + b^{\frac{2}{3}}\right) - \sqrt{3}b^{\frac{1}{3}}\log\left(-\sqrt{3}(\right.}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
1/4*(sqrt(3)*b^(1/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d
*x + c))^(2/3) + b^(2/3)) - sqrt(3)*b^(1/3)*log(-sqrt(3)*(b*tan(d*x + c))^
(1/3)*b^(1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3)) + 2*b^(1/3)*arctan((sqrt
(3)*b^(1/3) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 2*b^(1/3)*arctan(-(sqrt
(3)*b^(1/3) - 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 4*b^(1/3)*arctan((b*tan
(d*x + c))^(1/3)/b^(1/3)))/(b*d)
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{\frac{\sqrt{3}|b|^{\frac{1}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}} + (b\tan(dx+c))^{\frac{2}{3}} + |b|^{\frac{2}{3}}\right)}{b} - \frac{\sqrt{3}|b|^{\frac{1}{3}}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}} + (b\tan\right.}{b}}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
1/4*(sqrt(3)*abs(b)^(1/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3)
+ (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/b - sqrt(3)*abs(b)^(1/3)*log(-sqr
t(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) + abs(b)
^(2/3))/b + 2*abs(b)^(1/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*tan(d*x + c
))^(1/3))/abs(b)^(1/3))/b + 2*abs(b)^(1/3)*arctan(-(sqrt(3)*abs(b)^(1/3) -
 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/b + 4*abs(b)^(1/3)*arctan((b*tan(
d*x + c))^(1/3)/abs(b)^(1/3))/b)/d
```

## Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{(-1)^{1/6}\,\mathrm{atan}\left(\frac{(-1)^{5/6}\,(b\tan(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{b^{2/3}\,d}$$
$$-\frac{(-1)^{1/6}\,\ln\left(\left((-1)^{1/6}\,b^{1/3}-2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)}{2\,b^{2/3}\,d}$$
$$-\frac{(-1)^{1/6}\,\ln\left(\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}-(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)}{2\,b^{2/3}\,d}$$
$$+\frac{(-1)^{1/6}\,\ln\left(\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\right)}{b^{2/3}\,d}$$
$$+\frac{(-1)^{1/6}\,\ln\left(\left(2\,(b\tan(c+d\,x))^{1/3}-(-1)^{1/6}\,b^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\right)}{b^{2/3}\,d}$$

input
```
int(1/(b*tan(c + d*x))^(2/3),x)
```

output
```
((-1)^(1/6)*atan(((-1)^(5/6)*(b*tan(c + d*x))^(1/3)*1i)/b^(1/3))*1i)/(b^(2
/3)*d) - ((-1)^(1/6)*log((-1)^(1/6)*b^(1/3) - 2*(b*tan(c + d*x))^(1/3) + (
-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(2/3)*d) - ((-1)^(
1/6)*log((-1)^(1/6)*b^(1/3) + 2*(b*tan(c + d*x))^(1/3) - (-1)^(2/3)*3^(1/2
)*b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*b^(2/3)*d) + ((-1)^(1/6)*log((-1)^(1
/6)*b^(1/3) + 2*(b*tan(c + d*x))^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(
1/2)*1i)/4 + 1/4))/(b^(2/3)*d) + ((-1)^(1/6)*log(2*(b*tan(c + d*x))^(1/3)
- (-1)^(1/6)*b^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/4 - 1/4))
/(b^(2/3)*d)
```

## Reduce [F]

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{\int \frac{1}{\tan(dx+c)^{\frac{2}{3}}}dx}{b^{\frac{2}{3}}}$$

input
```
int(1/(b*tan(d*x+c))^(2/3),x)
```

output
```
int(1/tan(c + d*x)**(2/3),x)/b**(2/3)
```

# 3.22 $\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 333
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 334
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . . 334
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 339
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 340
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 340
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 341
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 341
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 342
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 343


## Optimal result

Integrand size = 12, antiderivative size = 245

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} + \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d} - \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d} - \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

output
```
-arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/b^(4/3)/d-1/2*arctan(-3^(1/2)+2*(b*t
an(d*x+c))^(1/3)/b^(1/3))/b^(4/3)/d-1/2*arctan(3^(1/2)+2*(b*tan(d*x+c))^(1
/3)/b^(1/3))/b^(4/3)/d-1/4*3^(1/2)*ln(b^(2/3)-3^(1/2)*b^(1/3)*(b*tan(d*x+c
))^(1/3)+(b*tan(d*x+c))^(2/3))/b^(4/3)/d+1/4*3^(1/2)*ln(b^(2/3)+3^(1/2)*b^
(1/3)*(b*tan(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))/b^(4/3)/d-3/b/d/(b*tan(d*
x+c))^(1/3)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.04

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \frac{-6-i\log\left(1-i\sqrt[6]{\tan^2(c+dx)}\right)\sqrt[6]{\tan^2(c+dx)}+i\log\left(1+i\sqrt[6]{\tan^2(c+dx)}\right.}{}$$

input
```
Integrate[(b*Tan[c + d*x])^(-4/3),x]
```

output
```
(-6 - I*Log[1 - I*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) + I*Log[1
 + I*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) - (-1)^(1/6)*Log[1 - (
-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) + (-1)^(1/6)*Log[
1 + (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) - (-1)^(5/6)
*Log[1 - (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) + (-1)^
(5/6)*Log[1 + (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6))/(
2*b*d*(b*Tan[c + d*x])^(1/3))
```

## Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3955, 3042, 3957, 266, 824, 27, 216, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3955}$$

$$-\frac{\int (b\tan(c+dx))^{2/3}dx}{b^2}-\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int (b\tan(c+dx))^{2/3}dx}{b^2}-\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$$\Big\downarrow \; 3957$$

$$-\frac{\int \frac{(b\tan(c+dx))^{2/3}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{bd}-\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$$\Big\downarrow \; 266$$

$$-\frac{3\int \frac{b^4\tan^4(c+dx)}{b^6\tan^6(c+dx)+b^2}d\sqrt[3]{b\tan(c+dx)}}{bd}-\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$$\Big\downarrow \; 824$$

$$-\frac{3\left(\frac{1}{3}\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}+\frac{\int -\frac{\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{2\left(b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}\right)}d\sqrt[3]{b\tan(c+dx)}}{3\sqrt[3]{b}}+\frac{\int -\frac{\sqrt[3]{b}+\cdots}{2\left(b^2\tan^2(c+\cdots\right.}}{\cdots}\right.}{bd}$$

$$\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{3\left(\frac{1}{3}\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{\int \frac{\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\,t\cdots}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4\cdots}}}{\cdots}\right.}{bd}$$

$$\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$$\Big\downarrow \; 216$$

$$-\frac{3\left(-\frac{\int \frac{\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}+\frac{\text{arcta}\cdots}{\cdots}\right.}{bd}$$

$$\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{3\left(-\frac{-\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+a}}{6\sqrt[3]{b}}\right.}{\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}\right.}{\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{3\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-\frac{1}{3}}d\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}}}{6\sqrt[3]{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-\frac{1}{3}}}{bd}\right.}{\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{3\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}+\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)}{6\sqrt[3]{b}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}+2\sqrt[3]{b\tan}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}}}{bd}\right.}{\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{3\left(\frac{\arctan(b^{2/3}\tan(c+dx))}{3\sqrt[3]{b}}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)-\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}+b^2\tan^2(c+dx)\right)}{6\sqrt[3]{b}}-\frac{\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}b^{4/3}\cdots\right.}{\cdots}\right)}{bd}$$

$$\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x])^(-4/3),x]
```

output
```
(-3*(ArcTan[b^(2/3)*Tan[c + d*x]]/(3*b^(1/3)) - (ArcTan[Sqrt[3]*(1 - (2*b^
(2/3)*Tan[c + d*x])/Sqrt[3])] - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*b^(4/3)*Tan
[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(1/3)) - (-ArcTan[Sqrt[3]*(1 + (2
*b^(2/3)*Tan[c + d*x])/Sqrt[3])] + (Sqrt[3]*Log[b^(2/3) + Sqrt[3]*b^(4/3)*
Tan[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(1/3))))/(b*d) - 3/(b*d*(b*Tan
[c + d*x])^(1/3))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.87

| method | result |
|---|---|
| derivativedivides | $\frac{3b\left(-\frac{1}{b^{2}(b\tan(dx+c))^{\frac{1}{3}}}-\frac{-\frac{\sqrt{3}\left(b^{2}\right)^{\frac{5}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\left(b^{2}\right)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+\left(b^{2}\right)^{\frac{1}{3}}\right)}{12b^{2}}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{\left(b^{2}\right)^{\frac{1}{6}}}+\sqrt{3}\right.}{6\left(b^{2}\right)^{\frac{1}{6}}}}{}\right.}{}$ |
| default | $\frac{3b\left(-\frac{1}{b^{2}(b\tan(dx+c))^{\frac{1}{3}}}-\frac{-\frac{\sqrt{3}\left(b^{2}\right)^{\frac{5}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\left(b^{2}\right)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+\left(b^{2}\right)^{\frac{1}{3}}\right)}{12b^{2}}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{\left(b^{2}\right)^{\frac{1}{6}}}+\sqrt{3}\right.}{6\left(b^{2}\right)^{\frac{1}{6}}}}{}\right.}{}$ |

input
```
int(1/(b*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/d*b*(-1/b^2/(b*tan(d*x+c))^(1/3)-1/b^2*(-1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln
((b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3)
)+1/6/(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)+3^(1/2))+1/12/
b^2*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c))^(2/3)-3^(1/2)*(b^2)^(1/6)*(b*tan
(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/
(b^2)^(1/6)-3^(1/2))+1/3/(b^2)^(1/6)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/
6))))
```

### Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(187) = 374.

Time = 0.11 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.76

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx =$$
$$-\frac{2\,b^2d\left(-\frac{1}{b^8d^6}\right)^{\frac{1}{6}}\log\left(b^7d^5\left(-\frac{1}{b^8d^6}\right)^{\frac{5}{6}}+(b\tan(dx+c))^{\frac{1}{3}}\right)\tan(dx+c)-2\,b^2d\left(-\frac{1}{b^8d^6}\right)^{\frac{1}{6}}\log\left(-b^7d^5\left(-\frac{1}{b^8d^6}\right)^{\frac{5}{6}}\right.}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
-1/4*(2*b^2*d*(-1/(b^8*d^6))^(1/6)*log(b^7*d^5*(-1/(b^8*d^6))^(5/6) + (b*t
an(d*x + c))^(1/3))*tan(d*x + c) - 2*b^2*d*(-1/(b^8*d^6))^(1/6)*log(-b^7*d
^5*(-1/(b^8*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) - (sqrt(-3)
*b^2*d - b^2*d)*(-1/(b^8*d^6))^(1/6)*log(1/2*(sqrt(-3)*b^7*d^5 + b^7*d^5)*
(-1/(b^8*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) + (sqrt(-3)*b^
2*d - b^2*d)*(-1/(b^8*d^6))^(1/6)*log(-1/2*(sqrt(-3)*b^7*d^5 + b^7*d^5)*(-
1/(b^8*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) - (sqrt(-3)*b^2*
d + b^2*d)*(-1/(b^8*d^6))^(1/6)*log(1/2*(sqrt(-3)*b^7*d^5 - b^7*d^5)*(-1/(
b^8*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) + (sqrt(-3)*b^2*d +
 b^2*d)*(-1/(b^8*d^6))^(1/6)*log(-1/2*(sqrt(-3)*b^7*d^5 - b^7*d^5)*(-1/(b^
8*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) + 12*(b*tan(d*x + c))
^(2/3))/(b^2*d*tan(d*x + c))
```

### Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\tan(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(4/3),x)
```

output
```
Integral((b*tan(c + d*x))**(-4/3), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.74

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \frac{\frac{\sqrt{3}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}} - \frac{\sqrt{3}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+\right.}{b^{\frac{1}{3}}}}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
1/4*(sqrt(3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d*x + c))
^(2/3) + b^(2/3))/b^(1/3) - sqrt(3)*log(-sqrt(3)*(b*tan(d*x + c))^(1/3)*b^
(1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3))/b^(1/3) - 2*arctan((sqrt(3)*b^(1
/3) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 2*arctan(-(sqrt(3)*b^(1
/3) - 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 4*arctan((b*tan(d*x + c
))^(1/3)/b^(1/3))/b^(1/3) - 12/(b*tan(d*x + c))^(1/3))/(b*d)
```

## Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \frac{1}{4}b\left(\frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{b^4d} - \frac{\sqrt{3}|b|^{\frac{5}{3}}}{}\right.$$

input
```
integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
1/4*b*(sqrt(3)*abs(b)^(5/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3
) + (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^4*d) - sqrt(3)*abs(b)^(5/3)*
log(-sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3)
+ abs(b)^(2/3))/(b^4*d) - 2*abs(b)^(5/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*
(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b^4*d) - 2*abs(b)^(5/3)*arctan(-(sq
rt(3)*abs(b)^(1/3) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b^4*d) - 4*a
bs(b)^(5/3)*arctan((b*tan(d*x + c))^(1/3)/abs(b)^(1/3))/(b^4*d) - 12/((b*t
an(d*x + c))^(1/3)*b^2*d))
```

## Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.13

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = -\frac{3}{b\,d\,(b\tan(c+d\,x))^{1/3}} - \frac{(-1)^{1/6}\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,(b\tan(c+d\,x))^{1/3}}{b^{1/3}}\right)\,1\mathrm{i}}{b^{4/3}\,d} - \frac{(-1)^{1/6}\,\ln\left(972\,b^{12}\,d^6 - 972\,(-1)^{1/6}\,b^{35/3}\,d^6\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(b\tan(c+d\,x))^{1/3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{4/3}\,d} - \frac{(-1)^{1/6}\,\ln\left(972\,b^{12}\,d^6 - 972\,(-1)^{1/6}\,b^{35/3}\,d^6\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(b\tan(c+d\,x))^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{4/3}\,d} + \frac{(-1)^{1/6}\,\ln\left(972\,b^{12}\,d^6 + 1944\,(-1)^{1/6}\,b^{35/3}\,d^6\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)(b\tan(c+d\,x))^{1/3}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{4/3}\,d} + \frac{(-1)^{1/6}\,\ln\left(972\,b^{12}\,d^6 + 1944\,(-1)^{1/6}\,b^{35/3}\,d^6\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)(b\tan(c+d\,x))^{1/3}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{4/3}\,d}$$

input
```
int(1/(b*tan(c + d*x))^(4/3),x)
```

output
```
((-1)^(1/6)*log(972*b^12*d^6 + 1944*(-1)^(1/6)*b^(35/3)*d^6*((3^(1/2)*1i)/
4 - 1/4)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/4 - 1/4))/(b^(4/3)*d) - ((-
1)^(1/6)*atan(((-1)^(2/3)*(b*tan(c + d*x))^(1/3))/b^(1/3))*1i)/(b^(4/3)*d)
 - ((-1)^(1/6)*log(972*b^12*d^6 - 972*(-1)^(1/6)*b^(35/3)*d^6*((3^(1/2)*1i
)/2 - 1/2)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*b^(4/3)*d) -
 ((-1)^(1/6)*log(972*b^12*d^6 - 972*(-1)^(1/6)*b^(35/3)*d^6*((3^(1/2)*1i)/
2 + 1/2)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(4/3)*d) - 3
/(b*d*(b*tan(c + d*x))^(1/3)) + ((-1)^(1/6)*log(972*b^12*d^6 + 1944*(-1)^(
1/6)*b^(35/3)*d^6*((3^(1/2)*1i)/4 + 1/4)*(b*tan(c + d*x))^(1/3))*((3^(1/2)
*1i)/4 + 1/4))/(b^(4/3)*d)
```

## Reduce [F]

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \frac{\int \frac{1}{\tan(dx+c)^{\frac{4}{3}}}dx}{b^{\frac{4}{3}}}$$

input
```
int(1/(b*tan(d*x+c))^(4/3),x)
```

output
```
int(1/(tan(c + d*x)**(1/3)*tan(c + d*x)),x)/(b**(1/3)*b)
```

# 3.23 $\int (b\tan(c+dx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 344
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 344
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 345
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 346
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 346
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 347
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 347
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 347
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 348
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 348

## Optimal result

Integrand size = 10, antiderivative size = 50

$$\int (b\tan(c+dx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(c+dx)\right)(b\tan(c+dx))^{1+n}}{bd(1+n)}$$

output
```
hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(d*x+c)^2)*(b*tan(d*x+c))^(1+n)/b
/d/(1+n)
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int (b\tan(c+dx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(c+dx)\right)\tan(c+dx)(b\tan(c+dx))^n}{d(1+n)}$$

input
```
Integrate[(b*Tan[c + d*x])^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x]*
(b*Tan[c + d*x])^n)/(d*(1 + n))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\int \frac{(b\tan(c+dx))^n}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{(b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(c+dx)\right)}{bd(n+1)}$$

input
```
Int[(b*Tan[c + d*x])^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[c + d*x]^2]*(b*Tan[c + d*
x])^(1 + n))/(b*d*(1 + n))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [F]

$$\int (b\tan (dx+c))^{n} dx$$

input
```
int((b*tan(d*x+c))^n,x)
```

output
```
int((b*tan(d*x+c))^n,x)
```

## Fricas [F]

$$\int (b\tan(c+dx))^n \, dx = \int (b\tan (dx+c))^{n} \, dx$$

input
```
integrate((b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c))^n, x)
```

## Sympy [F]

$$\int (b\tan(c+dx))^n \, dx = \int (b\tan{(c+dx)})^n \, dx$$

input
```
integrate((b*tan(d*x+c))**n,x)
```

output
```
Integral((b*tan(c + d*x))**n, x)
```

## Maxima [F]

$$\int (b\tan(c+dx))^n \, dx = \int (b\tan{(dx+c)})^n \, dx$$

input
```
integrate((b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c))^n, x)
```

## Giac [F]

$$\int (b\tan(c+dx))^n \, dx = \int (b\tan{(dx+c)})^n \, dx$$

input
```
integrate((b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c))^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\tan(c+dx))^n\,dx = \int (b\tan(c+d\,x))^n\,dx$$

input
```
int((b*tan(c + d*x))^n,x)
```

output
```
int((b*tan(c + d*x))^n, x)
```

## Reduce [F]

$$\int (b\tan(c+dx))^n\,dx = b^n\left(\int \tan(dx+c)^n\,dx\right)$$

input
```
int((b*tan(d*x+c))^n,x)
```

output
```
b**n*int(tan(c + d*x)**n,x)
```

# 3.24 $\int \left(b\tan^2(c+dx)\right)^{5/2} dx$

Optimal result . . . . . 349
Mathematica [A] (verified) . . . . . 349
Rubi [A] (verified) . . . . . 350
Maple [A] (verified) . . . . . 352
Fricas [A] (verification not implemented) . . . . . 352
Sympy [F] . . . . . 353
Maxima [A] (verification not implemented) . . . . . 353
Giac [A] (verification not implemented) . . . . . 353
Mupad [F(-1)] . . . . . 354
Reduce [B] (verification not implemented) . . . . . 354

## Optimal result

Integrand size = 14, antiderivative size = 98

$$\int \left(b\tan^2(c+dx)\right)^{5/2} dx = -\frac{b^2\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d} - \frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d}$$

output
```
-b^2*cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^2)^(1/2)/d-1/2*b^2*tan(d*x+c)
*(b*tan(d*x+c)^2)^(1/2)/d+1/4*b^2*tan(d*x+c)^3*(b*tan(d*x+c)^2)^(1/2)/d
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

$$\int \left(b\tan^2(c+dx)\right)^{5/2} dx = \frac{b^2\cot(c+dx)\left(-4\log(\cos(c+dx)) - 4\sec^2(c+dx) + \sec^4(c+dx)\right)\sqrt{b\tan^2(c+dx)}}{4d}$$

input
```
Integrate[(b*Tan[c + d*x]^2)^(5/2),x]
```

output
```
(b^2*Cot[c + d*x]*(-4*Log[Cos[c + d*x]] - 4*Sec[c + d*x]^2 + Sec[c + d*x]^
4)*Sqrt[b*Tan[c + d*x]^2])/(4*d)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^2(c+dx)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{4141}$$

$$b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\int \tan^5(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\int \tan(c+dx)^5dx$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\frac{\tan^4(c+dx)}{4d}-\int \tan^3(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\frac{\tan^4(c+dx)}{4d}-\int \tan(c+dx)^3dx\right)$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\int \tan(c+dx)dx+\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2 \cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\int \tan(c+dx)dx+\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3956}$$

$$b^2 \cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}-\frac{\log(\cos(c+dx))}{d}\right)$$

input
```
Int[(b*Tan[c + d*x]^2)^(5/2),x]
```

output
```
b^2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^2]*(-(Log[Cos[c + d*x]]/d) - Tan[c +
d*x]^2/(2*d) + Tan[c + d*x]^4/(4*d))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

| method | result |
|---|---|
| derivativedivides | $\frac{\left(b\tan(dx+c)^2\right)^{\frac{5}{2}}\left(\tan(dx+c)^4-2\tan(dx+c)^2+2\ln\left(1+\tan(dx+c)^2\right)\right)}{4d\tan(dx+c)^5}$ |
| default | $\frac{\left(b\tan(dx+c)^2\right)^{\frac{5}{2}}\left(\tan(dx+c)^4-2\tan(dx+c)^2+2\ln\left(1+\tan(dx+c)^2\right)\right)}{4d\tan(dx+c)^5}$ |
| risch | $\frac{b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1}-\frac{2b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{4ib^2\sqrt{-\frac{b\left(e^{2i(dx+c)}-\right.}{\left(e^{2i(dx+c)}+1\right.}}}{\left(e^{2i(dx-\right.}}$ |

input
```
int((b*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(b*tan(d*x+c)^2)^(5/2)*(tan(d*x+c)^4-2*tan(d*x+c)^2+2*ln(1+tan(d*x+c
)^2))/tan(d*x+c)^5
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx=\frac{\left(b^2\tan(dx+c)^4-2\,b^2\tan(dx+c)^2-2\,b^2\log\left(\frac{1}{\tan(dx+c)^2+1}\right)-3\,b^2\right)\sqrt{b\tan(dx+c)^2}}{4\,d\tan(dx+c)}$$

input
```
integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(b^2*tan(d*x + c)^4 - 2*b^2*tan(d*x + c)^2 - 2*b^2*log(1/(tan(d*x + c)
^2 + 1)) - 3*b^2)*sqrt(b*tan(d*x + c)^2)/(d*tan(d*x + c))
```

## Sympy [F]

$$\int \left(b\tan^2(c+dx)\right)^{5/2} dx = \int \left(b\tan^2{(c+dx)}\right)^{\frac{5}{2}} dx$$

input
```
integrate((b*tan(d*x+c)**2)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**2)**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.48

$$\int \left(b\tan^2(c+dx)\right)^{5/2} dx = \frac{b^{\frac{5}{2}}\tan{(dx+c)}^4 - 2\,b^{\frac{5}{2}}\tan{(dx+c)}^2 + 2\,b^{\frac{5}{2}}\log\left(\tan{(dx+c)}^2+1\right)}{4\,d}$$

input
```
integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/4*(b^(5/2)*tan(d*x + c)^4 - 2*b^(5/2)*tan(d*x + c)^2 + 2*b^(5/2)*log(tan
(d*x + c)^2 + 1))/d
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int \left(b\tan^2(c+dx)\right)^{5/2} dx = \frac{1}{4}\,b^{\frac{5}{2}}\left(\frac{2\,\log\left(\tan{(dx+c)}^2+1\right)}{d} + \frac{d\tan{(dx+c)}^4 - 2\,d\tan{(dx+c)}^2}{d^2}\right)\mathrm{sgn}(\tan{(dx+c)})$$

input
```
integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
1/4*b^(5/2)*(2*log(tan(d*x + c)^2 + 1)/d + (d*tan(d*x + c)^4 - 2*d*tan(d*x
 + c)^2)/d^2)*sgn(tan(d*x + c))
```

## Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^2(c+dx)\right)^{5/2} dx = \int \left(b\tan(c+d\,x)^2\right)^{5/2} dx$$

input
```
int((b*tan(c + d*x)^2)^(5/2),x)
```

output
```
int((b*tan(c + d*x)^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.43

$$\int \left(b\tan^2(c+dx)\right)^{5/2} dx = \frac{\sqrt{b}\,b^2\left(2\log\left(\tan\left(dx+c\right)^2+1\right)+\tan\left(dx+c\right)^4-2\tan\left(dx+c\right)^2\right)}{4d}$$

input
```
int((b*tan(d*x+c)^2)^(5/2),x)
```

output
```
(sqrt(b)*b**2*(2*log(tan(c + d*x)**2 + 1) + tan(c + d*x)**4 - 2*tan(c + d*
x)**2))/(4*d)
```

# 3.25 $\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx$

Optimal result . . . . . 355
Mathematica [A] (verified) . . . . . 355
Rubi [A] (verified) . . . . . 356
Maple [A] (verified) . . . . . 358
Fricas [A] (verification not implemented) . . . . . 358
Sympy [F] . . . . . 359
Maxima [A] (verification not implemented) . . . . . 359
Giac [A] (verification not implemented) . . . . . 359
Mupad [F(-1)] . . . . . 360
Reduce [B] (verification not implemented) . . . . . 360

## Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \frac{b\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d} + \frac{b\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}$$

output
```
b*cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^2)^(1/2)/d+1/2*b*tan(d*x+c)*(b*t
an(d*x+c)^2)^(1/2)/d
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \frac{\cot^3(c+dx)\left(2\log(\cos(c+dx))+\sec^2(c+dx)\right)\left(b\tan^2(c+dx)\right)^{3/2}}{2d}$$

input
```
Integrate[(b*Tan[c + d*x]^2)^(3/2),x]
```

output
```
(Cot[c + d*x]^3*(2*Log[Cos[c + d*x]] + Sec[c + d*x]^2)*(b*Tan[c + d*x]^2)^
(3/2))/(2*d)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4141, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(b\tan(c+dx)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4141}$$

$$b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\int \tan^3(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\int \tan(c+dx)^3dx$$

$$\Big\downarrow \text{ 3954}$$

$$b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\frac{\tan^2(c+dx)}{2d}-\int \tan(c+dx)dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\frac{\tan^2(c+dx)}{2d}-\int \tan(c+dx)dx\right)$$

$$\Big\downarrow \text{ 3956}$$

$$b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}\right)$$

input
```
Int[(b*Tan[c + d*x]^2)^(3/2),x]
```

output
```
b*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^2]*(Log[Cos[c + d*x]]/d + Tan[c + d*x]^
2/(2*d))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| derivativedivides | $-\frac{\left(b\tan(dx+c)^2\right)^{\frac{3}{2}}\left(-\tan(dx+c)^2+\ln\left(1+\tan(dx+c)^2\right)\right)}{2d\tan(dx+c)^3}$ |
| default | $-\frac{\left(b\tan(dx+c)^2\right)^{\frac{3}{2}}\left(-\tan(dx+c)^2+\ln\left(1+\tan(dx+c)^2\right)\right)}{2d\tan(dx+c)^3}$ |
| risch | $\frac{b\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(ie^{4i(dx+c)}\ln(e^{2i(dx+c)}+1)+e^{4i(dx+c)}dx+2\,e^{4i(dx+c)}c+2ie^{2i(dx+c)}\ln(e^{2i(dx+c)}+1)+2\,e^{2i(dx}}{(e^{2i(dx+c)}-1)\,(e^{2i(dx+c)}+1)\,d}$ |

input
```
int((b*tan(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(b*tan(d*x+c)^2)^(3/2)*(-tan(d*x+c)^2+ln(1+tan(d*x+c)^2))/tan(d*x+c
)^3
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \frac{\left(b\tan\left(dx+c\right)^2+b\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+b\right)\sqrt{b\tan\left(dx+c\right)^2}}{2\,d\tan\left(dx+c\right)}$$

input
```
integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(b*tan(d*x + c)^2 + b*log(1/(tan(d*x + c)^2 + 1)) + b)*sqrt(b*tan(d*x
+ c)^2)/(d*tan(d*x + c))
```

## Sympy [F]

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \int \left(b\tan^2\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)**2)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**2)**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \frac{b^{\frac{3}{2}}\tan\left(dx+c\right)^2 - b^{\frac{3}{2}}\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(b^(3/2)*tan(d*x + c)^2 - b^(3/2)*log(tan(d*x + c)^2 + 1))/d
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \frac{1}{2}\,b^{\frac{3}{2}}\left(\frac{\tan\left(dx+c\right)^2}{d} - \frac{\log\left(\tan\left(dx+c\right)^2+1\right)}{d}\right)\operatorname{sgn}(\tan\left(dx+c\right))$$

input
```
integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*b^(3/2)*(tan(d*x + c)^2/d - log(tan(d*x + c)^2 + 1)/d)*sgn(tan(d*x + c
))
```

## Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^2(c+dx)\right)^{3/2} dx = \int \left(b\tan(c+d\,x)^2\right)^{3/2} dx$$

input
```
int((b*tan(c + d*x)^2)^(3/2),x)
```

output
```
int((b*tan(c + d*x)^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.49

$$\int \left(b\tan^2(c+dx)\right)^{3/2} dx = \frac{\sqrt{b}\,b\left(-\log\left(\tan\left(dx+c\right)^2+1\right)+\tan\left(dx+c\right)^2\right)}{2d}$$

input
```
int((b*tan(d*x+c)^2)^(3/2),x)
```

output
```
(sqrt(b)*b*( - log(tan(c + d*x)**2 + 1) + tan(c + d*x)**2))/(2*d)
```

## 3.26 $\int \sqrt{b \tan^2(c+dx)}\,dx$

Optimal result . . . . . 361
Mathematica [A] (verified) . . . . . 361
Rubi [A] (verified) . . . . . 362
Maple [A] (verified) . . . . . 363
Fricas [A] (verification not implemented) . . . . . 364
Sympy [F] . . . . . 364
Maxima [A] (verification not implemented) . . . . . 364
Giac [A] (verification not implemented) . . . . . 365
Mupad [F(-1)] . . . . . 365
Reduce [B] (verification not implemented) . . . . . 365

### Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \sqrt{b \tan^2(c+dx)}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{b \tan^2(c+dx)}}{d}$$

output
```
-cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^2)^(1/2)/d
```

### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sqrt{b \tan^2(c+dx)}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{b \tan^2(c+dx)}}{d}$$

input
```
Integrate[Sqrt[b*Tan[c + d*x]^2],x]
```

output
```
-((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d)
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4141, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4141}$$

$$\cot(c+dx)\sqrt{b\tan^2(c+dx)}\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cot(c+dx)\sqrt{b\tan^2(c+dx)}\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\cot(c+dx)\sqrt{b\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

input
```
Int[Sqrt[b*Tan[c + d*x]^2],x]
```

output
```
-((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| derivativedivides | $\frac{\sqrt{b\tan(dx+c)^{2}}\,\ln\left(1+\tan(dx+c)^{2}\right)}{2d\tan(dx+c)}$ |
| default | $\frac{\sqrt{b\tan(dx+c)^{2}}\,\ln\left(1+\tan(dx+c)^{2}\right)}{2d\tan(dx+c)}$ |
| risch | $\frac{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^{2}}{\left(e^{2i(dx+c)}+1\right)^{2}}}\,\left(e^{2i(dx+c)}+1\right)x}{e^{2i(dx+c)}-1}-\frac{2\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^{2}}{\left(e^{2i(dx+c)}+1\right)^{2}}}\,\left(e^{2i(dx+c)}+1\right)(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{i\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^{2}}{\left(e^{2i(dx+c)}+1\right)^{2}}}\,\left(e^{2i\right.}}{\left(e^{2i(dx\right.}}$ |

input
```
int((b*tan(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(b*tan(d*x+c)^2)^(1/2)/tan(d*x+c)*ln(1+tan(d*x+c)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \sqrt{b\tan^2(c+dx)}\,dx = -\frac{\sqrt{b\tan\left(dx+c\right)^2}\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d\tan\left(dx+c\right)}$$

input
```
integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(b*tan(d*x + c)^2)*log(1/(tan(d*x + c)^2 + 1))/(d*tan(d*x + c))
```

## Sympy [F]

$$\int \sqrt{b\tan^2(c+dx)}\,dx = \int \sqrt{b\tan^2\left(c+dx\right)}\,dx$$

input
```
integrate((b*tan(d*x+c)**2)**(1/2),x)
```

output
```
Integral(sqrt(b*tan(c + d*x)**2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \sqrt{b\tan^2(c+dx)}\,dx = \frac{\sqrt{b}\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b)*log(tan(d*x + c)^2 + 1)/d
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \sqrt{b\tan^2(c+dx)}\,dx = \frac{\sqrt{b}\log\left(\tan\left(dx+c\right)^2+1\right)\operatorname{sgn}(\tan\left(dx+c\right))}{2\,d}$$

input
```
integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b)*log(tan(d*x + c)^2 + 1)*sgn(tan(d*x + c))/d
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^2(c+dx)}\,dx = \int \sqrt{b\tan\left(c+d\,x\right)^2}\,dx$$

input
```
int((b*tan(c + d*x)^2)^(1/2),x)
```

output
```
int((b*tan(c + d*x)^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.56

$$\int \sqrt{b\tan^2(c+dx)}\,dx = \frac{\sqrt{b}\log\left(\tan\left(dx+c\right)^2+1\right)}{2d}$$

input
```
int((b*tan(d*x+c)^2)^(1/2),x)
```

output
```
(sqrt(b)*log(tan(c + d*x)**2 + 1))/(2*d)
```

# 3.27 $\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx$


Optimal result . . . . . 366
Mathematica [A] (verified) . . . . . 366
Rubi [A] (verified) . . . . . 367
Maple [A] (verified) . . . . . 368
Fricas [A] (verification not implemented) . . . . . 369
Sympy [F] . . . . . 369
Maxima [A] (verification not implemented) . . . . . 370
Giac [B] (verification not implemented) . . . . . 370
Mupad [B] (verification not implemented) . . . . . 370
Reduce [B] (verification not implemented) . . . . . 371


## Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{b\tan^2(c+dx)}}$$

output
```
ln(sin(d*x+c))*tan(d*x+c)/d/(b*tan(d*x+c)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{b\tan^2(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[c + d*x]^2],x]
```

output
```
(Log[Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*x]^2])
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4141, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{b\tan(c+dx)^2}}dx$$

$$\Big\downarrow \text{ 4141}$$

$$\frac{\tan(c+dx)\int \cot(c+dx)dx}{\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\tan(c+dx)\log(-\sin(c+dx))}{d\sqrt{b\tan^2(c+dx)}}$$

input
```
Int[1/Sqrt[b*Tan[c + d*x]^2],x]
```

output
```
(Log[-Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*x]^2])
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

| method | result |
|---|---|
| derivativedivides | $\frac{\tan(dx+c)\left(2\ln(\tan(dx+c))-\ln\left(1+\tan(dx+c)^2\right)\right)}{2d\sqrt{b\tan(dx+c)^2}}$ |
| default | $\frac{\tan(dx+c)\left(2\ln(\tan(dx+c))-\ln\left(1+\tan(dx+c)^2\right)\right)}{2d\sqrt{b\tan(dx+c)^2}}$ |
| risch | $\frac{(e^{2i(dx+c)}-1)x}{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{2i(dx+c)}+1)}-\frac{2(e^{2i(dx+c)}-1)(dx+c)}{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{2i(dx+c)}+1)d}-\frac{i(e^{2i(dx+c)}-1)\ln(e^{2i(dx+c)}-1)}{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{2i(dx+c)}+1}$ |

input
```
int(1/(b*tan(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*tan(d*x+c)*(2*ln(tan(d*x+c))-ln(1+tan(d*x+c)^2))/(b*tan(d*x+c)^2)^(1
/2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx=\frac{\sqrt{b\tan{(dx+c)}^2}\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)}{2\,bd\tan{(dx+c)}}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(b*tan(d*x + c)^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))/(b*d*t
an(d*x + c))
```

## Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\tan^2{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(c + d*x)**2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = -\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{b}} - \frac{2\log(\tan(dx+c))}{\sqrt{b}}}{2\,d}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(log(tan(d*x + c)^2 + 1)/sqrt(b) - 2*log(tan(d*x + c))/sqrt(b))/d
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(29) = 58.

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = -\frac{1}{2}\sqrt{b}\left(\frac{\log\left(\tan\left(dx+c\right)^2+1\right)}{bd\mathrm{sgn}\left(\tan\left(dx+c\right)\right)} - \frac{\log\left(\tan\left(dx+c\right)^2\right)}{bd\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}\right)$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(b)*(log(tan(d*x + c)^2 + 1)/(b*d*sgn(tan(d*x + c))) - log(tan(d*
x + c)^2)/(b*d*sgn(tan(d*x + c))))
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{-b}\tan(c+d\,x)}{\sqrt{b\tan(c+d\,x)^2}}\right)}{\sqrt{-b}\,d}$$

input
```
int(1/(b*tan(c + d*x)^2)^(1/2),x)
```

output
```
atan(((-b)^(1/2)*tan(c + d*x))/(b*tan(c + d*x)^2)^(1/2))/((-b)^(1/2)*d)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{\sqrt{b}\left(-\log\left(\tan\left(dx+c\right)^2+1\right)+2\log(\tan\left(dx+c\right))\right)}{2bd}$$

input
```
int(1/(b*tan(d*x+c)^2)^(1/2),x)
```

output
```
(sqrt(b)*( - log(tan(c + d*x)**2 + 1) + 2*log(tan(c + d*x))))/(2*b*d)
```

# 3.28 $\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx$

Optimal result . . . . . . . . . . 372
Mathematica [A] (verified) . . . . . . . . . . 372
Rubi [A] (verified) . . . . . . . . . . 373
Maple [A] (verified) . . . . . . . . . . 375
Fricas [A] (verification not implemented) . . . . . . . . . . 375
Sympy [F] . . . . . . . . . . 376
Maxima [A] (verification not implemented) . . . . . . . . . . 376
Giac [A] (verification not implemented) . . . . . . . . . . 377
Mupad [F(-1)] . . . . . . . . . . 377
Reduce [B] (verification not implemented) . . . . . . . . . . 377

## Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx=-\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}}-\frac{\log(\sin(c+dx))\tan(c+dx)}{bd\sqrt{b\tan^2(c+dx)}}$$

output
```
-1/2*cot(d*x+c)/b/d/(b*tan(d*x+c)^2)^(1/2)-ln(sin(d*x+c))*tan(d*x+c)/b/d/(
b*tan(d*x+c)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx=-\frac{\left(\csc^2(c+dx)+2\log(\sin(c+dx))\right)\tan^3(c+dx)}{2d\left(b\tan^2(c+dx)\right)^{3/2}}$$

input
```
Integrate[(b*Tan[c + d*x]^2)^(-3/2),x]
```

output
```
-1/2*((Csc[c + d*x]^2 + 2*Log[Sin[c + d*x]])*Tan[c + d*x]^3)/(d*(b*Tan[c +
 d*x]^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 4141, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\tan(c+dx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan(c+dx)\int \cot^3(c+dx)dx}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)^3dx}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^3dx}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^2(c+dx)}{2d}-\int -\cot(c+dx)dx\right)}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\left(\int \cot(c+dx)dx+\frac{\cot^2(c+dx)}{2d}\right)}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tan(c+dx)\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\cot^2(c+dx)}{2d}\right)}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^2(c+dx)}{2d}-\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx\right)}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \; 3956$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^2(c+dx)}{2d}+\frac{\log(-\sin(c+dx))}{d}\right)}{b\sqrt{b\tan^2(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^2)^(-3/2),x]
```

output
```
-(((Cot[c + d*x]^2/(2*d) + Log[-Sin[c + d*x]]/d)*Tan[c + d*x])/(b*Sqrt[b*T
an[c + d*x]^2]))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $-\frac{\tan(dx+c)\left(2\ln(\tan(dx+c))\tan(dx+c)^{2}-\ln\left(1+\tan(dx+c)^{2}\right)\tan(dx+c)^{2}+1\right)}{2d\left(b\tan(dx+c)^{2}\right)^{\frac{3}{2}}}$ |
| default | $-\frac{\tan(dx+c)\left(2\ln(\tan(dx+c))\tan(dx+c)^{2}-\ln\left(1+\tan(dx+c)^{2}\right)\tan(dx+c)^{2}+1\right)}{2d\left(b\tan(dx+c)^{2}\right)^{\frac{3}{2}}}$ |
| risch | $\frac{i e^{4i(dx+c)}\ln\left(e^{2i(dx+c)}-1\right)+e^{4i(dx+c)}dx+2\,e^{4i(dx+c)}c-2i e^{2i(dx+c)}\ln\left(e^{2i(dx+c)}-1\right)-2\,e^{2i(dx+c)}dx-2i e^{2i(dx+c)}-4\,e}{b\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^{2}}{\left(e^{2i(dx+c)}+1\right)^{2}}}\,d}$ |

input
```
int(1/(b*tan(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*tan(d*x+c)*(2*ln(tan(d*x+c))*tan(d*x+c)^2-ln(1+tan(d*x+c)^2)*tan(d*
x+c)^2+1)/(b*tan(d*x+c)^2)^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = -\frac{\sqrt{b\tan\left(dx+c\right)^{2}}\left(\log\left(\frac{\tan(dx+c)^{2}}{\tan(dx+c)^{2}+1}\right)\tan\left(dx+c\right)^{2}+\tan\left(dx+c\right)^{2}+1\right)}{2\,b^{2}d\tan\left(dx+c\right)^{3}}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(b*tan(d*x + c)^2)*(log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(
d*x + c)^2 + tan(d*x + c)^2 + 1)/(b^2*d*tan(d*x + c)^3)
```

## Sympy [F]

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan^2\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**2)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**2)**(-3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = \frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{b^{\frac{3}{2}}} - \frac{2\log(\tan(dx+c))}{b^{\frac{3}{2}}} - \frac{1}{b^{\frac{3}{2}}\tan(dx+c)^2}}{2d}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(log(tan(d*x + c)^2 + 1)/b^(3/2) - 2*log(tan(d*x + c))/b^(3/2) - 1/(b^
(3/2)*tan(d*x + c)^2))/d
```

## Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = \frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{bd\mathrm{sgn}(\tan(dx+c))} - \frac{\log\left(\tan(dx+c)^2\right)}{bd\mathrm{sgn}(\tan(dx+c))} + \frac{\tan(dx+c)^2-1}{bd\mathrm{sgn}(\tan(dx+c))\tan(dx+c)^2}}{2\sqrt{b}}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(log(tan(d*x + c)^2 + 1)/(b*d*sgn(tan(d*x + c))) - log(tan(d*x + c)^2)
/(b*d*sgn(tan(d*x + c))) + (tan(d*x + c)^2 - 1)/(b*d*sgn(tan(d*x + c))*tan
(d*x + c)^2))/sqrt(b)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^2\right)^{3/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^2)^(3/2),x)
```

output
```
int(1/(b*tan(c + d*x)^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = \frac{\sqrt{b}\left(\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)^2 - 2\log\left(\tan\left(dx+c\right)\right)\tan\left(dx+c\right)^2 - \right.}{2\tan\left(dx+c\right)^2 b^2 d}$$

input
```
int(1/(b*tan(d*x+c)^2)^(3/2),x)
```

output
```
(sqrt(b)*(log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2 - 2*log(tan(c + d*x))*t
an(c + d*x)**2 - 1))/(2*tan(c + d*x)**2*b**2*d)
```

# 3.29 $\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 379
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 379
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 380
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 383
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 383
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 384
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . 384
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 384
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 385
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . 385

## Optimal result

Integrand size = 14, antiderivative size = 97

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = \frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{b^2d\sqrt{b\tan^2(c+dx)}}$$

output
```
1/2*cot(d*x+c)/b^2/d/(b*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/b^2/d/(b*tan(
d*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/b^2/d/(b*tan(d*x+c)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = -\frac{\cot(c+dx)\left(-4\csc^2(c+dx)+\csc^4(c+dx)-4\log(\sin(c+dx))\right)\sqrt{b\tan^2(c+dx)}}{4b^3d}$$

input
```
Integrate[(b*Tan[c + d*x]^2)^(-5/2),x]
```

output
```
-1/4*(Cot[c + d*x]*(-4*Csc[c + d*x]^2 + Csc[c + d*x]^4 - 4*Log[Sin[c + d*x
]])*Sqrt[b*Tan[c + d*x]^2])/(b^3*d)
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {3042, 4141, 3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\tan(c+dx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan(c+dx)\int \cot^5(c+dx)dx}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)^5dx}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^5dx}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^4(c+dx)}{4d}-\int -\cot^3(c+dx)dx\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\left(\int \cot^3(c+dx)dx+\frac{\cot^4(c+dx)}{4d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tan(c+dx)\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\cot^4(c+dx)}{4d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^4(c+dx)}{4d}-\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^3dx\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$-\frac{\tan(c+dx)\left(\int -\cot(c+dx)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\left(-\int \cot(c+dx)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tan(c+dx)\left(-\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\tan(c+dx)\left(\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}-\frac{\log(-\sin(c+dx))}{d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^2)^(-5/2),x]
```

output
```
-(((-1/2*Cot[c + d*x]^2/d + Cot[c + d*x]^4/(4*d) - Log[-Sin[c + d*x]]/d)*T
an[c + d*x])/(b^2*Sqrt[b*Tan[c + d*x]^2]))
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

| method | result |
|---|---|
| derivativedivides | $\frac{\tan(dx+c)\left(4\ln(\tan(dx+c))\tan(dx+c)^4-2\ln\left(1+\tan(dx+c)^2\right)\tan(dx+c)^4+2\tan(dx+c)^2-1\right)}{4d\left(b\tan(dx+c)^2\right)^{\frac{5}{2}}}$ |
| default | $\frac{\tan(dx+c)\left(4\ln(\tan(dx+c))\tan(dx+c)^4-2\ln\left(1+\tan(dx+c)^2\right)\tan(dx+c)^4+2\tan(dx+c)^2-1\right)}{4d\left(b\tan(dx+c)^2\right)^{\frac{5}{2}}}$ |
| risch | $\frac{(e^{2i(dx+c)}-1)x}{b^2(e^{2i(dx+c)}+1)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}-\frac{2(e^{2i(dx+c)}-1)(dx+c)}{b^2(e^{2i(dx+c)}+1)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}+\frac{4i(e^{6i(dx+c)}-e^{4i(dx+}}{b^2\left(e^{2i(dx+c)}-1\right)^3(e^{2i(dx+c)}+1}$ |

input
```
int(1/(b*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*tan(d*x+c)*(4*ln(tan(d*x+c))*tan(d*x+c)^4-2*ln(1+tan(d*x+c)^2)*tan(d
*x+c)^4+2*tan(d*x+c)^2-1)/(b*tan(d*x+c)^2)^(5/2)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1}{(b\tan^2(c+dx))^{5/2}}\,dx = \frac{\left(2\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^4+3\tan(dx+c)^4+2\tan(dx+c)^2-1\right)\sqrt{}}{4\,b^3d\tan(dx+c)^5}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(2*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^4 + 3*tan(d*x
 + c)^4 + 2*tan(d*x + c)^2 - 1)*sqrt(b*tan(d*x + c)^2)/(b^3*d*tan(d*x + c)
^5)
```

## Sympy [F]

$$\int \frac{1}{(b\tan^2(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan^2{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**2)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**2)**(-5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int \frac{1}{(b\tan^2(c+dx))^{5/2}}\,dx = -\frac{\frac{2\,\log\left(\tan(dx+c)^2+1\right)}{b^{\frac{5}{2}}} - \frac{4\,\log(\tan(dx+c))}{b^{\frac{5}{2}}} - \frac{2\,\sqrt{b}\tan(dx+c)^2-\sqrt{b}}{b^3\tan(dx+c)^4}}{4\,d}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(2*log(tan(d*x + c)^2 + 1)/b^(5/2) - 4*log(tan(d*x + c))/b^(5/2) - (2
*sqrt(b)*tan(d*x + c)^2 - sqrt(b))/(b^3*tan(d*x + c)^4))/d
```

## Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{1}{(b\tan^2(c+dx))^{5/2}}\,dx = -\frac{\frac{2\,\log\left(\tan(dx+c)^2+1\right)}{bd\mathrm{sgn}(\tan(dx+c))} - \frac{2\,\log\left(\tan(dx+c)^2\right)}{bd\mathrm{sgn}(\tan(dx+c))} + \frac{3\,\tan(dx+c)^4-2\,\tan(dx+c)^2+1}{bd\mathrm{sgn}(\tan(dx+c))\tan(dx+c)^4}}{4\,b^{\frac{3}{2}}}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/4*(2*log(tan(d*x + c)^2 + 1)/(b*d*sgn(tan(d*x + c))) - 2*log(tan(d*x +
c)^2)/(b*d*sgn(tan(d*x + c))) + (3*tan(d*x + c)^4 - 2*tan(d*x + c)^2 + 1)/
(b*d*sgn(tan(d*x + c))*tan(d*x + c)^4))/b^(3/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\tan\left(c+d\,x\right)^2\right)^{5/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^2)^(5/2),x)
```

output
```
int(1/(b*tan(c + d*x)^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx=\frac{\sqrt{b}\left(-2\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)^4+4\log(\tan\left(dx+c\right))\tan\left(dx+c\right)^4\right.}{4\tan\left(dx+c\right)^4b^3d}$$

input
```
int(1/(b*tan(d*x+c)^2)^(5/2),x)
```

output
```
(sqrt(b)*( - 2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4 + 4*log(tan(c + d*
x))*tan(c + d*x)**4 + 2*tan(c + d*x)**2 - 1))/(4*tan(c + d*x)**4*b**3*d)
```

# 3.30 $\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx$

Optimal result . . . 386
Mathematica [A] (verified) . . . 387
Rubi [A] (verified) . . . 387
Maple [A] (verified) . . . 393
Fricas [A] (verification not implemented) . . . 394
Sympy [F] . . . 394
Maxima [A] (verification not implemented) . . . 395
Giac [A] (verification not implemented) . . . 395
Mupad [F(-1)] . . . 396
Reduce [F] . . . 396

## Optimal result

Integrand size = 14, antiderivative size = 298

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx = -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{b^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d}$$

output
```
-2*b^2*cot(d*x+c)*(b*tan(d*x+c)^3)^(1/2)/d+1/2*b^2*arctan(-1+2^(1/2)*tan(d
*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)*2^(1/2)/d/tan(d*x+c)^(3/2)+1/2*b^2*arc
tan(1+2^(1/2)*tan(d*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)*2^(1/2)/d/tan(d*x+c
)^(3/2)+1/2*b^2*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*(b*tan(d*
x+c)^3)^(1/2)*2^(1/2)/d/tan(d*x+c)^(3/2)+2/5*b^2*tan(d*x+c)*(b*tan(d*x+c)^
3)^(1/2)/d-2/9*b^2*tan(d*x+c)^3*(b*tan(d*x+c)^3)^(1/2)/d+2/13*b^2*tan(d*x+
c)^5*(b*tan(d*x+c)^3)^(1/2)/d
```

## Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.68

$$\int (b\tan^3(c+dx))^{5/2}\,dx = \frac{(b\tan^3(c+dx))^{5/2}\left(-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\mathrm{ta}\right.}{2\sqrt{2}}\right.}{}$$

input
```
Integrate[(b*Tan[c + d*x]^3)^(5/2),x]
```

output
```
((b*Tan[c + d*x]^3)^(5/2)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2
]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt
[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]] + Tan[c + d*x]]/(2*Sqrt[2]) - 2*Sqrt[Tan[c + d*x]] + (2*Tan[c + d*x]
^(5/2))/5 - (2*Tan[c + d*x]^(9/2))/9 + (2*Tan[c + d*x]^(13/2))/13))/(d*Tan
[c + d*x]^(15/2))
```

## Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.78, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan^3(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (b\tan(c+dx)^3)^{5/2}\,dx$$

$$\Big\downarrow \; 4141$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\int \tan^{\frac{15}{2}}(c+dx)dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\int \tan(c+dx)^{15/2}dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\int \tan^{\frac{11}{2}}(c+dx)dx\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\int \tan(c+dx)^{11/2}dx\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\int \tan^{\frac{7}{2}}(c+dx)dx+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\int \tan(c+dx)^{7/2}dx+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(-\int \tan^{\frac{3}{2}}(c+dx)dx+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(-\int \tan(c+dx)^{3/2}dx+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\int\frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5d}-\frac{2\sqrt{\tan(c+dx)}}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\int\frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5d}-\frac{2\sqrt{\tan(c+dx)}}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 3957

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{\int\frac{1}{\sqrt{\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5d}-\frac{2\sqrt{\tan(c+dx)}}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 266

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\int\frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5d}-\frac{2\sqrt{\tan(c+dx)}}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 755

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+2\tan^{\frac{5}{2}}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 1476

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}}\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 1082

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 217

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1479

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\right.}}{}\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 25

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c\right.}}{\sqrt{2}}\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\right.}}{\sqrt{}}\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1103

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\cdots\right.}{2\sqrt{\cdots}}\right.\right.}{d}\cdots\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(b*Tan[c + d*x]^3)^(5/2),x]
```

output
```
(b^2*Sqrt[b*Tan[c + d*x]^3]*((2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[
1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]))/2))/d - (2*Sqrt[Tan[c + d*x
]])/d + (2*Tan[c + d*x]^(5/2))/(5*d) - (2*Tan[c + d*x]^(9/2))/(9*d) + (2*T
an[c + d*x]^(13/2))/(13*d)))/Tan[c + d*x]^(3/2)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| derivativedivides | $\frac{\left(b\tan(dx+c)^3\right)^{\frac{5}{2}}\left(360(b\tan(dx+c))^{\frac{13}{2}}-520b^2(b\tan(dx+c))^{\frac{9}{2}}+585b^6\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{}}\right.\right.}{}$ |
| default | $\frac{\left(b\tan(dx+c)^3\right)^{\frac{5}{2}}\left(360(b\tan(dx+c))^{\frac{13}{2}}-520b^2(b\tan(dx+c))^{\frac{9}{2}}+585b^6\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{}}\right.\right.}{}$ |

input
```
int((b*tan(d*x+c)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2340/d*(b*tan(d*x+c)^3)^(5/2)*(360*(b*tan(d*x+c))^(13/2)-520*b^2*(b*tan(
d*x+c))^(9/2)+585*b^6*(b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*
tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*
x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*b^6*(b^2)^(1/4)*2^(1/2)*arctan((2^(
1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+1170*b^6*(b^2)^(1/4)*2
^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+936*
b^4*(b*tan(d*x+c))^(5/2)-4680*b^6*(b*tan(d*x+c))^(1/2))/tan(d*x+c)^5/(b*ta
n(d*x+c))^(5/2)/b^4
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.04

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx = \frac{1170\sqrt{2}b^{\frac{5}{2}}\arctan\left(\frac{b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)^3}\sqrt{b}}{b\tan(dx+c)}\right)\tan\left(dx+c\right)+1170\sqrt{2}b^{\frac{5}{2}}\arctan\left(-\frac{b\tan(dx\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/2340*(1170*sqrt(2)*b^(5/2)*arctan((b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d
*x + c)^3)*sqrt(b))/(b*tan(d*x + c)))*tan(d*x + c) + 1170*sqrt(2)*b^(5/2)*
arctan(-(b*tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c)^3)*sqrt(b))/(b*tan(d
*x + c)))*tan(d*x + c) + 585*sqrt(2)*b^(5/2)*log((b*tan(d*x + c)^2 + b*tan
(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c)^3)*sqrt(b))/tan(d*x + c))*tan(d*x
+ c) - 585*sqrt(2)*b^(5/2)*log((b*tan(d*x + c)^2 + b*tan(d*x + c) - sqrt(2
)*sqrt(b*tan(d*x + c)^3)*sqrt(b))/tan(d*x + c))*tan(d*x + c) + 8*(45*b^2*t
an(d*x + c)^6 - 65*b^2*tan(d*x + c)^4 + 117*b^2*tan(d*x + c)^2 - 585*b^2)*
sqrt(b*tan(d*x + c)^3))/(d*tan(d*x + c))
```

## Sympy [F]

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx = \int \left(b\tan^3\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)**3)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**3)**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.60

$$\int (b\tan^3(c+dx))^{5/2}\,dx = \frac{360\,b^{\frac{5}{2}}\tan(dx+c)^{\frac{13}{2}} - 520\,b^{\frac{5}{2}}\tan(dx+c)^{\frac{9}{2}} + 936\,b^{\frac{5}{2}}\tan(dx+c)^{\frac{5}{2}} + 585\left(2\sqrt{2}\sqrt{b}\arctan\left(\right.\right.}{}$$

input
```
integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="maxima")
```

output
```
1/2340*(360*b^(5/2)*tan(d*x + c)^(13/2) - 520*b^(5/2)*tan(d*x + c)^(9/2) +
 936*b^(5/2)*tan(d*x + c)^(5/2) + 585*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2
)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan
(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(d*x
 + c)) + tan(d*x + c) + 1))*b^2 - 4680*b^(5/2)*sqrt(tan(d*x + c)))/d
```

## Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.97

$$\int (b\tan^3(c+dx))^{5/2}\,dx = \frac{1}{2340}b^2\left(\frac{1170\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d} + \frac{1170\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{}\right.}{d}\right.$$

input
```
integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="giac")
```

output
```
1/2340*b^2*(1170*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs
(b)) + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 1170*sqrt(2)*sqrt(abs(b))
*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(
abs(b)))/d + 585*sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*
tan(d*x + c))*sqrt(abs(b)) + abs(b))/d - 585*sqrt(2)*sqrt(abs(b))*log(b*ta
n(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d + 8*(45
*sqrt(b*tan(d*x + c))*b^78*d^12*tan(d*x + c)^6 - 65*sqrt(b*tan(d*x + c))*b
^78*d^12*tan(d*x + c)^4 + 117*sqrt(b*tan(d*x + c))*b^78*d^12*tan(d*x + c)^
2 - 585*sqrt(b*tan(d*x + c))*b^78*d^12)/(b^78*d^13))*sgn(tan(d*x + c))
```

## Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx=\int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^3\right)}^{5/2}\,dx$$

input
```
int((b*tan(c + d*x)^3)^(5/2),x)
```

output
```
int((b*tan(c + d*x)^3)^(5/2), x)
```

## Reduce [F]

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx=\frac{\sqrt{b}\,b^2\left(90\sqrt{\tan\left(dx+c\right)}\,\tan\left(dx+c\right)^6-130\sqrt{\tan\left(dx+c\right)}\,\tan\left(dx+c\right)^4+234\sqrt{\tan\left(dx+\right.}\right.}{585d}$$

input
```
int((b*tan(d*x+c)^3)^(5/2),x)
```

output
```
(sqrt(b)*b**2*(90*sqrt(tan(c + d*x))*tan(c + d*x)**6 - 130*sqrt(tan(c + d*
x))*tan(c + d*x)**4 + 234*sqrt(tan(c + d*x))*tan(c + d*x)**2 - 1170*sqrt(t
an(c + d*x)) + 585*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*d))/(585*d)
```

# 3.31 $\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx$


Optimal result . . . . . 397
Mathematica [A] (verified) . . . . . 398
Rubi [A] (verified) . . . . . 398
Maple [A] (verified) . . . . . 403
Fricas [A] (verification not implemented) . . . . . 404
Sympy [F] . . . . . 404
Maxima [A] (verification not implemented) . . . . . 405
Giac [A] (verification not implemented) . . . . . 405
Mupad [F(-1)] . . . . . 406
Reduce [F] . . . . . 406


## Optimal result

Integrand size = 14, antiderivative size = 223

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx = -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} - \frac{b\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{b\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{b\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d}$$

output
```
-2/3*b*(b*tan(d*x+c)^3)^(1/2)/d+1/2*b*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*
(b*tan(d*x+c)^3)^(1/2)*2^(1/2)/d/tan(d*x+c)^(3/2)+1/2*b*arctan(1+2^(1/2)*t
an(d*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)*2^(1/2)/d/tan(d*x+c)^(3/2)-1/2*b*a
rctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*(b*tan(d*x+c)^3)^(1/2)*2^(
1/2)/d/tan(d*x+c)^(3/2)+2/7*b*tan(d*x+c)^2*(b*tan(d*x+c)^3)^(1/2)/d
```

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.52

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx = \frac{b\sqrt{b\tan^3(c+dx)}\left(21\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan(c+dx)} - 21\text{arctanh}\left(\sqrt[4]{-\tan^2\right.\right.}{21d\tan^{\frac{7}{4}}(c+dx)}$$

input

```
Integrate[(b*Tan[c + d*x]^3)^(3/2),x]
```

output

```
(b*Sqrt[b*Tan[c + d*x]^3]*(21*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*
x])^(1/4) - 21*ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(1/4) + 2*
Tan[c + d*x]^(7/4)*(-7 + 3*Tan[c + d*x]^2)))/(21*d*Tan[c + d*x]^(7/4))
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.89, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.214$, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^3\right)^{3/2}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{b\sqrt{b\tan^3(c+dx)}\int \tan^{\frac{9}{2}}(c+dx)dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan^3(c+dx)}\int \tan(c+dx)^{9/2}dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3954

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\int \tan^{\frac{5}{2}}(c+dx)dx\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\int \tan(c+dx)^{5/2}dx\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3954

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\int \sqrt{\tan(c+dx)}dx+\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\int \sqrt{\tan(c+dx)}dx+\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3957

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{\int \frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 266

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 826

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}+\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1476

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}\int\frac{1-\tan(c+d}{\tan^2(c+dx)}\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 1082

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 217

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}+\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 1479

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan}\right.}{\sqrt{2}}\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 25

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c\right.}{\sqrt{2}}\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{t\right.}}{\sqrt{}}\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1103

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\right.}{2\sqrt{2}}\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(b*Tan[c + d*x]^3)^(3/2),x]
```

output
```
(b*Sqrt[b*Tan[c + d*x]^3]*((2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/S
qrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqr
t[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]))/2))/d - (2*Tan[c + d*x]^(3/2))
/(3*d) + (2*Tan[c + d*x]^(7/2))/(7*d)))/Tan[c + d*x]^(3/2)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| derivativedivides | $\frac{\left(b\tan(dx+c)^3\right)^{\frac{3}{2}}\left(24(b\tan(dx+c))^{\frac{7}{2}}\left(b^2\right)^{\frac{1}{4}}+21b^4\sqrt{2}\,\ln\left(-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+42b^4\sqrt{2}\,\text{arcta}}{84d\tan(dx+c)^3(b\tan(dx-}$ |
| default | $\frac{\left(b\tan(dx+c)^3\right)^{\frac{3}{2}}\left(24(b\tan(dx+c))^{\frac{7}{2}}\left(b^2\right)^{\frac{1}{4}}+21b^4\sqrt{2}\,\ln\left(-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+42b^4\sqrt{2}\,\text{arcta}}{84d\tan(dx+c)^3(b\tan(dx-}$ |

input
```
int((b*tan(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/84/d*(b*tan(d*x+c)^3)^(3/2)*(24*(b*tan(d*x+c))^(7/2)*(b^2)^(1/4)+21*b^4*
2^(1/2)*ln(-((b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(
1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))
+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1
/4))+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2
)^(1/4))-56*b^2*(b*tan(d*x+c))^(3/2)*(b^2)^(1/4))/tan(d*x+c)^3/(b*tan(d*x+
c))^(3/2)/b^2/(b^2)^(1/4)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.11

$$\int (b\tan^3(c+dx))^{3/2}\,dx = \frac{42\sqrt{2}b^{\frac{3}{2}}\arctan\left(\frac{b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)^3}\sqrt{b}}{b\tan(dx+c)}\right)+42\sqrt{2}b^{\frac{3}{2}}\arctan\left(-\frac{b\tan(dx+c)-\sqrt{2}\sqrt{b\tan(dx+c)^3}}{b\tan(dx+c)}\right.}{}$$

input
```
integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

output
```
1/84*(42*sqrt(2)*b^(3/2)*arctan((b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x +
 c)^3)*sqrt(b))/(b*tan(d*x + c))) + 42*sqrt(2)*b^(3/2)*arctan(-(b*tan(d*x
+ c) - sqrt(2)*sqrt(b*tan(d*x + c)^3)*sqrt(b))/(b*tan(d*x + c))) - 21*sqrt
(2)*b^(3/2)*log((b*tan(d*x + c)^2 + b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*
x + c)^3)*sqrt(b))/tan(d*x + c)) + 21*sqrt(2)*b^(3/2)*log((b*tan(d*x + c)^
2 + b*tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c)^3)*sqrt(b))/tan(d*x + c))
 + 8*sqrt(b*tan(d*x + c)^3)*(3*b*tan(d*x + c)^2 - 7*b))/d
```

## Sympy [F]

$$\int (b\tan^3(c+dx))^{3/2}\,dx = \int \left(b\tan^3(c+dx)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)**3)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**3)**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.63

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx = \frac{24\,b^{\frac{3}{2}}\tan\left(dx+c\right)^{\frac{7}{2}} - 56\,b^{\frac{3}{2}}\tan\left(dx+c\right)^{\frac{3}{2}} + 21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)\right.}{}$$

input
```
integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

output
```
1/84*(24*b^(3/2)*tan(d*x + c)^(7/2) - 56*b^(3/2)*tan(d*x + c)^(3/2) + 21*(
2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*log(sqrt(
2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(tan(
d*x + c)) + tan(d*x + c) + 1))*b^(3/2))/d
```

## Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.13

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx = \frac{1}{84}\,b\left(\frac{42\,\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{bd} + \frac{42\,\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan\right.}\right.}{2\sqrt{|b|}}\right.}{bd}\right.$$

input
```
integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="giac")
```

output
```
1/84*b*(42*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 42*sqrt(2)*abs(b)^(3/2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(
b)))/(b*d) - 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*t
an(d*x + c))*sqrt(abs(b)) + abs(b))/(b*d) + 21*sqrt(2)*abs(b)^(3/2)*log(b*
tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b*d) +
 8*(3*sqrt(b*tan(d*x + c))*b^21*d^6*tan(d*x + c)^3 - 7*sqrt(b*tan(d*x + c)
)*b^21*d^6*tan(d*x + c))/(b^21*d^7))*sgn(tan(d*x + c))
```

## Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx=\int \left(b\tan(c+d\,x)^3\right)^{3/2}\,dx$$

input
```
int((b*tan(c + d*x)^3)^(3/2),x)
```

output
```
int((b*tan(c + d*x)^3)^(3/2), x)
```

## Reduce [F]

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx=\sqrt{b}\left(\int \sqrt{\tan\left(dx+c\right)}\,\tan\left(dx+c\right)^4dx\right)b$$

input
```
int((b*tan(d*x+c)^3)^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(tan(c + d*x))*tan(c + d*x)**4,x)*b
```

# 3.32 $\int \sqrt{b\tan^3(c+dx)}\,dx$

Optimal result . . . . . 407
Mathematica [A] (verified) . . . . . 408
Rubi [A] (verified) . . . . . 408
Maple [A] (verified) . . . . . 413
Fricas [A] (verification not implemented) . . . . . 414
Sympy [F] . . . . . 414
Maxima [A] (verification not implemented) . . . . . 415
Giac [A] (verification not implemented) . . . . . 415
Mupad [F(-1)] . . . . . 416
Reduce [F] . . . . . 416

## Optimal result

Integrand size = 14, antiderivative size = 193

$$\int \sqrt{b\tan^3(c+dx)}\,dx = \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}$$

output
```
2*cot(d*x+c)*(b*tan(d*x+c)^3)^(1/2)/d-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/
2))*(b*tan(d*x+c)^3)^(1/2)*2^(1/2)/d/tan(d*x+c)^(3/2)-1/2*arctan(1+2^(1/2)
*tan(d*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)*2^(1/2)/d/tan(d*x+c)^(3/2)-1/2*a
rctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*(b*tan(d*x+c)^3)^(1/2)*2^(
1/2)/d/tan(d*x+c)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.84

$$\int \sqrt{b\tan^3(c+dx)}\,dx$$
$$= \frac{\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c-\right.}{2\sqrt{2}}}{d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[b*Tan[c + d*x]^3],x]
```

output
```
((ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqr
t[2]) + 2*Sqrt[Tan[c + d*x]])*Sqrt[b*Tan[c + d*x]^3])/(d*Tan[c + d*x]^(3/2
))
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {3042, 4141, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan^3(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \sqrt{b\tan(c+dx)^3}dx$$
$$\Big\downarrow \; 4141$$
$$\frac{\sqrt{b\tan^3(c+dx)}\int \tan^{\frac{3}{2}}(c+dx)dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{b\tan^3(c+dx)}\int \tan(c+dx)^{3/2}dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3954$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\int \frac{1}{\sqrt{\tan(c+dx)}}dx\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\int \frac{1}{\sqrt{\tan(c+dx)}}dx\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3957$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{\int \frac{1}{\sqrt{\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 266$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\int \frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 755$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 1476$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{\cdots}{\tan(c+dx)\cdots}\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 1082$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\cdots}\right)}{\sqrt{2}}\cdots\right)\right)}{d}\cdots\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 217

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1479

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\cdots\right)}{d}\cdots\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 25

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\cdots\right)\right)}{d}\cdots\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\cdots\right)}{d}\cdots\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1103

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\cdots\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[Sqrt[b*Tan[c + d*x]^3],x]
```

output
```
(((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]]/(2*Sqrt[2]))/2))/d + (2*Sqrt[Tan[c + d*x]])/d)*Sqrt[b*Tan[c + d*x]
^3])/Tan[c + d*x]^(3/2)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| derivativedivides | $-\frac{\sqrt{b\tan(dx+c)^3}\left((b^2)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2(b^2)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}+(\cdots}{(b^2)^{\frac{1}{4}}}\right.\right.}{4d\tan(dx+c)\sqrt{b\tan(dx+c)}}$ |
| default | $-\frac{\sqrt{b\tan(dx+c)^3}\left((b^2)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2(b^2)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}+(\cdots}{(b^2)^{\frac{1}{4}}}\right.\right.}{4d\tan(dx+c)\sqrt{b\tan(dx+c)}}$ |

input
```
int((b*tan(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(b*tan(d*x+c)^3)^(1/2)*((b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)+(b^2)^
(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*
(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*(b^2)^(1/4)*2^(1/2)*arctan((2
^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*(b^2)^(1/4)*2^(1/2
)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))-8*(b*tan(
d*x+c))^(1/2))/tan(d*x+c)/(b*tan(d*x+c))^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.37

$$\int \sqrt{b\tan^3(c+dx)}\,dx =$$
$$-\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)^3}\sqrt{b}}{b\tan(dx+c)}\right)\tan(dx+c)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{b\tan(dx+c)-\sqrt{2}\sqrt{b\tan(dx+c)}}{b\tan(dx+c)}\right.}{}$$

input
```
integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*sqrt(b)*arctan((b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x +
c)^3)*sqrt(b))/(b*tan(d*x + c)))*tan(d*x + c) + 2*sqrt(2)*sqrt(b)*arctan(-
(b*tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c)^3)*sqrt(b))/(b*tan(d*x + c))
)*tan(d*x + c) + sqrt(2)*sqrt(b)*log((b*tan(d*x + c)^2 + b*tan(d*x + c) +
sqrt(2)*sqrt(b*tan(d*x + c)^3)*sqrt(b))/tan(d*x + c))*tan(d*x + c) - sqrt(
2)*sqrt(b)*log((b*tan(d*x + c)^2 + b*tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x
 + c)^3)*sqrt(b))/tan(d*x + c))*tan(d*x + c) - 8*sqrt(b*tan(d*x + c)^3))/(
d*tan(d*x + c))
```

## Sympy [F]

$$\int \sqrt{b\tan^3(c+dx)}\,dx = \int \sqrt{b\tan^3{(c+dx)}}\,dx$$

input
```
integrate((b*tan(d*x+c)**3)**(1/2),x)
```

output
```
Integral(sqrt(b*tan(c + d*x)**3), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.69

$$\int \sqrt{b\tan^3(c+dx)}\,dx = -\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)$$

input

```
integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

output

```
-1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))
)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))
)) + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) -
sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*sq
rt(b)*sqrt(tan(d*x + c)))/d
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.01

$$\int \sqrt{b\tan^3(c+dx)}\,dx = -\frac{1}{4}\left(\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\right.$$

$$+c))$$

input

```
integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*
sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 2*sqrt(2)*sqrt(abs(b))*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d +
 sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sq
rt(abs(b)) + abs(b))/d - sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) - sqrt(2)
*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d - 8*sqrt(b*tan(d*x + c))/d)
*sgn(tan(d*x + c))
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^3(c+dx)}\,dx = \int \sqrt{b\tan\left(c+d\,x\right)^3}\,dx$$

input
```
int((b*tan(c + d*x)^3)^(1/2),x)
```

output
```
int((b*tan(c + d*x)^3)^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{b\tan^3(c+dx)}\,dx = \frac{\sqrt{b}\left(2\sqrt{\tan\left(dx+c\right)}-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)d\right)}{d}$$

input
```
int((b*tan(d*x+c)^3)^(1/2),x)
```

output
```
(sqrt(b)*(2*sqrt(tan(c + d*x)) - int(sqrt(tan(c + d*x))/tan(c + d*x),x)*d)
)/d
```

# 3.33 $\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$

Optimal result . . . . . . . . . . 417
Mathematica [A] (verified) . . . . . . . . . . 418
Rubi [A] (verified) . . . . . . . . . . 418
Maple [A] (verified) . . . . . . . . . . 423
Fricas [A] (verification not implemented) . . . . . . . . . . 424
Sympy [F] . . . . . . . . . . 424
Maxima [A] (verification not implemented) . . . . . . . . . . 425
Giac [A] (verification not implemented) . . . . . . . . . . 425
Mupad [F(-1)] . . . . . . . . . . 426
Reduce [F] . . . . . . . . . . 426

## Optimal result

Integrand size = 14, antiderivative size = 192

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}}$$

output
```
-2*tan(d*x+c)/d/(b*tan(d*x+c)^3)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1
/2))*tan(d*x+c)^(3/2)*2^(1/2)/d/(b*tan(d*x+c)^3)^(1/2)-1/2*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*2^(1/2)/d/(b*tan(d*x+c)^3)^(1/2)+1/2*
arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*tan(d*x+c)^(3/2)*2^(1/2)/
d/(b*tan(d*x+c)^3)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$$
$$= \frac{\tan(c+dx)\left(-2-\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}+\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-t\right.}{d\sqrt{b\tan^3(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[c + d*x]^3],x]
```

output
```
(Tan[c + d*x]*(-2 - ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4
) + ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4)))/(d*Sqrt[b*T
an[c + d*x]^3])
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.93, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {3042, 4141, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt{b\tan(c+dx)^3}}dx$$
$$\Big\downarrow \text{ 4141}$$
$$\frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}dx}{\sqrt{b\tan^3(c+dx)}}$$

$$\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan(c+dx)^{3/2}}dx}{\sqrt{b\tan^3(c+dx)}}$$

$$\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \sqrt{\tan(c+dx)}dx-\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{\sqrt{b\tan^3(c+dx)}}$$

$$\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \sqrt{\tan(c+dx)}dx-\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{\sqrt{b\tan^3(c+dx)}}$$

$$\downarrow \text{3957}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{\int \frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{\sqrt{b\tan^3(c+dx)}}$$

$$\downarrow \text{266}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{\sqrt{b\tan^3(c+dx)}}$$

$$\downarrow \text{826}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{\sqrt{b\tan^3(c+dx)}}$$

$$\downarrow \text{1476}$$

$$\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}\right.}{d}\right.$$

$$\sqrt{b\tan^3(c+dx)}$$

$$\downarrow \text{1082}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\right.}{\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 217

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 1479

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+\right.\right.}{\sqrt{2}}\right.}{d}\right.}{\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 25

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+d\right.\right.}{\sqrt{2}}\right.}{d}\right.}{\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 27

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan\right.\right.}{\sqrt{2}}\right.}{d}\right.}{\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 1103

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right.\right.}{d}\right.}{\sqrt{b\tan^3(c+dx)}}$$

input
```
Int[1/Sqrt[b*Tan[c + d*x]^3],x]
```

output
```
(((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2]))/2))/d - 2/(d*Sqrt[Tan[c + d*x]]))*Tan[c + d*x]^(3/2))/
Sqrt[b*Tan[c + d*x]^3]
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| derivativedivides | $-\frac{\tan(dx+c)\left(\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\ln\left(-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{}\cdots}{}\right.\cdots}{4d\sqrt{b\tan(dx+c)^3}\,\left(b^2\right)^{\frac{1}{4}}}$ |
| default | $-\frac{\tan(dx+c)\left(\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\ln\left(-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{}\cdots}{}\right.\cdots}{4d\sqrt{b\tan(dx+c)^3}\,\left(b^2\right)^{\frac{1}{4}}}$ |

input
```
int(1/(b*tan(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*tan(d*x+c)*(2^(1/2)*(b*tan(d*x+c))^(1/2)*ln(-((b^2)^(1/4)*(b*tan(d*
x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b
*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*2^(1/2)*(b*tan(d*x+c))^(1/2)*ar
ctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*2^(1/2)*(b*
tan(d*x+c))^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^
(1/4))+8*(b^2)^(1/4))/(b*tan(d*x+c)^3)^(1/2)/(b^2)^(1/4)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = -\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\frac{\sqrt{2}\sqrt{b\tan(dx+c)^3}}{\sqrt{b}}+\tan(dx+c)}{\tan(dx+c)}\right)\tan(dx+c)^2+2\sqrt{2}\sqrt{b}\arctan\left(\frac{\frac{\sqrt{2}\sqrt{b\tan(dx+c)^3}}{\sqrt{b}}-\tan(dx+c)}{\tan(dx+c)}\right)\tan$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*sqrt(b)*arctan((sqrt(2)*sqrt(b*tan(d*x + c)^3)/sqrt(b) + t
an(d*x + c))/tan(d*x + c))*tan(d*x + c)^2 + 2*sqrt(2)*sqrt(b)*arctan((sqrt
(2)*sqrt(b*tan(d*x + c)^3)/sqrt(b) - tan(d*x + c))/tan(d*x + c))*tan(d*x +
 c)^2 - sqrt(2)*sqrt(b)*log((tan(d*x + c)^2 + sqrt(2)*sqrt(b*tan(d*x + c)^
3)/sqrt(b) + tan(d*x + c))/tan(d*x + c))*tan(d*x + c)^2 + sqrt(2)*sqrt(b)*
log((tan(d*x + c)^2 - sqrt(2)*sqrt(b*tan(d*x + c)^3)/sqrt(b) + tan(d*x + c
))/tan(d*x + c))*tan(d*x + c)^2 + 8*sqrt(b*tan(d*x + c)^3))/(b*d*tan(d*x +
 c)^2)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan^3{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(c + d*x)**3), x)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}\log}{\sqrt{b}}}{4d}$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*lo
g(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sq
rt(tan(d*x + c)) + tan(d*x + c) + 1))/sqrt(b) + 8/(sqrt(b)*sqrt(tan(d*x +
c))))/d
```

## Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = -\frac{1}{4}b^2\left(\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4d\,\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}+\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4d\,\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}-\right.$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="giac")
```

output
```
-1/4*b^2*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b))
+ 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^4*d*sgn(tan(d*x + c))) + 2*sqrt
(2)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(
d*x + c)))/sqrt(abs(b)))/(b^4*d*sgn(tan(d*x + c))) - sqrt(2)*abs(b)^(3/2)*
log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(
b^4*d*sgn(tan(d*x + c))) + sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) - sqrt(
2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^4*d*sgn(tan(d*x + c))) +
 8/(sqrt(b*tan(d*x + c))*b^2*d*sgn(tan(d*x + c))))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,{\tan\left(c+d\,x\right)}^3}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^3)^(1/2),x)
```

output
```
int(1/(b*tan(c + d*x)^3)^(1/2), x)
```

## Reduce [F]

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)}{b}$$

input
```
int(1/(b*tan(d*x+c)^3)^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x))/b
```

# 3.34 $\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 427
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 428
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 428
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 433
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 434
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 434
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 435
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 435
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 436
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 436

## Optimal result

Integrand size = 14, antiderivative size = 232

$$\begin{aligned}\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx &= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}}-\frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}}\\ &-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}\\ &+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}\\ &+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}\end{aligned}$$

output
```
2/3/b/d/(b*tan(d*x+c)^3)^(1/2)-2/7*cot(d*x+c)^2/b/d/(b*tan(d*x+c)^3)^(1/2)
+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*2^(1/2)/b/d/(b*t
an(d*x+c)^3)^(1/2)+1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)
*2^(1/2)/b/d/(b*tan(d*x+c)^3)^(1/2)+1/2*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(
1+tan(d*x+c)))*tan(d*x+c)^(3/2)*2^(1/2)/b/d/(b*tan(d*x+c)^3)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx = \frac{14-6\cot^2(c+dx)-21\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\left(-\tan^2(c+dx)\right)^{3/4}-21}{21bd\sqrt{b\tan^3(c+dx)}}$$

input
```
Integrate[(b*Tan[c + d*x]^3)^(-3/2),x]
```

output
```
(14 - 6*Cot[c + d*x]^2 - 21*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]
^2)^(3/4) - 21*ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(3/4))/(
21*b*d*Sqrt[b*Tan[c + d*x]^3])
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.86, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.214, Rules used = {3042, 4141, 3042, 3955, 3042, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\tan(c+dx)^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{9}{2}}(c+dx)}dx}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan(c+dx)^{9/2}}dx}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \frac{1}{\tan(c+dx)^{5/2}}dx-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\int \frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\int \frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{\int \frac{1}{\sqrt{\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}+\frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\int \frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{755}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}+\frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\right.\right.}{d}}{b\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 1082

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)\right)}{d}+\right.}{b\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 217

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}+\frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{}{7d\,\mathrm{t}}\right.}{b\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 1479

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+\right.}{\sqrt{2}}\right.\right.}{d}\right.}{b\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 25

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+\right.}{\sqrt{2}}\right.\right.}{d}\right.}{b\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 27

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+d}{\sqrt{2}}\right.\right.}{d}\right.}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(\right.}{2\sqrt{2}}\right.\right.}{d}\right.}{b\sqrt{b\tan^3(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^3)^(-3/2),x]
```

output
```
(((2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt
[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2]))/2))/d - 2/(7*d*Tan[c + d*x]^(7/2)) + 2/(3*d*Tan[c + d*
x]^(3/2)))*Tan[c + d*x]^(3/2))/(b*Sqrt[b*Tan[c + d*x]^3])
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| derivativedivides | $\frac{\tan(dx+c)\left(21\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,(b\tan(dx+c))^{\frac{7}{2}}\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,(b\tan(dx+c))^{\frac{7}{2}}\right.}{84d\,b^4\left(\right.}$ |
| default | $\frac{\tan(dx+c)\left(21\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,(b\tan(dx+c))^{\frac{7}{2}}\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,(b\tan(dx+c))^{\frac{7}{2}}\right.}{84d\,b^4\left(\right.}$ |

input
```
int(1/(b*tan(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/84/d*tan(d*x+c)/b^4*(21*(b^2)^(1/4)*2^(1/2)*(b*tan(d*x+c))^(7/2)*ln((b*t
an(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x
+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+42*(b^2)^(1/4)*
2^(1/2)*(b*tan(d*x+c))^(7/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1
/4))/(b^2)^(1/4))+42*(b^2)^(1/4)*2^(1/2)*(b*tan(d*x+c))^(7/2)*arctan((2^(1
/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+56*b^4*tan(d*x+c)^2-24*
b^4)/(b*tan(d*x+c)^3)^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx = \frac{42\,\sqrt{2}\sqrt{b}\arctan\left(\frac{\frac{\sqrt{2}\sqrt{b\tan(dx+c)^3}}{\sqrt{b}}+\tan(dx+c)}{\tan(dx+c)}\right)\tan\left(dx+c\right)^5+42\,\sqrt{2}\sqrt{b}\arctan\left(\cdots\right.}{\cdots}$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

output
```
1/84*(42*sqrt(2)*sqrt(b)*arctan((sqrt(2)*sqrt(b*tan(d*x + c)^3)/sqrt(b) +
tan(d*x + c))/tan(d*x + c))*tan(d*x + c)^5 + 42*sqrt(2)*sqrt(b)*arctan((sq
rt(2)*sqrt(b*tan(d*x + c)^3)/sqrt(b) - tan(d*x + c))/tan(d*x + c))*tan(d*x
 + c)^5 + 21*sqrt(2)*sqrt(b)*log((tan(d*x + c)^2 + sqrt(2)*sqrt(b*tan(d*x
+ c)^3)/sqrt(b) + tan(d*x + c))/tan(d*x + c))*tan(d*x + c)^5 - 21*sqrt(2)*
sqrt(b)*log((tan(d*x + c)^2 - sqrt(2)*sqrt(b*tan(d*x + c)^3)/sqrt(b) + tan
(d*x + c))/tan(d*x + c))*tan(d*x + c)^5 + 8*sqrt(b*tan(d*x + c)^3)*(7*tan(
d*x + c)^2 - 3))/(b^2*d*tan(d*x + c)^5)
```

## Sympy [F]

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan^3\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**3)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**3)**(-3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx = \frac{\frac{21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{t\right.}}{b^{\frac{3}{2}}}}{}$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

output
```
1/84*(21*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) +
 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)
*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)
*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/b^(3/2) + 8*(21*sqrt(tan(d*x + c)
) + 7/tan(d*x + c)^(3/2) - 3/tan(d*x + c)^(7/2))/b^(3/2) - 168*sqrt(tan(d*
x + c))/b^(3/2))/d
```

## Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx = \frac{1}{84}b^4\left(\frac{42\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^6 d\,\mathrm{sgn}\left(\tan\left(dx+c\right)\right)} + \frac{42\sqrt{2}\sqrt{|b|}\arctan\left(-\right.}{b^6 d\,\mathrm{sgn}\left(\mathrm{ta}\right.}\right.$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="giac")
```

output
```
1/84*b^4*(42*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b))
 + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^6*d*sgn(tan(d*x + c))) + 42*sq
rt(2)*sqrt(abs(b))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*ta
n(d*x + c)))/sqrt(abs(b)))/(b^6*d*sgn(tan(d*x + c))) + 21*sqrt(2)*sqrt(abs
(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(
b))/(b^6*d*sgn(tan(d*x + c))) - 21*sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c)
 - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^6*d*sgn(tan(d*x
+ c))) + 8*(7*b^2*tan(d*x + c)^2 - 3*b^2)/(sqrt(b*tan(d*x + c))*b^7*d*sgn(
tan(d*x + c))*tan(d*x + c)^3))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\tan\left(c+d\,x\right)^3\right)^{3/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^3)^(3/2),x)
```

output
```
int(1/(b*tan(c + d*x)^3)^(3/2), x)
```

## Reduce [F]

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx=\frac{\sqrt{b}\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^5}dx\right)}{b^2}$$

input
```
int(1/(b*tan(d*x+c)^3)^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(tan(c + d*x))/tan(c + d*x)**5,x))/b**2
```

# 3.35 $\int \frac{1}{(b\tan^3(c+dx))^{5/2}}\,dx$

Optimal result . . . . . 437
Mathematica [A] (warning: unable to verify) . . . . . 438
Rubi [A] (verified) . . . . . 438
Maple [A] (verified) . . . . . 444
Fricas [A] (verification not implemented) . . . . . 445
Sympy [F] . . . . . 445
Maxima [A] (verification not implemented) . . . . . 446
Giac [A] (verification not implemented) . . . . . 446
Mupad [F(-1)] . . . . . 447
Reduce [F] . . . . . 447

## Optimal result

Integrand size = 14, antiderivative size = 299

$$\begin{aligned}\int \frac{1}{(b\tan^3(c+dx))^{5/2}}\,dx &= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}}\\&+\frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}}-\frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}}\\&+\frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}\\&+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}\\&-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}\end{aligned}$$

output
```
-2/5*cot(d*x+c)/b^2/d/(b*tan(d*x+c)^3)^(1/2)+2/9*cot(d*x+c)^3/b^2/d/(b*tan
(d*x+c)^3)^(1/2)-2/13*cot(d*x+c)^5/b^2/d/(b*tan(d*x+c)^3)^(1/2)+2*tan(d*x+
c)/b^2/d/(b*tan(d*x+c)^3)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*ta
n(d*x+c)^(3/2)*2^(1/2)/b^2/d/(b*tan(d*x+c)^3)^(1/2)+1/2*arctan(1+2^(1/2)*t
an(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*2^(1/2)/b^2/d/(b*tan(d*x+c)^3)^(1/2)-1/2
*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*tan(d*x+c)^(3/2)*2^(1/2)
/b^2/d/(b*tan(d*x+c)^3)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.46

$$\int \frac{1}{(b\tan^3(c+dx))^{5/2}}\,dx = \frac{-234\cot(c+dx)+130\cot^3(c+dx)-90\cot^5(c+dx)+585\text{arctanh}\left(\sqrt[4]{-\tan\right.}$$

input
```
Integrate[(b*Tan[c + d*x]^3)^(-5/2),x]
```

output
```
(-234*Cot[c + d*x] + 130*Cot[c + d*x]^3 - 90*Cot[c + d*x]^5 + 585*ArcTanh[
(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(5/4)*Tan[c + d*x]^(1/4) + 1170*T
an[c + d*x] + 585*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(1/4)*Ta
n[c + d*x]^(5/4))/(585*b^2*d*Sqrt[b*Tan[c + d*x]^3])
```

## Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.78, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 4141, 3042, 3955, 3042, 3955, 3042, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan^3(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(b\tan(c+dx)^3)^{5/2}}dx$$

$$\Big\downarrow \text{ 4141}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{15}{2}}(c+dx)}dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan(c+dx)^{15/2}}dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \frac{1}{\tan^{\frac{11}{2}}(c+dx)}dx-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \frac{1}{\tan(c+dx)^{11/2}}dx-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)}dx+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\int \frac{1}{\tan(c+dx)^{7/2}}dx+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \frac{1}{\tan(c+dx)^{3/2}}dx-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\int \sqrt{\tan(c+dx)}dx-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}+\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\int \sqrt{\tan(c+dx)}dx-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}+\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 3957

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{\int \frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}+\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 266

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}+\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 826

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(\ldots}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 1476

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\ldots\right.}{d}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 1082

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\ldots\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 217

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{}{9d\tan}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 1479

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}\right.}{\sqrt{2}}\right.\right.}{d}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 25

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}\right.}{\sqrt{2}}\right.\right.}{d}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 27

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+\right.}{\sqrt{2}}\right.\right.}{d}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 1103

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(\right.}{2\sqrt{2}}\right.\right.}{d}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^3)^(-5/2),x]
```

output
```
(((2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt
[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] +
 Tan[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]]/(2*Sqrt[2]))/2))/d - 2/(13*d*Tan[c + d*x]^(13/2)) + 2/(9*d*Tan[c + d
*x]^(9/2)) - 2/(5*d*Tan[c + d*x]^(5/2)) + 2/(d*Sqrt[Tan[c + d*x]]))*Tan[c
+ d*x]^(3/2))/(b^2*Sqrt[b*Tan[c + d*x]^3])
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| derivativedivides | $\frac{\tan(dx+c)\left(585\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\,\text{arcta}}{}$ |
| default | $\frac{\tan(dx+c)\left(585\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\,\text{arcta}}{}$ |

input
```
int(1/(b*tan(d*x+c)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2340/d*tan(d*x+c)/b^6*(585*2^(1/2)*(b*tan(d*x+c))^(13/2)*ln(-((b^2)^(1/4
)*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2))/(b*tan(d*x+c)+(b^
2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*2^(1/2)*(b*tan(d*
x+c))^(13/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4)
)+1170*2^(1/2)*(b*tan(d*x+c))^(13/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-
(b^2)^(1/4))/(b^2)^(1/4))+4680*b^6*tan(d*x+c)^6*(b^2)^(1/4)-936*b^6*tan(d*
x+c)^4*(b^2)^(1/4)+520*b^6*tan(d*x+c)^2*(b^2)^(1/4)-360*b^6*(b^2)^(1/4))/(
b*tan(d*x+c)^3)^(5/2)/(b^2)^(1/4)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \frac{1170\,\sqrt{2}\sqrt{b}\arctan\left(\frac{\frac{\sqrt{2}\sqrt{b\tan(dx+c)^3}}{\sqrt{b}}+\tan(dx+c)}{\tan(dx+c)}\right)\tan\left(dx+c\right)^8+1170\,\sqrt{2}\sqrt{b}\,\mathrm{arctan}$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/2340*(1170*sqrt(2)*sqrt(b)*arctan((sqrt(2)*sqrt(b*tan(d*x + c)^3)/sqrt(b
) + tan(d*x + c))/tan(d*x + c))*tan(d*x + c)^8 + 1170*sqrt(2)*sqrt(b)*arct
an((sqrt(2)*sqrt(b*tan(d*x + c)^3)/sqrt(b) - tan(d*x + c))/tan(d*x + c))*t
an(d*x + c)^8 - 585*sqrt(2)*sqrt(b)*log((tan(d*x + c)^2 + sqrt(2)*sqrt(b*t
an(d*x + c)^3)/sqrt(b) + tan(d*x + c))/tan(d*x + c))*tan(d*x + c)^8 + 585*
sqrt(2)*sqrt(b)*log((tan(d*x + c)^2 - sqrt(2)*sqrt(b*tan(d*x + c)^3)/sqrt(
b) + tan(d*x + c))/tan(d*x + c))*tan(d*x + c)^8 + 8*(585*tan(d*x + c)^6 -
117*tan(d*x + c)^4 + 65*tan(d*x + c)^2 - 45)*sqrt(b*tan(d*x + c)^3))/(b^3*
d*tan(d*x + c)^8)
```

## Sympy [F]

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan^3\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**3)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**3)**(-5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \frac{585\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\right.}\right.}{b^{\frac{5}{2}}}$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="maxima")
```

output
```
1/2340*(585*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt
(2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt
(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/b^(5/2) + 8*(585*sqrt(b)/sqrt(
tan(d*x + c)) - 117*sqrt(b)/tan(d*x + c)^(5/2) + 65*sqrt(b)/tan(d*x + c)^(
9/2) - 45*sqrt(b)/tan(d*x + c)^(13/2))/b^3)/d
```

## Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \frac{1}{2340}b^6\left(\frac{1170\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^{10}d\operatorname{sgn}\left(\tan\left(dx+c\right)\right)}+\frac{1170\sqrt{2}|b|^{\frac{3}{2}}\arctan}{b^{10}d\operatorname{sg}}\right.$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="giac")
```

output
```
1/2340*b^6*(1170*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs
(b)) + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^10*d*sgn(tan(d*x + c))) +
1170*sqrt(2)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sq
rt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^10*d*sgn(tan(d*x + c))) - 585*sqrt(2)
*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b
)) + abs(b))/(b^10*d*sgn(tan(d*x + c))) + 585*sqrt(2)*abs(b)^(3/2)*log(b*t
an(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^10*d*
sgn(tan(d*x + c))) + 8*(585*b^6*tan(d*x + c)^6 - 117*b^6*tan(d*x + c)^4 +
65*b^6*tan(d*x + c)^2 - 45*b^6)/(sqrt(b*tan(d*x + c))*b^14*d*sgn(tan(d*x +
 c))*tan(d*x + c)^6))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\tan\left(c+d\,x\right)^3\right)^{5/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^3)^(5/2),x)
```

output
```
int(1/(b*tan(c + d*x)^3)^(5/2), x)
```

## Reduce [F]

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx=\frac{\sqrt{b}\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^8}dx\right)}{b^3}$$

input
```
int(1/(b*tan(d*x+c)^3)^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(tan(c + d*x))/tan(c + d*x)**8,x))/b**3
```

# 3.36 $\int \left(b \tan^4(c+dx)\right)^{5/2} dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 448
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 449
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 449
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 452
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . 452
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 453
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . 453
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . 453
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 454
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . 454

## Optimal result

Integrand size = 14, antiderivative size = 182

$$\int \left(b \tan^4(c+dx)\right)^{5/2} dx = \frac{b^2 \cot(c+dx)\sqrt{b \tan^4(c+dx)}}{d} - b^2 x \cot^2(c+dx)\sqrt{b \tan^4(c+dx)} - \frac{b^2 \tan(c+dx)\sqrt{b \tan^4(c+dx)}}{3d} + \frac{b^2 \tan^3(c+dx)\sqrt{b \tan^4(c+dx)}}{5d} - \frac{b^2 \tan^5(c+dx)\sqrt{b \tan^4(c+dx)}}{7d} + \frac{b^2 \tan^7(c+dx)\sqrt{b \tan^4(c+dx)}}{9d}$$

output
```
b^2*cot(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d-b^2*x*cot(d*x+c)^2*(tan(d*x+c)^4*b
)^(1/2)-1/3*b^2*tan(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d+1/5*b^2*tan(d*x+c)^3*(
tan(d*x+c)^4*b)^(1/2)/d-1/7*b^2*tan(d*x+c)^5*(tan(d*x+c)^4*b)^(1/2)/d+1/9*
b^2*tan(d*x+c)^7*(tan(d*x+c)^4*b)^(1/2)/d
```

## Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.47

$$\int \left(b\tan^4(c+dx)\right)^{5/2} dx = \frac{\cot(c+dx)\left(35-45\cot^2(c+dx)+63\cot^4(c+dx)-105\cot^6(c+dx)+315\cot^8(c+dx)-\right.}{315d}$$

input
```
Integrate[(b*Tan[c + d*x]^4)^(5/2),x]
```

output
```
(Cot[c + d*x]*(35 - 45*Cot[c + d*x]^2 + 63*Cot[c + d*x]^4 - 105*Cot[c + d*
x]^6 + 315*Cot[c + d*x]^8 - 315*ArcTan[Tan[c + d*x]]*Cot[c + d*x]^9)*(b*Ta
n[c + d*x]^4)^(5/2))/(315*d)
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.55, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^4(c+dx)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^4\right)^{5/2} dx$$

$$\Big\downarrow \text{4141}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\int \tan^{10}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\int \tan(c+dx)^{10}dx$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\frac{\tan^9(c+dx)}{9d}-\int \tan^8(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\frac{\tan^9(c+dx)}{9d}-\int \tan(c+dx)^8dx\right)$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\int \tan^6(c+dx)dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\int \tan(c+dx)^6dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(-\int \tan^4(c+dx)dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}+\frac{\tan^5(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(-\int \tan(c+dx)^4dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}+\frac{\tan^5(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\int \tan^2(c+dx)dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\int \tan(c+dx)^2dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(-\int 1dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}\right)$$

$$\Big\downarrow \text{24}$$

$$b^2\left(\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}-x\right)\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

input
```
Int[(b*Tan[c + d*x]^4)^(5/2),x]
```

output
```
b^2*Cot[c + d*x]^2*Sqrt[b*Tan[c + d*x]^4]*(-x + Tan[c + d*x]/d - Tan[c + d
*x]^3/(3*d) + Tan[c + d*x]^5/(5*d) - Tan[c + d*x]^7/(7*d) + Tan[c + d*x]^9
/(9*d))
```

## Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.46

| method | result |
|---|---|
| derivativedivides | $-\frac{\left(b\tan(dx+c)^4\right)^{\frac{5}{2}}\left(-35\tan(dx+c)^9+45\tan(dx+c)^7-63\tan(dx+c)^5+105\tan(dx+c)^3+315\arctan(\tan(dx+c))-31\right.}{315d\tan(dx+c)^{10}}$ |
| default | $-\frac{\left(b\tan(dx+c)^4\right)^{\frac{5}{2}}\left(-35\tan(dx+c)^9+45\tan(dx+c)^7-63\tan(dx+c)^5+105\tan(dx+c)^3+315\arctan(\tan(dx+c))-31\right.}{315d\tan(dx+c)^{10}}$ |
| risch | $\frac{b^2\left(e^{2i(dx+c)}+1\right)^2\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\,x}{\left(e^{2i(dx+c)}-1\right)^2}-\frac{2ib^2\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\left(1575\,e^{16i(dx+c)}+6300\,e^{14i(dx+c)}+21000\,e^{12i(dx+\right.}{315(e^{2}}$ |

input
```
int((b*tan(d*x+c)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/315/d*(b*tan(d*x+c)^4)^(5/2)*(-35*tan(d*x+c)^9+45*tan(d*x+c)^7-63*tan(d
*x+c)^5+105*tan(d*x+c)^3+315*arctan(tan(d*x+c))-315*tan(d*x+c))/tan(d*x+c)
^10
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.53

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx=\frac{\left(35\,b^2\tan\left(dx+c\right)^9-45\,b^2\tan\left(dx+c\right)^7+63\,b^2\tan\left(dx+c\right)^5-105\,b^2\tan\left(dx+c\right)^3-315\right.}{315\,d\tan\left(dx+c\right)^2}$$

input
```
integrate((b*tan(d*x+c)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/315*(35*b^2*tan(d*x + c)^9 - 45*b^2*tan(d*x + c)^7 + 63*b^2*tan(d*x + c)
^5 - 105*b^2*tan(d*x + c)^3 - 315*b^2*d*x + 315*b^2*tan(d*x + c))*sqrt(b*t
an(d*x + c)^4)/(d*tan(d*x + c)^2)
```

## Sympy [F]

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx = \int \left(b\tan^4\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)**4)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**4)**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.43

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx = \frac{35\,b^{\frac{5}{2}}\tan\left(dx+c\right)^9 - 45\,b^{\frac{5}{2}}\tan\left(dx+c\right)^7 + 63\,b^{\frac{5}{2}}\tan\left(dx+c\right)^5 - 105\,b^{\frac{5}{2}}\tan\left(dx+c\right)^3 - 315}{315\,d}$$

input
```
integrate((b*tan(d*x+c)^4)^(5/2),x, algorithm="maxima")
```

output
```
1/315*(35*b^(5/2)*tan(d*x + c)^9 - 45*b^(5/2)*tan(d*x + c)^7 + 63*b^(5/2)*
tan(d*x + c)^5 - 105*b^(5/2)*tan(d*x + c)^3 - 315*(d*x + c)*b^(5/2) + 315*
b^(5/2)*tan(d*x + c))/d
```

## Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.47

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx = -\frac{1}{315}\,b^{\frac{5}{2}}\left(\frac{315\,(dx+c)}{d} - \frac{35\,d^8\tan\left(dx+c\right)^9 - 45\,d^8\tan\left(dx+c\right)^7 + 63\,d^8\tan\left(dx+c\right)^5 - 105\,d^8\tan\left(dx+\right.}{d^9}\right.$$

input
```
integrate((b*tan(d*x+c)^4)^(5/2),x, algorithm="giac")
```

output
```
-1/315*b^(5/2)*(315*(d*x + c)/d - (35*d^8*tan(d*x + c)^9 - 45*d^8*tan(d*x
+ c)^7 + 63*d^8*tan(d*x + c)^5 - 105*d^8*tan(d*x + c)^3 + 315*d^8*tan(d*x
+ c))/d^9)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx=\int \left(b\tan(c+d\,x)^4\right)^{5/2}\,dx$$

input
```
int((b*tan(c + d*x)^4)^(5/2),x)
```

output
```
int((b*tan(c + d*x)^4)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.35

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx=\frac{\sqrt{b}\,b^2\left(35\tan(dx+c)^9-45\tan(dx+c)^7+63\tan(dx+c)^5-105\tan(dx+c)^3+315\tan(\right.}{315d}$$

input
```
int((b*tan(d*x+c)^4)^(5/2),x)
```

output
```
(sqrt(b)*b**2*(35*tan(c + d*x)**9 - 45*tan(c + d*x)**7 + 63*tan(c + d*x)**
5 - 105*tan(c + d*x)**3 + 315*tan(c + d*x) - 315*d*x))/(315*d)
```

# 3.37 $\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 455
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 455
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 456
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 458
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 458
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 459
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 459
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 460
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 460
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 460

## Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx = \frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - bx\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} - \frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d}$$

output
```
b*cot(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d-b*x*cot(d*x+c)^2*(tan(d*x+c)^4*b)^(1
/2)-1/3*b*tan(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d+1/5*b*tan(d*x+c)^3*(tan(d*x+
c)^4*b)^(1/2)/d
```

## Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.60

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx = \frac{\cot(c+dx)\left(3-5\cot^2(c+dx)+15\cot^4(c+dx)-15\arctan(\tan(c+dx))\cot^5(c+dx)\right)(b}{15d}$$

input
```
Integrate[(b*Tan[c + d*x]^4)^(3/2),x]
```

output
```
(Cot[c + d*x]*(3 - 5*Cot[c + d*x]^2 + 15*Cot[c + d*x]^4 - 15*ArcTan[Tan[c
+ d*x]]*Cot[c + d*x]^5)*(b*Tan[c + d*x]^4)^(3/2))/(15*d)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^4(c+dx)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^4\right)^{3/2} dx$$

$$\Big\downarrow \text{4141}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\int \tan^6(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\int \tan(c+dx)^6dx$$

$$\Big\downarrow \text{3954}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\frac{\tan^5(c+dx)}{5d}-\int \tan^4(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\frac{\tan^5(c+dx)}{5d}-\int \tan(c+dx)^4dx\right)$$

$$\Big\downarrow \text{3954}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\int \tan^2(c+dx)dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\int \tan(c+dx)^2dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{ 3954}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(-\int 1dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}\right)$$

$$\Big\downarrow \text{ 24}$$

$$b\left(\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}-x\right)\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

input
```
Int[(b*Tan[c + d*x]^4)^(3/2),x]
```

output
```
b*Cot[c + d*x]^2*Sqrt[b*Tan[c + d*x]^4]*(-x + Tan[c + d*x]/d - Tan[c + d*x
]^3/(3*d) + Tan[c + d*x]^5/(5*d))
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

| method | result |
|---|---|
| derivativedivides | $-\frac{\left(b\tan(dx+c)^4\right)^{\frac{3}{2}}\left(-3\tan(dx+c)^5+5\tan(dx+c)^3+15\arctan(\tan(dx+c))-15\tan(dx+c)\right)}{15d\tan(dx+c)^6}$ |
| default | $-\frac{\left(b\tan(dx+c)^4\right)^{\frac{3}{2}}\left(-3\tan(dx+c)^5+5\tan(dx+c)^3+15\arctan(\tan(dx+c))-15\tan(dx+c)\right)}{15d\tan(dx+c)^6}$ |
| risch | $\frac{b\left(e^{2i(dx+c)}+1\right)^2\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\,x}{\left(e^{2i(dx+c)}-1\right)^2}-\frac{2ib\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\left(45\,e^{8i(dx+c)}+90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}+70\,e^{2i(dx}\right.}{15\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)^3d}$ |

input
```
int((b*tan(d*x+c)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d*(b*tan(d*x+c)^4)^(3/2)*(-3*tan(d*x+c)^5+5*tan(d*x+c)^3+15*arctan(t
an(d*x+c))-15*tan(d*x+c))/tan(d*x+c)^6
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx=\frac{\left(3\,b\tan\left(dx+c\right)^5-5\,b\tan\left(dx+c\right)^3-15\,bdx+15\,b\tan\left(dx+c\right)\right)\sqrt{b\tan\left(dx+c\right)^4}}{15\,d\tan\left(dx+c\right)^2}$$

input
```
integrate((b*tan(d*x+c)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(3*b*tan(d*x + c)^5 - 5*b*tan(d*x + c)^3 - 15*b*d*x + 15*b*tan(d*x +
c))*sqrt(b*tan(d*x + c)^4)/(d*tan(d*x + c)^2)
```

## Sympy [F]

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx=\int \left(b\tan^4{(c+dx)}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)**4)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**4)**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.48

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx=\frac{3\,b^{\frac{3}{2}}\tan{(dx+c)}^5-5\,b^{\frac{3}{2}}\tan{(dx+c)}^3-15\,(dx+c)b^{\frac{3}{2}}+15\,b^{\frac{3}{2}}\tan{(dx+c)}}{15\,d}$$

input
```
integrate((b*tan(d*x+c)^4)^(3/2),x, algorithm="maxima")
```

output
```
1/15*(3*b^(3/2)*tan(d*x + c)^5 - 5*b^(3/2)*tan(d*x + c)^3 - 15*(d*x + c)*b
^(3/2) + 15*b^(3/2)*tan(d*x + c))/d
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.54

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx = -\frac{1}{15}\,b^{\frac{3}{2}}\left(\frac{15\,(dx+c)}{d}-\frac{3\,d^4\tan\left(dx+c\right)^5-5\,d^4\tan\left(dx+c\right)^3+15\,d^4\tan\left(dx+c\right)}{d^5}\right)$$

input
```
integrate((b*tan(d*x+c)^4)^(3/2),x, algorithm="giac")
```

output
```
-1/15*b^(3/2)*(15*(d*x + c)/d - (3*d^4*tan(d*x + c)^5 - 5*d^4*tan(d*x + c)
^3 + 15*d^4*tan(d*x + c))/d^5)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx = \int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^4\right)}^{3/2} \,d x$$

input
```
int((b*tan(c + d*x)^4)^(3/2),x)
```

output
```
int((b*tan(c + d*x)^4)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.37

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx = \frac{\sqrt{b}\,b\left(3\tan\left(dx+c\right)^5-5\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)-15dx\right)}{15d}$$

input
```
int((b*tan(d*x+c)^4)^(3/2),x)
```

output
```
(sqrt(b)*b*(3*tan(c + d*x)**5 - 5*tan(c + d*x)**3 + 15*tan(c + d*x) - 15*d
*x))/(15*d)
```

## 3.38 $\int \sqrt{b\tan^4(c+dx)}\,dx$

Optimal result . . . . . 462
Mathematica [A] (verified) . . . . . 462
Rubi [A] (verified) . . . . . 463
Maple [A] (verified) . . . . . 464
Fricas [A] (verification not implemented) . . . . . 465
Sympy [F] . . . . . 465
Maxima [A] (verification not implemented) . . . . . 465
Giac [A] (verification not implemented) . . . . . 466
Mupad [F(-1)] . . . . . 466
Reduce [B] (verification not implemented) . . . . . 466

### Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \sqrt{b\tan^4(c+dx)}\,dx = \frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

output
```
cot(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d-x*cot(d*x+c)^2*(tan(d*x+c)^4*b)^(1/2)
```

### Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \sqrt{b\tan^4(c+dx)}\,dx = -\frac{\cot(c+dx)(-1+\arctan(\tan(c+dx))\cot(c+dx))\sqrt{b\tan^4(c+dx)}}{d}$$

input
```
Integrate[Sqrt[b*Tan[c + d*x]^4],x]
```

output
```
-((Cot[c + d*x]*(-1 + ArcTan[Tan[c + d*x]]*Cot[c + d*x])*Sqrt[b*Tan[c + d*
x]^4])/d)
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4141, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4141}$$

$$\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\int \tan^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\int \tan(c+dx)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\frac{\tan(c+dx)}{d}-\int 1dx\right)$$

$$\Big\downarrow \text{24}$$

$$\left(\frac{\tan(c+dx)}{d}-x\right)\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

input
```
Int[Sqrt[b*Tan[c + d*x]^4],x]
```

output
```
Cot[c + d*x]^2*Sqrt[b*Tan[c + d*x]^4]*(-x + Tan[c + d*x]/d)
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\sqrt{b\tan(dx+c)^4}\,(-\tan(dx+c)+\arctan(\tan(dx+c)))}{d\tan(dx+c)^2}$ | 42 |
| default | $-\frac{\sqrt{b\tan(dx+c)^4}\,(-\tan(dx+c)+\arctan(\tan(dx+c)))}{d\tan(dx+c)^2}$ | 42 |
| risch | $\frac{\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\,\left(e^{2i(dx+c)}+1\right)^2 x}{\left(e^{2i(dx+c)}-1\right)^2}-\frac{2i\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\,\left(e^{2i(dx+c)}+1\right)}{\left(e^{2i(dx+c)}-1\right)^2 d}$ | 120 |

input
```
int((b*tan(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(b*tan(d*x+c)^4)^(1/2)*(-tan(d*x+c)+arctan(tan(d*x+c)))/tan(d*x+c)^2
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \sqrt{b\tan^4(c+dx)}\,dx = -\frac{\sqrt{b\tan{(dx+c)}^4}(dx-\tan{(dx+c)})}{d\tan{(dx+c)}^2}$$

input
```
integrate((b*tan(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(b*tan(d*x + c)^4)*(d*x - tan(d*x + c))/(d*tan(d*x + c)^2)
```

## Sympy [F]

$$\int \sqrt{b\tan^4(c+dx)}\,dx = \int \sqrt{b\tan^4{(c+dx)}}\,dx$$

input
```
integrate((b*tan(d*x+c)**4)**(1/2),x)
```

output
```
Integral(sqrt(b*tan(c + d*x)**4), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int \sqrt{b\tan^4(c+dx)}\,dx = -\frac{(dx+c)\sqrt{b}-\sqrt{b}\tan{(dx+c)}}{d}$$

input
```
integrate((b*tan(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
-((d*x + c)*sqrt(b) - sqrt(b)*tan(d*x + c))/d
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int \sqrt{b\tan^4(c+dx)}\,dx = -\sqrt{b}\left(\frac{dx+c}{d} - \frac{\tan(dx+c)}{d}\right)$$

input
```
integrate((b*tan(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(b)*((d*x + c)/d - tan(d*x + c)/d)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^4(c+dx)}\,dx = \int \sqrt{b\tan(c+d\,x)^4}\,dx$$

input
```
int((b*tan(c + d*x)^4)^(1/2),x)
```

output
```
int((b*tan(c + d*x)^4)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.34

$$\int \sqrt{b\tan^4(c+dx)}\,dx = \frac{\sqrt{b}\,(\tan(dx+c) - dx)}{d}$$

input
```
int((b*tan(d*x+c)^4)^(1/2),x)
```

output
```
(sqrt(b)*(tan(c + d*x) - d*x))/d
```

# 3.39 $\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 467
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 467
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 468
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 469
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 470
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 470
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 470
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 471
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 471
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 471

## Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)}}$$

output
```
-tan(d*x+c)/d/(tan(d*x+c)^4*b)^(1/2)-x*tan(d*x+c)^2/(tan(d*x+c)^4*b)^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[c + d*x]^4],x]
```

output
```
-((Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*Sqrt[
b*Tan[c + d*x]^4]))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4141, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\tan(c+dx)^4}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan^2(c+dx)\int \cot^2(c+dx)dx}{\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(c+dx)\int \tan\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(c+dx)\left(-\int 1dx-\frac{\cot(c+dx)}{d}\right)}{\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^2(c+dx)\left(-\frac{\cot(c+dx)}{d}-x\right)}{\sqrt{b\tan^4(c+dx)}}$$

input
```
Int[1/Sqrt[b*Tan[c + d*x]^4],x]
```

output
```
((-x - Cot[c + d*x]/d)*Tan[c + d*x]^2)/Sqrt[b*Tan[c + d*x]^4]
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\tan(dx+c)\left(\arctan(\tan(dx+c))\tan(dx+c)+1\right)}{d\sqrt{b\tan(dx+c)^{4}}}$ | 40 |
| default | $-\frac{\tan(dx+c)\left(\arctan(\tan(dx+c))\tan(dx+c)+1\right)}{d\sqrt{b\tan(dx+c)^{4}}}$ | 40 |
| risch | $\frac{\left(e^{2i(dx+c)}-1\right)^{2}x}{\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^{4}}{\left(e^{2i(dx+c)}+1\right)^{4}}}\,\left(e^{2i(dx+c)}+1\right)^{2}}+\frac{2i\left(e^{2i(dx+c)}-1\right)}{\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^{4}}{\left(e^{2i(dx+c)}+1\right)^{4}}}\,\left(e^{2i(dx+c)}+1\right)^{2}d}$ | 120 |

input
```
int(1/(b*tan(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*tan(d*x+c)*(arctan(tan(d*x+c))*tan(d*x+c)+1)/(b*tan(d*x+c)^4)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\sqrt{b\tan{(dx+c)}^4}(dx\tan{(dx+c)}+1)}{bd\tan{(dx+c)}^3}$$

input
```
integrate(1/(b*tan(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(b*tan(d*x + c)^4)*(d*x*tan(d*x + c) + 1)/(b*d*tan(d*x + c)^3)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan^4{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(c + d*x)**4), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\frac{dx+c}{\sqrt{b}}+\frac{1}{\sqrt{b}\tan(dx+c)}}{d}$$

input
```
integrate(1/(b*tan(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
-((d*x + c)/sqrt(b) + 1/(sqrt(b)*tan(d*x + c)))/d
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\sqrt{b}\left(\frac{dx+c}{bd}+\frac{1}{bd\tan{(dx+c)}}\right)$$

input
```
integrate(1/(b*tan(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(b)*((d*x + c)/(b*d) + 1/(b*d*tan(d*x + c)))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan{(c+d\,x)}^4}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^4)^(1/2),x)
```

output
```
int(1/(b*tan(c + d*x)^4)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\sqrt{b}\,(\tan{(dx+c)}\,dx+1)}{\tan{(dx+c)}\,bd}$$

input
```
int(1/(b*tan(d*x+c)^4)^(1/2),x)
```

output
```
( - sqrt(b)*(tan(c + d*x)*d*x + 1))/(tan(c + d*x)*b*d)
```

# 3.40 $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 472
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 472
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 473
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 475
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . 475
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 476
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 476
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . 477
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 477
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . 477

## Optimal result

Integrand size = 14, antiderivative size = 119

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx=\frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}}-\frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}}-\frac{x\tan^2(c+dx)}{b\sqrt{b\tan^4(c+dx)}}$$

output
```
1/3*cot(d*x+c)/b/d/(tan(d*x+c)^4*b)^(1/2)-1/5*cot(d*x+c)^3/b/d/(tan(d*x+c)
^4*b)^(1/2)-tan(d*x+c)/b/d/(tan(d*x+c)^4*b)^(1/2)-x*tan(d*x+c)^2/b/(tan(d*
x+c)^4*b)^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)\tan(c+dx)}{5d\left(b\tan^4(c+dx)\right)^{3/2}}$$

input
```
Integrate[(b*Tan[c + d*x]^4)^(-3/2),x]
```

output
```
-1/5*(Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*(
b*Tan[c + d*x]^4)^(3/2))
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.60, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\tan(c+dx)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan^2(c+dx)\int \cot^6(c+dx)dx}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(c+dx)\int \tan\left(c+dx+\frac{\pi}{2}\right)^6dx}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(c+dx)\left(-\int \cot^4(c+dx)dx-\frac{\cot^5(c+dx)}{5d}\right)}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(c+dx)\left(-\int \tan\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{\cot^5(c+dx)}{5d}\right)}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(c+dx)\left(\int \cot^2(c+dx)dx - \frac{\cot^5(c+dx)}{5d} + \frac{\cot^3(c+dx)}{3d}\right)}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(c+dx)\left(\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{\cot^5(c+dx)}{5d} + \frac{\cot^3(c+dx)}{3d}\right)}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(c+dx)\left(-\int 1dx - \frac{\cot^5(c+dx)}{5d} + \frac{\cot^3(c+dx)}{3d} - \frac{\cot(c+dx)}{d}\right)}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^2(c+dx)\left(-\frac{\cot^5(c+dx)}{5d} + \frac{\cot^3(c+dx)}{3d} - \frac{\cot(c+dx)}{d} - x\right)}{b\sqrt{b\tan^4(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^4)^(-3/2),x]
```

output
```
((-x - Cot[c + d*x]/d + Cot[c + d*x]^3/(3*d) - Cot[c + d*x]^5/(5*d))*Tan[c
 + d*x]^2)/(b*Sqrt[b*Tan[c + d*x]^4])
```

## Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.53

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\tan(dx+c)\left(15\arctan(\tan(dx+c))\tan(dx+c)^{5}+15\tan(dx+c)^{4}-5\tan(dx+c)^{2}+3\right)}{15d\left(b\tan(dx+c)^{4}\right)^{\frac{3}{2}}}$ | 63 |
| default | $-\frac{\tan(dx+c)\left(15\arctan(\tan(dx+c))\tan(dx+c)^{5}+15\tan(dx+c)^{4}-5\tan(dx+c)^{2}+3\right)}{15d\left(b\tan(dx+c)^{4}\right)^{\frac{3}{2}}}$ | 63 |
| risch | $\frac{\left(e^{2i(dx+c)}-1\right)^{2}x}{b\left(e^{2i(dx+c)}+1\right)^{2}\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^{4}}{\left(e^{2i(dx+c)}+1\right)^{4}}}}+\frac{2i\left(45\,e^{8i(dx+c)}-90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}-70\,e^{2i(dx+c)}+23\right)}{15b\left(e^{2i(dx+c)}-1\right)^{3}\left(e^{2i(dx+c)}+1\right)^{2}\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^{4}}{\left(e^{2i(dx+c)}+1\right)^{4}}}\,d}$ | 174 |

input
```
int(1/(b*tan(d*x+c)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d*tan(d*x+c)*(15*arctan(tan(d*x+c))*tan(d*x+c)^5+15*tan(d*x+c)^4-5*t
an(d*x+c)^2+3)/(b*tan(d*x+c)^4)^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.52

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = -\frac{\left(15\,dx\tan\left(dx+c\right)^{5}+15\,\tan\left(dx+c\right)^{4}-5\,\tan\left(dx+c\right)^{2}+3\right)\sqrt{b\tan\left(dx+c\right)^{4}}}{15\,b^{2}d\tan\left(dx+c\right)^{7}}$$

input
```
integrate(1/(b*tan(d*x+c)^4)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(15*d*x*tan(d*x + c)^5 + 15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)*s
qrt(b*tan(d*x + c)^4)/(b^2*d*tan(d*x + c)^7)
```

## Sympy [F]

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan^4{(c+dx)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**4)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**4)**(-3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = -\frac{\frac{15\,(dx+c)}{b^{\frac{3}{2}}} + \frac{15\,\tan(dx+c)^4 - 5\,\tan(dx+c)^2 + 3}{b^{\frac{3}{2}}\tan(dx+c)^5}}{15\,d}$$

input
```
integrate(1/(b*tan(d*x+c)^4)^(3/2),x, algorithm="maxima")
```

output
```
-1/15*(15*(d*x + c)/b^(3/2) + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/(
b^(3/2)*tan(d*x + c)^5))/d
```

## Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.47

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = -\frac{\frac{15\,(dx+c)}{bd}+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{bd\tan(dx+c)^5}}{15\,\sqrt{b}}$$

input
```
integrate(1/(b*tan(d*x+c)^4)^(3/2),x, algorithm="giac")
```

output
```
-1/15*(15*(d*x + c)/(b*d) + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/(b*
d*tan(d*x + c)^5))/sqrt(b)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^4\right)^{3/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^4)^(3/2),x)
```

output
```
int(1/(b*tan(c + d*x)^4)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = \frac{\sqrt{b}\left(-15\tan\left(dx+c\right)^5 dx-15\tan\left(dx+c\right)^4+5\tan\left(dx+c\right)^2-3\right)}{15\tan\left(dx+c\right)^5 b^2 d}$$

input
```
int(1/(b*tan(d*x+c)^4)^(3/2),x)
```

output
```
(sqrt(b)*( - 15*tan(c + d*x)**5*d*x - 15*tan(c + d*x)**4 + 5*tan(c + d*x)*
*2 - 3))/(15*tan(c + d*x)**5*b**2*d)
```

# 3.41 $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx$

Optimal result . . . . . 478
Mathematica [C] (verified) . . . . . 479
Rubi [A] (verified) . . . . . 479
Maple [A] (verified) . . . . . 482
Fricas [A] (verification not implemented) . . . . . 482
Sympy [F] . . . . . 483
Maxima [A] (verification not implemented) . . . . . 483
Giac [A] (verification not implemented) . . . . . 483
Mupad [F(-1)] . . . . . 484
Reduce [B] (verification not implemented) . . . . . 484

## Optimal result

Integrand size = 14, antiderivative size = 183

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx=\frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}}+\frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\tan(c+dx)}{b^2d\sqrt{b\tan^4(c+dx)}}-\frac{x\tan^2(c+dx)}{b^2\sqrt{b\tan^4(c+dx)}}$$

output

```
1/3*cot(d*x+c)/b^2/d/(tan(d*x+c)^4*b)^(1/2)-1/5*cot(d*x+c)^3/b^2/d/(tan(d*
x+c)^4*b)^(1/2)+1/7*cot(d*x+c)^5/b^2/d/(tan(d*x+c)^4*b)^(1/2)-1/9*cot(d*x+
c)^7/b^2/d/(tan(d*x+c)^4*b)^(1/2)-tan(d*x+c)/b^2/d/(tan(d*x+c)^4*b)^(1/2)-
x*tan(d*x+c)^2/b^2/(tan(d*x+c)^4*b)^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.25

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(c+dx)\right)\tan(c+dx)}{9d\left(b\tan^4(c+dx)\right)^{5/2}}$$

input
```
Integrate[(b*Tan[c + d*x]^4)^(-5/2),x]
```

output
```
-1/9*(Hypergeometric2F1[-9/2, 1, -7/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*(
b*Tan[c + d*x]^4)^(5/2))
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.55, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(b\tan(c+dx)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4141}$$

$$\frac{\tan^2(c+dx)\int \cot^{10}(c+dx)dx}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^2(c+dx)\int \tan\left(c+dx+\frac{\pi}{2}\right)^{10}dx}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(c+dx)\left(-\int \cot^8(c+dx)dx-\frac{\cot^9(c+dx)}{9d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(c+dx)\left(-\int \tan\left(c+dx+\frac{\pi}{2}\right)^8dx-\frac{\cot^9(c+dx)}{9d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(c+dx)\left(\int \cot^6(c+dx)dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(c+dx)\left(\int \tan\left(c+dx+\frac{\pi}{2}\right)^6dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(c+dx)\left(-\int \cot^4(c+dx)dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}-\frac{\cot^5(c+dx)}{5d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(c+dx)\left(-\int \tan\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}-\frac{\cot^5(c+dx)}{5d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(c+dx)\left(\int \cot^2(c+dx)dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(c+dx)\left(\int \tan\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(c+dx)\left(-\int 1dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}-\frac{\cot(c+dx)}{d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^2(c+dx)\left(-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}-\frac{\cot(c+dx)}{d}-x\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^4)^(-5/2),x]
```

output
```
((-x - Cot[c + d*x]/d + Cot[c + d*x]^3/(3*d) - Cot[c + d*x]^5/(5*d) + Cot[
c + d*x]^7/(7*d) - Cot[c + d*x]^9/(9*d))*Tan[c + d*x]^2)/(b^2*Sqrt[b*Tan[c
 + d*x]^4])
```

## Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

| method | result |
|---|---|
| derivativedivides | $-\frac{\tan(dx+c)\left(315\arctan(\tan(dx+c))\tan(dx+c)^{9}+315\tan(dx+c)^{8}-105\tan(dx+c)^{6}+63\tan(dx+c)^{4}-45\tan(dx+c)^{2}\right.}{315d\left(b\tan(dx+c)^{4}\right)^{\frac{5}{2}}}$ |
| default | $-\frac{\tan(dx+c)\left(315\arctan(\tan(dx+c))\tan(dx+c)^{9}+315\tan(dx+c)^{8}-105\tan(dx+c)^{6}+63\tan(dx+c)^{4}-45\tan(dx+c)^{2}\right.}{315d\left(b\tan(dx+c)^{4}\right)^{\frac{5}{2}}}$ |
| risch | $\frac{\left(e^{2i(dx+c)}-1\right)^{2}x}{b^{2}\left(e^{2i(dx+c)}+1\right)^{2}\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^{4}}{\left(e^{2i(dx+c)}+1\right)^{4}}}}+\frac{2i\left(1575\,e^{16i(dx+c)}-6300\,e^{14i(dx+c)}+21000\,e^{12i(dx+c)}-31500\,e^{10i(dx+c)}+3\right.}{315b^{2}\left(e^{2i(dx+c)}-1\right)^{7}\left(e^{2i(dx+c}\right.}$ |

input
```
int(1/(b*tan(d*x+c)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/315/d*tan(d*x+c)*(315*arctan(tan(d*x+c))*tan(d*x+c)^9+315*tan(d*x+c)^8-
105*tan(d*x+c)^6+63*tan(d*x+c)^4-45*tan(d*x+c)^2+35)/(b*tan(d*x+c)^4)^(5/2
)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(b\tan^{4}(c+dx)\right)^{5/2}}\,dx =$$

$$-\frac{\left(315\,dx\tan\left(dx+c\right)^{9}+315\,\tan\left(dx+c\right)^{8}-105\,\tan\left(dx+c\right)^{6}+63\,\tan\left(dx+c\right)^{4}-45\,\tan\left(dx+c\right)^{2}+3\right.}{315\,b^{3}d\tan\left(dx+c\right)^{11}}$$

input
```
integrate(1/(b*tan(d*x+c)^4)^(5/2),x, algorithm="fricas")
```

output
```
-1/315*(315*d*x*tan(d*x + c)^9 + 315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 +
 63*tan(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)*sqrt(b*tan(d*x + c)^4)/(b^3*d
*tan(d*x + c)^11)
```

## Sympy [F]

$$\int \frac{1}{(b\tan^4(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\tan^4\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**4)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**4)**(-5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int \frac{1}{(b\tan^4(c+dx))^{5/2}}\,dx = -\frac{\frac{315\,(dx+c)}{b^{\frac{5}{2}}} + \frac{315\,\tan(dx+c)^8 - 105\,\tan(dx+c)^6 + 63\,\tan(dx+c)^4 - 45\,\tan(dx+c)^2 + 35}{b^{\frac{5}{2}}\tan(dx+c)^9}}{315\,d}$$

input
```
integrate(1/(b*tan(d*x+c)^4)^(5/2),x, algorithm="maxima")
```

output
```
-1/315*(315*(d*x + c)/b^(5/2) + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 +
 63*tan(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/(b^(5/2)*tan(d*x + c)^9))/d
```

## Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.42

$$\int \frac{1}{(b\tan^4(c+dx))^{5/2}}\,dx = -\frac{\frac{315\,(dx+c)}{bd} + \frac{315\,\tan(dx+c)^8 - 105\,\tan(dx+c)^6 + 63\,\tan(dx+c)^4 - 45\,\tan(dx+c)^2 + 35}{bd\tan(dx+c)^9}}{315\,b^{\frac{3}{2}}}$$

input
```
integrate(1/(b*tan(d*x+c)^4)^(5/2),x, algorithm="giac")
```

output
```
-1/315*(315*(d*x + c)/(b*d) + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 + 6
3*tan(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/(b*d*tan(d*x + c)^9))/b^(3/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx=\int \frac{1}{\left(b\tan\left(c+d\,x\right)^4\right)^{5/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^4)^(5/2),x)
```

output
```
int(1/(b*tan(c + d*x)^4)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx=\frac{\sqrt{b}\left(-315\tan\left(dx+c\right)^9dx-315\tan\left(dx+c\right)^8+105\tan\left(dx+c\right)^6-63\tan\left(d\right.\right.}{315\tan\left(dx+c\right)^9b^3d}$$

input
```
int(1/(b*tan(d*x+c)^4)^(5/2),x)
```

output
```
(sqrt(b)*( - 315*tan(c + d*x)**9*d*x - 315*tan(c + d*x)**8 + 105*tan(c + d
*x)**6 - 63*tan(c + d*x)**4 + 45*tan(c + d*x)**2 - 35))/(315*tan(c + d*x)*
*9*b**3*d)
```

# 3.42 $\int (b\tan^p(c+dx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 485
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 485
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 486
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 487
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 488
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 488
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 488
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 489
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 489
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 489

## Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (b\tan^p(c+dx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^p(c+dx)\right)^n}{d(1+np)}$$

output
```
hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(
d*x+c)^p)^n/d/(n*p+1)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int (b\tan^p(c+dx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^p(c+dx)\right)^n}{d(1+np)}$$

input
```
Integrate[(b*Tan[c + d*x]^p)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]*(b*Tan[c + d*x]^p)^n)/(d*(1 + n*p))
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan^p(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (b\tan(c+dx)^p)^n\,dx$$

$$\Big\downarrow \text{ 4142}$$

$$\tan^{-np}(c+dx)\,(b\tan^p(c+dx))^n\int \tan^{np}(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\tan^{-np}(c+dx)\,(b\tan^p(c+dx))^n\int \tan(c+dx)^{np}dx$$

$$\Big\downarrow \text{ 3957}$$

$$\frac{\tan^{-np}(c+dx)\,(b\tan^p(c+dx))^n\int \frac{\tan^{np}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{\tan(c+dx)\,(b\tan^p(c+dx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(c+dx)\right)}{d(np+1)}$$

input
```
Int[(b*Tan[c + d*x]^p)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]*(b*Tan[c + d*x]^p)^n)/(d*(1 + n*p))
```

**Defintions of rubi rules used**

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

## Maple [F]

$$\int \left(b\tan\left(dx+c\right)^{p}\right)^{n}dx$$

input
```
int((b*tan(d*x+c)^p)^n,x)
```

output
```
int((b*tan(d*x+c)^p)^n,x)
```

## Fricas [F]

$$\int (b\tan^p(c+dx))^n \, dx = \int (b\tan{(dx+c)}^p)^n \, dx$$

input
```
integrate((b*tan(d*x+c)^p)^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c)^p)^n, x)
```

## Sympy [F]

$$\int (b\tan^p(c+dx))^n \, dx = \int (b\tan^p{(c+dx)})^n \, dx$$

input
```
integrate((b*tan(d*x+c)**p)**n,x)
```

output
```
Integral((b*tan(c + d*x)**p)**n, x)
```

## Maxima [F]

$$\int (b\tan^p(c+dx))^n \, dx = \int (b\tan{(dx+c)}^p)^n \, dx$$

input
```
integrate((b*tan(d*x+c)^p)^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^p)^n, x)
```

## Giac [F]

$$\int (b\tan^p(c+dx))^n \, dx = \int (b\tan{(dx+c)}^p)^n \, dx$$

input
```
integrate((b*tan(d*x+c)^p)^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^p)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\tan^p(c+dx))^n \, dx = \int (b\tan(c+d\,x)^p)^n \, dx$$

input
```
int((b*tan(c + d*x)^p)^n,x)
```

output
```
int((b*tan(c + d*x)^p)^n, x)
```

## Reduce [F]

$$\int (b\tan^p(c+dx))^n \, dx = b^n \left( \int \tan{(dx+c)}^{np} \, dx \right)$$

input
```
int((b*tan(d*x+c)^p)^n,x)
```

output
```
b**n*int(tan(c + d*x)**(n*p),x)
```

## 3.43 $\int \left(b\tan^2(c+dx)\right)^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 490
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 490
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 491
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 492
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 493
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 493
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 493
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 494
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 494
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 494

### Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \left(b\tan^2(c+dx)\right)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+2n),\frac{1}{2}(3+2n),-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^2(c+dx)\right)^n}{d(1+2n)}$$

output
```
hypergeom([1, 1/2+n],[3/2+n],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(d*x+c)^2)^n/
d/(1+2*n)
```

### Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \left(b\tan^2(c+dx)\right)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^2(c+dx)\right)^n}{d(1+2n)}$$

input
```
Integrate[(b*Tan[c + d*x]^2)^n,x]
```

output
```
(Hypergeometric2F1[1, 1/2 + n, 3/2 + n, -Tan[c + d*x]^2]*Tan[c + d*x]*(b*T
an[c + d*x]^2)^n)/(d*(1 + 2*n))
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4141, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^2(c+dx)\right)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^2\right)^n\,dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-2n}(c+dx)\left(b\tan^2(c+dx)\right)^n\int \tan^{2n}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-2n}(c+dx)\left(b\tan^2(c+dx)\right)^n\int \tan(c+dx)^{2n}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{-2n}(c+dx)\left(b\tan^2(c+dx)\right)^n\int \frac{\tan^{2n}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(c+dx)\left(b\tan^2(c+dx)\right)^n\text{Hypergeometric2F1}\left(1,\frac{1}{2}(2n+1),\frac{1}{2}(2n+3),-\tan^2(c+dx)\right)}{d(2n+1)}$$

input
```
Int[(b*Tan[c + d*x]^2)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + 2*n)/2, (3 + 2*n)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]*(b*Tan[c + d*x]^2)^n)/(d*(1 + 2*n))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [F]

$$\int \left(b \tan \left(dx+c\right)^{2}\right)^{n} dx$$

input
```
int((b*tan(d*x+c)^2)^n,x)
```

output
```
int((b*tan(d*x+c)^2)^n,x)
```

## Fricas [F]

$$\int \left(b\tan^2(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^2\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)^2)^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c)^2)^n, x)
```

## Sympy [F]

$$\int \left(b\tan^2(c+dx)\right)^n\,dx = \int \left(b\tan^2\left(c+dx\right)\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)**2)**n,x)
```

output
```
Integral((b*tan(c + d*x)**2)**n, x)
```

## Maxima [F]

$$\int \left(b\tan^2(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^2\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)^2)^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^2)^n, x)
```

## Giac [F]

$$\int \left(b \tan^2(c+dx)\right)^n \, dx = \int \left(b \tan \left(dx+c\right)^2\right)^n \, dx$$

input
```
integrate((b*tan(d*x+c)^2)^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^2)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(b \tan^2(c+dx)\right)^n \, dx = \int \left(b \tan(c+d\,x)^2\right)^n \, dx$$

input
```
int((b*tan(c + d*x)^2)^n,x)
```

output
```
int((b*tan(c + d*x)^2)^n, x)
```

## Reduce [F]

$$\int \left(b \tan^2(c+dx)\right)^n \, dx = b^n \left(\int \tan \left(dx+c\right)^{2n} dx\right)$$

input
```
int((b*tan(d*x+c)^2)^n,x)
```

output
```
b**n*int(tan(c + d*x)**(2*n),x)
```

## 3.44 $\int \left(b\tan^3(c+dx)\right)^n\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 495
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 495
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 496
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 497
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 498
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 498
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 498
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 499
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 499
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 499


### Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \left(b\tan^3(c+dx)\right)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+3n),\frac{3(1+n)}{2},-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^3(c+dx)\right)^n}{d(1+3n)}$$

output
```
hypergeom([1, 1/2+3/2*n],[3/2+3/2*n],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(d*x+
c)^3)^n/d/(1+3*n)
```

### Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \left(b\tan^3(c+dx)\right)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+3n),\frac{3(1+n)}{2},-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^3(c+dx)\right)^n}{d(1+3n)}$$

input
```
Integrate[(b*Tan[c + d*x]^3)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, -Tan[c + d*x]^2]*Tan[c +
 d*x]*(b*Tan[c + d*x]^3)^n)/(d*(1 + 3*n))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4141, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^3(c+dx)\right)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^3\right)^n\,dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-3n}(c+dx)\left(b\tan^3(c+dx)\right)^n\int \tan^{3n}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-3n}(c+dx)\left(b\tan^3(c+dx)\right)^n\int \tan(c+dx)^{3n}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{-3n}(c+dx)\left(b\tan^3(c+dx)\right)^n\int \frac{\tan^{3n}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(c+dx)\left(b\tan^3(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(3n+1),\frac{3(n+1)}{2},-\tan^2(c+dx)\right)}{d(3n+1)}$$

input
```
Int[(b*Tan[c + d*x]^3)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, -Tan[c + d*x]^2]*Tan[c +
 d*x]*(b*Tan[c + d*x]^3)^n)/(d*(1 + 3*n))
```

**Defintions of rubi rules used**

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [F]

$$\int \left(b \tan \left(dx+c\right)^{3}\right)^{n} dx$$

input
```
int((b*tan(d*x+c)^3)^n,x)
```

output
```
int((b*tan(d*x+c)^3)^n,x)
```

## Fricas [F]

$$\int \left(b\tan^3(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^3\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)^3)^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c)^3)^n, x)
```

## Sympy [F]

$$\int \left(b\tan^3(c+dx)\right)^n\,dx = \int \left(b\tan^3\left(c+dx\right)\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)**3)**n,x)
```

output
```
Integral((b*tan(c + d*x)**3)**n, x)
```

## Maxima [F]

$$\int \left(b\tan^3(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^3\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)^3)^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^3)^n, x)
```

## Giac [F]

$$\int \left(b\tan^3(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^3\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)^3)^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^3)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^3(c+dx)\right)^n\,dx = \int \left(b\tan(c+d\,x)^3\right)^n\,dx$$

input
```
int((b*tan(c + d*x)^3)^n,x)
```

output
```
int((b*tan(c + d*x)^3)^n, x)
```

## Reduce [F]

$$\int \left(b\tan^3(c+dx)\right)^n\,dx = b^n\left(\int \tan\left(dx+c\right)^{3n}\,dx\right)$$

input
```
int((b*tan(d*x+c)^3)^n,x)
```

output
```
b**n*int(tan(c + d*x)**(3*n),x)
```

## 3.45 $\int \left(b\tan^4(c+dx)\right)^n\,dx$

Optimal result . . . 500
Mathematica [A] (verified) . . . 500
Rubi [A] (verified) . . . 501
Maple [F] . . . 502
Fricas [F] . . . 503
Sympy [F] . . . 503
Maxima [F] . . . 503
Giac [F] . . . 504
Mupad [F(-1)] . . . 504
Reduce [F] . . . 504

### Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \left(b\tan^4(c+dx)\right)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+4n),\frac{1}{2}(3+4n),-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^4(c+dx)\right)^n}{d(1+4n)}$$

output
```
hypergeom([1, 1/2+2*n],[3/2+2*n],-tan(d*x+c)^2)*tan(d*x+c)*(tan(d*x+c)^4*b
)^n/d/(1+4*n)
```

### Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \left(b\tan^4(c+dx)\right)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}+2n,\frac{3}{2}+2n,-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^4(c+dx)\right)^n}{d(1+4n)}$$

input
```
Integrate[(b*Tan[c + d*x]^4)^n,x]
```

output
```
(Hypergeometric2F1[1, 1/2 + 2*n, 3/2 + 2*n, -Tan[c + d*x]^2]*Tan[c + d*x]*
(b*Tan[c + d*x]^4)^n)/(d*(1 + 4*n))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4141, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^4(c+dx)\right)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^4\right)^n dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-4n}(c+dx)\left(b\tan^4(c+dx)\right)^n\int \tan^{4n}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-4n}(c+dx)\left(b\tan^4(c+dx)\right)^n\int \tan(c+dx)^{4n}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{-4n}(c+dx)\left(b\tan^4(c+dx)\right)^n\int \frac{\tan^{4n}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(c+dx)\left(b\tan^4(c+dx)\right)^n\text{Hypergeometric2F1}\left(1,\frac{1}{2}(4n+1),\frac{1}{2}(4n+3),-\tan^2(c+dx)\right)}{d(4n+1)}$$

input
```
Int[(b*Tan[c + d*x]^4)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + 4*n)/2, (3 + 4*n)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]*(b*Tan[c + d*x]^4)^n)/(d*(1 + 4*n))
```

**Defintions of rubi rules used**

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

## Maple [F]

$$\int \left(b \tan \left(dx + c\right)^{4}\right)^{n} dx$$

input
```
int((b*tan(d*x+c)^4)^n,x)
```

output
```
int((b*tan(d*x+c)^4)^n,x)
```

## Fricas [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx=\int \left(b\tan\left(dx+c\right)^4\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)^4)^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c)^4)^n, x)
```

## Sympy [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx=\int \left(b\tan^4\left(c+dx\right)\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)**4)**n,x)
```

output
```
Integral((b*tan(c + d*x)**4)**n, x)
```

## Maxima [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx=\int \left(b\tan\left(dx+c\right)^4\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)^4)^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^4)^n, x)
```

## Giac [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^4\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)^4)^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^4)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^4(c+dx)\right)^n\,dx = \int \left(b\tan(c+d\,x)^4\right)^n\,dx$$

input
```
int((b*tan(c + d*x)^4)^n,x)
```

output
```
int((b*tan(c + d*x)^4)^n, x)
```

## Reduce [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx = b^n\left(\int \tan\left(dx+c\right)^{4n}dx\right)$$

input
```
int((b*tan(d*x+c)^4)^n,x)
```

output
```
b**n*int(tan(c + d*x)**(4*n),x)
```

# 3.46 $\int (b\tan^p(c+dx))^{5/2}\,dx$

Optimal result . . . . . 505
Mathematica [A] (verified) . . . . . 505
Rubi [A] (verified) . . . . . 506
Maple [F] . . . . . 507
Fricas [F(-2)] . . . . . 508
Sympy [F] . . . . . 508
Maxima [F] . . . . . 508
Giac [F] . . . . . 509
Mupad [F(-1)] . . . . . 509
Reduce [F] . . . . . 509

## Optimal result

Integrand size = 14, antiderivative size = 71

$$\int (b\tan^p(c+dx))^{5/2}\,dx=\frac{2b^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+5p),\frac{1}{4}(6+5p),-\tan^2(c+dx)\right)\tan^{1+2p}(c+dx)\sqrt{b\tan^p(c}{d(2+5p)}$$

output
```
2*b^2*hypergeom([1, 1/2+5/4*p],[3/2+5/4*p],-tan(d*x+c)^2)*tan(d*x+c)^(1+2*
p)*(b*tan(d*x+c)^p)^(1/2)/d/(2+5*p)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int (b\tan^p(c+dx))^{5/2}\,dx=\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+5p),\frac{1}{4}(6+5p),-\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^{5/}}{d\left(1+\frac{5p}{2}\right)}$$

input
```
Integrate[(b*Tan[c + d*x]^p)^(5/2),x]
```

output
```
(Hypergeometric2F1[1, (2 + 5*p)/4, (6 + 5*p)/4, -Tan[c + d*x]^2]*Tan[c + d
*x]*(b*Tan[c + d*x]^p)^(5/2))/(d*(1 + (5*p)/2))
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^p(c+dx)\right)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^p\right)^{5/2}dx$$

$$\Big\downarrow \text{4142}$$

$$b^2\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \tan^{\frac{5p}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \tan(c+dx)^{5p/2}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b^2\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \frac{\tan^{\frac{5p}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{2b^2\tan^{2p+1}(c+dx)\sqrt{b\tan^p(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(5p+2),\frac{1}{4}(5p+6),-\tan^2(c+dx)\right)}{d(5p+2)}$$

input
```
Int[(b*Tan[c + d*x]^p)^(5/2),x]
```

output
```
(2*b^2*Hypergeometric2F1[1, (2 + 5*p)/4, (6 + 5*p)/4, -Tan[c + d*x]^2]*Tan
[c + d*x]^(1 + 2*p)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + 5*p))
```

**Defintions of rubi rules used**

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

## Maple [F]

$$\int (b\tan (dx+c)^{p})^{\frac{5}{2}}\, dx$$

input
```
int((b*tan(d*x+c)^p)^(5/2),x)
```

output
```
int((b*tan(d*x+c)^p)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

## Sympy [F]

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \int (b\tan^p{(c+dx)})^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)**p)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**p)**(5/2), x)
```

## Maxima [F]

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \int (b\tan{(dx+c)}^p)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^p)^(5/2), x)
```

## Giac [F]

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \int (b\tan(dx+c)^p)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^p)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \int (b\tan(c+d\,x)^p)^{5/2}\,dx$$

input
```
int((b*tan(c + d*x)^p)^(5/2),x)
```

output
```
int((b*tan(c + d*x)^p)^(5/2), x)
```

## Reduce [F]

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \sqrt{b}\left(\int \tan(dx+c)^{\frac{5p}{2}}\,dx\right)b^2$$

input
```
int((b*tan(d*x+c)^p)^(5/2),x)
```

output
```
sqrt(b)*int(tan(c + d*x)**((5*p)/2),x)*b**2
```

# 3.47 $\int (b\tan^p(c+dx))^{3/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 510
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . 510
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 511
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 512
Fricas [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 513
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 513
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 513
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 514
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 514
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 514

## Optimal result

Integrand size = 14, antiderivative size = 65

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \frac{2b\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+3p),\frac{3(2+p)}{4},-\tan^2(c+dx)\right)\tan^{1+p}(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+3p)}$$

output
```
2*b*hypergeom([1, 1/2+3/4*p],[3/2+3/4*p],-tan(d*x+c)^2)*tan(d*x+c)^(p+1)*(
b*tan(d*x+c)^p)^(1/2)/d/(2+3*p)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+3p),\frac{3(2+p)}{4},-\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^{3/2}}{d\left(1+\frac{3p}{2}\right)}$$

input
```
Integrate[(b*Tan[c + d*x]^p)^(3/2),x]
```

output
```
(Hypergeometric2F1[1, (2 + 3*p)/4, (3*(2 + p))/4, -Tan[c + d*x]^2]*Tan[c +
 d*x]*(b*Tan[c + d*x]^p)^(3/2))/(d*(1 + (3*p)/2))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan^p(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx)^p)^{3/2}dx$$

$$\Big\downarrow \text{4142}$$

$$b\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \tan^{\frac{3p}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \tan(c+dx)^{3p/2}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \frac{\tan^{\frac{3p}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{2b\tan^{p+1}(c+dx)\sqrt{b\tan^p(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(3p+2),\frac{3(p+2)}{4},-\tan^2(c+dx)\right)}{d(3p+2)}$$

input
```
Int[(b*Tan[c + d*x]^p)^(3/2),x]
```

output
```
(2*b*Hypergeometric2F1[1, (2 + 3*p)/4, (3*(2 + p))/4, -Tan[c + d*x]^2]*Tan
[c + d*x]^(1 + p)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + 3*p))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

## Maple [F]

$$\int (b\tan (dx+c)^{p})^{\frac{3}{2}}\, dx$$

input
```
int((b*tan(d*x+c)^p)^(3/2),x)
```

output
```
int((b*tan(d*x+c)^p)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

## Sympy [F]

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \int (b\tan^p{(c+dx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)**p)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**p)**(3/2), x)
```

## Maxima [F]

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \int (b\tan{(dx+c)}^p)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^p)^(3/2), x)
```

## Giac [F]

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \int (b\tan(dx+c)^p)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^p)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \int (b\tan(c+d\,x)^p)^{3/2}\,dx$$

input
```
int((b*tan(c + d*x)^p)^(3/2),x)
```

output
```
int((b*tan(c + d*x)^p)^(3/2), x)
```

## Reduce [F]

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \sqrt{b}\left(\int \tan(dx+c)^{\frac{3p}{2}}\,dx\right)b$$

input
```
int((b*tan(d*x+c)^p)^(3/2),x)
```

output
```
sqrt(b)*int(tan(c + d*x)**((3*p)/2),x)*b
```

# 3.48 $\int \sqrt{b \tan^p(c+dx)}\,dx$

Optimal result . . . . . 515
Mathematica [A] (verified) . . . . . 515
Rubi [A] (verified) . . . . . 516
Maple [F] . . . . . 517
Fricas [F(-2)] . . . . . 518
Sympy [F] . . . . . 518
Maxima [F] . . . . . 518
Giac [F] . . . . . 519
Mupad [F(-1)] . . . . . 519
Reduce [F] . . . . . 519

## Optimal result

Integrand size = 14, antiderivative size = 56

$$\int \sqrt{b \tan^p(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2+p}{4},\frac{6+p}{4},-\tan^2(c+dx)\right)\tan(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+p)}$$

output
```
2*hypergeom([1, 1/2+1/4*p],[3/2+1/4*p],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(d*
x+c)^p)^(1/2)/d/(2+p)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \sqrt{b \tan^p(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2+p}{4},\frac{6+p}{4},-\tan^2(c+dx)\right)\tan(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+p)}$$

input
```
Integrate[Sqrt[b*Tan[c + d*x]^p],x]
```

output
```
(2*Hypergeometric2F1[1, (2 + p)/4, (6 + p)/4, -Tan[c + d*x]^2]*Tan[c + d*x
]*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + p))
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan^p(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(c+dx)^p}dx$$

$$\Big\downarrow \text{4142}$$

$$\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \tan^{\frac{p}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \tan(c+dx)^{p/2}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \frac{\tan^{\frac{p}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{2\tan^{\frac{p+2}{2}-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{p+2}{4},\frac{p+6}{4},-\tan^2(c+dx)\right)}{d(p+2)}$$

input
```
Int[Sqrt[b*Tan[c + d*x]^p],x]
```

output
```
(2*Hypergeometric2F1[1, (2 + p)/4, (6 + p)/4, -Tan[c + d*x]^2]*Tan[c + d*x
]^(-1/2*p + (2 + p)/2)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + p))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

## Maple [F]

$$\int \sqrt{b \tan (dx + c)^{p}}dx$$

input
```
int((b*tan(d*x+c)^p)^(1/2),x)
```

output
```
int((b*tan(d*x+c)^p)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

## Sympy [F]

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \int \sqrt{b\tan^p{(c+dx)}}\,dx$$

input
```
integrate((b*tan(d*x+c)**p)**(1/2),x)
```

output
```
Integral(sqrt(b*tan(c + d*x)**p), x)
```

## Maxima [F]

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \int \sqrt{b\tan{(dx+c)}^p}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(d*x + c)^p), x)
```

## Giac [F]

$$\int \sqrt{b \tan^p(c+dx)}\,dx = \int \sqrt{b \tan{(dx+c)}^p}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(d*x + c)^p), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{b \tan^p(c+dx)}\,dx = \int \sqrt{b \tan{(c+d\,x)}^p}\,dx$$

input
```
int((b*tan(c + d*x)^p)^(1/2),x)
```

output
```
int((b*tan(c + d*x)^p)^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{b \tan^p(c+dx)}\,dx = \sqrt{b}\left(\int \tan{(dx+c)}^{\frac{p}{2}}\,dx\right)$$

input
```
int((b*tan(d*x+c)^p)^(1/2),x)
```

output
```
sqrt(b)*int(tan(c + d*x)**(p/2),x)
```

# 3.49 $\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx$


Optimal result . . . 520
Mathematica [A] (verified) . . . 520
Rubi [A] (verified) . . . 521
Maple [F] . . . 522
Fricas [F(-2)] . . . 523
Sympy [F] . . . 523
Maxima [F] . . . 523
Giac [F] . . . 524
Mupad [F(-1)] . . . 524
Reduce [F] . . . 524


## Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx=\frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2-p}{4},\frac{6-p}{4},-\tan^2(c+dx)\right)\tan(c+dx)}{d(2-p)\sqrt{b\tan^p(c+dx)}}$$

output
```
2*hypergeom([1, 1/2-1/4*p],[3/2-1/4*p],-tan(d*x+c)^2)*tan(d*x+c)/d/(2-p)/(
b*tan(d*x+c)^p)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2-p}{4},\frac{6-p}{4},-\tan^2(c+dx)\right)\tan(c+dx)}{d(-2+p)\sqrt{b\tan^p(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[c + d*x]^p],x]
```

output
```
(-2*Hypergeometric2F1[1, (2 - p)/4, (6 - p)/4, -Tan[c + d*x]^2]*Tan[c + d*
x])/(d*(-2 + p)*Sqrt[b*Tan[c + d*x]^p])
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\tan(c+dx)^p}}dx$$

$$\Big\downarrow \text{4142}$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{p}{2}}(c+dx)dx}{\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \tan(c+dx)^{-p/2}dx}{\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \frac{\tan^{-\frac{p}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \text{278}$$

$$\frac{2\tan(c+dx)\text{Hypergeometric2F1}\left(1,\frac{2-p}{4},\frac{6-p}{4},-\tan^2(c+dx)\right)}{d(2-p)\sqrt{b\tan^p(c+dx)}}$$

input
```
Int[1/Sqrt[b*Tan[c + d*x]^p],x]
```

output
```
(2*Hypergeometric2F1[1, (2 - p)/4, (6 - p)/4, -Tan[c + d*x]^2]*Tan[c + d*x
])/(d*(2 - p)*Sqrt[b*Tan[c + d*x]^p])
```

**Defintions of rubi rules used**

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

## Maple [F]

$$\int \frac{1}{\sqrt{b \tan (dx + c)^{p}}}dx$$

input
```
int(1/(b*tan(d*x+c)^p)^(1/2),x)
```

output
```
int(1/(b*tan(d*x+c)^p)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

## Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**p)**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(c + d*x)**p), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan(dx+c)^p}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*tan(d*x + c)^p), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan(dx+c)^p}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*tan(d*x + c)^p), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan(c+d\,x)^p}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^p)^(1/2),x)
```

output
```
int(1/(b*tan(c + d*x)^p)^(1/2), x)
```

## Reduce [F]

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{1}{\tan(dx+c)^{\frac{p}{2}}}dx\right)}{b}$$

input
```
int(1/(b*tan(d*x+c)^p)^(1/2),x)
```

output
```
(sqrt(b)*int(1/tan(c + d*x)**(p/2),x))/b
```

# 3.50 $\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 525
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 525
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 526
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 527
Fricas [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 528
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 528
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 528
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 529
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 529
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 529

## Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3p),\frac{3(2-p)}{4},-\tan^2(c+dx)\right)\tan^{1-p}(c+dx)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}$$

output
```
2*hypergeom([1, 1/2-3/4*p],[3/2-3/4*p],-tan(d*x+c)^2)*tan(d*x+c)^(1-p)/b/d
/(2-3*p)/(b*tan(d*x+c)^p)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3p),-\frac{3}{4}(-2+p),-\tan^2(c+dx)\right)\tan(c+dx)}{d\left(1-\frac{3p}{2}\right)(b\tan^p(c+dx))^{3/2}}$$

input
```
Integrate[(b*Tan[c + d*x]^p)^(-3/2),x]
```

output
```
(Hypergeometric2F1[1, (2 - 3*p)/4, (-3*(-2 + p))/4, -Tan[c + d*x]^2]*Tan[c
 + d*x])/(d*(1 - (3*p)/2)*(b*Tan[c + d*x]^p)^(3/2))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{(b\tan(c+dx)^p)^{3/2}}dx$$
$$\Big\downarrow \; 4142$$
$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{3p}{2}}(c+dx)dx}{b\sqrt{b\tan^p(c+dx)}}$$
$$\Big\downarrow \; 3042$$
$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \tan(c+dx)^{-3p/2}dx}{b\sqrt{b\tan^p(c+dx)}}$$
$$\Big\downarrow \; 3957$$
$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \frac{\tan^{-\frac{3p}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{bd\sqrt{b\tan^p(c+dx)}}$$
$$\Big\downarrow \; 278$$
$$\frac{2\tan^{1-p}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3p),\frac{3(2-p)}{4},-\tan^2(c+dx)\right)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^p)^(-3/2),x]
```

output
```
(2*Hypergeometric2F1[1, (2 - 3*p)/4, (3*(2 - p))/4, -Tan[c + d*x]^2]*Tan[c
 + d*x]^(1 - p))/(b*d*(2 - 3*p)*Sqrt[b*Tan[c + d*x]^p])
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

## Maple [F]

$$\int \frac{1}{(b\tan(dx+c)^p)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*tan(d*x+c)^p)^(3/2),x)
```

output
```
int(1/(b*tan(d*x+c)^p)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

## Sympy [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tan^p{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**p)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**p)**(-3/2), x)
```

## Maxima [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tan{(dx+c)}^p)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^p)^(-3/2), x)
```

## Giac [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tan{(dx+c)}^p)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^p)^(-3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tan{(c+d\,x)}^p)^{3/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^p)^(3/2),x)
```

output
```
int(1/(b*tan(c + d*x)^p)^(3/2), x)
```

## Reduce [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{1}{\tan(dx+c)^{\frac{3p}{2}}}dx\right)}{b^2}$$

input
```
int(1/(b*tan(d*x+c)^p)^(3/2),x)
```

output
```
(sqrt(b)*int(1/tan(c + d*x)**((3*p)/2),x))/b**2
```

# 3.51 $\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 530
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 530
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 531
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 532
Fricas [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 533
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 533
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 533
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 534
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 534
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 534

## Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx=\frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5p),\frac{1}{4}(6-5p),-\tan^2(c+dx)\right)\tan^{1-2p}(c+dx)}{b^2d(2-5p)\sqrt{b\tan^p(c+dx)}}$$

output
```
2*hypergeom([1, 1/2-5/4*p],[3/2-5/4*p],-tan(d*x+c)^2)*tan(d*x+c)^(1-2*p)/b
^2/d/(2-5*p)/(b*tan(d*x+c)^p)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx=\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5p),\frac{1}{4}(6-5p),-\tan^2(c+dx)\right)\tan(c+dx)}{d\left(1-\frac{5p}{2}\right)(b\tan^p(c+dx))^{5/2}}$$

input
```
Integrate[(b*Tan[c + d*x]^p)^(-5/2),x]
```

output
```
(Hypergeometric2F1[1, (2 - 5*p)/4, (6 - 5*p)/4, -Tan[c + d*x]^2]*Tan[c + d
*x])/(d*(1 - (5*p)/2)*(b*Tan[c + d*x]^p)^(5/2))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(c+dx)^p)^{5/2}}dx$$

$$\Big\downarrow \text{4142}$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{5p}{2}}(c+dx)dx}{b^2\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \tan(c+dx)^{-5p/2}dx}{b^2\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \frac{\tan^{-\frac{5p}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{b^2d\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \text{278}$$

$$\frac{2\tan^{1-2p}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5p),\frac{1}{4}(6-5p),-\tan^2(c+dx)\right)}{b^2d(2-5p)\sqrt{b\tan^p(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^p)^(-5/2),x]
```

output
```
(2*Hypergeometric2F1[1, (2 - 5*p)/4, (6 - 5*p)/4, -Tan[c + d*x]^2]*Tan[c +
 d*x]^(1 - 2*p))/(b^2*d*(2 - 5*p)*Sqrt[b*Tan[c + d*x]^p])
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

## Maple [F]

$$\int \frac{1}{(b\tan(dx+c)^p)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*tan(d*x+c)^p)^(5/2),x)
```

output
```
int(1/(b*tan(d*x+c)^p)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

## Sympy [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan^p(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**p)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**p)**(-5/2), x)
```

## Maxima [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan(dx+c)^p)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^p)^(-5/2), x)
```

## Giac [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan{(dx+c)}^p)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^p)^(-5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan{(c+d\,x)}^p)^{5/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^p)^(5/2),x)
```

output
```
int(1/(b*tan(c + d*x)^p)^(5/2), x)
```

## Reduce [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{1}{\tan(dx+c)^{\frac{5p}{2}}}dx\right)}{b^3}$$

input
```
int(1/(b*tan(d*x+c)^p)^(5/2),x)
```

output
```
(sqrt(b)*int(1/tan(c + d*x)**((5*p)/2),x))/b**3
```

# 3.52 $\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 535
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 535
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 536
Maple [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . 537
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 538
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 538
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 538
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 539
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 539
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . 539

## Optimal result

Integrand size = 14, antiderivative size = 32

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}$$

output
```
-cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^p)^(1/p)/d
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}$$

input
```
Integrate[(b*Tan[c + d*x]^p)^(p^(-1)),x]
```

output
```
-((Cot[c + d*x]*Log[Cos[c + d*x]]*(b*Tan[c + d*x]^p)^(p^(-1)))/d)
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4142, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx)^p)^{\frac{1}{p}}dx$$

$$\Big\downarrow \text{4142}$$

$$\cot(c+dx)\left(b\tan^p(c+dx)\right)^{\frac{1}{p}}\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cot(c+dx)\left(b\tan^p(c+dx)\right)^{\frac{1}{p}}\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\cot(c+dx)\log(\cos(c+dx))\left(b\tan^p(c+dx)\right)^{\frac{1}{p}}}{d}$$

input
```
Int[(b*Tan[c + d*x]^p)^p^(-1),x]
```

output
```
-((Cot[c + d*x]*Log[Cos[c + d*x]]*(b*Tan[c + d*x]^p)^p^(-1))/d)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.48 (sec) , antiderivative size = 5979, normalized size of antiderivative = 186.84

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 5979 |

input
```
int((b*tan(d*x+c)^p)^(1/p),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = -\frac{b^{\left(\frac{1}{p}\right)}\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="fricas")
```

output
```
-1/2*b^(1/p)*log(1/(tan(d*x + c)^2 + 1))/d
```

## Sympy [F]

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = \int (b\tan^p{(c+dx)})^{\frac{1}{p}}\,dx$$

input
```
integrate((b*tan(d*x+c)**p)**(1/p),x)
```

output
```
Integral((b*tan(c + d*x)**p)**(1/p), x)
```

## Maxima [F]

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = \int (b\tan{(dx+c)}^p)^{\left(\frac{1}{p}\right)}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^p)^(1/p), x)
```

## Giac [F]

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = \int (b\tan(dx+c)^p)^{\left(\frac{1}{p}\right)}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^p)^(1/p), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = \int (b\tan(c+d\,x)^p)^{1/p}\,dx$$

input
```
int((b*tan(c + d*x)^p)^(1/p),x)
```

output
```
int((b*tan(c + d*x)^p)^(1/p), x)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = \frac{b^{\frac{1}{p}}\log\left(\tan(dx+c)^2+1\right)}{2d}$$

input
```
int((b*tan(d*x+c)^p)^(1/p),x)
```

output
```
(b**(1/p)*log(tan(c + d*x)**2 + 1))/(2*d)
```

# 3.53 $\int (a(b\tan(c+dx))^p)^n\,dx$

Optimal result . . . 540
Mathematica [A] (verified) . . . 540
Rubi [A] (verified) . . . 541
Maple [F] . . . 542
Fricas [F] . . . 543
Sympy [F] . . . 543
Maxima [F] . . . 543
Giac [F] . . . 544
Mupad [F(-1)] . . . 544
Reduce [F] . . . 544

## Optimal result

Integrand size = 14, antiderivative size = 61

$$\int (a(b\tan(c+dx))^p)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(c+dx)\right)\tan(c+dx)\,(a(b\tan(c+dx))^p)^n}{d(1+np)}$$

output
```
hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(d*x+c)^2)*tan(d*x+c)*(a*(b*t
an(d*x+c))^p)^n/d/(n*p+1)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int (a(b\tan(c+dx))^p)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(c+dx)\right)\tan(c+dx)\,(a(b\tan(c+dx))^p)^n}{d(1+np)}$$

input
```
Integrate[(a*(b*Tan[c + d*x])^p)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]*(a*(b*Tan[c + d*x])^p)^n)/(d*(1 + n*p))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a(b\tan(c+dx))^p)^n\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a(b\tan(c+dx))^p)^n dx$$

$$\Big\downarrow \text{4142}$$

$$(b\tan(c+dx))^{-np}(a(b\tan(c+dx))^p)^n\int (b\tan(c+dx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(b\tan(c+dx))^{-np}(a(b\tan(c+dx))^p)^n\int (b\tan(c+dx))^{np}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b(b\tan(c+dx))^{-np}(a(b\tan(c+dx))^p)^n\int \frac{(b\tan(c+dx))^{np}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(c+dx)\right)(a(b\tan(c+dx))^p)^n}{d(np+1)}$$

input
```
Int[(a*(b*Tan[c + d*x])^p)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]*(a*(b*Tan[c + d*x])^p)^n)/(d*(1 + n*p))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

## Maple [F]

$$\int (a(b\tan (dx+c))^p)^n\,dx$$

input
```
int((a*(b*tan(d*x+c))^p)^n,x)
```

output
```
int((a*(b*tan(d*x+c))^p)^n,x)
```

## Fricas [F]

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int \left(\left(b\tan\left(dx+c\right)\right)^p a\right)^n \, dx$$

input
```
integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="fricas")
```

output
```
integral(((b*tan(d*x + c))^p*a)^n, x)
```

## Sympy [F]

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int \left(a\left(b\tan\left(c+dx\right)\right)^p\right)^n \, dx$$

input
```
integrate((a*(b*tan(d*x+c))**p)**n,x)
```

output
```
Integral((a*(b*tan(c + d*x))**p)**n, x)
```

## Maxima [F]

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int \left(\left(b\tan\left(dx+c\right)\right)^p a\right)^n \, dx$$

input
```
integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="maxima")
```

output
```
integrate(((b*tan(d*x + c))^p*a)^n, x)
```

## Giac [F]

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int ((b\tan(dx+c))^p a)^n \, dx$$

input
```
integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="giac")
```

output
```
integrate(((b*tan(d*x + c))^p*a)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int (a\,(b\tan(c+d\,x))^p)^n \, dx$$

input
```
int((a*(b*tan(c + d*x))^p)^n,x)
```

output
```
int((a*(b*tan(c + d*x))^p)^n, x)
```

## Reduce [F]

$$\int (a(b\tan(c+dx))^p)^n \, dx = b^{np} a^n \left(\int \tan(dx+c)^{np} \, dx\right)$$

input
```
int((a*(b*tan(d*x+c))^p)^n,x)
```

output
```
b**(n*p)*a**n*int(tan(c + d*x)**(n*p),x)
```

# 3.54 $\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result . . . . . 545
Mathematica [A] (verified) . . . . . 546
Rubi [A] (warning: unable to verify) . . . . . 546
Maple [B] (verified) . . . . . 550
Fricas [B] (verification not implemented) . . . . . 551
Sympy [F] . . . . . 552
Maxima [A] (verification not implemented) . . . . . 552
Giac [A] (verification not implemented) . . . . . 553
Mupad [F(-1)] . . . . . 554
Reduce [F] . . . . . 554

## Optimal result

Integrand size = 21, antiderivative size = 204

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{21\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{21\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{21\sqrt{d}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{32\sqrt{2}b} - \frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3}$$

output
```
-21/64*d^(1/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b+21
/64*d^(1/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b-21/64
*d^(1/2)*arctanh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a))
)*2^(1/2)/b-7/16*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d-1/4*cos(b*x+a)^4*(d
*tan(b*x+a))^(7/2)/b/d^3
```

## Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$
$$-\frac{\left(21\arcsin(\cos(a+bx)-\sin(a+bx))\csc(a+bx)\sqrt{\sin(2(a+bx))}+21\csc(a+bx)\log\left(\cos(a+bx)\right.\right.}{}$$

input
```
Integrate[Sin[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/64*((21*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x]*Sqrt[Sin[2*(a
+ b*x)]] + 21*Csc[a + b*x]*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a
 + b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + 18*Sin[2*(a + b*x)] - 2*Sin[4*(a + b*x
)])*Sqrt[d*Tan[a + b*x]])/b
```

## Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.23, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3071, 252, 252, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \sin(a+bx)^4\sqrt{d\tan(a+bx)}dx$$
$$\Big\downarrow \ 3071$$
$$\frac{d\int \frac{(d\tan(a+bx))^{9/2}}{(\tan^2(a+bx)d^2+d^2)^3}d(d\tan(a+bx))}{b}$$
$$\Big\downarrow \ 252$$

$$\frac{d\left(\frac{7}{8}\int \frac{(d\tan(a+bx))^{5/2}}{\left(\tan^2(a+bx)d^2+d^2\right)^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{7/2}}{4\left(d^2\tan^2(a+bx)+d^2\right)^2}\right)}{b}$$

$$\Big\downarrow \ 252$$

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{4}\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{3/2}}{2\left(d^2\tan^2(a+bx)+d^2\right)}\right)-\frac{(d\tan(a+bx))^{7/2}}{4\left(d^2\tan^2(a+bx)+d^2\right)^2}\right)}{b}$$

$$\Big\downarrow \ 266$$

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{(d\tan(a+bx))^{3/2}}{2\left(d^2\tan^2(a+bx)+d^2\right)}\right)-\frac{(d\tan(a+bx))^{7/2}}{4\left(d^2\tan^2(a+bx)+d^2\right)^2}\right)}{b}$$

$$\Big\downarrow \ 826$$

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(a+bx))^{3/2}}{2\left(d^2\tan^2(a+bx)+d^2\right)}\right)-\frac{(\cdots}{4(d^{\cdots}}\cdots}{b}$$

$$\Big\downarrow \ 1476$$

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+\cdots}{b}$$

$$\Big\downarrow \ 1082$$

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{\cdots}{b}$$

$$\Big\downarrow \ 217$$

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\cdots}{2(d^{2}\cdots}\cdots}{b}$$

$$\Big\downarrow \ 1479$$

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\mathrm{ar}\cdots}{\cdots}\cdots}{b}$$

$$\Big\downarrow \ 25$$

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\text{arct}}{}\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\text{arcta}}{}\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \; 1103$$

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\text{log}}{}\right.\right.\right.\right.}{b}$$

input

```
Int[Sin[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

output

```
(d*(-1/4*(d*Tan[a + b*x])^(7/2)/(d^2 + d^2*Tan[a + b*x]^2)^2 + (7*((3*((-(
ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + S
qrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/
2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2
]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/2 -
(d*Tan[a + b*x])^(3/2)/(2*(d^2 + d^2*Tan[a + b*x]^2))))/8))/b
```

## Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(158) = 316$.

Time = 4.26 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.44

| method | result |
|---|---|
| default | $-\frac{\left(-21\ln\left(-\frac{\cot(bx+a)\cos(bx+a)-2\cot(bx+a)-2\sin(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}-2\cos(bx+a)+\csc(bx+a)-\sin(bx+a)+2}{-1+\cos(bx+a)}\right)+21\ln\left(-\right.\right.}{}$ |

input
```
int(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/128/b*(-21*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-2*sin
(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x
+a)+2)/(-1+cos(b*x+a)))+21*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b
*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b
*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))+42*arctan((-sin(b*x+a)*(-2*sin(b*x+a)
*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))-42*arct
an((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a
)-1)/(-1+cos(b*x+a)))+(-4*(4*cos(b*x+a)^2-11)*(cos(b*x+a)+1)*sin(b*x+a)+72
*cos(b*x+a)*(cos(b*x+a)^2-1)*(cos(b*x+a)+1))*(-2*sin(b*x+a)*cos(b*x+a)/(co
s(b*x+a)+1)^2)^(1/2)+cos(b*x+a)*(72*cos(b*x+a)+72)*2^(1/2)*sin(b*x+a)^2*(-
sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2))*cos(b*x+a)*(d*tan(b*x+a))^(
1/2)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(cos(b*x+a)+1)*2^(1/2
)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(158) = 316$.

Time = 0.14 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.22

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$= \frac{16\left(4\cos(bx+a)^3 - 11\cos(bx+a)\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\sin(bx+a) - 42\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx}{d\cos(bx+a)-d\sin(bx+}\right.}{}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
1/256*(16*(4*cos(b*x + a)^3 - 11*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x
 + a))*sin(b*x + a) - 42*sqrt(2)*sqrt(d)*arctan(-sqrt(2)*sqrt(d)*sqrt(d*si
n(b*x + a)/cos(b*x + a))*cos(b*x + a)/(d*cos(b*x + a) - d*sin(b*x + a))) -
 21*sqrt(2)*sqrt(d)*arctan(1/2*(2*d*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(
b*x + a) + sqrt(2)*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 2*d)/(d*cos
(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a) - d)) - 21*sqrt(2)*sqrt(d)*arcta
n(-1/2*(2*d*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) - sqrt(2)*sqrt(
d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 2*d)/(d*cos(b*x + a)^2 + d*cos(b*x
+ a)*sin(b*x + a) - d)) - 21*sqrt(2)*sqrt(d)*log(4*d*cos(b*x + a)*sin(b*x
+ a) + 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d)*sqrt
(d*sin(b*x + a)/cos(b*x + a)) + d) + 21*sqrt(2)*sqrt(d)*log(4*d*cos(b*x +
a)*sin(b*x + a) - 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a))*s
qrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + d))/b
```

## Sympy [F]

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(a+bx)}}\sin^4{(a+bx)}\,dx$$

input
```
integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*sin(a + b*x)**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.10

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$= \frac{21\,d^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+\right.}{\sqrt{d}}\right.}{128\,bd^5}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output

```
1/128*(21*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x +
a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan
(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 8*(11*(d*
tan(b*x + a))^(7/2)*d^6 + 7*(d*tan(b*x + a))^(3/2)*d^8)/(d^4*tan(b*x + a)^
4 + 2*d^4*tan(b*x + a)^2 + d^4))/(b*d^5)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.20

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= \frac{\frac{42\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b} + \frac{42\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b} - \frac{21\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\right.}{b}}{}$$

input

```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output

```
1/128*(42*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 42*sqrt(2)*abs(d)^(3/2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/
b - 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x +
a))*sqrt(abs(d)) + abs(d))/b + 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a)
- sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 8*(11*sqrt(d*tan
(b*x + a))*d^5*tan(b*x + a)^3 + 7*sqrt(d*tan(b*x + a))*d^5*tan(b*x + a))/(
(d^2*tan(b*x + a)^2 + d^2)^2*b))/d
```

## Mupad [F(-1)]

Timed out.

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sin\left(a+b\,x\right)^4\,\sqrt{d\tan\left(a+b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^4*(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^4*(d*tan(a + b*x))^(1/2), x)
```

## Reduce [F]

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(bx+a\right)}\,\sin\left(bx+a\right)^4 dx\right)$$

input
```
int(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*sin(a + b*x)**4,x)
```

# 3.55 $\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result . . . . . 555
Mathematica [A] (verified) . . . . . 556
Rubi [A] (warning: unable to verify) . . . . . 556
Maple [B] (verified) . . . . . 560
Fricas [B] (verification not implemented) . . . . . 561
Sympy [F] . . . . . 562
Maxima [A] (verification not implemented) . . . . . 562
Giac [A] (verification not implemented) . . . . . 563
Mupad [F(-1)] . . . . . 563
Reduce [F] . . . . . 564

## Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{3\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} + \frac{3\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{3\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{4\sqrt{2}b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd}$$

output
```
-3/8*d^(1/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b+3/8*
d^(1/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b-3/8*d^(1/
2)*arctanh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))*2^(1
/2)/b-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d
```

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.60

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$
$$-\frac{\left(3\arcsin(\cos(a+bx)-\sin(a+bx))\csc(a+bx)+3\csc(a+bx)\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{s\ldots}\right.\right.}{8b}$$

input
```
Integrate[Sin[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/8*((3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x] + 3*Csc[a + b*x]
*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]] + 2*Sqrt[Sin[2*
(a + b*x)]])*Sqrt[Sin[2*(a + b*x)]]*Sqrt[d*Tan[a + b*x]])/b
```

## Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3071, 252, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin(a+bx)^2\sqrt{d\tan(a+bx)}dx$$
$$\Big\downarrow \text{ 3071}$$
$$\frac{d\int \frac{(d\tan(a+bx))^{5/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))}{b}$$
$$\Big\downarrow \text{ 252}$$

$$\frac{d\left(\frac{3}{4}\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{3/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{d\left(\frac{3}{2}\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{(d\tan(a+bx))^{3/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{826}$$

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(a+bx))^{3/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx}\right.\right.\right.}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\,t}\right.\right.}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(}{2(d^2\tan^2}\right.}{b}$$

$$\Big\downarrow \text{1479}$$

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan}{}\right.\right.\right.}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan(}{}\right.\right.\right.}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}}{}\right.\right.\right.}{b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{\right.}}{}\right.\right.\right.}{b}$$

input
```
Int[Sin[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*((3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + A
rcTan[1 + Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sq
rt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log
[d + Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]
))/2))/2 - (d*Tan[a + b*x])^(3/2)/(2*(d^2 + d^2*Tan[a + b*x]^2))))/b
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(132) = 264$.

Time = 0.54 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.32

| method | result |
|---|---|
| default | $-\frac{\left(-3\ln\left(-\frac{\cot(bx+a)\cos(bx+a)-2\cot(bx+a)-2\sin(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}-2\cos(bx+a)+\csc(bx+a)-\sin(bx+a)+2}{-1+\cos(bx+a)}\right)+3\ln\left(-\cdots\right.\right.}{}$ |

input
```
int(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/b*(-3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-2*sin(b
*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a
)+2)/(-1+cos(b*x+a)))+3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+
a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+
a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))+6*arctan((-sin(b*x+a)*(-2*sin(b*x+a)*cos
(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))-6*arctan((s
in(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/
(-1+cos(b*x+a)))+(4*cos(b*x+a)+4)*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(co
s(b*x+a)+1)^2)^(1/2))*cos(b*x+a)*(d*tan(b*x+a))^(1/2)/(cos(b*x+a)+1)/(-sin
(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(132) = 264$.

Time = 0.13 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.53

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{16\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)\sin(bx+a)+6\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{d\cos(bx+a)-d\sin(bx+a)}\right)+3\sqrt{2}\sqrt{d}\,\mathrm{arcta}}{}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-1/32*(16*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)*sin(b*x + a) + 6*
sqrt(2)*sqrt(d)*arctan(-sqrt(2)*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a))*
cos(b*x + a)/(d*cos(b*x + a) - d*sin(b*x + a))) + 3*sqrt(2)*sqrt(d)*arctan
(1/2*(2*d*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) + sqrt(2)*sqrt(d)
*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 2*d)/(d*cos(b*x + a)^2 + d*cos(b*x +
a)*sin(b*x + a) - d)) + 3*sqrt(2)*sqrt(d)*arctan(-1/2*(2*d*cos(b*x + a)^2
- 2*d*cos(b*x + a)*sin(b*x + a) - sqrt(2)*sqrt(d)*sqrt(d*sin(b*x + a)/cos(
b*x + a)) - 2*d)/(d*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a) - d)) + 3
*sqrt(2)*sqrt(d)*log(4*d*cos(b*x + a)*sin(b*x + a) + 2*sqrt(2)*(cos(b*x +
a)^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)
) + d) - 3*sqrt(2)*sqrt(d)*log(4*d*cos(b*x + a)*sin(b*x + a) - 2*sqrt(2)*(
cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d)*sqrt(d*sin(b*x + a)/co
s(b*x + a)) + d))/b
```

## Sympy [F]

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(a+bx)}\sin^2(a+bx)\,dx$$

input
```
integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*sin(a + b*x)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.11

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$= \frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\right.}{\sqrt{d}}}{16\,bd^3}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a)
 + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b
*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 8*(d*tan(b*
x + a))^(3/2)*d^4/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.26

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$

$$-\frac{\frac{8\,\sqrt{d\tan(bx+a)}d^3\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)b} - \frac{6\,\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\,\sqrt{d\tan(bx+a)}\right)}{2\,\sqrt{|d|}}\right)}{b} - \frac{6\,\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\,\sqrt{d\tan(bx+a)}\right)}{2\,\sqrt{|d|}}\right)}{b} + }{16\,d}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-1/16*(8*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)
*b) - 6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*
sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b - 6*sqrt(2)*abs(d)^(3/2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b +
 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*
sqrt(abs(d)) + abs(d))/b - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) - sqr
t(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b)/d
```

## Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sin\left(a+b\,x\right)^2\,\sqrt{d\tan\left(a+b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^2*(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^2*(d*tan(a + b*x))^(1/2), x)
```

## Reduce [F]

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\sin(bx+a)^2\,dx\right)$$

input
```
int(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*sin(a + b*x)**2,x)
```

# 3.56 $\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$


Optimal result . . . 565
Mathematica [A] (verified) . . . 565
Rubi [A] (verified) . . . 566
Maple [A] (verified) . . . 567
Fricas [B] (verification not implemented) . . . 567
Sympy [F] . . . 568
Maxima [A] (verification not implemented) . . . 568
Giac [A] (verification not implemented) . . . 568
Mupad [B] (verification not implemented) . . . 569
Reduce [F] . . . 569


## Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

output
```
-2*d/b/(d*tan(b*x+a))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{1}{(d\tan(a+bx))^{3/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{2d}{b\sqrt{d\tan(bx+a)}}$ | 17 |
| default | $-\frac{2d}{b\sqrt{d\tan(bx+a)}}$ | 17 |

input
```
int(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*d/b/(d*tan(b*x+a))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{b\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)/(b*sin(b*x + a))
```

## Sympy [F]

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(a+bx)}\csc^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\sqrt{d\tan(bx+a)}}{b\tan(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(d*tan(b*x + a))/(b*tan(b*x + a))
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d}{\sqrt{d\tan(bx+a)}b}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2*d/(sqrt(d*tan(b*x + a))*b)
```

## Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{b\sin\left(a+b\,x\right)^2}$$

input
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^2,x)
```

output
```
-(sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2))/(b
*sin(a + b*x)^2)
```

## Reduce [F]

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(bx+a\right)}\,\csc\left(bx+a\right)^2dx\right)$$

input
```
int(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**2,x)
```

# 3.57 $\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result . . . . . 570
Mathematica [A] (verified) . . . . . 570
Rubi [A] (verified) . . . . . 571
Maple [A] (verified) . . . . . 572
Fricas [A] (verification not implemented) . . . . . 573
Sympy [F] . . . . . 573
Maxima [A] (verification not implemented) . . . . . 573
Giac [A] (verification not implemented) . . . . . 574
Mupad [B] (verification not implemented) . . . . . 574
Reduce [F] . . . . . 575

## Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

output
```
-2/5*d^3/b/(d*tan(b*x+a))^(5/2)-2*d/b/(d*tan(b*x+a))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2d(4+\csc^2(a+bx))}{5b\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d*(4 + Csc[a + b*x]^2))/(5*b*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^4}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{\tan^2(a+bx)d^2+d^2}{(d\tan(a+bx))^{7/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{d\int \left(\frac{d^2}{(d\tan(a+bx))^{7/2}}+\frac{1}{(d\tan(a+bx))^{3/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\left(-\frac{2d^2}{5(d\tan(a+bx))^{5/2}}-\frac{2}{\sqrt{d\tan(a+bx)}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*((-2*d^2)/(5*(d*Tan[a + b*x])^(5/2)) - 2/Sqrt[d*Tan[a + b*x]]))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

| method | result | size |
|---|---|---|
| default | $\frac{2\sqrt{d\tan(bx+a)}\left(4\cot(bx+a)^3-5\cot(bx+a)\csc(bx+a)^2\right)}{5b}$ | 43 |

input
```
int(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/b*(d*tan(b*x+a))^(1/2)*(4*cot(b*x+a)^3-5*cot(b*x+a)*csc(b*x+a)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(4\cos\left(bx+a\right)^3 - 5\cos\left(bx+a\right)\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{5\left(b\cos\left(bx+a\right)^2 - b\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(4*cos(b*x + a)^3 - 5*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))
/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

## Sympy [F]

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan\left(a+bx\right)}\csc^4\left(a+bx\right)dx$$

input
```
integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(5\,d^2\tan\left(bx+a\right)^2 + d^2\right)d}{5\,\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}b}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/5*(5*d^2*tan(b*x + a)^2 + d^2)*d/((d*tan(b*x + a))^(5/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(5\,d^4\tan\left(bx+a\right)^2+d^4\right)}{5\,\sqrt{d\tan\left(bx+a\right)}bd^3\tan\left(bx+a\right)^2}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2/5*(5*d^4*tan(b*x + a)^2 + d^4)/(sqrt(d*tan(b*x + a))*b*d^3*tan(b*x + a)
^2)
```

## Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.49

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$= \frac{8\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\left(e^{a\,2i+b\,x\,2i}\,2i+e^{a\,4i+b\,x\,4i}\,2i-e^{a\,6i+b\,x\,6i}\,1i-i\right)}{5\,b\left(e^{a\,2i+b\,x\,2i}-1\right)^3}$$

input
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^4,x)
```

output
```
(8*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*(exp
(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*2i - exp(a*6i + b*x*6i)*1i - 1i))/
(5*b*(exp(a*2i + b*x*2i) - 1)^3)
```

## Reduce [F]

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\csc(bx+a)^4\,dx\right)$$

input

```
int(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x)
```

output

```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**4,x)
```

# 3.58 $\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result . . . . . 576
Mathematica [A] (verified) . . . . . 576
Rubi [A] (verified) . . . . . 577
Maple [A] (verified) . . . . . 578
Fricas [A] (verification not implemented) . . . . . 579
Sympy [F(-1)] . . . . . 579
Maxima [A] (verification not implemented) . . . . . 579
Giac [A] (verification not implemented) . . . . . 580
Mupad [B] (verification not implemented) . . . . . 580
Reduce [F] . . . . . 581

## Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d^5}{9b(d\tan(a+bx))^{9/2}} - \frac{4d^3}{5b(d\tan(a+bx))^{5/2}} - \frac{2d}{b\sqrt{d\tan(a+bx)}}$$

output
```
-2/9*d^5/b/(d*tan(b*x+a))^(9/2)-4/5*d^3/b/(d*tan(b*x+a))^(5/2)-2*d/b/(d*ta
n(b*x+a))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{2d(-21+20\cos(2(a+bx))-4\cos(4(a+bx)))\csc^4(a+bx)}{45b\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^6*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(2*d*(-21 + 20*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4)/(45*
b*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^6}dx$$
$$\Big\downarrow \text{3071}$$
$$\frac{d\int \frac{\left(\tan^2(a+bx)d^2+d^2\right)^2}{(d\tan(a+bx))^{11/2}}d(d\tan(a+bx))}{b}$$
$$\Big\downarrow \text{244}$$
$$\frac{d\int \left(\frac{d^4}{(d\tan(a+bx))^{11/2}}+\frac{2d^2}{(d\tan(a+bx))^{7/2}}+\frac{1}{(d\tan(a+bx))^{3/2}}\right)d(d\tan(a+bx))}{b}$$
$$\Big\downarrow \text{2009}$$
$$\frac{d\left(-\frac{2d^4}{9(d\tan(a+bx))^{9/2}}-\frac{4d^2}{5(d\tan(a+bx))^{5/2}}-\frac{2}{\sqrt{d\tan(a+bx)}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^6*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*((-2*d^4)/(9*(d*Tan[a + b*x])^(9/2)) - (4*d^2)/(5*(d*Tan[a + b*x])^(5/2
)) - 2/Sqrt[d*Tan[a + b*x]]))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $-\frac{2 \cot(bx+a) \csc(bx+a)^{4} \left(32 \cos(bx+a)^{4}-72 \cos(bx+a)^{2}+45\right) \sqrt{d \tan(bx+a)}}{45 b}$ | 52 |

input
```
int(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45/b*cot(b*x+a)*csc(b*x+a)^4*(32*cos(b*x+a)^4-72*cos(b*x+a)^2+45)*(d*ta
n(b*x+a))^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(32\cos\left(bx+a\right)^5-72\cos\left(bx+a\right)^3+45\cos\left(bx+a\right)\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45\left(b\cos\left(bx+a\right)^4-2\,b\cos\left(bx+a\right)^2+b\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(32*cos(b*x + a)^5 - 72*cos(b*x + a)^3 + 45*cos(b*x + a))*sqrt(d*sin
(b*x + a)/cos(b*x + a))/((b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)*sin(b
*x + a))
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(1/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(45\,d^4\tan\left(bx+a\right)^4+18\,d^4\tan\left(bx+a\right)^2+5\,d^4\right)d}{45\,\left(d\tan\left(bx+a\right)\right)^{\frac{9}{2}}\,b}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/45*(45*d^4*tan(b*x + a)^4 + 18*d^4*tan(b*x + a)^2 + 5*d^4)*d/((d*tan(b*
x + a))^(9/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(45\,d^6\tan\left(bx+a\right)^4+18\,d^6\tan\left(bx+a\right)^2+5\,d^6\right)}{45\,\sqrt{d\tan\left(bx+a\right)}bd^5\tan\left(bx+a\right)^4}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2/45*(45*d^6*tan(b*x + a)^4 + 18*d^6*tan(b*x + a)^2 + 5*d^6)/(sqrt(d*tan(
b*x + a))*b*d^5*tan(b*x + a)^4)
```

## Mupad [B] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 356, normalized size of antiderivative = 5.65

$$\begin{aligned}\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = &-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{45\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}\\&+\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{45\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2}\\&-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{15\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3}\\&-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{9\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^4}\\&-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^5}\end{aligned}$$

input
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^6,x)
```

output
```
((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2)*64i)/(45*b*(exp(a*2i + b*x*2i) - 1)^2) - ((exp(a*2i + b
*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(
1/2)*64i)/(45*b*(exp(a*2i + b*x*2i) - 1)) - ((exp(a*2i + b*x*2i) + 1)*(-(d
*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*32i)/(15*b*
(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b
*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*64i)/(9*b*(exp(a*2i + b*x
*2i) - 1)^4) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i)
)/(exp(a*2i + b*x*2i) + 1))^(1/2)*32i)/(9*b*(exp(a*2i + b*x*2i) - 1)^5)
```

## Reduce [F]

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(bx+a\right)}\,\csc\left(bx+a\right)^6 dx\right)$$

input
```
int(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**6,x)
```

# 3.59 $\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 582
Mathematica [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . 582
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 583
Maple [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . 585
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 586
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 587
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 587
Giac [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 587
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 588
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 588

## Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}} - \frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{5\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{12b}$$

output
```
-5/6*d*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)-1/3*d*sin(b*x+a)^3/b/(d*tan(b*x+a
))^(1/2)+5/12*csc(b*x+a)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2
*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.61 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.32

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{\cos(2(a+bx))\sec(a+bx)\left(-5\sqrt[4]{-1}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sec^2(a+bx)+(-\right.}{6b\sqrt{\sec^2(a+bx)}\sqrt{\tan(a+bx)}\left(-1+\tan^2\right.}$$

input
```
Integrate[Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/6*(Cos[2*(a + b*x)]*Sec[a + b*x]*(-5*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1
)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sec[a + b*x]^2 + (-6 + Cos[2*(a + b*x)])*
Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[a + b*x]])*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Sec
[a + b*x]^2]*Sqrt[Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3078, 3042, 3078, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{5}{6}\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{5}{6}\left(\frac{1}{2}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{1}{2}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{5}{6}\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{5}{6}\left(\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5}{6}\left(\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{2b}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/3*(d*Sin[a + b*x]^3)/(b*Sqrt[d*Tan[a + b*x]]) + (5*(-((d*Sin[a + b*x])/
(b*Sqrt[d*Tan[a + b*x]])) + (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqr
t[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(2*b)))/6
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.19 (sec) , antiderivative size = 1585, normalized size of antiderivative = 15.10

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1585 |

input
```
int(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/96/b*(d*tan(b*x+a))^(1/2)*(2^(1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)
+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-2*sin(
b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+
a)+2)/(-1+cos(b*x+a)))*(6*cos(b*x+a)^2+6*cos(b*x+a)-3*cot(b*x+a)-3*csc(b*x
+a))+(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b
*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2
)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))*(-12*cos(b*
x+a)^2-12*cos(b*x+a))+2^(1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^
(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-2*sin(b*x+a)*
cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(
-1+cos(b*x+a)))*(-6*cos(b*x+a)^2-6*cos(b*x+a)+3*cot(b*x+a)+3*csc(b*x+a))+(
-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-
2*cot(b*x+a)+2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2
)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))*(12*cos(b*x+a)^2+
12*cos(b*x+a))+2^(1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*a
rctan((-sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b
*x+a)-1)/(-1+cos(b*x+a)))*(12*cos(b*x+a)^2+12*cos(b*x+a)-6*cot(b*x+a)-6*cs
c(b*x+a))+arctan((-sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^
(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1
)^2)^(1/2)*(-24*cos(b*x+a)^2-24*cos(b*x+a))+2^(1/2)*(-2*sin(b*x+a)*cos(...
```

## Fricas [F]

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan\left(bx+a\right)}\sin\left(bx+a\right)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(b*x + a))*sin(b*x + a)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

## Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sin\left(a+b\,x\right)^3\,\sqrt{d\tan\left(a+b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2), x)
```

## Reduce [F]

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(bx+a\right)}\,\sin\left(bx+a\right)^3 dx\right)$$

input
```
int(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*sin(a + b*x)**3,x)
```

# 3.60 $\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 589
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 589
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 590
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 592
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 592
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 593
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 593
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 593
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 594
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 594

## Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{2b}$$

output
```
-d*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)+1/2*csc(b*x+a)*InverseJacobiAM(a-1/4*
Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{\cos(a+bx)\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a+bx)}}{b}$$

input
```
Integrate[Sin[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(Cos[a + b*x]*(-1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt
[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3078, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{1}{2}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{2b}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sin[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

output
```
-((d*Sin[a + b*x])/(b*Sqrt[d*Tan[a + b*x]])) + (Csc[a + b*x]*EllipticF[a -
 Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(2*b)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.63

| method | result |
|---|---|
| default | $\frac{\left(-\frac{\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1},\frac{\sqrt{2}}{2}\right)(-\cot(bx+}{2}}{b}$ |

input
```
int(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(
1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1
/2),1/2*2^(1/2))*(-cot(b*x+a)-csc(b*x+a))-cos(b*x+a))*(d*tan(b*x+a))^(1/2)
```

## Fricas [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan\left(bx+a\right)}\sin\left(bx+a\right)\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

## Sympy [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(a+bx)}\sin(a+bx)\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*sin(a + b*x), x)
```

## Maxima [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\sin(bx+a)\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

## Giac [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\sin(bx+a)\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sin\left(a+b\,x\right)\,\sqrt{d\tan\left(a+b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)*(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)*(d*tan(a + b*x))^(1/2), x)
```

## Reduce [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(bx+a\right)}\,\sin\left(bx+a\right)dx\right)$$

input
```
int(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*sin(a + b*x),x)
```

# 3.61 $\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$


Optimal result . . . 595
Mathematica [C] (verified) . . . 595
Rubi [A] (verified) . . . 596
Maple [B] (verified) . . . 597
Fricas [C] (verification not implemented) . . . 598
Sympy [F] . . . 598
Maxima [F] . . . 599
Giac [F] . . . 599
Mupad [F(-1)] . . . 599
Reduce [F] . . . 600


## Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{b}$$

output
```
csc(b*x+a)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*t
an(b*x+a))^(1/2)/b
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\sqrt[4]{-1}\cos(a+bx)\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}}{b\sqrt{\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*(-1)^(1/4)*Cos[a + b*x]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*
x]]], -1]*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Tan[a + b*x]]
)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{b}$$

input
```
Int[Csc[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*T
an[a + b*x]])/b
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(43) = 86$.

Time = 0.71 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.28

| method | result |
|---|---|
| default | $\frac{\sqrt{d\tan(bx+a)}\,\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{\csc(bx+a)-c\right.}{b}$ |

input
```
int(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(d*tan(b*x+a))^(1/2)*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*
cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-c
ot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(cot(b*x+a)+csc(b*x+a))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{\sqrt{i\,d}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)+\sqrt{-i\,d}F(\arcsin\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)|-}{b}$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + sqrt(-
I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1))/b
```

## Sympy [F]

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan\left(a+bx\right)}\csc\left(a+bx\right)dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*csc(a + b*x), x)
```

## Maxima [F]

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(b*x + a))*csc(b*x + a), x)
```

## Giac [F]

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(b*x + a))*csc(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \frac{\sqrt{d\tan(a+b\,x)}}{\sin(a+b\,x)}\,dx$$

input
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x),x)
```

output
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x), x)
```

## Reduce [F]

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx=\sqrt{d}\left(\int\sqrt{\tan\left(bx+a\right)}\,\csc\left(bx+a\right)dx\right)$$

input
```
int(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x),x)
```

# 3.62 $\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$


Optimal result . . . . . 601
Mathematica [C] (warning: unable to verify) . . . . . 601
Rubi [A] (verified) . . . . . 602
Maple [A] (verified) . . . . . 604
Fricas [C] (verification not implemented) . . . . . 604
Sympy [F] . . . . . 605
Maxima [F] . . . . . 605
Giac [F] . . . . . 606
Mupad [F(-1)] . . . . . 606
Reduce [F] . . . . . 606


## Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b}$$

output
```
-2/3*d*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)+2/3*csc(b*x+a)*InverseJacobiAM(a-
1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.49

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{2\cos(2(a+bx))\csc^3(a+bx)(d\tan(a+bx))^{3/2}\left(\sqrt{\sec^2(a+bx)}+2\sqrt[4]{-1}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan\right.\right.\right.}{3bd\sqrt{\sec^2(a+bx)}\left(-1+\tan^2(a+bx)\right)}$$

input
```
Integrate[Csc[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(2*Cos[2*(a + b*x)]*Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2)*(Sqrt[Sec[a + b*
x]^2] + 2*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -
1]*Tan[a + b*x]^(3/2)))/(3*b*d*Sqrt[Sec[a + b*x]^2]*(-1 + Tan[a + b*x]^2))
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3079, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \textbf{3079}$$

$$\frac{2}{3}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2}{3}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \textbf{3081}$$

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\downarrow \text{3053}$$

$$\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\downarrow \text{3042}$$

$$\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\downarrow \text{3120}$$

$$\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d*Csc[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF
[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.61

| method | result |
|---|---|
| default | $\frac{2\sqrt{d\tan(bx+a)}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{\csc(bx+a)}\right.\right.}{3b}$ |

input
```
int(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*(d*tan(b*x+a))^(1/2)*((csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)
+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a
)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(cot(b*x+a)+csc(b*x+a))-cot(b*x+a)*csc(
b*x+a))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$

$$-\frac{2\left(\left(\cos\left(bx+a\right)^2-1\right)\sqrt{i\,d}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)+\left(\cos\left(bx+a\right)^2-1\right)\sqrt{-i\,d}F\right.}{3\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*((cos(b*x + a)^2 - 1)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*si
n(b*x + a)), -1) + (cos(b*x + a)^2 - 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b
*x + a) - I*sin(b*x + a)), -1) - sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x
 + a))/(b*cos(b*x + a)^2 - b)
```

## Sympy [F]

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(a+bx)}}\csc^3{(a+bx)}\,dx$$

input
```
integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**3, x)
```

## Maxima [F]

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(bx+a)}}\csc{(bx+a)}^3\,dx$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^3, x)
```

## Giac [F]

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx=\int \sqrt{d\tan\left(bx+a\right)}\csc\left(bx+a\right)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx=\int \frac{\sqrt{d\tan\left(a+b\,x\right)}}{\sin\left(a+b\,x\right)^3}\,dx$$

input
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^3,x)
```

output
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^3, x)
```

## Reduce [F]

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx=\sqrt{d}\,\left(\int \sqrt{\tan\left(bx+a\right)}\,\csc\left(bx+a\right)^3dx\right)$$

input
```
int(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**3,x)
```

# 3.63 $\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx$


Optimal result . . . 607
Mathematica [C] (warning: unable to verify) . . . 607
Rubi [A] (verified) . . . 608
Maple [A] (verified) . . . 611
Fricas [C] (verification not implemented) . . . 611
Sympy [F] . . . 612
Maxima [F] . . . 612
Giac [F] . . . 612
Mupad [F(-1)] . . . 613
Reduce [F] . . . 613


## Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{4\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{7b}$$

output
```
-4/7*d*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-2/7*d*csc(b*x+a)^3/b/(d*tan(b*x+a
))^(1/2)+4/7*csc(b*x+a)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*
a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d\cos(2(a+bx))\csc^3(a+bx)\left((-2+\cos(2(a+bx)))\sec^2(a+bx)^{3/2}-4\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{\right.\right.\right.}{7b\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}\left(-1+\tan^2(a+bx)\right)}$$

input
```
Integrate[Csc[a + b*x]^5*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d*Cos[2*(a + b*x)]*Csc[a + b*x]^3*((-2 + Cos[2*(a + b*x)])*(Sec[a + b*
x]^2)^(3/2) - 4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x
]]], -1]*Tan[a + b*x]^(7/2)))/(7*b*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x
]]*(-1 + Tan[a + b*x]^2))
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3079, 3042, 3079, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^5}dx$$

$$\Big\downarrow \text{ 3079}$$

$$\frac{6}{7}\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{6}{7}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^3}dx-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 3079}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$\big\downarrow$ 3081

$$\frac{6}{7}\left(\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$\big\downarrow$ 3042

$$\frac{6}{7}\left(\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$\big\downarrow$ 3053

$$\frac{6}{7}\left(\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$\big\downarrow$ 3042

$$\frac{6}{7}\left(\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$\big\downarrow$ 3120

$$\frac{6}{7}\left(\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]^5*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d*Csc[a + b*x]^3)/(7*b*Sqrt[d*Tan[a + b*x]]) + (6*((-2*d*Csc[a + b*x])
/(3*b*Sqrt[d*Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]
*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)))/7
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| default | $-\frac{2\sqrt{d\tan(bx+a)}\,\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{\csc(bx+\right.}\right.}{7b}$ |

input
```
int(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7/b*(d*tan(b*x+a))^(1/2)*((csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a
)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+
a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-2*cot(b*x+a)-2*csc(b*x+a))+cot(b*x+a
)*csc(b*x+a)^3*(-2*cos(b*x+a)^2+3))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.50

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$
$$-\frac{2\left(2\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\sqrt{i\,d}F(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+2\left(\cos(\right.\right.}{7\left(b\,c\right.}$$

input
```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/7*(2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(I*d)*elliptic_f(arcsi
n(cos(b*x + a) + I*sin(b*x + a)), -1) + 2*(cos(b*x + a)^4 - 2*cos(b*x + a)
^2 + 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) -
 (2*cos(b*x + a)^3 - 3*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b
*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

## Sympy [F]

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(a+bx)}\csc^5(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**5*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**5, x)
```

## Maxima [F]

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\csc(bx+a)^5\,dx$$

input
```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^5, x)
```

## Giac [F]

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\csc(bx+a)^5\,dx$$

input
```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \frac{\sqrt{d\tan(a+b\,x)}}{\sin(a+b\,x)^5}\,dx$$

input
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^5,x)
```

output
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^5, x)
```

## Reduce [F]

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\csc(bx+a)^5\,dx\right)$$

input
```
int(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**5,x)
```

# 3.64 $\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$


Optimal result . . . . . 614
Mathematica [A] (verified) . . . . . 615
Rubi [A] (warning: unable to verify) . . . . . 615
Maple [B] (warning: unable to verify) . . . . . 620
Fricas [B] (verification not implemented) . . . . . 621
Sympy [F(-1)] . . . . . 621
Maxima [A] (verification not implemented) . . . . . 622
Giac [A] (verification not implemented) . . . . . 622
Mupad [F(-1)] . . . . . 623
Reduce [F] . . . . . 623


## Optimal result

Integrand size = 21, antiderivative size = 224

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{45d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{45d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{45d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{32\sqrt{2}b} + \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3}$$

output
```
45/64*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b-45/
64*d^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b-45/64*
d^(3/2)*arctanh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))
*2^(1/2)/b+45/16*d*(d*tan(b*x+a))^(1/2)/b-9/16*cos(b*x+a)^2*(d*tan(b*x+a))
^(5/2)/b/d-1/4*cos(b*x+a)^4*(d*tan(b*x+a))^(9/2)/b/d^3
```

## Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.55

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{d\csc(a+bx)\left(-143\sin(a+bx)-45\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}+45\log\left(\cos(a\right.\right.}{}$$

input
```
Integrate[Sin[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/64*(d*Csc[a + b*x]*(-143*Sin[a + b*x] - 45*ArcSin[Cos[a + b*x] - Sin[a
+ b*x]]*Sqrt[Sin[2*(a + b*x)]] + 45*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt
[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]] - 14*Sin[3*(a + b*x)] + Sin[5*(
a + b*x)])*Sqrt[d*Tan[a + b*x]])/b
```

## Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.24, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3071, 252, 252, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(a+bx)^4(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \ 3071$$

$$\frac{d\int \frac{(d\tan(a+bx))^{11/2}}{(\tan^2(a+bx)d^2+d^2)^3}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \ 252$$

$$\frac{d\left(\frac{9}{8}\int \frac{(d\tan(a+bx))^{7/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{9/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\Big\downarrow$ 252

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\int \frac{(d\tan(a+bx))^{3/2}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{9/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\Big\downarrow$ 262

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-d^2\int \frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)d^2+d^2)}d(d\tan(a+bx))\right)-\frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a-}{4(d^2\tan^2(a}}{b}$$

$\Big\downarrow$ 266

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\int \frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{9/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\Big\downarrow$ 755

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)\right)-\frac{(d\tan(a+bx)}{2(d^2\tan^2(a+bx}}{b}$$

$\Big\downarrow$ 1476

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d}}{2d}\right.\right.\right.}{b}$$

$\Big\downarrow$ 1082

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\tan}{d^4\tan^4(}}{}\right.\right.\right.}{b}$$

$\Big\downarrow$ 217

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)\right.\right.}{b}$$

$\downarrow$ 1479

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d}}{2\sqrt{2}\sqrt{d}}}{2d}\right.\right.\right.\right.}{b}$$

$\downarrow$ 25

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a}}{2\sqrt{2}\sqrt{d}}}{2d}\right.\right.\right.\right.}{b}$$

$\downarrow$ 27

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a}}{2\sqrt{d}}}{2d}\right.\right.\right.\right.}{b}$$

$\downarrow$ 1103

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}\right.\right.\right.\right.}{b}$$

input
```
Int[Sin[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*(-1/4*(d*Tan[a + b*x])^(9/2)/(d^2 + d^2*Tan[a + b*x]^2)^2 + (9*(-1/2*(d
*Tan[a + b*x])^(5/2)/(d^2 + d^2*Tan[a + b*x]^2) + (5*(-2*d^2*((-(ArcTan[1
- Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sq
rt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/
2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*
d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)) + 2
*Sqrt[d*Tan[a + b*x]]))/4))/8))/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. $2(174) = 348$.

Time = 11.26 (sec) , antiderivative size = 1375, normalized size of antiderivative = 6.14

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1375 |

input

```
int(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/256/b*(d*tan(b*x+a))^(1/2)*d*(2^(1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x
+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-2*s
in(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b
*x+a)+2)/(-1+cos(b*x+a)))*(-180*cos(b*x+a)^2-180*cos(b*x+a)-45*csc(b*x+a)*
(4*cos(b*x+a)^4-4*cos(b*x+a)^2-1)*(cos(b*x+a)+1))+csc(b*x+a)*ln(-(cot(b*x+
a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x
+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))*(-s
in(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(360*cos(b*x+a)^5+360*cos(b*x
+a)^4-360*cos(b*x+a)^3+360*cos(b*x+a)^2*sin(b*x+a)-360*cos(b*x+a)^2+360*si
n(b*x+a)*cos(b*x+a))+2^(1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-2*sin(b*x+a)*c
os(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(-
1+cos(b*x+a)))*(180*cos(b*x+a)^2+180*cos(b*x+a)+45*csc(b*x+a)*(4*cos(b*x+a
)^4-4*cos(b*x+a)^2-1)*(cos(b*x+a)+1))+csc(b*x+a)*ln(-(cot(b*x+a)*cos(b*x+a
)-2*cot(b*x+a)+2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1
/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))*(-sin(b*x+a)*co
s(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(-360*cos(b*x+a)^5-360*cos(b*x+a)^4+360*c
os(b*x+a)^3-360*cos(b*x+a)^2*sin(b*x+a)+360*cos(b*x+a)^2-360*sin(b*x+a)*co
s(b*x+a))+2^(1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan
((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+...
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(174) = 348$.

Time = 0.22 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.01

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{90\sqrt{2}d^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}(\cos(bx+a)-\sin(bx+a))}{2\sqrt{d}\sin(bx+a)}\right)-45\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{2\,d\cos(bx+a)^2-2\,d\cos(bx+a)\sin(bx+a)+\sqrt{}}{2\left(d\cos(bx+a)^2+d\cos(bx+a)\sin\right.}\right.}{}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/256*(90*sqrt(2)*d^(3/2)*arctan(-1/2*sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x
 + a))*(cos(b*x + a) - sin(b*x + a))/(sqrt(d)*sin(b*x + a))) - 45*sqrt(2)*
d^(3/2)*arctan(1/2*(2*d*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) + s
qrt(2)*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 2*d)/(d*cos(b*x + a)^2
+ d*cos(b*x + a)*sin(b*x + a) - d)) - 45*sqrt(2)*d^(3/2)*arctan(-1/2*(2*d*
cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) - sqrt(2)*sqrt(d)*sqrt(d*si
n(b*x + a)/cos(b*x + a)) - 2*d)/(d*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x
 + a) - d)) + 45*sqrt(2)*d^(3/2)*log(4*d*cos(b*x + a)*sin(b*x + a) + 2*sqr
t(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d)*sqrt(d*sin(b*x +
 a)/cos(b*x + a)) + d) - 45*sqrt(2)*d^(3/2)*log(4*d*cos(b*x + a)*sin(b*x +
 a) - 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d)*sqrt(
d*sin(b*x + a)/cos(b*x + a)) + d) + 16*(4*d*cos(b*x + a)^4 - 17*d*cos(b*x
+ a)^2 - 32*d)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/b
```

## Sympy [F(-1)]

Timed out.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{90\sqrt{2}d^{\frac{13}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+90\sqrt{2}d^{\frac{13}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+45\sqrt{2}d^{\frac{13}{2}}\,\mathrm{lo}}{}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-1/128*(90*sqrt(2)*d^(13/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d
*tan(b*x + a)))/sqrt(d)) + 90*sqrt(2)*d^(13/2)*arctan(-1/2*sqrt(2)*(sqrt(2
)*sqrt(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 45*sqrt(2)*d^(13/2)*log(d*t
an(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 45*sqrt(2)*d^(13
/2)*log(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 256*s
qrt(d*tan(b*x + a))*d^6 - 8*(17*(d*tan(b*x + a))^(5/2)*d^8 + 13*sqrt(d*tan
(b*x + a))*d^10)/(d^4*tan(b*x + a)^4 + 2*d^4*tan(b*x + a)^2 + d^4))/(b*d^5
)
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.20

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{\frac{90\sqrt{2}d^2\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{90\sqrt{2}d^2\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{45\sqrt{2}d^2\sqrt{|d|}\log\left(d\tan(bx+\right.}{}}{}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-1/128*(90*sqrt(2)*d^2*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d
)) + 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 90*sqrt(2)*d^2*sqrt(abs(d))
*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(
abs(d)))/b + 45*sqrt(2)*d^2*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt
(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 45*sqrt(2)*d^2*sqrt(abs(d))*lo
g(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b -
 256*sqrt(d*tan(b*x + a))*d^2/b - 8*(17*sqrt(d*tan(b*x + a))*d^6*tan(b*x +
 a)^2 + 13*sqrt(d*tan(b*x + a))*d^6)/((d^2*tan(b*x + a)^2 + d^2)^2*b))/d
```

## Mupad [F(-1)]

Timed out.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx=\int \sin\left(a+b\,x\right)^4\,{\left(d\,\tan(a+b\,x)\right)}^{3/2}\,dx$$

input
```
int(sin(a + b*x)^4*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^4*(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx=\sqrt{d}\,\left(\int \sqrt{\tan\left(bx+a\right)}\,\sin\left(bx+a\right)^4\tan\left(bx+a\right)dx\right)d$$

input
```
int(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*sin(a + b*x)**4*tan(a + b*x),x)*d
```

# 3.65 $\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . . . . . . . . 624
Mathematica [A] (verified) . . . . . . . . . . 625
Rubi [A] (warning: unable to verify) . . . . . . . . . . 625
Maple [B] (warning: unable to verify) . . . . . . . . . . 630
Fricas [B] (verification not implemented) . . . . . . . . . . 631
Sympy [F(-1)] . . . . . . . . . . 631
Maxima [A] (verification not implemented) . . . . . . . . . . 632
Giac [A] (verification not implemented) . . . . . . . . . . 632
Mupad [F(-1)] . . . . . . . . . . 633
Reduce [F] . . . . . . . . . . 633

## Optimal result

Integrand size = 21, antiderivative size = 194

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{5d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{5d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{5d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{4\sqrt{2}b} + \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd}$$

output
```
5/8*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b-5/8*d
^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b-5/8*d^(3/2
)*arctanh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))*2^(1/
2)/b+5/2*d*(d*tan(b*x+a))^(1/2)/b-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(5/2)/b/
d
```

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.58

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d\csc(a+bx)\left(17\sin(a+bx)+5\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-5\log\right.}{}$$

input
```
Integrate[Sin[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*Csc[a + b*x]*(17*Sin[a + b*x] + 5*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*S
qrt[Sin[2*(a + b*x)]] - 5*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a
+ b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])
/(8*b)
```

## Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.23, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {3042, 3071, 252, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{7/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{d\left(\frac{5}{4}\int \frac{(d\tan(a+bx))^{3/2}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx)) - \frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 262

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)} - d^2\int \frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)d^2+d^2)}d(d\tan(a+bx))\right) - \frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 266

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)} - 2d^2\int \frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right) - \frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 755

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)} - 2d^2\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d} + \frac{\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)\right) - \frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d}$$

$\downarrow$ 1476

$$d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)} - 2d^2\left(\frac{\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan}}{2d}\right.\right.\right.$$
$$b$$

$\downarrow$ 1082

$$d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)} - 2d^2\left(\frac{\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}} - \frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d} + \frac{\int \frac{d-d^2\tan^2(a}{d^4\tan^4(a+bx}\right.\right.\right.$$
$$b$$

$\downarrow$ 217

$$d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)} - 2d^2\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d} + \frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)\right) -$$
$$b$$

$\downarrow$ 1479

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}\right.\right.\right.}{b}$$

$\downarrow$ 25

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}\right.\right.\right.}{b}$$

$\downarrow$ 27

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}}{2d}\right.\right.\right.}{b}$$

$\downarrow$ 1103

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log}{}}{2d}\right.\right.\right.}{b}$$

input
```
Int[Sin[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*(-1/2*(d*Tan[a + b*x])^(5/2)/(d^2 + d^2*Tan[a + b*x]^2) + (5*(-2*d^2*((
-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 +
 Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sq
rt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d
 + Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))
/(2*d)) + 2*Sqrt[d*Tan[a + b*x]]))/4))/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(148) = 296$.

Time = 6.93 (sec) , antiderivative size = 1065, normalized size of antiderivative = 5.49

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1065 |

input
```
int(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/b*(d*tan(b*x+a))^(1/2)*d*(2^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*
x+a)+2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(
b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))*(-2*sin(b*x+a)*cos(b*x+a)
/(cos(b*x+a)+1)^2)^(1/2)*(10*cos(b*x+a)^2+10*cos(b*x+a)-5*cot(b*x+a)-5*csc
(b*x+a))+ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-2*sin(b*x+
a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)+2
)/(-1+cos(b*x+a)))*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(-20*co
s(b*x+a)^2-20*cos(b*x+a))+2^(1/2)*ln((2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+
a)/(cos(b*x+a)+1)^2)^(1/2)-cot(b*x+a)*cos(b*x+a)+sin(b*x+a)+2*cos(b*x+a)-c
sc(b*x+a)+2*cot(b*x+a)-2)/(-1+cos(b*x+a)))*(-2*sin(b*x+a)*cos(b*x+a)/(cos(
b*x+a)+1)^2)^(1/2)*(-10*cos(b*x+a)^2-10*cos(b*x+a)+5*cot(b*x+a)+5*csc(b*x+
a))+ln((2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cot
(b*x+a)*cos(b*x+a)+sin(b*x+a)+2*cos(b*x+a)-csc(b*x+a)+2*cot(b*x+a)-2)/(-1+
cos(b*x+a)))*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(20*cos(b*x+a
)^2+20*cos(b*x+a))+2^(1/2)*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(c
os(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))*(-2*sin(b*x+a)*cos(b*
x+a)/(cos(b*x+a)+1)^2)^(1/2)*(20*cos(b*x+a)^2+20*cos(b*x+a)-10*cot(b*x+a)-
10*csc(b*x+a))+arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+
a)+1)^2)^(1/2)*(-40*cos(b*x+a)^2-40*cos(b*x+a))+2^(1/2)*arctan((sin(b*x...
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(148) = 296.

Time = 0.15 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.26

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{10\sqrt{2}d^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}(\cos(bx+a)-\sin(bx+a))}{2\sqrt{d}\sin(bx+a)}\right)-5\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{2\,d\cos(bx+a)^2-2\,d\cos(bx+a)\sin(bx+a)+\sqrt{2}}{2\left(d\cos(bx+a)^2+d\cos(bx+a)\sin(\right.}\right.}{}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/32*(10*sqrt(2)*d^(3/2)*arctan(-1/2*sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x
+ a))*(cos(b*x + a) - sin(b*x + a))/(sqrt(d)*sin(b*x + a))) - 5*sqrt(2)*d^
(3/2)*arctan(1/2*(2*d*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) + sqr
t(2)*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 2*d)/(d*cos(b*x + a)^2 +
d*cos(b*x + a)*sin(b*x + a) - d)) - 5*sqrt(2)*d^(3/2)*arctan(-1/2*(2*d*cos
(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) - sqrt(2)*sqrt(d)*sqrt(d*sin(b
*x + a)/cos(b*x + a)) - 2*d)/(d*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x +
a) - d)) + 5*sqrt(2)*d^(3/2)*log(4*d*cos(b*x + a)*sin(b*x + a) + 2*sqrt(2)
*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d)*sqrt(d*sin(b*x + a)/
cos(b*x + a)) + d) - 5*sqrt(2)*d^(3/2)*log(4*d*cos(b*x + a)*sin(b*x + a) -
 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d)*sqrt(d*sin
(b*x + a)/cos(b*x + a)) + d) - 16*(d*cos(b*x + a)^2 + 4*d)*sqrt(d*sin(b*x
+ a)/cos(b*x + a)))/b
```

## Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.05

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{10\sqrt{2}d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{9}{2}}\log\Big(}{}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-1/16*(10*sqrt(2)*d^(9/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*t
an(b*x + a)))/sqrt(d)) + 10*sqrt(2)*d^(9/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 5*sqrt(2)*d^(9/2)*log(d*tan(b*
x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 5*sqrt(2)*d^(9/2)*log
(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan
(b*x + a))*d^6/(d^2*tan(b*x + a)^2 + d^2) - 32*sqrt(d*tan(b*x + a))*d^4)/(
b*d^3)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.25

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{\frac{10\sqrt{2}d^2\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{10\sqrt{2}d^2\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{5\sqrt{2}d^2\sqrt{|d|}\log\Big(d\tan(bx+a}{}}{1}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-1/16*(10*sqrt(2)*d^2*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)
) + 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 10*sqrt(2)*d^2*sqrt(abs(d))*
arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(a
bs(d)))/b + 5*sqrt(2)*d^2*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d
*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 5*sqrt(2)*d^2*sqrt(abs(d))*log(d
*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 8*
sqrt(d*tan(b*x + a))*d^4/((d^2*tan(b*x + a)^2 + d^2)*b) - 32*sqrt(d*tan(b*
x + a))*d^2/b)/d
```

## Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \sin{(a+b\,x)}^2\,{(d\tan(a+b\,x))}^{3/2}\,dx$$

input
```
int(sin(a + b*x)^2*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^2*(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan{(bx+a)}}\,\sin{(bx+a)}^2\tan{(bx+a)}\,dx\right)d$$

input
```
int(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*sin(a + b*x)**2*tan(a + b*x),x)*d
```

# 3.66 $\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . . . 634
Mathematica [A] (verified) . . . . . 634
Rubi [A] (verified) . . . . . 635
Maple [A] (verified) . . . . . 636
Fricas [A] (verification not implemented) . . . . . 636
Sympy [F(-1)] . . . . . 637
Maxima [A] (verification not implemented) . . . . . 637
Giac [A] (verification not implemented) . . . . . 637
Mupad [B] (verification not implemented) . . . . . 638
Reduce [F] . . . . . 638

## Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

output
```
2*d*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

input
```
Integrate[Csc[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*Sqrt[d*Tan[a + b*x]])/b
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{1}{\sqrt{d\tan(a+bx)}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

input
```
Int[Csc[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*Sqrt[d*Tan[a + b*x]])/b
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2d\sqrt{d\tan(bx+a)}}{b}$ | 17 |
| default | $\frac{2d\sqrt{d\tan(bx+a)}}{b}$ | 17 |

input
```
int(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*d*(d*tan(b*x+a))^(1/2)/b
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,d\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{b}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2*d*sqrt(d*sin(b*x + a)/cos(b*x + a))/b
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,(d\tan(bx+a))^{\frac{3}{2}}}{b\tan(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2*(d*tan(b*x + a))^(3/2)/(b*tan(b*x + a))
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,\sqrt{d\tan(bx+a)}d}{b}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2*sqrt(d*tan(b*x + a))*d/b
```

## Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{b}$$

input
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^2,x)
```

output
```
(2*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/b
```

## Reduce [F]

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\csc(bx+a)^2\tan(bx+a)\,dx\right)d$$

input
```
int(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**2*tan(a + b*x),x)*d
```

# 3.67 $\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$


Optimal result . . . 639
Mathematica [A] (verified) . . . 639
Rubi [A] (verified) . . . 640
Maple [A] (verified) . . . 641
Fricas [A] (verification not implemented) . . . 642
Sympy [F(-1)] . . . 642
Maxima [A] (verification not implemented) . . . 642
Giac [A] (verification not implemented) . . . 643
Mupad [B] (verification not implemented) . . . 643
Reduce [F] . . . 644


## Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^3}{3b(d\tan(a+bx))^{3/2}}+\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

output
```
-2/3*d^3/b/(d*tan(b*x+a))^(3/2)+2*d*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d(-4+\csc^2(a+bx))\sqrt{d\tan(a+bx)}}{3b}$$

input
```
Integrate[Csc[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*d*(-4 + Csc[a + b*x]^2)*Sqrt[d*Tan[a + b*x]])/(3*b)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sin(a+bx)^4}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\tan^2(a+bx)d^2+d^2}{(d\tan(a+bx))^{5/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^2}{(d\tan(a+bx))^{5/2}}+\frac{1}{\sqrt{d\tan(a+bx)}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(2\sqrt{d\tan(a+bx)}-\frac{2d^2}{3(d\tan(a+bx))^{3/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*((-2*d^2)/(3*(d*Tan[a + b*x])^(3/2)) + 2*Sqrt[d*Tan[a + b*x]]))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $-\frac{2\sqrt{d\tan(bx+a)}\, d\left(4\cot(bx+a)^{2}-3\csc(bx+a)^{2}\right)}{3b}$ | 38 |

input
```
int(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b*(d*tan(b*x+a))^(1/2)*d*(4*cot(b*x+a)^2-3*csc(b*x+a)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(4\,d\cos\left(bx+a\right)^2-3\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{3\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(4*d*cos(b*x + a)^2 - 3*d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*
x + a)^2 - b)
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\,d^3\left(\frac{1}{(d\tan(bx+a))^{\frac{3}{2}}}-\frac{3\sqrt{d\tan(bx+a)}}{d^2}\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/3*d^3*(1/(d*tan(b*x + a))^(3/2) - 3*sqrt(d*tan(b*x + a))/d^2)/b
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(\frac{3\sqrt{d\tan(bx+a)}d^2}{b} - \frac{d^3}{\sqrt{d\tan(bx+a)}b\tan(bx+a)}\right)}{3\,d}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/3*(3*sqrt(d*tan(b*x + a))*d^2/b - d^3/(sqrt(d*tan(b*x + a))*b*tan(b*x +
a)))/d
```

## Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.44

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{8\,d\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}\,\left(11\,\cos\left(2\,a+2\,b\,x\right)-5\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)-7\right)}{3\,b\,\left(15\,\cos\left(2\,a+2\,b\,x\right)-6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)-10\right)}$$

input
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^4,x)
```

output
```
(8*d*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)*(11*cos(2*a + 2*b
*x) - 5*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 7))/(3*b*(15*cos(2*a + 2*b*x
) - 6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 10))
```

## Reduce [F]

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\csc(bx+a)^4\tan(bx+a)\,dx\right)d$$

input
```
int(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**4*tan(a + b*x),x)*d
```

# 3.68 $\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$


Optimal result . . . 645
Mathematica [A] (verified) . . . 645
Rubi [A] (verified) . . . 646
Maple [A] (verified) . . . 647
Fricas [A] (verification not implemented) . . . 648
Sympy [F(-1)] . . . 648
Maxima [A] (verification not implemented) . . . 648
Giac [A] (verification not implemented) . . . 649
Mupad [B] (verification not implemented) . . . 649
Reduce [F] . . . 650


## Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx=-\frac{2d^5}{7b(d\tan(a+bx))^{7/2}}-\frac{4d^3}{3b(d\tan(a+bx))^{3/2}}+\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

output
```
-2/7*d^5/b/(d*tan(b*x+a))^(7/2)-4/3*d^3/b/(d*tan(b*x+a))^(3/2)+2*d*(d*tan(
b*x+a))^(1/2)/b
```

## Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx=-\frac{2d(-32+8\csc^2(a+bx)+3\csc^4(a+bx))\sqrt{d\tan(a+bx)}}{21b}$$

input
```
Integrate[Csc[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*d*(-32 + 8*Csc[a + b*x]^2 + 3*Csc[a + b*x]^4)*Sqrt[d*Tan[a + b*x]])/(2
1*b)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sin(a+bx)^6}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\left(\tan^2(a+bx)d^2+d^2\right)^2}{(d\tan(a+bx))^{9/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^4}{(d\tan(a+bx))^{9/2}}+\frac{2d^2}{(d\tan(a+bx))^{5/2}}+\frac{1}{\sqrt{d\tan(a+bx)}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^4}{7(d\tan(a+bx))^{7/2}}-\frac{4d^2}{3(d\tan(a+bx))^{3/2}}+2\sqrt{d\tan(a+bx)}\right)}{b}$$

input
```
Int[Csc[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*((-2*d^4)/(7*(d*Tan[a + b*x])^(7/2)) - (4*d^2)/(3*(d*Tan[a + b*x])^(3/2
)) + 2*Sqrt[d*Tan[a + b*x]]))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

| method | result | size |
|---|---|---|
| default | $\frac{2 \csc(bx+a)^4 \left(32 \cos(bx+a)^4-56 \cos(bx+a)^2+21\right) \sqrt{d \tan(bx+a)}\, d}{21b}$ | 47 |

input
```
int(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/b*csc(b*x+a)^4*(32*cos(b*x+a)^4-56*cos(b*x+a)^2+21)*(d*tan(b*x+a))^(1
/2)*d
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(32\,d\cos(bx+a)^4 - 56\,d\cos(bx+a)^2 + 21\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{21\left(b\cos(bx+a)^4 - 2\,b\cos(bx+a)^2 + b\right)}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/21*(32*d*cos(b*x + a)^4 - 56*d*cos(b*x + a)^2 + 21*d)*sqrt(d*sin(b*x + a
)/cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,d^5\left(\frac{21\sqrt{d\tan(bx+a)}}{d^4} - \frac{14\,d^2\tan(bx+a)^2+3\,d^2}{(d\tan(bx+a))^{\frac{7}{2}}d^2}\right)}{21\,b}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/21*d^5*(21*sqrt(d*tan(b*x + a))/d^4 - (14*d^2*tan(b*x + a)^2 + 3*d^2)/((
d*tan(b*x + a))^(7/2)*d^2))/b
```

## Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(\frac{21\sqrt{d\tan(bx+a)}d^2}{b} - \frac{14\,d^6\tan(bx+a)^2+3\,d^6}{\sqrt{d\tan(bx+a)}bd^3\tan(bx+a)^3}\right)}{21\,d}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/21*(21*sqrt(d*tan(b*x + a))*d^2/b - (14*d^6*tan(b*x + a)^2 + 3*d^6)/(sqr
t(d*tan(b*x + a))*b*d^3*tan(b*x + a)^3))/d
```

## Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.63

$$\begin{aligned}\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = &-\frac{\left(\frac{20\,d}{21\,b} - \frac{64\,d\,e^{a\,2i+b\,x\,2i}}{21\,b}\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{e^{a\,2i+b\,x\,2i}-1} \\ &+ \frac{20\,d\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{21\,b\left(e^{a\,2i+b\,x\,2i}-1\right)^2} - \frac{24\,d\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{7\,b\left(e^{a\,2i+b\,x\,2i}-1\right)^3} \\ &- \frac{16\,d\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{7\,b\left(e^{a\,2i+b\,x\,2i}-1\right)^4}\end{aligned}$$

input
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^6,x)
```

output
```
(20*d*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2
i + b*x*2i) + 1))^(1/2))/(21*b*(exp(a*2i + b*x*2i) - 1)^2) - (((20*d)/(21*
b) - (64*d*exp(a*2i + b*x*2i))/(21*b))*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/
(exp(a*2i + b*x*2i) + 1))^(1/2))/(exp(a*2i + b*x*2i) - 1) - (24*d*(exp(a*2
i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) +
1))^(1/2))/(7*b*(exp(a*2i + b*x*2i) - 1)^3) - (16*d*(exp(a*2i + b*x*2i) +
1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(7*
b*(exp(a*2i + b*x*2i) - 1)^4)
```

## Reduce [F]

$$\int \csc^6(a + bx)(d\tan(a + bx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx + a)}\,\csc(bx + a)^6\tan(bx + a)\,dx\right)d$$

input
```
int(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**6*tan(a + b*x),x)*d
```

# 3.69 $\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 651
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 651
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 652
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 655
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 655
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 656
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 656
Giac [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 656
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 657
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 657

## Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} - \frac{7d^2E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

output
```
7/3*d^3*sin(b*x+a)^3/b/(d*tan(b*x+a))^(3/2)+7/2*d^2*EllipticE(cos(a+1/4*Pi
+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*
sin(b*x+a)^3*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.50 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\left(-28\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sec(a+bx)+2\cos(a+bx)(13+\cos(2(a\right.}{12b\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
((-28*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x] + 2*C
os[a + b*x]*(13 + Cos[2*(a + b*x)])*Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])
^(3/2))/(12*b*Sqrt[Sec[a + b*x]^2])
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3074, 3042, 3078, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3074}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\int \frac{\sin(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

$$\Big\downarrow \text{3052}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

input
```
Int[Sin[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/b - 7*d^2*(-1/3*(d*Sin[a + b*x]^
3)/(b*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x]
)/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]]))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(99) = 198$.

Time = 1.40 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.62

| method | result |
|---|---|
| default | $-\frac{\csc(bx+a)\left((21\cos(bx+a)+21)\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\;\mathrm{Elliptic}\right.}{}$ |

input
```
int(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/b*csc(b*x+a)*((21*cos(b*x+a)+21)*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2
*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF
((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))+(-42*cos(b*x+a)-42)*(csc(b*x
+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+
cot(b*x+a))^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))+4
*cos(b*x+a)^4-22*cos(b*x+a)^2+42*cos(b*x+a)-24)*(-2*sin(b*x+a)*cos(b*x+a)/
(cos(b*x+a)+1)^2)^(1/2)*(d*tan(b*x+a))^(1/2)*d/(-sin(b*x+a)*cos(b*x+a)/(co
s(b*x+a)+1)^2)^(1/2)*2^(1/2)
```

## Fricas [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(d*cos(b*x + a)^2 - d)*sqrt(d*tan(b*x + a))*sin(b*x + a)*tan(b*x
 + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sin(b*x + a)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

## Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \sin\left(a+b\,x\right)^3\,{\left(d\,\mathrm{tan}\left(a+b\,x\right)\right)}^{3/2}\,dx$$

input
```
int(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(bx+a\right)}\,\sin\left(bx+a\right)^3\tan\left(bx+a\right)dx\right)d$$

input
```
int(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*sin(a + b*x)**3*tan(a + b*x),x)*d
```

# 3.70 $\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 658
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 658
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 659
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 661
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 661
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 662
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 662
Giac [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 662
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 663
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 663

## Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{3d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

output
```
3*d^2*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/
2)/(d*tan(b*x+a))^(1/2)+2*d*sin(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\cos(a+bx)\left(-1+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)(d\tan(a+bx))^{3/2}}{b}$$

input
```
Integrate[Sin[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x]*(-1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*S
qrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/b
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3074, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{ 3074}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-3d^2\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-3d^2\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{ 3081}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 3052}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sin[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-3*d^2*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x
]]*Sqrt[d*Tan[a + b*x]]) + (2*d*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(71) = 142$.

Time = 1.05 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.66

| method | result |
|---|---|
| default | $\frac{\csc(bx+a)\left((6\cos(bx+a)+6)\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{EllipticE}\left(\sqrt{\right.}\right.}{}$ |

input
```
int(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*csc(b*x+a)*((6*cos(b*x+a)+6)*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc
(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticE((cs
c(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))+(-3*cos(b*x+a)-3)*(csc(b*x+a)-co
t(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*
x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))+2*cos(b
*x+a)^2-6*cos(b*x+a)+4)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*
(d*tan(b*x+a))^(1/2)*d/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(
1/2)
```

## Fricas [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)\ dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*d*sin(b*x + a)*tan(b*x + a), x)
```

## Sympy [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(a+bx)})^{\frac{3}{2}}\sin{(a+bx)}\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sin(a + b*x), x)
```

## Maxima [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(bx+a)})^{\frac{3}{2}}\sin{(bx+a)}\;dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sin(b*x + a), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

## Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx=\int \sin\left(a+b\,x\right)\,\left(d\tan(a+b\,x)\right)^{3/2}\,dx$$

input
```
int(sin(a + b*x)*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)*(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx=\sqrt{d}\left(\int \sqrt{\tan\left(bx+a\right)}\,\sin\left(bx+a\right)\tan\left(bx+a\right)dx\right)d$$

input
```
int(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*sin(a + b*x)*tan(a + b*x),x)*d
```

# 3.71 $\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . . . . . . . . 664
Mathematica [C] (verified) . . . . . . . . . . 664
Rubi [A] (verified) . . . . . . . . . . 665
Maple [B] (verified) . . . . . . . . . . 667
Fricas [C] (verification not implemented) . . . . . . . . . . 668
Sympy [F] . . . . . . . . . . 668
Maxima [F] . . . . . . . . . . 669
Giac [F] . . . . . . . . . . 669
Mupad [F(-1)] . . . . . . . . . . 669
Reduce [F] . . . . . . . . . . 670

## Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

output
```
2*d^2*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/
2)/(d*tan(b*x+a))^(1/2)+2*d*sin(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\cos(a+bx)\left(-3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Integrate[Csc[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x]*(-3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]
*Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/(3*b)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3073, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3073}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-2d^2\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-2d^2\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*d^2*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x
]]*Sqrt[d*Tan[a + b*x]]) + (2*d*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(71) = 142$.

Time = 0.97 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.51

| method | result |
|---|---|
| default | $\frac{\csc(bx+a)\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,(-\cos(bx+a)-1)\,\text{EllipticF}\left(\sqrt{\ldots}\right.\right.}{}$ |

input
```
int(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*csc(b*x+a)*((csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x
+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*(-cos(b*x+a)-1)*EllipticF((csc
(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))+2*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)
*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*(cos(
b*x+a)+1)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*cos(b*x
+a)+2)*(d*tan(b*x+a))^(1/2)*d*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^
(1/2)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.79

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{-i\sqrt{i\,d}dE(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+i\sqrt{-i\,d}dE(\arcsin(\cos(bx+a)-i}{}$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(I*d)*d*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + I*
sqrt(-I*d)*d*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) + I*sqr
t(I*d)*d*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) - I*sqrt(-I
*d)*d*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) + 2*d*sqrt(d*s
in(b*x + a)/cos(b*x + a))*sin(b*x + a))/b
```

## Sympy [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(a+bx))^{\frac{3}{2}}\csc(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*csc(a + b*x), x)
```

## Maxima [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a), x)
```

## Giac [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\sin(a+b\,x)}\,dx$$

input
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x),x)
```

output
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x), x)
```

## Reduce [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\csc(bx+a)\tan(bx+a)\,dx\right)d$$

input
```
int(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)*tan(a + b*x),x)*d
```

# 3.72 $\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 671
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 671
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 672
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 675
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . 675
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 676
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 676
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 677
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 677
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 677

## Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{4d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

output
```
-4*d^2*cos(b*x+a)/b/(d*tan(b*x+a))^(1/2)+4*d^2*EllipticE(cos(a+1/4*Pi+b*x)
,2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*csc(b
*x+a)*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.54 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\cos(a+bx)\left(-6+3\csc^2(a+bx)+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)(d}{3b}$$

input
```
Integrate[Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x]*(-6 + 3*Csc[a + b*x]^2 + 4*Hypergeometric2F1[3/4, 3/2, 7/
4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/(3*b)
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.35, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3073, 3042, 3081, 3042, 3050, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{3073}$$

$$2d^2\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$2d^2\int \frac{1}{\sin(a+bx)\sqrt{d\tan(a+bx)}}dx+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin(a+bx)^{3/2}}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3050}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\left(-2\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\left(-2\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\left(-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}E\left(a+bx-\frac{\pi}{4}|2\right)}{b\sqrt{\sin(2a+2bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

input
```
Int[Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d^2*Sqrt[Sin[a + b*x]]*((-2*Cos[a + b*x]^(3/2))/(b*Sqrt[Sin[a + b*x]])
- (2*Sqrt[Cos[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[Sin[a + b*x]])/(
b*Sqrt[Sin[2*a + 2*b*x]])))/(Sqrt[Cos[a + b*x]]*Sqrt[d*Tan[a + b*x]]) + (2
*d*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(95) = 190$.

Time = 0.97 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.08

| method | result |
|---|---|
| default | $\frac{\left(-4\cos(bx+a)+2+(4\cos(bx+a)+4)\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{Elli}\right.}{}$ |

input
```
int(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-4*cos(b*x+a)+2+(4*cos(b*x+a)+4)*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*
csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticE(
(csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))+(-2*cos(b*x+a)-2)*(csc(b*x+a)
-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot
(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2)))*csc
(b*x+a)*(d*tan(b*x+a))^(1/2)*d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.73

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{2\left(i\sqrt{i}\,dd E(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)\sin\left(bx+a\right)-i\sqrt{-i}\,dd E(\arcsin\left(\cos\left(bx+a\right)-\right.\right.}{}$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(I*d)*d*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*si
n(b*x + a) - I*sqrt(-I*d)*d*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a
)), -1)*sin(b*x + a) - I*sqrt(I*d)*d*elliptic_f(arcsin(cos(b*x + a) + I*si
n(b*x + a)), -1)*sin(b*x + a) + I*sqrt(-I*d)*d*elliptic_f(arcsin(cos(b*x +
 a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (2*d*cos(b*x + a)^2 - d)*sqrt(d*
sin(b*x + a)/cos(b*x + a)))/(b*sin(b*x + a))
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

## Giac [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\sin(a+b\,x)^3}\,dx$$

input
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^3,x)
```

output
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^3, x)
```

## Reduce [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\csc(bx+a)^3\tan(bx+a)\,dx\right)d$$

input
```
int(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**3*tan(a + b*x),x)*d
```

# 3.73 $\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result . . . . . 678
Mathematica [A] (verified) . . . . . 679
Rubi [A] (warning: unable to verify) . . . . . 679
Maple [B] (warning: unable to verify) . . . . . 684
Fricas [B] (verification not implemented) . . . . . 684
Sympy [F(-1)] . . . . . 685
Maxima [A] (verification not implemented) . . . . . 686
Giac [A] (verification not implemented) . . . . . 686
Mupad [F(-1)] . . . . . 687
Reduce [F] . . . . . 687

## Optimal result

Integrand size = 21, antiderivative size = 224

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{77d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{77d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{77d^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{32\sqrt{2}b} + \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3}$$

output
```
77/64*d^(5/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b-77/
64*d^(5/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b+77/64*
d^(5/2)*arctanh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))
*2^(1/2)/b+77/48*d*(d*tan(b*x+a))^(3/2)/b-11/16*cos(b*x+a)^2*(d*tan(b*x+a)
)^(7/2)/b/d-1/4*cos(b*x+a)^4*(d*tan(b*x+a))^(11/2)/b/d^3
```

## Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.63

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{d\Big(128+204\cos^2(a+bx)+231\arcsin(\cos(a+bx)-\sin(a+bx))\cot(a+bx)\csc(a+bx)\sqrt{}$$

input
```
Integrate[Sin[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*(128 + 204*Cos[a + b*x]^2 + 231*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Cot
[a + b*x]*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]] + 231*Cot[a + b*x]*Csc[a + b
*x]*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(
a + b*x)]] - 6*Cot[a + b*x]*Sin[4*(a + b*x)])*(d*Tan[a + b*x])^(3/2))/(192
*b)
```

## Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.22, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3071, 252, 252, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4(d\tan(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{13/2}}{(\tan^2(a+bx)d^2+d^2)^3}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{d\left(\frac{11}{8}\int \frac{(d\tan(a+bx))^{9/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{11/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 252

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\int \frac{(d\tan(a+bx))^{5/2}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{7/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{11/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 262

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-d^2\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))\right)-\frac{(d\tan(a+bx))^{7/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{11/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 266

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(a+bx))^{7/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{11/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 826

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)\right.\right.\right.}{b}$$

$\downarrow$ 1476

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}\ldots}\right.\right.\right.\right.\right.}{}$$

$\downarrow$ 1082

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+\ldots\right)}{\sqrt{2}\sqrt{d}}\right.\right.\right.\right.\right.}{b}$$

$\downarrow$ 217

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+\ldots)}{d^4\tan^4(a+bx)\ldots}\right.\right.\right.\right.}{b}$$

$\downarrow$ 1479

$$d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right.\right.\right.\right.\right.$$

$\downarrow$ 25

$$d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right.\right.\right.\right.\right.$$

$\downarrow$ 27

$$d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)}}{2\sqrt{d}}\right.\right.\right.\right.\right.$$

$\downarrow$ 1103

$$d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan\right.}{}\right.\right.\right.\right.\right.$$

input
```
Int[Sin[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*(-1/4*(d*Tan[a + b*x])^(11/2)/(d^2 + d^2*Tan[a + b*x]^2)^2 + (11*(-1/2*
(d*Tan[a + b*x])^(7/2)/(d^2 + d^2*Tan[a + b*x]^2) + (7*(-2*d^2*((-(ArcTan[
1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*
Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Tan[
a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/
2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2) + (2*(d*Tan[
a + b*x])^(3/2))/3))/4))/8))/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(174) = 348$.

Time = 30.46 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.20

| method | result |
|---|---|
| default | $\frac{\sqrt{d\tan(bx+a)}\left(231\ln\left(-2\cot(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}-2\csc(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}-2\cot(bx+a)+2\right)\sqrt{-\frac{\sin(bx\cdots}{(\cos\cdots}}\right.}{}$ |

input
```
int(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/b*(d*tan(b*x+a))^(1/2)*(231*ln(-2*cot(b*x+a)*(-2*sin(b*x+a)*cos(b*x+
a)/(cos(b*x+a)+1)^2)^(1/2)-2*csc(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x
+a)+1)^2)^(1/2)-2*cot(b*x+a)+2)*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^
(1/2)*(cot(b*x+a)+csc(b*x+a))+231*ln(2*cot(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a
)/(cos(b*x+a)+1)^2)^(1/2)+2*csc(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+
a)+1)^2)^(1/2)-2*cot(b*x+a)+2)*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)*(-cot(b*x+a)-csc(b*x+a))+462*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*
x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*(-sin(b*x+a)*c
os(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(cot(b*x+a)+csc(b*x+a))+462*arctan((-sin
(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-
1+cos(b*x+a)))*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(-cot(b*x+a
)-csc(b*x+a))+4*tan(b*x+a)*(32-12*cos(b*x+a)^4+57*cos(b*x+a)^2)*2^(1/2))*d
^2*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(174) = 348$.

Time = 0.20 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.25

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{462\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{d\cos(bx+a)-d\sin(bx+a)}\right)\cos(bx+a)+231\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{2d\cos(bx+a)^2-2\cdots}{2\left(d\cos\cdots\right.}\right.}{}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/768*(462*sqrt(2)*d^(5/2)*arctan(-sqrt(2)*sqrt(d)*sqrt(d*sin(b*x + a)/cos
(b*x + a))*cos(b*x + a)/(d*cos(b*x + a) - d*sin(b*x + a)))*cos(b*x + a) +
231*sqrt(2)*d^(5/2)*arctan(1/2*(2*d*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(
b*x + a) + sqrt(2)*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 2*d)/(d*cos
(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a) - d))*cos(b*x + a) + 231*sqrt(2)
*d^(5/2)*arctan(-1/2*(2*d*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) -
 sqrt(2)*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 2*d)/(d*cos(b*x + a)^
2 + d*cos(b*x + a)*sin(b*x + a) - d))*cos(b*x + a) + 231*sqrt(2)*d^(5/2)*c
os(b*x + a)*log(4*d*cos(b*x + a)*sin(b*x + a) + 2*sqrt(2)*(cos(b*x + a)^2
+ cos(b*x + a)*sin(b*x + a))*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + d
) - 231*sqrt(2)*d^(5/2)*cos(b*x + a)*log(4*d*cos(b*x + a)*sin(b*x + a) - 2
*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d)*sqrt(d*sin(b
*x + a)/cos(b*x + a)) + d) - 16*(12*d^2*cos(b*x + a)^4 - 57*d^2*cos(b*x +
a)^2 - 32*d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))*sin(b*x + a))/(b*cos(b*x
+ a))
```

## Sympy [F(-1)]

Timed out.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.07

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{231\,d^8\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\right.}{\sqrt{d}}\right.}{}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-1/384*(231*d^8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*
tan(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x
+ a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*t
an(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 256*(d*
tan(b*x + a))^(3/2)*d^6 - 24*(19*(d*tan(b*x + a))^(7/2)*d^8 + 15*(d*tan(b*
x + a))^(3/2)*d^10)/(d^4*tan(b*x + a)^4 + 2*d^4*tan(b*x + a)^2 + d^4))/(b*
d^5)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.24

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{1}{384}\,d^2\left(\frac{462\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd}+\frac{462\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right.}{bd}\right.$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-1/384*d^2*(462*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(
d)) + 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) + 462*sqrt(2)*abs(d)^(3/
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqr
t(abs(d)))/(b*d) - 231*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*s
qrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) + 231*sqrt(2)*abs(d)^(3/2
)*log(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))
/(b*d) - 256*sqrt(d*tan(b*x + a))*tan(b*x + a)/b - 24*(19*sqrt(d*tan(b*x +
 a))*d^4*tan(b*x + a)^3 + 15*sqrt(d*tan(b*x + a))*d^4*tan(b*x + a))/((d^2*
tan(b*x + a)^2 + d^2)^2*b))
```

## Mupad [F(-1)]

Timed out.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \sin\left(a+b\,x\right)^4\,{\left(d\tan(a+b\,x)\right)}^{5/2}\,dx$$

input
```
int(sin(a + b*x)^4*(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^4*(d*tan(a + b*x))^(5/2), x)
```

## Reduce [F]

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(bx+a\right)}\,\sin\left(bx+a\right)^4\tan\left(bx+a\right)^2dx\right)d^2$$

input
```
int(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*sin(a + b*x)**4*tan(a + b*x)**2,x)*d**2
```

# 3.74 $\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 688
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 689
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . 689
Maple [B] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . 693
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . 694
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 695
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . 695
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 696
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 697
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 697


## Optimal result

Integrand size = 21, antiderivative size = 194

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{7d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{7d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} + \frac{7d^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{4\sqrt{2}b} + \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd}$$

output
```
7/8*d^(5/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b-7/8*d
^(5/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b+7/8*d^(5/2
)*arctanh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))*2^(1/
2)/b+7/6*d*(d*tan(b*x+a))^(3/2)/b-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(7/2)/b/
d
```

## Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.65

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{d\Big(16+12\cos^2(a+bx)+21\arcsin(\cos(a+bx)-\sin(a+bx))\cot(a+bx)\csc(a+bx)\sqrt{\sin(}$$

input
```
Integrate[Sin[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*(16 + 12*Cos[a + b*x]^2 + 21*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Cot[a
+ b*x]*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]] + 21*Cot[a + b*x]*Csc[a + b*x]*
Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a +
b*x)]])*(d*Tan[a + b*x])^(3/2))/(24*b)
```

## Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3071, 252, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(a+bx)^2(d\tan(a+bx))^{5/2}dx$$

$$\Big\downarrow \; 3071$$

$$\frac{d\int \frac{(d\tan(a+bx))^{9/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \; 252$$

$$\frac{d\left(\frac{7}{4}\int \frac{(d\tan(a+bx))^{5/2}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{7/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 262

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-d^2\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))\right)-\frac{(d\tan(a+bx))^{7/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 266

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(a+bx))^{7/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 826

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)\right)-\right.}{b}$$

$\downarrow$ 1476

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}}\right.\right.\right.\right.}{b}$$

$\downarrow$ 1082

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right)\right.\right.\right.}{b}$$

$\downarrow$ 217

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\right.\right.\right.}{b}$$

$\downarrow$ 1479

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+}}{2\sqrt{2}\sqrt{d}}\right.\right.\right.\right.}{b}$$

$\downarrow$ 25

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}}{2\sqrt{2}\sqrt{d}}\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}}{2\sqrt{d}}\right.\right.\right.\right.}{b}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+\right.}{2}\right.\right.\right.\right.}{b}$$

input
```
Int[Sin[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*(-1/2*(d*Tan[a + b*x])^(7/2)/(d^2 + d^2*Tan[a + b*x]^2) + (7*(-2*d^2*((
-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 +
 Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(
3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt
[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2) + (
2*(d*Tan[a + b*x])^(3/2))/3))/4))/b
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(148) = 296$.

Time = 5.28 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.52

| method | result |
|---|---|
| default | $-\frac{\sqrt{d\tan(bx+a)}\left(21\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\ \ln\left(2\cot(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}+2\csc(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}-2\,\mathrm{co}\right.\right.}{}$ |

input
```
int(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48/b*(d*tan(b*x+a))^(1/2)*(21*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)
^(1/2)*ln(2*cot(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+2
*csc(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cot(b*x+a)
+2)*(cot(b*x+a)+csc(b*x+a))+21*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)*ln(-2*cot(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*
csc(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cot(b*x+a)+
2)*(-cot(b*x+a)-csc(b*x+a))+42*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+
cos(b*x+a)-1)/(-1+cos(b*x+a)))*(-cot(b*x+a)-csc(b*x+a))+42*(-sin(b*x+a)*co
s(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x
+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))*(-cot(b*x+a)-cs
c(b*x+a))+4*tan(b*x+a)*(-4-3*cos(b*x+a)^2)*2^(1/2))*d^2*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(148) = 296$.

Time = 0.20 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.53

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{42\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{d\cos(bx+a)-d\sin(bx+a)}\right)\cos(bx+a)+21\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{2\,d\cos(bx+a)^2-2\,d\,c}{2\left(d\cos(b\right.}\right.}{}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/96*(42*sqrt(2)*d^(5/2)*arctan(-sqrt(2)*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b
*x + a))*cos(b*x + a)/(d*cos(b*x + a) - d*sin(b*x + a)))*cos(b*x + a) + 21
*sqrt(2)*d^(5/2)*arctan(1/2*(2*d*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x
 + a) + sqrt(2)*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 2*d)/(d*cos(b*
x + a)^2 + d*cos(b*x + a)*sin(b*x + a) - d))*cos(b*x + a) + 21*sqrt(2)*d^(
5/2)*arctan(-1/2*(2*d*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) - sqr
t(2)*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 2*d)/(d*cos(b*x + a)^2 +
d*cos(b*x + a)*sin(b*x + a) - d))*cos(b*x + a) + 21*sqrt(2)*d^(5/2)*cos(b*
x + a)*log(4*d*cos(b*x + a)*sin(b*x + a) + 2*sqrt(2)*(cos(b*x + a)^2 + cos
(b*x + a)*sin(b*x + a))*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + d) - 2
1*sqrt(2)*d^(5/2)*cos(b*x + a)*log(4*d*cos(b*x + a)*sin(b*x + a) - 2*sqrt(
2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d)*sqrt(d*sin(b*x + a
)/cos(b*x + a)) + d) + 16*(3*d^2*cos(b*x + a)^2 + 4*d^2)*sqrt(d*sin(b*x +
a)/cos(b*x + a))*sin(b*x + a))/(b*cos(b*x + a))
```

## Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.08

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{21\,d^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\right.}{\sqrt{d}}\right.}{48\,bd^3}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-1/48*(21*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x +
a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan
(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 24*(d*tan
(b*x + a))^(3/2)*d^6/(d^2*tan(b*x + a)^2 + d^2) - 32*(d*tan(b*x + a))^(3/2
)*d^4)/(b*d^3)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.30

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{1}{48}\left(\frac{24\sqrt{d\tan(bx+a)}d^2\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)b} - \frac{42\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd} - \cdots\right.$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
1/48*(24*sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)
*b) - 42*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) - 42*sqrt(2)*abs(d)^(3/2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)
))/(b*d) + 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan
(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 21*sqrt(2)*abs(d)^(3/2)*log(d*ta
n(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) + 3
2*sqrt(d*tan(b*x + a))*tan(b*x + a)/b)*d^2
```

## Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \sin\left(a+b\,x\right)^2\,{\left(d\,\tan(a+b\,x)\right)}^{5/2}\,dx$$

input
```
int(sin(a + b*x)^2*(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^2*(d*tan(a + b*x))^(5/2), x)
```

## Reduce [F]

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(bx+a\right)}\,\sin\left(bx+a\right)^2\tan\left(bx+a\right)^2 dx\right) d^2$$

input
```
int(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*sin(a + b*x)**2*tan(a + b*x)**2,x)*d**2
```

# 3.75 $\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 698
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 698
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 699
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 700
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . 700
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 701
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 701
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 701
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 702
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 702

## Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx=\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

output
```
2/3*d*(d*tan(b*x+a))^(3/2)/b
```

## Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx=\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Integrate[Csc[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \sqrt{d\tan(a+bx)}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Int[Csc[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2d(d\tan(bx+a))^{\frac{3}{2}}}{3b}$ | 17 |
| default | $\frac{2d(d\tan(bx+a))^{\frac{3}{2}}}{3b}$ | 17 |

input
```
int(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*d*(d*tan(b*x+a))^(3/2)/b
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,d^2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\,\sin{(bx+a)}}{3\,b\cos{(bx+a)}}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/3*d^2*sqrt(d*sin(b*x + a)/cos(b*x + a))*sin(b*x + a)/(b*cos(b*x + a))
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,(d\tan{(bx+a)})^{\frac{5}{2}}}{3\,b\tan{(bx+a)}}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/3*(d*tan(b*x + a))^(5/2)/(b*tan(b*x + a))
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,\sqrt{d\tan{(bx+a)}}d^2\tan{(bx+a)}}{3\,b}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2/3*sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)/b
```

## Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.80

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,d^2\,\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{3\,b\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^2,x)
```

output
```
(2*d^2*sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2
))/(3*b*(cos(2*a + 2*b*x) + 1))
```

## Reduce [F]

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(bx+a\right)}\,\csc\left(bx+a\right)^2\tan\left(bx+a\right)^2dx\right)d^2$$

input
```
int(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**2*tan(a + b*x)**2,x)*d**2
```

# 3.76 $\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result . . . 703
Mathematica [A] (verified) . . . 703
Rubi [A] (verified) . . . 704
Maple [A] (verified) . . . 705
Fricas [A] (verification not implemented) . . . 706
Sympy [F(-1)] . . . 706
Maxima [A] (verification not implemented) . . . 706
Giac [A] (verification not implemented) . . . 707
Mupad [B] (verification not implemented) . . . 707
Reduce [F] . . . 708

## Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

output
```
-2*d^3/b/(d*tan(b*x+a))^(1/2)+2/3*d*(d*tan(b*x+a))^(3/2)/b
```

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2d(-1+3\cot^2(a+bx))\,(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Integrate[Csc[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*d*(-1 + 3*Cot[a + b*x]^2)*(d*Tan[a + b*x])^(3/2))/(3*b)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)^4}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\tan^2(a+bx)d^2+d^2}{(d\tan(a+bx))^{3/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^2}{(d\tan(a+bx))^{3/2}}+\sqrt{d\tan(a+bx)}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-\frac{2d^2}{\sqrt{d\tan(a+bx)}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*((-2*d^2)/Sqrt[d*Tan[a + b*x]] + (2*(d*Tan[a + b*x])^(3/2))/3))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

| method | result | size |
|---|---|---|
| default | $-\frac{2\sqrt{d\tan(bx+a)}\, d^{2}(4\cot(bx+a)-\sec(bx+a)\csc(bx+a))}{3b}$ | 42 |

input
```
int(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b*(d*tan(b*x+a))^(1/2)*d^2*(4*cot(b*x+a)-sec(b*x+a)*csc(b*x+a))
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2\left(4\,d^2\cos\left(bx+a\right)^2-d^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{3\,b\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(4*d^2*cos(b*x + a)^2 - d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos
(b*x + a)*sin(b*x + a))
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2\,d^3\left(\frac{3}{\sqrt{d\tan(bx+a)}}-\frac{(d\tan(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/3*d^3*(3/sqrt(d*tan(b*x + a)) - (d*tan(b*x + a))^(3/2)/d^2)/b
```

## Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2}{3}\,d^2\left(\frac{\sqrt{d\tan\left(bx+a\right)}\tan\left(bx+a\right)}{b} - \frac{3\,d}{\sqrt{d\tan\left(bx+a\right)}b}\right)$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2/3*d^2*(sqrt(d*tan(b*x + a))*tan(b*x + a)/b - 3*d/(sqrt(d*tan(b*x + a))*b
))
```

## Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{4\,d^2\,\left(\sin\left(2\,a+2\,b\,x\right)+\sin\left(4\,a+4\,b\,x\right)\right)\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{3\,b\sin\left(2\,a+2\,b\,x\right)^2}$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^4,x)
```

output
```
-(4*d^2*(sin(2*a + 2*b*x) + sin(4*a + 4*b*x))*((d*sin(2*a + 2*b*x))/(cos(2
*a + 2*b*x) + 1))^(1/2))/(3*b*sin(2*a + 2*b*x)^2)
```

## Reduce [F]

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\csc(bx+a)^4\tan(bx+a)^2\,dx\right)d^2$$

input
```
int(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**4*tan(a + b*x)**2,x)*d**2
```

# 3.77 $\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result . . . 709
Mathematica [A] (verified) . . . 709
Rubi [A] (verified) . . . 710
Maple [A] (verified) . . . 711
Fricas [A] (verification not implemented) . . . 712
Sympy [F(-1)] . . . 712
Maxima [A] (verification not implemented) . . . 712
Giac [A] (verification not implemented) . . . 713
Mupad [B] (verification not implemented) . . . 713
Reduce [F] . . . 714

## Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx=-\frac{2d^5}{5b(d\tan(a+bx))^{5/2}}-\frac{4d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

output
```
-2/5*d^5/b/(d*tan(b*x+a))^(5/2)-4*d^3/b/(d*tan(b*x+a))^(1/2)+2/3*d*(d*tan(
b*x+a))^(3/2)/b
```

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx=-\frac{2d\left(-5+3\cot^2(a+bx)\left(9+\csc^2(a+bx)\right)\right)(d\tan(a+bx))^{3/2}}{15b}$$

input
```
Integrate[Csc[a + b*x]^6*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*d*(-5 + 3*Cot[a + b*x]^2*(9 + Csc[a + b*x]^2))*(d*Tan[a + b*x])^(3/2))
/(15*b)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)^6}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\left(\tan^2(a+bx)d^2+d^2\right)^2}{(d\tan(a+bx))^{7/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^4}{(d\tan(a+bx))^{7/2}}+\frac{2d^2}{(d\tan(a+bx))^{3/2}}+\sqrt{d\tan(a+bx)}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^4}{5(d\tan(a+bx))^{5/2}}-\frac{4d^2}{\sqrt{d\tan(a+bx)}}+\frac{2}{3}(d\tan(a+bx))^{3/2}\right)}{b}$$

input
```
Int[Csc[a + b*x]^6*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*((-2*d^4)/(5*(d*Tan[a + b*x])^(5/2)) - (4*d^2)/Sqrt[d*Tan[a + b*x]] + (
2*(d*Tan[a + b*x])^(3/2))/3))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 67.62 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

| method | result | size |
|---|---|---|
| default | $\frac{2\sec(bx+a)\csc(bx+a)^{3}\left(32\cos(bx+a)^{4}-40\cos(bx+a)^{2}+5\right)\sqrt{d\tan(bx+a)}\,d^{2}}{15b}$ | 55 |

input
```
int(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/b*sec(b*x+a)*csc(b*x+a)^3*(32*cos(b*x+a)^4-40*cos(b*x+a)^2+5)*(d*tan(
b*x+a))^(1/2)*d^2
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2\left(32\,d^2\cos\left(bx+a\right)^4-40\,d^2\cos\left(bx+a\right)^2+5\,d^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{15\left(b\cos\left(bx+a\right)^3-b\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(32*d^2*cos(b*x + a)^4 - 40*d^2*cos(b*x + a)^2 + 5*d^2)*sqrt(d*sin(b
*x + a)/cos(b*x + a))/((b*cos(b*x + a)^3 - b*cos(b*x + a))*sin(b*x + a))
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,d^5\left(\frac{5\,(d\tan(bx+a))^{\frac{3}{2}}}{d^4}-\frac{3\left(10\,d^2\tan(bx+a)^2+d^2\right)}{(d\tan(bx+a))^{\frac{5}{2}}\,d^2}\right)}{15\,b}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/15*d^5*(5*(d*tan(b*x + a))^(3/2)/d^4 - 3*(10*d^2*tan(b*x + a)^2 + d^2)/(
(d*tan(b*x + a))^(5/2)*d^2))/b
```

## Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2}{15}\,d^2\left(\frac{5\,\sqrt{d\tan{(bx+a)}}\tan{(bx+a)}}{b} - \frac{3\,\left(10\,d^3\tan{(bx+a)}^2+d^3\right)}{\sqrt{d\tan{(bx+a)}}bd^2\tan{(bx+a)}^2}\right)$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2/15*d^2*(5*sqrt(d*tan(b*x + a))*tan(b*x + a)/b - 3*(10*d^3*tan(b*x + a)^2
 + d^3)/(sqrt(d*tan(b*x + a))*b*d^2*tan(b*x + a)^2))
```

## Mupad [B] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.13

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{32\,d^2\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,2\mathrm{i}-\mathrm{e}^{a\,8\mathrm{i}+b\,x\,8\mathrm{i}}\,2\mathrm{i}-2\mathrm{i}\right)}{15\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^6,x)
```

output
```
(32*d^2*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)
*(exp(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*x*6i)*2i -
exp(a*8i + b*x*8i)*2i - 2i))/(15*b*(exp(a*2i + b*x*2i) - 1)^3*(exp(a*2i +
b*x*2i) + 1))
```

## Reduce [F]

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx=\sqrt{d}\left(\int\sqrt{\tan(bx+a)}\,\csc(bx+a)^6\tan(bx+a)^2\,dx\right)d^2$$

input
```
int(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**6*tan(a + b*x)**2,x)*d**2
```

# 3.78 $\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 715
Mathematica [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . 715
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 716
Maple [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . 719
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 720
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 721
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 721
Giac [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 721
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 722
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 722

## Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{5d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{4b} + \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

output
```
5/2*d^3*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)+d^3*sin(b*x+a)^3/b/(d*tan(b*x+a)
)^(1/2)-5/4*d^2*csc(b*x+a)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sin(2*b*x
+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*d*sin(b*x+a)^3*(d*tan(b*x+a))^(3/2)
/b
```

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(120\sqrt[4]{-1}\cos(2(a+bx))\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)+(2\right.}{48b\tan^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Sin[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

output
```
-1/48*(Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(120*(-1)^(1/4)*Cos[2*(a + b*x)]*
EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1] + (22 + 77*Cos[2*(
a + b*x)] + 22*Cos[4*(a + b*x)] - Cos[6*(a + b*x)])*Sqrt[Sec[a + b*x]^2]*S
qrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(5/2))/(b*Tan[a + b*x]^(3/2)*(-1 + Tan
[a + b*x]^2))
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3074, 3042, 3078, 3042, 3078, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3(d\tan(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3074}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-3d^2\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-3d^2\int\sin(a+bx)^3\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-3d^2\left(\frac{5}{6}\int\sin(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-3d^2\left(\frac{5}{6}\int\sin(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)$$

$$\Big\downarrow \text{3078}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-3d^2\left(\frac{5}{6}\left(\frac{1}{2}\int\csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-3d^2\left(\frac{5}{6}\left(\frac{1}{2}\int\frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-3d^2\left(\frac{5}{6}\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-3d^2\left(\frac{5}{6}\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)$$

$$\Big\downarrow \text{3053}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-$$
$$3d^2\left(\frac{5}{6}\left(\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a-}{3b\sqrt{d\tan(a}}\right.$$

$\downarrow$ 3042

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-$$
$$3d^2\left(\frac{5}{6}\left(\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a-}{3b\sqrt{d\tan(a}}\right.$$

$\downarrow$ 3120

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-$$
$$3d^2\left(\frac{5}{6}\left(\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{2b}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a}{3b\sqrt{d\tan}}\right.$$

input
```
Int[Sin[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d*Sin[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b) - 3*d^2*(-1/3*(d*Sin[a +
 b*x]^3)/(b*Sqrt[d*Tan[a + b*x]]) + (5*(-((d*Sin[a + b*x])/(b*Sqrt[d*Tan[a
 + b*x]])) + (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b
*x]]*Sqrt[d*Tan[a + b*x]])/(2*b)))/6)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] && !(GtQ[m, 1] && !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 1532, normalized size of antiderivative = 11.18

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1532 |

input
```
int(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/b*(d*tan(b*x+a))^(1/2)*(3*2^(1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x
+a)+1)^2)^(1/2)*ln(-2*cot(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^
2)^(1/2)-2*csc(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*
cot(b*x+a)+2)*(2*cos(b*x+a)^2-cot(b*x+a)-csc(b*x+a)+2*cos(b*x+a))+12*ln(-2
*cot(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*csc(b*x+a)
*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cot(b*x+a)+2)*(-sin(b
*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(-cos(b*x+a)^2-cos(b*x+a))+3*2^(1
/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(2*cot(b*x+a)*(-2*
sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+2*csc(b*x+a)*(-2*sin(b*x+a)*
cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cot(b*x+a)+2)*(-2*cos(b*x+a)^2+cot(b*
x+a)+csc(b*x+a)-2*cos(b*x+a))+12*ln(2*cot(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)
/(cos(b*x+a)+1)^2)^(1/2)+2*csc(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a
)+1)^2)^(1/2)-2*cot(b*x+a)+2)*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1
/2)*(cos(b*x+a)^2+cos(b*x+a))+6*2^(1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x
+a)+1)^2)^(1/2)*arctan((-sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*(2*cos(b*x+a)^2-cot(b*x+a)-csc(
b*x+a)+2*cos(b*x+a))+24*arctan((-sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*(-sin(b*x+a)*cos(b*x+a)
/(cos(b*x+a)+1)^2)^(1/2)*(-cos(b*x+a)^2-cos(b*x+a))+6*2^(1/2)*(-2*sin(b*x+
a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*(-2*sin(b*x+a)...
```

## Fricas [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(-(d^2*cos(b*x + a)^2 - d^2)*sqrt(d*tan(b*x + a))*sin(b*x + a)*tan
(b*x + a)^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*sin(b*x + a)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

## Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \sin(a+b\,x)^3\,(d\tan(a+b\,x))^{5/2}\,dx$$

input
```
int(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2), x)
```

## Reduce [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\sin(bx+a)^3\tan(bx+a)^2\,dx\right)d^2$$

input
```
int(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*sin(a + b*x)**3*tan(a + b*x)**2,x)*d**2
```

# 3.79 $\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result . . . 723
Mathematica [C] (verified) . . . 723
Rubi [A] (verified) . . . 724
Maple [A] (verified) . . . 727
Fricas [F] . . . 727
Sympy [F(-1)] . . . 728
Maxima [F] . . . 728
Giac [F(-2)] . . . 728
Mupad [F(-1)] . . . 729
Reduce [F] . . . 729

## Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}} - \frac{5d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{6b} + \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

output
```
5/3*d^3*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)-5/6*d^2*csc(b*x+a)*InverseJacobi
AM(a-1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*d
*sin(b*x+a)*(d*tan(b*x+a))^(3/2)/b
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.23

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{\cos(2(a+bx))\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(10\sqrt[4]{-1}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)+(7-\right.}{6b\tan^{\frac{3}{2}}(a+bx)\left(-1+\tan^2(a+bx)\right)}$$

input
```
Integrate[Sin[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

output
```
-1/6*(Cos[2*(a + b*x)]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(10*(-1)^(1/4)*El
lipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1] + (7 + 3*Cos[2*(a +
b*x)])*Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(5/2))/(b
*Tan[a + b*x]^(3/2)*(-1 + Tan[a + b*x]^2))
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 3074, 3042, 3078, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{ 3074}$$

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{ 3078}$$

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{1}{2}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{1}{2}\int\frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

$\downarrow$ 3081

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

$\downarrow$ 3042

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

$\downarrow$ 3053

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

$\downarrow$ 3042

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

$\downarrow$ 3120

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{2b}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

input
```
Int[Sin[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d*Sin[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b) - (5*d^2*(-((d*Sin[a + b*x
])/(b*Sqrt[d*Tan[a + b*x]])) + (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*
Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(2*b)))/3
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.21

| method | result |
|---|---|
| default | $\frac{\Big(\cos(bx+a)+\frac{2\sec(bx+a)}{3}+\frac{\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\mathrm{EllipticF}\Big(\sqrt{\csc(bx+a)-\cot(}}{6}}{b}$ |

input
```
int(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(cos(b*x+a)+2/3*sec(b*x+a)+1/6*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc
(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((cs
c(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-5*cot(b*x+a)-5*csc(b*x+a)))*(d
*tan(b*x+a))^(1/2)*d^2
```

## Fricas [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx=\int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*d^2*sin(b*x + a)*tan(b*x + a)^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*sin(b*x + a), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

## Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx=\int \sin\left(a+b\,x\right)\,\left(d\tan(a+b\,x)\right)^{5/2}\,dx$$

input
```
int(sin(a + b*x)*(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)*(d*tan(a + b*x))^(5/2), x)
```

## Reduce [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx=\sqrt{d}\left(\int \sqrt{\tan\left(bx+a\right)}\,\sin\left(bx+a\right)\tan\left(bx+a\right)^{2}dx\right)d^{2}$$

input
```
int(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*sin(a + b*x)*tan(a + b*x)**2,x)*d**2
```

# 3.80 $\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 730
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 730
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 731
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 733
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . 734
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 734
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 735
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 735
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 735
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 736

## Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

output
```
-1/3*d^2*csc(b*x+a)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(
1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*d*csc(b*x+a)*(d*tan(b*x+a))^(3/2)/b
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d^2\cos(a+bx)\left(\sec^2(a+bx)-\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right.}{3b}$$

input
```
Integrate[Csc[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d^2*Cos[a + b*x]*(Sec[a + b*x]^2 - Hypergeometric2F1[1/4, 1/2, 5/4, -Ta
n[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3074, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3074}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{1}{3}d^2\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{1}{3}d^2\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{1}{3}d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{1}{3}d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}$$

input
```
Int[Csc[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

output
```
-1/3*(d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]
*Sqrt[d*Tan[a + b*x]])/b + (2*d*Csc[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| default | $\frac{\sqrt{d\tan(bx+a)}\, d^{2} \left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\, \sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\, \sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{EllipticF}\left(\sqrt{\csc(bx+a}\right.\right.}{3b}$ |

input
```
int(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*(d*tan(b*x+a))^(1/2)*d^2*((csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*
x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b
*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)-csc(b*x+a))+2*sec(b*x+
a))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{\sqrt{i\,d}d^2\cos\left(bx+a\right)F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\mid-1)+\sqrt{-i\,d}d^2\cos\left(bx+a\right)F(a}{3\,b\cos\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(I*d)*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x
 + a)), -1) + sqrt(-I*d)*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) -
 I*sin(b*x + a)), -1) + 2*d^2*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*
x + a))
```

## Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a), x)
```

## Giac [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \frac{(d\tan(a+b\,x))^{5/2}}{\sin(a+b\,x)}\,dx$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x),x)
```

output
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x), x)
```

## Reduce [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\csc(bx+a)\tan(bx+a)^2\,dx\right)d^2$$

input
```
int(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)*tan(a + b*x)**2,x)*d**2
```

# 3.81 $\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$


Optimal result . . . . . 737
Mathematica [C] (verified) . . . . . 737
Rubi [A] (verified) . . . . . 738
Maple [A] (verified) . . . . . 740
Fricas [C] (verification not implemented) . . . . . 741
Sympy [F(-1)] . . . . . 741
Maxima [F] . . . . . 742
Giac [F] . . . . . 742
Mupad [F(-1)] . . . . . 742
Reduce [F] . . . . . 743


## Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

output
```
2/3*d^2*csc(b*x+a)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(1
/2)*(d*tan(b*x+a))^(1/2)/b+2/3*d*csc(b*x+a)*(d*tan(b*x+a))^(3/2)/b
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.48 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d^2\cos(a+bx)\left(\sec^2(a+bx)+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+b}\right.}{3b}$$

input
```
Integrate[Csc[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d^2*Cos[a + b*x]*(Sec[a + b*x]^2 + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -
Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3073, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{3073}$$

$$\frac{2}{3}d^2\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}d^2\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2}{3}d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Int[Csc[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sq
rt[d*Tan[a + b*x]])/(3*b) + (2*d*Csc[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b
)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| default | $\frac{2\sqrt{d\tan(bx+a)}\,d^2\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{EllipticF}\left(\sqrt{\csc(bx+\right.\right.}{3b}$ |

input
```
int(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*(d*tan(b*x+a))^(1/2)*d^2*((csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*
x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b
*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(cot(b*x+a)+csc(b*x+a))+sec(b*x+a))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$
$$-\frac{2\left(\sqrt{i\,d}d^2\cos(bx+a)\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+\sqrt{-i\,d}d^2\cos(bx+a)\,F(\arcsin(\text{co}\right.}{3\,b\cos(bx+a)}$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(I*d)*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*
x + a)), -1) + sqrt(-I*d)*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a)
- I*sin(b*x + a)), -1) - d^2*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x
 + a))
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

## Giac [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \frac{(d\tan(a+b\,x))^{5/2}}{\sin(a+b\,x)^3}\,dx$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^3,x)
```

output
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^3, x)
```

## Reduce [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\csc(bx+a)^3\tan(bx+a)^2\,dx\right)d^2$$

input
```
int(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**3*tan(a + b*x)**2,x)*d**2
```

# 3.82 $\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 744
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . 744
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 745
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 748
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . 748
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 749
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 749
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 750
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 750
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 750


## Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{4d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

output
```
-4/3*d^3*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)+4/3*d^2*csc(b*x+a)*InverseJacob
iAM(a-1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*
d*csc(b*x+a)^3*(d*tan(b*x+a))^(3/2)/b
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2d\csc^3(a+bx)\left(\cos(2(a+bx))\sqrt{\sec^2(a+bx)} + 2\sqrt[4]{-1}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\text{s}\right.}{3b\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^5*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*d*Csc[a + b*x]^3*(Cos[2*(a + b*x)]*Sqrt[Sec[a + b*x]^2] + 2*(-1)^(1/4)
*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sin[2*(a + b*x)]*
Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(3*b*Sqrt[Sec[a + b*x]^2])
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3073, 3042, 3079, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)^5}dx$$

$$\Big\downarrow \; 3073$$

$$2d^2\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \; 3042$$

$$2d^2\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^3}dx+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \; 3079$$

$$2d^2\left(\frac{2}{3}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \; 3042$$

$$2d^2\left(\frac{2}{3}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3081

$$2d^2\left(\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3042

$$2d^2\left(\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3053

$$2d^2\left(\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3042

$$2d^2\left(\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3120

$$2d^2\left(\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Int[Csc[a + b*x]^5*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d*Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b) + 2*d^2*((-2*d*Csc[a + b
*x])/(3*b*Sqrt[d*Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x
, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 10.32 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| default | $\frac{2\sqrt{d\tan(bx+a)}\,d^2\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{EllipticF}\left(\sqrt{\csc(bx+\right.\right.}{3b}$ |

input
```
int(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*(d*tan(b*x+a))^(1/2)*d^2*((csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*
x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b
*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(2*cot(b*x+a)+2*csc(b*x+a))-2*cot(b
*x+a)*csc(b*x+a)+sec(b*x+a)*csc(b*x+a)^2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.47

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{2\left(2\left(d^2\cos(bx+a)^3-d^2\cos(bx+a)\right)\sqrt{i\,d}F(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+2\left(d^2\cos(bx\right.\right.}{3\left(b\cos(bx\right.}$$

input
```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*sqrt(I*d)*elliptic_f(arcsi
n(cos(b*x + a) + I*sin(b*x + a)), -1) + 2*(d^2*cos(b*x + a)^3 - d^2*cos(b*
x + a))*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) -
 (2*d^2*cos(b*x + a)^2 - d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*
x + a)^3 - b*cos(b*x + a))
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**5*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^5\,dx$$

input
```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^5, x)
```

## Giac [F]

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx=\int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^5\,dx$$

input
```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx=\int \frac{(d\tan(a+b\,x))^{5/2}}{\sin(a+b\,x)^5}\,dx$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^5,x)
```

output
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^5, x)
```

## Reduce [F]

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx=\sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\csc(bx+a)^5\tan(bx+a)^2\,dx\right)d^2$$

input
```
int(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**5*tan(a + b*x)**2,x)*d**2
```

# 3.83 $\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal result . . . 751
Mathematica [C] (warning: unable to verify) . . . 751
Rubi [A] (verified) . . . 752
Maple [A] (verified) . . . 755
Fricas [C] (verification not implemented) . . . 756
Sympy [F(-1)] . . . 756
Maxima [F] . . . 757
Giac [F] . . . 757
Mupad [F(-1)] . . . 757
Reduce [F] . . . 758

## Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}} - \frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{40d^2\csc(a+bx)\,\mathrm{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

output
```
-40/21*d^3*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-20/21*d^3*csc(b*x+a)^3/b/(d*t
an(b*x+a))^(1/2)+40/21*d^2*csc(b*x+a)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2)
)*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*d*csc(b*x+a)^5*(d*tan(b*
x+a))^(3/2)/b
```

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.93

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$
$$-\frac{d^2\csc(a+bx)\left((1+10\cos(2(a+bx))-5\cos(4(a+bx)))\csc^3(a+bx)\sec(a+bx)\sqrt{\sec^2(a+bx)}+80\sqrt[4]{\ }\right.}{21b\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^7*(d*Tan[a + b*x])^(5/2),x]
```

output
```
-1/21*(d^2*Csc[a + b*x]*((1 + 10*Cos[2*(a + b*x)] - 5*Cos[4*(a + b*x)])*Cs
c[a + b*x]^3*Sec[a + b*x]*Sqrt[Sec[a + b*x]^2] + 80*(-1)^(1/4)*EllipticF[I
*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Tan[a + b*x]])*Sqrt[d*Ta
n[a + b*x]])/(b*Sqrt[Sec[a + b*x]^2])
```

## Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3073, 3042, 3079, 3042, 3079, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)^7}dx$$

$$\Big\downarrow \text{3073}$$

$$\frac{10}{3}d^2\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{3}d^2\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^5}dx+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\downarrow$ 3079

$$\frac{10}{3}d^2\left(\frac{6}{7}\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\downarrow$ 3042

$$\frac{10}{3}d^2\left(\frac{6}{7}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^3}dx-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\downarrow$ 3079

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2}{3}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\downarrow$ 3042

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2}{3}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\downarrow$ 3081

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\right.$$
$$\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\downarrow$ 3042

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\right.$$
$$\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\downarrow$ 3053

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3}{7b\sqrt{d\tan}}\right.$$
$$\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3042

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3}{7b\sqrt{d\tan}}\right.$$
$$\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3120

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\,c}{7b\sqrt{}}\right.$$
$$\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Int[Csc[a + b*x]^7*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d*Csc[a + b*x]^5*(d*Tan[a + b*x])^(3/2))/(3*b) + (10*d^2*((-2*d*Csc[a +
 b*x]^3)/(7*b*Sqrt[d*Tan[a + b*x]]) + (6*((-2*d*Csc[a + b*x])/(3*b*Sqrt[d*
Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a
 + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)))/7))/3
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 231.62 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.14

| method | result |
|---|---|
| default | $\frac{\left(-\frac{2 \sin(bx+a)^3 \cos(bx+a)(-20\cos(bx+a)-20)\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{EllipticF}\left(\right.}{21}\right.}{b}$ |

input
```
int(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-2/21*sin(b*x+a)^3*cos(b*x+a)*(-20*cos(b*x+a)-20)*(csc(b*x+a)-cot(b*x
+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))
^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))+40/21*cos(b*
x+a)^4-20/7*cos(b*x+a)^2+2/3)*sec(b*x+a)*csc(b*x+a)^4*(d*tan(b*x+a))^(1/2)
*d^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.49

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{2\left(20\left(d^2\cos\left(bx+a\right)^5-2\,d^2\cos\left(bx+a\right)^3+d^2\cos\left(bx+a\right)\right)\sqrt{i\,d}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)}{}$$

input
```
integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/21*(20*(d^2*cos(b*x + a)^5 - 2*d^2*cos(b*x + a)^3 + d^2*cos(b*x + a))*s
qrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + 20*(d^2*c
os(b*x + a)^5 - 2*d^2*cos(b*x + a)^3 + d^2*cos(b*x + a))*sqrt(-I*d)*ellipt
ic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - (20*d^2*cos(b*x + a)^4 -
 30*d^2*cos(b*x + a)^2 + 7*d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(
b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**7*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^7\,dx$$

input
```
integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^7, x)
```

## Giac [F]

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^7\,dx$$

input
```
integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^7, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \frac{(d\tan(a+b\,x))^{5/2}}{\sin(a+b\,x)^7}\,dx$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^7,x)
```

output
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^7, x)
```

## Reduce [F]

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\csc(bx+a)^7\tan(bx+a)^2\,dx\right)d^2$$

input
```
int(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*csc(a + b*x)**7*tan(a + b*x)**2,x)*d**2
```

# 3.84 $\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 759
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 760
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . 760
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 764
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . 765
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 766
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . 766
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . 767
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 768
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 768


## Optimal result

Integrand size = 21, antiderivative size = 204

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}}+\frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}}+\frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{32\sqrt{2}b\sqrt{d}}-\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd}-\frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3}$$

output
```
-5/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(1/2)+5/6
4*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(1/2)+5/64*ar
ctanh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))*2^(1/2)/b
/d^(1/2)-5/16*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d-1/4*cos(b*x+a)^4*(d*ta
n(b*x+a))^(5/2)/b/d^3
```

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sec(a+bx)\left(-7\sin(a+bx)-5\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}+5\log\left(\cos(a+bx)\right.\right.}{64b\sqrt{d\tan(a+b}}$$

input
```
Integrate[Sin[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(Sec[a + b*x]*(-7*Sin[a + b*x] - 5*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqr
t[Sin[2*(a + b*x)]] + 5*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a +
b*x)]]]*Sqrt[Sin[2*(a + b*x)]] - 6*Sin[3*(a + b*x)] + Sin[5*(a + b*x)]))/(
64*b*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3071, 252, 252, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(a+bx)^4}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \; 3071$$

$$\frac{d\int \frac{(d\tan(a+bx))^{7/2}}{(\tan^2(a+bx)d^2+d^2)^3}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \; 252$$

$$\frac{d\left(\frac{5}{8}\int \frac{(d\tan(a+bx))^{3/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{5/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{4}\int \frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)d^2+d^2)}d(d\tan(a+bx))-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{5/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{2}\int \frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{5/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\Big\downarrow \text{755}$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{5/2}}{4(d^2\tan^2(a+bx)+d^2)}\right.}{b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2d}+\int \frac{d-d^2\tan^2(a+b}{d^4\tan^4(a+bx)+}\right.\right.\right.}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)-\right.\right.}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\right.}{b}$$

$$\Big\downarrow \text{1479}$$

$$d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a-\right.}{\sqrt{2}\sqrt{d}}}{\ }\right.\right.\right.}{b}$$

$\downarrow$ 25

$$d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+\right.}{\sqrt{2}\sqrt{d}}}{\ }\right.\right.\right.}{b}$$

$\downarrow$ 27

$$d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+\right.}{\sqrt{2}\sqrt{d}}}{\ }\right.\right.\right.}{b}$$

$\downarrow$ 1103

$$d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.\right.\right.}{b}$$

input
```
Int[Sin[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*(-1/4*(d*Tan[a + b*x])^(5/2)/(d^2 + d^2*Tan[a + b*x]^2)^2 + (5*(((-(Arc
Tan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt
[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]
*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sq
rt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d
))/2 - Sqrt[d*Tan[a + b*x]]/(2*(d^2 + d^2*Tan[a + b*x]^2))))/8))/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(158) = 316$.

Time = 7.86 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.69

| method | result |
|---|---|
| default | $\frac{\left(-5\ln\left(-\frac{\cot(bx+a)\cos(bx+a)-2\cot(bx+a)+2\sin(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}-2\cos(bx+a)+\csc(bx+a)-\sin(bx+a)+2}{-1+\cos(bx+a)}\right)\sin(bx+a)+5\right.}{}$ |

input
```
int(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/128/b*(-5*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-2*sin(b
*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a
)+2)/(-1+cos(b*x+a)))*sin(b*x+a)+5*ln((2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x
+a)/(cos(b*x+a)+1)^2)^(1/2)-cot(b*x+a)*cos(b*x+a)+sin(b*x+a)+2*cos(b*x+a)-
csc(b*x+a)+2*cot(b*x+a)-2)/(-1+cos(b*x+a)))*sin(b*x+a)-10*arctan((sin(b*x+
a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos
(b*x+a)))*sin(b*x+a)-10*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(
b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*sin(b*x+a)+cos(b*x+a)*(-
80*cos(b*x+a)^2+(80*sin(b*x+a)-80)*cos(b*x+a)+80*sin(b*x+a))*sin(b*x+a)^2*
(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)*(4*(24*cos(b*
x+a)^2-29)*(cos(b*x+a)+1)*sin(b*x+a)-80*cos(b*x+a)*(cos(b*x+a)^2-1)*(cos(b
*x+a)+1))*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2))/(-sin(b
*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(cos(b*x+a)+1)/(d*tan(b*x+a))^(1/
2)*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(158) = 316$.

Time = 0.14 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.10

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$
$$= \frac{10\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}(\cos(bx+a)-\sin(bx+a))}{2\sqrt{d}\sin(bx+a)}\right)-5\sqrt{2}\sqrt{d}\arctan\left(\frac{2\cos(bx+a)^2-2\cos(bx+a)\sin(bx+a)+\frac{\sqrt{2}}{\cdots}}{2\left(\cos(bx+a)^2+\cos(bx+a)\sin(bx+\cdots\right.}\right.}{\cdots}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
1/256*(10*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x
+ a))*(cos(b*x + a) - sin(b*x + a))/(sqrt(d)*sin(b*x + a))) - 5*sqrt(2)*sq
rt(d)*arctan(1/2*(2*cos(b*x + a)^2 - 2*cos(b*x + a)*sin(b*x + a) + sqrt(2)
*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) - 2)/(cos(b*x + a)^2 + cos(b*x
+ a)*sin(b*x + a) - 1)) - 5*sqrt(2)*sqrt(d)*arctan(-1/2*(2*cos(b*x + a)^2
- 2*cos(b*x + a)*sin(b*x + a) - sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x + a))/
sqrt(d) - 2)/(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a) - 1)) + 5*sqrt(2)
*sqrt(d)*log(4*cos(b*x + a)*sin(b*x + a) + 2*sqrt(2)*(cos(b*x + a)^2 + cos
(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) + 1) - 5
*sqrt(2)*sqrt(d)*log(4*cos(b*x + a)*sin(b*x + a) - 2*sqrt(2)*(cos(b*x + a)
^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d)
+ 1) + 16*(4*cos(b*x + a)^4 - 9*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*
x + a)))/(b*d)
```

## Sympy [F]

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin^4{(a+bx)}}{\sqrt{d\tan{(a+bx)}}}\,dx$$

input
```
integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sin(a + b*x)**4/sqrt(d*tan(a + b*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.08

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{10\sqrt{2}d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{9}{2}}\log\left(\right.}{}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
1/128*(10*sqrt(2)*d^(9/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*t
an(b*x + a)))/sqrt(d)) + 10*sqrt(2)*d^(9/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 5*sqrt(2)*d^(9/2)*log(d*tan(b*
x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 5*sqrt(2)*d^(9/2)*log
(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*(9*(d*tan(
b*x + a))^(5/2)*d^6 + 5*sqrt(d*tan(b*x + a))*d^8)/(d^4*tan(b*x + a)^4 + 2*
d^4*tan(b*x + a)^2 + d^4))/(b*d^5)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.21

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{5\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd} + \frac{5\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd} + \frac{5\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{128\,bd} - \frac{5\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{128\,bd} - \frac{9\sqrt{d\tan\left(bx+a\right)}d^3\tan\left(bx+a\right)^2+5\sqrt{d\tan\left(bx+a\right)}d^3}{16\left(d^2\tan\left(bx+a\right)^2+d^2\right)^2b}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
5/64*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqr
t(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) + 5/64*sqrt(2)*sqrt(abs(d))*arctan(
-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))
/(b*d) + 5/128*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*ta
n(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 5/128*sqrt(2)*sqrt(abs(d))*log(
d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d)
 - 1/16*(9*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)^2 + 5*sqrt(d*tan(b*x + a)
)*d^3)/((d^2*tan(b*x + a)^2 + d^2)^2*b)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=\int \frac{\sin\left(a+b\,x\right)^4}{\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

input
```
int(sin(a + b*x)^4/(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^4/(d*tan(a + b*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=\frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)^4}{\tan(bx+a)}dx\right)}{d}$$

input
```
int(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x)**4)/tan(a + b*x),x))/d
```

# 3.85 $\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 769
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 769
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . 770
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 774
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . 775
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 776
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . 776
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 777
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 777
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 778

## Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}} + \frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{4\sqrt{2}b\sqrt{d}} - \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd}$$

output
```
-1/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(1/2)+1/8*
arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(1/2)+1/8*arcta
nh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))*2^(1/2)/b/d^
(1/2)-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d
```

## Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.63

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{\sec(a+bx)\left(\sin(a+bx)+\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-\log\left(\cos(a+bx)+\text{si}\right.\right.}{8b\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/8*(Sec[a + b*x]*(Sin[a + b*x] + ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqr
t[Sin[2*(a + b*x)]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*
x)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)]))/(b*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3071, 252, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{3/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{d\left(\frac{1}{4}\int \frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)d^2+d^2)}d(d\tan(a+bx))-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{d\left(\frac{1}{2}\int \frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{755}$$

$$\frac{d\left(\frac{1}{2}\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 1476

$$\frac{d\left(\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d}{2d}\right.\right.}{b}$$

$\downarrow$ 1082

$$\frac{d\left(\frac{1}{2}\left(\frac{\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)-\frac{}{2(d^2}\right.}{b}$$

$\downarrow$ 217

$$\frac{d\left(\frac{1}{2}\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 1479

$$\frac{d\left(\frac{1}{2}\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+\right.}{\sqrt{2}\sqrt{d}}\right.\right.}{b}$$

$\downarrow$ 25

$$\frac{d\left(\frac{1}{2}\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}\right.\right.}{b}$$

$\downarrow$ 27

$$d\left(\frac{1}{2}\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}\right.\right.$$
$$\overline{\qquad\qquad b\qquad\qquad}$$

$\downarrow$ 1103

$$d\left(\frac{1}{2}\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+b\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.\right.$$
$$\overline{\qquad\qquad b\qquad\qquad}$$

input
```
Int[Sin[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*(((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcT
an[1 + Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[
d - Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d]) +
 Log[d + Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqr
t[d]))/(2*d))/2 - Sqrt[d*Tan[a + b*x]]/(2*(d^2 + d^2*Tan[a + b*x]^2))))/b
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(132) = 264$.

Time = 1.82 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.32

| method | result |
|---|---|
| default | $-\frac{\sin(bx+a)\left(\cos(bx+a)(4\cos(bx+a)+4)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}+2\arctan\left(\frac{\sin(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}-\cos(bx+a)+1}{-1+\cos(bx+a)}\right)+\mathrm{l}\cdots\right.}{\cdots}$ |

input
```
int(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/b*sin(b*x+a)*(cos(b*x+a)*(4*cos(b*x+a)+4)*(-2*sin(b*x+a)*cos(b*x+a)/
(cos(b*x+a)+1)^2)^(1/2)+2*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(co
s(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))+ln(-(cot(b*x+a)*cos(b*
x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)
^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))-ln((2*sin(b*
x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cot(b*x+a)*cos(b*x+
a)+sin(b*x+a)+2*cos(b*x+a)-csc(b*x+a)+2*cot(b*x+a)-2)/(-1+cos(b*x+a)))+2*a
rctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*
x+a)-1)/(-1+cos(b*x+a))))/(cos(b*x+a)+1)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+
a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1/2)*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(132) = 264$.

Time = 0.14 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.37

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx =$$

$$-\frac{16\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)^2 - 2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}(\cos(bx+a)-\sin(bx+a))}{2\sqrt{d}\sin(bx+a)}\right) + \sqrt{2}\sqrt{d}\arctan\left(2\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-1/32*(16*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2 - 2*sqrt(2)*sqr
t(d)*arctan(-1/2*sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x + a))*(cos(b*x + a) -
 sin(b*x + a))/(sqrt(d)*sin(b*x + a))) + sqrt(2)*sqrt(d)*arctan(1/2*(2*cos
(b*x + a)^2 - 2*cos(b*x + a)*sin(b*x + a) + sqrt(2)*sqrt(d*sin(b*x + a)/co
s(b*x + a))/sqrt(d) - 2)/(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a) - 1))
 + sqrt(2)*sqrt(d)*arctan(-1/2*(2*cos(b*x + a)^2 - 2*cos(b*x + a)*sin(b*x
+ a) - sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) - 2)/(cos(b*x + a
)^2 + cos(b*x + a)*sin(b*x + a) - 1)) - sqrt(2)*sqrt(d)*log(4*cos(b*x + a)
*sin(b*x + a) + 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a))*sqr
t(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) + 1) + sqrt(2)*sqrt(d)*log(4*cos(b*
x + a)*sin(b*x + a) - 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a
))*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) + 1))/(b*d)
```

## Sympy [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sin(a + b*x)**2/sqrt(d*tan(a + b*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.08

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+\sqrt{2}d^{\frac{5}{2}}\log\left(d\tan\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(2*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d)) + 2*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + sqrt(2)*d^(5/2)*log(d*tan(b*x + a
) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - sqrt(2)*d^(5/2)*log(d*tan(
b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x +
a))*d^4/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.25

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd} + \frac{\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd} + \frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{16\,bd} - \frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{16\,bd} - \frac{\sqrt{d\tan\left(bx+a\right)}d}{2\left(d^2\tan\left(bx+a\right)^2+d^2\right)b}$$

input

```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output

```
1/8*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) + 1/8*sqrt(2)*sqrt(abs(d))*arctan(-1
/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(
b*d) + 1/16*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b
*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 1/16*sqrt(2)*sqrt(abs(d))*log(d*ta
n(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 1
/2*sqrt(d*tan(b*x + a))*d/((d^2*tan(b*x + a)^2 + d^2)*b)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

input

```
int(sin(a + b*x)^2/(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^2/(d*tan(a + b*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)^2}{\tan(bx+a)}dx\right)}{d}$$

input
```
int(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x)**2)/tan(a + b*x),x))/d
```

# 3.86 $\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 779
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 779
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 780
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 781
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . 781
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 782
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 782
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . 782
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 783
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 783

## Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

output
```
-2/3*d/b/(d*tan(b*x+a))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Csc[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{1}{(d\tan(a+bx))^{5/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

input
```
Int[Csc[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{2d}{3b(d\tan(bx+a))^{\frac{3}{2}}}$ | 17 |
| default | $-\frac{2d}{3b(d\tan(bx+a))^{\frac{3}{2}}}$ | 17 |

input
```
int(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d/b/(d*tan(b*x+a))^(3/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(16) = 32.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos{(bx+a)}^2}{3\left(bd\cos{(bx+a)}^2 - bd\right)}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2/(b*d*cos(b*x + a)^2 -
 b*d)
```

## Sympy [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc^2{(a+bx)}}{\sqrt{d\tan{(a+bx)}}}\,dx$$

input
```
integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)**2/sqrt(d*tan(a + b*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2}{3\sqrt{d\tan{(bx+a)}}b\tan{(bx+a)}}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/3/(sqrt(d*tan(b*x + a))*b*tan(b*x + a))
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2}{3\sqrt{d\tan{(bx+a)}}b\tan{(bx+a)}}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2/3/(sqrt(d*tan(b*x + a))*b*tan(b*x + a))
```

## Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.10

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}\,(\cos(2\,a+2\,b\,x)+2\,\cos(4\,a+4\,b\,x)-\cos(6\,a+6\,b\,x)-2)}{3\,b\,d\,(15\,\cos(2\,a+2\,b\,x)-6\,\cos(4\,a+4\,b\,x)+\cos(6\,a+6\,b\,x)-10)}$$

input
```
int(1/(sin(a + b*x)^2*(d*tan(a + b*x))^(1/2)),x)
```

output
```
-(2*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)*(cos(2*a + 2*b*x)
+ 2*cos(4*a + 4*b*x) - cos(6*a + 6*b*x) - 2))/(3*b*d*(15*cos(2*a + 2*b*x)
- 6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 10))
```

## Reduce [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)^2}{\tan(bx+a)}dx\right)}{d}$$

input
```
int(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x)**2)/tan(a + b*x),x))/d
```

# 3.87 $\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 784
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 784
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 785
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 786
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 787
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 787
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 787
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 788
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 788
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 789

## Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d^3}{7b(d\tan(a+bx))^{7/2}} - \frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

output
```
-2/7*d^3/b/(d*tan(b*x+a))^(7/2)-2/3*d/b/(d*tan(b*x+a))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2d(-5+2\cos(2(a+bx)))\csc^2(a+bx)}{21b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Csc[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(2*d*(-5 + 2*Cos[2*(a + b*x)])*Csc[a + b*x]^2)/(21*b*(d*Tan[a + b*x])^(3/2
))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^4\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\tan^2(a+bx)d^2+d^2}{(d\tan(a+bx))^{9/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^2}{(d\tan(a+bx))^{9/2}}+\frac{1}{(d\tan(a+bx))^{5/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^2}{7(d\tan(a+bx))^{7/2}}-\frac{2}{3(d\tan(a+bx))^{3/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*((-2*d^2)/(7*(d*Tan[a + b*x])^(7/2)) - 2/(3*(d*Tan[a + b*x])^(3/2))))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(35) = 70$.

Time = 0.68 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.28

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(4\cot(bx+a)^3-7\cot(bx+a)\csc(bx+a)^2\right)\sqrt{2}}{21b\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\sqrt{d\tan(bx+a)}}$ | 98 |

input
```
int(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/21/b*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(-sin(b*x+a)*cos(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1/2)*(4*cot(b*x+a)^3-7*cot(
b*x+a)*csc(b*x+a)^2)*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\left(4\cos\left(bx+a\right)^4-7\cos\left(bx+a\right)^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{21\left(bd\cos\left(bx+a\right)^4-2\,bd\cos\left(bx+a\right)^2+bd\right)}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/21*(4*cos(b*x + a)^4 - 7*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a
))/(b*d*cos(b*x + a)^4 - 2*b*d*cos(b*x + a)^2 + b*d)
```

## Sympy [F]

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc^4\left(a+bx\right)}{\sqrt{d\tan\left(a+bx\right)}}\,dx$$

input
```
integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)**4/sqrt(d*tan(a + b*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(7\,d^2\tan\left(bx+a\right)^2+3\,d^2\right)d}{21\,\left(d\tan\left(bx+a\right)\right)^{\frac{7}{2}}b}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/21*(7*d^2*tan(b*x + a)^2 + 3*d^2)*d/((d*tan(b*x + a))^(7/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(7\,d^3\tan\left(bx+a\right)^2+3\,d^3\right)}{21\,\sqrt{d\tan\left(bx+a\right)}bd^3\tan\left(bx+a\right)^3}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2/21*(7*d^3*tan(b*x + a)^2 + 3*d^3)/(sqrt(d*tan(b*x + a))*b*d^3*tan(b*x +
 a)^3)
```

## Mupad [B] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 530, normalized size of antiderivative = 12.33

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(a + b*x)^4*(d*tan(a + b*x))^(1/2)),x)
```

output
```
(344*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
 + b*x*2i) + 1))^(1/2))/(105*b*d*(exp(a*2i + b*x*2i) - 1)) + (40*(exp(a*2i
 + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1
))^(1/2))/(21*b*d*(exp(a*2i + b*x*2i) - 1)^2) + (24*(exp(a*2i + b*x*2i) +
1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(35
*b*d*(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2
i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*304i)/(105*b*d*(exp(
a*2i + b*x*2i)*1i - 1i)) + (16*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b
*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(7*b*d*(exp(a*2i + b*x*2
i)*1i - 1i)^2) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1
i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*144i)/(35*b*d*(exp(a*2i + b*x*2i)*1i -
 1i)^3) - (16*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/
(exp(a*2i + b*x*2i) + 1))^(1/2))/(7*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^4)
```

## Reduce [F]

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)^4}{\tan(bx+a)}dx\right)}{d}$$

input
```
int(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x)**4)/tan(a + b*x),x))/d
```

# 3.88 $\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 790
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 790
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 791
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 792
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . 793
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 793
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . 793
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . 794
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 794
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 795

## Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d^5}{11b(d\tan(a+bx))^{11/2}} - \frac{4d^3}{7b(d\tan(a+bx))^{7/2}} - \frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

output
```
-2/11*d^5/b/(d*tan(b*x+a))^(11/2)-4/7*d^3/b/(d*tan(b*x+a))^(7/2)-2/3*d/b/(
d*tan(b*x+a))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2d(-45+28\cos(2(a+bx))-4\cos(4(a+bx)))\csc^4(a+bx)}{231b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Csc[a + b*x]^6/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(2*d*(-45 + 28*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4)/(231
*b*(d*Tan[a + b*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^6\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\left(\tan^2(a+bx)d^2+d^2\right)^2}{(d\tan(a+bx))^{13/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^4}{(d\tan(a+bx))^{13/2}}+\frac{2d^2}{(d\tan(a+bx))^{9/2}}+\frac{1}{(d\tan(a+bx))^{5/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^4}{11(d\tan(a+bx))^{11/2}}-\frac{4d^2}{7(d\tan(a+bx))^{7/2}}-\frac{2}{3(d\tan(a+bx))^{3/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^6/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*((-2*d^4)/(11*(d*Tan[a + b*x])^(11/2)) - (4*d^2)/(7*(d*Tan[a + b*x])^(7
/2)) - 2/(3*(d*Tan[a + b*x])^(3/2))))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

| method | result | size |
|---|---|---|
| default | $-\frac{\cot(bx+a)\csc(bx+a)^4\left(32\cos(bx+a)^4-88\cos(bx+a)^2+77\right)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\,\sqrt{2}}{231b\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\,\sqrt{d\tan(bx+a)}}$ | 107 |

input
```
int(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/231/b*cot(b*x+a)*csc(b*x+a)^4*(32*cos(b*x+a)^4-88*cos(b*x+a)^2+77)*(-2*
sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(-sin(b*x+a)*cos(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1/2)*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\left(32\cos\left(bx+a\right)^6-88\cos\left(bx+a\right)^4+77\cos\left(bx+a\right)^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{231\left(bd\cos\left(bx+a\right)^6-3\,bd\cos\left(bx+a\right)^4+3\,bd\cos\left(bx+a\right)^2-bd\right)}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/231*(32*cos(b*x + a)^6 - 88*cos(b*x + a)^4 + 77*cos(b*x + a)^2)*sqrt(d*s
in(b*x + a)/cos(b*x + a))/(b*d*cos(b*x + a)^6 - 3*b*d*cos(b*x + a)^4 + 3*b
*d*cos(b*x + a)^2 - b*d)
```

## Sympy [F]

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc^6\left(a+bx\right)}{\sqrt{d\tan\left(a+bx\right)}}\,dx$$

input
```
integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)**6/sqrt(d*tan(a + b*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(77\,d^4\tan\left(bx+a\right)^4+66\,d^4\tan\left(bx+a\right)^2+21\,d^4\right)d}{231\,\left(d\tan\left(bx+a\right)\right)^{\frac{11}{2}}\,b}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/231*(77*d^4*tan(b*x + a)^4 + 66*d^4*tan(b*x + a)^2 + 21*d^4)*d/((d*tan(
b*x + a))^(11/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(77\,d^5\tan\left(bx+a\right)^4+66\,d^5\tan\left(bx+a\right)^2+21\,d^5\right)}{231\,\sqrt{d\tan\left(bx+a\right)}bd^5\tan\left(bx+a\right)^5}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2/231*(77*d^5*tan(b*x + a)^4 + 66*d^5*tan(b*x + a)^2 + 21*d^5)/(sqrt(d*ta
n(b*x + a))*b*d^5*tan(b*x + a)^5)
```

## Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 831, normalized size of antiderivative = 12.78

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(a + b*x)^6*(d*tan(a + b*x))^(1/2)),x)
```

output
```
((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2)*44864i)/(10395*b*d*(exp(a*2i + b*x*2i)*1i - 1i)) - (128
*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2))/(35*b*d*(exp(a*2i + b*x*2i) - 1)^2) - (7136*(exp(a*2i
+ b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1)
)^(1/2))/(1155*b*d*(exp(a*2i + b*x*2i) - 1)^3) - (1216*(exp(a*2i + b*x*2i)
 + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/
(231*b*d*(exp(a*2i + b*x*2i) - 1)^4) - (160*(exp(a*2i + b*x*2i) + 1)*(-(d*
(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(99*b*d*(ex
p(a*2i + b*x*2i) - 1)^5) - (41984*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i
+ b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(10395*b*d*(exp(a*2i
+ b*x*2i) - 1)) - (3904*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*
1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(1155*b*d*(exp(a*2i + b*x*2i)*1
i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/
(exp(a*2i + b*x*2i) + 1))^(1/2)*1088i)/(165*b*d*(exp(a*2i + b*x*2i)*1i - 1
i)^3) + (320*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2))/(21*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^4) +
((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2)*1600i)/(99*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (64*(e
xp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b...
```

## Reduce [F]

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)^6}{\tan(bx+a)}dx\right)}{d}$$

input
```
int(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x)**6)/tan(a + b*x),x))/d
```

# 3.89 $\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 796
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 796
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 797
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 799
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 800
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 800
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 801
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 801
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 801
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 802

## Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}} - \frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}} + \frac{7E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{20b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-7/30*d*sin(b*x+a)^3/b/(d*tan(b*x+a))^(3/2)-1/5*d*sin(b*x+a)^5/b/(d*tan(b*
x+a))^(3/2)-7/20*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b
*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.77 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sin(a+bx)\left(-20\sin(2(a+bx))+3\sin(4(a+bx))+28\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\phantom{x}}\right.}{120b\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^5/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(Sin[a + b*x]*(-20*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + 28*Hypergeometr
ic2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]))/
(120*b*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3078, 3042, 3078, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^5}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{7}{10}\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\int \frac{\sin(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{7}{10}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{7}{10}\left(\frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{7}{10}\left(\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7}{10}\left(\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]^5/Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/5*(d*Sin[a + b*x]^5)/(b*(d*Tan[a + b*x])^(3/2)) + (7*(-1/3*(d*Sin[a + b
*x]^3)/(b*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x, 2]*Sin[a +
b*x])/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])))/10
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(94) = 188$.

Time = 1.26 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| default | $-\frac{\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{EllipticF}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,(-21-21\sec(}{120}$ |

input
```
int(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/120*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^
(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(csc(b*x+a)-c
ot(b*x+a)+1)^(1/2)*(-21-21*sec(b*x+a))-1/120*(-2*csc(b*x+a)+2*cot(b*x+a)+2
)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*Ell
ipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(42+42*sec(b*x+a))-1/5
*cos(b*x+a)^5+19/30*cos(b*x+a)^3-47/60*cos(b*x+a)+7/20)/(d*tan(b*x+a))^(1/
2)
```

## Fricas [F]

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^5}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

input
```
integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*tan(b*x + a))*sin(
b*x + a)/(d*tan(b*x + a)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**5/(d*tan(b*x+a))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)^5}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^5/sqrt(d*tan(b*x + a)), x)
```

## Giac [F]

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)^5}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^5/sqrt(d*tan(b*x + a)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(a+b\,x)^5}{\sqrt{d\tan(a+b\,x)}}\,dx$$

input
```
int(sin(a + b*x)^5/(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^5/(d*tan(a + b*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)^5}{\tan(bx+a)}dx\right)}{d}$$

input
```
int(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x)**5)/tan(a + b*x),x))/d
```

# 3.90 $\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$


Optimal result . . . . . . . . . . 803
Mathematica [C] (verified) . . . . . . . . . . 803
Rubi [A] (verified) . . . . . . . . . . 804
Maple [B] (verified) . . . . . . . . . . 806
Fricas [F] . . . . . . . . . . 807
Sympy [F(-1)] . . . . . . . . . . 807
Maxima [F] . . . . . . . . . . 807
Giac [F] . . . . . . . . . . 808
Mupad [F(-1)] . . . . . . . . . . 808
Reduce [F] . . . . . . . . . . 808


## Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} + \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-1/3*d*sin(b*x+a)^3/b/(d*tan(b*x+a))^(3/2)-1/2*EllipticE(cos(a+1/4*Pi+b*x)
,2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.24

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$
$$= \frac{\sqrt{d\tan(a+bx)}\left(-\sqrt{\sec^2(a+bx)}(\sin(a+bx)+\sin(3(a+bx)))+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^{2}\right.\right.}{12bd\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(Sin[a + b*x] + Sin[3*(a + b
*x)])) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*
Tan[a + b*x]))/(12*b*d*Sqrt[Sec[a + b*x]^2])
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3078, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/3*(d*Sin[a + b*x]^3)/(b*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 +
 b*x, 2]*Sin[a + b*x])/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(70) = 140$.

Time = 0.90 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.73

| method | result |
|---|---|
| default | $\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \text{EllipticE}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1},\right.$ |

input
```
int(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/b/(d*tan(b*x+a))^(1/2)*((csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a
)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticE((csc(b*x+
a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-6-6*sec(b*x+a))+(csc(b*x+a)-cot(b*x+
a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^
(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(3+3*sec(b*x+
a))+4*cos(b*x+a)^3-10*cos(b*x+a)+6)
```

## Fricas [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d*tan(b*
x + a)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

## Giac [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)^3}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(a+b\,x)^3}{\sqrt{d\tan(a+b\,x)}}\,dx$$

input
```
int(sin(a + b*x)^3/(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^3/(d*tan(a + b*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)^3}{\tan(bx+a)}dx\right)}{d}$$

input
```
int(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x)**3)/tan(a + b*x),x))/d
```

# 3.91 $\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 809
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 809
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 810
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 811
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 812
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 812
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 813
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 813
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 813
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 814

## Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d
*tan(b*x+a))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\sin(a+bx)\sqrt{d\tan(a+bx)}}{3bd}$$

input
```
Integrate[Sin[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*
Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b*d)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sin[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt
[d*Tan[a + b*x]])
```

**Defintions of rubi rules used**

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(45) = 90$.

Time = 0.75 (sec) , antiderivative size = 207, normalized size of antiderivative = 4.40

| method | result |
|---|---|
| default | $-\frac{\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \operatorname{EllipticE}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}, \frac{\sqrt{2}}{2}\right)(2+2\sec(bx+$ |

input
```
int(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(
1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1
/2),1/2*2^(1/2))*(2+2*sec(b*x+a))-1/2*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*
csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF(
(csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-1-sec(b*x+a))+1-cos(b*x+a))
/(d*tan(b*x+a))^(1/2)
```

## Fricas [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d*tan(b*x + a)), x)
```

## Sympy [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(a+bx\right)}{\sqrt{d\tan\left(a+bx\right)}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sin(a + b*x)/sqrt(d*tan(a + b*x)), x)
```

## Maxima [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

## Giac [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(a+b\,x)}{\sqrt{d\tan(a+b\,x)}}\,dx$$

input
```
int(sin(a + b*x)/(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)/(d*tan(a + b*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)}{\tan(bx+a)}dx\right)}{d}$$

input
```
int(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x))/tan(a + b*x),x))/d
```

# 3.92 $\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 815
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 815
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 816
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 818
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 819
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 819
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 820
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 820
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 820
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 821

## Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-2*cos(b*x+a)/b/(d*tan(b*x+a))^(1/2)+2*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2)
)*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\cos(a+bx)\left(3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{3b\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*Cos[a + b*x]*(3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*
Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(3*b*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.51, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3081, 3042, 3050, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin(a+bx)^{3/2}}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3050}$$

$$\frac{\sqrt{\sin(a+bx)}\left(-2\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\left(-2\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sqrt{\sin(a+bx)}\left(-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}E\left(a+bx-\frac{\pi}{4}|2\right)}{b\sqrt{\sin(2a+2bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(Sqrt[Sin[a + b*x]]*((-2*Cos[a + b*x]^(3/2))/(b*Sqrt[Sin[a + b*x]]) - (2*S
qrt[Cos[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt
[Sin[2*a + 2*b*x]])))/(Sqrt[Cos[a + b*x]]*Sqrt[d*Tan[a + b*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(67) = 134$.

Time $= 0.72$ (sec) , antiderivative size $= 281$, normalized size of antiderivative $= 3.90$

| method | result |
|---|---|
| default | $-\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(-1+2\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{Ellipt}\right.}{}$ |

input
```
int(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(-1+2*(csc(b*x+a)-c
ot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b
*x+a))^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))-(csc(b
*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a
)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))
+csc(b*x+a)^2*(1-cos(b*x+a))^2)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^
(1/2)/(csc(b*x+a)^2*(1-cos(b*x+a))^2-1)/(d*tan(b*x+a))^(1/2)*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.35

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx =$$
$$-\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)^2 + i\sqrt{i\,d}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a) - i\sqrt{-i}$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2 + I*sqrt(I*d)*ellipti
c_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) - I*sqrt(-I*d)
*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - I*sq
rt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a)
 + I*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(
b*x + a))/(b*d*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)/sqrt(d*tan(a + b*x)), x)
```

## Maxima [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc(bx+a)}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

## Giac [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc(bx+a)}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{1}{\sin(a+b\,x)\,\sqrt{d\tan(a+b\,x)}}\,dx$$

input
```
int(1/(sin(a + b*x)*(d*tan(a + b*x))^(1/2)),x)
```

output
```
int(1/(sin(a + b*x)*(d*tan(a + b*x))^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)}{\tan(bx+a)}dx\right)}{d}$$

input
```
int(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x))/tan(a + b*x),x))/d
```

# 3.93 $\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal result . . . . . . . . . . 822
Mathematica [C] (verified) . . . . . . . . . . 822
Rubi [A] (verified) . . . . . . . . . . 823
Maple [B] (verified) . . . . . . . . . . 826
Fricas [C] (verification not implemented) . . . . . . . . . . 826
Sympy [F] . . . . . . . . . . 827
Maxima [F] . . . . . . . . . . 827
Giac [F] . . . . . . . . . . 828
Mupad [F(-1)] . . . . . . . . . . 828
Reduce [F] . . . . . . . . . . 828

## Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}} - \frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}} - \frac{4E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-2/5*d*csc(b*x+a)/b/(d*tan(b*x+a))^(3/2)-4/5*cos(b*x+a)/b/(d*tan(b*x+a))^(
1/2)+4/5*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^
(1/2)/(d*tan(b*x+a))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.59 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{6(-2+\cos(2(a+bx)))\cot(a+bx)\csc(a+bx)\sqrt{\sec^2(a+bx)} - 8\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a\right.}{15b\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(6*(-2 + Cos[2*(a + b*x)])*Cot[a + b*x]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]
- 8*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a +
 b*x]^2)/(15*b*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3079, 3042, 3081, 3042, 3050, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3079}$$

$$\frac{2}{5}\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \frac{1}{\sin(a+bx)\sqrt{d\tan(a+bx)}}dx-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin(a+bx)^{3/2}}dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3050}$$

$$\frac{2\sqrt{\sin(a+bx)}\left(-2\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(a+bx)}\left(-2\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sqrt{\sin(a+bx)}\left(-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}E\left(a+bx-\frac{\pi}{4}|2\right)}{b\sqrt{\sin(2a+2bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

input

```
Int[Csc[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

output

```
(-2*d*Csc[a + b*x])/(5*b*(d*Tan[a + b*x])^(3/2)) + (2*Sqrt[Sin[a + b*x]]*(
(-2*Cos[a + b*x]^(3/2))/(b*Sqrt[Sin[a + b*x]]) - (2*Sqrt[Cos[a + b*x]]*Ell
ipticE[a - Pi/4 + b*x, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[Sin[2*a + 2*b*x]])))
/(5*Sqrt[Cos[a + b*x]]*Sqrt[d*Tan[a + b*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(89) = 178.

Time = 0.87 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.59

| method | result |
|---|---|
| default | $-\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \text{EllipticE}\left(\sqrt{\right.\right.}}{}$ |

input
```
int(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/b*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(-sin(b*x+a)*cos(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1/2)*((csc(b*x+a)-cot(b*x+a
)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(
1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-2-2*sec(b*x+
a))+(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(
-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/
2*2^(1/2))*(1+sec(b*x+a))+2+cot(b*x+a)*csc(b*x+a))*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.32

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx =$$
$$-\frac{2\left(\left(i\cos(bx+a)^2-i\right)\sqrt{i\,d}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)+\left(-i\cos(bx+\right.\right.}{}$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/5*((I*cos(b*x + a)^2 - I)*sqrt(I*d)*elliptic_e(arcsin(cos(b*x + a) + I*
sin(b*x + a)), -1)*sin(b*x + a) + (-I*cos(b*x + a)^2 + I)*sqrt(-I*d)*ellip
tic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (-I*cos(b*
x + a)^2 + I)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)),
-1)*sin(b*x + a) + (I*cos(b*x + a)^2 - I)*sqrt(-I*d)*elliptic_f(arcsin(cos
(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (2*cos(b*x + a)^4 - 3*cos(
b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((b*d*cos(b*x + a)^2 - b*d)
*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

input
```
integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)**3/sqrt(d*tan(a + b*x)), x)
```

## Maxima [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc(bx+a)^3}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

## Giac [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=\int \frac{\csc\left(bx+a\right)^3}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=\int \frac{1}{\sin\left(a+b\,x\right)^3\,\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

input
```
int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2)),x)
```

output
```
int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=\frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)^3}{\tan(bx+a)}dx\right)}{d}$$

input
```
int(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x)**3)/tan(a + b*x),x))/d
```

# 3.94 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result . . . 829
Mathematica [A] (verified) . . . 830
Rubi [A] (warning: unable to verify) . . . 830
Maple [B] (verified) . . . 835
Fricas [B] (verification not implemented) . . . 835
Sympy [F] . . . 836
Maxima [A] (verification not implemented) . . . 836
Giac [A] (verification not implemented) . . . 837
Mupad [F(-1)] . . . 838
Reduce [F] . . . 838

## Optimal result

Integrand size = 21, antiderivative size = 204

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} - \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{32\sqrt{2}bd^{3/2}} + \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3}$$

output
```
-3/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(3/2)+3/6
4*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(3/2)-3/64*ar
ctanh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))*2^(1/2)/b
/d^(3/2)+3/16*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d^3-1/4*cos(b*x+a)^4*(d*
tan(b*x+a))^(3/2)/b/d^3
```

## Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.60

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\csc(a+bx)\left(\cos(a+bx)-2\cos(3(a+bx))+\cos(5(a+bx))-3\arcsin(\cos(a\right.}{}$$

input
```
Integrate[Sin[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Csc[a + b*x]*(Cos[a + b*x] - 2*Cos[3*(a + b*x)] + Cos[5*(a + b*x)] - 3*Ar
cSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[Sin[2*(a + b*x)]] - 3*Log[Cos[a + b
*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]])*Sqrt[
d*Tan[a + b*x]])/(64*b*d^2)
```

## Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3071, 252, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^4}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{5/2}}{(\tan^2(a+bx)d^2+d^2)^3}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{d\left(\frac{3}{8}\int \frac{\sqrt{d\tan(a+bx)}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{3/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\Big\downarrow$ 253

$$\frac{d\left(\frac{3}{8}\left(\frac{\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))}{4d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{3/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\Big\downarrow$ 266

$$\frac{d\left(\frac{3}{8}\left(\frac{\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{3/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\Big\downarrow$ 826

$$\frac{d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{3/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right.}{b}$$

$\Big\downarrow$ 1476

$$\frac{d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+}{d^4\tan^4(a+bx)}}{2d^2}\right.\right.}{b}$$

$\Big\downarrow$ 1082

$$\frac{d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{}{2d}\right.\right.}{b}$$

$\Big\downarrow$ 217

$$\frac{d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{}{4}\right.}{b}$$

$\Big\downarrow$ 1479

$$d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a\cdots\right.}{\sqrt{2}\sqrt{d}}\cdots\right.}{2d^2}\cdots\right.\right.}{b}$$

$$\Big\downarrow \; 25$$

$$d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+\cdots\right.}{\sqrt{2}\sqrt{d}}\cdots\right.}{2d^2}\cdots\right.\right.}{b}$$

$$\Big\downarrow \; 27$$

$$d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+\cdots\right.}{\sqrt{2}\sqrt{d}}\cdots\right.}{2d^2}\cdots\right.\right.}{b}$$

$$\Big\downarrow \; 1103$$

$$d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan\cdots\right.}{2\sqrt{2}\sqrt{d}}\cdots\right.}{2d^2}\cdots\right.\right.}{b}$$

input

```
Int[Sin[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*(-1/4*(d*Tan[a + b*x])^(3/2)/(d^2 + d^2*Tan[a + b*x]^2)^2 + (3*(((-(Arc
Tan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt
[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*
Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d
^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2)/(2*d^2)
+ (d*Tan[a + b*x])^(3/2)/(2*d^2*(d^2 + d^2*Tan[a + b*x]^2))))/8))/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(158) = 316.

Time = 6.42 (sec) , antiderivative size = 519, normalized size of antiderivative = 2.54

| method | result |
|---|---|
| default | $-\frac{\left(-3\ln\left(-\frac{\cot(bx+a)\cos(bx+a)-2\cot(bx+a)-2\sin(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}-2\cos(bx+a)+\csc(bx+a)-\sin(bx+a)+2}{-1+\cos(bx+a)}\right)\sin(bx+a)+}{}$ |

input
```
int(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/128/b*(-3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-2*sin(
b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+
a)+2)/(-1+cos(b*x+a)))*sin(b*x+a)+3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a
)+2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x
+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))*sin(b*x+a)+6*arctan((-sin(b*
x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+c
os(b*x+a)))*sin(b*x+a)-6*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*sin(b*x+a)+(-12+16*cos(
b*x+a)^3+(8*sin(b*x+a)+16)*cos(b*x+a)^2+(8*sin(b*x+a)-12)*cos(b*x+a))*sin(
b*x+a)^2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+sin(b*x+a)^3*co
s(b*x+a)*(-8*cos(b*x+a)-8)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^
2)^(1/2))/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(cos(b*x+a)+1)/d
/(d*tan(b*x+a))^(1/2)*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(158) = 316.

Time = 0.15 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.12

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$

$$-\frac{16\left(4\cos(bx+a)^3-3\cos(bx+a)\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\sin(bx+a)+6\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{\sqrt{d}(\cos(bx+a)-\sin(bx+a))}\right)}{}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/256*(16*(4*cos(b*x + a)^3 - 3*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x
 + a))*sin(b*x + a) + 6*sqrt(2)*sqrt(d)*arctan(-sqrt(2)*sqrt(d*sin(b*x + a
)/cos(b*x + a))*cos(b*x + a)/(sqrt(d)*(cos(b*x + a) - sin(b*x + a)))) + 3*
sqrt(2)*sqrt(d)*arctan(1/2*(2*cos(b*x + a)^2 - 2*cos(b*x + a)*sin(b*x + a)
 + sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) - 2)/(cos(b*x + a)^2
+ cos(b*x + a)*sin(b*x + a) - 1)) + 3*sqrt(2)*sqrt(d)*arctan(-1/2*(2*cos(b
*x + a)^2 - 2*cos(b*x + a)*sin(b*x + a) - sqrt(2)*sqrt(d*sin(b*x + a)/cos(
b*x + a))/sqrt(d) - 2)/(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a) - 1)) +
 3*sqrt(2)*sqrt(d)*log(4*cos(b*x + a)*sin(b*x + a) + 2*sqrt(2)*(cos(b*x +
a)^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d
) + 1) - 3*sqrt(2)*sqrt(d)*log(4*cos(b*x + a)*sin(b*x + a) - 2*sqrt(2)*(co
s(b*x + a)^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)
)/sqrt(d) + 1))/(b*d^2)
```

## Sympy [F]

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin^4{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sin(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.10

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\,\mathrm{lo}\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
1/128*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a
) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(
b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) + 8*(3*(d*ta
n(b*x + a))^(7/2)*d^4 - (d*tan(b*x + a))^(3/2)*d^6)/(d^4*tan(b*x + a)^4 +
2*d^4*tan(b*x + a)^2 + d^4))/(b*d^5)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.26

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2} + \frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2} - \frac{3\cdots}{\cdots}}{\cdots}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/128*(6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 6*sqrt(2)*abs(d)^(3/2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d
)))/(b*d^2) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*t
an(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) + 3*sqrt(2)*abs(d)^(3/2)*log(d
*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2
) + 8*(3*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)^3 - sqrt(d*tan(b*x + a))*d^
3*tan(b*x + a))/((d^2*tan(b*x + a)^2 + d^2)^2*b))/d
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^4}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^4/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^4/(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)^4}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x)**4)/tan(a + b*x)**2,x))/d**2
```

# 3.95 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 839
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 840
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . . . 840
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 845
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 845
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 846
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 846
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 847
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 847
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 848

## Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{4\sqrt{2}bd^{3/2}} + \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3}$$

output
```
-1/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(3/2)+1/8*
arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(3/2)-1/8*arcta
nh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))*2^(1/2)/b/d^
(3/2)+1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d^3
```

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.60

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$
$$-\frac{\left(\arcsin(\cos(a+bx)-\sin(a+bx))\csc(a+bx)+\csc(a+bx)\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a}{8bd^2}$$

input
```
Integrate[Sin[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/8*((ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x] + Csc[a + b*x]*Log
[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]] - 2*Sqrt[Sin[2*(a +
 b*x)]])*Sqrt[Sin[2*(a + b*x)]]*Sqrt[d*Tan[a + b*x]])/(b*d^2)
```

## Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3071, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(a+bx)^2}{(d\tan(a+bx))^{3/2}}dx$$
$$\Big\downarrow \; 3071$$
$$\frac{d\int \frac{\sqrt{d\tan(a+bx)}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))}{b}$$
$$\Big\downarrow \; 253$$

$$\frac{d\left(\frac{\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))}{4d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{d\left(\frac{\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{d\left(\frac{\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{d\left(\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d}{2d^2}}{b}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{d\left(\frac{\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d}{2d^2(d^2}\right.}{b}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{d\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{d\left(\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)\cdot\right.}{\sqrt{2}\sqrt{d}}\right.}{2d^2}\right.}{b}$$

↓ 25

$$\frac{d\left(\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+\right.}{\sqrt{2}\sqrt{d}}\right.}{2d^2}\right.}{b}$$

↓ 27

$$\frac{d\left(\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+\right.}{\sqrt{2}\sqrt{d}}\right.}{2d^2}\right.}{b}$$

↓ 1103

$$\frac{d\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+\right.}{2\sqrt{2}\sqrt{d}}\right.}{2d^2}\right.}{b}$$

input
```
Int[Sin[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*(((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcT
an[1 + Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[
2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d
+ Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/
2)/(2*d^2) + (d*Tan[a + b*x])^(3/2)/(2*d^2*(d^2 + d^2*Tan[a + b*x]^2))))/b
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(132) = 264.

Time = 6.24 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.45

| method | result |
|---|---|
| default | $-\frac{\left(\ln\left(-\frac{\cot(bx+a)\cos(bx+a)-2\cot(bx+a)+2\sin(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}-2\cos(bx+a)+\csc(bx+a)-\sin(bx+a)+2}{-1+\cos(bx+a)}\right)\sin(bx+a)-\ln}{}$ |

input
```
int(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/b*(ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-2*sin(b*x+
a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)+2
)/(-1+cos(b*x+a)))*sin(b*x+a)-ln((2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(
cos(b*x+a)+1)^2)^(1/2)-cot(b*x+a)*cos(b*x+a)+sin(b*x+a)+2*cos(b*x+a)-csc(b
*x+a)+2*cot(b*x+a)-2)/(-1+cos(b*x+a)))*sin(b*x+a)-2*arctan((sin(b*x+a)*(-2
*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a
)))*sin(b*x+a)+(-4*cos(b*x+a)-4)*sin(b*x+a)^2*(-2*sin(b*x+a)*cos(b*x+a)/(c
os(b*x+a)+1)^2)^(1/2)-2*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(
b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*sin(b*x+a))/(cos(b*x+a)+
1)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/d/(d*tan(b*x+a))^(1/2)*
2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(132) = 264.

Time = 0.12 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.41

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{16\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)\sin(bx+a) - 2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+}{\sqrt{d}(\cos(bx+a)-\sin(bx+a}\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/32*(16*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)*sin(b*x + a) - 2*s
qrt(2)*sqrt(d)*arctan(-sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x +
 a)/(sqrt(d)*(cos(b*x + a) - sin(b*x + a)))) - sqrt(2)*sqrt(d)*arctan(1/2*
(2*cos(b*x + a)^2 - 2*cos(b*x + a)*sin(b*x + a) + sqrt(2)*sqrt(d*sin(b*x +
 a)/cos(b*x + a))/sqrt(d) - 2)/(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a)
 - 1)) - sqrt(2)*sqrt(d)*arctan(-1/2*(2*cos(b*x + a)^2 - 2*cos(b*x + a)*si
n(b*x + a) - sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) - 2)/(cos(b
*x + a)^2 + cos(b*x + a)*sin(b*x + a) - 1)) - sqrt(2)*sqrt(d)*log(4*cos(b*
x + a)*sin(b*x + a) + 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a
))*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) + 1) + sqrt(2)*sqrt(d)*log(4*
cos(b*x + a)*sin(b*x + a) - 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b
*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) + 1))/(b*d^2)
```

## Sympy [F]

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sin(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.11

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(\right.}{}\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
1/16*(d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b*
x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a) +
 sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x
 + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) + 8*(d*tan(b*x
+ a))^(3/2)*d^2/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.31

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\frac{8\sqrt{d\tan(bx+a)}d\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)b} + \frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2} + \frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\cdots}}{\cdots}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/16*(8*sqrt(d*tan(b*x + a))*d*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)*b)
 + 2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqr
t(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 2*sqrt(2)*abs(d)^(3/2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/
(b*d^2) - sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x
 + a))*sqrt(abs(d)) + abs(d))/(b*d^2) + sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x
 + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2))/d
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^2/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^2/(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)^2}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x)**2)/tan(a + b*x)**2,x))/d**2
```

# 3.96 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$


Optimal result . . . . . 849
Mathematica [A] (verified) . . . . . 849
Rubi [A] (verified) . . . . . 850
Maple [A] (verified) . . . . . 851
Fricas [B] (verification not implemented) . . . . . 851
Sympy [F] . . . . . 852
Maxima [A] (verification not implemented) . . . . . 852
Giac [A] (verification not implemented) . . . . . 852
Mupad [B] (verification not implemented) . . . . . 853
Reduce [F] . . . . . 853


## Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

output
```
-2/5*d/b/(d*tan(b*x+a))^(5/2)
```

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

input
```
Integrate[Csc[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)^2(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{1}{(d\tan(a+bx))^{7/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

input
```
Int[Csc[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{2d}{5b(d\tan(bx+a))^{\frac{5}{2}}}$ | 17 |
| default | $-\frac{2d}{5b(d\tan(bx+a))^{\frac{5}{2}}}$ | 17 |

input
```
int(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*d/b/(d*tan(b*x+a))^(5/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(16) = 32.

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos\left(bx+a\right)^3}{5\left(bd^2\cos\left(bx+a\right)^2-bd^2\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^3/((b*d^2*cos(b*x + a)^
2 - b*d^2)*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc^2{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{5\,(d\tan{(bx+a)})^{\frac{3}{2}}\,b\tan{(bx+a)}}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/5/((d*tan(b*x + a))^(3/2)*b*tan(b*x + a))
```

## Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{5\,\sqrt{d\tan{(bx+a)}}bd\tan{(bx+a)}^2}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2/5/(sqrt(d*tan(b*x + a))*b*d*tan(b*x + a)^2)
```

## Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 381, normalized size of antiderivative = 19.05

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,14i}{5\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}-1\right)} - \frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,8i}{15\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}-1\right)^2} - \frac{16\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{5\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)} - \frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,32i}{15\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2} + \frac{8\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{5\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3}$$

input
```
int(1/(sin(a + b*x)^2*(d*tan(a + b*x))^(3/2)),x)
```

output
```
(8*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i +
 b*x*2i) + 1))^(1/2))/(5*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^3) - ((exp(a*2
i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) +
1))^(1/2)*8i)/(15*b*d^2*(exp(a*2i + b*x*2i) - 1)^2) - (16*(exp(a*2i + b*x*
2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2
))/(5*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)) - ((exp(a*2i + b*x*2i) + 1)*(-(d
*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*32i)/(15*b*
d^2*(exp(a*2i + b*x*2i)*1i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(
a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*14i)/(5*b*d^2*(ex
p(a*2i + b*x*2i) - 1))
```

## Reduce [F]

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)^2}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x)**2)/tan(a + b*x)**2,x))/d**2
```

# 3.97 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 854
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 854
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 855
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 856
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . 857
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 857
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 857
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 858
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 858
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 859

## Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d^3}{9b(d\tan(a+bx))^{9/2}} - \frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

output
```
-2/9*d^3/b/(d*tan(b*x+a))^(9/2)-2/5*d/b/(d*tan(b*x+a))^(5/2)
```

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2(4+\csc^2(a+bx)-5\csc^4(a+bx))}{45bd\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*(4 + Csc[a + b*x]^2 - 5*Csc[a + b*x]^4))/(45*b*d*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)^4(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{\tan^2(a+bx)d^2+d^2}{(d\tan(a+bx))^{11/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{d\int \left(\frac{d^2}{(d\tan(a+bx))^{11/2}}+\frac{1}{(d\tan(a+bx))^{7/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\left(-\frac{2d^2}{9(d\tan(a+bx))^{9/2}}-\frac{2}{5(d\tan(a+bx))^{5/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*((-2*d^2)/(9*(d*Tan[a + b*x])^(9/2)) - 2/(5*(d*Tan[a + b*x])^(5/2))))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

| method | result | size |
|---|---|---|
| default | $\frac{\frac{8\cot(bx+a)^4}{45}-\frac{2\cot(bx+a)^2\csc(bx+a)^2}{5}}{bd\sqrt{d\tan(bx+a)}}$ | 48 |

input
```
int(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/b/d/(d*tan(b*x+a))^(1/2)*(4*cot(b*x+a)^4-9*cot(b*x+a)^2*csc(b*x+a)^2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(35) = 70$.

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\left(4\cos(bx+a)^5 - 9\cos(bx+a)^3\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45\left(bd^2\cos(bx+a)^4 - 2\,bd^2\cos(bx+a)^2 + bd^2\right)\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/45*(4*cos(b*x + a)^5 - 9*cos(b*x + a)^3)*sqrt(d*sin(b*x + a)/cos(b*x + a
))/((b*d^2*cos(b*x + a)^4 - 2*b*d^2*cos(b*x + a)^2 + b*d^2)*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(9\,d^2\tan(bx+a)^2 + 5\,d^2\right)d}{45\,(d\tan(bx+a))^{\frac{9}{2}}\,b}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/45*(9*d^2*tan(b*x + a)^2 + 5*d^2)*d/((d*tan(b*x + a))^(9/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(9\,d^4\tan\left(bx+a\right)^2+5\,d^4\right)}{45\,\sqrt{d\tan\left(bx+a\right)}bd^5\tan\left(bx+a\right)^4}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2/45*(9*d^4*tan(b*x + a)^2 + 5*d^4)/(sqrt(d*tan(b*x + a))*b*d^5*tan(b*x +
 a)^4)
```

## Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 684, normalized size of antiderivative = 15.91

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(a + b*x)^4*(d*tan(a + b*x))^(3/2)),x)
```

output
```
((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2)*6088i)/(945*b*d^2*(exp(a*2i + b*x*2i) - 1)) + ((exp(a*2
i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) +
1))^(1/2)*4024i)/(945*b*d^2*(exp(a*2i + b*x*2i) - 1)^2) + ((exp(a*2i + b*x
*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/
2)*200i)/(63*b*d^2*(exp(a*2i + b*x*2i) - 1)^3) + ((exp(a*2i + b*x*2i) + 1)
*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*64i)/(
63*b*d^2*(exp(a*2i + b*x*2i) - 1)^4) + (1184*(exp(a*2i + b*x*2i) + 1)*(-(d
*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(189*b*d^2
*(exp(a*2i + b*x*2i)*1i - 1i)) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i
+ b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*4192i)/(945*b*d^2*(exp
(a*2i + b*x*2i)*1i - 1i)^2) - (2176*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2
i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(315*b*d^2*(exp(a*2
i + b*x*2i)*1i - 1i)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2
i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*512i)/(63*b*d^2*(exp(a*2i + b
*x*2i)*1i - 1i)^4) + (32*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)
*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(9*b*d^2*(exp(a*2i + b*x*2i)*1
i - 1i)^5)
```

## Reduce [F]

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)^4}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x)**4)/tan(a + b*x)**2,x))/d**2
```

# 3.98 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 860
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 860
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 861
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 862
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . 863
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 863
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 863
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . 864
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 864
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 865


## Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d^5}{13b(d\tan(a+bx))^{13/2}} - \frac{4d^3}{9b(d\tan(a+bx))^{9/2}} - \frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

output
```
-2/13*d^5/b/(d*tan(b*x+a))^(13/2)-4/9*d^3/b/(d*tan(b*x+a))^(9/2)-2/5*d/b/(
d*tan(b*x+a))^(5/2)
```

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{64+16\csc^2(a+bx)+10\csc^4(a+bx)-90\csc^6(a+bx)}{585bd\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(64 + 16*Csc[a + b*x]^2 + 10*Csc[a + b*x]^4 - 90*Csc[a + b*x]^6)/(585*b*d*
Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)^6(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{\left(\tan^2(a+bx)d^2+d^2\right)^2}{(d\tan(a+bx))^{15/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{d\int \left(\frac{d^4}{(d\tan(a+bx))^{15/2}}+\frac{2d^2}{(d\tan(a+bx))^{11/2}}+\frac{1}{(d\tan(a+bx))^{7/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\left(-\frac{2d^4}{13(d\tan(a+bx))^{13/2}}-\frac{4d^2}{9(d\tan(a+bx))^{9/2}}-\frac{2}{5(d\tan(a+bx))^{5/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*((-2*d^4)/(13*(d*Tan[a + b*x])^(13/2)) - (4*d^2)/(9*(d*Tan[a + b*x])^(9
/2)) - 2/(5*(d*Tan[a + b*x])^(5/2))))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| default | $-\frac{2 \cot \left(b x +a \right)^{2} \csc \left(b x +a \right)^{4} \left(32 \cos \left(b x +a \right)^{4}-104 \cos \left(b x +a \right)^{2}+117\right)}{585 b d \sqrt{d \tan \left(b x +a \right)}}$ | 57 |

input
```
int(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/585/b*cot(b*x+a)^2*csc(b*x+a)^4*(32*cos(b*x+a)^4-104*cos(b*x+a)^2+117)/
d/(d*tan(b*x+a))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(53) = 106$.

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\left(32\cos(bx+a)^7 - 104\cos(bx+a)^5 + 117\cos(bx+a)^3\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{585\left(bd^2\cos(bx+a)^6 - 3\,bd^2\cos(bx+a)^4 + 3\,bd^2\cos(bx+a)^2 - bd^2\right)\sin(bx+}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/585*(32*cos(b*x + a)^7 - 104*cos(b*x + a)^5 + 117*cos(b*x + a)^3)*sqrt(d
*sin(b*x + a)/cos(b*x + a))/((b*d^2*cos(b*x + a)^6 - 3*b*d^2*cos(b*x + a)^
4 + 3*b*d^2*cos(b*x + a)^2 - b*d^2)*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)**6/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(117\,d^4\tan(bx+a)^4 + 130\,d^4\tan(bx+a)^2 + 45\,d^4\right)d}{585\,(d\tan(bx+a))^{\frac{13}{2}}\,b}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/585*(117*d^4*tan(b*x + a)^4 + 130*d^4*tan(b*x + a)^2 + 45*d^4)*d/((d*ta
n(b*x + a))^(13/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(117\,d^6\tan\left(bx+a\right)^4+130\,d^6\tan\left(bx+a\right)^2+45\,d^6\right)}{585\,\sqrt{d\tan\left(bx+a\right)}bd^7\tan\left(bx+a\right)^6}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2/585*(117*d^6*tan(b*x + a)^4 + 130*d^6*tan(b*x + a)^2 + 45*d^6)/(sqrt(d*
tan(b*x + a))*b*d^7*tan(b*x + a)^6)
```

## Mupad [B] (verification not implemented)

Time = 11.63 (sec) , antiderivative size = 987, normalized size of antiderivative = 15.18

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(a + b*x)^6*(d*tan(a + b*x))^(3/2)),x)
```

output
```
(128*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
 + b*x*2i) + 1))^(1/2))/(11*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^3) - ((exp(
a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i)
 + 1))^(1/2)*294464i)/(45045*b*d^2*(exp(a*2i + b*x*2i) - 1)^2) - ((exp(a*2
i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) +
1))^(1/2)*24608i)/(2145*b*d^2*(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b
*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(
1/2)*135104i)/(9009*b*d^2*(exp(a*2i + b*x*2i) - 1)^4) - ((exp(a*2i + b*x*2
i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)
*13088i)/(1287*b*d^2*(exp(a*2i + b*x*2i) - 1)^5) - ((exp(a*2i + b*x*2i) +
1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*384i
)/(143*b*d^2*(exp(a*2i + b*x*2i) - 1)^6) - (55808*(exp(a*2i + b*x*2i) + 1)
*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(6435
*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp
(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*7424i)/(1155*b*d
^2*(exp(a*2i + b*x*2i)*1i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a
*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*18368i)/(2145*b*d^
2*(exp(a*2i + b*x*2i) - 1)) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b
*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*228736i)/(9009*b*d^2*(exp
(a*2i + b*x*2i)*1i - 1i)^4) - (17152*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp...
```

## Reduce [F]

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)^6}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x)**6)/tan(a + b*x)**2,x))/d**2
```

# 3.99 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$


Optimal result . . . . . 866
Mathematica [C] (verified) . . . . . 866
Rubi [A] (verified) . . . . . 867
Maple [C] (warning: unable to verify) . . . . . 870
Fricas [F] . . . . . 871
Sympy [F(-1)] . . . . . 871
Maxima [F] . . . . . 871
Giac [F] . . . . . 872
Mupad [F(-1)] . . . . . 872
Reduce [F] . . . . . 872


## Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\csc(a+bx)\,\mathrm{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{12bd^2}$$

output
```
-1/6*sin(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+1/3*sin(b*x+a)^3/b/d/(d*tan(b*x+a
))^(1/2)+1/12*csc(b*x+a)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2
*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b/d^2
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\csc(a+bx)\left(\sqrt{\sec^2(a+bx)}\sin(4(a+bx))+4\sqrt[4]{-1}\,\mathrm{EllipticF}\left(i\,\mathrm{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sqrt{\mathrm{ta}}\right.}{24bd^2\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/24*(Csc[a + b*x]*(Sqrt[Sec[a + b*x]^2]*Sin[4*(a + b*x)] + 4*(-1)^(1/4)*
EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Tan[a + b*x]]
)*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sec[a + b*x]^2])
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3076, 3042, 3078, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\frac{1}{2}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{2b}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sin[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
Sin[a + b*x]^3/(3*b*d*Sqrt[d*Tan[a + b*x]]) + (-((d*Sin[a + b*x])/(b*Sqrt[
d*Tan[a + b*x]])) + (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*
a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(2*b))/(6*d^2)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.21 (sec) , antiderivative size = 1510, normalized size of antiderivative = 13.48

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1510 |

input
```
int(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/b/(d*tan(b*x+a))^(1/2)/d*(2^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b
*x+a)+2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos
(b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))*(-2*sin(b*x+a)*cos(b*x+a
)/(cos(b*x+a)+1)^2)^(1/2)*(sin(b*x+a)*(-6*cos(b*x+a)-6)+3+3*sec(b*x+a))+si
n(b*x+a)*(12*cos(b*x+a)+12)*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2
)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-2*sin(b*x+a)*cos(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1+c
os(b*x+a)))+2^(1/2)*ln((2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)
+1)^2)^(1/2)-cot(b*x+a)*cos(b*x+a)+sin(b*x+a)+2*cos(b*x+a)-csc(b*x+a)+2*co
t(b*x+a)-2)/(-1+cos(b*x+a)))*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)*(sin(b*x+a)*(6*cos(b*x+a)+6)-3-3*sec(b*x+a))+sin(b*x+a)*(-12*cos(b*x+
a)-12)*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln((2*sin(b*x+a)*(-
2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cot(b*x+a)*cos(b*x+a)+sin(
b*x+a)+2*cos(b*x+a)-csc(b*x+a)+2*cot(b*x+a)-2)/(-1+cos(b*x+a)))+2^(1/2)*ar
ctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x
+a)+1)/(-1+cos(b*x+a)))*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*
(sin(b*x+a)*(-12*cos(b*x+a)-12)+6+6*sec(b*x+a))+sin(b*x+a)*(24*cos(b*x+a)+
24)*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*(-2
*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a
)))+2^(1/2)*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)...
```

## Fricas [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^2*tan(
b*x + a)^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

## Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(a+b\,x)^3}{(d\tan(a+b\,x))^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^3/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^3/(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)^3}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x)**3)/tan(a + b*x)**2,x))/d**2
```

# 3.100 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 873
Mathematica [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . 873
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 874
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 876
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 877
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 877
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 877
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 878
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 878
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 878

## Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2bd\sqrt{d\tan(a+bx)}}$$

output
```
sin(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+1/2*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/
2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/d/(d*tan(b*x+a))^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.59

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\cos(2(a+bx))\sec(a+bx)\left(\sqrt[4]{-1}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right.\right.}{b\sqrt{\sec^2(a+bx)}(d\tan(a+bx))^{3/2}\,(-}$$

input
```
Integrate[Sin[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Cos[2*(a + b*x)]*Sec[a + b*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*
Sqrt[Tan[a + b*x]]], -1]*Sec[a + b*x]^2 - Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[a
+ b*x]])*Tan[a + b*x]^(3/2))/(b*Sqrt[Sec[a + b*x]^2]*(d*Tan[a + b*x])^(3/2
)*(-1 + Tan[a + b*x]^2))
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.43, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3082, 3042, 3049, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\sqrt{\sin(a+bx)}\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sqrt{\sin(a+bx)}}dx}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\int \frac{\cos(a+bx)^{3/2}}{\sqrt{\sin(a+bx)}}dx}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3049}$$

$$\frac{\sqrt{\sin(a+bx)}\left(\frac{1}{2}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx+\frac{\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}{b}\right)}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\left(\frac{1}{2}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx+\frac{\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}{b}\right)}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{\sqrt{\sin(a+bx)}\left(\frac{\sqrt{\sin(2a+2bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}+\frac{\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}{b}\right)}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\left(\frac{\sqrt{\sin(2a+2bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}+\frac{\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}{b}\right)}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(a+bx)}\left(\frac{\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}{b}+\frac{\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}\right)}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

input

```
Int[Sin[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output

```
(Sqrt[Sin[a + b*x]]*((Sqrt[Cos[a + b*x]]*Sqrt[Sin[a + b*x]])/b + (Elliptic
F[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[Cos[a + b*x]]*Sqrt[
Sin[a + b*x]])))/(d*Sqrt[Cos[a + b*x]]*Sqrt[d*Tan[a + b*x]])
```

### Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3049

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.44

| method | result |
|---|---|
| default | $\frac{\frac{\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \operatorname{EllipticF}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1},\frac{\sqrt{2}}{2}\right)(1+\sec(bx+a))}{2}}{b\sqrt{d\tan(bx+a)}\,d}$ |

input
```
int(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1
/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/
2),1/2*2^(1/2))*(1+sec(b*x+a))+sin(b*x+a))/(d*tan(b*x+a))^(1/2)/d
```

## Fricas [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^2*tan(b*x + a)^2), x)
```

## Sympy [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sin(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

## Giac [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(a+b\,x)}{(d\tan(a+b\,x))^{3/2}}\,dx$$

input
```
int(sin(a + b*x)/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)/(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x))/tan(a + b*x)**2,x))/d**2
```

# 3.101 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 879
Mathematica [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . 879
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 880
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 882
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . 882
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 883
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 883
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 884
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 884
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 884

## Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}} - \frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3bd^2}$$

output
```
-2/3*csc(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)-1/3*csc(b*x+a)*InverseJacobiAM(a-
1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b/d^2
```

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.34

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\cos(2(a+bx))\sec(a+bx)\sqrt{\sec^2(a+bx)}\left(\sqrt{\sec^2(a+bx)}-\sqrt[4]{-1}\,\text{EllipticF}\left(i\right.\right.}{3b(d\tan(a+bx))^{3/2}\left(-1+\tan^2(a\right.}$$

input
```
Integrate[Csc[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*Cos[2*(a + b*x)]*Sec[a + b*x]*Sqrt[Sec[a + b*x]^2]*(Sqrt[Sec[a + b*x]^2
] - (-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Tan
[a + b*x]^(3/2)))/(3*b*(d*Tan[a + b*x])^(3/2)*(-1 + Tan[a + b*x]^2))
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3077, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx}{3d^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx}{3d^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3d^2\sqrt{\sin(a+bx)}}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3d^2\sqrt{\sin(a+bx)}}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3bd^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Csc[a + b*x])/(3*b*d*Sqrt[d*Tan[a + b*x]]) - (Csc[a + b*x]*EllipticF[a
 - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b*d^2)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| default | $-\frac{\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\mathrm{EllipticF}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\right.}{3b\sqrt{d\tan(bx+a)}\,d}$ |

input
```
int(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(d*tan(b*x+a))^(1/2)/d*((csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x
+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*
x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(1+sec(b*x+a))+2*csc(b*x+a))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\left(\cos{(bx+a)}^2-1\right)\sqrt{i\,d}F(\arcsin{(\cos{(bx+a)}+i\,\sin{(bx+a)})}\,|-1)+(\cos{(ba}}{3\,(bd^2\,c}$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/3*((cos(b*x + a)^2 - 1)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin
(b*x + a)), -1) + (cos(b*x + a)^2 - 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*
x + a) - I*sin(b*x + a)), -1) + 2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*
x + a))/(b*d^2*cos(b*x + a)^2 - b*d^2)
```

## Sympy [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

## Giac [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\sin(a+b\,x)\,(d\tan(a+b\,x))^{3/2}}\,dx$$

input
```
int(1/(sin(a + b*x)*(d*tan(a + b*x))^(3/2)),x)
```

output
```
int(1/(sin(a + b*x)*(d*tan(a + b*x))^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x))/tan(a + b*x)**2,x))/d**2
```

# 3.102 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result . . . 885
Mathematica [C] (warning: unable to verify) . . . 885
Rubi [A] (verified) . . . 886
Maple [A] (verified) . . . 889
Fricas [C] (verification not implemented) . . . 889
Sympy [F] . . . 890
Maxima [F] . . . 890
Giac [F] . . . 890
Mupad [F(-1)] . . . 891
Reduce [F] . . . 891

## Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}} - \frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}} - \frac{2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{21bd^2}$$

output
```
2/21*csc(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)-2/7*csc(b*x+a)^3/b/d/(d*tan(b*x+a
))^(1/2)-2/21*csc(b*x+a)*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2
*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b/d^2
```

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\csc^3(a+bx)\left((1+10\cos(2(a+bx))+\cos(4(a+bx)))\sec^2(a+bx)^{3/2}-8\sqrt[4]{-1}\right.}{42bd\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a}}$$

input
```
Integrate[Csc[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Csc[a + b*x]^3*((1 + 10*Cos[2*(a + b*x)] + Cos[4*(a + b*x)])*(Sec[a + b*x
]^2)^(3/2) - 8*(-1)^(1/4)*Cos[2*(a + b*x)]*EllipticF[I*ArcSinh[(-1)^(1/4)*
Sqrt[Tan[a + b*x]]], -1]*Tan[a + b*x]^(7/2)))/(42*b*d*Sqrt[Sec[a + b*x]^2]
*Sqrt[d*Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 3077, 3042, 3079, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}dx}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^3}dx}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3079}$$

$$-\frac{\frac{2}{3}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2}{3}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3053

$$-\frac{\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3120

$$-\frac{\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

input

```
Int[Csc[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output

```
(-2*Csc[a + b*x]^3)/(7*b*d*Sqrt[d*Tan[a + b*x]]) - ((-2*d*Csc[a + b*x])/(3
*b*Sqrt[d*Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sq
rt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b))/(7*d^2)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.18

| method | result |
|---|---|
| default | $\frac{\frac{2\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1},\frac{\sqrt{2}}{2}\right)(-1-\sec(bx+}{21}}{b\sqrt{d\tan(bx+a)}\,d}$ |

input
```
int(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(2/21*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(
1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1
/2),1/2*2^(1/2))*(-1-sec(b*x+a))+2/21*csc(b*x+a)^3*(-cos(b*x+a)^2-2))/(d*t
an(b*x+a))^(1/2)/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.46

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\left(\left(\cos{(bx+a)}^4 - 2\cos{(bx+a)}^2 + 1\right)\sqrt{i\,d}F(\arcsin{(\cos{(bx+a)} + i\sin{(bx+}}{}$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/21*((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(I*d)*elliptic_f(arcsin(
cos(b*x + a) + I*sin(b*x + a)), -1) + (cos(b*x + a)^4 - 2*cos(b*x + a)^2 +
 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - (co
s(b*x + a)^3 + 2*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*d^2*c
os(b*x + a)^4 - 2*b*d^2*cos(b*x + a)^2 + b*d^2)
```

## Sympy [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)**3/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

## Giac [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\sin(a+b\,x)^3\,(d\tan(a+b\,x))^{3/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2)),x)
```

output
```
int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)^3}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x)**3)/tan(a + b*x)**2,x))/d**2
```

# 3.103 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 892
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 893
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . 893
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 898
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 899
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 899
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 900
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 900
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 901
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 901

## Optimal result

Integrand size = 21, antiderivative size = 204

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{32\sqrt{2}bd^{5/2}} + \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3}$$

output
```
-3/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(5/2)+3/6
4*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(5/2)+3/64*ar
ctanh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))*2^(1/2)/b
/d^(5/2)+1/16*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d^3-1/4*cos(b*x+a)^4*(d*
tan(b*x+a))^(1/2)/b/d^3
```

## Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.60

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{\csc(a+bx)\left(\sin(a+bx)+3\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-3\log\left(\cos(a+bx)+\text{s}\right.\right.}{64b\text{d}}$$

input
```
Integrate[Sin[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

output
```
-1/64*(Csc[a + b*x]*(Sin[a + b*x] + 3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*
Sqrt[Sin[2*(a + b*x)]] - 3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a
 + b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + 2*Sin[3*(a + b*x)] + Sin[5*(a + b*x)])
*Sqrt[d*Tan[a + b*x]])/(b*d^3)
```

## Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.29, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3071, 252, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)^4}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{3/2}}{(\tan^2(a+bx)d^2+d^2)^3}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{d\left(\frac{1}{8}\int \frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))-\frac{\sqrt{d\tan(a+bx)}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\Big\downarrow$ 253

$$\frac{d\left(\frac{1}{8}\left(\frac{3\int \frac{1}{\sqrt{d\tan(a+bx)}\left(\tan^2(a+bx)d^2+d^2\right)}d(d\tan(a+bx))}{4d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{\sqrt{d\tan(a+bx)}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\Big\downarrow$ 266

$$\frac{d\left(\frac{1}{8}\left(\frac{3\int \frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{\sqrt{d\tan(a+bx)}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\Big\downarrow$ 755

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{2d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{\sqrt{d\tan(a+bx)}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\Big\downarrow$ 1476

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{2d^2}\cdots\right.\right.}{b}$$

$\Big\downarrow$ 1082

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{2d^2}+\frac{\sqrt{\cdots}}{2d^2(d\cdots}\cdots\right.\right.}{b}$$

$\Big\downarrow$ 217

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{2d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{}{4(d^2}\right.}{b}$$

$\downarrow$ 1479

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx\right.}{\sqrt{2}\sqrt{d}}}{}\right.}{2d^2}\right.\right.}{b}$$

$\downarrow$ 25

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2}\right.}{2d^2}\right.\right.}{b}$$

$\downarrow$ 27

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2}\right.}{2d^2}\right.\right.}{b}$$

$\downarrow$ 1103

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{2d^2}\right.\right.}{b}$$

input
```
Int[Sin[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*(-1/4*Sqrt[d*Tan[a + b*x]]/(d^2 + d^2*Tan[a + b*x]^2)^2 + ((3*((-(ArcTa
n[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2
]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d
^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt
[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d))
)/(2*d^2) + Sqrt[d*Tan[a + b*x]]/(2*d^2*(d^2 + d^2*Tan[a + b*x]^2)))/8))/b
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(158) = 316$.

Time = 8.61 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.70

| method | result |
|---|---|
| default | $-\frac{\left(-3\ln\left(-\frac{\cot(bx+a)\cos(bx+a)-2\cot(bx+a)-2\sin(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}-2\cos(bx+a)+\csc(bx+a)-\sin(bx+a)+2}{-1+\cos(bx+a)}\right)\sin(bx+a)+\cdots}{\cdots}$ |

input
```
int(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/128/b*(-3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-2*sin(
b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+
a)+2)/(-1+cos(b*x+a)))*sin(b*x+a)+3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a
)+2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x
+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))*sin(b*x+a)-6*arctan((-sin(b*
x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+c
os(b*x+a)))*sin(b*x+a)+6*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*sin(b*x+a)+cos(b*x+a)*(
48*cos(b*x+a)^2+(-48*sin(b*x+a)+48)*cos(b*x+a)-48*sin(b*x+a))*sin(b*x+a)^2
*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)*(-4*(8*cos(b
*x+a)^2-11)*(cos(b*x+a)+1)*sin(b*x+a)+48*cos(b*x+a)*(cos(b*x+a)^2-1)*(cos(
b*x+a)+1))*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2))/(-sin(
b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(cos(b*x+a)+1)/d^2/(d*tan(b*x+a)
)^(1/2)*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(158) = 316$.

Time = 0.13 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.10

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{6\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}(\cos(bx+a)-\sin(bx+a))}{2\sqrt{d}\sin(bx+a)}\right)-3\sqrt{2}\sqrt{d}\arctan\left(\frac{2\cos(bx+\cdots}{2\cdots}\right.}{\cdots}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/256*(6*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x +
 a))*(cos(b*x + a) - sin(b*x + a))/(sqrt(d)*sin(b*x + a))) - 3*sqrt(2)*sqr
t(d)*arctan(1/2*(2*cos(b*x + a)^2 - 2*cos(b*x + a)*sin(b*x + a) + sqrt(2)*
sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) - 2)/(cos(b*x + a)^2 + cos(b*x +
 a)*sin(b*x + a) - 1)) - 3*sqrt(2)*sqrt(d)*arctan(-1/2*(2*cos(b*x + a)^2 -
 2*cos(b*x + a)*sin(b*x + a) - sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x + a))/s
qrt(d) - 2)/(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a) - 1)) + 3*sqrt(2)*
sqrt(d)*log(4*cos(b*x + a)*sin(b*x + a) + 2*sqrt(2)*(cos(b*x + a)^2 + cos(
b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) + 1) - 3*
sqrt(2)*sqrt(d)*log(4*cos(b*x + a)*sin(b*x + a) - 2*sqrt(2)*(cos(b*x + a)^
2 + cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) +
 1) - 16*(4*cos(b*x + a)^4 - cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x +
 a)))/(b*d^3)
```

## Sympy [F]

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(sin(a + b*x)**4/(d*tan(a + b*x))**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.07

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{6\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+6\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(b}\right.}{2\sqrt{d}}\right.}{}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
1/128*(6*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(b*x + a)))/sqrt(d)) + 6*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 3*sqrt(2)*d^(5/2)*log(d*tan(b*x
+ a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 3*sqrt(2)*d^(5/2)*log(d
*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) + 8*((d*tan(b*x
+ a))^(5/2)*d^4 - 3*sqrt(d*tan(b*x + a))*d^6)/(d^4*tan(b*x + a)^4 + 2*d^4*
tan(b*x + a)^2 + d^4))/(b*d^5)
```

## Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= \frac{3\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd^3} \\ &+ \frac{3\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd^3} \\ &+ \frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{128\,bd^3} \\ &- \frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{128\,bd^3} \\ &+ \frac{\sqrt{d\tan\left(bx+a\right)}d^2\tan\left(bx+a\right)^2-3\sqrt{d\tan\left(bx+a\right)}d^2}{16\left(d^2\tan\left(bx+a\right)^2+d^2\right)^2bd}\end{aligned}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
3/64*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqr
t(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^3) + 3/64*sqrt(2)*sqrt(abs(d))*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)
))/(b*d^3) + 3/128*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(
d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^3) - 3/128*sqrt(2)*sqrt(abs(d)
)*log(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))
/(b*d^3) + 1/16*(sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)^2 - 3*sqrt(d*tan(b*
x + a))*d^2)/((d^2*tan(b*x + a)^2 + d^2)^2*b*d)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^4}{\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^4/(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^4/(d*tan(a + b*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)^4}{\tan(bx+a)^3}dx\right)}{d^3}$$

input
```
int(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x)**4)/tan(a + b*x)**3,x))/d**3
```

# 3.104 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$


Optimal result . . . 902
Mathematica [A] (verified) . . . 903
Rubi [A] (warning: unable to verify) . . . 903
Maple [B] (verified) . . . 908
Fricas [B] (verification not implemented) . . . 908
Sympy [F] . . . 909
Maxima [A] (verification not implemented) . . . 909
Giac [A] (verification not implemented) . . . 910
Mupad [F(-1)] . . . 911
Reduce [F] . . . 911


## Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{4\sqrt{2}bd^{5/2}} + \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3}$$

output
```
-3/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(5/2)+3/8*
arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(5/2)+3/8*arcta
nh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))*2^(1/2)/b/d^
(5/2)+1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d^3
```

## Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.65

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\csc(a+bx)\left(\sin(a+bx) - 3\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}\right.}{}$$

input
```
Integrate[Sin[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(Csc[a + b*x]*(Sin[a + b*x] - 3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[S
in[2*(a + b*x)]] + 3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x
)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/(8*b
*d^3)
```

## Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.29, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3071, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{1}{\sqrt{d\tan(a+bx)}\left(\tan^2(a+bx)d^2+d^2\right)^2}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{253}$$

$$\frac{d\left(\frac{3\int \frac{1}{\sqrt{d\tan(a+bx)}\left(\tan^2(a+bx)d^2+d^2\right)}d(d\tan(a+bx))}{4d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 266

$$\frac{d\left(\frac{3\int \frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 755

$$\frac{d\left(\frac{3\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{2d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 1476

$$\frac{d\left(\frac{3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a}{2d}\right.}{2d^2}\right.}{b}$$

$\downarrow$ 1082

$$\frac{d\left(\frac{3\left(\frac{\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{2d^2}+\frac{\sqrt{d\,ta}}{2d^2(d^2\tan}\right.}{b}$$

$\downarrow$ 217

$$\frac{d\left(\frac{3\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{2d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 1479

$$d\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\cdots}{\cdots}\right)}{2d^2}\cdots\right)}{b}$$

$\downarrow$ 25

$$d\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\cdots\right)}{2d^2}\cdots\right)}{b}$$

$\downarrow$ 27

$$d\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\cdots\right)}{2d^2}\cdots\right)}{b}$$

$\downarrow$ 1103

$$d\left(\frac{3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+\cdots\right)}{2\sqrt{2}\sqrt{d}}}{2d}\cdots\right)}{2d^2}\cdots\right)}{b}$$

input
```
Int[Sin[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*((3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + A
rcTan[1 + Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*L
og[d - Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d]
) + Log[d + Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*
Sqrt[d]))/(2*d)))/(2*d^2) + Sqrt[d*Tan[a + b*x]]/(2*d^2*(d^2 + d^2*Tan[a +
 b*x]^2))))/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(132) = 264.

Time = 7.98 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.34

| method | result |
|---|---|
| default | $\frac{\sin(bx+a)\left(6\arctan\left(\frac{-\sin(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}+\cos(bx+a)-1}{-1+\cos(bx+a)}\right)-6\arctan\left(\frac{\sin(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}+\cos(bx+a)-}{-1+\cos(bx+a)}\right.}{}$ |

input
```
int(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/b*sin(b*x+a)*(6*arctan((-sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*
x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))-6*arctan((sin(b*x+a)*(-2*s
in(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a))
)+3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-2*sin(b*x+a)*co
s(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1
+cos(b*x+a)))-3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-2*s
in(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b
*x+a)+2)/(-1+cos(b*x+a)))+cos(b*x+a)*(4*cos(b*x+a)+4)*(-2*sin(b*x+a)*cos(b
*x+a)/(cos(b*x+a)+1)^2)^(1/2))/(cos(b*x+a)+1)/(-sin(b*x+a)*cos(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)/d^2/(d*tan(b*x+a))^(1/2)*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(132) = 264.

Time = 0.15 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.39

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{16\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)^2+6\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}(\cos(bx+a)-\sin(bx+a))}{2\sqrt{d}\sin(bx+a)}\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(16*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2 + 6*sqrt(2)*sqrt
(d)*arctan(-1/2*sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x + a))*(cos(b*x + a) -
sin(b*x + a))/(sqrt(d)*sin(b*x + a))) - 3*sqrt(2)*sqrt(d)*arctan(1/2*(2*co
s(b*x + a)^2 - 2*cos(b*x + a)*sin(b*x + a) + sqrt(2)*sqrt(d*sin(b*x + a)/c
os(b*x + a))/sqrt(d) - 2)/(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a) - 1)
) - 3*sqrt(2)*sqrt(d)*arctan(-1/2*(2*cos(b*x + a)^2 - 2*cos(b*x + a)*sin(b
*x + a) - sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) - 2)/(cos(b*x
+ a)^2 + cos(b*x + a)*sin(b*x + a) - 1)) + 3*sqrt(2)*sqrt(d)*log(4*cos(b*x
 + a)*sin(b*x + a) + 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a)
)*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) + 1) - 3*sqrt(2)*sqrt(d)*log(4
*cos(b*x + a)*sin(b*x + a) - 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(
b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) + 1))/(b*d^3)
```

## Sympy [F]

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin^2{(a+bx)}}{(d\tan{(a+bx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(sin(a + b*x)**2/(d*tan(a + b*x))**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right) + 6\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\cdots}\right)}{2\sqrt{d}}\cdots\right.}{\cdots}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
1/16*(6*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d)) + 6*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 3*sqrt(2)*sqrt(d)*log(d*tan(b*x +
 a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 3*sqrt(2)*sqrt(d)*log(d*
tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) + 8*sqrt(d*tan(b*
x + a))*d^2/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.26

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{3\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd^3} + \frac{3\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd^3} + \frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{16\,bd^3} - \frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{16\,bd^3} + \frac{\sqrt{d\tan(bx+a)}}{2\left(d^2\tan(bx+a)^2+d^2\right)bd}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
3/8*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^3) + 3/8*sqrt(2)*sqrt(abs(d))*arctan(
-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))
/(b*d^3) + 3/16*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*t
an(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^3) - 3/16*sqrt(2)*sqrt(abs(d))*lo
g(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*
d^3) + 1/2*sqrt(d*tan(b*x + a))/((d^2*tan(b*x + a)^2 + d^2)*b*d)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^2/(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^2/(d*tan(a + b*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)^2}{\tan(bx+a)^3}dx\right)}{d^3}$$

input
```
int(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x)**2)/tan(a + b*x)**3,x))/d**3
```

# 3.105 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result . . . 912
Mathematica [A] (verified) . . . 912
Rubi [A] (verified) . . . 913
Maple [A] (verified) . . . 914
Fricas [B] (verification not implemented) . . . 914
Sympy [F] . . . 915
Maxima [A] (verification not implemented) . . . 915
Giac [A] (verification not implemented) . . . 915
Mupad [B] (verification not implemented) . . . 916
Reduce [F] . . . 916

## Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

output
```
-2/7*d/b/(d*tan(b*x+a))^(7/2)
```

## Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

input
```
Integrate[Csc[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{1}{(d\tan(a+bx))^{9/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

input
```
Int[Csc[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{2d}{7b(d\tan(bx+a))^{\frac{7}{2}}}$ | 17 |
| default | $-\frac{2d}{7b(d\tan(bx+a))^{\frac{7}{2}}}$ | 17 |

input
```
int(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*d/b/(d*tan(b*x+a))^(7/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.15

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)^4}{7\left(bd^3\cos(bx+a)^4-2\,bd^3\cos(bx+a)^2+bd^3\right)}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/7*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^4/(b*d^3*cos(b*x + a)^
4 - 2*b*d^3*cos(b*x + a)^2 + b*d^3)
```

## Sympy [F]

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(csc(a + b*x)**2/(d*tan(a + b*x))**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2}{7\,(d\tan(bx+a))^{\frac{5}{2}}\,b\tan(bx+a)}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/7/((d*tan(b*x + a))^(5/2)*b*tan(b*x + a))
```

## Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2}{7\sqrt{d\tan(bx+a)}bd^2\tan(bx+a)^3}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/7/(sqrt(d*tan(b*x + a))*b*d^2*tan(b*x + a)^3)
```

## Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 530, normalized size of antiderivative = 26.50

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(a + b*x)^2*(d*tan(a + b*x))^(5/2)),x)
```

output
```
(46*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
+ b*x*2i) + 1))^(1/2))/(7*b*d^3*(exp(a*2i + b*x*2i) - 1)) + (12*(exp(a*2i
+ b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1)
)^(1/2))/(5*b*d^3*(exp(a*2i + b*x*2i) - 1)^2) + (24*(exp(a*2i + b*x*2i) +
1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(35
*b*d^3*(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a
*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*48i)/(7*b*d^3*(exp
(a*2i + b*x*2i)*1i - 1i)) + (144*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i +
 b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(35*b*d^3*(exp(a*2i +
b*x*2i)*1i - 1i)^2) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1
i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*144i)/(35*b*d^3*(exp(a*2i + b*x*2
i)*1i - 1i)^3) - (16*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i
- 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(7*b*d^3*(exp(a*2i + b*x*2i)*1i -
1i)^4)
```

## Reduce [F]

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)^2}{\tan(bx+a)^3}dx\right)}{d^3}$$

input
```
int(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x)**2)/tan(a + b*x)**3,x))/d**3
```

# 3.106 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 917
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 917
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 918
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 919
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 920
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 920
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 921
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 921
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 921
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 922

## Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2d^3}{11b(d\tan(a+bx))^{11/2}} - \frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

output
```
-2/11*d^3/b/(d*tan(b*x+a))^(11/2)-2/7*d/b/(d*tan(b*x+a))^(7/2)
```

## Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2(-9+2\cos(2(a+bx)))\cot^4(a+bx)\csc^2(a+bx)\sqrt{d\tan(a+bx)}}{77bd^3}$$

input
```
Integrate[Csc[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*(-9 + 2*Cos[2*(a + b*x)])*Cot[a + b*x]^4*Csc[a + b*x]^2*Sqrt[d*Tan[a + 
b*x]])/(77*b*d^3)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)^4(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{\tan^2(a+bx)d^2+d^2}{(d\tan(a+bx))^{13/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{d\int \left(\frac{d^2}{(d\tan(a+bx))^{13/2}}+\frac{1}{(d\tan(a+bx))^{9/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\left(-\frac{2d^2}{11(d\tan(a+bx))^{11/2}}-\frac{2}{7(d\tan(a+bx))^{7/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*((-2*d^2)/(11*(d*Tan[a + b*x])^(11/2)) - 2/(7*(d*Tan[a + b*x])^(7/2))))
/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(35) = 70$.

Time = 0.78 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(4\cot(bx+a)^5-11\cot(bx+a)^3\csc(bx+a)^2\right)\sqrt{2}}{77b\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\,\sqrt{d\tan(bx+a)}\,d^2}$ | 103 |

input
```
int(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/77/b*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(-sin(b*x+a)*cos(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1/2)/d^2*(4*cot(b*x+a)^5-11
*cot(b*x+a)^3*csc(b*x+a)^2)*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(35) = 70$.

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.12

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(4\cos\left(bx+a\right)^6-11\cos\left(bx+a\right)^4\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{77\left(bd^3\cos\left(bx+a\right)^6-3\,bd^3\cos\left(bx+a\right)^4+3\,bd^3\cos\left(bx+a\right)^2-bd^3\right)}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/77*(4*cos(b*x + a)^6 - 11*cos(b*x + a)^4)*sqrt(d*sin(b*x + a)/cos(b*x +
 a))/(b*d^3*cos(b*x + a)^6 - 3*b*d^3*cos(b*x + a)^4 + 3*b*d^3*cos(b*x + a)
^2 - b*d^3)
```

## Sympy [F]

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc^4\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(csc(a + b*x)**4/(d*tan(a + b*x))**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(11\,d^2\tan\left(bx+a\right)^2+7\,d^2\right)d}{77\,\left(d\tan\left(bx+a\right)\right)^{\frac{11}{2}}b}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/77*(11*d^2*tan(b*x + a)^2 + 7*d^2)*d/((d*tan(b*x + a))^(11/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(11\,d^3\tan\left(bx+a\right)^2+7\,d^3\right)}{77\,\sqrt{d\tan\left(bx+a\right)}bd^5\tan\left(bx+a\right)^5}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/77*(11*d^3*tan(b*x + a)^2 + 7*d^3)/(sqrt(d*tan(b*x + a))*b*d^5*tan(b*x
+ a)^5)
```

## Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 831, normalized size of antiderivative = 19.33

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(a + b*x)^4*(d*tan(a + b*x))^(5/2)),x)
```

output
```
((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2)*2048i)/(165*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)) - (7768
*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2))/(945*b*d^3*(exp(a*2i + b*x*2i) - 1)^2) - (4232*(exp(a*
2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) +
 1))^(1/2))/(495*b*d^3*(exp(a*2i + b*x*2i) - 1)^3) - (1328*(exp(a*2i + b*x
*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/
2))/(231*b*d^3*(exp(a*2i + b*x*2i) - 1)^4) - (160*(exp(a*2i + b*x*2i) + 1)
*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(99*b
*d^3*(exp(a*2i + b*x*2i) - 1)^5) - (14456*(exp(a*2i + b*x*2i) + 1)*(-(d*(e
xp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(1155*b*d^3*(
exp(a*2i + b*x*2i) - 1)) - (86528*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i
+ b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(10395*b*d^3*(exp(a*2
i + b*x*2i)*1i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2
i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*3904i)/(315*b*d^3*(exp(a*2i +
 b*x*2i)*1i - 1i)^3) + (4160*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x
*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(231*b*d^3*(exp(a*2i + b*x
*2i)*1i - 1i)^4) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i -
 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*1600i)/(99*b*d^3*(exp(a*2i + b*x*2i)
*1i - 1i)^5) - (64*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i...
```

## Reduce [F]

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)^4}{\tan(bx+a)^3}dx\right)}{d^3}$$

input
```
int(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x)**4)/tan(a + b*x)**3,x))/d**3
```

# 3.107 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 923
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 923
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 924
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 925
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 926
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 926
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 927
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 927
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 927
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 928


## Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2d^5}{15b(d\tan(a+bx))^{15/2}} - \frac{4d^3}{11b(d\tan(a+bx))^{11/2}} - \frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

output
```
-2/15*d^5/b/(d*tan(b*x+a))^(15/2)-4/11*d^3/b/(d*tan(b*x+a))^(11/2)-2/7*d/b
/(d*tan(b*x+a))^(7/2)
```

## Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2(-117+44\cos(2(a+bx))-4\cos(4(a+bx)))\cot^4(a+bx)\csc^4(a+bx)\sqrt{d\,\mathrm{ta}}}{1155bd^3}$$

input
```
Integrate[Csc[a + b*x]^6/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*(-117 + 44*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Cot[a + b*x]^4*Csc[a
+ b*x]^4*Sqrt[d*Tan[a + b*x]])/(1155*b*d^3)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^6(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\left(\tan^2(a+bx)d^2+d^2\right)^2}{(d\tan(a+bx))^{17/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^4}{(d\tan(a+bx))^{17/2}}+\frac{2d^2}{(d\tan(a+bx))^{13/2}}+\frac{1}{(d\tan(a+bx))^{9/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^4}{15(d\tan(a+bx))^{15/2}}-\frac{4d^2}{11(d\tan(a+bx))^{11/2}}-\frac{2}{7(d\tan(a+bx))^{7/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^6/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*((-2*d^4)/(15*(d*Tan[a + b*x])^(15/2)) - (4*d^2)/(11*(d*Tan[a + b*x])^(
11/2)) - 2/(7*(d*Tan[a + b*x])^(7/2))))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(53) = 106$.

Time = 0.92 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.72

| method | result | size |
|---|---|---|
| default | $-\frac{\cot(bx+a)^{3}\csc(bx+a)^{4}\left(32\cos(bx+a)^{4}-120\cos(bx+a)^{2}+165\right)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^{2}}}\,\sqrt{2}}{1155b\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^{2}}}\,\sqrt{d\tan(bx+a)}\,d^{2}}$ | 112 |

input
```
int(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1155/b*cot(b*x+a)^3*csc(b*x+a)^4*(32*cos(b*x+a)^4-120*cos(b*x+a)^2+165)
*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(-sin(b*x+a)*cos(b*x+a)
/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1/2)/d^2*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(53) = 106$.

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.75

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(32\cos\left(bx+a\right)^8-120\cos\left(bx+a\right)^6+165\cos\left(bx+a\right)^4\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{1155\left(bd^3\cos\left(bx+a\right)^8-4\,bd^3\cos\left(bx+a\right)^6+6\,bd^3\cos\left(bx+a\right)^4-4\,bd^3\cos\left(bx+a\right)^2+bd^3\right)}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/1155*(32*cos(b*x + a)^8 - 120*cos(b*x + a)^6 + 165*cos(b*x + a)^4)*sqrt
(d*sin(b*x + a)/cos(b*x + a))/(b*d^3*cos(b*x + a)^8 - 4*b*d^3*cos(b*x + a)
^6 + 6*b*d^3*cos(b*x + a)^4 - 4*b*d^3*cos(b*x + a)^2 + b*d^3)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(165\,d^4\tan\left(bx+a\right)^4+210\,d^4\tan\left(bx+a\right)^2+77\,d^4\right)d}{1155\,\left(d\tan\left(bx+a\right)\right)^{\frac{15}{2}}b}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/1155*(165*d^4*tan(b*x + a)^4 + 210*d^4*tan(b*x + a)^2 + 77*d^4)*d/((d*t
an(b*x + a))^(15/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(165\,d^5\tan\left(bx+a\right)^4+210\,d^5\tan\left(bx+a\right)^2+77\,d^5\right)}{1155\,\sqrt{d\tan\left(bx+a\right)}bd^7\tan\left(bx+a\right)^7}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/1155*(165*d^5*tan(b*x + a)^4 + 210*d^5*tan(b*x + a)^2 + 77*d^5)/(sqrt(d
*tan(b*x + a))*b*d^7*tan(b*x + a)^7)
```

## Mupad [B] (verification not implemented)

Time = 11.12 (sec) , antiderivative size = 1132, normalized size of antiderivative = 17.42

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(a + b*x)^6*(d*tan(a + b*x))^(5/2)),x)
```

output
```
(199232*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a
*2i + b*x*2i) + 1))^(1/2))/(12285*b*d^3*(exp(a*2i + b*x*2i) - 1)) + (15813
76*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i +
 b*x*2i) + 1))^(1/2))/(135135*b*d^3*(exp(a*2i + b*x*2i) - 1)^2) + (4539104
*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2))/(225225*b*d^3*(exp(a*2i + b*x*2i) - 1)^3) + (1152*(exp
(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i
) + 1))^(1/2))/(35*b*d^3*(exp(a*2i + b*x*2i) - 1)^4) + (74528*(exp(a*2i +
b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^
(1/2))/(2145*b*d^3*(exp(a*2i + b*x*2i) - 1)^5) + (1088*(exp(a*2i + b*x*2i)
 + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/
(55*b*d^3*(exp(a*2i + b*x*2i) - 1)^6) + (896*(exp(a*2i + b*x*2i) + 1)*(-(d
*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(195*b*d^3
*(exp(a*2i + b*x*2i) - 1)^7) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i +
b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*439808i)/(27027*b*d^3*(e
xp(a*2i + b*x*2i)*1i - 1i)) + (1573888*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(
a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(135135*b*d^3*(e
xp(a*2i + b*x*2i)*1i - 1i)^2) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i +
 b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*4557824i)/(225225*b*d^3
*(exp(a*2i + b*x*2i)*1i - 1i)^3) - (7168*(exp(a*2i + b*x*2i) + 1)*(-(d*...
```

## Reduce [F]

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=\frac{\sqrt{d}\left(\int\frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)^6}{\tan(bx+a)^3}dx\right)}{d^3}$$

input
```
int(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x)**6)/tan(a + b*x)**3,x))/d**3
```

# 3.108 $\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$


Optimal result . . . . . . . . . . 929
Mathematica [C] (verified) . . . . . . . . . . 929
Rubi [A] (verified) . . . . . . . . . . 930
Maple [A] (verified) . . . . . . . . . . 933
Fricas [F] . . . . . . . . . . 933
Sympy [F(-1)] . . . . . . . . . . 934
Maxima [F] . . . . . . . . . . 934
Giac [F] . . . . . . . . . . 934
Mupad [F(-1)] . . . . . . . . . . 935
Reduce [F] . . . . . . . . . . 935


## Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{40bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-1/20*sin(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)-3/70*sin(b*x+a)^5/b/d/(d*tan(b
*x+a))^(3/2)+1/7*sin(b*x+a)^7/b/d/(d*tan(b*x+a))^(3/2)-3/40*EllipticE(cos(
a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a)
)^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.49 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d\tan(a+bx)}\left(-\sqrt{\sec^2(a+bx)}(15\sin(a+bx)+29\sin(3(a+bx))+9\sin(5(\right.}{}$$

input
```
Integrate[Sin[a + b*x]^7/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(15*Sin[a + b*x] + 29*Sin[3*
(a + b*x)] + 9*Sin[5*(a + b*x)] - 5*Sin[7*(a + b*x)])) + 112*Hypergeometri
c2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a + b*x]))/(2240*b*d
^3*Sqrt[Sec[a + b*x]^2])
```

## Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3076, 3042, 3078, 3042, 3078, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^7}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{3\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sin(a+bx)^5}{\sqrt{d\tan(a+bx)}}dx}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{3\left(\frac{7}{10}\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7}{10}\int \frac{\sin(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3078

$$\frac{3\left(\frac{7}{10}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{3\left(\frac{7}{10}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3081

$$\frac{3\left(\frac{7}{10}\left(\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{3\left(\frac{7}{10}\left(\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3052

$$\frac{3\left(\frac{7}{10}\left(\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{3\left(\frac{7}{10}\left(\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3119

$$\frac{3\left(\frac{7}{10}\left(\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]^7/(d*Tan[a + b*x])^(5/2),x]
```

output
```
Sin[a + b*x]^7/(7*b*d*(d*Tan[a + b*x])^(3/2)) + (3*(-1/5*(d*Sin[a + b*x]^5
)/(b*(d*Tan[a + b*x])^(3/2)) + (7*(-1/3*(d*Sin[a + b*x]^3)/(b*(d*Tan[a + b
*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(2*b*Sqrt[Sin[2*
a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])))/10))/(14*d^2)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.67

| method | result |
|---|---|
| default | $-\frac{\frac{\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\sqrt{\csc(bx+a)-\cot(bx+a)+1}\ \mathrm{EllipticE}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1},\frac{\sqrt{2}}{2}\right)(42+42\sec(bx}{560}}{}$ |

input
```
int(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/560*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^
(1/2)*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^
(1/2),1/2*2^(1/2))*(42+42*sec(b*x+a))-1/560*(-2*csc(b*x+a)+2*cot(b*x+a)+2)
^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^
(1/2),1/2*2^(1/2))*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-21-21*sec(b*x+a))-1/7
*cos(b*x+a)^7+27/70*cos(b*x+a)^5-41/140*cos(b*x+a)^3-1/40*cos(b*x+a)+3/40)
/(d*tan(b*x+a))^(1/2)/d^2
```

## Fricas [F]

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=\int \frac{\sin(bx+a)^7}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^6 - 3*cos(b*x + a)^4 + 3*cos(b*x + a)^2 - 1)*sqrt(
d*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b*x + a)^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**7/(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^7}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^7/(d*tan(b*x + a))^(5/2), x)
```

## Giac [F]

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^7}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^7/(d*tan(b*x + a))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^7}{\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^7/(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^7/(d*tan(a + b*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)^7}{\tan(bx+a)^3}dx\right)}{d^3}$$

input
```
int(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x)**7)/tan(a + b*x)**3,x))/d**3
```

# 3.109 $\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 936
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 936
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 937
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 939
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 940
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 940
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 941
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 941
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 941
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 942

## Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}} + \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{20bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-1/10*sin(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)+1/5*sin(b*x+a)^5/b/d/(d*tan(b*
x+a))^(3/2)-3/20*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin
(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.67 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d\tan(a+bx)}\left(-\sqrt{\sec^2(a+bx)}(\sin(3(a+bx))+\sin(5(a+bx)))+8\,\text{Hyperg}\right.}{80bd^3\sqrt{\sec^2(a+b}}$$

input
```
Integrate[Sin[a + b*x]^5/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(Sin[3*(a + b*x)] + Sin[5*(a
 + b*x)])) + 8*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b
*x]*Tan[a + b*x]))/(80*b*d^3*Sqrt[Sec[a + b*x]^2])
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3076, 3042, 3078, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^5}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{3\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sin(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{3\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{3\left(\frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \; 3052$$

$$\frac{3\left(\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \; 3119$$

$$\frac{3\left(\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]^5/(d*Tan[a + b*x])^(5/2),x]
```

output
```
Sin[a + b*x]^5/(5*b*d*(d*Tan[a + b*x])^(3/2)) + (3*(-1/3*(d*Sin[a + b*x]^3
)/(b*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])
/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])))/(10*d^2)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(101) = 202$.

Time = 1.41 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.02

| method | result |
|---|---|
| default | $\frac{\frac{\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{EllipticE}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}, \frac{\sqrt{2}}{2}\right)(-6-6\sec(bx+}{40}}{}$ |

input
```
int(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/40*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(
1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1
/2),1/2*2^(1/2))*(-6-6*sec(b*x+a))+1/40*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-
2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*Elliptic
F((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(3+3*sec(b*x+a))+1/5*cos(b*
x+a)^5-3/10*cos(b*x+a)^3-1/20*cos(b*x+a)+3/20)/(d*tan(b*x+a))^(1/2)/d^2
```

## Fricas [F]

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^5}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*tan(b*x + a))*sin(
b*x + a)/(d^3*tan(b*x + a)^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**5/(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^5/(d*tan(b*x + a))^(5/2), x)
```

## Giac [F]

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^5/(d*tan(b*x + a))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^5}{\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^5/(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^5/(d*tan(a + b*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)^5}{\tan(bx+a)^3}dx\right)}{d^3}$$

input
```
int(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x)**5)/tan(a + b*x)**3,x))/d**3
```

# 3.110 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result . . . . . 943
Mathematica [C] (verified) . . . . . 943
Rubi [A] (verified) . . . . . 944
Maple [B] (verified) . . . . . 946
Fricas [F] . . . . . 947
Sympy [F(-1)] . . . . . 947
Maxima [F] . . . . . 947
Giac [F] . . . . . 948
Mupad [F(-1)] . . . . . 948
Reduce [F] . . . . . 948

## Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}+\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
1/3*sin(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)-1/2*EllipticE(cos(a+1/4*Pi+b*x),
2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.97 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=\frac{\sqrt{d\tan(a+bx)}\left(\sqrt{\sec^2(a+bx)}(\sin(a+bx)+\sin(3(a+bx)))+4\,\text{Hypergeome}\right.}{12bd^3\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(Sqrt[d*Tan[a + b*x]]*(Sqrt[Sec[a + b*x]^2]*(Sin[a + b*x] + Sin[3*(a + b*x
)]) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan
[a + b*x]))/(12*b*d^3*Sqrt[Sec[a + b*x]^2])
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3076, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{2d^2}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{2d^2}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$\big\downarrow$ 3119

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

output
```
Sin[a + b*x]^3/(3*b*d*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x,
 2]*Sin[a + b*x])/(2*b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(75) = 150$.

Time = 0.99 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.61

| method | result |
|---|---|
| default | $-\frac{\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\operatorname{EllipticE}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\right.}{}$ |

input
```
int(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/b/(d*tan(b*x+a))^(1/2)/d^2*((csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(
b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticE((csc
(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(6+6*sec(b*x+a))+(csc(b*x+a)-cot(
b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+
a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-3-3*sec
(b*x+a))+4*cos(b*x+a)^3+2*cos(b*x+a)-6)
```

## Fricas [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^3*tan(
b*x + a)^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

## Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^3/(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^3/(d*tan(a + b*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sin(bx+a)^3}{\tan(bx+a)^3}dx\right)}{d^3}$$

input
```
int(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sin(a + b*x)**3)/tan(a + b*x)**3,x))/d**3
```

# 3.111 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 949
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . 949
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 950
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 952
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 953
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 953
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 953
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 954
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 954
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 954


## Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}} - \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-2*sin(b*x+a)/b/d/(d*tan(b*x+a))^(3/2)+3*EllipticE(cos(a+1/4*Pi+b*x),2^(1/
2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.67 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\cos(a+bx)\left(1+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{bd^2\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*Cos[a + b*x]*(1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sq
rt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(b*d^2*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3077, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{3\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{d^2}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{d^2}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{3\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{3\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*Sin[a + b*x])/(b*d*(d*Tan[a + b*x])^(3/2)) - (3*EllipticE[a - Pi/4 + b
*x, 2]*Sin[a + b*x])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(73) = 146$.

Time = 0.85 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.38

| method | result |
|---|---|
| default | $\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{EllipticE}\left(\sqrt{\mathrm{cs}\right.\right.}{}$ |

input
```
int(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1/
2)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/d^2*((csc(b*x+a)-cot(b*
x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a)
)^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(6+6*sec(b*
x+a))+(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)
*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),
1/2*2^(1/2))*(-3-3*sec(b*x+a))+2*cos(b*x+a)-6)*2^(1/2)
```

## Fricas [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b*x + a)^3), x)
```

## Sympy [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(sin(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

## Maxima [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

## Giac [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=\int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=\int \frac{\sin(a+b\,x)}{(d\tan(a+b\,x))^{5/2}}\,dx$$

input
```
int(sin(a + b*x)/(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)/(d*tan(a + b*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=\frac{\sqrt{d}\left(-4\sqrt{\tan(bx+a)}\,\cos(bx+a)\tan(bx+a)-2\sqrt{\tan(bx+a)}\,\sin(bx+a)\right.}{}$$

input
```
int(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*( - 4*sqrt(tan(a + b*x))*cos(a + b*x)*tan(a + b*x) - 2*sqrt(tan(a
 + b*x))*sin(a + b*x)*tan(a + b*x)**2 - 2*sqrt(tan(a + b*x))*sin(a + b*x)
- 6*int((sqrt(tan(a + b*x))*sin(a + b*x))/tan(a + b*x),x)*tan(a + b*x)**2*
b + int(sqrt(tan(a + b*x))*sin(a + b*x)*tan(a + b*x),x)*tan(a + b*x)**2*b)
)/(3*tan(a + b*x)**2*b*d**3)
```

# 3.112 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 955
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 955
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 956
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 959
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 959
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 960
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 960
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 961
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 961
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 961

## Optimal result

Integrand size = 19, antiderivative size = 110

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{5bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-2/5*csc(b*x+a)/b/d/(d*tan(b*x+a))^(3/2)+6/5*cos(b*x+a)/b/d^2/(d*tan(b*x+a
))^(1/2)-6/5*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b
*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.86 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2\left(2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sec^2(a+bx)-(3-4\csc^2(a-\cdots\right.}{5bd^3\sqrt{\sec^2(a+b\cdots}}$$

input
```
Integrate[Csc[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*(2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 - (
3 - 4*Csc[a + b*x]^2 + Csc[a + b*x]^4)*Sqrt[Sec[a + b*x]^2])*Sin[a + b*x]*
Sqrt[d*Tan[a + b*x]])/(5*b*d^3*Sqrt[Sec[a + b*x]^2])
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.31, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 3077, 3042, 3081, 3042, 3050, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3077}$$

$$-\frac{3\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{5d^2}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\int \frac{1}{\sin(a+bx)\sqrt{d\tan(a+bx)}}dx}{5d^2}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 3081}$$

$$-\frac{3\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}dx}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin(a+bx)^{3/2}}dx}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 3050}$$

$$-\frac{3\sqrt{\sin(a+bx)}\left(-2\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{\sin(a+bx)}\left(-2\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{3\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3\sqrt{\sin(a+bx)}\left(-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}E\left(a+bx-\frac{\pi}{4}|2\right)}{b\sqrt{\sin(2a+2bx)}}\right)}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

input

```
Int[Csc[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

output

```
(-2*Csc[a + b*x])/(5*b*d*(d*Tan[a + b*x])^(3/2)) - (3*Sqrt[Sin[a + b*x]]*(
(-2*Cos[a + b*x]^(3/2))/(b*Sqrt[Sin[a + b*x]]) - (2*Sqrt[Cos[a + b*x]]*Ell
ipticE[a - Pi/4 + b*x, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[Sin[2*a + 2*b*x]])))
/(5*d^2*Sqrt[Cos[a + b*x]]*Sqrt[d*Tan[a + b*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(97) = 194.

Time = 0.88 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.45

| method | result |
|---|---|
| default | $\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{cs\right.\right.}{}$ |

input
```
int(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/10/b*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(-sin(b*x+a)*cos(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1/2)/d^2*((csc(b*x+a)-cot(b
*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a
))^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-6-6*sec(
b*x+a))+(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/
2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2
),1/2*2^(1/2))*(3+3*sec(b*x+a))+6-2*cot(b*x+a)*csc(b*x+a))*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.24

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx =$$

$$-\frac{3\left(-i\cos\left(bx+a\right)^2+i\right)\sqrt{i\,d}E(\arcsin\left(\cos\left(bx+a\right)+i\sin\left(bx+a\right)\right)\,|-1)\sin\left(bx+a\right)+3\left(i\cos\left(bx+a\right)\right.}{}$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-1/5*(3*(-I*cos(b*x + a)^2 + I)*sqrt(I*d)*elliptic_e(arcsin(cos(b*x + a) +
 I*sin(b*x + a)), -1)*sin(b*x + a) + 3*(I*cos(b*x + a)^2 - I)*sqrt(-I*d)*e
lliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + 3*(I*c
os(b*x + a)^2 - I)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x +
a)), -1)*sin(b*x + a) + 3*(-I*cos(b*x + a)^2 + I)*sqrt(-I*d)*elliptic_f(ar
csin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - 2*(3*cos(b*x + a)^
4 - 2*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((b*d^3*cos(b*x +
 a)^2 - b*d^3)*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(csc(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

## Maxima [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

## Giac [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{1}{\sin(a+b\,x)\,\left(d\tan(a+b\,x)\right)^{5/2}}\,dx$$

input
```
int(1/(sin(a + b*x)*(d*tan(a + b*x))^(5/2)),x)
```

output
```
int(1/(sin(a + b*x)*(d*tan(a + b*x))^(5/2)), x)
```

## Reduce [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)}{\tan(bx+a)^3}dx\right)}{d^3}$$

input
```
int(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x))/tan(a + b*x)**3,x))/d**3
```

# 3.113 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 962
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 962
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 963
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 966
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . 967
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 968
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 968
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 968
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 969
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 969

## Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{4E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{15bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
2/15*csc(b*x+a)/b/d/(d*tan(b*x+a))^(3/2)-2/9*csc(b*x+a)^3/b/d/(d*tan(b*x+a
))^(3/2)+4/15*cos(b*x+a)/b/d^2/(d*tan(b*x+a))^(1/2)-4/15*EllipticE(cos(a+1
/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(
1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2\left(4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sec^2(a+bx)+(-6+3\csc^2(a\right.}{45bd^3\sqrt{}}$$

input
```
Integrate[Csc[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*(4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 + (
-6 + 3*Csc[a + b*x]^2 + 8*Csc[a + b*x]^4 - 5*Csc[a + b*x]^6)*Sqrt[Sec[a +
b*x]^2])*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/(45*b*d^3*Sqrt[Sec[a + b*x]^2]
)
```

## Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3077, 3042, 3079, 3042, 3081, 3042, 3050, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{1}{\sin(a+bx)^3\sqrt{d\tan(a+bx)}}dx}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3079}$$

$$-\frac{\frac{2}{5}\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2}{5}\int \frac{1}{\sin(a+bx)\sqrt{d\tan(a+bx)}}dx-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$\Big\downarrow$ 3081

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin(a+bx)^{3/2}}dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$\Big\downarrow$ 3050

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\left(-2\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\left(-2\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$\Big\downarrow$ 3052

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \; 3119$$

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\left(-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

input
```
Int[Csc[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*Csc[a + b*x]^3)/(9*b*d*(d*Tan[a + b*x])^(3/2)) - ((-2*d*Csc[a + b*x])/
(5*b*(d*Tan[a + b*x])^(3/2)) + (2*Sqrt[Sin[a + b*x]]*((-2*Cos[a + b*x]^(3/
2))/(b*Sqrt[Sin[a + b*x]]) - (2*Sqrt[Cos[a + b*x]]*EllipticE[a - Pi/4 + b*
x, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[Sin[2*a + 2*b*x]])))/(5*Sqrt[Cos[a + b*x
]]*Sqrt[d*Tan[a + b*x]]))/(3*d^2)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(123) = 246$.

Time = 1.00 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.04

| method | result |
|---|---|
| default | $-\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{\right.\right.}{}$ |

input
```
int(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45/b*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(-sin(b*x+a)*cos
(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/d^2/(d*tan(b*x+a))^(1/2)*((csc(b*x+a)-cot(
b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+
a))^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(6+6*sec(
b*x+a))+(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/
2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2
),1/2*2^(1/2))*(-3-3*sec(b*x+a))-6-3*cot(b*x+a)*csc(b*x+a)+5*cot(b*x+a)*cs
c(b*x+a)^3)*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.21

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx =$$
$$-\frac{2\left(3\left(-i\cos(bx+a)^4+2i\cos(bx+a)^2-i\right)\sqrt{i\,d}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/45*(3*(-I*cos(b*x + a)^4 + 2*I*cos(b*x + a)^2 - I)*sqrt(I*d)*elliptic_e
(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 3*(I*cos(b*x +
a)^4 - 2*I*cos(b*x + a)^2 + I)*sqrt(-I*d)*elliptic_e(arcsin(cos(b*x + a) -
 I*sin(b*x + a)), -1)*sin(b*x + a) + 3*(I*cos(b*x + a)^4 - 2*I*cos(b*x + a
)^2 + I)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*s
in(b*x + a) + 3*(-I*cos(b*x + a)^4 + 2*I*cos(b*x + a)^2 - I)*sqrt(-I*d)*el
liptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - (6*cos(
b*x + a)^6 - 15*cos(b*x + a)^4 + 4*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos
(b*x + a)))/((b*d^3*cos(b*x + a)^4 - 2*b*d^3*cos(b*x + a)^2 + b*d^3)*sin(b
*x + a))
```

## Sympy [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc^3{(a+bx)}}{(d\tan{(a+bx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(csc(a + b*x)**3/(d*tan(a + b*x))**(5/2), x)
```

## Maxima [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc{(bx+a)}^3}{(d\tan{(bx+a)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

## Giac [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc{(bx+a)}^3}{(d\tan{(bx+a)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^3\,\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2)),x)
```

output
```
int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2)), x)
```

## Reduce [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\csc(bx+a)^3}{\tan(bx+a)^3}dx\right)}{d^3}$$

input
```
int(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*csc(a + b*x)**3)/tan(a + b*x)**3,x))/d**3
```

# 3.114 $\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal result . . . 970
Mathematica [A] (verified) . . . 970
Rubi [A] (verified) . . . 971
Maple [B] (verified) . . . 972
Fricas [A] (verification not implemented) . . . 973
Sympy [F(-1)] . . . 973
Maxima [F] . . . 974
Giac [F] . . . 974
Mupad [B] (verification not implemented) . . . 974
Reduce [F] . . . 975

## Optimal result

Integrand size = 25, antiderivative size = 68

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

output
```
-8/5*a^2*b*(a*sin(f*x+e))^(1/2)/f/(b*tan(f*x+e))^(1/2)-2/5*b*(a*sin(f*x+e)
)^(5/2)/f/(b*tan(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{a^2\sqrt{a\sin(e+fx)}(8\cot(e+fx)+\sin(2(e+fx)))\sqrt{b\tan(e+fx)}}{5f}$$

input
```
Integrate[(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
-1/5*(a^2*Sqrt[a*Sin[e + f*x]]*(8*Cot[e + f*x] + Sin[2*(e + f*x)])*Sqrt[b*
Tan[e + f*x]])/f
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{4}{5}a^2\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a^2\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3069}$$

$$-\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-8*a^2*b*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[b*Tan[e + f*x]]) - (2*b*(a*Sin[e
 + f*x])^(5/2))/(5*f*Sqrt[b*Tan[e + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(56) = 112$.

Time = 1.76 (sec) , antiderivative size = 340, normalized size of antiderivative = 5.00

| method | result |
|---|---|
| default | $-\frac{\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\left(-16\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+5\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^{2}}}\ \ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^{2}}}+2\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^{2}}}-\cos}{1+\cos(fx+e)}\right.\right.}{}$ |

input
```
int((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/f*sec(1/2*f*x+1/2*e)*csc(1/2*f*x+1/2*e)*(-16*cos(1/2*f*x+1/2*e)^6+5*(
-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+co
s(f*x+e)))*cos(1/2*f*x+1/2*e)^2-5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*ln(
2*(2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+e)))*cos(1/2*f*x+1/2*e)^2+24*co
s(1/2*f*x+1/2*e)^4+8*cos(1/2*f*x+1/2*e)^2-8)*(sin(1/2*f*x+1/2*e)*cos(1/2*f
*x+1/2*e)*a)^(1/2)*a^2*(cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)/(2*cos(1/2*f
*x+1/2*e)^2-1)*b)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \frac{2\left(a^2\cos(fx+e)^3-5a^2\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{5f\sin(fx+e)}$$

input
```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/5*(a^2*cos(f*x + e)^3 - 5*a^2*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*
sin(f*x + e)/cos(f*x + e))/(f*sin(f*x + e))
```

## Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(5/2)*(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \int (a\sin(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \int (a\sin(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \frac{a^2\sqrt{a\sin(e+fx)}\,(18\sin(2e+2fx)-\sin(4e+4fx))\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}{10f\,(\cos(2e+2fx)-1)}$$

input
```
int((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(1/2),x)
```

output
```
(a^2*(a*sin(e + f*x))^(1/2)*(18*sin(2*e + 2*f*x) - sin(4*e + 4*f*x))*((b*s
in(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(10*f*(cos(2*e + 2*f*x) -
1))
```

## Reduce [F]

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx=\sqrt{b}\,\sqrt{a}\left(\int\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)^2\,dx\right)a^2$$

input
```
int((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*sqrt(a)*int(sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x)**2,
x)*a**2
```

# 3.115 $\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal result . . . 976
Mathematica [A] (verified) . . . 976
Rubi [A] (verified) . . . 977
Maple [C] (verified) . . . 979
Fricas [C] (verification not implemented) . . . 979
Sympy [F(-1)] . . . 980
Maxima [F] . . . 980
Giac [F] . . . 980
Mupad [F(-1)] . . . 981
Reduce [F] . . . 981

## Optimal result

Integrand size = 25, antiderivative size = 88

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}} + \frac{4a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}$$

output
```
-2/3*b*(a*sin(f*x+e))^(3/2)/f/(b*tan(f*x+e))^(1/2)+4/3*a^2*cos(f*x+e)^(1/2
)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(a*sin(f*x
+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 6.74 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{2ab\sqrt{a\sin(e+fx)}\left(-2\,\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)+\sqrt[4]{\cos^2(e+fx)}\sin(e+fx)\right)}{3f\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*a*b*Sqrt[a*Sin[e + f*x]]*(-2*EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (C
os[e + f*x]^2)^(1/4)*Sin[e + f*x]))/(3*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan
[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3078, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{2}{3}a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{3\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{4a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*Sqrt[b*Tan[e + f*x]]) + (4*a^2*Sqrt[Cos
[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin
[e + f*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

| method | result |
|---|---|
| default | $\frac{2\sqrt{a\sin(fx+e)}\,a\sqrt{b\tan(fx+e)}\,\left(i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\operatorname{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)(2\cot(fx+e)+2\csc(fx+\right.}{3f}$ |

input
```
int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/f*(a*sin(f*x+e))^(1/2)*a*(b*tan(f*x+e))^(1/2)*(I*(1/(1+cos(f*x+e)))^(1
/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-csc(f*x+e)+cot(f*x+e))
,I)*(2*cot(f*x+e)+2*csc(f*x+e))-cos(f*x+e))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx =$$
$$-\frac{2\left(\sqrt{a\sin(fx+e)}a\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)-2\sqrt{\frac{1}{2}}\sqrt{-ab}a\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\right.}{3f}$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e
) - 2*sqrt(1/2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) + I*s
in(f*x + e)) - 2*sqrt(1/2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x
 + e) - I*sin(f*x + e)))/f
```

## Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx=\int (a\,\sin\left(e+f\,x\right))^{3/2}\,\sqrt{b\tan\left(e+f\,x\right)}\,dx$$

input
```
int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx=\sqrt{b}\,\sqrt{a}\left(\int\sqrt{\tan\left(fx+e\right)}\,\sqrt{\sin\left(fx+e\right)}\,\sin\left(fx+e\right)dx\right)a$$

input
```
int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*sqrt(a)*int(sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x),x)*
a
```

# 3.116 $\int \sqrt{a \sin(e+fx)}\sqrt{b \tan(e+fx)}\,dx$

Optimal result . . . 982
Mathematica [A] (verified) . . . 982
Rubi [A] (verified) . . . 983
Maple [C] (verified) . . . 984
Fricas [A] (verification not implemented) . . . 984
Sympy [F] . . . 985
Maxima [F] . . . 985
Giac [F] . . . 985
Mupad [B] (verification not implemented) . . . 986
Reduce [F] . . . 986

## Optimal result

Integrand size = 25, antiderivative size = 30

$$\int \sqrt{a \sin(e+fx)}\sqrt{b \tan(e+fx)}\,dx = -\frac{2b\sqrt{a \sin(e+fx)}}{f\sqrt{b \tan(e+fx)}}$$

output
```
-2*b*(a*sin(f*x+e))^(1/2)/f/(b*tan(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \sqrt{a \sin(e+fx)}\sqrt{b \tan(e+fx)}\,dx = -\frac{2b\sqrt{a \sin(e+fx)}}{f\sqrt{b \tan(e+fx)}}$$

input
```
Integrate[Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \sin(e+fx)}\sqrt{b \tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a \sin(e+fx)}\sqrt{b \tan(e+fx)}dx$$

$$\Big\downarrow \text{3069}$$

$$-\frac{2b\sqrt{a \sin(e+fx)}}{f\sqrt{b \tan(e+fx)}}$$

input
```
Int[Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.37

| method | result |
|---|---|
| risch | $-\frac{2i\sqrt{a\sin(fx+e)}\sqrt{-\frac{ib\left(e^{2i(fx+e)}-1\right)}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)}{\left(e^{2i(fx+e)}-1\right)f}$ |
| default | $-\frac{\cot(fx+e)\left(4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+4\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+4\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}-2\cos\cdots}{1+\cos(fx+e)}\right.\right.}{2f(1+\cos(fx+e))\sqrt{-\frac{\cdots}{(1\cdots}}}$ |

input
```
int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(a*sin(f*x+e))^(1/2)/(exp(2*I*(f*x+e))-1)*(-I*b*(exp(2*I*(f*x+e))-1)/
(exp(2*I*(f*x+e))+1))^(1/2)*(exp(2*I*(f*x+e))+1)/f
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = -\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{f\sin(fx+e)}$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/(f*
sin(f*x + e))
```

## Sympy [F]

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{a\sin{(e+fx)}}\sqrt{b\tan{(e+fx)}}\,dx$$

input
```
integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*sin(e + f*x))*sqrt(b*tan(e + f*x)), x)
```

## Maxima [F]

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{a\sin{(fx+e)}}\sqrt{b\tan{(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{a\sin{(fx+e)}}\sqrt{b\tan{(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.00

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \frac{\sin(2e+2fx)\ \sqrt{a\ \sin(e+fx)}\ \sqrt{\frac{b\ \sin(2e+2fx)}{2\cos(e+fx)^2}}}{f\ \left(\cos(e+fx)^2-1\right)}$$

input
```
int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(1/2),x)
```

output
```
(sin(2*e + 2*f*x)*(a*sin(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(2*cos(e +
f*x)^2))^(1/2))/(f*(cos(e + f*x)^2 - 1))
```

## Reduce [F]

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \sqrt{b}\,\sqrt{a}\left(\int \sqrt{\tan(fx+e)}\ \sqrt{\sin(fx+e)}dx\right)$$

input
```
int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*sqrt(a)*int(sqrt(tan(e + f*x))*sqrt(sin(e + f*x)),x)
```

# 3.117 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal result . . . 987
Mathematica [A] (verified) . . . 987
Rubi [A] (verified) . . . 988
Maple [C] (verified) . . . 989
Fricas [C] (verification not implemented) . . . 990
Sympy [F] . . . 990
Maxima [F] . . . 991
Giac [F] . . . 991
Mupad [F(-1)] . . . 991
Reduce [F] . . . 992

## Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

output
```
2*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*tan(f*x+e))^(
1/2)/f/(a*sin(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\cos(e+fx)\,\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)\sqrt{b\tan(e+fx)}}{f\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

output
```
(2*Cos[e + f*x]*EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sqrt[b*Tan[e + f*x]])
/(f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

output
```
(2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*S
qrt[a*Sin[e + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

| method | result | size |
|---|---|---|
| default | $-\frac{2 i \operatorname{EllipticF}\left(i\left(\csc \left(f x+e\right)-\cot \left(f x+e\right)\right), i\right) \sqrt{\frac{\cos \left(f x+e\right)}{1+\cos \left(f x+e\right)}}\, \sqrt{b \tan \left(f x+e\right)}}{f \sqrt{a \sin \left(f x+e\right)}\, \sqrt{\frac{1}{1+\cos \left(f x+e\right)}}}$ | 79 |

input
```
int((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I/f/(a*sin(f*x+e))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)/(1/(1+cos(f*x+e)))^(1/2)*(b*tan(f*x+e))^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\left(\sqrt{\frac{1}{2}}\sqrt{-ab}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+\sqrt{\frac{1}{2}}\sqrt{-ab}\text{weierstrassPInverse}(\right.}{af}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(1/2)*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*
x + e)) + sqrt(1/2)*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I
*sin(f*x + e)))/(a*f)
```

## Sympy [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x))/sqrt(a*sin(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

## Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{\sqrt{a\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}}{\sin(fx+e)}dx\right)}{a}$$

input
```
int((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x)))/sin(e + f*x),
x))/a
```

# 3.118 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . 993
Mathematica [A] (verified) . . . . . . . . . . 993
Rubi [A] (verified) . . . . . . . . . . 994
Maple [A] (verified) . . . . . . . . . . 997
Fricas [B] (verification not implemented) . . . . . . . . . . 997
Sympy [F] . . . . . . . . . . 998
Maxima [F] . . . . . . . . . . 998
Giac [F] . . . . . . . . . . 999
Mupad [F(-1)] . . . . . . . . . . 999
Reduce [F] . . . . . . . . . . 999

## Optimal result

Integrand size = 25, antiderivative size = 107

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}} - \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}}$$

output

```
-arctan(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a/f/(a*sin
(f*x+e))^(1/2)-arctanh(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(
1/2)/a/f/(a*sin(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{b\left(\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

output
```
-((b*(ArcTan[(Cos[e + f*x]^2)^(1/4)] + ArcTanh[(Cos[e + f*x]^2)^(1/4)])*Sq
rt[a*Sin[e + f*x]])/(a^2*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]]))
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3081, 27, 3042, 3045, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b \tan(e+fx)}}{(a \sin(e+fx))^{3/2}} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{b \tan(e+fx)}}{(a \sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3081}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b \tan(e+fx)} \int \frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}dx}{\sqrt{a \sin(e+fx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b \tan(e+fx)} \int \frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}dx}{a\sqrt{a \sin(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b \tan(e+fx)} \int \frac{1}{\sqrt{\cos(e+fx)}\sin(e+fx)}dx}{a\sqrt{a \sin(e+fx)}}$$

$$\Big\downarrow \text{ 3045}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b \tan(e+fx)} \int \frac{1}{\sqrt{\cos(e+fx)}(1-\cos^2(e+fx))}d\cos(e+fx)}{af\sqrt{a \sin(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{1-\cos^2(e+fx)}d\sqrt{\cos(e+fx)}}{af\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{756}$$

$$-\frac{2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}+\frac{1}{2}\int \frac{1}{\cos(e+fx)+1}d\sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}+\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{af\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\right)}{af\sqrt{a\sin(e+fx)}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

output
```
(-2*(ArcTan[Sqrt[Cos[e + f*x]]]/2 + ArcTanh[Sqrt[Cos[e + f*x]]]/2)*Sqrt[Co
s[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(a*f*Sqrt[a*Sin[e + f*x]])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

## Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.53

| method | result |
|---|---|
| default | $\frac{\left(\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}}\right)-\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+4\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}-2\cos(fx+e)+2}{1+\cos(fx+e)}\right)\right)\sqrt{b\tan(fx+e)}\,\cos(f}{2f(1+\cos(fx+e))a\sqrt{a\sin(fx+e)}\,\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}}$ |

input
```
int((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(arctan(1/2/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2))-ln(2*(2*cos(f*x+e)
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)-cos(f*x+e)+1)/(1+cos(f*x+e))))*(b*tan(f*x+e))^(1/2)*cos(f*x+e)/(1+cos(f
*x+e))/a/(a*sin(f*x+e))^(1/2)/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(91) = 182.

Time = 0.26 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.86

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \left[\frac{2\sqrt{-\frac{b}{a}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{b}{a}}\cos(fx+e)}{(b\cos(fx+e)+b)\sin(fx+e)}\right)+\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(fx+e)^3}{}\right.}{4af}\right.$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f
*x + e))*sqrt(-b/a)*cos(f*x + e)/((b*cos(f*x + e) + b)*sin(f*x + e))) + sq
rt(-b/a)*log(-(b*cos(f*x + e)^3 + 4*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x +
e)/cos(f*x + e))*sqrt(-b/a)*cos(f*x + e)*sin(f*x + e) - 5*b*cos(f*x + e)^2
 - 5*b*cos(f*x + e) + b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x +
e) + 1)))/(a*f), 1/4*(2*sqrt(b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin
(f*x + e)/cos(f*x + e))*sqrt(b/a)*cos(f*x + e)/((b*cos(f*x + e) - b)*sin(f
*x + e))) + sqrt(b/a)*log((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*
x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/a) - (b*cos(f*x + e)^2 +
6*b*cos(f*x + e) + b)*sin(f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)
*sin(f*x + e))))/(a*f)]
```

## Sympy [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan\left(e+fx\right)}}{\left(a\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x))/(a*sin(e + f*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx=\frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}}{\sin(fx+e)^2}dx\right)}{a^2}$$

input
```
int((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x)))/sin(e + f*x)*
*2,x))/a**2
```

# 3.119 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$


Optimal result . . . 1000
Mathematica [A] (warning: unable to verify) . . . 1000
Rubi [A] (verified) . . . 1001
Maple [C] (verified) . . . 1003
Fricas [C] (verification not implemented) . . . 1003
Sympy [F(-1)] . . . 1004
Maxima [F] . . . 1004
Giac [F] . . . 1004
Mupad [F(-1)] . . . 1005
Reduce [F] . . . 1005


## Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = -\frac{b}{a^2 f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{a^2 f\sqrt{a\sin(e+fx)}}$$

output
```
-b/a^2/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)+cos(f*x+e)^(1/2)*Invers
eJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*tan(f*x+e))^(1/2)/a^2/f/(a*sin(f*x+e))
^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{b\left(-\sqrt[4]{\cos^2(e+fx)} + \text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)\sin(e+fx)\right)}{a^2 f\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

output
```
(b*(-(Cos[e + f*x]^2)^(1/4) + EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e +
 f*x]))/(a^2*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e +
f*x]])
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3079, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3079}$$

$$\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2a^2}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2a^2}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{2a^2\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a^2\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

output
```
-(b/(a^2*f*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) + (Sqrt[Cos[e + f*x
]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f
*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{b\tan(fx+e)}\left(i(1+\cos(fx+e))\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\mathrm{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)-\cot(fx+e)\right)}{f\sqrt{a\sin(fx+e)}\,a^2}$ | 100 |

input
```
int((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)/a^2*(I*(1+cos(f*x+e))*(1/(1+
cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-csc(f*x
+e)+cot(f*x+e)),I)-cot(f*x+e))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.57

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\sqrt{-ab}\left(\cos(fx+e)^2-1\right)\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(f}{}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
(sqrt(1/2)*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*weierstrassPInverse(-4, 0, cos(
f*x + e) + I*sin(f*x + e)) + sqrt(1/2)*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*wei
erstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + sqrt(a*sin(f*x +
e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/(a^3*f*cos(f*x + e)^2
- a^3*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{(a\sin(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

## Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{(a\sin(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}}{\sin(fx+e)^3}dx\right)}{a^3}$$

input
```
int((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x)))/sin(e + f*x)*
*3,x))/a**3
```

# 3.120 $\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal result . . . 1006
Mathematica [C] (verified) . . . 1006
Rubi [A] (verified) . . . 1007
Maple [C] (verified) . . . 1009
Fricas [C] (verification not implemented) . . . 1010
Sympy [F(-1)] . . . 1011
Maxima [F] . . . 1011
Giac [F(-2)] . . . 1011
Mupad [F(-1)] . . . 1012
Reduce [F] . . . 1012

## Optimal result

Integrand size = 25, antiderivative size = 126

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx=-\frac{24a^2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f}-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

output
```
-24/5*a^2*b^2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f
/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)+12/5*a^2*b*(a*sin(f*x+e))^(1/2)*(b*
tan(f*x+e))^(1/2)/f-2/5*b*(a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2)/f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.97 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx=\frac{a^2b\left(\cos^2(e+fx)^{3/4}(11+\cos(2(e+fx)))-12\cos^2(e+fx)\,\text{Hypergeometr}\right.}{5f\cos^2(e+fx)}$$

input
```
Integrate[(a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(a^2*b*((Cos[e + f*x]^2)^(3/4)*(11 + Cos[2*(e + f*x)]) - 12*Cos[e + f*x]^2
*Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sq
rt[b*Tan[e + f*x]])/(5*f*(Cos[e + f*x]^2)^(3/4))
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3078, 3042, 3074, 3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{6}{5}a^2\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}dx-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}a^2\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}dx-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

$$\Big\downarrow \text{3074}$$

$$\frac{6}{5}a^2\left(\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-2b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx\right)-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}a^2\left(\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-2b^2\int\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx\right)-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

$\Big\downarrow$ 3081

$$\frac{6}{5}a^2\left(\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{2b^2\sqrt{a\sin(e+fx)}\int\sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

$\Big\downarrow$ 3042

$$\frac{6}{5}a^2\left(\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{2b^2\sqrt{a\sin(e+fx)}\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

$\Big\downarrow$ 3119

$$\frac{6}{5}a^2\left(\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

input

```
Int[(a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

output

```
(-2*b*(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]])/(5*f) + (6*a^2*((-4*b^2
*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*x]]*Sqr
t[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])/f))/5
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.81

| method | result |
|---|---|
| default | $\frac{2\sqrt{a\sin(fx+e)}\,\sqrt{b\tan(fx+e)}\,a^2b\left(\cos(fx+e)^3+\cos(fx+e)^2-7\cos(fx+e)+5+i\csc(fx+e)\left(12\cos(fx+e)^2+24\cos(fx+e)+12\right)\mathrm{E}\right.}{}$ |

input
```
int((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/f*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)*a^2*b/(1+cos(f*x+e))*(cos(
f*x+e)^3+cos(f*x+e)^2-7*cos(f*x+e)+5+I*csc(f*x+e)*(12*cos(f*x+e)^2+24*cos(
f*x+e)+12)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(1+cos(f*x+e)))^(1/2)
*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+I*csc(f*x+e)*(-12*cos(f*x+e)^2-24*cos(f
*x+e)-12)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(1+cos(f*x+e)))^(1/2)*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.01

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \frac{2\left(12\sqrt{\frac{1}{2}}\sqrt{-ab}a^2b\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(f$$

input
```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/5*(12*sqrt(1/2)*sqrt(-a*b)*a^2*b*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 12*sqrt(1/2)*sqrt(-a*b)*a^2*b
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*
x + e))) + (a^2*b*cos(f*x + e)^2 + 5*a^2*b)*sqrt(a*sin(f*x + e))*sqrt(b*si
n(f*x + e)/cos(f*x + e)))/f
```

## Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(5/2)*(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (a\sin(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (a\,\sin(e+f\,x))^{5/2}\,(b\tan(e+f\,x))^{3/2}\,dx$$

input
```
int((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \sqrt{b}\,\sqrt{a}\left(\int \sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)^2\tan(fx+e)\,dx\right)a^2b$$

input
```
int((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x)
```

output
```
sqrt(b)*sqrt(a)*int(sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x)**2*
tan(e + f*x),x)*a**2*b
```

# 3.121 $\int (a \sin(e+fx))^{3/2}(b \tan(e+fx))^{3/2}\,dx$

Optimal result . . . 1013
Mathematica [A] (verified) . . . 1013
Rubi [A] (verified) . . . 1014
Maple [B] (verified) . . . 1015
Fricas [A] (verification not implemented) . . . 1016
Sympy [F(-1)] . . . 1016
Maxima [F] . . . 1017
Giac [F(-2)] . . . 1017
Mupad [B] (verification not implemented) . . . 1017
Reduce [F] . . . 1018

## Optimal result

Integrand size = 25, antiderivative size = 68

$$\int (a \sin(e+fx))^{3/2}(b \tan(e+fx))^{3/2}\,dx = \frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}} - \frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

output
```
8/3*a^2*b*(b*tan(f*x+e))^(1/2)/f/(a*sin(f*x+e))^(1/2)-2/3*b*(a*sin(f*x+e))
^(3/2)*(b*tan(f*x+e))^(1/2)/f
```

## Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

$$\int (a \sin(e+fx))^{3/2}(b \tan(e+fx))^{3/2}\,dx = \frac{a^2b(7+\cos(2(e+fx)))\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(a^2*b*(7 + Cos[2*(e + f*x)])*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*
x]])
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{4}{3}a^2\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a^2\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

$$\Big\downarrow \text{3069}$$

$$\frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

input
```
Int[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(8*a^2*b*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x]]) - (2*b*(a*Sin[e
+ f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(3*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(56) = 112$.

Time = 2.20 (sec) , antiderivative size = 285, normalized size of antiderivative = 4.19

| method | result |
|---|---|
| default | $\left(\frac{\left(\frac{3\cos(fx+e)}{2}+\frac{3}{2}\right)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+2\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}-\cos(fx+e)+1}{1+\cos(fx+e)}\right)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}}{6}+\frac{\left(-\frac{3\cos(fx+e)}{2}-\frac{3}{2}\right)\cdots}{\cdots}\right.$ |

input
```
int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/6*(3/2*cos(f*x+e)+3/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+
e)))*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+1/6*(-3/2*cos(f*x+e)-3/2)*ln(2*(
2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+e)))*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)+4/3*cos(1/2*f*x+1/2*e)^4-2/3*cos(f*x+e)+2/3)*sec(1/2*f*x+1/2*e)*cs
c(1/2*f*x+1/2*e)*(a*sin(f*x+e))^(1/2)*a*b*(b*sin(f*x+e)/cos(f*x+e))^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{2\left(ab\cos\left(fx+e\right)^{2}+3\,ab\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{3\,f\sin\left(fx+e\right)}$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a*b*cos(f*x + e)^2 + 3*a*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/
cos(f*x + e))/(f*sin(f*x + e))
```

## Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx=\int (a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

## Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx=\frac{a\,b\,(13\sin(e+f\,x)+\sin(3\,e+3\,f\,x))\,\sqrt{a\sin(e+f\,x)}\,\sqrt{\frac{b\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{6\,f\sin(e+f\,x)^2}$$

input
```
int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(3/2),x)
```

output
```
(a*b*(13*sin(e + f*x) + sin(3*e + 3*f*x))*(a*sin(e + f*x))^(1/2)*((b*sin(2
*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(6*f*sin(e + f*x)^2)
```

## Reduce [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx=\sqrt{b}\,\sqrt{a}\left(\int \sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)\tan(fx+e)\,dx\right)ab$$

input
```
int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x)
```

output
```
sqrt(b)*sqrt(a)*int(sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x)*tan
(e + f*x),x)*a*b
```

# 3.122 $\int \sqrt{a \sin(e+fx)}(b \tan(e+fx))^{3/2}\,dx$


Optimal result . . . 1019
Mathematica [C] (verified) . . . 1019
Rubi [A] (verified) . . . 1020
Maple [C] (verified) . . . 1022
Fricas [C] (verification not implemented) . . . 1022
Sympy [F(-1)] . . . 1023
Maxima [F] . . . 1023
Giac [F] . . . 1023
Mupad [F(-1)] . . . 1024
Reduce [F] . . . 1024


## Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \sqrt{a \sin(e+fx)}(b \tan(e+fx))^{3/2}\,dx = -\frac{4b^2 E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a \sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b \tan(e+fx)}}+\frac{2b\sqrt{a \sin(e+fx)}\sqrt{b \tan(e+fx)}}{f}$$

output
```
-4*b^2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*
x+e)^(1/2)/(b*tan(f*x+e))^(1/2)+2*b*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1
/2)/f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.67 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \sqrt{a \sin(e+fx)}(b \tan(e+fx))^{3/2}\,dx = \frac{2b\left(\cos^2(e+fx)^{3/4}-\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\right)\sqrt{a \sin(e+f}}{f\cos^2(e+fx)^{3/4}}$$

input
```
Integrate[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*b*((Cos[e + f*x]^2)^(3/4) - Cos[e + f*x]^2*Hypergeometric2F1[1/4, 1/2,
3/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(f*(Cos[e
 + f*x]^2)^(3/4))
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3074, 3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3074}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-2b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-2b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{2b^2\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{2b^2\sqrt{a\sin(e+fx)}\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-4*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*
x]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]
)/f
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.71 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.48

| method | result |
|---|---|
| default | $-\frac{\sqrt{a\sin(fx+e)}\left(2i\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\text{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)-2i\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\text{Ellipt}\right.}{f(1-}$ |

input
```
int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(a*sin(f*x+e))^(1/2)*(2*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)-2*I*(1/(1+cos(f*x+e
)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*
x+e)),I)+csc(f*x+e)^3*(1-cos(f*x+e))^3)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*
tan(f*x+e)*b*(b*tan(f*x+e))^(1/2)/(1-cos(f*x+e))^2*sin(f*x+e)^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \frac{2\left(2\sqrt{\frac{1}{2}}\sqrt{-ab}b\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+\right.}{}$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2*(2*sqrt(1/2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(f*x + e) + I*sin(f*x + e))) + 2*sqrt(1/2)*sqrt(-a*b)*b*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + s
qrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))/f
```

## Sympy [F(-1)]

Timed out.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int \sqrt{a\sin(fx+e)}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int \sqrt{a\sin(fx+e)}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx=\int \sqrt{a\,\sin\left(e+f\,x\right)}\,{\left(b\tan(e+f\,x)\right)}^{3/2}\,dx$$

input
```
int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx=\frac{\sqrt{b}\,\sqrt{a}\,b\left(2\sqrt{\tan\left(fx+e\right)}\,\sqrt{\sin\left(fx+e\right)}-\left(\int\frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\cos(fx+e)}{\sin(fx+e)}dx\right)f-\left(\int\frac{\sqrt{\tan(\ldots}}{\ldots}\right.\ldots}{f}$$

input
```
int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*b*(2*sqrt(tan(e + f*x))*sqrt(sin(e + f*x)) - int((sqrt(ta
n(e + f*x))*sqrt(sin(e + f*x))*cos(e + f*x))/sin(e + f*x),x)*f - int((sqrt
(tan(e + f*x))*sqrt(sin(e + f*x)))/tan(e + f*x),x)*f))/f
```

# 3.123 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx$


Optimal result . . . . . 1025
Mathematica [A] (verified) . . . . . 1025
Rubi [A] (verified) . . . . . 1026
Maple [B] (verified) . . . . . 1027
Fricas [A] (verification not implemented) . . . . . 1027
Sympy [F(-1)] . . . . . 1028
Maxima [F] . . . . . 1028
Giac [F] . . . . . 1028
Mupad [B] (verification not implemented) . . . . . 1029
Reduce [F] . . . . . 1029


## Optimal result

Integrand size = 25, antiderivative size = 30

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

output
```
2*b*(b*tan(f*x+e))^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/Sqrt[a*Sin[e + f*x]],x]
```

output
```
(2*b*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[a*Sin[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3069}$$

$$\frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/Sqrt[a*Sin[e + f*x]],x]
```

output
```
(2*b*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[a*Sin[e + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(26) = 52$.

Time = 2.18 (sec) , antiderivative size = 253, normalized size of antiderivative = 8.43

| method | result |
|---|---|
| default | $\frac{\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\,b\left(8\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+2\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}-\cos(fx+e)+1}{1+\cos(fx+e)}\right)\left(-2+\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right.}{4f\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}\sqrt{a\sin(fx+}}$ |

input
```
int((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*(b*sin(f*x+e)/cos(f*x+e))^(1/2)*b/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)/(a*sin(f*x+e))^(1/2)*(8*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+ln((2*cos(
f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e))^
2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+e)))*(-2+sec(1/2*f*x+1/2*e)^2)+ln(2*(2*c
os(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+e)))*(2-sec(1/2*f*x+1/2*e)^2))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\sqrt{a\sin(fx+e)}\,b\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{af\sin(fx+e)}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e))/(a*f*sin(f*x +
e))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/sqrt(a*sin(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/sqrt(a*sin(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\,b\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{2\cos(e+f\,x)^2}}}{f\,\sqrt{a\,\sin\left(e+f\,x\right)}}$$

input
```
int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(1/2),x)
```

output
```
(2*b*((b*sin(2*e + 2*f*x))/(2*cos(e + f*x)^2))^(1/2))/(f*(a*sin(e + f*x))^
(1/2))
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\tan(fx+e)}{\sin(fx+e)}dx\right)b}{a}$$

input
```
int((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*tan(e + f*x))/
sin(e + f*x),x)*b)/a
```

# 3.124 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . 1030
Mathematica [C] (verified) . . . . . . . . . . 1030
Rubi [A] (verified) . . . . . . . . . . 1031
Maple [C] (verified) . . . . . . . . . . 1033
Fricas [C] (verification not implemented) . . . . . . . . . . 1033
Sympy [F(-1)] . . . . . . . . . . 1034
Maxima [F] . . . . . . . . . . 1034
Giac [F(-2)] . . . . . . . . . . 1034
Mupad [F(-1)] . . . . . . . . . . 1035
Reduce [F] . . . . . . . . . . 1035

## Optimal result

Integrand size = 25, antiderivative size = 90

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}$$

output
```
-2*b^2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/a^2/f/co
s(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)+2*b*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e)
)^(1/2)/a^2/f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.70 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \frac{\left(2\cos(e+fx)\cos^2(e+fx)^{3/4}-\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2\right.\right.}{af\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(3/2),x]
```

output
```
((2*Cos[e + f*x]*(Cos[e + f*x]^2)^(3/4) - Cos[e + f*x]^3*Hypergeometric2F1
[1/4, 1/2, 3/2, Sin[e + f*x]^2])*(b*Tan[e + f*x])^(3/2))/(a*f*(Cos[e + f*x
]^2)^(3/4)*Sqrt[a*Sin[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3073, 3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3073}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{a^2}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{b^2\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}dx}{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{b^2\sqrt{a\sin(e+fx)}\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(3/2),x]
```

output
```
(-2*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(a^2*f*Sqrt[Cos[e
+ f*x]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f
*x]])/(a^2*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.19

| method | result |
|---|---|
| default | $-\frac{2\left(i\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\operatorname{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)+i\left(-\cos(fx+e)^2-2\right.}{f(1+\cos(fx+e))a\sqrt{a}}$ |

input
```
int((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f*(I*(cos(f*x+e)^2+2*cos(f*x+e)+1)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-csc(f*x+e)+cot(f*x+e)),I)+I*(-cos(f*x
+e)^2-2*cos(f*x+e)-1)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*EllipticE(I*(-csc(f*x+e)+cot(f*x+e)),I)-sin(f*x+e))*b*(b*tan(f*x+e)
)^(1/2)/(1+cos(f*x+e))/a/(a*sin(f*x+e))^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \frac{2\left(\sqrt{\frac{1}{2}}\sqrt{-ab}b\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+\right.}{}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2*(sqrt(1/2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(f*x + e) + I*sin(f*x + e))) + sqrt(1/2)*sqrt(-a*b)*b*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + sqrt(
a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^2*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx=\int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(3/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{3/2}}{(a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\tan(fx+e)}{\sin(fx+e)^2}dx\right)b}{a^2}$$

input
```
int((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*tan(e + f*x))/
sin(e + f*x)**2,x)*b)/a**2
```

# 3.125 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1036
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1036
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1037
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1040
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . 1041
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1041
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1042
Giac [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1042
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1042
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1043

## Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{b^2\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{b^2\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

output
```
b^2*arctan(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f/cos(f*x+e)^(1/2)/(
b*tan(f*x+e))^(1/2)-b^2*arctanh(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3
/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)+2*b*(b*tan(f*x+e))^(1/2)/a^2/f/(a
*sin(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.72

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{b\left(\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)\cos^2(e+fx) - \text{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)\cos^2(e+\right.}{a^2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(5/2),x]
```

output
```
(b*(ArcTan[(Cos[e + f*x]^2)^(1/4)]*Cos[e + f*x]^2 - ArcTanh[(Cos[e + f*x]^
2)^(1/4)]*Cos[e + f*x]^2 + 2*(Cos[e + f*x]^2)^(3/4))*Sqrt[b*Tan[e + f*x]])
/(a^2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3073, 3042, 3081, 27, 3042, 3045, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3073}$$

$$\frac{b^2\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}dx}{a^2}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}dx}{a^2}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{b^2\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}dx}{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}\csc(e+fx)dx}{a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}}{\sin(e+fx)}dx}{a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3045}$$

$$\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{b^2\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}}{1-\cos^2(e+fx)}d\cos(e+fx)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{2b^2\sqrt{a\sin(e+fx)}\int \frac{\cos(e+fx)}{1-\cos^2(e+fx)}d\sqrt{\cos(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{2b^2\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\int \frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}-\frac{1}{2}\int \frac{1}{\cos(e+fx)+1}d\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{2b^2\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\int \frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}-\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{2b^2\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(5/2),x]
```

output
```
(-2*b^2*(-1/2*ArcTan[Sqrt[Cos[e + f*x]]] + ArcTanh[Sqrt[Cos[e + f*x]]]/2)*
Sqrt[a*Sin[e + f*x]])/(a^3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]) + (2
*b*Sqrt[b*Tan[e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x]])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

## Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.44

| method | result |
|---|---|
| default | $\frac{\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\, b \left(8\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}}\right)\left(2-\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)+\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+4\sqrt{\cdots}}{1+\cos(\cdots}\right.\right.}{4f\,a^2\sqrt{a\sin(fx+e)}\,\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}}$ |

input
```
int((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*(b*sin(f*x+e)/cos(f*x+e))^(1/2)*b/a^2/(a*sin(f*x+e))^(1/2)/(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)*(8*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+arctan
(1/2/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2))*(2-sec(1/2*f*x+1/2*e)^2)+ln(2*(
2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+e)))*(2-sec(1/2*f*x+1/2*e)^2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(125) = 250$.

Time = 0.40 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.61

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \left[\frac{2\,ab\sqrt{-\frac{b}{a}}\arctan\left(\frac{2\,\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{b}{a}}\cos(fx+e)}{(b\cos(fx+e)+b)\sin(fx+e)}\right)\sin(fx+e)+ab\sqrt{-\frac{b}{a}}\right.$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*sqrt(-b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/c
os(f*x + e))*sqrt(-b/a)*cos(f*x + e)/((b*cos(f*x + e) + b)*sin(f*x + e)))*
sin(f*x + e) + a*b*sqrt(-b/a)*log(-(b*cos(f*x + e)^3 - 4*sqrt(a*sin(f*x +
e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/a)*cos(f*x + e)*sin(f*x + e)
 - 5*b*cos(f*x + e)^2 - 5*b*cos(f*x + e) + b)/(cos(f*x + e)^3 + 3*cos(f*x
+ e)^2 + 3*cos(f*x + e) + 1))*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*b*sqrt
(b*sin(f*x + e)/cos(f*x + e)))/(a^3*f*sin(f*x + e)), -1/4*(2*a*b*sqrt(b/a)
*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/a)
*cos(f*x + e)/((b*cos(f*x + e) - b)*sin(f*x + e)))*sin(f*x + e) - a*b*sqrt
(b/a)*log((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*s
in(f*x + e)/cos(f*x + e))*sqrt(b/a) - (b*cos(f*x + e)^2 + 6*b*cos(f*x + e)
 + b)*sin(f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*
sin(f*x + e) - 8*sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))
/(a^3*f*sin(f*x + e))]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(a\sin(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{3/2}}{(a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\tan(fx+e)}{\sin(fx+e)^3}dx\right)b}{a^3}$$

input
```
int((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*tan(e + f*x))/
sin(e + f*x)**3,x)*b)/a**3
```

# 3.126 $\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx$


Optimal result . . . 1044
Mathematica [C] (verified) . . . 1044
Rubi [A] (verified) . . . 1045
Maple [C] (verified) . . . 1047
Fricas [C] (verification not implemented) . . . 1048
Sympy [F(-1)] . . . 1048
Maxima [F] . . . 1049
Giac [F] . . . 1049
Mupad [F(-1)] . . . 1049
Reduce [F] . . . 1050


## Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}} - \frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}} + \frac{8a^4E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{15f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
-4/15*a^2*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(3/2)-2/9*b*(a*sin(f*x+e
))^(9/2)/f/(b*tan(f*x+e))^(3/2)+8/15*a^4*EllipticE(sin(1/2*f*x+1/2*e),2^(1
/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.61 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{a^4\left(\cos^2(e+fx)^{3/4}(-17+5\cos(2(e+fx)))+12\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\text{s}\right.\right.}{90f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(9/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(a^4*((Cos[e + f*x]^2)^(3/4)*(-17 + 5*Cos[2*(e + f*x)]) + 12*Hypergeometri
c2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)]
)/(90*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3078, 3042, 3078, 3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{2}{3}a^2\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a^2\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{2}{3}a^2\left(\frac{2}{5}a^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a^2\left(\frac{2}{5}a^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2}{3}a^2\left(\frac{2a^2\sqrt{a\sin(e+fx)}\int\sqrt{\cos(e+fx)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a^2\left(\frac{2a^2\sqrt{a\sin(e+fx)}\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2}{3}a^2\left(\frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

input
```
Int[(a*Sin[e + f*x])^(9/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(9/2))/(9*f*(b*Tan[e + f*x])^(3/2)) + (2*a^2*((-2*b
*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(3/2)) + (4*a^2*EllipticE[(
e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e
+ f*x]])))/3
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.43 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.83

| method | result |
|---|---|
| default | $\frac{2\sqrt{a\sin(fx+e)}\,a^4\left(\sin(fx+e)\left(5\cos(fx+e)^4+5\cos(fx+e)^3-11\cos(fx+e)^2-11\cos(fx+e)+12\right)+12i\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\sqrt{\frac{1}{1+\cos(fx+\ldots}}}{}$ |

input
```
int((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/f*(a*sin(f*x+e))^(1/2)*a^4/(1+cos(f*x+e))/(b*tan(f*x+e))^(1/2)*(sin(f
*x+e)*(5*cos(f*x+e)^4+5*cos(f*x+e)^3-11*cos(f*x+e)^2-11*cos(f*x+e)+12)+12*
I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*(2+cos(f*x+e)
+sec(f*x+e))*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-12*I*(cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*Ellipt
icF(I*(csc(f*x+e)-cot(f*x+e)),I))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx =$$
$$-\frac{2\left(12\sqrt{\frac{1}{2}}\sqrt{-ab}a^4\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+12\right.}{}$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(12*sqrt(1/2)*sqrt(-a*b)*a^4*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 12*sqrt(1/2)*sqrt(-a*b)*a^4*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x
+ e))) - (5*a^4*cos(f*x + e)^4 - 11*a^4*cos(f*x + e)^2)*sqrt(a*sin(f*x + e
))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(b*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{9}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(9/2)/sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{9}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(9/2)/sqrt(b*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\,\sin(e+f\,x))^{9/2}}{\sqrt{b\tan(e+f\,x)}}\,dx$$

input
```
int((a*sin(e + f*x))^(9/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(9/2)/(b*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)^4}{\tan(fx+e)}dx\right)a^4}{b}$$

input
```
int((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x)**
4)/tan(e + f*x),x)*a**4)/b
```

# 3.127 $\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1051
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1051
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1052
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1053
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1054
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1054
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1054
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1055
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1055
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1055

## Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}} - \frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

output
```
-8/21*a^2*b*(a*sin(f*x+e))^(3/2)/f/(b*tan(f*x+e))^(3/2)-2/7*b*(a*sin(f*x+e
))^(7/2)/f/(b*tan(f*x+e))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{a^3\cos(e+fx)(-11+3\cos(2(e+fx)))\sqrt{a\sin(e+fx)}}{21f\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(a^3*Cos[e + f*x]*(-11 + 3*Cos[2*(e + f*x)])*Sqrt[a*Sin[e + f*x]])/(21*f*S
qrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{4}{7}a^2\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{7}a^2\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3069}$$

$$-\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

input
```
Int[(a*Sin[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-8*a^2*b*(a*Sin[e + f*x])^(3/2))/(21*f*(b*Tan[e + f*x])^(3/2)) - (2*b*(a*
Sin[e + f*x])^(7/2))/(7*f*(b*Tan[e + f*x])^(3/2))
```

**Defintions of rubi rules used**

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

## Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

| method | result | size |
|---|---|---|
| default | $\frac{2\sqrt{a\sin(fx+e)}\,a^{3}\left(3\cos(fx+e)^{3}-7\cos(fx+e)\right)}{21f\sqrt{b\tan(fx+e)}}$ | 48 |

input
```
int((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/f*(a*sin(f*x+e))^(1/2)*a^3/(b*tan(f*x+e))^(1/2)*(3*cos(f*x+e)^3-7*cos
(f*x+e))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2\,\left(3\,a^{3}\cos\left(fx+e\right)^{4}-7\,a^{3}\cos\left(fx+e\right)^{2}\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{21\,bf\sin\left(fx+e\right)}$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/21*(3*a^3*cos(f*x + e)^4 - 7*a^3*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sq
rt(b*sin(f*x + e)/cos(f*x + e))/(b*f*sin(f*x + e))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(7/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{7}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.29

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{a^3\sqrt{a\sin(e+fx)}\sqrt{-\frac{b\sin(2e+2fx)}{2\sin(e+fx)^2-2}}\,(22\sin(e+fx)+19\sin(3e+3fx)-3\sin(5e+5fx))}{168\,b\,f\sin(e+fx)^2}$$

input
```
int((a*sin(e + f*x))^(7/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
-(a^3*(a*sin(e + f*x))^(1/2)*(-(b*sin(2*e + 2*f*x))/(2*sin(e + f*x)^2 - 2)
)^(1/2)*(22*sin(e + f*x) + 19*sin(3*e + 3*f*x) - 3*sin(5*e + 5*f*x)))/(168
*b*f*sin(e + f*x)^2)
```

## Reduce [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)^3}{\tan(fx+e)}dx\right)a^3}{b}$$

input
```
int((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x)**
3)/tan(e + f*x),x)*a**3)/b
```

# 3.128 $\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$


Optimal result . . . 1057
Mathematica [C] (verified) . . . 1057
Rubi [A] (verified) . . . 1058
Maple [C] (verified) . . . 1060
Fricas [C] (verification not implemented) . . . 1060
Sympy [F(-1)] . . . 1061
Maxima [F] . . . 1061
Giac [F] . . . 1061
Mupad [F(-1)] . . . 1062
Reduce [F] . . . 1062


## Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}} + \frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
-2/5*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(3/2)+4/5*a^2*EllipticE(sin(1
/2*f*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+
e))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.65 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{a^2\left(\cos^2(e+fx)^{3/4} - \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\right)\sqrt{a\sin(e+fx)}\sin(2(e+fx))}{5f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
-1/5*(a^2*((Cos[e + f*x]^2)^(3/4) - Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e
 + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)])/(f*(Cos[e + f*x]^2)^(3/
4)*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3078, 3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{2}{5}a^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}a^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2a^2\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\sqrt{a\sin(e+fx)}\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

input
```
Int[(a*Sin[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(3/2)) + (4*a^2*Ellipt
icE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*T
an[e + f*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.03 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.28

| method | result |
|---|---|
| default | $-\frac{2\sqrt{a\sin(fx+e)}\,a^2\left(\sin(fx+e)\left(\cos(fx+e)^2+\cos(fx+e)-2\right)+2i\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\sqrt{\frac{1}{1+\cos(fx+e)}}\,(2+\cos(fx+e)+\sec(fx+e))\,\text{Ellip}}{5f(1+\cos(f}$ |

input
```
int((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/f*(a*sin(f*x+e))^(1/2)*a^2/(1+cos(f*x+e))/(b*tan(f*x+e))^(1/2)*(sin(f
*x+e)*(cos(f*x+e)^2+cos(f*x+e)-2)+2*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*EllipticF(I*(csc(f*x+e)-c
ot(f*x+e)),I)-2*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*(2+cos(f*x+e)+sec(f*x+e))*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx =$$
$$-\frac{2\left(\sqrt{a\sin(fx+e)}a^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)^2+2\sqrt{\frac{1}{2}}\sqrt{-ab}a^2\text{weierstrassZeta}(-4,0,\text{weierstrassPInvers}}{}$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(sqrt(a*sin(f*x + e))*a^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x +
 e)^2 + 2*sqrt(1/2)*sqrt(-a*b)*a^2*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 2*sqrt(1/2)*sqrt(-a*b)*a^2*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x +
 e))))/(b*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\ \sin\left(e+f\,x\right))^{5/2}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int((a*sin(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)^2}{\tan(fx+e)}dx\right)a^2}{b}$$

input
```
int((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x)**
2)/tan(e + f*x),x)*a**2)/b
```

# 3.129 $\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result . . . 1063
Mathematica [A] (verified) . . . 1063
Rubi [A] (verified) . . . 1064
Maple [A] (verified) . . . 1065
Fricas [B] (verification not implemented) . . . 1065
Sympy [F(-1)] . . . 1065
Maxima [F] . . . 1066
Giac [F] . . . 1066
Mupad [B] (verification not implemented) . . . 1066
Reduce [F] . . . 1067

## Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

output
```
-2/3*b*(a*sin(f*x+e))^(3/2)/f/(b*tan(f*x+e))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

input
```
Integrate[(a*Sin[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*(b*Tan[e + f*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3069}$$

$$-\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

input
```
Int[(a*Sin[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*(b*Tan[e + f*x])^(3/2))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

## Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

| method | result | size |
|---|---|---|
| default | $-\frac{2\sqrt{a\sin(fx+e)}\, a\cos(fx+e)}{3f\sqrt{b\tan(fx+e)}}$ | 33 |

input
```
int((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/f*(a*sin(f*x+e))^(1/2)*a*cos(f*x+e)/(b*tan(f*x+e))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(26) = 52.

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2\sqrt{a\sin(fx+e)}a\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)^2}{3\,bf\sin(fx+e)}$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)
^2/(b*f*sin(f*x + e))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.16

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{a\sqrt{a\sin(e+fx)}\,(\sin(e+fx)+\sin(3e+3fx))\,\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}{6\,b\,f\,\sin(e+fx)^2}$$

input
```
int((a*sin(e + f*x))^(3/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
-(a*(a*sin(e + f*x))^(1/2)*(sin(e + f*x) + sin(3*e + 3*f*x))*((b*sin(2*e +
 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(6*b*f*sin(e + f*x)^2)
```

## Reduce [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)}{\tan(fx+e)}dx\right)a}{b}$$

input
```
int((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x))/
tan(e + f*x),x)*a)/b
```

# 3.130 $\int \frac{\sqrt{a \sin(e+fx)}}{\sqrt{b \tan(e+fx)}}\, dx$

Optimal result . . . 1068
Mathematica [C] (verified) . . . 1068
Rubi [A] (verified) . . . 1069
Maple [C] (verified) . . . 1070
Fricas [C] (verification not implemented) . . . 1071
Sympy [F] . . . 1071
Maxima [F] . . . 1072
Giac [F] . . . 1072
Mupad [F(-1)] . . . 1072
Reduce [F] . . . 1073

## Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{\sqrt{a \sin(e+fx)}}{\sqrt{b \tan(e+fx)}}\, dx = \frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a \sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b \tan(e+fx)}}$$

output
```
2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^
(1/2)/(b*tan(f*x+e))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{a \sin(e+fx)}}{\sqrt{b \tan(e+fx)}}\, dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sqrt{a \sin(e+fx)}\sin(2(e+fx))}{2f\cos^2(e+fx)^{3/4}\sqrt{b \tan(e+fx)}}$$

input
```
Integrate[Sqrt[a*Sin[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*Sin
[2*(e + f*x)])/(2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}(e+fx)|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[Sqrt[a*Sin[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*x]]*S
qrt[b*Tan[e + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.74

| method | result |
|---|---|
| default | $-\frac{2\sqrt{a\sin(fx+e)}\left(i\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\sqrt{\frac{1}{1+\cos(fx+e)}}\,(2+\cos(fx+e)+\sec(fx+e))\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)+i\sqrt{\frac{1}{1+\cos(f}}\right.}{f(1+\cos(fx+e))\sqrt{b\tan(fx-}}$ |
| risch | $-\frac{i\sqrt{2}\sqrt{-ia\left(e^{2i(fx+e)}-1\right)e^{-i(fx+e)}}}{f\sqrt{-\frac{ib\left(e^{2i(fx+e)}-1\right)}{e^{2i(fx+e)}+1}}}-\frac{i\left(-\frac{2\left(ab\,e^{2i(fx+e)}+ab\right)}{ab\sqrt{e^{i(fx+e)}\left(ab\,e^{2i(fx+e)}+ab\right)}}+\frac{i\sqrt{-i\left(e^{i(fx+e)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(fx+e)}-i\right)}\sqrt{ie^{i(fx+e)}}\left(\right.}{\sqrt{c}}\right.}{}$ |

input
```
int((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f*(a*sin(f*x+e))^(1/2)/(1+cos(f*x+e))/(b*tan(f*x+e))^(1/2)*(I*(1/(1+cos
(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(csc(f*x+e)-
cot(f*x+e)),I)*(2+cos(f*x+e)+sec(f*x+e))+I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(-cos(f
*x+e)-2-sec(f*x+e))-sin(f*x+e))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2\left(\sqrt{\frac{1}{2}}\sqrt{-ab}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+\sqrt{\frac{1}{2}}\sqrt{\phantom{x}}\right.}{bf}$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(1/2)*sqrt(-a*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(f*x + e) + I*sin(f*x + e))) + sqrt(1/2)*sqrt(-a*b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b*f)
```

## Sympy [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{a\sin\left(e+fx\right)}}{\sqrt{b\tan\left(e+fx\right)}}\,dx$$

input
```
integrate((a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*sin(e + f*x))/sqrt(b*tan(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{a\sin(fx+e)}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{a\sin(fx+e)}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sin(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{a\,\sin(e+f\,x)}}{\sqrt{b\tan(e+f\,x)}}\,dx$$

input
```
int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{\sqrt{a \sin(e+fx)}}{\sqrt{b \tan(e+fx)}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}}{\tan(fx+e)}dx\right)}{b}$$

input
```
int((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x)))/tan(e + f*x),
x))/b
```

# 3.131 $\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$

Optimal result . . . 1074
Mathematica [A] (verified) . . . 1074
Rubi [A] (verified) . . . 1075
Maple [A] (verified) . . . 1078
Fricas [B] (verification not implemented) . . . 1078
Sympy [F] . . . 1079
Maxima [F] . . . 1079
Giac [F] . . . 1080
Mupad [F(-1)] . . . 1080
Reduce [F] . . . 1080

## Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
arctan(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a/f/cos(f*x+e)^(1/2)/(b*tan(
f*x+e))^(1/2)-arctanh(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a/f/cos(f*x+e
)^(1/2)/(b*tan(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{\left(\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right) - \operatorname{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)\sin(2(e+fx))}{2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[1/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
((ArcTan[(Cos[e + f*x]^2)^(1/4)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)])*Sin[2*
(e + f*x)])/(2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e
+ f*x]])
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.69, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3081, 27, 3042, 3045, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}\csc(e+fx)dx}{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}}{\sin(e+fx)}dx}{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}}{1-\cos^2(e+fx)}d\cos(e+fx)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2\sqrt{a\sin(e+fx)}\int\frac{\cos(e+fx)}{1-\cos^2(e+fx)}d\sqrt{\cos(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{827}$$

$$-\frac{2\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}-\frac{1}{2}\int\frac{1}{\cos(e+fx)+1}d\sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}-\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[1/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(-2*(-1/2*ArcTan[Sqrt[Cos[e + f*x]]] + ArcTanh[Sqrt[Cos[e + f*x]]]/2)*Sqrt
[a*Sin[e + f*x]])/(a*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

## Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.48

| method | result |
|---|---|
| default | $-\frac{\left(\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}}\right)+\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+2\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}-\cos(fx+e)+1}{1+\cos(fx+e)}\right)\right)(-\csc(fx+e)+\cot(}{2f\sqrt{a\sin(fx+e)}\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}\sqrt{b\tan(fx+e)}}$ |

input
```
int(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*(arctan(1/2/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2))+ln((2*cos(f*x+e)*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-cos(f*x+e)+1)/(1+cos(f*x+e))))/(a*sin(f*x+e))^(1/2)/(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)/(b*tan(f*x+e))^(1/2)*(-csc(f*x+e)+cot(f*x+e))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(90) = 180.

Time = 0.28 (sec) , antiderivative size = 419, normalized size of antiderivative = 3.95

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$$

$$= \left[\frac{2\sqrt{-ab}\arctan\left(\frac{2\sqrt{-ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{(ab\cos(fx+e)+ab)\sin(fx+e)}\right)-\sqrt{-ab}\log\left(-\frac{ab\cos(fx+e)^3-5\,ab\cos(fx+e)^2+4\sqrt{-ab}\sqrt{a}}{\cos(fx+e)^3}\right.}{4\,abf}\right.$$

$$-\frac{2\sqrt{ab}\arctan\left(\frac{2\sqrt{ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{(ab\cos(fx+e)-ab)\sin(fx+e)}\right)-\sqrt{ab}\log\left(\frac{4\sqrt{ab}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin}{\cos}}}{\left(\cos(fx+e)^2-2\cos\right.}\right.}{4\,abf}$$

input
```
integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
[1/4*(2*sqrt(-a*b)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x
 + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) + a*b)*sin(f*x + e)))
- sqrt(-a*b)*log(-(a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b
)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) - 5*a*b*cos(f*x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 +
3*cos(f*x + e) + 1)))/(a*b*f), -1/4*(2*sqrt(a*b)*arctan(2*sqrt(a*b)*sqrt(a
*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*
x + e) - a*b)*sin(f*x + e))) - sqrt(a*b)*log((4*sqrt(a*b)*(cos(f*x + e)^2
+ cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)) - (
a*b*cos(f*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x + e))/((cos(f*x + e
)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e))))/(a*b*f)]
```

## Sympy [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(e+fx\right)}\sqrt{b\tan\left(e+fx\right)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*sin(e + f*x))*sqrt(b*tan(e + f*x))), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(1/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(1/2)), x)
```

## Reduce [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}}{\sin(fx+e)\tan(fx+e)}dx\right)}{ab}$$

input
```
int(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x)))/(sin(e + f*x)
*tan(e + f*x)),x))/(a*b)
```

# 3.132 $\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1081
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1081
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1082
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1084
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1084
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1085
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1085
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1085
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1086
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1086

## Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}} - \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
-b*(a*sin(f*x+e))^(1/2)/a^2/f/(b*tan(f*x+e))^(3/2)-EllipticE(sin(1/2*f*x+1
/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/a^2/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^
(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.74 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b\sqrt{a\sin(e+fx)}\left(2\cos^2(e+fx)^{3/4}+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin^2(e+fx)\right)}{2a^2f\cos^2(e+fx)^{3/4}(b\tan(e+fx))^{3/2}}$$

input
```
Integrate[1/((a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
-1/2*(b*Sqrt[a*Sin[e + f*x]]*(2*(Cos[e + f*x]^2)^(3/4) + Hypergeometric2F1
[1/4, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x]^2))/(a^2*f*(Cos[e + f*x]^2)^(
3/4)*(b*Tan[e + f*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3079, 3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3079}$$

$$-\frac{\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{2a^2}-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{2a^2}-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}dx}{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{a\sin(e+fx)}\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3119}$$

$$-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[1/((a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
-((b*Sqrt[a*Sin[e + f*x]])/(a^2*f*(b*Tan[e + f*x])^(3/2))) - (EllipticE[(e
 + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(a^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e
 + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.97

| method | result |
|---|---|
| default | $-\frac{i\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\sqrt{\frac{1}{1+\cos(fx+e)}}\,\mathrm{EllipticE}(i(\csc(fx+e)-\cot(fx+e)),i)(\sin(fx+e)+\tan(fx+e))+i\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\sqrt{\frac{1}{1+\cos(fx+e)}}}{f\sqrt{b\tan(fx+e)}\sqrt{a\sin(fx+e)}\,a}$ |

input
```
int(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)/a*(I*(cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I
)*(sin(f*x+e)+tan(f*x+e))+I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*
x+e)))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(-sin(f*x+e)-tan(f*x+e
))+1)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.67

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)^2+\sqrt{\frac{1}{2}}\sqrt{-ab}(\cos(fx}{}$$

input
```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
(sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)^2 + s
qrt(1/2)*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(1/2)*sqrt(-a*b)*(c
os(f*x + e)^2 - 1)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f
*x + e) - I*sin(f*x + e))))/(a^2*b*f*cos(f*x + e)^2 - a^2*b*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

## Giac [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\,\sin\left(e+f\,x\right))^{3/2}\,\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}}{\sin(fx+e)^2\tan(fx+e)}dx\right)}{a^2b}$$

input
```
int(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x)))/(sin(e + f*x)
**2*tan(e + f*x)),x))/(a**2*b)
```

# 3.133 $\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal result . . . . . 1087
Mathematica [A] (verified) . . . . . 1088
Rubi [A] (verified) . . . . . 1088
Maple [B] (verified) . . . . . 1091
Fricas [B] (verification not implemented) . . . . . 1092
Sympy [F(-1)] . . . . . 1093
Maxima [F] . . . . . 1093
Giac [F] . . . . . 1093
Mupad [F(-1)] . . . . . 1094
Reduce [F] . . . . . 1094

## Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}} + \frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
-1/2*b/a^2/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2)+1/4*arctan(cos(f*x+
e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)
-1/4*arctanh(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f/cos(f*x+e)^(1/2)
/(b*tan(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{-4\cos^2(e+fx)^{3/4}\cot(e+fx)+\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)\sin(\cdots}{8a^2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e\cdots}}$$

input
```
Integrate[1/((a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(-4*(Cos[e + f*x]^2)^(3/4)*Cot[e + f*x] + ArcTan[(Cos[e + f*x]^2)^(1/4)]*S
in[2*(e + f*x)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)]*Sin[2*(e + f*x)])/(8*a^2
*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3079, 3042, 3081, 27, 3042, 3045, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3079}$$

$$\frac{\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}dx}{4a^2} - \frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}dx}{4a^2} - \frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{a\sin(e+fx)}\int\frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}dx}{4a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a\sin(e+fx)}\int\sqrt{\cos(e+fx)}\csc(e+fx)dx}{4a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\sin(e+fx)}\int\frac{\sqrt{\cos(e+fx)}}{\sin(e+fx)}dx}{4a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\sqrt{a\sin(e+fx)}\int\frac{\sqrt{\cos(e+fx)}}{1-\cos^2(e+fx)}d\cos(e+fx)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{\sqrt{a\sin(e+fx)}\int\frac{\cos(e+fx)}{1-\cos^2(e+fx)}d\sqrt{\cos(e+fx)}}{2a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{827}$$

$$-\frac{\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}-\frac{1}{2}\int\frac{1}{\cos(e+fx)+1}d\sqrt{\cos(e+fx)}\right)}{2a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}-\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{2a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{2a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

input
```
Int[1/((a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
-1/2*b/(a^2*f*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)) - ((-1/2*ArcTan
[Sqrt[Cos[e + f*x]]] + ArcTanh[Sqrt[Cos[e + f*x]]]/2)*Sqrt[a*Sin[e + f*x]]
)/(2*a^3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(120) = 240$.

Time = 0.87 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.01

| method | result |
|---|---|
| default | $\frac{\csc(fx+e)^3(1-\cos(fx+e))\left(\csc(fx+e)^2\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+2\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}-\cos(fx+e)+1}{1+\cos(fx+e)}\right)(1-\cos(fx+e))^2+}{2f\,a^2\sqrt{a\sin(fx+e)}\left(\csc(fx+e)^2(1-\cos(f\right.}$ |

input
```
int(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*csc(f*x+e)^3/a^2/(a*sin(f*x+e))^(1/2)*(1-cos(f*x+e))/(csc(f*x+e)^2*(
1-cos(f*x+e))^2+1)^2*(csc(f*x+e)^2*ln((2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)+2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+cos
(f*x+e)))*(1-cos(f*x+e))^2+csc(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2))*(1-cos(f*x+e))^2+2*csc(f*x+e)^2*(1-cos(f*x+e))^2*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(1/2)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2))/(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)/(b*tan(f*x+e))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(120) = 240$.

Time = 0.32 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.14

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
[1/16*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*
x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e)
+ a*b)*sin(f*x + e)))*sin(f*x + e) - sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-
(a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*sqrt(a*sin(f*x +
 e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 5*a*b*c
os(f*x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) + 1
))*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))
*cos(f*x + e)^2)/((a^3*b*f*cos(f*x + e)^2 - a^3*b*f)*sin(f*x + e)), -1/16*
(2*sqrt(a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(a*b)*sqrt(a*sin(f*x + e))*
sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) - a*b)*s
in(f*x + e)))*sin(f*x + e) - sqrt(a*b)*(cos(f*x + e)^2 - 1)*log((4*sqrt(a*
b)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e
)/cos(f*x + e)) - (a*b*cos(f*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x
+ e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x + e) -
 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)^2)/
((a^3*b*f*cos(f*x + e)^2 - a^3*b*f)*sin(f*x + e))]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

## Giac [F]

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\left(a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(1/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(1/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}}{\sin(fx+e)^3\tan(fx+e)}dx\right)}{a^3b}$$

input
```
int(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x)))/(sin(e + f*x)
**3*tan(e + f*x)),x))/(a**3*b)
```

# 3.134 $\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result . . . 1095
Mathematica [A] (verified) . . . 1095
Rubi [A] (verified) . . . 1096
Maple [A] (verified) . . . 1098
Fricas [A] (verification not implemented) . . . 1098
Sympy [F(-1)] . . . 1099
Maxima [F] . . . 1099
Giac [F(-1)] . . . 1100
Mupad [B] (verification not implemented) . . . 1100
Reduce [F] . . . 1101

## Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{64a^6\sqrt{a\sin(e+fx)}}{585bf\sqrt{b\tan(e+fx)}} - \frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

output
```
-64/585*a^6*(a*sin(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)-16/585*a^4*(a*si
n(f*x+e))^(5/2)/b/f/(b*tan(f*x+e))^(1/2)-2/117*a^2*(a*sin(f*x+e))^(9/2)/b/
f/(b*tan(f*x+e))^(1/2)+2/13*(a*sin(f*x+e))^(13/2)/b/f/(b*tan(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.46

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^6\cos^2(e+fx)(-551+340\cos(2(e+fx))-45\cos(4(e+fx)))\sqrt{a\sin(e+f}}{2340bf\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(13/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(a^6*Cos[e + f*x]^2*(-551 + 340*Cos[2*(e + f*x)] - 45*Cos[4*(e + f*x)])*Sq
rt[a*Sin[e + f*x]])/(2340*b*f*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3076, 3042, 3078, 3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{a^2\int (a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}dx}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int (a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}dx}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{a^2\left(\frac{8}{9}a^2\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{9/2}}{9f\sqrt{b\tan(e+fx)}}\right)}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{8}{9}a^2\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{9/2}}{9f\sqrt{b\tan(e+fx)}}\right)}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{a^2\left(\frac{8}{9}a^2\left(\frac{4}{5}a^2\int\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f\sqrt{b\tan(e+fx)}}\right)}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{a^2\left(\frac{8}{9}a^2\left(\frac{4}{5}a^2\int\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f\sqrt{b\tan(e+fx)}}\right)}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

$\Big\downarrow$ 3069

$$\frac{a^2\left(\frac{8}{9}a^2\left(-\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f\sqrt{b\tan(e+fx)}}\right)}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(13/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(a*Sin[e + f*x])^(13/2))/(13*b*f*Sqrt[b*Tan[e + f*x]]) + (a^2*((-2*b*(a
*Sin[e + f*x])^(9/2))/(9*f*Sqrt[b*Tan[e + f*x]]) + (8*a^2*((-8*a^2*b*Sqrt[
a*Sin[e + f*x]])/(5*f*Sqrt[b*Tan[e + f*x]]) - (2*b*(a*Sin[e + f*x])^(5/2))
/(5*f*Sqrt[b*Tan[e + f*x]])))/9))/(13*b^2)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

## Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.43

| method | result | size |
|---|---|---|
| default | $-\frac{2\sqrt{a\sin(fx+e)}\, a^{6}\left(45\cos(fx+e)^{6}-130\cos(fx+e)^{4}+117\cos(fx+e)^{2}\right)}{585 f b\sqrt{b\tan(fx+e)}}$ | 63 |

input
```
int((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/585/f*(a*sin(f*x+e))^(1/2)*a^6/b/(b*tan(f*x+e))^(1/2)*(45*cos(f*x+e)^6-
130*cos(f*x+e)^4+117*cos(f*x+e)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.58

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(45\,a^{6}\cos\left(fx+e\right)^{7}-130\,a^{6}\cos\left(fx+e\right)^{5}+117\,a^{6}\cos\left(fx+e\right)^{3}\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{585\,b^{2}f\sin\left(fx+e\right)}$$

input
```
integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

output
```
-2/585*(45*a^6*cos(f*x + e)^7 - 130*a^6*cos(f*x + e)^5 + 117*a^6*cos(f*x +
 e)^3)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))/(b^2*f*sin(f
*x + e))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(13/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{13}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((a*sin(f*x + e))^(13/2)/(b*tan(f*x + e))^(3/2), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [B] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.03

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{(\cos(7e+7fx)-\sin(7e+7fx)\,1\mathrm{i})\,\sqrt{\frac{b\,(\sin(2e+2fx)-\cos(2e+2fx)\,1\mathrm{i}+1\mathrm{i})}{\cos(2e+2fx)+1+\sin(2e+2fx)\,1\mathrm{i}}}\,\left(\frac{a^6\,\cos(3\right.}{}$$

input
```
int((a*sin(e + f*x))^(13/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
((cos(7*e + 7*f*x) - sin(7*e + 7*f*x)*1i)*((b*(sin(2*e + 2*f*x) - cos(2*e
+ 2*f*x)*1i + 1i))/(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))^(1/2)*((a
^6*cos(3*e + 3*f*x)*(a*sin(e + f*x))^(1/2)*(cos(7*e + 7*f*x) + sin(7*e + 7
*f*x)*1i)*217i)/(9360*b^2*f) - (a^6*cos(5*e + 5*f*x)*(a*sin(e + f*x))^(1/2
)*(cos(7*e + 7*f*x) + sin(7*e + 7*f*x)*1i)*41i)/(1872*b^2*f) + (a^6*cos(7*
e + 7*f*x)*(a*sin(e + f*x))^(1/2)*(cos(7*e + 7*f*x) + sin(7*e + 7*f*x)*1i)
*1i)/(208*b^2*f) + (a^6*cos(e + f*x)*(a*sin(e + f*x))^(1/2)*(cos(7*e + 7*f
*x) + sin(7*e + 7*f*x)*1i)*1991i)/(9360*b^2*f))*1i)/(2*sin(e + f*x))
```

## Reduce [F]

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)^{6}}{\tan(fx+e)^{2}}dx\right)a^{6}}{b^{2}}$$

input
```
int((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x)**
6)/tan(e + f*x)**2,x)*a**6)/b**2
```

# 3.135 $\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1102
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1102
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1103
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1105
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . 1105
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1105
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1106
Giac [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1106
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 1106
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1107

## Optimal result

Integrand size = 25, antiderivative size = 109

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{8a^4\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

output
```
-8/45*a^4*(a*sin(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)-2/45*a^2*(a*sin(f*
x+e))^(5/2)/b/f/(b*tan(f*x+e))^(1/2)+2/9*(a*sin(f*x+e))^(9/2)/b/f/(b*tan(f
*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^4\cos^2(e+fx)(-13+5\cos(2(e+fx)))\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(9/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(a^4*Cos[e + f*x]^2*(-13 + 5*Cos[2*(e + f*x)])*Sqrt[a*Sin[e + f*x]])/(45*b
*f*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3076, 3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{a^2\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx}{9b^2}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx}{9b^2}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{a^2\left(\frac{4}{5}a^2\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\right)}{9b^2}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{4}{5}a^2\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\right)}{9b^2}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{a^2\left(-\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\right)}{9b^2}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(9/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(a*Sin[e + f*x])^(9/2))/(9*b*f*Sqrt[b*Tan[e + f*x]]) + (a^2*((-8*a^2*b*
Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[b*Tan[e + f*x]]) - (2*b*(a*Sin[e + f*x])^(
5/2))/(5*f*Sqrt[b*Tan[e + f*x]])))/(9*b^2)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

## Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.49

| method | result | size |
|---|---|---|
| default | $\frac{2\sqrt{a\sin(fx+e)}\, a^4 \left(5\cos(fx+e)^4-9\cos(fx+e)^2\right)}{45 f b\sqrt{b\tan(fx+e)}}$ | 53 |

input
```
int((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/f*(a*sin(f*x+e))^(1/2)*a^4/b/(b*tan(f*x+e))^(1/2)*(5*cos(f*x+e)^4-9*c
os(f*x+e)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.65

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(5\,a^4\cos\left(fx+e\right)^5-9\,a^4\cos\left(fx+e\right)^3\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{45\,b^2 f\sin\left(fx+e\right)}$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/45*(5*a^4*cos(f*x + e)^5 - 9*a^4*cos(f*x + e)^3)*sqrt(a*sin(f*x + e))*sq
rt(b*sin(f*x + e)/cos(f*x + e))/(b^2*f*sin(f*x + e))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{9}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(9/2)/(b*tan(f*x + e))^(3/2), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^4\sqrt{a\sin(e+fx)}\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}\,(47\sin(2e+2fx)+16\sin(4e+4fx)-}{360\,b^2 f\,(\cos(2e+2fx)-1)}$$

input
```
int((a*sin(e + f*x))^(9/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
(a^4*(a*sin(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^
(1/2)*(47*sin(2*e + 2*f*x) + 16*sin(4*e + 4*f*x) - 5*sin(6*e + 6*f*x)))/(3
60*b^2*f*(cos(2*e + 2*f*x) - 1))
```

## Reduce [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)^4}{\tan(fx+e)^2}dx\right)a^4}{b^2}$$

input
```
int((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x)**
4)/tan(e + f*x)**2,x)*a**4)/b**2
```

# 3.136 $\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$


Optimal result . . . 1108
Mathematica [A] (verified) . . . 1108
Rubi [A] (verified) . . . 1109
Maple [A] (verified) . . . 1110
Fricas [B] (verification not implemented) . . . 1110
Sympy [F(-1)] . . . 1110
Maxima [F] . . . 1111
Giac [F(-1)] . . . 1111
Mupad [B] (verification not implemented) . . . 1111
Reduce [F] . . . 1112


## Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

output
```
-2/5*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(5/2)
```

## Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2a^2\cos^2(e+fx)\sqrt{a\sin(e+fx)}}{5bf\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*a^2*Cos[e + f*x]^2*Sqrt[a*Sin[e + f*x]])/(5*b*f*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{3069}$$
$$-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

input
```
Int[(a*Sin[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(5/2))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

## Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

| method | result | size |
|---|---|---|
| default | $-\frac{2\sqrt{a\sin(fx+e)}\,a^{2}\cos(fx+e)^{2}}{5fb\sqrt{b\tan(fx+e)}}$ | 40 |

input
```
int((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/f*(a*sin(f*x+e))^(1/2)*a^2*cos(f*x+e)^2/b/(b*tan(f*x+e))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{a\sin(fx+e)}a^{2}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)^{3}}{5\,b^{2}f\sin(fx+e)}$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(a*sin(f*x + e))*a^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x +
e)^3/(b^2*f*sin(f*x + e))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.53

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^2\,\sqrt{a\,\sin(e+f\,x)}\,(2\,\sin(2\,e+2\,f\,x)+\sin(4\,e+4\,f\,x))\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{10\,b^2\,f\,\left(\cos(2\,e+2\,f\,x)-1\right)}$$

input
```
int((a*sin(e + f*x))^(5/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
(a^2*(a*sin(e + f*x))^(1/2)*(2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*((b*si
n(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(10*b^2*f*(cos(2*e + 2*f*x)
 - 1))
```

## Reduce [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)^2}{\tan(fx+e)^2}dx\right)a^2}{b^2}$$

input
```
int((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x)**
2)/tan(e + f*x)**2,x)*a**2)/b**2
```

# 3.137 $\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result . . . 1113
Mathematica [A] (verified) . . . 1114
Rubi [A] (verified) . . . 1114
Maple [A] (verified) . . . 1117
Fricas [B] (verification not implemented) . . . 1118
Sympy [F] . . . 1119
Maxima [F] . . . 1119
Giac [F] . . . 1119
Mupad [F(-1)] . . . 1120
Reduce [F] . . . 1120

## Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{a\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}} - \frac{a\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}$$

output
```
2*(a*sin(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)-a*arctan(cos(f*x+e)^(1/2))
*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)-a*arctan
h(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x
+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\left(-\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)-\operatorname{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)+2\sqrt[4]{\cos^2(e+fx)}\right)}{bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[Sqrt[a*Sin[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

output
```
((-ArcTan[(Cos[e + f*x]^2)^(1/4)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)] + 2*(C
os[e + f*x]^2)^(1/4))*Sqrt[a*Sin[e + f*x]])/(b*f*(Cos[e + f*x]^2)^(1/4)*Sq
rt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3075, 3042, 3081, 27, 3042, 3045, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3075}$$

$$\frac{a^2\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}dx}{b^2}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}dx}{b^2}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}dx}{b^2\sqrt{a\sin(e+fx)}}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}dx}{b^2\sqrt{a\sin(e+fx)}}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}\sin(e+fx)}dx}{b^2\sqrt{a\sin(e+fx)}}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3045}$$

$$\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}(1-\cos^2(e+fx))}d\cos(e+fx)}{b^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{2a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{1-\cos^2(e+fx)}d\sqrt{\cos(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{756}$$

$$\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{2a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}+\frac{1}{2}\int\frac{1}{\cos(e+fx)+1}d\sqrt{\cos(e+fx)}\right)}{b^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{2a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}+\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{b^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{2a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\right)}{b^2f\sqrt{a\sin(e+fx)}}$$

input
```
Int[Sqrt[a*Sin[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*Sqrt[a*Sin[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]) - (2*a*(ArcTan[Sqrt[Co
s[e + f*x]]]/2 + ArcTanh[Sqrt[Cos[e + f*x]]]/2)*Sqrt[Cos[e + f*x]]*Sqrt[b*
Tan[e + f*x]])/(b^2*f*Sqrt[a*Sin[e + f*x]])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3075
```
Int[Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]/((b_.)*tan[(e_.) + (f_.)*(x_)])^(3/
2), x_Symbol] :> Simp[2*(Sqrt[a*Sin[e + f*x]]/(b*f*Sqrt[b*Tan[e + f*x]])),
x] + Simp[a^2/b^2   Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2), x], x]
 /; FreeQ[{a, b, e, f}, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

## Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| default | $-\frac{\sqrt{a\sin(fx+e)}\left(-8\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^2\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+2\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}-\cos(fx+e)+1}{1+\cos(fx+e)}\right)-}{4f\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}\,b\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}$ |

input
```
int((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*(a*sin(f*x+e))^(1/2)/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)/b/(b*sin(
f*x+e)/cos(f*x+e))^(1/2)*(-8*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+sec(1/2*
f*x+1/2*e)^2*ln((2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+e)))-sec(1/2*f*x+
1/2*e)^2*arctan(1/2/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(121) = 242$.

Time = 0.27 (sec) , antiderivative size = 529, normalized size of antiderivative = 3.75

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \left[\frac{2\,b\sqrt{-\frac{a}{b}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{a}{b}}\cos(fx+e)}{(a\cos(fx+e)+a)\sin(fx+e)}\right)\sin(fx+e)+b\sqrt{-\frac{a}{b}}\,\mathrm{lo}}{}\right.$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*b*sqrt(-a/b)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos
(f*x + e))*sqrt(-a/b)*cos(f*x + e)/((a*cos(f*x + e) + a)*sin(f*x + e)))*si
n(f*x + e) + b*sqrt(-a/b)*log(-(a*cos(f*x + e)^3 + 4*sqrt(a*sin(f*x + e))*
sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-a/b)*cos(f*x + e)*sin(f*x + e) - 5
*a*cos(f*x + e)^2 - 5*a*cos(f*x + e) + a)/(cos(f*x + e)^3 + 3*cos(f*x + e)
^2 + 3*cos(f*x + e) + 1))*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin
(f*x + e)/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x + e)), 1/4*(2*b*sqrt(
a/b)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(
a/b)*cos(f*x + e)/((a*cos(f*x + e) - a)*sin(f*x + e)))*sin(f*x + e) + b*sq
rt(a/b)*log((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b
*sin(f*x + e)/cos(f*x + e))*sqrt(a/b) - (a*cos(f*x + e)^2 + 6*a*cos(f*x +
e) + a)*sin(f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e))
)*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*
cos(f*x + e))/(b^2*f*sin(f*x + e))]
```

## Sympy [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin{\left(e+fx \right)}}}{\left(b\tan{\left(e+fx \right)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a*sin(e + f*x))/(b*tan(e + f*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sin(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a\,\sin\left(e+f\,x\right)}}{{\left(b\tan\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx=\frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}}{\tan(fx+e)^2}dx\right)}{b^2}$$

input
```
int((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x)))/tan(e + f*x)*
*2,x))/b**2
```

# 3.138 $\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$


Optimal result . . . . . . 1121
Mathematica [A] (verified) . . . . . . 1122
Rubi [A] (verified) . . . . . . 1122
Maple [B] (verified) . . . . . . 1126
Fricas [B] (verification not implemented) . . . . . . 1126
Sympy [F(-1)] . . . . . . 1127
Maxima [F] . . . . . . 1127
Giac [F(-2)] . . . . . . 1128
Mupad [F(-1)] . . . . . . 1128
Reduce [F] . . . . . . 1129


## Optimal result

Integrand size = 25, antiderivative size = 151

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} + \frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}} + \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}}$$

output

```
-1/2/b/f/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2)+1/4*arctan(cos(f*x+e)^(
1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a/b^2/f/(a*sin(f*x+e))^(1/2)+1
/4*arctanh(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a/b^2/f
/(a*sin(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\left(\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)-2\sqrt[4]{\ }\right.}{4bf\sqrt[4]{\cos^2(e+fx)}(a\sin(e+fx))^{3/2}}$$

input
```
Integrate[1/((a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
((ArcTan[(Cos[e + f*x]^2)^(1/4)] + ArcTanh[(Cos[e + f*x]^2)^(1/4)] - 2*(Co
s[e + f*x]^2)^(1/4)*Csc[e + f*x]^2)*Sin[e + f*x]^2)/(4*b*f*(Cos[e + f*x]^2
)^(1/4)*(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3077, 3042, 3081, 27, 3042, 3045, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3077$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}dx}{4b^2}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}dx}{4b^2}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}dx}{4b^2\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}dx}{4ab^2\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sin(e+fx)}dx}{4ab^2\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3045}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}(1-\cos^2(e+fx))}d\cos(e+fx)}{4ab^2f\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{1-\cos^2(e+fx)}d\sqrt{\cos(e+fx)}}{2ab^2f\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{756}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}+\frac{1}{2}\int \frac{1}{\cos(e+fx)+1}d\sqrt{\cos(e+fx)}\right)}{2ab^2f\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}+\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{2ab^2f\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\right)}{2ab^2f\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

input
```
Int[1/((a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-1/2*1/(b*f*(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]) + ((ArcTan[Sqrt[C
os[e + f*x]]]/2 + ArcTanh[Sqrt[Cos[e + f*x]]]/2)*Sqrt[Cos[e + f*x]]*Sqrt[b
*Tan[e + f*x]])/(2*a*b^2*f*Sqrt[a*Sin[e + f*x]])
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(125) = 250.

Time = 0.88 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.91

| method | result |
|---|---|
| default | $\frac{\csc(fx+e)\left(\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}}\right)-\cos(fx+e)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}+2\sqrt{-\frac{\cos(fx+e)}{(1+\cos(fx+e))^2}}-\cos(fx}{1+\cos(fx+e)}\right.\right.}{8f\sqrt{b\tan(fx+e)}\,\sqrt{a\,\mathrm{si}}}$ |

input
```
int(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f*csc(f*x+e)*(cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2))-cos(f*x+e)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+e)))-4*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-arctan(1/2/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2))+ln((2*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(1+cos(f*x+e))))/(b*tan(f*x+e))^(1/2)
/(a*sin(f*x+e))^(1/2)/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)/a/b
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(125) = 250.

Time = 0.38 (sec) , antiderivative size = 608, normalized size of antiderivative = 4.03

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
[-1/16*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f
*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e)
 + a*b)*sin(f*x + e)))*sin(f*x + e) + sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(
-(a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*sqrt(a*sin(f*x
+ e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 5*a*b*
cos(f*x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) +
1))*sin(f*x + e) - 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)
)*cos(f*x + e))/((a^2*b^2*f*cos(f*x + e)^2 - a^2*b^2*f)*sin(f*x + e)), -1/
16*(2*sqrt(a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(a*b)*sqrt(a*sin(f*x + e
))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) - a*b
)*sin(f*x + e)))*sin(f*x + e) - sqrt(a*b)*(cos(f*x + e)^2 - 1)*log(-(4*sqr
t(a*b)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x
 + e)/cos(f*x + e)) + (a*b*cos(f*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(
f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x +
e) - 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)
)/((a^2*b^2*f*cos(f*x + e)^2 - a^2*b^2*f)*sin(f*x + e))]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\,\sin(e+f\,x))^{3/2}\,(b\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(3/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}}{\sin(fx+e)^2\tan(fx+e)^2}dx\right)}{a^2b^2}$$

input
```
int(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x)))/(sin(e + f*x)
**2*tan(e + f*x)**2),x))/(a**2*b**2)
```

# 3.139 $\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx$


Optimal result . . . 1130
Mathematica [A] (warning: unable to verify) . . . 1130
Rubi [A] (verified) . . . 1131
Maple [C] (verified) . . . 1133
Fricas [C] (verification not implemented) . . . 1134
Sympy [F(-1)] . . . 1134
Maxima [F] . . . 1135
Giac [F(-1)] . . . 1135
Mupad [F(-1)] . . . 1135
Reduce [F] . . . 1136


## Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}} + \frac{8a^6\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{77b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-4/77*a^4*(a*sin(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(1/2)-2/77*a^2*(a*sin(f*
x+e))^(7/2)/b/f/(b*tan(f*x+e))^(1/2)+2/11*(a*sin(f*x+e))^(11/2)/b/f/(b*tan
(f*x+e))^(1/2)+8/77*a^6*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(
1/2))*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.71

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^5\left(\sqrt[4]{\cos^2(e+fx)}(-22\cos(e+fx)-17\cos(3(e+fx))+7\cos(5(e+fx)))+\cdots\right.}{616f\sqrt[4]{\cos^2(e+\cdots}}$$

input
```
Integrate[(a*Sin[e + f*x])^(11/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(a^5*((Cos[e + f*x]^2)^(1/4)*(-22*Cos[e + f*x] - 17*Cos[3*(e + f*x)] + 7*C
os[5*(e + f*x)]) + 64*Cot[e + f*x]*EllipticF[ArcSin[Sin[e + f*x]]/2, 2])*S
qrt[a*Sin[e + f*x]]*Tan[e + f*x]^2)/(616*f*(Cos[e + f*x]^2)^(1/4)*(b*Tan[e
 + f*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3076, 3042, 3078, 3042, 3078, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{a^2\int (a\sin(e+fx))^{7/2}\sqrt{b\tan(e+fx)}dx}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int (a\sin(e+fx))^{7/2}\sqrt{b\tan(e+fx)}dx}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{a^2\left(\frac{6}{7}a^2\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{7/2}}{7f\sqrt{b\tan(e+fx)}}\right)}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{6}{7}a^2\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{7/2}}{7f\sqrt{b\tan(e+fx)}}\right)}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{a^2\left(\frac{6}{7}a^2\left(\frac{2}{3}a^2\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{7/2}}{7f\sqrt{b\tan(e+fx)}}\right)}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{a^2\left(\frac{6}{7}a^2\left(\frac{2}{3}a^2\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{7/2}}{7f\sqrt{b\tan(e+fx)}}\right)}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

$\Big\downarrow$ 3081

$$\frac{a^2\left(\frac{6}{7}a^2\left(\frac{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx}{3\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{7/2}}{7f\sqrt{b\tan(e+fx)}}\right)}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{a^2\left(\frac{6}{7}a^2\left(\frac{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{7/2}}{7f\sqrt{b\tan(e+fx)}}\right)}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

$\Big\downarrow$ 3120

$$\frac{a^2\left(\frac{6}{7}a^2\left(\frac{4a^2\sqrt{\cos(e+fx)}\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{7/2}}{7f\sqrt{b\tan(e+fx)}}\right)}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(11/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(a*Sin[e + f*x])^(11/2))/(11*b*f*Sqrt[b*Tan[e + f*x]]) + (a^2*((-2*b*(a
*Sin[e + f*x])^(7/2))/(7*f*Sqrt[b*Tan[e + f*x]]) + (6*a^2*((-2*b*(a*Sin[e
+ f*x])^(3/2))/(3*f*Sqrt[b*Tan[e + f*x]]) + (4*a^2*Sqrt[Cos[e + f*x]]*Elli
pticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x]])))/7
))/(11*b^2)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.55 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.76

| method | result |
|---|---|
| default | $\frac{2\sqrt{a\sin(fx+e)}\,a^5\left(\sin(fx+e)\left(7\cos(fx+e)^4-13\cos(fx+e)^2+4\right)+i\sqrt{\frac{1}{1+\cos(fx+e)}}\ \text{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(f}{1+\cos}}\right.}{77f\sqrt{b\tan(fx+e)}\,b}$ |

input
```
int((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/77/f*(a*sin(f*x+e))^(1/2)*a^5/(b*tan(f*x+e))^(1/2)/b*(sin(f*x+e)*(7*cos(
f*x+e)^4-13*cos(f*x+e)^2+4)+I*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(csc(f*
x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-4-4*sec(f*x+e)))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(4\sqrt{\frac{1}{2}}\sqrt{-ab}a^5\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+4\right.}{}$$

input
```
integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

output
```
2/77*(4*sqrt(1/2)*sqrt(-a*b)*a^5*weierstrassPInverse(-4, 0, cos(f*x + e) +
 I*sin(f*x + e)) + 4*sqrt(1/2)*sqrt(-a*b)*a^5*weierstrassPInverse(-4, 0, c
os(f*x + e) - I*sin(f*x + e)) + (7*a^5*cos(f*x + e)^5 - 13*a^5*cos(f*x + e
)^3 + 4*a^5*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x
 + e)))/(b^2*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(11/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{11}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((a*sin(f*x + e))^(11/2)/(b*tan(f*x + e))^(3/2), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\,\sin(e+f\,x))^{11/2}}{(b\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(11/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(11/2)/(b*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)^5}{\tan(fx+e)^2}dx\right)a^5}{b^2}$$

input
```
int((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x)**
5)/tan(e + f*x)**2,x)*a**5)/b**2
```

# 3.140 $\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx$


Optimal result . . . . . 1137
Mathematica [A] (verified) . . . . . 1137
Rubi [A] (verified) . . . . . 1138
Maple [C] (verified) . . . . . 1140
Fricas [C] (verification not implemented) . . . . . 1141
Sympy [F(-1)] . . . . . 1141
Maxima [F] . . . . . 1141
Giac [F(-1)] . . . . . 1142
Mupad [F(-1)] . . . . . 1142
Reduce [F] . . . . . 1142


## Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}} + \frac{4a^4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{21b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-2/21*a^2*(a*sin(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(1/2)+2/7*(a*sin(f*x+e))
^(7/2)/b/f/(b*tan(f*x+e))^(1/2)+4/21*a^4*cos(f*x+e)^(1/2)*InverseJacobiAM(
1/2*f*x+1/2*e,2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.75

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^3\sqrt{a\sin(e+fx)}\left(8\,\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right) + \sqrt[4]{\cos^2(e+fx)}(5\sin\right.}{42bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(7/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(a^3*Sqrt[a*Sin[e + f*x]]*(8*EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Cos[e
 + f*x]^2)^(1/4)*(5*Sin[e + f*x] - 3*Sin[3*(e + f*x)])))/(42*b*f*(Cos[e +
f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3076, 3042, 3078, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{a^2\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}dx}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}dx}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{a^2\left(\frac{2}{3}a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{2}{3}a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{a^2\left(\frac{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx}{3\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a^2\left(\frac{4a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(7/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(a*Sin[e + f*x])^(7/2))/(7*b*f*Sqrt[b*Tan[e + f*x]]) + (a^2*((-2*b*(a*S
in[e + f*x])^(3/2))/(3*f*Sqrt[b*Tan[e + f*x]]) + (4*a^2*Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x]]
)))/(7*b^2)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| default | $-\frac{2\sqrt{a\sin(fx+e)}\,a^{3}\left(\sin(fx+e)\left(3\cos(fx+e)^{2}-2\right)+i\sqrt{\frac{1}{1+\cos(fx+e)}}\ \text{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\ (2+2\,\text{se}\right.}{21f\sqrt{b\tan(fx+e)}\,b}$ |

input
```
int((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21/f*(a*sin(f*x+e))^(1/2)*a^3/(b*tan(f*x+e))^(1/2)/b*(sin(f*x+e)*(3*cos
(f*x+e)^2-2)+I*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)
),I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(2+2*sec(f*x+e)))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(2\sqrt{\frac{1}{2}}\sqrt{-ab}a^3\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+2\,\sqrt{}\right.}{}$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/21*(2*sqrt(1/2)*sqrt(-a*b)*a^3*weierstrassPInverse(-4, 0, cos(f*x + e) +
 I*sin(f*x + e)) + 2*sqrt(1/2)*sqrt(-a*b)*a^3*weierstrassPInverse(-4, 0, c
os(f*x + e) - I*sin(f*x + e)) - (3*a^3*cos(f*x + e)^3 - 2*a^3*cos(f*x + e)
)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(b^2*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(7/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(7/2)/(b*tan(f*x + e))^(3/2), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\,\sin(e+f\,x))^{7/2}}{(b\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(7/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(7/2)/(b*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)^3}{\tan(fx+e)^2}dx\right)a^3}{b^2}$$

input
```
int((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x)**
3)/tan(e + f*x)**2,x)*a**3)/b**2
```

# 3.141 $\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1143
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1143
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1144
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1146
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . 1146
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1147
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1147
Giac [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1147
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1148
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1148

## Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}} + \frac{2a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}}$$

output
```
2/3*(a*sin(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(1/2)+2/3*a^2*cos(f*x+e)^(1/2)
*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(
f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2a\sqrt{a\sin(e+fx)}\left(\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)+\sqrt[4]{\cos^2(e+fx)}\sin(e \cdots\right.}{3bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*a*Sqrt[a*Sin[e + f*x]]*(EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Cos[e +
 f*x]^2)^(1/4)*Sin[e + f*x]))/(3*b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e +
 f*x]])
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3076, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3b^2}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3b^2}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{3b^2\sqrt{a\sin(e+fx)}}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{a\sin(e+fx)}}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(a*Sin[e + f*x])^(3/2))/(3*b*f*Sqrt[b*Tan[e + f*x]]) + (2*a^2*Sqrt[Cos[
e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*b^2*f*Sqrt[a*
Sin[e + f*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10

| method | result | size |
|---|---|---|
| default | $-\frac{2\sqrt{a\sin(fx+e)}\,a\left(i\sqrt{\frac{1}{1+\cos(fx+e)}}\ \mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(1+\sec(fx+e))-\sin(fx+e)\right)}{3f\sqrt{b\tan(fx+e)}\,b}$ | 102 |

input
```
int((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/f*(a*sin(f*x+e))^(1/2)*a/(b*tan(f*x+e))^(1/2)/b*(I*(1/(1+cos(f*x+e)))
^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(1+sec(f*x+e))-sin(f*x+e))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(\sqrt{a\sin\left(fx+e\right)}a\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos\left(fx+e\right)+\sqrt{\frac{1}{2}}\sqrt{-ab}a\mathrm{weierstrassPInverse}(\right.}{}$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)
 + sqrt(1/2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(
f*x + e)) + sqrt(1/2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e)
 - I*sin(f*x + e)))/(b^2*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx=\int \frac{{\left(a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(b\,\tan\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx=\frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}\,\sin(fx+e)}{\tan(fx+e)^{2}}dx\right)a}{b^{2}}$$

input
```
int((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x))*sin(e + f*x))/
tan(e + f*x)**2,x)*a)/b**2
```

# 3.142 $\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$

Optimal result . . . 1149
Mathematica [A] (verified) . . . 1149
Rubi [A] (verified) . . . 1150
Maple [C] (verified) . . . 1152
Fricas [C] (verification not implemented) . . . 1152
Sympy [F(-1)] . . . 1153
Maxima [F] . . . 1153
Giac [F] . . . 1153
Mupad [F(-1)] . . . 1154
Reduce [F] . . . 1154

## Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-1/b/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)-cos(f*x+e)^(1/2)*InverseJ
acobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(
1/2)
```

## Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \frac{-\sqrt[4]{\cos^2(e+fx)} - \text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)\sin(e+}{bf\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[1/(Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
(-(Cos[e + f*x]^2)^(1/4) - EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e + f*
x])/(b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3077, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2b^2}-\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2b^2}-\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{2b^2\sqrt{a\sin(e+fx)}}-\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{2b^2\sqrt{a\sin(e+fx)}}-\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}-\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[1/(Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-(1/(b*f*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) - (Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[a*Sin[e + f*x
]])
```

**Defintions of rubi rules used**

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.19

| method | result | size |
|---|---|---|
| default | $\frac{i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,\operatorname{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)(-\sin(fx+e)-\tan(fx+e))-1}{f\sqrt{a\sin(fx+e)}\,\sqrt{b\tan(fx+e)}\,b}$ | 102 |

input
```
int(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b*(I*(1/(1+cos(f*x+e)))^(1/2
)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-csc(f*x+e)+cot(f*x+e)),I
)*(-sin(f*x+e)-tan(f*x+e))-1)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = -\frac{\sqrt{\frac{1}{2}}\sqrt{-ab}\left(\cos(fx+e)^2-1\right)\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+\sqrt{\frac{1}{2}}\sqrt{-ab}(\cos}{ab^2f\cos}$$

input
```
integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
-(sqrt(1/2)*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e)) + sqrt(1/2)*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*we
ierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - sqrt(a*sin(f*x +
 e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/(a*b^2*f*cos(f*x + e)
^2 - a*b^2*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a\,\sin\left(e+f\,x\right)}\,\left(b\tan\left(e+f\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2)), x)
```

## Reduce [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}}{\sin(fx+e)\tan(fx+e)^2}dx\right)}{a\,b^2}$$

input
```
int(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x)))/(sin(e + f*x)
*tan(e + f*x)**2),x))/(a*b**2)
```

# 3.143 $\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1155
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1155
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1156
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1158
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1159
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1159
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1160
Giac [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1160
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1160
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1161


## Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}+\frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{6a^2b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-1/3/b/f/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2)+1/6/a^2/b/f/(a*sin(f*x+
e))^(1/2)/(b*tan(f*x+e))^(1/2)-1/6*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*
x+1/2*e,2^(1/2))*(b*tan(f*x+e))^(1/2)/a^2/b^2/f/(a*sin(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(e+fx)}(1-2\csc^2(e+fx)) - \text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(\right.}{6a^2bf\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(}$$

input
```
Integrate[1/((a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
((Cos[e + f*x]^2)^(1/4)*(1 - 2*Csc[e + f*x]^2) - EllipticF[ArcSin[Sin[e +
f*x]]/2, 2]*Sin[e + f*x])/(6*a^2*b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e +
 f*x]]*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3077, 3042, 3079, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{3077}$$
$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$
$$\Big\downarrow \text{3079}$$
$$-\frac{\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2a^2}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2a^2}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx}{2a^2\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a^2\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{\sqrt{\cos(e+fx)}\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

input
```
Int[1/((a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-1/3*1/(b*f*(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) - (-(b/(a^2*f*Sqr
t[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) + (Sqrt[Cos[e + f*x]]*EllipticF[(
e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x]]))/(6*b^2)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{i\,\mathrm{EllipticF}(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(\sin(fx+e)+\tan(fx+e))-\cot(fx+e)^2-\csc(fx+e)^2}{6f\sqrt{a\sin(fx+e)}\,\sqrt{b\tan(fx+e)}\,a^2 b}$ | 121 |

input
```
int(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/a^2/b*(I*EllipticF(I*(csc(
f*x+e)-cot(f*x+e)),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(sin(f*x+e)+tan(f*x+e))-cot(f*x+e)^2-csc(f*x+e)^2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{\sqrt{\frac{1}{2}}\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\sqrt{-ab}\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))}{}$$

input
```
integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
-1/6*(sqrt(1/2)*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-a*b)*weierst
rassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + sqrt(1/2)*(cos(f*x +
e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x
 + e) - I*sin(f*x + e)) + (cos(f*x + e)^3 + cos(f*x + e))*sqrt(a*sin(f*x +
 e))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^3*b^2*f*cos(f*x + e)^4 - 2*a^3*
b^2*f*cos(f*x + e)^2 + a^3*b^2*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\,\sin(e+f\,x))^{5/2}\,(b\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}}{\sin(fx+e)^3\tan(fx+e)^2}dx\right)}{a^3b^2}$$

input
```
int(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x)))/(sin(e + f*x)
**3*tan(e + f*x)**2),x))/(a**3*b**2)
```

# 3.144 $\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1162
Mathematica [A] (warning: unable to verify) . . . . . . . . . . . . . . . . 1163
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1163
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1166
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . 1167
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1167
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1168
Giac [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1168
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1168
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1169


## Optimal result

Integrand size = 25, antiderivative size = 167

$$\begin{aligned}\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = &-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}\\ &+\frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\\ &+\frac{1}{12a^4bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\\ &-\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{12a^4b^2f\sqrt{a\sin(e+fx)}}\end{aligned}$$

output
```
-1/5/b/f/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2)+1/30/a^2/b/f/(a*sin(f*x
+e))^(5/2)/(b*tan(f*x+e))^(1/2)+1/12/a^4/b/f/(a*sin(f*x+e))^(1/2)/(b*tan(f
*x+e))^(1/2)-1/12*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*
(b*tan(f*x+e))^(1/2)/a^4/b^2/f/(a*sin(f*x+e))^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(e+fx)}(5+2\csc^2(e+fx)-12\csc^4(e+fx))-5\,\mathrm{Elli}}{60a^4bf\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+f}}$$

input
```
Integrate[1/((a*Sin[e + f*x])^(9/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
((Cos[e + f*x]^2)^(1/4)*(5 + 2*Csc[e + f*x]^2 - 12*Csc[e + f*x]^4) - 5*Ell
ipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e + f*x])/(60*a^4*b*f*(Cos[e + f*x]^
2)^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3077, 3042, 3079, 3042, 3079, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{9/2}}dx}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{9/2}}dx}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3079}$$

$$-\frac{\frac{5\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3079}$$

$$-\frac{\frac{5\left(\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2a^2}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\right)}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5\left(\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2a^2}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\right)}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\frac{5\left(\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{2a^2\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\right)}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5\left(\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a^2\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\right)}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \ 3120$$

$$-\frac{\frac{5\left(\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\right)}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

input
```
Int[1/((a*Sin[e + f*x])^(9/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-1/5*1/(b*f*(a*Sin[e + f*x])^(9/2)*Sqrt[b*Tan[e + f*x]]) - (-1/3*b/(a^2*f*
(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) + (5*(-(b/(a^2*f*Sqrt[a*Sin[e
 + f*x]]*Sqrt[b*Tan[e + f*x]])) + (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/
2, 2]*Sqrt[b*Tan[e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x]])))/(6*a^2))/(10*b^
2)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| default | $\frac{-5i\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,(\sin(fx+e)+\tan(fx+e))\,\mathrm{EllipticF}(i(-\csc(fx+e)+\cot(fx+e)),i)+\csc(fx+e)^4\left(5\cos(fx+e)^4-\right.}{60f\sqrt{a\sin(fx+e)}\,\sqrt{b\tan(fx+e)}\,a^4b}$ |

input
```
int(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/60/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/a^4/b*(-5*I*(1/(1+cos(f*x
+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(sin(f*x+e)+tan(f*x+e))*Elli
pticF(I*(-csc(f*x+e)+cot(f*x+e)),I)+csc(f*x+e)^4*(5*cos(f*x+e)^4-12*cos(f*
x+e)^2-5))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx =$$

$$-\frac{5\sqrt{\frac{1}{2}}\left(\cos(fx+e)^{6}-3\cos(fx+e)^{4}+3\cos(fx+e)^{2}-1\right)\sqrt{-ab}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+\ldots}{\ldots}$$

input
```
integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
-1/60*(5*sqrt(1/2)*(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 -
 1)*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) +
 5*sqrt(1/2)*(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sq
rt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + (5*co
s(f*x + e)^5 - 12*cos(f*x + e)^3 - 5*cos(f*x + e))*sqrt(a*sin(f*x + e))*sq
rt(b*sin(f*x + e)/cos(f*x + e)))/(a^5*b^2*f*cos(f*x + e)^6 - 3*a^5*b^2*f*c
os(f*x + e)^4 + 3*a^5*b^2*f*cos(f*x + e)^2 - a^5*b^2*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{9}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((a*sin(f*x + e))^(9/2)*(b*tan(f*x + e))^(3/2)), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(9/2)*(b*tan(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(9/2)*(b*tan(e + f*x))^(3/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sin(fx+e)}}{\sin(fx+e)^{5}\tan(fx+e)^{2}}dx\right)}{a^{5}b^{2}}$$

input
```
int(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(tan(e + f*x))*sqrt(sin(e + f*x)))/(sin(e + f*x)
**5*tan(e + f*x)**2),x))/(a**5*b**2)
```

# 3.145 $\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1170
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 1170
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1171
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1172
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1172
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1173
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1173
Giac [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1173
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1174
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1174


## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)(b\sin(e+fx}{17df}$$

output
```
6/17*(cos(f*x+e)^2)^(3/4)*hypergeom([3/4, 17/12],[29/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2)/d/f
```

## Mathematica [A] (verified)

Time = 11.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}\sin(2(e+f}{17f\sqrt[4]{\cos^2(e+fx)}}$$

input
```
Integrate[(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(3*Hypergeometric2F1[3/4, 17/12, 29/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(
4/3)*Sin[2*(e + f*x)]*Sqrt[d*Tan[e + f*x]])/(17*f*(Cos[e + f*x]^2)^(1/4))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{(b\sin(e+fx))^{11/6}}{\sqrt{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{(b\sin(e+fx))^{11/6}}{\sqrt{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{3/4}(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)}{17df}$$

input
```
Int[(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, 17/12, 29/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2))/(17*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (b\sin (fx+e))^{\frac{4}{3}} \sqrt{d\tan (fx+e)}dx$$

input
```
int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

output
```
int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

## Fricas [F]

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sin (fx+e))^{\frac{4}{3}} \sqrt{d\tan (fx+e)}\,dx$$

input
```
integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*sin(f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(4/3)*(d*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{4}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

## Giac [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sin(e+f\,x))^{4/3}\,\sqrt{d\tan(e+f\,x)}\,dx$$

input
```
int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(1/2),x)
```

output
```
int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \sqrt{d}\,b^{\frac{4}{3}}\left(\int \sqrt{\tan(fx+e)}\,\sin(fx+e)^{\frac{4}{3}}\,dx\right)$$

input
```
int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*b**(1/3)*int(sqrt(tan(e + f*x))*sin(e + f*x)**(1/3)*sin(e + f*x),x
)*b
```

# 3.146 $\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx$


Optimal result . . . . . 1175
Mathematica [A] (verified) . . . . . 1175
Rubi [A] (verified) . . . . . 1176
Maple [F] . . . . . 1177
Fricas [F] . . . . . 1177
Sympy [F] . . . . . 1178
Maxima [F] . . . . . 1178
Giac [F] . . . . . 1178
Mupad [F(-1)] . . . . . 1179
Reduce [F] . . . . . 1179


## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}}{11df}$$

output
```
6/11*(cos(f*x+e)^2)^(3/4)*hypergeom([3/4, 11/12],[23/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2)/d/f
```

## Mathematica [A] (verified)

Time = 10.82 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\sin(2(e+fx))\sqrt{d\tan(e+fx)}}{11f\sqrt[4]{\cos^2(e+fx)}}$$

input
```
Integrate[(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(3*Hypergeometric2F1[3/4, 11/12, 23/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(
1/3)*Sin[2*(e + f*x)]*Sqrt[d*Tan[e + f*x]])/(11*f*(Cos[e + f*x]^2)^(1/4))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{(b\sin(e+fx))^{5/6}}{\sqrt{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{(b\sin(e+fx))^{5/6}}{\sqrt{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{3/4}\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)}{11df}$$

input
```
Int[(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, 11/12, 23/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2))/(11*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (b\sin(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}dx$$

input
```
int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

output
```
int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

## Fricas [F]

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

## Sympy [F]

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int \sqrt[3]{b\sin{(e+fx)}}\sqrt{d\tan{(e+fx)}}\,dx$$

input
```
integrate((b*sin(f*x+e))**(1/3)*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral((b*sin(e + f*x))**(1/3)*sqrt(d*tan(e + f*x)), x)
```

## Maxima [F]

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

## Giac [F]

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\,\sin(e+f\,x))^{1/3}\,\sqrt{d\tan(e+f\,x)}\,dx$$

input
```
int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(1/2),x)
```

output
```
int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \sqrt{d}\,b^{\frac{1}{3}}\left(\int \sqrt{\tan(fx+e)}\,\sin(fx+e)^{\frac{1}{3}}\,dx\right)$$

input
```
int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*b**(1/3)*int(sqrt(tan(e + f*x))*sin(e + f*x)**(1/3),x)
```

# 3.147 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx$


Optimal result . . . 1180
Mathematica [A] (verified) . . . 1180
Rubi [A] (verified) . . . 1181
Maple [F] . . . 1182
Fricas [F] . . . 1182
Sympy [F] . . . 1183
Maxima [F] . . . 1183
Giac [F] . . . 1183
Mupad [F(-1)] . . . 1184
Reduce [F] . . . 1184


## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{7df\sqrt[3]{b\sin(e+fx)}}$$

output
```
6/7*(cos(f*x+e)^2)^(3/4)*hypergeom([7/12, 3/4],[19/12],sin(f*x+e)^2)*(d*ta
n(f*x+e))^(3/2)/d/f/(b*sin(f*x+e))^(1/3)
```

## Mathematica [A] (verified)

Time = 10.78 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{7df\sqrt[3]{b\sin(e+fx)}}$$

input
```
Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[7/12, 3/4, 19/12, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(3/2))/(7*d*f*(b*Sin[e + f*x])^(1/3))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \text{ 3082}$$

$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{\sqrt[6]{b\sin(e+fx)}}{\sqrt{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{\sqrt[6]{b\sin(e+fx)}}{\sqrt{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{6\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},\sin^2(e+fx)\right)}{7df\sqrt[3]{b\sin(e+fx)}}$$

input
```
Int[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[7/12, 3/4, 19/12, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(3/2))/(7*d*f*(b*Sin[e + f*x])^(1/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

## Maple [F]

$$\int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x)
```

output
```
int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b*sin(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\tan\left(e+fx\right)}}{\sqrt[3]{b\sin\left(e+fx\right)}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(1/2)/(b*sin(f*x+e))**(1/3),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))/(b*sin(e + f*x))**(1/3), x)
```

## Maxima [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{{\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3}} \,d x$$

input
```
int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(1/3),x)
```

output
```
int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx=\frac{\sqrt{d}\,\left(\int \frac{\sqrt{\tan(fx+e)}}{\sin(fx+e)^{\frac{1}{3}}}dx\right)}{b^{\frac{1}{3}}}$$

input
```
int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x)
```

output
```
(sqrt(d)*int(sqrt(tan(e + f*x))/sin(e + f*x)**(1/3),x))/b**(1/3)
```

# 3.148 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1185
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1185
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1186
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1187
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1187
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1188
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1188
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1188
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1189
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1189

## Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{df(b\sin(e+fx))^{4/3}}$$

output
```
6*(cos(f*x+e)^2)^(3/4)*hypergeom([1/12, 3/4],[13/12],sin(f*x+e)^2)*(d*tan(
f*x+e))^(3/2)/d/f/(b*sin(f*x+e))^(4/3)
```

## Mathematica [A] (verified)

Time = 10.87 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},\sin^2(e+fx)\right)\sin(2(e+fx))\sqrt{d\tan(e+fx)}}{f\sqrt[4]{\cos^2(e+fx)}(b\sin(e+fx))^{4/3}}$$

input
```
Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(4/3),x]
```

output
```
(3*Hypergeometric2F1[1/12, 3/4, 13/12, Sin[e + f*x]^2]*Sin[2*(e + f*x)]*Sq
rt[d*Tan[e + f*x]])/(f*(Cos[e + f*x]^2)^(1/4)*(b*Sin[e + f*x])^(4/3))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}dx$$
$$\Big\downarrow \text{3082}$$
$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{1}{\sqrt{\cos(e+fx)}(b\sin(e+fx))^{5/6}}dx}{d(b\sin(e+fx))^{3/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{1}{\sqrt{\cos(e+fx)}(b\sin(e+fx))^{5/6}}dx}{d(b\sin(e+fx))^{3/2}}$$
$$\Big\downarrow \text{3057}$$
$$\frac{6\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},\sin^2(e+fx)\right)}{df(b\sin(e+fx))^{4/3}}$$

input
```
Int[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(4/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[1/12, 3/4, 13/12, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(b*Sin[e + f*x])^(4/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{\sqrt{d \tan (fx+e)}}{(b \sin (fx+e))^{\frac{4}{3}}} dx$$

input
```
int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x)
```

output
```
int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x)
```

## Fricas [F]

$$\int \frac{\sqrt{d \tan(e+fx)}}{(b \sin(e+fx))^{4/3}} dx = \int \frac{\sqrt{d \tan (fx+e)}}{(b \sin (fx+e))^{\frac{4}{3}}} dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(-(b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b^2*cos(f*x + e)^2
- b^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx=\text{Timed out}$$

input
```
integrate((d*tan(f*x+e))**(1/2)/(b*sin(f*x+e))**(4/3),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx=\int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(4/3), x)
```

## Giac [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx=\int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{\left(b\,\sin\left(e+f\,x\right)\right)^{4/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(4/3),x)
```

output
```
int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(fx+e)}}{\sin(fx+e)^{\frac{4}{3}}}dx\right)}{b^{\frac{4}{3}}}$$

input
```
int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x)
```

output
```
(sqrt(d)*int(sqrt(tan(e + f*x))/(sin(e + f*x)**(1/3)*sin(e + f*x)),x))/(b*
*(1/3)*b)
```

# 3.149 $\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$


Optimal result . . . 1190
Mathematica [A] (verified) . . . 1190
Rubi [A] (verified) . . . 1191
Maple [F] . . . 1192
Fricas [F] . . . 1192
Sympy [F(-1)] . . . 1193
Maxima [F] . . . 1193
Giac [F(-2)] . . . 1193
Mupad [F(-1)] . . . 1194
Reduce [F] . . . 1194


## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)(b\sin(e+f\ldots}{23df}$$

output
```
6/23*(cos(f*x+e)^2)^(5/4)*hypergeom([5/4, 23/12],[35/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(5/2)/d/f
```

## Mathematica [A] (verified)

Time = 11.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = -\frac{2d\left(-1+\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\right)(b\sin(e+fx))^{4/3}\sqrt{d\tan(e+f\ldots}}{f}$$

input
```
Integrate[(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 11/12, 23/12, Si
n[e + f*x]^2])*(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]])/f
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{ 3082}$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{(b\sin(e+fx))^{17/6}}{\cos^{\frac{3}{2}}(e+fx)}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{(b\sin(e+fx))^{17/6}}{\cos(e+fx)^{3/2}}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{6\cos^2(e+fx)^{5/4}(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)}{23df}$$

input
```
Int[(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, 23/12, 35/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(5/2))/(23*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (b\sin(fx+e))^{\frac{4}{3}} (d\tan(fx+e))^{\frac{3}{2}} dx$$

input
```
int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

output
```
int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

## Fricas [F]

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sin(fx+e))^{\frac{4}{3}} (d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*d*sin(f*x + e)*tan(
f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(4/3)*(d*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sin(fx+e))^{\frac{4}{3}}\,(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{i,[2,2,2]%%%}+%%%{%%%{%%%{%%%{-i,[10]%%%}+%%%{5*i,[8]%%%}+
%%%{-10*i
```

## Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\,\sin\left(e+f\,x\right))^{4/3}\,\left(d\tan(e+f\,x)\right)^{3/2}\,dx$$

input
```
int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(3/2),x)
```

output
```
int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \sqrt{d}\,b^{\frac{4}{3}}\left(\int \sqrt{\tan\left(fx+e\right)}\,\sin\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)dx\right)d$$

input
```
int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

output
```
sqrt(d)*b**(1/3)*int(sqrt(tan(e + f*x))*sin(e + f*x)**(1/3)*sin(e + f*x)*t
an(e + f*x),x)*b*d
```

# 3.150 $\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1195
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 1195
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1196
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1197
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1197
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1198
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1198
Giac [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1198
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1199
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1199

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx}{17df}$$

output
```
6/17*(cos(f*x+e)^2)^(5/4)*hypergeom([5/4, 17/12],[29/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(5/2)/d/f
```

## Mathematica [A] (verified)

Time = 10.81 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = -\frac{2d\left(-1+\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{12},\frac{17}{12},\sin^2(e+fx)\right)\right)\sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}}{f}$$

input
```
Integrate[(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/12, 17/12, Sin
[e + f*x]^2])*(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]])/f
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{(b\sin(e+fx))^{11/6}}{\cos^{\frac{3}{2}}(e+fx)}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{(b\sin(e+fx))^{11/6}}{\cos(e+fx)^{3/2}}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{5/4}\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)}{17df}$$

input
```
Int[(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, 17/12, 29/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(5/2))/(17*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (b\sin(fx+e))^{\frac{1}{3}} (d\tan(fx+e))^{\frac{3}{2}} dx$$

input
```
int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

output
```
int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

## Fricas [F]

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\, dx = \int (b\sin(fx+e))^{\frac{1}{3}} (d\tan(fx+e))^{\frac{3}{2}}\, dx$$

input
```
integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(1/3)*(d*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\,(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

## Giac [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int (b\,\sin(e+f\,x))^{1/3}\,(d\tan(e+f\,x))^{3/2}\,dx$$

input
```
int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(3/2),x)
```

output
```
int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \frac{\sqrt{d}\,b^{\frac{1}{3}}d\left(6\sqrt{\tan(fx+e)}\,\sin(fx+e)^{\frac{1}{3}}-2\left(\int\frac{\sqrt{\tan(fx+e)}\,\cos(fx+e)}{\sin(fx+e)^{\frac{2}{3}}}dx\right)f-3\left(\int\frac{\sqrt{\tan(fx+e)}\,\sin}{\tan(fx+}\right.\right.}{3f}$$

input
```
int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*b**(1/3)*d*(6*sqrt(tan(e + f*x))*sin(e + f*x)**(1/3) - 2*int((sqr
t(tan(e + f*x))*sin(e + f*x)**(1/3)*cos(e + f*x))/sin(e + f*x),x)*f - 3*in
t((sqrt(tan(e + f*x))*sin(e + f*x)**(1/3))/tan(e + f*x),x)*f))/(3*f)
```

# 3.151 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1200
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1200
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1201
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1202
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1202
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1203
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1203
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1203
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1204
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1204

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx=\frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{25}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{13df\sqrt[3]{b\sin(e+fx)}}$$

output
```
6/13*(cos(f*x+e)^2)^(5/4)*hypergeom([13/12, 5/4],[25/12],sin(f*x+e)^2)*(d*
tan(f*x+e))^(5/2)/d/f/(b*sin(f*x+e))^(1/3)
```

## Mathematica [A] (verified)

Time = 10.82 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx=-\frac{2d\left(-1+\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{1}{4},\frac{13}{12},\sin^2(e+fx)\right)\right)\sqrt{d\tan(e+fx)}}{f\sqrt[3]{b\sin(e+fx)}}$$

input
```
Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/12, 1/4, 13/12, Sin
[e + f*x]^2])*Sqrt[d*Tan[e + f*x]])/(f*(b*Sin[e + f*x])^(1/3))
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{(b\sin(e+fx))^{7/6}}{\cos^{\frac{3}{2}}(e+fx)}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{(b\sin(e+fx))^{7/6}}{\cos(e+fx)^{3/2}}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{25}{12},\sin^2(e+fx)\right)}{13df\sqrt[3]{b\sin(e+fx)}}$$

input
```
Int[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[13/12, 5/4, 25/12, Sin[e + f*x
]^2]*(d*Tan[e + f*x])^(5/2))/(13*d*f*(b*Sin[e + f*x])^(1/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x)
```

output
```
int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b*sin
(f*x + e)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((d*tan(f*x+e))**(3/2)/(b*sin(f*x+e))**(1/3),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{{\left(d\,\tan\left(e+f\,x\right)\right)}^{3/2}}{{\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(1/3),x)
```

output
```
int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx=\frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(fx+e)}\,\tan(fx+e)}{\sin(fx+e)^{\frac{1}{3}}}dx\right)d}{b^{\frac{1}{3}}}$$

input
```
int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x)
```

output
```
(sqrt(d)*int((sqrt(tan(e + f*x))*tan(e + f*x))/sin(e + f*x)**(1/3),x)*d)/b
**(1/3)
```

# 3.152 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1205
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1205
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1206
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1207
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1207
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1208
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1208
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1208
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1209
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1209

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{19}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{7df(b\sin(e+fx))^{4/3}}$$

output
```
6/7*(cos(f*x+e)^2)^(5/4)*hypergeom([7/12, 5/4],[19/12],sin(f*x+e)^2)*(d*ta
n(f*x+e))^(5/2)/d/f/(b*sin(f*x+e))^(4/3)
```

## Mathematica [A] (verified)

Time = 10.86 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = -\frac{2d\left(-7+4\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)\right)(b\sin(e+fx))^{2/3}\sqrt{d\tan(e+fx)}}{7b^2f}$$

input
```
Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(4/3),x]
```

output
```
(-2*d*(-7 + 4*(Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 7/12, 19/12, S
in[e + f*x]^2])*(b*Sin[e + f*x])^(2/3)*Sqrt[d*Tan[e + f*x]])/(7*b^2*f)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{ 3082}$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{\sqrt[6]{b\sin(e+fx)}}{\cos^{\frac{3}{2}}(e+fx)}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{\sqrt[6]{b\sin(e+fx)}}{\cos(e+fx)^{3/2}}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{6\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{19}{12},\sin^2(e+fx)\right)}{7df(b\sin(e+fx))^{4/3}}$$

input
```
Int[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(4/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[7/12, 5/4, 19/12, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(5/2))/(7*d*f*(b*Sin[e + f*x])^(4/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{(d\tan (fx+e))^{\frac{3}{2}}}{(b\sin (fx+e))^{\frac{4}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x)
```

output
```
int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x)
```

## Fricas [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{(d\tan (fx+e))^{\frac{3}{2}}}{(b\sin (fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(-(b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b^2*
cos(f*x + e)^2 - b^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*tan(f*x+e))**(3/2)/(b*sin(f*x+e))**(4/3),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(4/3), x)
```

## Giac [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(e+f\,x))^{3/2}}{(b\,\sin{(e+f\,x)})^{4/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(4/3),x)
```

output
```
int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(fx+e)}\,\tan(fx+e)}{\sin(fx+e)^{\frac{4}{3}}}dx\right)d}{b^{\frac{4}{3}}}$$

input
```
int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x)
```

output
```
(sqrt(d)*int((sqrt(tan(e + f*x))*tan(e + f*x))/(sin(e + f*x)**(1/3)*sin(e
+ f*x)),x)*d)/(b**(1/3)*b)
```

# 3.153 $\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1210
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . 1210
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1211
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1212
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1212
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1213
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1213
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1213
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1214
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1214

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{6\cos^2(e+fx)^{7/6}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx}{17df}$$

output
```
6/17*(cos(f*x+e)^2)^(7/6)*hypergeom([7/6, 17/12],[29/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(7/3)/d/f
```

## Mathematica [A] (verified)

Time = 11.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = -\frac{3d\left(-1+\text{Hypergeometric2F1}\left(\frac{5}{12},\frac{5}{4},\frac{17}{12},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)\sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+f}}{f}$$

input
```
Integrate[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(-3*d*(-1 + Hypergeometric2F1[5/12, 5/4, 17/12, -Tan[e + f*x]^2]*(Sec[e +
f*x]^2)^(1/4))*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3))/f
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\int \frac{(b\sin(e+fx))^{11/6}}{\cos^{\frac{4}{3}}(e+fx)}dx}{d(b\sin(e+fx))^{7/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\int \frac{(b\sin(e+fx))^{11/6}}{\cos(e+fx)^{4/3}}dx}{d(b\sin(e+fx))^{7/3}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{7/6}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)}{17df}$$

input
```
Int[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 17/12, 29/12, Sin[e + f*x
]^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(7/3))/(17*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \sqrt{b\sin\left(fx+e\right)}\,\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}dx$$

input
```
int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

## Fricas [F]

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*d*tan(f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(1/2)*(d*tan(f*x+e))**(4/3),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sin\left(fx+e\right)}(d\tan\left(fx+e\right))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

## Giac [F]

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sin\left(fx+e\right)}(d\tan\left(fx+e\right))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\,\sin\left(e+f\,x\right)}\,\left(d\tan(e+f\,x)\right)^{4/3}\,dx$$

input
```
int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(4/3),x)
```

output
```
int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{d^{\frac{4}{3}}\sqrt{b}\left(6\tan\left(fx+e\right)^{\frac{1}{3}}\sqrt{\sin\left(fx+e\right)}-3\left(\int\frac{\tan(fx+e)^{\frac{1}{3}}\sqrt{\sin(fx+e)}\,\cos(fx+e)}{\sin(fx+e)}dx\right)f-2\left(\int\frac{\sqrt{\sin(\ldots}}{\tan(f\ldots}\right.\right.}{2f}$$

input
```
int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

output
```
(d**(1/3)*sqrt(b)*d*(6*tan(e + f*x)**(1/3)*sqrt(sin(e + f*x)) - 3*int((tan
(e + f*x)**(1/3)*sqrt(sin(e + f*x))*cos(e + f*x))/sin(e + f*x),x)*f - 2*in
t((tan(e + f*x)**(1/3)*sqrt(sin(e + f*x)))/tan(e + f*x),x)*f))/(2*f)
```

# 3.154 $\int \sqrt{b \sin(e+fx)} \sqrt[3]{d \tan(e+fx)}\, dx$


Optimal result . . . . . 1215
Mathematica [A] (verified) . . . . . 1215
Rubi [A] (verified) . . . . . 1216
Maple [F] . . . . . 1217
Fricas [F] . . . . . 1217
Sympy [F] . . . . . 1218
Maxima [F] . . . . . 1218
Giac [F] . . . . . 1218
Mupad [F(-1)] . . . . . 1219
Reduce [F] . . . . . 1219


## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt{b \sin(e+fx)} \sqrt[3]{d \tan(e+fx)}\, dx = \frac{6 \cos^2(e+fx)^{2/3} \text{Hypergeometric2F1}\left(\frac{2}{3}, \frac{11}{12}, \frac{23}{12}, \sin^2(e+fx)\right) \sqrt{b \sin(e+fx)} (d \tan(e+fx))^{4/3}}{11 d f}$$

output
```
6/11*(cos(f*x+e)^2)^(2/3)*hypergeom([2/3, 11/12],[23/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3)/d/f
```

## Mathematica [A] (verified)

Time = 10.84 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \sqrt{b \sin(e+fx)} \sqrt[3]{d \tan(e+fx)}\, dx = \frac{6 \text{Hypergeometric2F1}\left(\frac{11}{12}, \frac{5}{4}, \frac{23}{12}, -\tan^2(e+fx)\right) \sqrt[4]{\sec^2(e+fx)} \sqrt{b \sin(e+fx)} (d \tan(e+fx))^{4/3}}{11 d f}$$

input
```
Integrate[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*Hypergeometric2F1[11/12, 5/4, 23/12, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^
(1/4)*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(11*d*f)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 3082}$$

$$\frac{b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\int \frac{(b\sin(e+fx))^{5/6}}{\sqrt[3]{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{4/3}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\int \frac{(b\sin(e+fx))^{5/6}}{\sqrt[3]{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{4/3}}$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{6\cos^2(e+fx)^{2/3}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)}{11df}$$

input
```
Int[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[2/3, 11/12, 23/12, Sin[e + f*x
]^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(11*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \sqrt{b\sin\left(fx+e\right)}\,\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}dx$$

input
```
int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int \sqrt{b\sin(e+fx)}\,\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

## Sympy [F]

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sin\left(e+fx\right)}\sqrt[3]{d\tan\left(e+fx\right)}\,dx$$

input
```
integrate((b*sin(f*x+e))**(1/2)*(d*tan(f*x+e))**(1/3),x)
```

output
```
Integral(sqrt(b*sin(e + f*x))*(d*tan(e + f*x))**(1/3), x)
```

## Maxima [F]

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)}(d\tan\left(fx+e\right))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)}(d\tan\left(fx+e\right))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\,\sin\left(e+f\,x\right)}\,\left(d\tan(e+f\,x)\right)^{1/3}\,dx$$

input
```
int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(1/3),x)
```

output
```
int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = d^{\frac{1}{3}}\sqrt{b}\left(\int \tan\left(fx+e\right)^{\frac{1}{3}}\sqrt{\sin\left(fx+e\right)}dx\right)$$

input
```
int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

output
```
d**(1/3)*sqrt(b)*int(tan(e + f*x)**(1/3)*sqrt(sin(e + f*x)),x)
```

# 3.155 $\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1220
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1220
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1221
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1222
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1222
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1223
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1223
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1223
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1224
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1224

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{6\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}}{7df}$$

output
```
6/7*(cos(f*x+e)^2)^(1/3)*hypergeom([1/3, 7/12],[19/12],sin(f*x+e)^2)*(b*si
n(f*x+e))^(1/2)*(d*tan(f*x+e))^(2/3)/d/f
```

## Mathematica [A] (verified)

Time = 10.87 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{6\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{19}{12},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}}{7df}$$

input
```
Integrate[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*Hypergeometric2F1[7/12, 5/4, 19/12, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(
1/4)*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(7*d*f)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\int \sqrt[3]{\cos(e+fx)}\sqrt[6]{b\sin(e+fx)}dx}{d(b\sin(e+fx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\int \sqrt[3]{\cos(e+fx)}\sqrt[6]{b\sin(e+fx)}dx}{d(b\sin(e+fx))^{2/3}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\sqrt[3]{\cos^2(e+fx)}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)}{7df}$$

input
```
Int[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 7/12, 19/12, Sin[e + f*x]
^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(7*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{\sqrt{b \sin \left(f x+e\right)}}{\left(d \tan \left(f x+e\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int \frac{\sqrt{b \sin(e+fx)}}{\sqrt[3]{d \tan(e+fx)}}\,dx=\int \frac{\sqrt{b \sin \left(f x+e\right)}}{\left(d \tan \left(f x+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)/(d*tan(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx=\int \frac{\sqrt{b\sin{(e+fx)}}}{\sqrt[3]{d\tan{(e+fx)}}}\,dx$$

input
```
integrate((b*sin(f*x+e))**(1/2)/(d*tan(f*x+e))**(1/3),x)
```

output
```
Integral(sqrt(b*sin(e + f*x))/(d*tan(e + f*x))**(1/3), x)
```

## Maxima [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx=\int \frac{\sqrt{b\sin{(fx+e)}}}{(d\tan{(fx+e)})^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx=\int \frac{\sqrt{b\sin{(fx+e)}}}{(d\tan{(fx+e)})^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\,\sin\left(e+f\,x\right)}}{\left(d\tan\left(e+f\,x\right)\right)^{1/3}}\,dx$$

input
```
int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3),x)
```

output
```
int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}}{\tan(fx+e)^{\frac{1}{3}}}dx\right)}{d^{\frac{1}{3}}}$$

input
```
int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

output
```
(sqrt(b)*int(sqrt(sin(e + f*x))/tan(e + f*x)**(1/3),x))/d**(1/3)
```

# 3.156 $\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$


Optimal result . . . 1225
Mathematica [A] (verified) . . . 1225
Rubi [A] (verified) . . . 1226
Maple [F] . . . 1227
Fricas [F] . . . 1228
Sympy [F(-1)] . . . 1228
Maxima [F] . . . 1228
Giac [F] . . . 1229
Mupad [F(-1)] . . . 1229
Reduce [F] . . . 1229


## Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx=\frac{6\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{13}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}}{df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

output
```
6*hypergeom([-1/6, 1/12],[13/12],sin(f*x+e)^2)*(b*sin(f*x+e))^(1/2)/d/f/(c
os(f*x+e)^2)^(1/6)/(d*tan(f*x+e))^(1/3)
```

## Mathematica [A] (verified)

Time = 31.56 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx=\frac{3\sqrt[4]{\sec^2(e+fx)}\sqrt{b\sin(e+fx)}\left(13\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{1}{4},\frac{13}{12},-\tan^2(e+f\right.\right.}{}$$

input
```
Integrate[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(3*(Sec[e + f*x]^2)^(1/4)*Sqrt[b*Sin[e + f*x]]*(13*Hypergeometric2F1[1/12,
 1/4, 13/12, -Tan[e + f*x]^2] + 13*Hypergeometric2F1[1/12, 5/4, 13/12, -Ta
n[e + f*x]^2] - Hypergeometric2F1[13/12, 5/4, 25/12, -Tan[e + f*x]^2]*Tan[
e + f*x]^2))/(13*d*f*(d*Tan[e + f*x])^(1/3))
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{ 3082}$$

$$\frac{b\sqrt[3]{b\sin(e+fx)}\int \frac{\cos^{\frac{4}{3}}(e+fx)}{(b\sin(e+fx))^{5/6}}dx}{d\sqrt[3]{\cos(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt[3]{b\sin(e+fx)}\int \frac{\cos(e+fx)^{4/3}}{(b\sin(e+fx))^{5/6}}dx}{d\sqrt[3]{\cos(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{6\sqrt{b\sin(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{13}{12},\sin^2(e+fx)\right)}{df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

input
```
Int[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(6*Hypergeometric2F1[-1/6, 1/12, 13/12, Sin[e + f*x]^2]*Sqrt[b*Sin[e + f*x
]])/(d*f*(Cos[e + f*x]^2)^(1/6)*(d*Tan[e + f*x])^(1/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{\sqrt{b\sin \left(fx+e\right)}}{\left(d\tan \left(fx+e\right)\right)^{\frac{4}{3}}}dx$$

input
```
int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

## Fricas [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)/(d^2*tan(f*x + e)^2),
 x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(1/2)/(d*tan(f*x+e))**(4/3),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

## Giac [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sin\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\,\sin\left(e+f\,x\right)}}{\left(d\tan\left(e+f\,x\right)\right)^{4/3}}\,dx$$

input
```
int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3),x)
```

output
```
int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}}{\tan(fx+e)^{\frac{4}{3}}}dx\right)}{d^{\frac{4}{3}}}$$

input
```
int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

output
```
(sqrt(b)*int(sqrt(sin(e + f*x))/(tan(e + f*x)**(1/3)*tan(e + f*x)),x))/(d*
*(1/3)*d)
```

# 3.157 $\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$

Optimal result . . . . . . . . . . 1230
Mathematica [A] (verified) . . . . . . . . . . 1230
Rubi [A] (verified) . . . . . . . . . . 1231
Maple [F] . . . . . . . . . . 1232
Fricas [F] . . . . . . . . . . 1233
Sympy [F(-1)] . . . . . . . . . . 1233
Maxima [F] . . . . . . . . . . 1233
Giac [F] . . . . . . . . . . 1234
Mupad [F(-1)] . . . . . . . . . . 1234
Reduce [F] . . . . . . . . . . 1234

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{6\cos^2(e+fx)^{7/6}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)(b\sin(e+fx$$

output
```
6/23*(cos(f*x+e)^2)^(7/6)*hypergeom([7/6, 23/12],[35/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(7/3)/d/f
```

## Mathematica [A] (verified)

Time = 11.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{3d\left(-\text{Hypergeometric2F1}\left(\frac{11}{12},\frac{7}{4},\frac{23}{12},-\tan^2(e+fx)\right)\sec^2(e+fx)+\sqrt[4]{\sec}\right.}{f\sqrt[4]{\sec^2(e+fx)}}$$

input
```
Integrate[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(3*d*(-(Hypergeometric2F1[11/12, 7/4, 23/12, -Tan[e + f*x]^2]*Sec[e + f*x]
^2) + (Sec[e + f*x]^2)^(1/4))*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3
))/(f*(Sec[e + f*x]^2)^(1/4))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}dx$$

$$\Big\downarrow \; 3082$$

$$\frac{b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\int \frac{(b\sin(e+fx))^{17/6}}{\cos^{\frac{4}{3}}(e+fx)}dx}{d(b\sin(e+fx))^{7/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\int \frac{(b\sin(e+fx))^{17/6}}{\cos(e+fx)^{4/3}}dx}{d(b\sin(e+fx))^{7/3}}$$

$$\Big\downarrow \; 3057$$

$$\frac{6\cos^2(e+fx)^{7/6}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)}{23df}$$

input
```
Int[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 23/12, 35/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(7/3))/(23*d*f)
```

**Defintions of rubi rules used**

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (b\sin(fx+e))^{\frac{3}{2}} (d\tan(fx+e))^{\frac{4}{3}} dx$$

input
```
int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

## Fricas [F]

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*b*d*sin(f*x + e)*tan(
f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(3/2)*(d*tan(f*x+e))**(4/3),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

## Giac [F]

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx=\int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx=\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$$

input
```
int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(4/3),x)
```

output
```
int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx=d^{\frac{4}{3}}\sqrt{b}\left(\int \tan(fx+e)^{\frac{4}{3}}\sqrt{\sin(fx+e)}\,\sin(fx+e)\,dx\right)b$$

input
```
int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

output
```
d**(1/3)*sqrt(b)*int(tan(e + f*x)**(1/3)*sqrt(sin(e + f*x))*sin(e + f*x)*t
an(e + f*x),x)*b*d
```

# 3.158 $\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$

Optimal result . . . . . 1235
Mathematica [A] (verified) . . . . . 1235
Rubi [A] (verified) . . . . . 1236
Maple [F] . . . . . 1237
Fricas [F] . . . . . 1237
Sympy [F(-1)] . . . . . 1238
Maxima [F] . . . . . 1238
Giac [F(-1)] . . . . . 1238
Mupad [F(-1)] . . . . . 1239
Reduce [F] . . . . . 1239

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)(b\sin(e+fx}{17df}$$

output
```
6/17*(cos(f*x+e)^2)^(2/3)*hypergeom([2/3, 17/12],[29/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3)/d/f
```

## Mathematica [A] (verified)

Time = 10.97 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{6\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{17}{12},\frac{7}{4},\frac{29}{12},-\tan^2(e+fx)\right)\sec^2(e+fx)^{7/}}{17bf}$$

input
```
Integrate[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*Cos[e + f*x]*Hypergeometric2F1[17/12, 7/4, 29/12, -Tan[e + f*x]^2]*(Sec
[e + f*x]^2)^(7/4)*(b*Sin[e + f*x])^(5/2)*(d*Tan[e + f*x])^(1/3))/(17*b*f)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\int \frac{(b\sin(e+fx))^{11/6}}{\sqrt[3]{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{4/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\int \frac{(b\sin(e+fx))^{11/6}}{\sqrt[3]{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{4/3}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{2/3}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)}{17df}$$

input
```
Int[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[2/3, 17/12, 29/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3))/(17*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{1}{3}}dx$$

input
```
int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*b*sin(f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(3/2)*(d*tan(f*x+e))**(1/3),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}}\,(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

## Giac [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int \left(b\,\sin\left(e+f\,x\right)\right)^{3/2}\left(d\tan(e+f\,x)\right)^{1/3}\,dx$$

input
```
int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(1/3),x)
```

output
```
int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = d^{\frac{1}{3}}\sqrt{b}\left(\int \tan\left(fx+e\right)^{\frac{1}{3}}\sqrt{\sin\left(fx+e\right)}\,\sin\left(fx+e\right)dx\right)b$$

input
```
int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

output
```
d**(1/3)*sqrt(b)*int(tan(e + f*x)**(1/3)*sqrt(sin(e + f*x))*sin(e + f*x),x
)*b
```

# 3.159 $\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1240
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1240
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1241
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1242
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1242
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1243
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1243
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1243
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1244
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1244


## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{6\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{25}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(}{13df}$$

output
```
6/13*(cos(f*x+e)^2)^(1/3)*hypergeom([1/3, 13/12],[25/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(2/3)/d/f
```

## Mathematica [A] (verified)

Time = 11.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{2d\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)(b\sin(}{3f(d\tan(e+fx))^{4/3}}$$

input
```
Integrate[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(2*d*(-1 + Hypergeometric2F1[1/12, 3/4, 13/12, -Tan[e + f*x]^2]*(Sec[e + f
*x]^2)^(3/4))*(b*Sin[e + f*x])^(3/2))/(3*f*(d*Tan[e + f*x])^(4/3))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}dx$$
$$\Big\downarrow \; 3082$$
$$\frac{b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\int \sqrt[3]{\cos(e+fx)}(b\sin(e+fx))^{7/6}dx}{d(b\sin(e+fx))^{2/3}}$$
$$\Big\downarrow \; 3042$$
$$\frac{b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\int \sqrt[3]{\cos(e+fx)}(b\sin(e+fx))^{7/6}dx}{d(b\sin(e+fx))^{2/3}}$$
$$\Big\downarrow \; 3057$$
$$\frac{6\sqrt[3]{\cos^2(e+fx)}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{25}{12},\sin^2(e+fx)\right)}{13df}$$

input
```
Int[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 13/12, 25/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(2/3))/(13*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sin(f*x + e)/(d*tan
(f*x + e)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(3/2)/(d*tan(f*x+e))**(1/3),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\,\sin\left(e+f\,x\right))^{3/2}}{(d\tan\left(e+f\,x\right))^{1/3}}\,dx$$

input
```
int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3),x)
```

output
```
int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sin(fx+e)}{\tan(fx+e)^{\frac{1}{3}}}dx\right)b}{d^{\frac{1}{3}}}$$

input
```
int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sin(e + f*x))/tan(e + f*x)**(1/3),x)*b)/d
**(1/3)
```

# 3.160 $\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1245
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1245
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1246
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1247
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1247
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1248
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1248
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1248
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1249
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1249

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{6\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

output
```
6/7*hypergeom([-1/6, 7/12],[19/12],sin(f*x+e)^2)*(b*sin(f*x+e))^(3/2)/d/f/
(cos(f*x+e)^2)^(1/6)/(d*tan(f*x+e))^(1/3)
```

## Mathematica [A] (verified)

Time = 10.95 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{2\left(7+2\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)(b\sin(e}{21df\sqrt[3]{d\tan(e+fx)}}$$

input
```
Integrate[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(2*(7 + 2*Hypergeometric2F1[7/12, 3/4, 19/12, -Tan[e + f*x]^2]*(Sec[e + f*
x]^2)^(3/4))*(b*Sin[e + f*x])^(3/2))/(21*d*f*(d*Tan[e + f*x])^(1/3))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\sqrt[3]{b\sin(e+fx)}\int \cos^{\frac{4}{3}}(e+fx)\sqrt[6]{b\sin(e+fx)}dx}{d\sqrt[3]{\cos(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt[3]{b\sin(e+fx)}\int \cos(e+fx)^{4/3}\sqrt[6]{b\sin(e+fx)}dx}{d\sqrt[3]{\cos(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6(b\sin(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

input
```
Int[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(6*Hypergeometric2F1[-1/6, 7/12, 19/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(
3/2))/(7*d*f*(Cos[e + f*x]^2)^(1/6)*(d*Tan[e + f*x])^(1/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

## Fricas [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sin(f*x + e)/(d^2*t
an(f*x + e)^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(3/2)/(d*tan(f*x+e))**(4/3),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

## Giac [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\left(b\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(d\tan\left(e+f\,x\right)\right)^{4/3}}\,dx$$

input
```
int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3),x)
```

output
```
int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sin(fx+e)}{\tan(fx+e)^{\frac{4}{3}}}dx\right)b}{d^{\frac{4}{3}}}$$

input
```
int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sin(e + f*x))/(tan(e + f*x)**(1/3)*tan(e
+ f*x)),x)*b)/(d**(1/3)*d)
```

# 3.161 $\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx$


Optimal result . . . 1250
Mathematica [A] (verified) . . . 1250
Rubi [A] (verified) . . . 1251
Maple [F] . . . 1252
Fricas [F] . . . 1252
Sympy [F] . . . 1253
Maxima [F] . . . 1253
Giac [F] . . . 1253
Mupad [F(-1)] . . . 1254
Reduce [F] . . . 1254


## Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{4+m}{2},\frac{6+m}{2},\sin^2(e+fx)\right)(a\sin(e+fx))^{4+m}}{a^4f(4+m)}$$

output
```
hypergeom([2, 2+1/2*m],[3+1/2*m],sin(f*x+e)^2)*(a*sin(f*x+e))^(4+m)/a^4/f/
(4+m)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{4+m}{2},1+\frac{4+m}{2},\sin^2(e+fx)\right)\sin^4(e+fx)(a\sin(e+fx))^m}{f(4+m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
(Hypergeometric2F1[2, (4 + m)/2, 1 + (4 + m)/2, Sin[e + f*x]^2]*Sin[e + f*
x]^4*(a*Sin[e + f*x])^m)/(f*(4 + m))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3072, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)^3(a\sin(e+fx))^m dx$$

$$\Big\downarrow \; 3072$$

$$\frac{\int \frac{(a\sin(e+fx))^{m+3}}{\left(a^2-a^2\sin^2(e+fx)\right)^2}d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \; 278$$

$$\frac{(a\sin(e+fx))^{m+4}\text{Hypergeometric2F1}\left(2,\frac{m+4}{2},\frac{m+6}{2},\sin^2(e+fx)\right)}{a^4f(m+4)}$$

input
```
Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
(Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x
])^(4 + m))/(a^4*f*(4 + m))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

## Maple [F]

$$\int (a\sin (fx+e))^m \tan (fx+e)^3\,dx$$

input
```
int((a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

output
```
int((a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

## Fricas [F]

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \int (a\sin (fx+e))^m \tan (fx+e)^3\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

## Sympy [F]

$$\int (a\sin(e+fx))^m\tan^3(e+fx)\,dx=\int (a\sin{(e+fx)})^m\tan^3{(e+fx)}\,dx$$

input
```
integrate((a*sin(f*x+e))**m*tan(f*x+e)**3,x)
```

output
```
Integral((a*sin(e + f*x))**m*tan(e + f*x)**3, x)
```

## Maxima [F]

$$\int (a\sin(e+fx))^m\tan^3(e+fx)\,dx=\int (a\sin{(fx+e)})^m\tan{(fx+e)}^3\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

## Giac [F]

$$\int (a\sin(e+fx))^m\tan^3(e+fx)\,dx=\int (a\sin{(fx+e)})^m\tan{(fx+e)}^3\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3\,(a\,\sin(e+f\,x))^m\,dx$$

input
```
int(tan(e + f*x)^3*(a*sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^3*(a*sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = a^m\left(\int \sin(fx+e)^m \tan(fx+e)^3\,dx\right)$$

input
```
int((a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

output
```
a**m*int(sin(e + f*x)**m*tan(e + f*x)**3,x)
```

# 3.162 $\int (a\sin(e+fx))^m \tan(e+fx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1255
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 1255
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1256
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1257
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1257
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1258
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1258
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1258
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1259
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1259

## Optimal result

Integrand size = 17, antiderivative size = 48

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(e+fx)\right)(a\sin(e+fx))^{2+m}}{a^2 f(2+m)}$$

output
```
hypergeom([1, 1+1/2*m],[2+1/2*m],sin(f*x+e)^2)*(a*sin(f*x+e))^(2+m)/a^2/f/
(2+m)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},1+\frac{2+m}{2},\sin^2(e+fx)\right)\sin^2(e+fx)(a\sin(e+fx))^m}{f(2+m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

output
```
(Hypergeometric2F1[1, (2 + m)/2, 1 + (2 + m)/2, Sin[e + f*x]^2]*Sin[e + f*
x]^2*(a*Sin[e + f*x])^m)/(f*(2 + m))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3072, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{(a\sin(e+fx))^{m+1}}{a^2-a^2\sin^2(e+fx)}d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(a\sin(e+fx))^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},\sin^2(e+fx)\right)}{a^2f(m+2)}$$

input
```
Int[(a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

output
```
(Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x
])^(2 + m))/(a^2*f*(2 + m))
```

**Defintions of rubi rules used**

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

## Maple [F]

$$\int (a\sin(fx+e))^m \tan(fx+e)\,dx$$

input
```
int((a*sin(f*x+e))^m*tan(f*x+e),x)
```

output
```
int((a*sin(f*x+e))^m*tan(f*x+e),x)
```

## Fricas [F]

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx = \int (a\sin(fx+e))^m \tan(fx+e)\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*tan(f*x + e), x)
```

## Sympy [F]

$$\int (a\sin(e+fx))^m\tan(e+fx)\,dx=\int (a\sin{(e+fx)})^m\tan{(e+fx)}\,dx$$

input
```
integrate((a*sin(f*x+e))**m*tan(f*x+e),x)
```

output
```
Integral((a*sin(e + f*x))**m*tan(e + f*x), x)
```

## Maxima [F]

$$\int (a\sin(e+fx))^m\tan(e+fx)\,dx=\int (a\sin{(fx+e)})^m\tan{(fx+e)}\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e), x)
```

## Giac [F]

$$\int (a\sin(e+fx))^m\tan(e+fx)\,dx=\int (a\sin{(fx+e)})^m\tan{(fx+e)}\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m\tan(e+fx)\,dx = \int \tan(e+f\,x)\,\left(a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

input
```
int(tan(e + f*x)*(a*sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)*(a*sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int (a\sin(e+fx))^m\tan(e+fx)\,dx = a^m\left(\int \sin\left(fx+e\right)^m\tan\left(fx+e\right)dx\right)$$

input
```
int((a*sin(f*x+e))^m*tan(f*x+e),x)
```

output
```
a**m*int(sin(e + f*x)**m*tan(e + f*x),x)
```

# 3.163 $\int \cot(e+fx)(a\sin(e+fx))^m\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1260
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1260
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1261
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1262
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1262
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1263
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1263
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1263
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1264
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1264


## Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{(a\sin(e+fx))^m}{fm}$$

output
```
(a*sin(f*x+e))^m/f/m
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{(a\sin(e+fx))^m}{fm}$$

input
```
Integrate[Cot[e + f*x]*(a*Sin[e + f*x])^m,x]
```

output
```
(a*Sin[e + f*x])^m/(f*m)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3072, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^m}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int (a\sin(e+fx))^{m-1}d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{15}$$

$$\frac{(a\sin(e+fx))^m}{fm}$$

input
```
Int[Cot[e + f*x]*(a*Sin[e + f*x])^m,x]
```

output
```
(a*Sin[e + f*x])^m/(f*m)
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| derivativedivides | $\frac{(a\sin(fx+e))^m}{fm}$ |
| default | $\frac{(a\sin(fx+e))^m}{fm}$ |
| risch | $\frac{\left(\frac{1}{2}\right)^m a^m \left(e^{i(fx+e)}\right)^{-m} \left(e^{2i(fx+e)}-1\right)^m e^{\frac{i\pi m\left(-\operatorname{csgn}(ia\sin(fx+e))^3-\operatorname{csgn}(ia\sin(fx+e))^2\operatorname{csgn}(a\sin(fx+e))+\operatorname{csgn}(a\sin(fx+e}{}}}{}$ |

input
```
int(cot(f*x+e)*(a*sin(f*x+e))^m,x,method=_RETURNVERBOSE)
```

output
```
(a*sin(f*x+e))^m/f/m
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{(a\sin(fx+e))^m}{fm}$$

input
```
integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
(a*sin(f*x + e))^m/(f*m)
```

## Sympy [F]

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(e+fx))^m \cot(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*cot(e + f*x), x)
```

## Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{a^m \sin(fx+e)^m}{fm}$$

input
```
integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
a^m*sin(f*x + e)^m/(f*m)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{(a\sin(fx+e))^m}{fm}$$

input
```
integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
(a*sin(f*x + e))^m/(f*m)
```

## Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot(e + fx)(a\sin(e + fx))^m \, dx = \frac{(a \sin(e + f x))^m}{f m}$$

input
```
int(cot(e + f*x)*(a*sin(e + f*x))^m,x)
```

output
```
(a*sin(e + f*x))^m/(f*m)
```

## Reduce [F]

$$\int \cot(e + fx)(a\sin(e + fx))^m \, dx = a^m \left( \int \sin(fx + e)^m \cot(fx + e) \, dx \right)$$

input
```
int(cot(f*x+e)*(a*sin(f*x+e))^m,x)
```

output
```
a**m*int(sin(e + f*x)**m*cot(e + f*x),x)
```

# 3.164 $\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx$

Optimal result . . . . . . . . . . 1265
Mathematica [A] (verified) . . . . . . . . . . 1265
Rubi [A] (verified) . . . . . . . . . . 1266
Maple [C] (warning: unable to verify) . . . . . . . . . . 1267
Fricas [A] (verification not implemented) . . . . . . . . . . 1268
Sympy [F] . . . . . . . . . . 1269
Maxima [A] (verification not implemented) . . . . . . . . . . 1269
Giac [F] . . . . . . . . . . 1269
Mupad [B] (verification not implemented) . . . . . . . . . . 1270
Reduce [F] . . . . . . . . . . 1270

## Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx=-\frac{a^2(a\sin(e+fx))^{-2+m}}{f(2-m)}-\frac{(a\sin(e+fx))^m}{fm}$$

output
```
-a^2*(a*sin(f*x+e))^(-2+m)/f/(2-m)-(a*sin(f*x+e))^m/f/m
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx=\frac{(2-m+m\csc^2(e+fx))\,(a\sin(e+fx))^m}{f(-2+m)m}$$

input
```
Integrate[Cot[e + f*x]^3*(a*Sin[e + f*x])^m,x]
```

output
```
((2 - m + m*Csc[e + f*x]^2)*(a*Sin[e + f*x])^m)/(f*(-2 + m)*m)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3072, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx))^m}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \; 3072$$

$$\frac{\int (a\sin(e+fx))^{m-3}\left(a^2-a^2\sin^2(e+fx)\right)d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \; 244$$

$$\frac{\int \left(a^2(a\sin(e+fx))^{m-3}-(a\sin(e+fx))^{m-1}\right)d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{a^2(a\sin(e+fx))^{m-2}}{2-m}-\frac{(a\sin(e+fx))^m}{m}}{f}$$

input
```
Int[Cot[e + f*x]^3*(a*Sin[e + f*x])^m,x]
```

output
```
(-((a^2*(a*Sin[e + f*x])^(-2 + m))/(2 - m)) - (a*Sin[e + f*x])^m/m)/f
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.04 (sec) , antiderivative size = 2751, normalized size of antiderivative = 59.80

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2751 |

input
```
int(cot(f*x+e)^3*(a*sin(f*x+e))^m,x,method=_RETURNVERBOSE)
```

output
```
-1/(-2+m)/f/(exp(2*I*(f*x+e))-1)^2/m*exp(I*(f*x+e))^(-m)*(exp(2*I*(f*x+e))
-1)^m*(1/2)^m*a^m*(m*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*si
n(e))^3*Pi)*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2*P
i*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e)))*exp(1/2*I*m*Pi*csgn(a*sin(f*x
)*cos(e)+a*cos(f*x)*sin(e))^3)*exp(1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e)+a*cos
(f*x)*sin(e))^2*csgn(I*a))*exp(-1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e)+a*cos(f*
x)*sin(e))^2*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*exp(-1/2*I*m*Pi*csgn(a
*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*csgn(I*a)*csgn(sin(f*x)*cos(e)+cos(f*x
)*sin(e)))*exp(1/2*I*Pi*m*csgn(I*exp(2*I*(f*x+e))-I)*csgn(sin(f*x)*cos(e)+
cos(f*x)*sin(e))^2)*exp(1/2*I*Pi*m*csgn(I*exp(2*I*(f*x+e))-I)*csgn(sin(f*x
)*cos(e)+cos(f*x)*sin(e))*csgn(I*exp(-I*(f*x+e))))*exp(1/2*I*m*Pi*csgn(sin
(f*x)*cos(e)+cos(f*x)*sin(e))^3)*exp(1/2*I*Pi*m*csgn(sin(f*x)*cos(e)+cos(f
*x)*sin(e))^2*csgn(I*exp(-I*(f*x+e))))*exp(1/2*I*m*csgn(I*a*sin(f*x)*cos(e
)+I*a*cos(f*x)*sin(e))^2*Pi)*exp(1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(
f*x)*sin(e))*Pi*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e)))*exp(-1/2*I*Pi*m
)*exp(4*I*f*x)*exp(4*I*e)-2*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(
f*x)*sin(e))^3*Pi)*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(
e))^2*Pi*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e)))*exp(1/2*I*m*Pi*csgn(a*
sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^3)*exp(1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e
)+a*cos(f*x)*sin(e))^2*csgn(I*a))*exp(-1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e...
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = \frac{\left((m-2)\cos\left(fx+e\right)^2+2\right)\left(a\sin\left(fx+e\right)\right)^m}{fm^2-\left(fm^2-2\,fm\right)\cos\left(fx+e\right)^2-2\,fm}$$

input
```
integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
((m - 2)*cos(f*x + e)^2 + 2)*(a*sin(f*x + e))^m/(f*m^2 - (f*m^2 - 2*f*m)*c
os(f*x + e)^2 - 2*f*m)
```

## Sympy [F]

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(e+fx))^m\cot^3(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**3*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*cot(e + f*x)**3, x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = -\frac{\frac{a^m\sin(fx+e)^m}{m} - \frac{a^m\sin(fx+e)^m}{(m-2)\sin(fx+e)^2}}{f}$$

input
```
integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
-(a^m*sin(f*x + e)^m/m - a^m*sin(f*x + e)^m/((m - 2)*sin(f*x + e)^2))/f
```

## Giac [F]

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m\cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*cot(f*x + e)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.98

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = -\frac{(a\,\sin\left(e+f\,x\right))^m\,\left(m-4\sin\left(2\,e+2\,f\,x\right)^2+m\,\left(2\sin\left(2\,e+2\,f\,x\right)^2-1\right)+16\sin\left(e+f\,x\right)^2\right)}{f\,m\,\left(2\sin\left(2\,e+2\,f\,x\right)^2-8\sin\left(e+f\,x\right)^2\right)\,\left(m-2\right)}$$

input
```
int(cot(e + f*x)^3*(a*sin(e + f*x))^m,x)
```

output
```
-((a*sin(e + f*x))^m*(m - 4*sin(2*e + 2*f*x)^2 + m*(2*sin(2*e + 2*f*x)^2 -
 1) + 16*sin(e + f*x)^2))/(f*m*(2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2)*(
m - 2))
```

## Reduce [F]

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = a^m\left(\int \sin\left(fx+e\right)^m\cot\left(fx+e\right)^3dx\right)$$

input
```
int(cot(f*x+e)^3*(a*sin(f*x+e))^m,x)
```

output
```
a**m*int(sin(e + f*x)**m*cot(e + f*x)**3,x)
```

# 3.165 $\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx$

Optimal result . . . 1271
Mathematica [A] (verified) . . . 1271
Rubi [A] (verified) . . . 1272
Maple [C] (warning: unable to verify) . . . 1273
Fricas [A] (verification not implemented) . . . 1274
Sympy [F] . . . 1274
Maxima [A] (verification not implemented) . . . 1274
Giac [F] . . . 1275
Mupad [B] (verification not implemented) . . . 1275
Reduce [F] . . . 1276

## Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx=-\frac{a^4(a\sin(e+fx))^{-4+m}}{f(4-m)}+\frac{2a^2(a\sin(e+fx))^{-2+m}}{f(2-m)}+\frac{(a\sin(e+fx))^m}{fm}$$

output
```
-a^4*(a*sin(f*x+e))^(-4+m)/f/(4-m)+2*a^2*(a*sin(f*x+e))^(-2+m)/f/(2-m)+(a*
sin(f*x+e))^m/f/m
```

## Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx=\frac{(8-6m+m^2-2(-4+m)m\csc^2(e+fx)+(-2+m)m\csc^4(e+fx))\,(a\sin(e+fx))^m}{f(-4+m)(-2+m)m}$$

input
```
Integrate[Cot[e + f*x]^5*(a*Sin[e + f*x])^m,x]
```

output
```
((8 - 6*m + m^2 - 2*(-4 + m)*m*Csc[e + f*x]^2 + (-2 + m)*m*Csc[e + f*x]^4)
*(a*Sin[e + f*x])^m)/(f*(-4 + m)*(-2 + m)*m)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3072, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^m}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int (a\sin(e+fx))^{m-5}\left(a^2-a^2\sin^2(e+fx)\right)^2d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(a^4(a\sin(e+fx))^{m-5}-2a^2(a\sin(e+fx))^{m-3}+(a\sin(e+fx))^{m-1}\right)d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^4(a\sin(e+fx))^{m-4}}{4-m}+\frac{2a^2(a\sin(e+fx))^{m-2}}{2-m}+\frac{(a\sin(e+fx))^m}{m}}{f}$$

input
```
Int[Cot[e + f*x]^5*(a*Sin[e + f*x])^m,x]
```

output
```
(-((a^4*(a*Sin[e + f*x])^(-4 + m))/(4 - m)) + (2*a^2*(a*Sin[e + f*x])^(-2
+ m))/(2 - m) + (a*Sin[e + f*x])^m/m)/f
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.85 (sec) , antiderivative size = 6931, normalized size of antiderivative = 96.26

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 6931 |

input
```
int(cot(f*x+e)^5*(a*sin(f*x+e))^m,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.56

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \frac{\left((m^2-6\,m+8)\cos\left(fx+e\right)^4+4\,(m-4)\cos\left(fx+e\right)^2+8\right)\left(a\sin\left(fx+e\right)\right)^m}{(fm^3-6\,fm^2+8\,fm)\cos\left(fx+e\right)^4+fm^3-6\,fm^2-2\,(fm^3-6\,fm^2+8\,fm)\cos\left(fx+e\right)^2+8\,fm}$$

input
```
integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
((m^2 - 6*m + 8)*cos(f*x + e)^4 + 4*(m - 4)*cos(f*x + e)^2 + 8)*(a*sin(f*x
 + e))^m/((f*m^3 - 6*f*m^2 + 8*f*m)*cos(f*x + e)^4 + f*m^3 - 6*f*m^2 - 2*(
f*m^3 - 6*f*m^2 + 8*f*m)*cos(f*x + e)^2 + 8*f*m)
```

## Sympy [F]

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \int \left(a\sin\left(e+fx\right)\right)^m\cot^5\left(e+fx\right)\,dx$$

input
```
integrate(cot(f*x+e)**5*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*cot(e + f*x)**5, x)
```

## Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \frac{\frac{a^m\sin(fx+e)^m}{m}-\frac{2\,a^m\sin(fx+e)^m}{(m-2)\sin(fx+e)^2}+\frac{a^m\sin(fx+e)^m}{(m-4)\sin(fx+e)^4}}{f}$$

input
```
integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
(a^m*sin(f*x + e)^m/m - 2*a^m*sin(f*x + e)^m/((m - 2)*sin(f*x + e)^2) + a^
m*sin(f*x + e)^m/((m - 4)*sin(f*x + e)^4))/f
```

## Giac [F]

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx=\int (a\sin(fx+e))^m\cot(fx+e)^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*cot(f*x + e)^5, x)
```

## Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.04

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx=$$
$$-\frac{(a\sin(e+fx))^m\left(2\sin(2e+2fx)^2+\sin(4e+4fx)\,1\mathrm{i}-1\right)\left(-\frac{2\left(2\sin(2e+2fx)^2-1\right)\left(-2\sin(2e+2fx)^2+\right.}{fm}\right.}{}$$

input
```
int(cot(e + f*x)^5*(a*sin(e + f*x))^m,x)
```

output
```
-((a*sin(e + f*x))^m*(sin(4*e + 4*f*x)*1i + 2*sin(2*e + 2*f*x)^2 - 1)*(((s
in(4*e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1)*(6*m^2 - 4*m + 48))/(f*m*(m
^2 - 6*m + 8)) - (2*(2*sin(2*e + 2*f*x)^2 - 1)*(sin(4*e + 4*f*x)*1i - 2*si
n(2*e + 2*f*x)^2 + 1))/(f*m) + (2*(2*sin(e + f*x)^2 - 1)*(sin(4*e + 4*f*x)
*1i - 2*sin(2*e + 2*f*x)^2 + 1)*(8*m - 4*m^2 + 32))/(f*m*(m^2 - 6*m + 8)))
)/(16*sin(e + f*x)^4)
```

## Reduce [F]

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = a^m\left(\int \sin(fx+e)^m\cot(fx+e)^5\,dx\right)$$

input
```
int(cot(f*x+e)^5*(a*sin(f*x+e))^m,x)
```

output
```
a**m*int(sin(e + f*x)**m*cot(e + f*x)**5,x)
```

# 3.166 $\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1277
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1277
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1278
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1279
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1279
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1280
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1280
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1280
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1281
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1281

## Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7+m}{2},\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{5+m}}{a^5f(5+m)}$$

output
```
(cos(f*x+e)^2)^(1/2)*hypergeom([5/2, 5/2+1/2*m],[7/2+1/2*m],sin(f*x+e)^2)*
sec(f*x+e)*(a*sin(f*x+e))^(5+m)/a^5/f/(5+m)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7+m}{2},\sin^2(e+fx)\right)\sin^4(e+fx)(a\sin(e+fx))^m\tan(e+f}{f(5+m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[5/2, (5 + m)/2, (7 + m)/2, Sin[e +
 f*x]^2]*Sin[e + f*x]^4*(a*Sin[e + f*x])^m*Tan[e + f*x])/(f*(5 + m))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3080, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3080}$$

$$\frac{\int \sec^4(e+fx)(a\sin(e+fx))^{m+4}dx}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(a\sin(e+fx))^{m+4}}{\cos(e+fx)^4}dx}{a^4}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)(a\sin(e+fx))^{m+5}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+5}{2},\frac{m+7}{2},\sin^2(e+fx)\right)}{a^5f(m+5)}$$

input
```
Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[5/2, (5 + m)/2, (7 + m)/2, Sin[e +
 f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3080
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Simp[1/a^n   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /
; FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

## Maple [F]

$$\int (a\sin (fx+e))^m \tan (fx+e)^4 \, dx$$

input
```
int((a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

output
```
int((a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

## Fricas [F]

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \int (a\sin (fx+e))^m \tan (fx+e)^4 \, dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

## Sympy [F]

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \int (a\sin{(e+fx)})^m \tan^4{(e+fx)}\,dx$$

input
```
integrate((a*sin(f*x+e))**m*tan(f*x+e)**4,x)
```

output
```
Integral((a*sin(e + f*x))**m*tan(e + f*x)**4, x)
```

## Maxima [F]

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \int (a\sin{(fx+e)})^m \tan{(fx+e)}^4 \,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

## Giac [F]

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \int (a\sin{(fx+e)})^m \tan{(fx+e)}^4 \,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\,(a\,\sin(e+f\,x))^m\,dx$$

input
```
int(tan(e + f*x)^4*(a*sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^4*(a*sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx = a^m\left(\int \sin(fx+e)^m\tan(fx+e)^4\,dx\right)$$

input
```
int((a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

output
```
a**m*int(sin(e + f*x)**m*tan(e + f*x)**4,x)
```

# 3.167 $\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1282
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1282
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1283
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1284
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1284
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1285
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1285
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1285
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1286
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1286


## Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx$$
$$= \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{3+m}}{a^3f(3+m)}$$

output
```
(cos(f*x+e)^2)^(1/2)*hypergeom([3/2, 3/2+1/2*m],[5/2+1/2*m],sin(f*x+e)^2)*
sec(f*x+e)*(a*sin(f*x+e))^(3+m)/a^3/f/(3+m)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx$$
$$= \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(e+fx)\right)\sin^2(e+fx)(a\sin(e+fx))^m\tan(e+f}{f(3+m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[e +
 f*x]^2]*Sin[e + f*x]^2*(a*Sin[e + f*x])^m*Tan[e + f*x])/(f*(3 + m))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3080, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3080}$$

$$\frac{\int \sec^2(e+fx)(a\sin(e+fx))^{m+2}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(a\sin(e+fx))^{m+2}}{\cos(e+fx)^2}dx}{a^2}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)(a\sin(e+fx))^{m+3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+3}{2},\frac{m+5}{2},\sin^2(e+fx)\right)}{a^3f(m+3)}$$

input
```
Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[e +
 f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3080
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Simp[1/a^n   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /
; FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

## Maple [F]

$$\int (a\sin (fx+e))^m \tan (fx+e)^2\,dx$$

input
```
int((a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

output
```
int((a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

## Fricas [F]

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx = \int (a\sin (fx+e))^m \tan (fx+e)^2\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

## Sympy [F]

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx = \int (a\sin(e+fx))^m \tan^2(e+fx)\,dx$$

input
```
integrate((a*sin(f*x+e))**m*tan(f*x+e)**2,x)
```

output
```
Integral((a*sin(e + f*x))**m*tan(e + f*x)**2, x)
```

## Maxima [F]

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx = \int (a\sin(fx+e))^m \tan(fx+e)^2\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

## Giac [F]

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx = \int (a\sin(fx+e))^m \tan(fx+e)^2\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx=\int \tan(e+f\,x)^2\,(a\,\sin(e+f\,x))^m\,dx$$

input
```
int(tan(e + f*x)^2*(a*sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^2*(a*sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx=a^m\left(\int \sin(fx+e)^m\tan(fx+e)^2\,dx\right)$$

input
```
int((a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

output
```
a**m*int(sin(e + f*x)**m*tan(e + f*x)**2,x)
```

# 3.168 $\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1287
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1287
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1288
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1289
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1289
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1290
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1290
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1290
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1291
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1291


## Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sin^2(e+fx)\right)(a\sin(e+fx))^{-1+m}}{f(1-m)\sqrt{\cos^2(e+fx)}}$$

output
```
-a*cos(f*x+e)*hypergeom([-1/2, -1/2+1/2*m],[1/2+1/2*m],sin(f*x+e)^2)*(a*si
n(f*x+e))^(-1+m)/f/(1-m)/(cos(f*x+e)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \frac{a\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{-1-}}{f(-1+m)}$$

input
```
Integrate[Cot[e + f*x]^2*(a*Sin[e + f*x])^m,x]
```

output
```
(a*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin
[e + f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(-1 + m))/(f*(-1 + m))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3080, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^m}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{3080}$$

$$a^2\int \cos^2(e+fx)(a\sin(e+fx))^{m-2}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \cos(e+fx)^2(a\sin(e+fx))^{m-2}dx$$

$$\Big\downarrow \text{3057}$$

$$-\frac{a\cos(e+fx)(a\sin(e+fx))^{m-1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-1}{2},\frac{m+1}{2},\sin^2(e+fx)\right)}{f(1-m)\sqrt{\cos^2(e+fx)}}$$

input
```
Int[Cot[e + f*x]^2*(a*Sin[e + f*x])^m,x]
```

output
```
-((a*Cos[e + f*x]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[e + f
*x]^2]*(a*Sin[e + f*x])^(-1 + m))/(f*(1 - m)*Sqrt[Cos[e + f*x]^2]))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3080
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Simp[1/a^n   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /
; FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

## Maple [F]

$$\int \cot (fx+e)^2 (a\sin (fx+e))^m dx$$

input
```
int(cot(f*x+e)^2*(a*sin(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^2*(a*sin(f*x+e))^m,x)
```

## Fricas [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m dx = \int (a\sin (fx+e))^m \cot (fx+e)^2 dx$$

input
```
integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

## Sympy [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(e+fx))^m \cot^2(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**2*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*cot(e + f*x)**2, x)
```

## Maxima [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m \cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

## Giac [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m \cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int \cot(e+f\,x)^2\,(a\,\sin(e+f\,x))^m\,dx$$

input
```
int(cot(e + f*x)^2*(a*sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^2*(a*sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = a^m\left(\int \sin(fx+e)^m\cot(fx+e)^2\,dx\right)$$

input
```
int(cot(f*x+e)^2*(a*sin(f*x+e))^m,x)
```

output
```
a**m*int(sin(e + f*x)**m*cot(e + f*x)**2,x)
```

# 3.169 $\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1292
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1292
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1293
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1294
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1294
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1295
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1295
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1295
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1296
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1296

## Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a^3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(e+fx)\right)(a\sin(e+fx))^{-3+m}}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

output
```
-a^3*cos(f*x+e)*hypergeom([-3/2, -3/2+1/2*m],[-1/2+1/2*m],sin(f*x+e)^2)*(a
*sin(f*x+e))^(-3+m)/f/(3-m)/(cos(f*x+e)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \frac{\sqrt{\cos^2(e+fx)}\csc^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(e+fx)\right)\sec(e+fx}{f(-3+m)}$$

input
```
Integrate[Cot[e + f*x]^4*(a*Sin[e + f*x])^m,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Csc[e + f*x]^3*Hypergeometric2F1[-3/2, (-3 + m)/2, (
-1 + m)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^m)/(f*(-3 + m))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3080, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^m}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{3080}$$

$$a^4\int \cos^4(e+fx)(a\sin(e+fx))^{m-4}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4\int \cos(e+fx)^4(a\sin(e+fx))^{m-4}dx$$

$$\Big\downarrow \text{3057}$$

$$-\frac{a^3\cos(e+fx)(a\sin(e+fx))^{m-3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-3}{2},\frac{m-1}{2},\sin^2(e+fx)\right)}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

input
```
Int[Cot[e + f*x]^4*(a*Sin[e + f*x])^m,x]
```

output
```
-((a^3*Cos[e + f*x]*Hypergeometric2F1[-3/2, (-3 + m)/2, (-1 + m)/2, Sin[e
+ f*x]^2]*(a*Sin[e + f*x])^(-3 + m))/(f*(3 - m)*Sqrt[Cos[e + f*x]^2]))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3080
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Simp[1/a^n   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /
; FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

## Maple [F]

$$\int \cot (fx+e)^4 (a\sin (fx+e))^m\,dx$$

input
```
int(cot(f*x+e)^4*(a*sin(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^4*(a*sin(f*x+e))^m,x)
```

## Fricas [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin (fx+e))^m \cot (fx+e)^4 \,dx$$

input
```
integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

## Sympy [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin{(e+fx)})^m \cot^4{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**4*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*cot(e + f*x)**4, x)
```

## Maxima [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin{(fx+e)})^m \cot{(fx+e)}^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

## Giac [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin{(fx+e)})^m \cot{(fx+e)}^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int \cot(e+f\,x)^4\,(a\,\sin(e+f\,x))^m\,dx$$

input
```
int(cot(e + f*x)^4*(a*sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^4*(a*sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = a^m\left(\int \sin(fx+e)^m\cot(fx+e)^4\,dx\right)$$

input
```
int(cot(f*x+e)^4*(a*sin(f*x+e))^m,x)
```

output
```
a**m*int(sin(e + f*x)**m*cot(e + f*x)**4,x)
```

# 3.170 $\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx$

Optimal result . . . 1297
Mathematica [A] (verified) . . . 1297
Rubi [A] (verified) . . . 1298
Maple [F] . . . 1299
Fricas [F] . . . 1300
Sympy [F(-1)] . . . 1300
Maxima [F] . . . 1300
Giac [F] . . . 1301
Mupad [F(-1)] . . . 1301
Reduce [F] . . . 1301

## Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx=\frac{2\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(5+2m),\frac{1}{4}(9+2m),\sin^2(e+fx)\right)(a\sin(e+fx$$
$$\overline{bf(5+2m)}$$

output
```
2*(cos(f*x+e)^2)^(5/4)*hypergeom([5/4, 5/4+1/2*m],[9/4+1/2*m],sin(f*x+e)^2
)*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(5/2)/b/f/(5+2*m)
```

## Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx=\frac{2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1}{4}(5+2m),\frac{1}{4}(9+2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}(a\sin(e$$
$$\overline{bf(5+2m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*Hypergeometric2F1[(2 + m)/2, (5 + 2*m)/4, (9 + 2*m)/4, -Tan[e + f*x]^2]
*(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(5/2))/(b*f*(5
 + 2*m))
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{3/2}(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (b\tan(e+fx))^{3/2}(a\sin(e+fx))^m dx$$

$$\Big\downarrow \; 3082$$

$$\frac{a\cos^{\frac{5}{2}}(e+fx)(b\tan(e+fx))^{5/2}\int \frac{(a\sin(e+fx))^{m+\frac{3}{2}}}{\cos^{\frac{3}{2}}(e+fx)}dx}{b(a\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\cos^{\frac{5}{2}}(e+fx)(b\tan(e+fx))^{5/2}\int \frac{(a\sin(e+fx))^{m+\frac{3}{2}}}{\cos(e+fx)^{3/2}}dx}{b(a\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3057$$

$$\frac{2\cos^2(e+fx)^{5/4}(b\tan(e+fx))^{5/2}(a\sin(e+fx))^m \text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(2m+5),\frac{1}{4}(2m+9),\sin^2(e+fx)\right)}{bf(2m+5)}$$

input
```
Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, (5 + 2*m)/4, (9 + 2*m)/4,
 Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(5/2))/(b*f*(5 + 2*m)
)
```

**Defintions of rubi rules used**

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (a\sin(fx+e))^m (b\tan(fx+e))^{\frac{3}{2}} dx$$

input
```
int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x)
```

output
```
int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x)
```

## Fricas [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx=\int (b\tan(fx+e))^{\frac{3}{2}}\,(a\sin(fx+e))^m\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m*b*tan(f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx=\int (b\tan(fx+e))^{\frac{3}{2}}\,(a\sin(fx+e))^m\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)*(a*sin(f*x + e))^m, x)
```

## Giac [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx=\int (b\tan(fx+e))^{\frac{3}{2}}(a\sin(fx+e))^m\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)*(a*sin(f*x + e))^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx=\int (a\,\sin(e+f\,x))^m\,(b\tan(e+f\,x))^{3/2}\,dx$$

input
```
int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx=\frac{\sqrt{b}\,a^m b\left(2\sqrt{\tan(fx+e)}\,\sin(fx+e)^m-2\left(\int\frac{\sqrt{\tan(fx+e)}\,\sin(fx+e)^m\cos(fx+e)}{\sin(fx+e)}dx\right)fm-\left(\int\frac{\sqrt{\mathrm{ta}}}{}\right.\right.}{f}$$

input
```
int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*a**m*b*(2*sqrt(tan(e + f*x))*sin(e + f*x)**m - 2*int((sqrt(tan(e
+ f*x))*sin(e + f*x)**m*cos(e + f*x))/sin(e + f*x),x)*f*m - int((sqrt(tan(
e + f*x))*sin(e + f*x)**m)/tan(e + f*x),x)*f))/f
```

# 3.171 $\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1302
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1302
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1303
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1304
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1305
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1305
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1305
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1306
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1306
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1306


## Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(3+2m),\frac{1}{4}(7+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m(b\tan(e}{bf(3+2m)}$$

output
```
2*(cos(f*x+e)^2)^(3/4)*hypergeom([3/4, 3/4+1/2*m],[7/4+1/2*m],sin(f*x+e)^2
)*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2)/b/f/(3+2*m)
```

## Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1}{4}(3+2m),\frac{1}{4}(7+2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}(a\sin(e+fx))^m(b}{bf(3+2m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[(2 + m)/2, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]
*(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2))/(b*f*(3
 + 2*m))
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan(e+fx)}(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(e+fx)}(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{a\cos^{\frac{3}{2}}(e+fx)(b\tan(e+fx))^{3/2}\int \frac{(a\sin(e+fx))^{m+\frac{1}{2}}}{\sqrt{\cos(e+fx)}}dx}{b(a\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\cos^{\frac{3}{2}}(e+fx)(b\tan(e+fx))^{3/2}\int \frac{(a\sin(e+fx))^{m+\frac{1}{2}}}{\sqrt{\cos(e+fx)}}dx}{b(a\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{2\cos^2(e+fx)^{3/4}(b\tan(e+fx))^{3/2}(a\sin(e+fx))^m \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(2m+3),\frac{1}{4}(2m+7),\sin^2(e+fx)\right.}{bf(2m+3)}$$

input
```
Int[(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, (3 + 2*m)/4, (7 + 2*m)/4,
 Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2))/(b*f*(3 + 2*m)
)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (a\sin(fx+e))^m \sqrt{b\tan(fx+e)}dx$$

input
```
int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x)
```

output
```
int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x)
```

## Fricas [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan(fx+e)}(a\sin(fx+e))^m\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

## Sympy [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx = \int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx$$

input
```
integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral((a*sin(e + f*x))**m*sqrt(b*tan(e + f*x)), x)
```

## Maxima [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan(fx+e)}(a\sin(fx+e))^m\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

## Giac [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx=\int \sqrt{b\tan\left(fx+e\right)}(a\sin\left(fx+e\right))^m\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx=\int (a\,\sin\left(e+f\,x\right))^m\,\sqrt{b\tan\left(e+f\,x\right)}\,dx$$

input
```
int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx=\sqrt{b}\,a^m\left(\int \sqrt{\tan\left(fx+e\right)}\,\sin\left(fx+e\right)^m dx\right)$$

input
```
int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*a**m*int(sqrt(tan(e + f*x))*sin(e + f*x)**m,x)
```

# 3.172 $\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1307
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1307
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1308
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1309
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1309
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1310
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1310
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1310
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1311
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1311

## Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(1+2m),\frac{1}{4}(5+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m\sqrt{b\tan(e \cdots}{bf(1+2m)}$$

output
```
2*(cos(f*x+e)^2)^(1/4)*hypergeom([1/4, 1/4+1/2*m],[5/4+1/2*m],sin(f*x+e)^2
)*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2)/b/f/(1+2*m)
```

## Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1}{4}(1+2m),\frac{1}{4}(5+2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}(a\sin(e+fx))^m\sqrt{\cdots}}{bf(1+2m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[(2 + m)/2, (1 + 2*m)/4, (5 + 2*m)/4, -Tan[e + f*x]^2]
*(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]])/(b*f*(1 +
 2*m))
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3082}$$

$$\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \sqrt{\cos(e+fx)}(a\sin(e+fx))^{m-\frac{1}{2}}dx}{b\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \sqrt{\cos(e+fx)}(a\sin(e+fx))^{m-\frac{1}{2}}dx}{b\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{2\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}(a\sin(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(2m+1),\frac{1}{4}(2m+5),\sin^2(e+fx)\right)}{bf(2m+1)}$$

input
```
Int[(a*Sin[e + f*x])^m/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + 2*m)/4, (5 + 2*m)/4,
 Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]])/(b*f*(1 + 2*m))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{(a\sin(fx+e))^m}{\sqrt{b\tan(fx+e)}}dx$$

input
```
int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x)
```

output
```
int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x)
```

## Fricas [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^m}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m/(b*tan(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin{(e+fx)})^m}{\sqrt{b\tan{(e+fx)}}}\,dx$$

input
```
integrate((a*sin(f*x+e))**m/(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral((a*sin(e + f*x))**m/sqrt(b*tan(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin{(fx+e)})^m}{\sqrt{b\tan{(fx+e)}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m/sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin{(fx+e)})^m}{\sqrt{b\tan{(fx+e)}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m/sqrt(b*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\,\sin\left(e+f\,x\right))^m}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{b}\,a^m\left(\int \frac{\sqrt{\tan(fx+e)}\,\sin(fx+e)^m}{\tan(fx+e)}dx\right)}{b}$$

input
```
int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*a**m*int((sqrt(tan(e + f*x))*sin(e + f*x)**m)/tan(e + f*x),x))/b
```

# 3.173 $\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1312
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1312
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1313
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1314
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1315
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1315
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1315
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1316
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1316
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1316


## Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(-1+2m),\frac{1}{4}(3+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
-2*hypergeom([-1/4, -1/4+1/2*m],[3/4+1/2*m],sin(f*x+e)^2)*(a*sin(f*x+e))^m
/b/f/(1-2*m)/(cos(f*x+e)^2)^(1/4)/(b*tan(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1}{4}(-1+2m),\frac{1}{4}(3+2m),-\tan^2(e+fx)\right)\sec^2(e+}{bf(-1+2m)\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^m/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*Hypergeometric2F1[(2 + m)/2, (-1 + 2*m)/4, (3 + 2*m)/4, -Tan[e + f*x]^2
]*(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m)/(b*f*(-1 + 2*m)*Sqrt[b*Tan[e
+ f*x]])
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3082}$$

$$\frac{a\sqrt{a\sin(e+fx)}\int \cos^{\frac{3}{2}}(e+fx)(a\sin(e+fx))^{m-\frac{3}{2}}dx}{b\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\sqrt{a\sin(e+fx)}\int \cos(e+fx)^{3/2}(a\sin(e+fx))^{m-\frac{3}{2}}dx}{b\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3057}$$

$$-\frac{2(a\sin(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(2m-1),\frac{1}{4}(2m+3),\sin^2(e+fx)\right)}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^m/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*Hypergeometric2F1[-1/4, (-1 + 2*m)/4, (3 + 2*m)/4, Sin[e + f*x]^2]*(a*
Sin[e + f*x])^m)/(b*f*(1 - 2*m)*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]
])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{(a\sin (fx+e))^m}{(b\tan (fx+e))^{\frac{3}{2}}}dx$$

input
```
int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x)
```

output
```
int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x)
```

## Fricas [F]

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin\left(fx+e\right))^m}{(b\tan\left(fx+e\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m/(b^2*tan(f*x + e)^2), x)
```

## Sympy [F]

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin\left(e+fx\right))^m}{(b\tan\left(e+fx\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))**m/(b*tan(f*x+e))**(3/2),x)
```

output
```
Integral((a*sin(e + f*x))**m/(b*tan(e + f*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin\left(fx+e\right))^m}{(b\tan\left(fx+e\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m/(b*tan(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^m}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m/(b*tan(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\,\sin(e+f\,x))^m}{(b\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,a^m\left(\int \frac{\sqrt{\tan(fx+e)}\,\sin(fx+e)^m}{\tan(fx+e)^2}dx\right)}{b^2}$$

input
```
int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*a**m*int((sqrt(tan(e + f*x))*sin(e + f*x)**m)/tan(e + f*x)**2,x))
/b**2
```

# 3.174 $\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$

Optimal result . . . 1317
Mathematica [C] (warning: unable to verify) . . . 1317
Rubi [A] (verified) . . . 1318
Maple [F] . . . 1319
Fricas [F] . . . 1320
Sympy [F] . . . 1320
Maxima [F] . . . 1320
Giac [F] . . . 1321
Mupad [F(-1)] . . . 1321
Reduce [F] . . . 1321

## Optimal result

Integrand size = 21, antiderivative size = 83

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$$
$$=\frac{\cos^2(e+fx)^{\frac{1+n}{2}}\operatorname{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\sin^2(e+fx)\right)(a\sin(e+fx))^m(}{bf(1+m+n)}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1/2+1/2*n, 1/2+1/2*m+1/2*n],[3/2+1/2
*m+1/2*n],sin(f*x+e)^2)*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(1+n)/b/f/(1+m+n)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.76 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.13

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$$
$$=\frac{(3+m-}{f(1+m+n)\left((3+m+n)\operatorname{AppellF1}\left(\frac{1}{2}(1+m+n),n,1+m,\frac{1}{2}(3+m+n),\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\right.\right.}$$

input
```
Integrate[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((3 + m + n)*AppellF1[(1 + m + n)/2, n, 1 + m, (3 + m + n)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(a*Sin[e + f*x])^m*(b*Tan[e + f*
x])^n)/(f*(1 + m + n)*((3 + m + n)*AppellF1[(1 + m + n)/2, n, 1 + m, (3 +
m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((1 + m)*AppellF1[(
3 + m + n)/2, n, 2 + m, (5 + m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] - n*AppellF1[(3 + m + n)/2, 1 + n, 1 + m, (5 + m + n)/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)(a\sin(e+fx))^{m+n}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}(a\sin(e+fx))^{m+n}dx}{b}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\cos^2(e+fx)^{\frac{n+1}{2}}(a\sin(e+fx))^m(b\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),\sin^2\right.}{bf(m+n+1)}$$

input
```
Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + m + n)/2,
(3 + m + n)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(1 + m + n))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (a\sin (fx+e))^m (b\tan (fx+e))^n \, dx$$

input
```
int((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
int((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\sin(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Sympy [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\sin(e+fx))^m\,(b\tan(e+fx))^n\,dx$$

input
```
integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((a*sin(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

## Maxima [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\sin(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Giac [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\sin(fx+e))^m(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\sin(e+f\,x))^m\,(b\tan(e+f\,x))^n\,dx$$

input
```
int((a*sin(e + f*x))^m*(b*tan(e + f*x))^n,x)
```

output
```
int((a*sin(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

## Reduce [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx=b^na^m\left(\int \tan(fx+e)^n\sin(fx+e)^m\,dx\right)$$

input
```
int((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
b**n*a**m*int(tan(e + f*x)**n*sin(e + f*x)**m,x)
```

# 3.175 $\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx$


Optimal result . . . 1322
Mathematica [A] (verified) . . . 1322
Rubi [A] (verified) . . . 1323
Maple [F] . . . 1324
Fricas [F] . . . 1324
Sympy [F] . . . 1325
Maxima [F] . . . 1325
Giac [F] . . . 1325
Mupad [F(-1)] . . . 1326
Reduce [F] . . . 1326


## Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(3,\frac{5+n}{2},\frac{7+n}{2},-\tan^2(e+fx)\right)(b\tan(e+fx))^{5+n}}{b^5f(5+n)}$$

output
```
hypergeom([3, 5/2+1/2*n],[7/2+1/2*n],-tan(f*x+e)^2)*(b*tan(f*x+e))^(5+n)/b
^5/f/(5+n)
```

## Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(3,\frac{5+n}{2},\frac{7+n}{2},-\tan^2(e+fx)\right)\tan^5(e+fx)(b\tan(e+fx))^n}{f(5+n)}$$

input
```
Integrate[Sin[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^
5*(b*Tan[e + f*x])^n)/(f*(5 + n))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3071, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^4(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{b\int \frac{(b\tan(e+fx))^{n+4}}{(\tan^2(e+fx)b^2+b^2)^3}d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(b\tan(e+fx))^{n+5}\text{Hypergeometric2F1}\left(3,\frac{n+5}{2},\frac{n+7}{2},-\tan^2(e+fx)\right)}{b^5f(n+5)}$$

input
```
Int[Sin[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*
x])^(5 + n))/(b^5*f*(5 + n))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [F]

$$\int \sin \left(fx+e\right)^{4} \left(b\tan \left(fx+e\right)\right)^{n}dx$$

input
```
int(sin(f*x+e)^4*(b*tan(f*x+e))^n,x)
```

output
```
int(sin(f*x+e)^4*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx=\int \left(b\tan \left(fx+e\right)\right)^{n}\sin \left(fx+e\right)^{4}\,dx$$

input
```
integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*tan(f*x + e))^n, x)
```

## Sympy [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan{(e+fx)})^n\sin^4{(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)**4*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*sin(e + f*x)**4, x)
```

## Maxima [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan{(fx+e)})^n\sin{(fx+e)}^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e)^4, x)
```

## Giac [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan{(fx+e)})^n\sin{(fx+e)}^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int \sin\left(e+f\,x\right)^4\,\left(b\tan(e+f\,x)\right)^n\,dx$$

input
```
int(sin(e + f*x)^4*(b*tan(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)^4*(b*tan(e + f*x))^n, x)
```

## Reduce [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = b^n\left(\int \tan\left(fx+e\right)^n\sin\left(fx+e\right)^4dx\right)$$

input
```
int(sin(f*x+e)^4*(b*tan(f*x+e))^n,x)
```

output
```
b**n*int(tan(e + f*x)**n*sin(e + f*x)**4,x)
```

# 3.176 $\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1327
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1327
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1328
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1329
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1329
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1330
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1330
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1330
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1331
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1331


## Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{3+n}{2},\frac{5+n}{2},-\tan^2(e+fx)\right)(b\tan(e+fx))^{3+n}}{b^3f(3+n)}$$

output
```
hypergeom([2, 3/2+1/2*n],[5/2+1/2*n],-tan(f*x+e)^2)*(b*tan(f*x+e))^(3+n)/b
^3/f/(3+n)
```

## Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{3+n}{2},\frac{5+n}{2},-\tan^2(e+fx)\right)\tan^3(e+fx)(b\tan(e+fx))^n}{f(3+n)}$$

input
```
Integrate[Sin[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^
3*(b*Tan[e + f*x])^n)/(f*(3 + n))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3071, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^2(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{b\int \frac{(b\tan(e+fx))^{n+2}}{\left(\tan^2(e+fx)b^2+b^2\right)^2}d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(b\tan(e+fx))^{n+3}\,\text{Hypergeometric2F1}\left(2,\frac{n+3}{2},\frac{n+5}{2},-\tan^2(e+fx)\right)}{b^3f(n+3)}$$

input
```
Int[Sin[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*
x])^(3 + n))/(b^3*f*(3 + n))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [F]

$$\int \sin \left(fx+e\right)^{2} \left(b \tan \left(fx+e\right)\right)^{n} dx$$

input
```
int(sin(f*x+e)^2*(b*tan(f*x+e))^n,x)
```

output
```
int(sin(f*x+e)^2*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \int \left(b \tan \left(fx+e\right)\right)^{n} \sin \left(fx+e\right)^{2}\,dx$$

input
```
integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e))^n, x)
```

## Sympy [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx=\int(b\tan{(e+fx)})^n\sin^2{(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)**2*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*sin(e + f*x)**2, x)
```

## Maxima [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx=\int(b\tan{(fx+e)})^n\sin{(fx+e)}^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e)^2, x)
```

## Giac [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx=\int(b\tan{(fx+e)})^n\sin{(fx+e)}^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx=\int \sin\left(e+f\,x\right)^2\,\left(b\tan(e+f\,x)\right)^n\,dx$$

input
```
int(sin(e + f*x)^2*(b*tan(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)^2*(b*tan(e + f*x))^n, x)
```

## Reduce [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx=b^n\left(\int \tan\left(fx+e\right)^n\sin\left(fx+e\right)^2dx\right)$$

input
```
int(sin(f*x+e)^2*(b*tan(f*x+e))^n,x)
```

output
```
b**n*int(tan(e + f*x)**n*sin(e + f*x)**2,x)
```

# 3.177 $\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result . . . 1332
Mathematica [A] (verified) . . . 1332
Rubi [A] (verified) . . . 1333
Maple [A] (verified) . . . 1334
Fricas [A] (verification not implemented) . . . 1334
Sympy [F] . . . 1335
Maxima [A] (verification not implemented) . . . 1335
Giac [F] . . . 1335
Mupad [B] (verification not implemented) . . . 1336
Reduce [F] . . . 1336

## Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = -\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

output
```
-b*(b*tan(f*x+e))^(-1+n)/f/(1-n)
```

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{b(b\tan(e+fx))^{-1+n}}{f(-1+n)}$$

input
```
Integrate[Csc[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

output
```
(b*(b*Tan[e + f*x])^(-1 + n))/(f*(-1 + n))
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(b\tan(e+fx))^n}{\sin(e+fx)^2}dx$$
$$\Big\downarrow \text{3071}$$
$$\frac{b\int (b\tan(e+fx))^{n-2}d(b\tan(e+fx))}{f}$$
$$\Big\downarrow \text{15}$$
$$-\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

input
```
Int[Csc[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

output
```
-((b*(b*Tan[e + f*x])^(-1 + n))/(f*(1 - n)))
```

### Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{e^{n\ln(b\tan(fx+e))}}{f(-1+n)\tan(fx+e)}$ | 30 |
| default | $\frac{e^{n\ln(b\tan(fx+e))}}{f(-1+n)\tan(fx+e)}$ | 30 |
| risch | Expression too large to display | 1750 |

input
```
int(csc(f*x+e)^2*(b*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

output
```
1/f/(-1+n)*exp(n*ln(b*tan(f*x+e)))/tan(f*x+e)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^n\cos(fx+e)}{(fn-f)\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
(b*sin(f*x + e)/cos(f*x + e))^n*cos(f*x + e)/((f*n - f)*sin(f*x + e))
```

## Sympy [F]

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(e+fx))^n \csc^2(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**2*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*csc(e + f*x)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{b^n \tan(fx+e)^n}{f(n-1)\tan(fx+e)}$$

input
```
integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
b^n*tan(f*x + e)^n/(f*(n - 1)*tan(f*x + e))
```

## Giac [F]

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\sin\left(2\,e+2\,f\,x\right)\,{\left(\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{2\,{\cos\left(e+f\,x\right)}^2}\right)}^n}{2\,f\,\left({\cos\left(e+f\,x\right)}^2-1\right)\,\left(n-1\right)}$$

input
```
int((b*tan(e + f*x))^n/sin(e + f*x)^2,x)
```

output
```
-(sin(2*e + 2*f*x)*((b*sin(2*e + 2*f*x))/(2*cos(e + f*x)^2))^n)/(2*f*(cos(
e + f*x)^2 - 1)*(n - 1))
```

## Reduce [F]

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = b^n\left(\int \tan\left(fx+e\right)^n\csc\left(fx+e\right)^2dx\right)$$

input
```
int(csc(f*x+e)^2*(b*tan(f*x+e))^n,x)
```

output
```
b**n*int(tan(e + f*x)**n*csc(e + f*x)**2,x)
```

# 3.178 $\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result . . . 1337
Mathematica [A] (verified) . . . 1337
Rubi [A] (verified) . . . 1338
Maple [C] (warning: unable to verify) . . . 1339
Fricas [A] (verification not implemented) . . . 1340
Sympy [F] . . . 1340
Maxima [A] (verification not implemented) . . . 1340
Giac [F] . . . 1341
Mupad [B] (verification not implemented) . . . 1341
Reduce [F] . . . 1342

## Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = -\frac{b^3(b\tan(e+fx))^{-3+n}}{f(3-n)} - \frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

output
```
-b^3*(b*tan(f*x+e))^(-3+n)/f/(3-n)-b*(b*tan(f*x+e))^(-1+n)/f/(1-n)
```

## Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{b(-2+n+\cos(2(e+fx)))\csc^2(e+fx)(b\tan(e+fx))^{-1+n}}{f(-3+n)(-1+n)}$$

input
```
Integrate[Csc[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

output
```
(b*(-2 + n + Cos[2*(e + f*x)])*Csc[e + f*x]^2*(b*Tan[e + f*x])^(-1 + n))/(
f*(-3 + n)*(-1 + n))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^n}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{b\int (b\tan(e+fx))^{n-4}\left(\tan^2(e+fx)b^2+b^2\right)d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left(b^2(b\tan(e+fx))^{n-4}+(b\tan(e+fx))^{n-2}\right)d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{b^2(b\tan(e+fx))^{n-3}}{3-n}-\frac{(b\tan(e+fx))^{n-1}}{1-n}\right)}{f}$$

input
```
Int[Csc[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

output
```
(b*(-((b^2*(b*Tan[e + f*x])^(-3 + n))/(3 - n)) - (b*Tan[e + f*x])^(-1 + n)
/(1 - n)))/f
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 20.09 (sec) , antiderivative size = 5281, normalized size of antiderivative = 99.64

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 5281 |

input
```
int(csc(f*x+e)^4*(b*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{\left(2\cos\left(fx+e\right)^3+(n-3)\cos\left(fx+e\right)\right)\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^n}{\left(fn^2-\left(fn^2-4\,fn+3\,f\right)\cos\left(fx+e\right)^2-4\,fn+3\,f\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
(2*cos(f*x + e)^3 + (n - 3)*cos(f*x + e))*(b*sin(f*x + e)/cos(f*x + e))^n/
((f*n^2 - (f*n^2 - 4*f*n + 3*f)*cos(f*x + e)^2 - 4*f*n + 3*f)*sin(f*x + e)
)
```

## Sympy [F]

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \int \left(b\tan\left(e+fx\right)\right)^n\csc^4\left(e+fx\right)\,dx$$

input
```
integrate(csc(f*x+e)**4*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*csc(e + f*x)**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{\frac{b^n\tan(fx+e)^n}{(n-1)\tan(fx+e)}+\frac{b^n\tan(fx+e)^n}{(n-3)\tan(fx+e)^3}}{f}$$

input
```
integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
(b^n*tan(f*x + e)^n/((n - 1)*tan(f*x + e)) + b^n*tan(f*x + e)^n/((n - 3)*t
an(f*x + e)^3))/f
```

## Giac [F]

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^4, x)
```

## Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.60

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = -\frac{2\left(-\frac{b\sin(2e+2fx)}{2\sin(e+fx)^2-2}\right)^n(9\sin(2e+2fx)-6\sin(4e+4fx)+\sin(6e+6fx)-4n\sin(2e+2fx)-}{f\left(30\sin(e+fx)^2-12\sin(2e+2fx)^2+2\sin(3e+3fx)^2\right)(n^2-4n+3)}$$

input
```
int((b*tan(e + f*x))^n/sin(e + f*x)^4,x)
```

output
```
-(2*(-(b*sin(2*e + 2*f*x))/(2*sin(e + f*x)^2 - 2))^n*(9*sin(2*e + 2*f*x) -
 6*sin(4*e + 4*f*x) + sin(6*e + 6*f*x) - 4*n*sin(2*e + 2*f*x) + 2*n*sin(4*
e + 4*f*x)))/(f*(2*sin(3*e + 3*f*x)^2 - 12*sin(2*e + 2*f*x)^2 + 30*sin(e +
 f*x)^2)*(n^2 - 4*n + 3))
```

## Reduce [F]

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = b^n\left(\int \tan\left(fx+e\right)^n\csc\left(fx+e\right)^4dx\right)$$

input
```
int(csc(f*x+e)^4*(b*tan(f*x+e))^n,x)
```

output
```
b**n*int(tan(e + f*x)**n*csc(e + f*x)**4,x)
```

# 3.179 $\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx$


Optimal result . . . 1343
Mathematica [A] (verified) . . . 1343
Rubi [A] (verified) . . . 1344
Maple [C] (warning: unable to verify) . . . 1345
Fricas [A] (verification not implemented) . . . 1346
Sympy [F(-1)] . . . 1346
Maxima [A] (verification not implemented) . . . 1346
Giac [F] . . . 1347
Mupad [F(-1)] . . . 1347
Reduce [F] . . . 1347


## Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = -\frac{b^5(b\tan(e+fx))^{-5+n}}{f(5-n)} - \frac{2b^3(b\tan(e+fx))^{-3+n}}{f(3-n)} - \frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

output
```
-b^5*(b*tan(f*x+e))^(-5+n)/f/(5-n)-2*b^3*(b*tan(f*x+e))^(-3+n)/f/(3-n)-b*(
b*tan(f*x+e))^(-1+n)/f/(1-n)
```

## Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \frac{b(8-6n+n^2+2(-3+n)\cos(2(e+fx))+\cos(4(e+fx)))\csc^4(e+fx)(b\tan(e+fx))^{-1+n}}{f(-5+n)(-3+n)(-1+n)}$$

input
```
Integrate[Csc[e + f*x]^6*(b*Tan[e + f*x])^n,x]
```

output
```
(b*(8 - 6*n + n^2 + 2*(-3 + n)*Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*Csc[e
+ f*x]^4*(b*Tan[e + f*x])^(-1 + n))/(f*(-5 + n)*(-3 + n)*(-1 + n))
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^n}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{b\int (b\tan(e+fx))^{n-6}\left(\tan^2(e+fx)b^2+b^2\right)^2d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left(b^4(b\tan(e+fx))^{n-6}+2b^2(b\tan(e+fx))^{n-4}+(b\tan(e+fx))^{n-2}\right)d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{b^4(b\tan(e+fx))^{n-5}}{5-n}-\frac{2b^2(b\tan(e+fx))^{n-3}}{3-n}-\frac{(b\tan(e+fx))^{n-1}}{1-n}\right)}{f}$$

input
```
Int[Csc[e + f*x]^6*(b*Tan[e + f*x])^n,x]
```

output
```
(b*(-((b^4*(b*Tan[e + f*x])^(-5 + n))/(5 - n)) - (2*b^2*(b*Tan[e + f*x])^(
-3 + n))/(3 - n) - (b*Tan[e + f*x])^(-1 + n)/(1 - n)))/f
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 156.64 (sec) , antiderivative size = 10580, normalized size of antiderivative = 132.25

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 10580 |

input
```
int(csc(f*x+e)^6*(b*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.80

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \frac{\left(8\cos(fx+e)^5+4(n-5)\cos(fx+e)^3+(n^2-8n+15)\cos(fx+e)\right)\left(\frac{b\sin(\ldots}{\cos(\ldots}\right.}{\left((fn^3-9fn^2+23fn-15f)\cos(fx+e)^4+fn^3-9fn^2-2(fn^3-9fn^2+23fn-15f)\cos(fx+\ldots\right.}$$

input
```
integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
(8*cos(f*x + e)^5 + 4*(n - 5)*cos(f*x + e)^3 + (n^2 - 8*n + 15)*cos(f*x +
e))*(b*sin(f*x + e)/cos(f*x + e))^n/(((f*n^3 - 9*f*n^2 + 23*f*n - 15*f)*co
s(f*x + e)^4 + f*n^3 - 9*f*n^2 - 2*(f*n^3 - 9*f*n^2 + 23*f*n - 15*f)*cos(f
*x + e)^2 + 23*f*n - 15*f)*sin(f*x + e))
```

## Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(b*tan(f*x+e))**n,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \frac{\frac{b^n\tan(fx+e)^n}{(n-1)\tan(fx+e)}+\frac{2b^n\tan(fx+e)^n}{(n-3)\tan(fx+e)^3}+\frac{b^n\tan(fx+e)^n}{(n-5)\tan(fx+e)^5}}{f}$$

input
```
integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
(b^n*tan(f*x + e)^n/((n - 1)*tan(f*x + e)) + 2*b^n*tan(f*x + e)^n/((n - 3)
*tan(f*x + e)^3) + b^n*tan(f*x + e)^n/((n - 5)*tan(f*x + e)^5))/f
```

## Giac [F]

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^6, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sin(e+f\,x)^6}\,dx$$

input
```
int((b*tan(e + f*x))^n/sin(e + f*x)^6,x)
```

output
```
int((b*tan(e + f*x))^n/sin(e + f*x)^6, x)
```

## Reduce [F]

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = b^n\left(\int \tan(fx+e)^n \csc(fx+e)^6\,dx\right)$$

input
```
int(csc(f*x+e)^6*(b*tan(f*x+e))^n,x)
```

output
```
b**n*int(tan(e + f*x)**n*csc(e + f*x)**6,x)
```

# 3.180 $\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result . . . . . . . . . . 1348
Mathematica [C] (warning: unable to verify) . . . . . . . . . . 1348
Rubi [A] (verified) . . . . . . . . . . 1349
Maple [F] . . . . . . . . . . 1350
Fricas [F] . . . . . . . . . . 1351
Sympy [F(-1)] . . . . . . . . . . 1351
Maxima [F] . . . . . . . . . . 1351
Giac [F] . . . . . . . . . . 1352
Mupad [F(-1)] . . . . . . . . . . 1352
Reduce [F] . . . . . . . . . . 1352

## Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{4+n}{2},\frac{6+n}{2},\sin^2(e+fx)\right)\sin^3(e+fx)(b\tan(e+fx))^{1+n}}{bf(4+n)}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([2+1/2*n, 1/2+1/2*n],[3+1/2*n],sin(f*
x+e)^2)*sin(f*x+e)^3*(b*tan(f*x+e))^(1+n)/b/f/(4+n)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 3.49 (sec) , antiderivative size = 456, normalized size of antiderivative = 5.85

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \frac{}{f(2+n)\left(-2(4+n)\text{AppellF1}\left(1+\frac{n}{2},n,4,2+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx\right.\right.}$$

input
```
Integrate[Sin[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

output
```
(4*(4 + n)*(AppellF1[1 + n/2, n, 3, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - AppellF1[1 + n/2, n, 4, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*Sin[e + f*x]^3*(b*Tan[e
 + f*x])^n)/(f*(2 + n)*(-2*(4 + n)*AppellF1[1 + n/2, n, 4, 2 + n/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(3*AppellF1[2 +
n/2, n, 4, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[
2 + n/2, n, 5, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*(-App
ellF1[2 + n/2, 1 + n, 3, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + AppellF1[2 + n/2, 1 + n, 4, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]))*(-1 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 3, 2 + n/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])))
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\sin^{-n-1}(e+fx)\cos^{n+1}(e+fx)(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)\sin^{n+3}(e+fx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{-n-1}(e+fx)\cos^{n+1}(e+fx)(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}\sin(e+fx)^{n+3}dx}{b}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sin^3(e+fx)\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+4}{2},\frac{n+6}{2},\sin^2(e+fx)\right)}{bf(n+4)}$$

input
```
Int[Sin[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (4 + n)/2, (6 +
 n)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*(b*Tan[e + f*x])^(1 + n))/(b*f*(4 +
n))
```

**Defintions of rubi rules used**

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \sin \left(fx+e\right)^{3} \left(b\tan \left(fx+e\right)\right)^{n}dx$$

input
```
int(sin(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

output
```
int(sin(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \sin(fx+e)^3\,dx$$

input
```
integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e))^n*sin(f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3*(b*tan(f*x+e))**n,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \sin(fx+e)^3\,dx$$

input
```
integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e)^3, x)
```

## Giac [F]

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(fx+e))^n\sin(fx+e)^3\,dx$$

input
```
integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx=\int \sin(e+f\,x)^3\,(b\tan(e+f\,x))^n\,dx$$

input
```
int(sin(e + f*x)^3*(b*tan(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)^3*(b*tan(e + f*x))^n, x)
```

## Reduce [F]

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx=b^n\left(\int \tan(fx+e)^n\sin(fx+e)^3\,dx\right)$$

input
```
int(sin(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

output
```
b**n*int(tan(e + f*x)**n*sin(e + f*x)**3,x)
```

# 3.181 $\int \sin(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1353
Mathematica [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . 1353
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1354
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1355
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1356
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1356
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1356
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1357
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1357
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1357

## Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)\sin(e+fx)(b\tan(e+fx))^{1+n}}{bf(2+n)}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[2+1/2*n],sin(f*
x+e)^2)*sin(f*x+e)*(b*tan(f*x+e))^(1+n)/b/f/(2+n)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.46 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.32

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \frac{8(4+n)\,\text{AppellF1}\left(1+\frac{n}{2},n,\right.}{f(2+n)\left(2\left(2\,\text{AppellF1}\left(2+\frac{n}{2},n,3,3+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-n\,\text{AppellF1}\left(2+\frac{n}{2}\right.\right.\right.}$$

input
```
Integrate[Sin[e + f*x]*(b*Tan[e + f*x])^n,x]
```

output
```
(8*(4 + n)*AppellF1[1 + n/2, n, 2, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Cos[(e + f*x)/2]^4*Sin[(e + f*x)/2]^2*(b*Tan[e + f*x])^n)/(f*(2
 + n)*(2*(2*AppellF1[2 + n/2, n, 3, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - n*AppellF1[2 + n/2, 1 + n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 2,
 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\sin^{-n-1}(e+fx)\cos^{n+1}(e+fx)(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)\sin^{n+1}(e+fx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{-n-1}(e+fx)\cos^{n+1}(e+fx)(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}\sin(e+fx)^{n+1}dx}{b}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+4}{2},\sin^2(e+fx)\right)}{bf(n+2)}$$

input
```
Int[Sin[e + f*x]*(b*Tan[e + f*x])^n,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (4 +
 n)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*Tan[e + f*x])^(1 + n))/(b*f*(2 + n)
)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \sin(fx+e)\,(b\tan(fx+e))^n\,dx$$

input
```
int(sin(f*x+e)*(b*tan(f*x+e))^n,x)
```

output
```
int(sin(f*x+e)*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(fx+e))^n\sin(fx+e)\,dx$$

input
```
integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e))^n*sin(f*x + e), x)
```

## Sympy [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(e+fx))^n\sin(e+fx)\,dx$$

input
```
integrate(sin(f*x+e)*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*sin(e + f*x), x)
```

## Maxima [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(fx+e))^n\sin(fx+e)\,dx$$

input
```
integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e), x)
```

## Giac [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(fx+e))^n\sin(fx+e)\,dx$$

input
```
integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx=\int \sin(e+f\,x)\;(b\tan(e+f\,x))^n\,dx$$

input
```
int(sin(e + f*x)*(b*tan(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)*(b*tan(e + f*x))^n, x)
```

## Reduce [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx=b^n\left(\int \tan(fx+e)^n\sin(fx+e)\,dx\right)$$

input
```
int(sin(f*x+e)*(b*tan(f*x+e))^n,x)
```

output
```
b**n*int(tan(e + f*x)**n*sin(e + f*x),x)
```

# 3.182 $\int \csc(e+fx)(b\tan(e+fx))^n\,dx$


Optimal result . . . 1358
Mathematica [A] (verified) . . . 1358
Rubi [A] (verified) . . . 1359
Maple [F] . . . 1360
Fricas [F] . . . 1360
Sympy [F] . . . 1361
Maxima [F] . . . 1361
Giac [F] . . . 1361
Mupad [F(-1)] . . . 1362
Reduce [F] . . . 1362


## Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{2-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

output
```
-cos(f*x+e)*hypergeom([1-1/2*n, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*ta
n(f*x+e))^n/f/(1-n)/((sin(f*x+e)^2)^(1/2*n))
```

## Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{n}{2},n,1+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n(b\tan(e+fx))^n}{fn}$$

input
```
Integrate[Csc[e + f*x]*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[
(e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(f*n)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3081, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^n}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{3081}$$

$$\sin^{-n}(e+fx)\cos^n(e+fx)(b\tan(e+fx))^n\int \cos^{-n}(e+fx)\sin^{n-1}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-n}(e+fx)\cos^n(e+fx)(b\tan(e+fx))^n\int \cos(e+fx)^{-n}\sin(e+fx)^{n-1}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{2-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Csc[e + f*x]*(b*Tan[e + f*x])^n,x]
```

output
```
-((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (2 - n)/2, (3 - n)/2, Cos[e +
 f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

## Maple [F]

$$\int \csc(fx+e)\,(b\tan(fx+e))^n\,dx$$

input
```
int(csc(f*x+e)*(b*tan(f*x+e))^n,x)
```

output
```
int(csc(f*x+e)*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\csc(fx+e)\,dx$$

input
```
integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e))^n*csc(f*x + e), x)
```

## Sympy [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan{(e+fx)})^n \csc{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*csc(e + f*x), x)
```

## Maxima [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan{(fx+e)})^n \csc{(fx+e)}\,dx$$

input
```
integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e), x)
```

## Giac [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan{(fx+e)})^n \csc{(fx+e)}\,dx$$

input
```
integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx=\int \frac{(b\tan(e+f\,x))^n}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((b*tan(e + f*x))^n/sin(e + f*x),x)
```

output
```
int((b*tan(e + f*x))^n/sin(e + f*x), x)
```

## Reduce [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx=b^n\left(\int \tan\left(fx+e\right)^n\csc\left(fx+e\right)dx\right)$$

input
```
int(csc(f*x+e)*(b*tan(f*x+e))^n,x)
```

output
```
b**n*int(tan(e + f*x)**n*csc(e + f*x),x)
```

# 3.183 $\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result . . . 1363
Mathematica [C] (warning: unable to verify) . . . 1363
Rubi [A] (verified) . . . 1364
Maple [F] . . . 1366
Fricas [F] . . . 1366
Sympy [F] . . . 1366
Maxima [F] . . . 1367
Giac [F] . . . 1367
Mupad [F(-1)] . . . 1367
Reduce [F] . . . 1368

## Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{4-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

output
```
-cos(f*x+e)*hypergeom([2-1/2*n, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*ta
n(f*x+e))^n/f/(1-n)/((sin(f*x+e)^2)^(1/2*n))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 14.00 (sec) , antiderivative size = 1242, normalized size of antiderivative = 15.92

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

output
```
(Cot[(e + f*x)/2]^2*Hypergeometric2F1[-1 + n/2, n, n/2, Tan[(e + f*x)/2]^2
]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(f*(-8 + 4*n)) +
 ((4 + n)*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sin[e + f*x]^2*(b*Tan[e + f*x])^n)/(4*f*(2 + n)*(2*(AppellF1[2 +
 n/2, n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1
[2 + n/2, 1 + n, 1, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1
 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]))) + (Hypergeometric2F1[1 + n
/2, n, 2 + n/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Ta
n[(e + f*x)/2]^2*(b*Tan[e + f*x])^n)/(f*(8 + 4*n)) + (Cot[(e + f*x)/2]*(Co
s[e + f*x]*Sec[(e + f*x)/2]^2)^n*((2 + n)*Hypergeometric2F1[n/2, n, 1 + n/
2, Tan[(e + f*x)/2]^2] - n*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^n*(b*Tan[e + f
*x])^n)/(8*f*n*(2 + n)*(((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^
2*((2 + n)*Hypergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x)/2]^2] - n*Appe
llF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[
(e + f*x)/2]^2)*Tan[e + f*x]^(-1 + n))/(2*(2 + n)) + ((Cos[e + f*x]*Sec[(e
 + f*x)/2]^2)^(-1 + n)*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])*((2 + n)*Hypergeometric2F1[n/2, n, 1
+ n/2, Tan[(e + f*x)/2]^2] - n*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e ...
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3081, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^n}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{3081}$$

$$\sin^{-n}(e+fx)\cos^n(e+fx)(b\tan(e+fx))^n\int \cos^{-n}(e+fx)\sin^{n-3}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-n}(e+fx)\cos^n(e+fx)(b\tan(e+fx))^n\int \cos(e+fx)^{-n}\sin(e+fx)^{n-3}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{4-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Csc[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

output
```
-((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (4 - n)/2, (3 - n)/2, Cos[e +
 f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

## Maple [F]

$$\int \csc(fx+e)^3 (b\tan(fx+e))^n\,dx$$

input
```
int(csc(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

output
```
int(csc(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

## Sympy [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(e+fx))^n \csc^3(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**3*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*csc(e + f*x)**3, x)
```

## Maxima [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

## Giac [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sin(e+f\,x)^3}\,dx$$

input
```
int((b*tan(e + f*x))^n/sin(e + f*x)^3,x)
```

output
```
int((b*tan(e + f*x))^n/sin(e + f*x)^3, x)
```

## Reduce [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = b^n\left(\int \tan(fx+e)^n\csc(fx+e)^3\,dx\right)$$

input
```
int(csc(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

output
```
b**n*int(tan(e + f*x)**n*csc(e + f*x)**3,x)
```

# 3.184 $\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1369
Mathematica [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . 1369
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1370
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1372
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1372
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1372
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1373
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1373
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1373
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1374

## Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{6-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

output
```
-cos(f*x+e)*hypergeom([3-1/2*n, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*ta
n(f*x+e))^n/f/(1-n)/((sin(f*x+e)^2)^(1/2*n))
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 14.92 (sec) , antiderivative size = 1516, normalized size of antiderivative = 19.44

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[e + f*x]^5*(b*Tan[e + f*x])^n,x]
```

output
```
(3*Cot[(e + f*x)/2]^2*Hypergeometric2F1[-1 + n/2, n, n/2, Tan[(e + f*x)/2]
^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(16*f*(-2 + n)
) + (Cot[(e + f*x)/2]^2*((-2 + n)*Cot[(e + f*x)/2]^2*Hypergeometric2F1[-2
+ n/2, n, -1 + n/2, Tan[(e + f*x)/2]^2] + (-4 + n)*Hypergeometric2F1[-1 +
n/2, n, n/2, Tan[(e + f*x)/2]^2])*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(b*T
an[e + f*x])^n)/(16*f*(-4 + n)*(-2 + n)) + (3*(4 + n)*AppellF1[1 + n/2, n,
 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]^2*(b*Ta
n[e + f*x])^n)/(16*f*(2 + n)*(2*(AppellF1[2 + n/2, n, 2, 3 + n/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[2 + n/2, 1 + n, 1, 3 + n/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x]) + (4 + n)*Ap
pellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1
 + Cos[e + f*x]))) + (3*Hypergeometric2F1[1 + n/2, n, 2 + n/2, Tan[(e + f*
x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^2*(b*Tan[e +
 f*x])^n)/(16*f*(2 + n)) + ((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f
*x)/2]^2*((4 + n)*Hypergeometric2F1[1 + n/2, n, 2 + n/2, Tan[(e + f*x)/2]^
2] + (2 + n)*Hypergeometric2F1[2 + n/2, n, 3 + n/2, Tan[(e + f*x)/2]^2]*Ta
n[(e + f*x)/2]^2)*(b*Tan[e + f*x])^n)/(16*f*(2 + n)*(4 + n)) + (9*Cot[(e +
 f*x)/2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*((2 + n)*Hypergeometric2F1[n/
2, n, 1 + n/2, Tan[(e + f*x)/2]^2] - n*AppellF1[1 + n/2, n, 1, 2 + n/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]...
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3081, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^n}{\sin(e+fx)^5}dx$$

$$\Big\downarrow \text{3081}$$

$$\sin^{-n}(e+fx)\cos^n(e+fx)(b\tan(e+fx))^n\int \cos^{-n}(e+fx)\sin^{n-5}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-n}(e+fx)\cos^n(e+fx)(b\tan(e+fx))^n\int\cos(e+fx)^{-n}\sin(e+fx)^{n-5}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{6-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Csc[e + f*x]^5*(b*Tan[e + f*x])^n,x]
```

output
```
-((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (6 - n)/2, (3 - n)/2, Cos[e +
 f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

## Maple [F]

$$\int \csc(fx+e)^5 (b\tan(fx+e))^n\,dx$$

input
```
int(csc(f*x+e)^5*(b*tan(f*x+e))^n,x)
```

output
```
int(csc(f*x+e)^5*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

## Sympy [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(e+fx))^n \csc^5(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**5*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*csc(e + f*x)**5, x)
```

## Maxima [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

## Giac [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sin(e+f\,x)^5}\,dx$$

input
```
int((b*tan(e + f*x))^n/sin(e + f*x)^5,x)
```

output
```
int((b*tan(e + f*x))^n/sin(e + f*x)^5, x)
```

## Reduce [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = b^n\left(\int \tan(fx+e)^n \csc(fx+e)^5\,dx\right)$$

input
```
int(csc(f*x+e)^5*(b*tan(f*x+e))^n,x)
```

output
```
b**n*int(tan(e + f*x)**n*csc(e + f*x)**5,x)
```

# 3.185 $\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx$


Optimal result . . . 1375
Mathematica [C] (warning: unable to verify) . . . 1375
Rubi [A] (verified) . . . 1376
Maple [F] . . . 1377
Fricas [F] . . . 1378
Sympy [F(-1)] . . . 1378
Maxima [F] . . . 1378
Giac [F] . . . 1379
Mupad [F(-1)] . . . 1379
Reduce [F] . . . 1379


## Optimal result

Integrand size = 23, antiderivative size = 89

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\sin^2(e+f$$
$$bf(5+2n)$$

output
```
2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1/2+1/2*n, 5/4+1/2*n],[9/4+1/2*n],
sin(f*x+e)^2)*(a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1+n)/b/f/(5+2*n)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 33.99 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.34

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \frac{8(9+}{f(5+2n)\left(2(9+2n)\,\text{AppellF1}\left(\frac{5}{4}+\frac{n}{2},n,\frac{5}{2},\frac{9}{4}+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\right.\right.}$$

input
```
Integrate[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^n,x]
```

output
```
(8*(9 + 2*n)*AppellF1[5/4 + n/2, n, 5/2, 9/4 + n/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*(a*Sin[e + f*x])^(3
/2)*(b*Tan[e + f*x])^n)/(f*(5 + 2*n)*(2*(9 + 2*n)*AppellF1[5/4 + n/2, n, 5
/2, 9/4 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2
 + 2*(5*AppellF1[9/4 + n/2, n, 7/2, 13/4 + n/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] - 2*n*AppellF1[9/4 + n/2, 1 + n, 5/2, 13/4 + n/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x])))
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n dx$$

$$\Big\downarrow \; 3082$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)(a\sin(e+fx))^{n+\frac{3}{2}}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}(a\sin(e+fx))^{n+\frac{3}{2}}dx}{b}$$

$$\Big\downarrow \; 3057$$

$$\frac{2(a\sin(e+fx))^{3/2}\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n+5),\frac{1}{4}(2n+9),\sin^2(e+\right.}{bf(2n+5)}$$

input
```
Int[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^n,x]
```

output
```
(2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (5 + 2*n)/4,
(9 + 2*n)/4, Sin[e + f*x]^2]*(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(1 +
n))/(b*f*(5 + 2*n))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (a \sin (fx + e))^{\frac{3}{2}} (b \tan (fx + e))^n \, dx$$

input
```
int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x)
```

output
```
int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n*a*sin(f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**n,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^n, x)
```

## Giac [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx=\int (a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx=\int (a\,\sin(e+f\,x))^{3/2}\,(b\tan(e+f\,x))^n\,dx$$

input
```
int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^n,x)
```

output
```
int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^n, x)
```

## Reduce [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx=b^n\sqrt{a}\left(\int \tan(fx+e)^n\sqrt{\sin(fx+e)}\,\sin(fx+e)\,dx\right)a$$

input
```
int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x)
```

output
```
b**n*sqrt(a)*int(tan(e + f*x)**n*sqrt(sin(e + f*x))*sin(e + f*x),x)*a
```

# 3.186 $\int \sqrt{a \sin(e+fx)}(b \tan(e+fx))^n \, dx$

Optimal result . . . 1380
Mathematica [A] (verified) . . . 1380
Rubi [A] (verified) . . . 1381
Maple [F] . . . 1382
Fricas [F] . . . 1382
Sympy [F] . . . 1383
Maxima [F] . . . 1383
Giac [F] . . . 1383
Mupad [F(-1)] . . . 1384
Reduce [F] . . . 1384

## Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \sqrt{a \sin(e+fx)}(b \tan(e+fx))^n \, dx = \frac{2 \cos^2(e+fx)^{\frac{1+n}{2}} \text{Hypergeometric2F1}\left(\frac{1+n}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \sin^2(e+fx)\right) \sqrt{a \sin(e+fx)}(b \tan(e$$
$$bf(3+2n)$$

output
```
2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1/2+1/2*n, 3/4+1/2*n],[7/4+1/2*n],
sin(f*x+e)^2)*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1+n)/b/f/(3+2*n)
```

## Mathematica [A] (verified)

Time = 11.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \sqrt{a \sin(e+fx)}(b \tan(e+fx))^n \, dx = \frac{\cos^2(e+fx)^{\frac{1}{2}(-1+n)} \text{Hypergeometric2F1}\left(\frac{1+n}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \sin^2(e+fx)\right) \sqrt{a \sin(e+fx)} \sin(2$$
$$f(3+2n)$$

input
```
Integrate[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^n,x]
```

output
```
((Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4, (
7 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)]*(b*Tan[e
 + f*x])^n)/(f*(3 + 2*n))
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n dx$$

$$\Big\downarrow \; 3082$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)(a\sin(e+fx))^{n+\frac{1}{2}}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}(a\sin(e+fx))^{n+\frac{1}{2}}dx}{b}$$

$$\Big\downarrow \; 3057$$

$$\frac{2\sqrt{a\sin(e+fx)}\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\sin^2(e+fx\right.}{bf(2n+3)}$$

input
```
Int[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^n,x]
```

output
```
(2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4,
(7 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(3 + 2*n))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \sqrt{a \sin(fx+e)}\, (b \tan(fx+e))^n \, dx$$

input
```
int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x)
```

output
```
int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int \sqrt{a \sin(e+fx)}(b \tan(e+fx))^n \, dx = \int \sqrt{a \sin(fx+e)}(b \tan(fx+e))^n \, dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

## Sympy [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a\sin{(e+fx)}}(b\tan{(e+fx)})^n\,dx$$

input
```
integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**n,x)
```

output
```
Integral(sqrt(a*sin(e + f*x))*(b*tan(e + f*x))**n, x)
```

## Maxima [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a\sin{(fx+e)}}(b\tan{(fx+e)})^n\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

## Giac [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a\sin{(fx+e)}}(b\tan{(fx+e)})^n\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx=\int \sqrt{a\,\sin\left(e+f\,x\right)}\,\left(b\tan(e+f\,x)\right)^n\,dx$$

input
```
int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^n,x)
```

output
```
int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^n, x)
```

## Reduce [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx=b^n\sqrt{a}\left(\int \tan\left(fx+e\right)^n\sqrt{\sin\left(fx+e\right)}dx\right)$$

input
```
int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x)
```

output
```
b**n*sqrt(a)*int(tan(e + f*x)**n*sqrt(sin(e + f*x)),x)
```

# 3.187 $\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1385
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1385
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1386
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1387
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1388
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1388
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1388
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1389
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1389
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1389

## Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1+2n)\sqrt{a\sin(e+fx)}}$$

output
```
2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1/2+1/2*n, 1/4+1/2*n],[5/4+1/2*n],
sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1+2*n)/(a*sin(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 11.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\sin^2(e+fx)\right)\sin(2(e+fx))(b\tan(}{(f+2fn)\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^n/Sqrt[a*Sin[e + f*x]],x]
```

output
```
((Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4, (
5 + 2*n)/4, Sin[e + f*x]^2]*Sin[2*(e + f*x)]*(b*Tan[e + f*x])^n)/((f + 2*f
*n)*Sqrt[a*Sin[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)(a\sin(e+fx))^{n-\frac{1}{2}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}(a\sin(e+fx))^{n-\frac{1}{2}}dx}{b}$$

$$\Big\downarrow \text{3057}$$

$$\frac{2\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n+1),\frac{1}{4}(2n+5),\sin^2(e+fx)\right)}{bf(2n+1)\sqrt{a\sin(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x])^n/Sqrt[a*Sin[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4,
(5 + 2*n)/4, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 + 2*n)*Sqrt
[a*Sin[e + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{(b\tan (fx+e))^n}{\sqrt{a\sin (fx+e)}}dx$$

input
```
int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x)
```

output
```
int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x)
```

## Fricas [F]

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^n}{\sqrt{a\sin(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n/(a*sin(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx$$

input
```
integrate((b*tan(f*x+e))**n/(a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((b*tan(e + f*x))**n/sqrt(a*sin(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^n}{\sqrt{a\sin(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n/sqrt(a*sin(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^n}{\sqrt{a\sin(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n/sqrt(a*sin(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sqrt{a\,\sin(e+f\,x)}}\,dx$$

input
```
int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \frac{b^n\sqrt{a}\left(\int \frac{\tan(fx+e)^n\sqrt{\sin(fx+e)}}{\sin(fx+e)}dx\right)}{a}$$

input
```
int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x)
```

output
```
(b**n*sqrt(a)*int((tan(e + f*x)**n*sqrt(sin(e + f*x)))/sin(e + f*x),x))/a
```

# 3.188 $\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal result . . . . . 1390
Mathematica [A] (verified) . . . . . 1390
Rubi [A] (verified) . . . . . 1391
Maple [F] . . . . . 1392
Fricas [F] . . . . . 1393
Sympy [F] . . . . . 1393
Maxima [F] . . . . . 1393
Giac [F] . . . . . 1394
Mupad [F(-1)] . . . . . 1394
Reduce [F] . . . . . 1394

## Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

output
```
-2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([-1/4+1/2*n, 1/2+1/2*n],[3/4+1/2*n
],sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1-2*n)/(a*sin(f*x+e))^(3/2)
```

## Mathematica [A] (verified)

Time = 9.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \frac{2b\cos^2(e+fx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\sin^2(e\right.}{a^2 f(-1+2n)}$$

input
```
Integrate[(b*Tan[e + f*x])^n/(a*Sin[e + f*x])^(3/2),x]
```

output
```
(2*b*(Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (-1 + 2*n)
/4, (3 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(-1
 + n))/(a^2*f*(-1 + 2*n))
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)(a\sin(e+fx))^{n-\frac{3}{2}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}(a\sin(e+fx))^{n-\frac{3}{2}}dx}{b}$$

$$\Big\downarrow \text{3057}$$

$$-\frac{2\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\sin^2(e+fx)\right)}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

input
```
Int[(b*Tan[e + f*x])^n/(a*Sin[e + f*x])^(3/2),x]
```

output
```
(-2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (-1 + 2*n)/4
, (3 + 2*n)/4, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - 2*n)*(a
*Sin[e + f*x])^(3/2))
```

**Defintions of rubi rules used**

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{(b\tan(fx+e))^n}{(a\sin(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x)
```

output
```
int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x)
```

## Fricas [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\left(b\tan\left(fx+e\right)\right)^n}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n/(a^2*cos(f*x + e)^2 - a^
2), x)
```

## Sympy [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\left(b\tan\left(e+fx\right)\right)^n}{\left(a\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**n/(a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((b*tan(e + f*x))**n/(a*sin(e + f*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\left(b\tan\left(fx+e\right)\right)^n}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n/(a*sin(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^n}{(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n/(a*sin(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+f\,x))^n}{(a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \frac{b^n\sqrt{a}\left(\int \frac{\tan(fx+e)^n\sqrt{\sin(fx+e)}}{\sin(fx+e)^2}dx\right)}{a^2}$$

input
```
int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x)
```

output
```
(b**n*sqrt(a)*int((tan(e + f*x)**n*sqrt(sin(e + f*x)))/sin(e + f*x)**2,x))
/a**2
```

# 3.189 $\int (a\cos(e+fx))^m (b\tan(e+fx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1395
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . 1395
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1396
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1397
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1398
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1398
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1398
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1399
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . 1399
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1399

## Optimal result

Integrand size = 21, antiderivative size = 86

$$\int (a\cos(e+fx))^m (b\tan(e+fx))^n\,dx = \frac{(a\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(1-m+n)} \text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1-m+n),\frac{3+n}{2},\sin^2(e+fx)\right)(b\,\mathrm{ta}}{bf(1+n)}$$

output
```
(a*cos(f*x+e))^m*(cos(f*x+e)^2)^(1/2-1/2*m+1/2*n)*hypergeom([1/2+1/2*n, 1/
2-1/2*m+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1+n)
```

## Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int (a\cos(e+fx))^m (b\tan(e+fx))^n\,dx = \frac{(a\cos(e+fx))^m \text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}\tan(e+fx)(b\,\mathrm{ta}}{f(1+n)}$$

input
```
Integrate[(a*Cos[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((a*Cos[e + f*x])^m*Hypergeometric2F1[(2 + m)/2, (1 + n)/2, (3 + n)/2, -Ta
n[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*Tan[e + f*x])^n)/(f*(
1 + n))
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3083, 3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3083}$$

$$(a\cos(e+fx))^m\left(\frac{\sec(e+fx)}{a}\right)^m\int\left(\frac{\sec(e+fx)}{a}\right)^{-m}(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3042}$$

$$(a\cos(e+fx))^m\left(\frac{\sec(e+fx)}{a}\right)^m\int\left(\frac{\sec(e+fx)}{a}\right)^{-m}(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{(a\cos(e+fx))^m(b\tan(e+fx))^{n+1}\cos^2(e+fx)^{\frac{1}{2}(-m+n+1)}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1),\frac{n+3}{2},\sin^2\right.}{bf(n+1)}$$

input
```
Int[(a*Cos[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((a*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + n)/2)*Hypergeometric2F1[(1
+ n)/2, (1 - m + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(1 + n))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3083
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (a\cos (fx+e))^m (b\tan (fx+e))^n \, dx$$

input
```
int((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
int((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\cos(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Sympy [F]

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\cos(e+fx))^m\,(b\tan(e+fx))^n\,dx$$

input
```
integrate((a*cos(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((a*cos(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

## Maxima [F]

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\cos(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Giac [F]

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\cos(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\cos(e+f\,x))^m\,(b\tan(e+f\,x))^n\,dx$$

input
```
int((a*cos(e + f*x))^m*(b*tan(e + f*x))^n,x)
```

output
```
int((a*cos(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

## Reduce [F]

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx=b^n a^m\left(\int \tan(fx+e)^n\cos(fx+e)^m\,dx\right)$$

input
```
int((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
b**n*a**m*int(tan(e + f*x)**n*cos(e + f*x)**m,x)
```

# 3.190 $\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx$


Optimal result . . . 1400
Mathematica [A] (verified) . . . 1400
Rubi [A] (verified) . . . 1401
Maple [F] . . . 1402
Fricas [F] . . . 1403
Sympy [F] . . . 1403
Maxima [F] . . . 1403
Giac [F] . . . 1404
Mupad [F(-1)] . . . 1404
Reduce [F] . . . 1404


## Optimal result

Integrand size = 21, antiderivative size = 63

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),-\tan^2(e+fx)\right)(a\tan(e+fx))^{1+m}(b\tan(e+fx)$$
$$af(1+m+n)$$

output
```
hypergeom([1, 1/2+1/2*m+1/2*n],[3/2+1/2*m+1/2*n],-tan(f*x+e)^2)*(a*tan(f*x
+e))^(1+m)*(b*tan(f*x+e))^n/a/f/(1+m+n)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),-\tan^2(e+fx)\right)\tan(e+fx)(a\tan(e+fx))^m(b\,\text{ta}$$
$$f(1+m+n)$$

input
```
Integrate[(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[e + f*x]^2]*Tan[e
 + f*x]*(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n)/(f*(1 + m + n))
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2034, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 2034}$$

$$(a\tan(e+fx))^{-n}(b\tan(e+fx))^n\int (a\tan(e+fx))^{m+n}dx$$

$$\Big\downarrow \text{ 3042}$$

$$(a\tan(e+fx))^{-n}(b\tan(e+fx))^n\int (a\tan(e+fx))^{m+n}dx$$

$$\Big\downarrow \text{ 3957}$$

$$\frac{a(a\tan(e+fx))^{-n}(b\tan(e+fx))^n\int \frac{(a\tan(e+fx))^{m+n}}{\tan^2(e+fx)a^2+a^2}d(a\tan(e+fx))}{f}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(a\tan(e+fx))^{m+1}(b\tan(e+fx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),-\tan^2(e+fx)\right)}{af(m+n+1)}$$

input
```
Int[(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[e + f*x]^2]*(a*Ta
n[e + f*x])^(1 + m)*(b*Tan[e + f*x])^n)/(a*f*(1 + m + n))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [F]

$$\int (a\tan (fx+e))^m (b\tan (fx+e))^n \, dx$$

input
```
int((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
int((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\tan{(fx+e)})^m\,(b\tan{(fx+e)})^n\,dx$$

input
```
integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Sympy [F]

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\tan{(e+fx)})^m\,(b\tan{(e+fx)})^n\,dx$$

input
```
integrate((a*tan(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((a*tan(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

## Maxima [F]

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\tan{(fx+e)})^m\,(b\tan{(fx+e)})^n\,dx$$

input
```
integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Giac [F]

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\tan(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\tan(e+f\,x))^m\,(b\tan(e+f\,x))^n\,dx$$

input
```
int((a*tan(e + f*x))^m*(b*tan(e + f*x))^n,x)
```

output
```
int((a*tan(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

## Reduce [F]

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx=b^n a^m\left(\int \tan(fx+e)^{m+n}\,dx\right)$$

input
```
int((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
b**n*a**m*int(tan(e + f*x)**(m + n),x)
```

# 3.191 $\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx$

Optimal result . . . . . 1405
Mathematica [A] (verified) . . . . . 1406
Rubi [A] (warning: unable to verify) . . . . . 1406
Maple [B] (warning: unable to verify) . . . . . 1411
Fricas [A] (verification not implemented) . . . . . 1412
Sympy [F] . . . . . 1413
Maxima [A] (verification not implemented) . . . . . 1413
Giac [F] . . . . . 1414
Mupad [B] (verification not implemented) . . . . . 1414
Reduce [F] . . . . . 1415

## Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f} + \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}}$$

output
```
1/2*d^(1/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d
^(1/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*d^(1/2
)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/
2)/f+2/5*d^3/f/(d*cot(f*x+e))^(5/2)-2*d/f/(d*cot(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.62

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx =$$
$$-\frac{\sqrt{d\cot(e+fx)}\left(-2+10\cot^2(e+fx)+5\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}\cot^{\frac{9}{4}}(e+fx)\right.}{5f}$$

input
```
Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^4,x]
```

output
```
-1/5*(Sqrt[d*Cot[e + f*x]]*(-2 + 10*Cot[e + f*x]^2 + 5*ArcTan[(-Cot[e + f*
x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4)*Cot[e + f*x]^(9/4) + 5*ArcTanh[(-Cot[e
+ f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(5/4))*Tan[e + f*x]^3)/f
```

## Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.30, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 2030, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\sqrt{d\cot(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}{\tan\left(e+fx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$d^4\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int\frac{1}{(d\cot(e+fx))^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int\frac{1}{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \text{3955}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int\sqrt{d\cot(e+fx)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \text{3957}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{\int\frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \text{266}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\int\frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \text{826}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \text{1476}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right.\right.}{df}}{d^2}\right.$$

$\downarrow$ 1082

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right.}{df}}{d^2}\right.$$

$\downarrow$ 217

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+}{d^2}\right.$$

$\downarrow$ 1479

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right.\right.}{df}}{d^2}\right.$$

$\downarrow$ 25

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right.\right.}{df}}{d^2}\right.$$

$\big\downarrow$ 27

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e\cdots}{2\sqrt{d}}\cdots}{df}\cdots}{d^2}\cdots\right.$$

$\big\downarrow$ 1103

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+\cdots}{2\sqrt{2}\sqrt{d}}\cdots}{df}\cdots}{d^2}\cdots\right.$$

input
```
Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^4,x]
```

output
```
d^4*(2/(5*d*f*(d*Cot[e + f*x])^(5/2)) - (2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2
*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[
1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*
d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + S
qrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))
/(d*f))/d^2)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(140) = 280$.

Time = 7.46 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.78

| method | result |
|---|---|
| default | $\frac{\sqrt{d\cot(fx+e)}\left(\sqrt{2}\left(-24\tan(fx+e)+4\tan(fx+e)\sec(fx+e)^2\right)+\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}\ \ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)+}{}\right.\right.}{}$ |

input
```
int((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/20/f*(d*cot(f*x+e))^(1/2)*(2^(1/2)*(-24*tan(f*x+e)+4*tan(f*x+e)*sec(f*x+
e)^2)+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*ln(-(cot(f*x+e)*cos(
f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^
2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(5+5*sec(f*
x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*ln(-(cot(f*x+e)*cos(
f*x+e)-2*cot(f*x+e)-2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^
2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(-5-5*sec(f
*x+e))+arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-cos(f*x+e)+1)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^
(1/2)*(10+10*sec(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*a
rctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+cos(f*
x+e)-1)/(cos(f*x+e)-1))*(10+10*sec(f*x+e)))*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.23

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx =$$

$$-\frac{10\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d}{d}\right)+10\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}-d}{d}\right)-5\sqrt{2}\sqrt{d}\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan}}}{}\right.}{}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
-1/20*(10*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d
)/d) + 10*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) - d
)/d) - 5*sqrt(2)*sqrt(d)*log((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e))*tan(f*x
 + e) + d*tan(f*x + e) + d)/tan(f*x + e)) + 5*sqrt(2)*sqrt(d)*log(-(sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e))*tan(f*x + e) - d*tan(f*x + e) - d)/tan(f*x
+ e)) - 8*(tan(f*x + e)^3 - 5*tan(f*x + e))*sqrt(d/tan(f*x + e)))/f
```

## Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx$$

input
```
integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**4,x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.18

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx =$$

$$-\frac{d^5\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\cdots\right)}{d^4}\cdots\right)}{20\,f}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
-1/20*d^5*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan
(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(
2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^
2 - 5*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

## Giac [F]

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{d\cot\left(fx+e\right)}\tan\left(fx+e\right)^4\,dx$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^4, x)
```

## Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.55

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = \frac{\frac{2d^3}{5}-\frac{2d^3}{\tan(e+fx)^2}}{f\left(\frac{d}{\tan(e+fx)}\right)^{5/2}} - \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f} + \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f}$$

input
```
int(tan(e + f*x)^4*(d*cot(e + f*x))^(1/2),x)
```

output
```
((2*d^3)/5 - (2*d^3)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1
/4)*d^(1/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(
1/4)*d^(1/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

## Reduce [F]

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\cot(fx+e)}\,\tan(fx+e)^4\,dx\right)$$

input
```
int((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x)
```

output
```
sqrt(d)*int(sqrt(cot(e + f*x))*tan(e + f*x)**4,x)
```

# 3.192 $\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$

Optimal result . . . . . 1416
Mathematica [A] (verified) . . . . . 1417
Rubi [A] (warning: unable to verify) . . . . . 1417
Maple [B] (warning: unable to verify) . . . . . 1422
Fricas [A] (verification not implemented) . . . . . 1423
Sympy [F] . . . . . 1424
Maxima [A] (verification not implemented) . . . . . 1424
Giac [F] . . . . . 1425
Mupad [B] (verification not implemented) . . . . . 1425
Reduce [F] . . . . . 1426

## Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f} + \frac{2d^2}{3f(d\cot(e+fx))^{3/2}}$$

output
```
-1/2*d^(1/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*
d^(1/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*d^(1/
2)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1
/2)/f+2/3*d^2/f/(d*cot(f*x+e))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.58

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx =$$
$$-\frac{\sqrt{d\cot(e+fx)}\left(-2+3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+\right.\right.}{3f}$$

input
```
Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^3,x]
```

output
```
-1/3*(Sqrt[d*Cot[e + f*x]]*(-2 + 3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e
 + f*x]^2)^(3/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3
/4))*Tan[e + f*x]^2)/f
```

## Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.32, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 25, 2030, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}{\tan\left(e+fx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$d^3\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \ 3955$$

$$d^3\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}dx}{d^2}\right)$$

$$\Big\downarrow \ 3042$$

$$d^3\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx}{d^2}\right)$$

$$\Big\downarrow \ 3957$$

$$d^3\left(\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \ 266$$

$$d^3\left(\frac{2\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \ 755$$

$$d^3\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \ 1476$$

$$d^3\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}\ldots}{2d}\ldots\right)}{df}\ldots\right.$$

$$\Big\downarrow \ 1082$$

$$d^3\left(\frac{2\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{\cdots}{3d\cdots}\right.$$

$\downarrow$ 217

$$d^3\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\downarrow$ 1479

$$d^3\left(\frac{2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)\cdots\right.}{\sqrt{2}\sqrt{d}}\cdots}{\cdots}\right.}{df}\right.$$

$\downarrow$ 25

$$d^3\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d\cdots}\right.}{df}\right.$$

$\downarrow$ 27

$$d^3\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\ldots}{2d\ldots}\right.}{df}\right.$$

$$\Big\downarrow \text{1103}$$

$$d^3\left(\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+f\ldots\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{df}\right.$$

input
```
Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^3,x]
```

output
```
d^3*(2/(3*d*f*(d*Cot[e + f*x])^(3/2)) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*
Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]
]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] +
d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*
x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/(d*f))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(124) = 248$.

Time = 8.30 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.99

| method | result |
|---|---|
| default | $-\frac{\sqrt{d\cot(fx+e)}\left(\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)-2\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+\csc(fx+e)-\sin(fx+e)-2\cos(fx+e)+2}{\cos(fx+e)-1}\right.\right.}{}$ |

input
```
int((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/12/f*(d*cot(f*x+e))^(1/2)*(ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*si
n(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(
f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+
e))^2)^(1/2)*(-3-3*sec(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+
cos(f*x+e)-1)/(cos(f*x+e)-1))*(-6-6*sec(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)
*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2
*cos(f*x+e)+2)/(cos(f*x+e)-1))*(3+3*sec(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)-1))*(-6-6*sec(f*x+e))-4*tan(f*x
+e)^2*2^(1/2))*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.32

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$$

$$= \frac{8\sqrt{\frac{d}{\tan(fx+e)}}\tan(fx+e)^2 + 6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d}{d}\right) + 6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}-d}{d}\right) + \cdots}{\cdots}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/12*(8*sqrt(d/tan(f*x + e))*tan(f*x + e)^2 + 6*sqrt(2)*sqrt(d)*arctan((sq
rt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d)/d) + 6*sqrt(2)*sqrt(d)*arctan((sqr
t(2)*sqrt(d)*sqrt(d/tan(f*x + e)) - d)/d) + 3*sqrt(2)*sqrt(d)*log((sqrt(2)
*sqrt(d)*sqrt(d/tan(f*x + e))*tan(f*x + e) + d*tan(f*x + e) + d)/tan(f*x +
 e)) - 3*sqrt(2)*sqrt(d)*log(-(sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e))*tan(f*
x + e) - d*tan(f*x + e) - d)/tan(f*x + e)))/f
```

## Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = \int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$$

input
```
integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**3,x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**3, x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.20

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$$
$$= \frac{d^4\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\sqrt{2}\,\mathrm{l}\cdots}{d^2}\right.}{12f}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/12*d^4*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2
*(d/tan(f*x + e))^(3/2)))/f
```

## Giac [F]

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = \int \sqrt{d\cot(fx+e)}\tan(fx+e)^3\,dx$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.53

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = \frac{2\,d^2}{3\,f\,\left(\frac{d}{\tan(e+f\,x)}\right)^{3/2}} - \frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} - \frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

input
```
int(tan(e + f*x)^3*(d*cot(e + f*x))^(1/2),x)
```

output
```
(2*d^2)/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4
)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(1/2)*atanh(((-1)
^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

## Reduce [F]

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\cot(fx+e)}\,\tan(fx+e)^3\,dx\right)$$

input
```
int((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x)
```

output
```
sqrt(d)*int(sqrt(cot(e + f*x))*tan(e + f*x)**3,x)
```

# 3.193 $\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$

Optimal result . . . . . . . . . . 1427
Mathematica [A] (verified) . . . . . . . . . . 1428
Rubi [A] (warning: unable to verify) . . . . . . . . . . 1428
Maple [B] (verified) . . . . . . . . . . 1433
Fricas [A] (verification not implemented) . . . . . . . . . . 1433
Sympy [F] . . . . . . . . . . 1434
Maxima [A] (verification not implemented) . . . . . . . . . . 1434
Giac [F] . . . . . . . . . . 1435
Mupad [B] (verification not implemented) . . . . . . . . . . 1435
Reduce [F] . . . . . . . . . . 1436

## Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f} + \frac{2d}{f\sqrt{d\cot(e+fx)}}$$

output
```
-1/2*d^(1/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*
d^(1/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d^(1/
2)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1
/2)/f+2*d/f/(d*cot(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$$
$$= \frac{d\left(2+\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}\right)}{f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^2,x]
```

output
```
(d*(2 + ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) - ArcTanh[
(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4)))/(f*Sqrt[d*Cot[e + f*x]]
)
```

## Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 2030, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}{\tan\left(e+fx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$d^2\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$d^2\left(\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int\sqrt{d\cot(e+fx)}dx}{d^2}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$d^2\left(\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}\right)$$

$$\Big\downarrow \textbf{3957}$$

$$d^2\left(\frac{\int\frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$$\Big\downarrow \textbf{266}$$

$$d^2\left(\frac{2\int\frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$$\Big\downarrow \textbf{826}$$

$$d^2\left(\frac{2\left(\frac{1}{2}\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$$\Big\downarrow \textbf{1476}$$

$$d^2\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+f\right.\right.\right.}{df}\right.$$

$$\Big\downarrow \textbf{1082}$$

$$d^2\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\right.}{df}\right.$$

$$\Big\downarrow \textbf{217}$$

$$d^2\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{}{df\sqrt{d}}\right.$$

$$\Big\downarrow \ 1479$$

$$d^2\left(\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan}{}\right.\right.}{df}\right.$$

$$\Big\downarrow \ 25$$

$$d^2\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan(}{}\right.\right.}{df}\right.$$

$$\Big\downarrow \ 27$$

$$d^2\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan(}{}\right.\right.}{df}\right.$$

$$\Big\downarrow \ 1103$$

$$d^2\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{}\right.}{}\right.\right.}{df}\right.$$

input
```
Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^2,x]
```

output
```
d^2*(2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[
e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(S
qrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f
*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Co
t[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/(d*f))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(122) = 244$.

Time = 7.26 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.94

| method | result |
|---|---|
| default | $-\frac{\sec(fx+e)\csc(fx+e)^2\sqrt{d\cot(fx+e)}\,(1-\cos(fx+e))\left(\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+2-2\cos}{1-\cos(fx+e)}\right.\right.}{}$ |

input
```
int((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*sec(f*x+e)*csc(f*x+e)^2*(d*cot(f*x+e))^(1/2)*(1-cos(f*x+e))*(ln(1/(
1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(-2*sin(f*x+e)*cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2-2*cos(f*x+e)-sin(f*x+e)))*(-2*sin(f*x+e)
*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2*arctan((sin(f*x+e)*(-2*sin(f*x+e)*co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)-1))*(-2*sin(f*x
+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-ln(-1/(1-cos(f*x+e))*(-csc(f*x+e)*(
1-cos(f*x+e))^2+2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)-2+2*cos(f*x+e)+sin(f*x+e)))*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^
2)^(1/2)+2*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^
(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-8*csc(f*x+e)+8*cot(f*x+e))*(1+cos(f*x+e))^2
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.32

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$$
$$= \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d}{d}\right)+2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}-d}{d}\right)-\sqrt{2}\sqrt{d}\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}\,\text{ta}}{\text{tan}}\right.}{4}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d)/d
) + 2*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) - d)/d)
 - sqrt(2)*sqrt(d)*log((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e))*tan(f*x + e)
+ d*tan(f*x + e) + d)/tan(f*x + e)) + sqrt(2)*sqrt(d)*log(-(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e))*tan(f*x + e) - d*tan(f*x + e) - d)/tan(f*x + e)) +
8*sqrt(d/tan(f*x + e))*tan(f*x + e))/f
```

## Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$$

input
```
integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**2,x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.22

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$$

$$= \frac{d^3\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log(}{}}{d^2}}{4f}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/4*d^3*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
 + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sq
rt(d/tan(f*x + e))))/f
```

## Giac [F]

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{d\cot(fx+e)}\tan(fx+e)^2\,dx$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = \frac{2d}{f\sqrt{\frac{d}{\tan(e+f\,x)}}} + \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{f} - \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{f}$$

input
```
int(tan(e + f*x)^2*(d*cot(e + f*x))^(1/2),x)
```

output
```
(2*d)/(f*(d/tan(e + f*x))^(1/2)) + ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d
/tan(e + f*x))^(1/2))/d^(1/2)))/f - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*
(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

## Reduce [F]

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\cot\left(fx+e\right)}\,\tan\left(fx+e\right)^2 dx\right)$$

input
```
int((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x)
```

output
```
sqrt(d)*int(sqrt(cot(e + f*x))*tan(e + f*x)**2,x)
```

# 3.194 $\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1437
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1438
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . 1438
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1442
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 1443
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1444
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 1444
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1445
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 1445
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1445

## Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f}$$

output
```
1/2*d^(1/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d
^(1/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d^(1/2
)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/
2)/f
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx$$
$$= \frac{d\sqrt{\cot(e+fx)}\Big(2\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+\log\left(1-\sqrt{2}\sqrt{c\right.}}{2\sqrt{2}f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x],x]
```

output
```
(d*Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan
[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Co
t[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt
[2]*f*Sqrt[d*Cot[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.33, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {3042, 25, 2030, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\sqrt{d\cot(e+fx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int -\frac{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}{\tan\left(e+fx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \ 25$$
$$-\int \frac{\sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$d\int \frac{1}{\sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{d^2\int \frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2d^2\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{755}$$

$$-\frac{2d^2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2d^2\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d}{2d}\right)}{f}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2d^2\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2d^2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{2d^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)-\right.}{\sqrt{2}\sqrt{d}}\right.}{f}$$

$\Big\downarrow$ 25

$$-\frac{2d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2c}\right.}{f}$$

$\Big\downarrow$ 27

$$-\frac{2d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2c}\right.}{f}$$

$\Big\downarrow$ 1103

$$-\frac{2d^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+f\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}$$

input

```
Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x],x]
```

output

```
(-2*d^2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*
Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d
]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]
*Sqrt[d]))/(2*d)))/f
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(106) = 212$.

Time = 9.19 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.52

| method | result |
|---|---|
| default | $-\frac{\sqrt{d\cot(fx+e)}\left(\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+2-2\cos(fx+e)-\sin(fx+e)}{1-\cos(fx+e)}\right)-2\arctan\left(\frac{\sin(fx-}{}\right.\right.}{}$ |

input
```
int((d*cot(f*x+e))^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*(d*cot(f*x+e))^(1/2)*(ln(1/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e)
)^2+2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2-2*cos
(f*x+e)-sin(f*x+e)))-2*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)-1))-ln(-1/(1-cos(f*x+e))*(-csc
(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)-2+2*cos(f*x+e)+sin(f*x+e)))-2*arctan((sin(f*x+e)*(-2*sin(f*x
+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1)))/(-si
n(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(csc(f*x+e)-cot(f*x+e))*2^(1/2
)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = -\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d}{d}\right)+2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}-d}{d}\right)+\sqrt{2}\sqrt{d}\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}}{}\right)}{4f}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d)/
d) + 2*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) - d)/d
) + sqrt(2)*sqrt(d)*log((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e))*tan(f*x + e)
 + d*tan(f*x + e) + d)/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-(sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e))*tan(f*x + e) - d*tan(f*x + e) - d)/tan(f*x + e)))/
f
```

## Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \int \sqrt{d\cot{(e+fx)}}\tan{(e+fx)}\,dx$$

input
```
integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e),x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*tan(e + f*x), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.22

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = -\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}} - \cdots\right)}{4f}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
-1/4*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
 + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

## Giac [F]

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \int \sqrt{d\cot(fx+e)}\tan(fx+e)\,dx$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*tan(f*x + e), x)
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} + \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

input
```
int(tan(e + f*x)*(d*cot(e + f*x))^(1/2),x)
```

output
```
((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/
f + ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))
*1i)/f
```

## Reduce [F]

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\cot(fx+e)}\,\tan(fx+e)\,dx\right)$$

input
```
int((d*cot(f*x+e))^(1/2)*tan(f*x+e),x)
```

output
```
sqrt(d)*int(sqrt(cot(e + f*x))*tan(e + f*x),x)
```

# 3.195 $\int \sqrt{d\cot(e+fx)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1447
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1448
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . 1448
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1452
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . 1452
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1453
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 1453
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1454
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 1454
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1455

## Optimal result

Integrand size = 12, antiderivative size = 136

$$\int \sqrt{d\cot(e+fx)}\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f}$$

output
```
1/2*d^(1/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d
^(1/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*d^(1/2
)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/
2)/f
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int \sqrt{d\cot(e+fx)}\,dx$$
$$=\frac{\left(-\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right)\sqrt[4]{-\cot(e+fx)}\sqrt{d\cot(e+fx)}}{f\cot^{\frac{3}{4}}(e+fx)}$$

input
```
Integrate[Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-ArcTan[(-Cot[e + f*x]^2)^(1/4)] + ArcTanh[(-Cot[e + f*x]^2)^(1/4)])*(-C
ot[e + f*x])^(1/4)*Sqrt[d*Cot[e + f*x]])/(f*Cot[e + f*x]^(3/4))
```

## Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\cot(e+fx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{ 3957}$$
$$-\frac{d\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}$$
$$\Big\downarrow \text{ 266}$$

$$-\frac{2d\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \ 826$$

$$-\frac{2d\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}$$

$$\Big\downarrow \ 1476$$

$$-\frac{2d\Bigg(\frac{1}{2}\Bigg(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx}}{f}$$

$$\Big\downarrow \ 1082$$

$$-\frac{2d\Bigg(\frac{1}{2}\Bigg(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\Bigg)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\,\mathrm{co}}}{f}$$

$$\Big\downarrow \ 217$$

$$-\frac{2d\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}$$

$$\Big\downarrow \ 1479$$

$$-\frac{2d\Bigg(\frac{1}{2}\Bigg(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\Bigg)+\frac{1}{2}\Bigg(\arctan\Big(}{f}$$

$$\Big\downarrow \ 25$$

$$-\frac{2d\Bigg(\frac{1}{2}\Bigg(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\Bigg)+\frac{1}{2}\Bigg(\arctan\Big(\sqrt{}}{f}$$

$$\Big\downarrow \ 27$$

$$-\frac{2d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}}{}\right.\right.}{f}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{2d\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}\right.}{}\right.\right.}{f}$$

input
```
Int[Sqrt[d*Cot[e + f*x]],x]
```

output
```
(-2*d*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + Ar
cTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqr
t[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[
d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d])
)/2))/f
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| derivativedivides | $-\frac{d\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{d\cot(fx+}}{\left(d^2\right)^{\frac{1}{4}}}\right.\right.}{4f\left(d^2\right)^{\frac{1}{4}}}$ |
| default | $-\frac{d\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{d\cot(fx+}}{\left(d^2\right)^{\frac{1}{4}}}\right.\right.}{4f\left(d^2\right)^{\frac{1}{4}}}$ |

input
```
int((d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*d/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^
(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*
2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(106) = 212.

Time = 0.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.10

$$\int\sqrt{d\cot(e+fx)}\,dx =$$

$$-\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}+d}{d}\right)+2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}-d}{d}\right)-\sqrt{2}\sqrt{d}\log\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{}$$

input
```
integrate((d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) +
 d)/sin(2*f*x + 2*e)) + d)/d) + 2*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d)*
sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) - d)/d) - sqrt(2)*sqrt(d)*
log((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2
*f*x + 2*e) + d*cos(2*f*x + 2*e) + d*sin(2*f*x + 2*e) + d)/sin(2*f*x + 2*e
)) + sqrt(2)*sqrt(d)*log(-(sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) + d)/s
in(2*f*x + 2*e))*sin(2*f*x + 2*e) - d*cos(2*f*x + 2*e) - d*sin(2*f*x + 2*e
) - d)/sin(2*f*x + 2*e)))/f
```

## Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot{(e+fx)}}\,dx$$

input
```
integrate((d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*cot(e + f*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.21

$$\int \sqrt{d\cot(e+fx)}\,dx = -\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx-}\right.}{\sqrt{d}}}{4f}$$

input
```
integrate((d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x +
 e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqr
t(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqr
t(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/f
```

## Giac [F]

$$\int \sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(fx+e)}\,dx$$

input
```
integrate((d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.37

$$\int \sqrt{d\cot(e+fx)}\,dx = -\frac{(-1)^{1/4}\sqrt{d}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\right)}{f}$$

input
```
int((d*cot(e + f*x))^(1/2),x)
```

output
```
-((-1)^(1/4)*d^(1/2)*(atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)) -
atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))))/f
```

## Reduce [F]

$$\int \sqrt{d\cot(e+fx)}\,dx = \sqrt{d}\left(\int \sqrt{\cot\left(fx+e\right)}dx\right)$$

input

```
int((d*cot(f*x+e))^(1/2),x)
```

output

```
sqrt(d)*int(sqrt(cot(e + f*x)),x)
```

# 3.196 $\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$


Optimal result . . . . . 1456
Mathematica [A] (verified) . . . . . 1457
Rubi [A] (warning: unable to verify) . . . . . 1457
Maple [A] (verified) . . . . . 1462
Fricas [B] (verification not implemented) . . . . . 1462
Sympy [F] . . . . . 1463
Maxima [A] (verification not implemented) . . . . . 1463
Giac [F] . . . . . 1464
Mupad [B] (verification not implemented) . . . . . 1464
Reduce [F] . . . . . 1465


## Optimal result

Integrand size = 19, antiderivative size = 153

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f} - \frac{2\sqrt{d\cot(e+fx)}}{f}$$

output
```
-1/2*d^(1/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*
d^(1/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*d^(1/
2)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1
/2)/f-2*(d*cot(f*x+e))^(1/2)/f
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.07

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx =$$

$$-\frac{(d\cot(e+fx))^{3/2}\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}+2\sqrt{\cot(e+fx)}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot(e+\cdots}{2\sqrt{\cdots}}\right.}{df\cot^{\frac{3}{2}}(e+fx)}$$

input
```
Integrate[Cot[e + f*x]*Sqrt[d*Cot[e + f*x]],x]
```

output
```
-(((d*Cot[e + f*x])^(3/2)*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2]
- ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] + 2*Sqrt[Cot[e + f*x]] +
Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2]) - Log[1 + S
qrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2])))/(d*f*Cot[e + f*x]^
(3/2)))
```

## Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.34, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (d\cot(e+fx))^{3/2}dx}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx}{d}$$

$$\Big\downarrow \text{3954}$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+f}{2d}}{f}}{d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e-}{f}}{d}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d}$$

$\Big\downarrow$ 1479

$$\frac{\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\cdots\right)}{f}\cdots}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arctan}\cdots}{2d}\cdots\right)}{f}\cdots}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arctan}\cdots}{2d}\cdots\right)}{f}\cdots}{d}$$

$\Big\downarrow$ 1103

$$\frac{\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}}{2d}\cdots\right)}{f}\cdots}{d}$$

input
```
Int[Cot[e + f*x]*Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-2*d*Sqrt[d*Cot[e + f*x]])/f + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot
[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(
Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2
*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x]
+ d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/f)/d
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $\frac{-2\sqrt{d\cot(fx+e)}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\cdots\right.\right.}{4}}{f}$ |
| default | $\frac{-2\sqrt{d\cot(fx+e)}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\cdots\right.\right.}{4}}{f}$ |

input
```
int(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2*(d*cot(f*x+e))^(1/2)+1/4*(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^
2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/
4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)
*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2
)+1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(121) = 242.

Time = 0.10 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.06

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$$
$$=\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}+d}{d}\right)+2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}-d}{d}\right)+\sqrt{2}\sqrt{d}\log\left(\frac{\sqrt{2}\sqrt{d}\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) +
d)/sin(2*f*x + 2*e)) + d)/d) + 2*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d)*s
qrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) - d)/d) + sqrt(2)*sqrt(d)*l
og((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*
f*x + 2*e) + d*cos(2*f*x + 2*e) + d*sin(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)
) - sqrt(2)*sqrt(d)*log(-(sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) + d)/si
n(2*f*x + 2*e))*sin(2*f*x + 2*e) - d*cos(2*f*x + 2*e) - d*sin(2*f*x + 2*e)
 - d)/sin(2*f*x + 2*e)) - 8*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)
))/f
```

## Sympy [F]

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot{(e+fx)}}\cot{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*cot(e + f*x), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.16

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\sqrt{\ldots}\right.}{4f}$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x +
e)))/f
```

## Giac [F]

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(fx+e)}\cot(fx+e)\,dx$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*cot(f*x + e), x)
```

## Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.48

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} - \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

input
```
int(cot(e + f*x)*(d*cot(e + f*x))^(1/2),x)
```

output
```
- (2*(d*cot(e + f*x))^(1/2))/f - ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*c
ot(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)
*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

## Reduce [F]

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{\sqrt{d}\left(-2\sqrt{\cot\left(fx+e\right)}-\left(\int \frac{\sqrt{\cot(fx+e)}}{\cot(fx+e)}dx\right)f\right)}{f}$$

input
```
int(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*( - 2*sqrt(cot(e + f*x)) - int(sqrt(cot(e + f*x))/cot(e + f*x),x)
*f))/f
```

# 3.197 $\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1466
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1467
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . 1467
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1472
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . 1472
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1473
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 1473
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1474
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 1474
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1475


## Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3df}$$

output
```
-1/2*d^(1/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*
d^(1/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d^(1/
2)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1
/2)/f-2/3*(d*cot(f*x+e))^(3/2)/d/f
```

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.63

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{\sqrt{d\cot(e+fx)}\left(-3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{}\right.}{3f\cot^{\frac{3}{4}}(e+fx)}$$

input
```
Integrate[Cot[e + f*x]^2*Sqrt[d*Cot[e + f*x]],x]
```

output
```
-1/3*(Sqrt[d*Cot[e + f*x]]*(-3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f
*x])^(1/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) + 2*
Cot[e + f*x]^(7/4)))/(f*Cot[e + f*x]^(3/4))
```

## Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.26, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (d\cot(e+fx))^{5/2}dx}{d^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx}{d^2}$$

$$\Big\downarrow \; 3954$$

$$\frac{d^2\left(-\int \sqrt{d\cot(e+fx)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^2}$$

$\Big\downarrow$ 3042

$$\frac{d^2\left(-\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^2}$$

$\Big\downarrow$ 3957

$$\frac{\frac{d^3\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^2}$$

$\Big\downarrow$ 266

$$\frac{\frac{2d^3\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^2}$$

$\Big\downarrow$ 826

$$\frac{\frac{2d^3\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^2}$$

$\Big\downarrow$ 1476

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}}{f}}{d^2}$$

$\Big\downarrow$ 1082

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-2d(}{d^2}$$

$\Big\downarrow$ 217

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^2}$$

$\Big\downarrow$ 1479

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx\right.}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\right.}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\right.}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+\right.}{2\sqrt{2}\sqrt{d}}\right.}{f}}{d^2}$$

input
```
Int[Cot[e + f*x]^2*Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(3/2))/(3*f) + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[
d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f
*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Co
t[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] +
 d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/f)/d^2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| derivativedivides | $-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{fd}$ |
| default | $-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{fd}$ |

input
```
int(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d*(1/3*(d*cot(f*x+e))^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*
x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(
d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^
2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+
e))^(1/2)+1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(124) = 248.

Time = 0.13 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.35

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$$
$$=\frac{6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}+d}{d}\right)\sin(2fx+2e)+6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}-d}{d}\right)\sin(2\ldots}{}$$

input
```
integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/12*(6*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) +
 d)/sin(2*f*x + 2*e)) + d)/d)*sin(2*f*x + 2*e) + 6*sqrt(2)*sqrt(d)*arctan(
(sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) - d)/d)*s
in(2*f*x + 2*e) - 3*sqrt(2)*sqrt(d)*log((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x
 + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e) + d*cos(2*f*x + 2*e) + d*s
in(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e) + 3*sqrt(2)*sqrt(d
)*log(-(sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*si
n(2*f*x + 2*e) - d*cos(2*f*x + 2*e) - d*sin(2*f*x + 2*e) - d)/sin(2*f*x +
2*e))*sin(2*f*x + 2*e) - 8*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))
*(cos(2*f*x + 2*e) + 1))/(f*sin(2*f*x + 2*e))
```

## Sympy [F]

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot\left(e+fx\right)}\cot^2\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**2*(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*cot(e + f*x)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.18

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{3\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx\cdots}\right.}{\sqrt{d}}\cdots\right.}{12\,df}$$

input
```
integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f
*x + e))^(3/2))/(d*f)
```

## Giac [F]

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot\left(fx+e\right)}\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*cot(f*x + e)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{f} - \frac{2\,(d\cot(e+f\,x))^{3/2}}{3\,d\,f} - \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{f}$$

input
```
int(cot(e + f*x)^2*(d*cot(e + f*x))^(1/2),x)
```

output
```
((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/f -
 (2*(d*cot(e + f*x))^(3/2))/(3*d*f) - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4
)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/f
```

## Reduce [F]

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \sqrt{d}\left(\int \sqrt{\cot\left(fx+e\right)}\,\cot\left(fx+e\right)^2 dx\right)$$

input
```
int(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(cot(e + f*x))*cot(e + f*x)**2,x)
```

# 3.198 $\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$

Optimal result . . . . . 1476
Mathematica [A] (verified) . . . . . 1477
Rubi [A] (warning: unable to verify) . . . . . 1477
Maple [A] (verified) . . . . . 1483
Fricas [B] (verification not implemented) . . . . . 1483
Sympy [F] . . . . . 1484
Maxima [A] (verification not implemented) . . . . . 1484
Giac [F] . . . . . 1485
Mupad [B] (verification not implemented) . . . . . 1485
Reduce [F] . . . . . 1486

## Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f} + \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f}$$

output
```
1/2*d^(1/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d
^(1/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d^(1/2
)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/
2)/f+2*(d*cot(f*x+e))^(1/2)/f-2/5*(d*cot(f*x+e))^(5/2)/d^2/f
```

## Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$$
$$= \frac{\sqrt{d\cot(e+fx)}\left(10\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-10\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+40\sqrt{c}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^3*Sqrt[d*Cot[e + f*x]],x]
```

output
```
(Sqrt[d*Cot[e + f*x]]*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] -
 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 40*Sqrt[Cot[e + f*x]]
 - 8*Cot[e + f*x]^(5/2) + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + C
ot[e + f*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]
]))/(20*f*Sqrt[Cot[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.31, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {2030, 3042, 3954, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$$
$$\Big\downarrow \text{2030}$$
$$\frac{\int (d\cot(e+fx))^{7/2}dx}{d^3}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx}{d^3}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-d^2\int (d\cot(e+fx))^{3/2}dx-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-d^2\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-d^2\left(d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-d^2\left(d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \text{3957}$$

$$\frac{-d^2\left(\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{-d^2\left(\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \text{755}$$

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \text{1476}$$

$$-d^2\left(\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d}}{2d}\right.}{f}\right.}{d^3}$$

$\downarrow$ 1082

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{}}{}\right.}{d^3}$$

$\downarrow$ 217

$$-d^2\left(\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e}{5}}{d^3}$$

$\downarrow$ 1479

$$-d^2\left(\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+f\right.}{\sqrt{2}\sqrt{d}}}{}\right.}{f}\right.}{d^3}$$

$\downarrow$ 25

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right.}{f}\right.}{d^3}$$

$\Big\downarrow$ 27

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right.}{f}\right.}{d^3}$$

$\Big\downarrow$ 1103

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e-\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}\right.}{d^3}$$

input
```
Int[Cot[e + f*x]^3*Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(5/2))/(5*f) - d^2*((-2*d*Sqrt[d*Cot[e + f*x]])/f
+ (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
 ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2
*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[
d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2
]*Sqrt[d]))/(2*d)))/f))/d^3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{5}{2}}}{5}-d^2\sqrt{d\cot(fx+e)}+\frac{d^2\left(d^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cdots}}{(\cdots}\right.\cdots\right.}{8}\cdots\right.}{f\,d^2}$ |
| default | $-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{5}{2}}}{5}-d^2\sqrt{d\cot(fx+e)}+\frac{d^2\left(d^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cdots}}{(\cdots}\right.\cdots\right.}{8}\cdots\right.}{f\,d^2}$ |

input
```
int(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d^2*(1/5*(d*cot(f*x+e))^(5/2)-d^2*(d*cot(f*x+e))^(1/2)+1/8*d^2*(d^2)^
(1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(
d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(
1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1
/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(139) = 278.

Time = 0.10 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.20

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx =$$

$$-\frac{10\sqrt{2}\sqrt{d}(\cos(2fx+2e)-1)\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}+d}{d}\right)+10\sqrt{2}\sqrt{d}(\cos(2fx+2e)-1)\,\mathrm{arcta}\cdots}{\cdots}$$

input
```
integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/20*(10*sqrt(2)*sqrt(d)*(cos(2*f*x + 2*e) - 1)*arctan((sqrt(2)*sqrt(d)*s
qrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) + d)/d) + 10*sqrt(2)*sqrt(d
)*(cos(2*f*x + 2*e) - 1)*arctan((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e)
+ d)/sin(2*f*x + 2*e)) - d)/d) + 5*sqrt(2)*sqrt(d)*(cos(2*f*x + 2*e) - 1)*
log((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2
*f*x + 2*e) + d*cos(2*f*x + 2*e) + d*sin(2*f*x + 2*e) + d)/sin(2*f*x + 2*e
)) - 5*sqrt(2)*sqrt(d)*(cos(2*f*x + 2*e) - 1)*log(-(sqrt(2)*sqrt(d)*sqrt((
d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e) - d*cos(2*f*x +
 2*e) - d*sin(2*f*x + 2*e) - d)/sin(2*f*x + 2*e)) - 16*sqrt((d*cos(2*f*x +
 2*e) + d)/sin(2*f*x + 2*e))*(3*cos(2*f*x + 2*e) - 2))/(f*cos(2*f*x + 2*e)
 - f)
```

## Sympy [F]

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(e+fx)}\cot^3(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**3*(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*cot(e + f*x)**3, x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.13

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx =$$
$$-\frac{10\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{5}{2}}\log\left(\right.}{}$$

input
```
integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/20*(10*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/t
an(f*x + e)))/sqrt(d)) + 10*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + 5*sqrt(2)*d^(5/2)*log(sqrt(2)*
sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 5*sqrt(2)*d^(5/2)*log
(-sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 40*d^2*sqrt
(d/tan(f*x + e)) + 8*(d/tan(f*x + e))^(5/2))/(d^2*f)
```

## Giac [F]

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx=\int \sqrt{d\cot\left(fx+e\right)}\cot\left(fx+e\right)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*cot(f*x + e)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.51

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx=\frac{2\sqrt{d\cot\left(e+f\,x\right)}}{f}-\frac{2\left(d\cot(e+f\,x)\right)^{5/2}}{5\,d^2\,f}+\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}+\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}\,1\mathrm{i}}{\sqrt{d}}\right)}{f}$$

input
```
int(cot(e + f*x)^3*(d*cot(e + f*x))^(1/2),x)
```

output
```
(2*(d*cot(e + f*x))^(1/2))/f - (2*(d*cot(e + f*x))^(5/2))/(5*d^2*f) + ((-1
)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f +
((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2)*1i)/d^(1/2)))/
f
```

## Reduce [F]

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$$
$$= \frac{\sqrt{d}\left(-2\sqrt{\cot(fx+e)}\,\cot(fx+e)^2+10\sqrt{\cot(fx+e)}+5\left(\int \frac{\sqrt{\cot(fx+e)}}{\cot(fx+e)}dx\right)f\right)}{5f}$$

input
```
int(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*( - 2*sqrt(cot(e + f*x))*cot(e + f*x)**2 + 10*sqrt(cot(e + f*x))
+ 5*int(sqrt(cot(e + f*x))/cot(e + f*x),x)*f))/(5*f)
```

# 3.199 $\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx$


Optimal result . . . 1487
Mathematica [A] (verified) . . . 1488
Rubi [A] (warning: unable to verify) . . . 1488
Maple [B] (warning: unable to verify) . . . 1493
Fricas [A] (verification not implemented) . . . 1494
Sympy [F(-1)] . . . 1495
Maxima [A] (verification not implemented) . . . 1495
Giac [F] . . . 1496
Mupad [B] (verification not implemented) . . . 1496
Reduce [F] . . . 1496


## Optimal result

Integrand size = 21, antiderivative size = 178

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f} + \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}}$$

output
```
1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d
^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*d^(3/2
)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/
2)/f+2/5*d^4/f/(d*cot(f*x+e))^(5/2)-2*d^2/f/(d*cot(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.61

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx =$$

$$-\frac{(d\cot(e+fx))^{3/2}\left(-2+10\cot^2(e+fx)+5\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}\cot^{\frac{9}{4}}(e+fx)}{5f}$$

input
```
Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^5,x]
```

output
```
-1/5*((d*Cot[e + f*x])^(3/2)*(-2 + 10*Cot[e + f*x]^2 + 5*ArcTan[(-Cot[e +
f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4)*Cot[e + f*x]^(9/4) + 5*ArcTanh[(-Cot[
e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(5/4))*Tan[e + f*x]^4)/f
```

## Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.29, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 25, 2030, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$d^5 \int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int \frac{1}{(d\cot(e+fx))^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int \frac{1}{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \text{3955}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \text{3957}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \text{266}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \text{826}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$\downarrow$ 1476

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cdots}\right.\right.}{df}}{d^2}\right.$$

$\downarrow$ 1082

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cdots}{d^4\cot^4(\cdots}\right.}{df}}{d^2}\right.$$

$\downarrow$ 217

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\cdots}{d^2}\right.$$

$\downarrow$ 1479

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(\cdots}}{2\sqrt{2}\sqrt{d}}\right.\right.}{df}}{d^2}\right.$$

$\downarrow$ 25

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e\cdots}}{2\sqrt{2}\sqrt{d}}\cdots}{df}\cdots}{d^2}\cdots$$

$$\Big\downarrow \; 27$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e\cdots}}{2\sqrt{d}}\cdots}{df}\cdots}{d^2}\cdots$$

$$\Big\downarrow \; 1103$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+\cdots\right.}{2\sqrt{2}\sqrt{d}}\cdots\right.}{df}\cdots}{d^2}\cdots$$

input

```
Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^5,x]
```

output

```
d^5*(2/(5*d*f*(d*Cot[e + f*x])^(5/2)) - (2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2
*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[
1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*
d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + S
qrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))
/(d*f))/d^2)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (warning: unable to verify)

Leave count of result is larger than twice the leaf count of optimal. 488 vs. $2(142) = 284$.

Time = 2.57 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.75

| method | result |
|---|---|
| default | $-\sqrt{d\cot(fx+e)}\,d\left(\sqrt{2}\left(24\tan(fx+e)-4\tan(fx+e)\sec(fx+e)^2\right)+\arctan\left(\frac{-\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+\cos(fx+e)-1}{\cos(fx+e)-1}\right)\sqrt{}\right.$ |

input
```
int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/20/f*(d*cot(f*x+e))^(1/2)*d*(2^(1/2)*(24*tan(f*x+e)-4*tan(f*x+e)*sec(f*
x+e)^2)+arctan((-sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1
/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)*(10+10*sec(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+cos(
f*x+e)-1)/(cos(f*x+e)-1))*(-10-10*sec(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+c
os(f*x+e))^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(
-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*c
os(f*x+e)+2)/(cos(f*x+e)-1))*(-5-5*sec(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+
cos(f*x+e))^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*sin(f*x+e)*
(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*
cos(f*x+e)+2)/(cos(f*x+e)-1))*(5+5*sec(f*x+e)))*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.24

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx =$$

$$-\frac{10\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d}{d}\right)+10\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}-d}{d}\right)-5\sqrt{2}d^{\frac{3}{2}}\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/20*(10*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d
)/d) + 10*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) - d
)/d) - 5*sqrt(2)*d^(3/2)*log((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e))*tan(f*x
 + e) + d*tan(f*x + e) + d)/tan(f*x + e)) + 5*sqrt(2)*d^(3/2)*log(-(sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e))*tan(f*x + e) - d*tan(f*x + e) - d)/tan(f*x
+ e)) - 8*(d*tan(f*x + e)^3 - 5*d*tan(f*x + e))*sqrt(d/tan(f*x + e)))/f
```

## Sympy [F(-1)]

Timed out.

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**5,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.16

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx =$$

$$-\frac{d^6\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\,\mathrm{lo}}{}\right.}{d^4}}{20f}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/20*d^6*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan
(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(
2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^
2 - 5*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

## Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)^5\,dx$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^5, x)
```

## Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.54

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \frac{\frac{2d^4}{5}-\frac{2d^4}{\tan(e+fx)^2}}{f\left(\frac{d}{\tan(e+fx)}\right)^{5/2}} - \frac{(-1)^{1/4}d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f} + \frac{(-1)^{1/4}d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f}$$

input
```
int(tan(e + f*x)^5*(d*cot(e + f*x))^(3/2),x)
```

output
```
((2*d^4)/5 - (2*d^4)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1
/4)*d^(3/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(
1/4)*d^(3/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

## Reduce [F]

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\cot(fx+e)}\cot(fx+e)\tan(fx+e)^5\,dx\right)d$$

input
```
int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x)
```

output
```
sqrt(d)*int(sqrt(cot(e + f*x))*cot(e + f*x)*tan(e + f*x)**5,x)*d
```

# 3.200 $\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1498
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1499
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . 1499
Maple [B] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . 1504
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 1505
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1505
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 1506
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1506
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1507
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1507

## Optimal result

Integrand size = 21, antiderivative size = 158

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f}+\frac{2d^3}{3f(d\cot(e+fx))^{3/2}}$$

output
```
-1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*
d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*d^(3/
2)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1
/2)/f+2/3*d^3/f/(d*cot(f*x+e))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.58

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx =$$

$$-\frac{(d\cot(e+fx))^{3/2}\left(-2+3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+f\right.\right.}{3f}$$

input
```
Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

output
```
-1/3*((d*Cot[e + f*x])^(3/2)*(-2 + 3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot
[e + f*x]^2)^(3/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^
(3/4))*Tan[e + f*x]^3)/f
```

## Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.32, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 2030, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{ 2030}$$

$$d^4\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3955}$$

$$d^4\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int\frac{1}{\sqrt{d\cot(e+fx)}}dx}{d^2}\right)$$

$$\Big\downarrow \ 3042$$

$$d^4\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int\frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx}{d^2}\right)$$

$$\Big\downarrow \ 3957$$

$$d^4\left(\frac{\int\frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \ 266$$

$$d^4\left(\frac{2\int\frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \ 755$$

$$d^4\left(\frac{2\left(\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \ 1476$$

$$d^4\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\right.}{2d}}{df}\right.$$

$$\Big\downarrow \ 1082$$

$$d^4\left(\frac{2\left(\frac{\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{}{3d}\right.$$

$$\Big\downarrow \; 217$$

$$d^4\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \; 1479$$

$$d^4\left(\frac{2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)\cdots\right.}{\sqrt{2}\sqrt{d}}\cdots}{\cdots}\cdots\right.}{df}\cdots\right.$$

$$\Big\downarrow \; 25$$

$$d^4\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d\cdots}\cdots\right.}{df}\cdots\right.$$

$$\Big\downarrow \; 27$$

$$d^4\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d\cdots}\cdots\right.}{df}\cdots\right.$$

$$\Big\downarrow \; 1103$$

$$d^4\left(\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+f\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{df}\right.$$

input
```
Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

output
```
d^4*(2/(3*d*f*(d*Cot[e + f*x])^(3/2)) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*
Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]
]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] +
d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*
x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/(d*f))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(124) = 248$.

Time = 1.48 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.99

| method | result |
|---|---|
| default | $\frac{d\sqrt{d\cot(fx+e)}\left(\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)-2\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+\csc(fx+e)-\sin(fx+e)-2\cos(fx+e)+2}{\cos(fx+e)-1}\right)\right.}{}$ |

input
```
int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/12/f*d*(d*cot(f*x+e))^(1/2)*(ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*s
in(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin
(f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)*(3+3*sec(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+
cos(f*x+e)-1)/(cos(f*x+e)-1))*(6+6*sec(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+
cos(f*x+e))^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*
(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*
cos(f*x+e)+2)/(cos(f*x+e)-1))*(-3-3*sec(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)*arctan((-sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-6-6*sec(f*x+e))+4*tan(f*
x+e)^2*2^(1/2))*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.32

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{8\,d\sqrt{\frac{d}{\tan(fx+e)}}\tan{(fx+e)}^2 + 6\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d}{d}\right) + 6\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx}}}{d}\right.}{}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/12*(8*d*sqrt(d/tan(f*x + e))*tan(f*x + e)^2 + 6*sqrt(2)*d^(3/2)*arctan((
sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d)/d) + 6*sqrt(2)*d^(3/2)*arctan((s
qrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) - d)/d) + 3*sqrt(2)*d^(3/2)*log((sqrt(
2)*sqrt(d)*sqrt(d/tan(f*x + e))*tan(f*x + e) + d*tan(f*x + e) + d)/tan(f*x
 + e)) - 3*sqrt(2)*d^(3/2)*log(-(sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e))*tan(
f*x + e) - d*tan(f*x + e) - d)/tan(f*x + e)))/f
```

## Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \int (d\cot{(e+fx)})^{\frac{3}{2}}\tan^4{(e+fx)}\,dx$$

input
```
integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**4,x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.20

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{d^5\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\cdots\right)}{d^2}\cdots\right)}{12f}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
1/12*d^5*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2
*(d/tan(f*x + e))^(3/2)))/f
```

## Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^4, x)
```

## Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.53

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{2\,d^3}{3\,f\,\left(\frac{d}{\tan(e+f\,x)}\right)^{3/2}} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

input
```
int(tan(e + f*x)^4*(d*cot(e + f*x))^(3/2),x)
```

output
```
(2*d^3)/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4
)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(3/2)*atanh(((-1)
^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

## Reduce [F]

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\cot(fx+e)}\,\cot(fx+e)\tan(fx+e)^4\,dx\right)d$$

input
```
int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x)
```

output
```
sqrt(d)*int(sqrt(cot(e + f*x))*cot(e + f*x)*tan(e + f*x)**4,x)*d
```

# 3.201 $\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx$

Optimal result . . . 1508
Mathematica [A] (verified) . . . 1509
Rubi [A] (warning: unable to verify) . . . 1509
Maple [B] (warning: unable to verify) . . . 1514
Fricas [A] (verification not implemented) . . . 1514
Sympy [F] . . . 1515
Maxima [A] (verification not implemented) . . . 1515
Giac [F] . . . 1516
Mupad [B] (verification not implemented) . . . 1516
Reduce [F] . . . 1517

## Optimal result

Integrand size = 21, antiderivative size = 157

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f}+\frac{2d^2}{f\sqrt{d\cot(e+fx)}}$$

output

```
-1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*
d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d^(3/
2)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1
/2)/f+2*d^2/f/(d*cot(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.52

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{d^2\left(2+\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2}\right.}{f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^3,x]
```

output
```
(d^2*(2 + ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) - ArcTan
h[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4)))/(f*Sqrt[d*Cot[e + f*x
]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.28, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 25, 2030, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$d^3 \int \frac{1}{\left(-d \tan \left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}dx$$

$\big\downarrow$ 3955

$$d^3 \left(\frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}\right)$$

$\big\downarrow$ 3042

$$d^3 \left(\frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}\right)$$

$\big\downarrow$ 3957

$$d^3\left(\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df} + \frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$\big\downarrow$ 266

$$d^3\left(\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$\big\downarrow$ 826

$$d^3\left(\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$\big\downarrow$ 1476

$$d^3\Bigg(\frac{2\Big(\frac{1}{2}\Big(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+f}{df}$$

$\big\downarrow$ 1082

$$d^3\Bigg(\frac{2\Big(\frac{1}{2}\Big(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\Big)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d}}{df}$$

$\big\downarrow$ 217

$$d^3\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{}{df\sqrt{d}}\right.$$

$\big\downarrow$ 1479

$$d^3\left(\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan}{}\right.\right.}{df}\right.$$

$\big\downarrow$ 25

$$d^3\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan(}{}\right.\right.}{df}\right.$$

$\big\downarrow$ 27

$$d^3\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan(}{}\right.\right.}{df}\right.$$

$\big\downarrow$ 1103

$$d^3\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{}\right.}{}\right.\right.}{df}\right.$$

input
```
Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^3,x]
```

output
```
d^3*(2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[
e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(S
qrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f
*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Co
t[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/(d*f))
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(124) = 248$.

Time = 2.32 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.04

| method | result |
|---|---|
| default | $\frac{\csc(fx+e)^4 d\sqrt{d\cot(fx+e)}\,(1-\cos(fx+e))^2 \left(\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+2-2\cos(fx+e)-\sin(}{1-\cos(fx+e)}\right.\right.}{}$ |

input
```
int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/f*csc(f*x+e)^4*d*(d*cot(f*x+e))^(1/2)*(1-cos(f*x+e))^2*(ln(1/(1-cos(f*
x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)+2-2*cos(f*x+e)-sin(f*x+e)))*(-2*sin(f*x+e)*cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-2*arctan((-sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e
)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-2*sin(f*x+e)*cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-ln(-1/(1-cos(f*x+e))*(-csc(f*x+e)*(1-cos(f
*x+e))^2+2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-2+
2*cos(f*x+e)+sin(f*x+e)))*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)+2*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+c
os(f*x+e)-1)/(cos(f*x+e)-1))*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)-8*csc(f*x+e)+8*cot(f*x+e))/(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(1+cos(f
*x+e))^2
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.31

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{2\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d}{d}\right)+2\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}-d}{d}\right)-\sqrt{2}d^{\frac{3}{2}}\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan}}}{}\right.}{}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d)/d
) + 2*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) - d)/d)
 - sqrt(2)*d^(3/2)*log((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e))*tan(f*x + e)
+ d*tan(f*x + e) + d)/tan(f*x + e)) + sqrt(2)*d^(3/2)*log(-(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e))*tan(f*x + e) - d*tan(f*x + e) - d)/tan(f*x + e)) +
8*d*sqrt(d/tan(f*x + e))*tan(f*x + e))/f
```

## Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\tan^3(e+fx)\,dx$$

input
```
integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**3,x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**3, x)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.20

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{d^4\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right.}{\sqrt{d}}}{d^2}\right.}{4f}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/4*d^4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
 + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sq
rt(d/tan(f*x + e))))/f
```

## Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx=\int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)^3\,dx$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.52

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx=\frac{2\,d^2}{f\,\sqrt{\frac{d}{\tan(e+f\,x)}}}+\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{f}-\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{f}$$

input
```
int(tan(e + f*x)^3*(d*cot(e + f*x))^(3/2),x)
```

output
```
(2*d^2)/(f*(d/tan(e + f*x))^(1/2)) + ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*
(d/tan(e + f*x))^(1/2))/d^(1/2)))/f - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4
)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

## Reduce [F]

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\cot(fx+e)}\,\cot(fx+e)\tan(fx+e)^3\,dx\right)d$$

input
```
int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x)
```

output
```
sqrt(d)*int(sqrt(cot(e + f*x))*cot(e + f*x)*tan(e + f*x)**3,x)*d
```

# 3.202 $\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx$

Optimal result . . . 1518
Mathematica [A] (verified) . . . 1518
Rubi [A] (warning: unable to verify) . . . 1519
Maple [B] (verified) . . . 1523
Fricas [A] (verification not implemented) . . . 1524
Sympy [F] . . . 1524
Maxima [A] (verification not implemented) . . . 1525
Giac [F] . . . 1525
Mupad [B] (verification not implemented) . . . 1526
Reduce [F] . . . 1526

## Optimal result

Integrand size = 21, antiderivative size = 137

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f}$$

output
```
1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d
^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d^(3/2
)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/
2)/f
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \frac{d^2\sqrt{\cot(e+fx)}\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+\log\left(1\right.\right.}{2\sqrt{2}f\sqrt{d\cot(e+}}$$

input
```
Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

output
```
(d^2*Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcT
an[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] +
Cot[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sq
rt[2]*f*Sqrt[d*Cot[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.33, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 2030, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$d^2\int \frac{1}{\sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{755}$$

$$-\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$\big\downarrow$ 1476

$$-\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$\big\downarrow$ 1082

$$-\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$\big\downarrow$ 217

$$-\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}$$

$\big\downarrow$ 1479

$$-\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)-\right.}{\sqrt{2}\sqrt{d}}}{}\right.}{f}$$

$\big\downarrow$ 25

$$-\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}\right.}{f}$$

$\big\downarrow$ 27

$$-\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}$$

$$\downarrow 1103$$

$$-\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+f\right.}{2\sqrt{2}\sqrt{d}}}{2d}$$

input
```
Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

output
```
(-2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*
Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d
]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]
*Sqrt[d]))/(2*d)))/f
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(106) = 212.

Time = 1.11 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.63

| method | result |
|---|---|
| default | $-\frac{\sin(fx+e)\left(\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)+2\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+\csc(fx+e)-\sin(fx+e)-2\cos(fx+e)+2}{\cos(fx+e)-1}\right)+2$ |

input
```
int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*sin(f*x+e)*(ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-
2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*co
s(f*x+e)+2)/(cos(f*x+e)-1))+2*arctan((-sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e
)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))-ln(-(cot(f*x+e)*co
s(f*x+e)-2*cot(f*x+e)-2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e)
)^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))-2*arctan(
(sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1
)/(cos(f*x+e)-1)))*(d*cot(f*x+e))^(1/2)*d/(1+cos(f*x+e))/(-sin(f*x+e)*cos(
f*x+e)/(1+cos(f*x+e))^2)^(1/2)*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx =$$

$$-\frac{2\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d}{d}\right)+2\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}-d}{d}\right)+\sqrt{2}d^{\frac{3}{2}}\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}\tan(}{\tan(f}\right.}{4f}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d)/
d) + 2*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) - d)/d
) + sqrt(2)*d^(3/2)*log((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e))*tan(f*x + e)
 + d*tan(f*x + e) + d)/tan(f*x + e)) - sqrt(2)*d^(3/2)*log(-(sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e))*tan(f*x + e) - d*tan(f*x + e) - d)/tan(f*x + e)))/
f
```

## Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \int (d\cot{(e+fx)})^{\frac{3}{2}}\tan^2{(e+fx)}\,dx$$

input
```
integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**2,x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.22

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx =$$

$$-\frac{d^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\cdots\right.}{d^{\frac{3}{2}}}\cdots\right.}{4f}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
-1/4*d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
 + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

## Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} + \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

input
```
int(tan(e + f*x)^2*(d*cot(e + f*x))^(3/2),x)
```

output
```
((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/
f + ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))
*1i)/f
```

## Reduce [F]

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\cot(fx+e)}\,\cot(fx+e)\tan(fx+e)^2\,dx\right)d$$

input
```
int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x)
```

output
```
sqrt(d)*int(sqrt(cot(e + f*x))*cot(e + f*x)*tan(e + f*x)**2,x)*d
```

# 3.203 $\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx$

Optimal result . . . . . 1527
Mathematica [A] (verified) . . . . . 1527
Rubi [A] (warning: unable to verify) . . . . . 1528
Maple [A] (verified) . . . . . 1532
Fricas [A] (verification not implemented) . . . . . 1532
Sympy [F] . . . . . 1533
Maxima [A] (verification not implemented) . . . . . 1533
Giac [F] . . . . . 1534
Mupad [B] (verification not implemented) . . . . . 1534
Reduce [F] . . . . . 1534

## Optimal result

Integrand size = 19, antiderivative size = 136

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx=\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f}$$

output
```
1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d
^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*d^(3/2
)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/
2)/f
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.53

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx=\frac{d\left(-\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)+\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right)\sqrt[4]{-\cot(e+fx)}\sqrt{d\cot(e+}}{f\cot^{\frac{3}{4}}(e+fx)}$$

input
```
Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x],x]
```

output
```
(d*(-ArcTan[(-Cot[e + f*x]^2)^(1/4)] + ArcTanh[(-Cot[e + f*x]^2)^(1/4)])*(
-Cot[e + f*x])^(1/4)*Sqrt[d*Cot[e + f*x]])/(f*Cot[e + f*x]^(3/4))
```

## Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.29, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {3042, 25, 2030, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\tan\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$d\int \sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{d^2\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2d^2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$\Big\downarrow$ 826

$$-\frac{2d^2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}$$

$\Big\downarrow$ 1476

$$-\frac{2d^2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+f\right.\right.}{f}$$

$\Big\downarrow$ 1082

$$-\frac{2d^2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\right.}{f}$$

$\Big\downarrow$ 217

$$-\frac{2d^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}$$

$\Big\downarrow$ 1479

$$-\frac{2d^2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan}{}\right.\right.}{f}$$

$\Big\downarrow$ 25

$$-\frac{2d^2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan(}{}\right.\right.}{f}$$

$\Big\downarrow$ 27

$$-\frac{2d^2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan(}{}\right.\right.}{f}$$

$\Big\downarrow$ 1103

$$-\frac{2d^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{}\right.}{}\right.\right.}{f}$$

input
```
Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x],x]
```

output
```
(-2*d^2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - S
qrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Lo
g[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d
]))/2))/f
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| default | $-\frac{d^{2}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}{d\cot(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)\right)}{4f\left(d^{2}\right)^{\frac{1}{4}}}$ |

input
```
int((d*cot(f*x+e))^(3/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/4*d^2/f/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e)
)^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2
)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+
1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.36

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx =$$

$$-\frac{2\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d}{d}\right)+2\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}-d}{d}\right)-\sqrt{2}d^{\frac{3}{2}}\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}\tan(}{\tan(f}\right.}{4f}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d)/
d) + 2*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) - d)/d
) - sqrt(2)*d^(3/2)*log((sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e))*tan(f*x + e)
 + d*tan(f*x + e) + d)/tan(f*x + e)) + sqrt(2)*d^(3/2)*log(-(sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e))*tan(f*x + e) - d*tan(f*x + e) - d)/tan(f*x + e)))/
f
```

## Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = \int (d\cot{(e+fx)})^{\frac{3}{2}}\tan{(e+fx)}\,dx$$

input
```
integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e),x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.23

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = -\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right.}{\sqrt{d}}\right.}{4f}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
-1/4*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
 + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/f
```

## Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx=\int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)\,dx$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e), x)
```

## Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.40

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = -\frac{(-1)^{1/4}\,d^{3/2}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\right)}{f}$$

input
```
int(tan(e + f*x)*(d*cot(e + f*x))^(3/2),x)
```

output
```
-((-1)^(1/4)*d^(3/2)*(atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)) -
atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))))/f
```

## Reduce [F]

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx=\sqrt{d}\left(\int \sqrt{\cot(fx+e)}\,\cot(fx+e)\tan(fx+e)\,dx\right)d$$

input
```
int((d*cot(f*x+e))^(3/2)*tan(f*x+e),x)
```

output

```
sqrt(d)*int(sqrt(cot(e + f*x))*cot(e + f*x)*tan(e + f*x),x)*d
```

# 3.204 $\int (d\cot(e+fx))^{3/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1536
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1537
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . . 1537
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1542
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1542
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1543
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1543
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1544
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1544
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1545

## Optimal result

Integrand size = 12, antiderivative size = 154

$$\int (d\cot(e+fx))^{3/2}\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f} - \frac{2d\sqrt{d\cot(e+fx)}}{f}$$

output
```
-1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*
d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*d^(3/
2)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1
/2)/f-2*d*(d*cot(f*x+e))^(1/2)/f
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\int (d\cot(e+fx))^{3/2}\,dx =$$
$$-\frac{(d\cot(e+fx))^{3/2}\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}+2\sqrt{\cot(e+fx)}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right.}{2\sqrt{2}}\right.}{f\cot^{\frac{3}{2}}(e+fx)}$$

input
```
Integrate[(d*Cot[e + f*x])^(3/2),x]
```

output
```
-(((d*Cot[e + f*x])^(3/2)*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2]
- ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] + 2*Sqrt[Cot[e + f*x]] +
Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2]) - Log[1 + S
qrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2])))/(f*Cot[e + f*x]^(3
/2)))
```

## Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.31, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3954}$$

$$d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{3957}$$

$$\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{266}$$

$$\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{755}$$

$$\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{}}{2d}\right.}{f}$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{217}$$

$$\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$\Big\downarrow$ 1479

$$\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\cdots\right.}{f}$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$\Big\downarrow$ 25

$$\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\right.}{f}$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$\Big\downarrow$ 27

$$\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\right.}{f}$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$\Big\downarrow$ 1103

$$\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)\cdots\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

input
```
Int[(d*Cot[e + f*x])^(3/2),x]
```

output
```
(-2*d*Sqrt[d*Cot[e + f*x]])/f + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[
e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(S
qrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*
Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] +
 d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/f
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $-\frac{2d\left(\sqrt{d\cot(fx+e)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arct}}{8}\right)}{f}$ |
| default | $-\frac{2d\left(\sqrt{d\cot(fx+e)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arct}}{8}\right)}{f}$ |

input
```
int((d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f*d*((d*cot(f*x+e))^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^
2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/
4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)
*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2
)+1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(122) = 244.

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.05

$$\int (d\cot(e+fx))^{3/2}\,dx = \frac{2\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}+d}{d}\right)+2\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}-d}{d}\right)+\sqrt{2}d^{\frac{3}{2}}\,\mathrm{lo}}{}$$

input
```
integrate((d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) +
d)/sin(2*f*x + 2*e)) + d)/d) + 2*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d)*s
qrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) - d)/d) + sqrt(2)*d^(3/2)*l
og((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*
f*x + 2*e) + d*cos(2*f*x + 2*e) + d*sin(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)
) - sqrt(2)*d^(3/2)*log(-(sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) + d)/si
n(2*f*x + 2*e))*sin(2*f*x + 2*e) - d*cos(2*f*x + 2*e) - d*sin(2*f*x + 2*e)
 - d)/sin(2*f*x + 2*e)) - 8*d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*
e)))/f
```

## Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\,dx=\int \left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*cot(f*x+e))**(3/2),x)
```

output
```
Integral((d*cot(e + f*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.16

$$\int (d\cot(e+fx))^{3/2}\,dx=\frac{\left(2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\cdots\right.}{\cdots}$$

input
```
integrate((d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x +
e)))*d/f
```

## Giac [F]

$$\int (d\cot(e+fx))^{3/2}\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.49

$$\int (d\cot(e+fx))^{3/2}\,dx = -\frac{2\,d\,\sqrt{d\cot(e+f\,x)}}{f} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

input
```
int((d*cot(e + f*x))^(3/2),x)
```

output
```
- (2*d*(d*cot(e + f*x))^(1/2))/f - ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d
*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/
4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

## Reduce [F]

$$\int (d\cot(e+fx))^{3/2}\,dx = \frac{\sqrt{d}\,d\left(-2\sqrt{\cot\left(fx+e\right)}-\left(\int \frac{\sqrt{\cot(fx+e)}}{\cot(fx+e)}dx\right)f\right)}{f}$$

input
```
int((d*cot(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*d*( - 2*sqrt(cot(e + f*x)) - int(sqrt(cot(e + f*x))/cot(e + f*x),
x)*f))/f
```

# 3.205 $\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx$


Optimal result . . . 1546
Mathematica [A] (verified) . . . 1547
Rubi [A] (warning: unable to verify) . . . 1547
Maple [A] (verified) . . . 1552
Fricas [B] (verification not implemented) . . . 1552
Sympy [F] . . . 1553
Maxima [A] (verification not implemented) . . . 1553
Giac [F] . . . 1554
Mupad [B] (verification not implemented) . . . 1554
Reduce [F] . . . 1555


## Optimal result

Integrand size = 19, antiderivative size = 156

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f}-\frac{2(d\cot(e+fx))^{3/2}}{3f}$$

output
```
-1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*
d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d^(3/
2)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1
/2)/f-2/3*(d*cot(f*x+e))^(3/2)/f
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.64

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = -\frac{(d\cot(e+fx))^{3/2}\left(-3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{}\right.}{3f\cot^{\frac{7}{4}}(e+fx)}$$

input
```
Integrate[Cot[e + f*x]*(d*Cot[e + f*x])^(3/2),x]
```

output
```
-1/3*((d*Cot[e + f*x])^(3/2)*(-3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e +
 f*x])^(1/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) +
2*Cot[e + f*x]^(7/4)))/(f*Cot[e + f*x]^(7/4))
```

## Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.29, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (d\cot(e+fx))^{5/2}dx}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx}{d}$$

$$\Big\downarrow \; 3954$$

$$\frac{d^2\left(-\int \sqrt{d\cot(e+fx)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d}$$

$\Big\downarrow$ 3042

$$\frac{d^2\left(-\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d}$$

$\Big\downarrow$ 3957

$$\frac{\frac{d^3\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d}$$

$\Big\downarrow$ 266

$$\frac{\frac{2d^3\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d}$$

$\Big\downarrow$ 826

$$\frac{\frac{2d^3\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d}$$

$\Big\downarrow$ 1476

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}\cdots\right)}{f}}{d}$$

$\Big\downarrow$ 1082

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-2d\cdots}{d}$$

$\Big\downarrow$ 217

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d}$$

$\Big\downarrow$ 1479

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx\right.\cdots}{\sqrt{2}\sqrt{d}}\cdots\right.\right.}{f}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\cdots\right.}{\sqrt{2}\sqrt{d}}\cdots\right.\right.}{f}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\cdots\right.}{\sqrt{2}\sqrt{d}}\cdots\right.\right.}{f}}{d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e\cdots\right.}{2\sqrt{2}\sqrt{d}}\cdots\right.\right.}{f}}{d}$$

input
```
Int[Cot[e + f*x]*(d*Cot[e + f*x])^(3/2),x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(3/2))/(3*f) + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[
d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f
*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Co
t[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] +
 d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/f)/d
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2(d\cot(fx+e))^{\frac{3}{2}}}{3}+\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.}{4\left(d^2\right)^{\frac{1}{4}}}}{f}$ |
| default | $\frac{-\frac{2(d\cot(fx+e))^{\frac{3}{2}}}{3}+\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.}{4\left(d^2\right)^{\frac{1}{4}}}}{f}$ |

input
```
int(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2/3*(d*cot(f*x+e))^(3/2)+1/4*d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+
e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^
2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)
^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e)
)^(1/2)+1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(121) = 242.

Time = 0.09 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.41

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{6\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}+d}{d}\right)\sin\left(2fx+2e\right)+6\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\cos(2fx+2e)+}{\sin(2fx+2e)}}}{d}\right.}{}$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/12*(6*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) +
 d)/sin(2*f*x + 2*e)) + d)/d)*sin(2*f*x + 2*e) + 6*sqrt(2)*d^(3/2)*arctan(
(sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) - d)/d)*s
in(2*f*x + 2*e) - 3*sqrt(2)*d^(3/2)*log((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x
 + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e) + d*cos(2*f*x + 2*e) + d*s
in(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e) + 3*sqrt(2)*d^(3/2
)*log(-(sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*si
n(2*f*x + 2*e) - d*cos(2*f*x + 2*e) - d*sin(2*f*x + 2*e) - d)/sin(2*f*x +
2*e))*sin(2*f*x + 2*e) - 8*(d*cos(2*f*x + 2*e) + d)*sqrt((d*cos(2*f*x + 2*
e) + d)/sin(2*f*x + 2*e)))/(f*sin(2*f*x + 2*e))
```

## Sympy [F]

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\cot(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*cot(e + f*x), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.18

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{3\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(f}}\right.}{\sqrt{}}\right.}{12\,f}$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f
*x + e))^(3/2))/f
```

## Giac [F]

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx=\int (d\cot(fx+e))^{\frac{3}{2}}\cot(fx+e)\,dx$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*cot(f*x + e), x)
```

## Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx=\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{f}-\frac{2\,(d\cot(e+f\,x))^{3/2}}{3\,f}-\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{f}$$

input
```
int(cot(e + f*x)*(d*cot(e + f*x))^(3/2),x)
```

output
```
((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/f -
 (2*(d*cot(e + f*x))^(3/2))/(3*f) - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)*
(d*cot(e + f*x))^(1/2))/d^(1/2)))/f
```

## Reduce [F]

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\cot\left(fx+e\right)}\,\cot\left(fx+e\right)^{2}dx\right)d$$

input
```
int(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(cot(e + f*x))*cot(e + f*x)**2,x)*d
```

# 3.206 $\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$


Optimal result . . . 1556
Mathematica [A] (verified) . . . 1557
Rubi [A] (warning: unable to verify) . . . 1557
Maple [A] (verified) . . . 1563
Fricas [B] (verification not implemented) . . . 1563
Sympy [F] . . . 1564
Maxima [A] (verification not implemented) . . . 1564
Giac [F] . . . 1565
Mupad [B] (verification not implemented) . . . 1565
Reduce [F] . . . 1566


## Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}f} + \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df}$$

output
```
1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d
^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d^(3/2
)*arctanh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/
2)/f+2*d*(d*cot(f*x+e))^(1/2)/f-2/5*(d*cot(f*x+e))^(5/2)/d/f
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.97

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx =$$
$$-\frac{(d\cot(e+fx))^{3/2}\left(-10\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)+10\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)-40\right.}{}$$

input
```
Integrate[Cot[e + f*x]^2*(d*Cot[e + f*x])^(3/2),x]
```

output
```
-1/20*((d*Cot[e + f*x])^(3/2)*(-10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e +
 f*x]]] + 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] - 40*Sqrt[Cot[
e + f*x]] + 8*Cot[e + f*x]^(5/2) - 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e +
f*x]] + Cot[e + f*x]] + 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot
[e + f*x]]))/(f*Cot[e + f*x]^(3/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.31, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {2030, 3042, 3954, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\cot(e+fx))^{7/2}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx}{d^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-d^2\int (d\cot(e+fx))^{3/2}dx-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-d^2\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-d^2\left(d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-d^2\left(d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{3957}$$

$$\frac{-d^2\left(\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{-d^2\left(\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{1476}$$

$$-d^2\left(\frac{2d^3\left(\frac{\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cdots}}{2d}\cdots\right)}{f}\cdots\right.$$
$$\overline{\qquad\qquad d^2\qquad\qquad}$$

$$\Big\downarrow \text{ 1082}$$

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{\cdots}}{\cdots}\cdots\right.$$
$$\overline{\qquad\qquad d^2\qquad\qquad}$$

$$\Big\downarrow \text{ 217}$$

$$-d^2\left(\frac{2d^3\left(\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e\cdots}{5\cdots}\cdots$$
$$\overline{\qquad\qquad d^2\qquad\qquad}$$

$$\Big\downarrow \text{ 1479}$$

$$-d^2\left(\frac{2d^3\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+f\cdots\right.}{\sqrt{2}\sqrt{d}}\cdots}{\cdots}\cdots\right)}{f}\cdots\right.$$
$$\overline{\qquad\qquad d^2\qquad\qquad}$$

$$\Big\downarrow \text{ 25}$$

$$-d^2\left(\frac{\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^2}\right.$$

$\downarrow$ 27

$$-d^2\left(\frac{\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^2}\right.$$

$\downarrow$ 1103

$$-d^2\left(\frac{\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}}{d^2}\right.$$

input
```
Int[Cot[e + f*x]^2*(d*Cot[e + f*x])^(3/2),x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(5/2))/(5*f) - d^2*((-2*d*Sqrt[d*Cot[e + f*x]])/f
+ (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
 ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2
*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[
d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2
]*Sqrt[d]))/(2*d)))/f))/d^2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{5}{2}}}{5}-d^{2}\sqrt{d\cot(fx+e)}+\frac{d^{2}(d^{2})^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^{2})^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^{2}}}{d\cot(fx+e)-(d^{2})^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d}}{(}\right.\right.}{8}\right.}{fd}$ |
| default | $-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{5}{2}}}{5}-d^{2}\sqrt{d\cot(fx+e)}+\frac{d^{2}(d^{2})^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^{2})^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^{2}}}{d\cot(fx+e)-(d^{2})^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d}}{(}\right.\right.}{8}\right.}{fd}$ |

input
```
int(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d*(1/5*(d*cot(f*x+e))^(5/2)-d^2*(d*cot(f*x+e))^(1/2)+1/8*d^2*(d^2)^(1
/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^
2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/
2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2
)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(140) = 280.

Time = 0.09 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.29

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx =$$

$$-\frac{10\sqrt{2}(d\cos(2fx+2e)-d)\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}+d}{d}\right)+10\sqrt{2}(d\cos(2fx+2e)-d)\sqrt{d}\,\mathrm{arcta}}{}$$

input
```
integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/20*(10*sqrt(2)*(d*cos(2*f*x + 2*e) - d)*sqrt(d)*arctan((sqrt(2)*sqrt(d)
*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) + d)/d) + 10*sqrt(2)*(d*c
os(2*f*x + 2*e) - d)*sqrt(d)*arctan((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2
*e) + d)/sin(2*f*x + 2*e)) - d)/d) + 5*sqrt(2)*(d*cos(2*f*x + 2*e) - d)*sq
rt(d)*log((sqrt(2)*sqrt(d)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))
*sin(2*f*x + 2*e) + d*cos(2*f*x + 2*e) + d*sin(2*f*x + 2*e) + d)/sin(2*f*x
 + 2*e)) - 5*sqrt(2)*(d*cos(2*f*x + 2*e) - d)*sqrt(d)*log(-(sqrt(2)*sqrt(d
)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e) - d*cos
(2*f*x + 2*e) - d*sin(2*f*x + 2*e) - d)/sin(2*f*x + 2*e)) - 16*(3*d*cos(2*
f*x + 2*e) - 2*d)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))/(f*cos(
2*f*x + 2*e) - f)
```

## Sympy [F]

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\cot^2(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**2*(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*cot(e + f*x)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx =$$

$$-\frac{10\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{5}{2}}\log\left(\sqrt{2}\right.}{}$$

input
```
integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-1/20*(10*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/t
an(f*x + e)))/sqrt(d)) + 10*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + 5*sqrt(2)*d^(5/2)*log(sqrt(2)*
sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 5*sqrt(2)*d^(5/2)*log
(-sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 40*d^2*sqrt
(d/tan(f*x + e)) + 8*(d/tan(f*x + e))^(5/2))/(d*f)
```

## Giac [F]

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx=\int (d\cot(fx+e))^{\frac{3}{2}}\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*cot(f*x + e)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.51

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx=\frac{2\,d\,\sqrt{d\cot(e+f\,x)}}{f}-\frac{2\,(d\cot(e+f\,x))^{5/2}}{5\,d\,f}+\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}+\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}\,1\mathrm{i}}{\sqrt{d}}\right)}{f}$$

input
```
int(cot(e + f*x)^2*(d*cot(e + f*x))^(3/2),x)
```

output
```
(2*d*(d*cot(e + f*x))^(1/2))/f - (2*(d*cot(e + f*x))^(5/2))/(5*d*f) + ((-1
)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f +
((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2)*1i)/d^(1/2)))/
f
```

## Reduce [F]

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{\sqrt{d}\,d\left(-2\sqrt{\cot(fx+e)}\,\cot(fx+e)^2+10\sqrt{\cot(fx+e)}+5\left(\int \frac{\sqrt{\cot(fx+e)}}{\cot(fx+e)}dx\right)f\right)}{5f}$$

input
```
int(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*d*( - 2*sqrt(cot(e + f*x))*cot(e + f*x)**2 + 10*sqrt(cot(e + f*x)
) + 5*int(sqrt(cot(e + f*x))/cot(e + f*x),x)*f))/(5*f)
```

# 3.207 $\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1567
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1568
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . 1568
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1573
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1574
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1575
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1575
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1576
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1576
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1577

## Optimal result

Integrand size = 21, antiderivative size = 175

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}\sqrt{d}f} + \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}}$$

output
```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f+1/2*arctan
h(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(1/
2)/f+2/5*d^2/f/(d*cot(f*x+e))^(5/2)-2/f/(d*cot(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.54

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= \frac{-5\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}+5\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}+2}{5f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

output
```
(-5*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) + 5*ArcTanh[(-
Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) + 2*(-5 + Tan[e + f*x]^2))/
(5*f*Sqrt[d*Cot[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.31, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 25, 2030, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^3\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3\sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}}dx$$

$$\Big\downarrow \; 2030$$

$$d^3 \int \frac{1}{\left(-d \tan \left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \; 3955$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int \frac{1}{(d\cot(e+fx))^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \; 3042$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int \frac{1}{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \; 3955$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \; 3042$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \; 3957$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \; 266$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \; 826$$

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$\downarrow$ 1476

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\,\mathrm{c}}\right)\right)}{df}}{d^2}\right)$$

$\downarrow$ 1082

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\,\mathrm{co}}{d^4\cot^4(}\right)}{df}}{d^2}\right)$$

$\downarrow$ 217

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+}{d^2}\right)$$

$\downarrow$ 1479

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(}}{2\sqrt{2}\sqrt{d}}\right)\right)}{df}}{d^2}\right)$$

$\downarrow$ 25

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e\cdots}{2\sqrt{2}\sqrt{d}}\cdots}{df}}{d^2}\cdots\right.$$

$$\Big\downarrow \; 27$$

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e\cdots}{2\sqrt{d}}\cdots}{df}}{d^2}\cdots\right.$$

$$\Big\downarrow \; 1103$$

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+\cdots\right.}{2\sqrt{2}\sqrt{d}}\cdots}{df}}{d^2}\cdots\right.$$

input
```
Int[Tan[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

output
```
d^3*(2/(5*d*f*(d*Cot[e + f*x])^(5/2)) - (2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2
*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[
1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*
d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + S
qrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))
/(d*f))/d^2)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(139) = 278$.

Time = 2.50 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.87

| method | result |
|---|---|
| default | $-\frac{\left(\sqrt{2}\left(24-4\sec(fx+e)^2\right)+\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}\ \arctan\left(\frac{\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+\cos(fx+e)-1}{\cos(fx+e)-1}\right)(-10\cot(fx+e)-$ |

input
```
int(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20/f/(d*cot(f*x+e))^(1/2)*(2^(1/2)*(24-4*sec(f*x+e)^2)+(-sin(f*x+e)*cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+
e)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-10*cot(f*x+e)-1
0*csc(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*ln(-(cot(f*x
+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(-5
*cot(f*x+e)-5*csc(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*
ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*cos(f*x+e)+2)/(cos(f*
x+e)-1))*(5*cot(f*x+e)+5*csc(f*x+e))+arctan((-sin(f*x+e)*(-2*sin(f*x+e)*co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-sin(f*x+e
)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(10*cot(f*x+e)+10*csc(f*x+e)))*2^(1/2
)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.17

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx =$$

$$-\frac{10\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d}{\tan(fx+e)}}}{\sqrt{d}}+1\right)+10\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d}{\tan(fx+e)}}}{\sqrt{d}}-1\right)-5\sqrt{2}\sqrt{d}\log\left(\frac{\sqrt{2}\sqrt{\frac{d}{\tan(fx+}}}{\sqrt{}}\right.}{}$$

input
```
integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/20*(10*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt(d/tan(f*x + e))/sqrt(d) + 1)
 + 10*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt(d/tan(f*x + e))/sqrt(d) - 1) - 5
*sqrt(2)*sqrt(d)*log((sqrt(2)*sqrt(d/tan(f*x + e))*tan(f*x + e)/sqrt(d) +
tan(f*x + e) + 1)/tan(f*x + e)) + 5*sqrt(2)*sqrt(d)*log(-(sqrt(2)*sqrt(d/t
an(f*x + e))*tan(f*x + e)/sqrt(d) - tan(f*x + e) - 1)/tan(f*x + e)) - 8*(t
an(f*x + e)^3 - 5*tan(f*x + e))*sqrt(d/tan(f*x + e)))/(d*f)
```

## Sympy [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(tan(e + f*x)**3/sqrt(d*cot(e + f*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.18

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx =$$

$$-\frac{d^4\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\cdots\right)}{d^4}\cdots\right)}{20\,f}$$

input
```
integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/20*d^4*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan
(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(
2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^
2 - 5*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

## Giac [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^3}{\sqrt{d\cot\left(fx+e\right)}}\,dx$$

input
```
integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^3/sqrt(d*cot(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.55

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\frac{2\,d^2}{5}-\frac{2\,d^2}{\tan(e+f\,x)^2}}{f\,\left(\frac{d}{\tan(e+f\,x)}\right)^{5/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f} + \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

input
```
int(tan(e + f*x)^3/(d*cot(e + f*x))^(1/2),x)
```

output
```
((2*d^2)/5 - (2*d^2)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1
/4)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f) + ((-1)
^(1/4)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

## Reduce [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cot(fx+e)}\,\tan(fx+e)^3}{\cot(fx+e)}dx\right)}{d}$$

input
```
int(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(cot(e + f*x))*tan(e + f*x)**3)/cot(e + f*x),x))/d
```

# 3.208 $\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1578
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1578
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . 1579
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1584
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . 1584
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1585
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 1585
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1586
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 1586
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1587

## Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}\sqrt{d}f}+\frac{2d}{3f(d\cot(e+fx))^{3/2}}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f+1/2*arcta
nh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(1
/2)/f+2/3*d/f/(d*cot(f*x+e))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.54

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{d\left(-2+3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))^{3/4}+3\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e$$
$$3f(d\cot(e+fx))^{3/2}$$

input
```
Integrate[Tan[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

output
```
-1/3*(d*(-2 + 3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4) +
3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4)))/(f*(d*Cot[e +
 f*x])^(3/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.34, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 2030, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^2\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$d^2\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$d^2\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}dx}{d^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx}{d^2}\right)$$

$$\Big\downarrow \text{3957}$$

$$d^2\left(\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\downarrow$ 266

$$d^2\left(\frac{2\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\downarrow$ 755

$$d^2\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\downarrow$ 1476

$$d^2\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}\cdots}{2d}\cdots\right.}{df}\cdots\right.$$

$\downarrow$ 1082

$$d^2\left(\frac{2\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{\cdots}{3d\cdots}\right.$$

$\downarrow$ 217

$$d^2\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\downarrow$ 1479

$$d^2\left(\frac{2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)\cdots\right.}{\sqrt{2}\sqrt{d}}\cdots\right.}{df}\cdots\right.$$

$\downarrow$ 25

$$d^2\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d\cdots}\right.}{df}\cdots\right.$$

$\downarrow$ 27

$$d^2\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d\cdots}\right.}{df}\cdots\right.$$

$\downarrow$ 1103

$$d^2\left(\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+f\cdots\right.}{2\sqrt{2}\sqrt{d}}}{2d}\cdots\right.}{df}\cdots\right.$$

input
```
Int[Tan[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

output
```
d^2*(2/(3*d*f*(d*Cot[e + f*x])^(3/2)) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*
Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]
]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] +
d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*
x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/(d*f))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(122) = 244$.

Time = 1.38 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.19

| method | result |
|---|---|
| default | $-\frac{\left(\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)-2\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+\csc(fx+e)-\sin(fx+e)-2\cos(fx+e)+2}{\cos(fx+e)-1}\right)\sqrt{-\frac{\sin(fx+e)}{(1+\cos(}}$ |

input
```
int(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/f/(d*cot(f*x+e))^(1/2)*(ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*si
n(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(
f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+
e))^2)^(1/2)*(-3*cot(f*x+e)-3*csc(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e)
)^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-6*cot(f*x+e)-6*csc(f*x+e))+(-si
n(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x+e)-2*c
ot(f*x+e)+2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+c
sc(f*x+e)-sin(f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(3*cot(f*x+e)+3*csc(f
*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*arctan((-sin(f*x+e)
*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+
e)-1))*(6*cot(f*x+e)+6*csc(f*x+e))-4*2^(1/2)*tan(f*x+e))*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.26

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= \frac{8\sqrt{\frac{d}{\tan(fx+e)}}\tan(fx+e)^2 + 6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d}{\tan(fx+e)}}}{\sqrt{d}}+1\right) + 6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d}{\tan(fx+e)}}}{\sqrt{d}}-1\right) +}{12\,df}$$

input
```
integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/12*(8*sqrt(d/tan(f*x + e))*tan(f*x + e)^2 + 6*sqrt(2)*sqrt(d)*arctan(sqr
t(2)*sqrt(d/tan(f*x + e))/sqrt(d) + 1) + 6*sqrt(2)*sqrt(d)*arctan(sqrt(2)*
sqrt(d/tan(f*x + e))/sqrt(d) - 1) + 3*sqrt(2)*sqrt(d)*log((sqrt(2)*sqrt(d/
tan(f*x + e))*tan(f*x + e)/sqrt(d) + tan(f*x + e) + 1)/tan(f*x + e)) - 3*s
qrt(2)*sqrt(d)*log(-(sqrt(2)*sqrt(d/tan(f*x + e))*tan(f*x + e)/sqrt(d) - t
an(f*x + e) - 1)/tan(f*x + e)))/(d*f)
```

## Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan^2{(e+fx)}}{\sqrt{d\cot{(e+fx)}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(tan(e + f*x)**2/sqrt(d*cot(e + f*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.22

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{d^3\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{2}\,\mathrm{l}\cdots}{\cdots}\right)}{d^2}\cdots\right)}{12\,f}$$

input
```
integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/12*d^3*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2
*(d/tan(f*x + e))^(3/2)))/f
```

## Giac [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{\sqrt{d\cot(fx+e)}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/sqrt(d*cot(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.52

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{2\,d}{3\,f\,\left(\frac{d}{\tan(e+f\,x)}\right)^{3/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f}$$

input
```
int(tan(e + f*x)^2/(d*cot(e + f*x))^(1/2),x)
```

output
```
(2*d)/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d/tan(e
 + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(
d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f)
```

## Reduce [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cot(fx+e)}\,\tan(fx+e)^2}{\cot(fx+e)}dx\right)}{d}$$

input
```
int(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(cot(e + f*x))*tan(e + f*x)**2)/cot(e + f*x),x))/d
```

# 3.209 $\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1588
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1588
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . 1589
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1593
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1594
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1595
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 1595
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1596
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1596
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1597

## Optimal result

Integrand size = 19, antiderivative size = 154

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}\sqrt{d}f}+\frac{2}{f\sqrt{d\cot(e+fx)}}$$

output

```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f-1/2*arcta
nh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(1
/2)/f+2/f/(d*cot(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.51

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{2+\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}}{f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

output
```
(2 + ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) - ArcTanh[(-C
ot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4))/(f*Sqrt[d*Cot[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.29, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 25, 2030, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$d\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3955}$$

$$d\left(\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$d\left(\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}\right)$$

$$\Big\downarrow \text{3957}$$

$$d\left(\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$$\Big\downarrow \text{266}$$

$$d\left(\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$$\Big\downarrow \text{826}$$

$$d\left(\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$$\Big\downarrow \text{1476}$$

$$d\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx}\right.\right.}{df}\right.$$

$$\Big\downarrow \text{1082}$$

$$d\left(\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\,c}\right.}{df}\right.$$

$$\Big\downarrow \text{217}$$

$$d\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{}{df\sqrt{d\,c}}\right.$$

$$\Big\downarrow \text{1479}$$

$$d\left(\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\right.}{}\right.\right.}{df}\right.$$

$\downarrow$ 25

$$d\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}}{}\right.\right.}{df}\right.$$

$\downarrow$ 27

$$d\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}}{}\right.\right.}{df}\right.$$

$\downarrow$ 1103

$$d\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{\right.}}{}\right.\right.}{df}\right.$$

input
```
Int[Tan[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

output
```
d*(2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e
+ f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqr
t[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x
]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[
e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/(d*f))
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(121) = 242$.

Time = 2.38 (sec) , antiderivative size = 859, normalized size of antiderivative = 5.58

| method | result | size |
|---|---|---|
| default | Expression too large to display | 859 |

input
```
int(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*csc(f*x+e)*(ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-
2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*co
s(f*x+e)+2)/(cos(f*x+e)-1))*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)-2*arctan((-sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+
e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-2*sin(f*x+e)*cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)-ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*si
n(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(
f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+2*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-2*sin(f*x+e)*cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f
*x+e)+2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f
*x+e)-sin(f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(-2*sin(f*x+e)*cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)-2*arctan((-sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-2*sin(f*x+e)*cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*sin
(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f
*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)+2*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e)
)^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-2*sin(f*x+e)*cos(f*x+e)/(1+c...
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.25

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d}{\tan(fx+e)}}}{\sqrt{d}}+1\right)+2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d}{\tan(fx+e)}}}{\sqrt{d}}-1\right)-\sqrt{2}\sqrt{d}\log\left(\frac{\frac{\sqrt{2}\sqrt{\frac{d}{\tan(fx+e)}}\tan(fx}{\sqrt{d}}}{\tan(fx}\right.}{4\,df}$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt(d/tan(f*x + e))/sqrt(d) + 1) +
2*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt(d/tan(f*x + e))/sqrt(d) - 1) - sqrt(
2)*sqrt(d)*log((sqrt(2)*sqrt(d/tan(f*x + e))*tan(f*x + e)/sqrt(d) + tan(f*
x + e) + 1)/tan(f*x + e)) + sqrt(2)*sqrt(d)*log(-(sqrt(2)*sqrt(d/tan(f*x +
 e))*tan(f*x + e)/sqrt(d) - tan(f*x + e) - 1)/tan(f*x + e)) + 8*sqrt(d/tan
(f*x + e))*tan(f*x + e))/(d*f)
```

## Sympy [F]

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan{(e+fx)}}{\sqrt{d\cot{(e+fx)}}}\,dx$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(tan(e + f*x)/sqrt(d*cot(e + f*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.23

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{d^2\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(\cdots\right.}{\cdots}}{d^2}\cdots\right.}{4f}$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/4*d^2*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
 + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sq
rt(d/tan(f*x + e))))/f
```

## Giac [F]

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan(fx+e)}{\sqrt{d\cot(fx+e)}}\,dx$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)/sqrt(d*cot(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.51

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{2}{f\sqrt{\frac{d}{\tan(e+fx)}}} + \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

input
```
int(tan(e + f*x)/(d*cot(e + f*x))^(1/2),x)
```

output
```
2/(f*(d/tan(e + f*x))^(1/2)) + ((-1)^(1/4)*atan(((-1)^(1/4)*(d/tan(e + f*x
))^(1/2))/d^(1/2)))/(d^(1/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d/tan(e +
 f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

## Reduce [F]

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cot(fx+e)}\,\tan(fx+e)}{\cot(fx+e)}dx\right)}{d}$$

input
```
int(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(cot(e + f*x))*tan(e + f*x))/cot(e + f*x),x))/d
```

# 3.210 $\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result . . . . . 1598
Mathematica [A] (verified) . . . . . 1598
Rubi [A] (warning: unable to verify) . . . . . 1599
Maple [A] (verified) . . . . . 1603
Fricas [B] (verification not implemented) . . . . . 1603
Sympy [F] . . . . . 1604
Maxima [A] (verification not implemented) . . . . . 1604
Giac [F] . . . . . 1605
Mupad [B] (verification not implemented) . . . . . 1605
Reduce [F] . . . . . 1606

## Optimal result

Integrand size = 12, antiderivative size = 137

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx=\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}\sqrt{d}f}$$

output
```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f-1/2*arctan
h(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(1/
2)/f
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\sqrt{\cot(e+fx)}\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+\log\left(1-\sqrt{2}\sqrt{\mathrm{co}\ldots}\right.\right.}{2\sqrt{2}f\sqrt{d\cot(e+fx)}}$$

[...]

input
```
Integrate[1/Sqrt[d*Cot[e + f*x]],x]
```

output
```
(Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[1
 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[
e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2
]*f*Sqrt[d*Cot[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.31, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{d\int \frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2d\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{755}$$

$$-\frac{2d\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2d\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$\Big\downarrow$ 1082

$$-\frac{2d\left(\frac{\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$\Big\downarrow$ 217

$$-\frac{2d\left(\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}$$

$\Big\downarrow$ 1479

$$-\frac{2d\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}$$

$\Big\downarrow$ 25

$$-\frac{2d\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}$$

$\Big\downarrow$ 27

$$-\frac{2d\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}$$

$\Big\downarrow$ 1103

$$-\frac{2d\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}$$

input
```
Int[1/Sqrt[d*Cot[e + f*x]],x]
```

output
```
(-2*d*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + Ar
cTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Lo
g[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d])
 + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*S
qrt[d]))/(2*d)))/f
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| derivativedivides | $-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{d\cot \cdots}}{(d^2 \cdots}\right.\cdots\right.}{4fd}$ |
| default | $-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{d\cot \cdots}}{(d^2 \cdots}\right.\cdots\right.}{4fd}$ |

input
```
int(1/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/d*(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^
(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*
2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(106) = 212$.

Time = 0.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx =$$

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}+1\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}-1\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(\frac{\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\sin(2fx+2e)}{\sqrt{d}}+\cos(2 \cdots}{\sin(2fx+2e)}\right.\cdots}{\sqrt{d}}\cdots}{4f}$$

input
```
integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2
*e))/sqrt(d) + 1)/sqrt(d) + 2*sqrt(2)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2
*e) + d)/sin(2*f*x + 2*e))/sqrt(d) - 1)/sqrt(d) + sqrt(2)*log((sqrt(2)*sqr
t((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(d) + co
s(2*f*x + 2*e) + sin(2*f*x + 2*e) + 1)/sin(2*f*x + 2*e))/sqrt(d) - sqrt(2)
*log(-(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x +
 2*e)/sqrt(d) - cos(2*f*x + 2*e) - sin(2*f*x + 2*e) - 1)/sin(2*f*x + 2*e))
/sqrt(d))/f
```

## Sympy [F]

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\cot{(e+fx)}}}\,dx$$

input
```
integrate(1/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(1/sqrt(d*cot(e + f*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right.}{d^{\frac{3}{2}}}\right.}{4f}$$

input
```
integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x +
 e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqr
t(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sqr
t(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

## Giac [F]

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\cot(fx+e)}}\,dx$$

input
```
integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(d*cot(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f} + \frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f}$$

input
```
int(1/(d*cot(e + f*x))^(1/2),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)
*f) + ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(
d^(1/2)*f)
```

## Reduce [F]

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cot(fx+e)}}{\cot(fx+e)}dx\right)}{d}$$

input
```
int(1/(d*cot(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int(sqrt(cot(e + f*x))/cot(e + f*x),x))/d
```

# 3.211 $\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result . . . 1607
Mathematica [A] (verified) . . . 1607
Rubi [A] (warning: unable to verify) . . . 1608
Maple [A] (verified) . . . 1612
Fricas [B] (verification not implemented) . . . 1612
Sympy [F] . . . 1613
Maxima [A] (verification not implemented) . . . 1613
Giac [F] . . . 1614
Mupad [B] (verification not implemented) . . . 1614
Reduce [F] . . . 1615

## Optimal result

Integrand size = 19, antiderivative size = 136

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}\sqrt{d}f}$$

output
```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f+1/2*arctan
h(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(1/
2)/f
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\left(-\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right)\sqrt[4]{-\cot(e+fx)}\sqrt{d\cot(e+fx)}}{df\cot^{\frac{3}{4}}(e+fx)}$$

input
```
Integrate[Cot[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-ArcTan[(-Cot[e + f*x]^2)^(1/4)] + ArcTanh[(-Cot[e + f*x]^2)^(1/4)])*(-C
ot[e + f*x])^(1/4)*Sqrt[d*Cot[e + f*x]])/(d*f*Cot[e + f*x]^(3/4))
```

## Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2030, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \sqrt{d\cot(e+fx)}dx}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d}$$

$$\Big\downarrow \text{ 3957}$$

$$-\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{ 826}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}$$

$$\downarrow \text{1476}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)\right.}{f}$$

$$\downarrow \text{1082}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot\right.}{f}$$

$$\downarrow \text{217}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}$$

$$\downarrow \text{1479}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}{\right.}\right.}{f}$$

$$\downarrow \text{25}$$

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2\right.}{\right.}\right.}{f}$$

$$\downarrow \text{27}$$

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2\right.}{\right.}\right.}{f}$$

$$\downarrow \text{1103}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d\right.}{}\right.}{f}$$

input
```
Int[Cot[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcT
an[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[
2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d
+ Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/
2))/f
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| derivativedivides | $-\frac{\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}{d\cot(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{d\cot(fx+e}}{\left(d^{2}\right)^{\frac{1}{4}}}\right.\right.}{4f\left(d^{2}\right)^{\frac{1}{4}}}$ |
| default | $-\frac{\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}{d\cot(fx+e)+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{d\cot(fx+e}}{\left(d^{2}\right)^{\frac{1}{4}}}\right.\right.}{4f\left(d^{2}\right)^{\frac{1}{4}}}$ |

input
```
int(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1
/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^
(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2
*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(106) = 212$.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.97

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx =$$

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}+1\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}-1\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\frac{\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\sin(2fx+2e)}{\sqrt{d}}+\cos(}{\sin(2fx+2e)}\right.}{\sqrt{d}}}{4f}$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2
*e))/sqrt(d) + 1)/sqrt(d) + 2*sqrt(2)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2
*e) + d)/sin(2*f*x + 2*e))/sqrt(d) - 1)/sqrt(d) - sqrt(2)*log((sqrt(2)*sqr
t((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(d) + co
s(2*f*x + 2*e) + sin(2*f*x + 2*e) + 1)/sin(2*f*x + 2*e))/sqrt(d) + sqrt(2)
*log(-(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x +
 2*e)/sqrt(d) - cos(2*f*x + 2*e) - sin(2*f*x + 2*e) - 1)/sin(2*f*x + 2*e))
/sqrt(d))/f
```

## Sympy [F]

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot{(e+fx)}}{\sqrt{d\cot{(e+fx)}}}\,dx$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(cot(e + f*x)/sqrt(d*cot(e + f*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}}{4f}$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e
)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(
d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/f
```

## Giac [F]

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot(fx+e)}{\sqrt{d\cot(fx+e)}}\,dx$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)/sqrt(d*cot(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.43

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

input
```
int(cot(e + f*x)/(d*cot(e + f*x))^(1/2),x)
```

output
```
((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f
) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2
)*f)
```

## Reduce [F]

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \sqrt{\cot\left(fx+e\right)}dx\right)}{d}$$

input
```
int(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int(sqrt(cot(e + f*x)),x))/d
```

# 3.212 $\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal result . . . . . 1616
Mathematica [A] (verified) . . . . . 1616
Rubi [A] (warning: unable to verify) . . . . . 1617
Maple [A] (verified) . . . . . 1622
Fricas [B] (verification not implemented) . . . . . 1622
Sympy [F] . . . . . 1623
Maxima [A] (verification not implemented) . . . . . 1623
Giac [F] . . . . . 1624
Mupad [B] (verification not implemented) . . . . . 1624
Reduce [F] . . . . . 1625

## Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}\sqrt{d}f}-\frac{2\sqrt{d\cot(e+fx)}}{df}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f+1/2*arcta
nh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(1
/2)/f-2*(d*cot(f*x+e))^(1/2)/d/f
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx=$$
$$-\frac{\sqrt{\cot(e+fx)}\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}+2\sqrt{\cot(e+fx)}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+\cdots\right.}{2\sqrt{2}}\right.}{f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

output
```
-((Sqrt[Cot[e + f*x]]*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] - Ar
cTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] + 2*Sqrt[Cot[e + f*x]] + Log[
1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[
2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2])))/(f*Sqrt[d*Cot[e + f*x]
]))
```

## Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.31, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\cot(e+fx))^{3/2}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx}{d^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^2}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^2}$$

$\big\downarrow$ 266

$$\frac{\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^2}$$

$\big\downarrow$ 755

$$\frac{\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^2}$$

$\big\downarrow$ 1476

$$\frac{\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+f\cdots}}{2d}\cdots\right.}{f}\cdots}{d^2}$$

$\big\downarrow$ 1082

$$\frac{\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e-\cdots}}{f}}{d^2}$$

$\big\downarrow$ 217

$$\frac{\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^2}$$

$\big\downarrow$ 1479

$$\frac{\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\cdots\right.}{f}\cdots}{d^2}$$

$\Big\downarrow$ 25

$$\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arctan}}{2d}\right.}{f}}{d^2}$$

$\Big\downarrow$ 27

$$\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arctan}}{2d}\right.}{f}}{d^2}$$

$\Big\downarrow$ 1103

$$\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}}{d^2}$$

input

```
Int[Cot[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

output

```
((-2*d*Sqrt[d*Cot[e + f*x]])/f + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot
[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(
Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2
*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x]
+ d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/f)/d^2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $-\frac{2\left(\sqrt{d\cot(fx+e)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arcta}}{8}\right.}{fd}$ |
| default | $-\frac{2\left(\sqrt{d\cot(fx+e)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arcta}}{8}\right.}{fd}$ |

input
```
int(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d*((d*cot(f*x+e))^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^
2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/
4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)
*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2
)+1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(124) = 248.

Time = 0.09 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.92

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}+1\right)+2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}-1\right)+\sqrt{2}\sqrt{d}\log\left(\cdots\right.}{\cdots}$$

input
```
integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*
f*x + 2*e))/sqrt(d) + 1) + 2*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt((d*cos(2*
f*x + 2*e) + d)/sin(2*f*x + 2*e))/sqrt(d) - 1) + sqrt(2)*sqrt(d)*log((sqrt
(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(
d) + cos(2*f*x + 2*e) + sin(2*f*x + 2*e) + 1)/sin(2*f*x + 2*e)) - sqrt(2)*
sqrt(d)*log(-(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(
2*f*x + 2*e)/sqrt(d) - cos(2*f*x + 2*e) - sin(2*f*x + 2*e) - 1)/sin(2*f*x
+ 2*e)) - 8*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))/(d*f)
```

## Sympy [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(cot(e + f*x)**2/sqrt(d*cot(e + f*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.16

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\sqrt{}\right.}{4\,df}$$

input
```
integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x +
e)))/(d*f)
```

## Giac [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot(fx+e)^2}{\sqrt{d\cot(fx+e)}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^2/sqrt(d*cot(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.49

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{2\sqrt{d\cot(e+f\,x)}}{d\,f} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f}$$

input
```
int(cot(e + f*x)^2/(d*cot(e + f*x))^(1/2),x)
```

output
```
- (2*(d*cot(e + f*x))^(1/2))/(d*f) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e
 + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(
d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f)
```

## Reduce [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\sqrt{d}\left(-2\sqrt{\cot(fx+e)}-\left(\int \frac{\sqrt{\cot(fx+e)}}{\cot(fx+e)}dx\right)f\right)}{df}$$

input
```
int(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*( - 2*sqrt(cot(e + f*x)) - int(sqrt(cot(e + f*x))/cot(e + f*x),x)
*f))/(d*f)
```

# 3.213 $\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1626
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1626
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . . . 1627
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1631
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1632
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1632
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1633
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1633
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1634
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1634


## Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}\sqrt{d}f} - \frac{2(d\cot(e+fx))^{3/2}}{3d^2f}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/f-1/2*arcta
nh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(1
/2)/f-2/3*(d*cot(f*x+e))^(3/2)/d^2/f
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.63

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\sqrt[4]{\cot(e+fx)}\left(-3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)} + 3\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\right.}{3f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

output
```
-1/3*(Cot[e + f*x]^(1/4)*(-3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x
])^(1/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) + 2*Co
t[e + f*x]^(7/4)))/(f*Sqrt[d*Cot[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.26, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\cot(e+fx))^{5/2}dx}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx}{d^3}$$

$$\Big\downarrow \text{3954}$$

$$\frac{d^2\left(-\int \sqrt{d\cot(e+fx)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\left(-\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^3}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\frac{d^3\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^3}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{2d^3\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^3}$$

$$\Big\downarrow \; 826$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^3}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}}{f}}{d^3}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(}{}}{d^3}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^3}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx\right.}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\right.}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d}\cot(e+fx)}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d}\cot(e+fx)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\right.}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^3}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e\right.}{2\sqrt{2}\sqrt{d}}\right.}{f}}{d^3}$$

input
```
Int[Cot[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(3/2))/(3*f) + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[
d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f
*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Co
t[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] +
 d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/f)/d^3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| derivativedivides | $-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\cdots\right)\right)}{8(d^2)^{\frac{1}{4}}}\right)}{f\,d^2}$ |
| default | $-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\cdots\right)\right)}{8(d^2)^{\frac{1}{4}}}\right)}{f\,d^2}$ |

input
```
int(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d^2*(1/3*(d*cot(f*x+e))^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(
f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)
+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(
d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*
x+e))^(1/2)+1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(124) = 248.

Time = 0.12 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.26

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= \frac{6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}+1\right)\sin(2fx+2e)+6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}-1\right)\sin(2\cdots}{\cdots}$$

input
```
integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/12*(6*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2
*f*x + 2*e))/sqrt(d) + 1)*sin(2*f*x + 2*e) + 6*sqrt(2)*sqrt(d)*arctan(sqrt
(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))/sqrt(d) - 1)*sin(2*f*x
 + 2*e) - 3*sqrt(2)*sqrt(d)*log((sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin
(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(d) + cos(2*f*x + 2*e) + sin(2*f*x + 2
*e) + 1)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e) + 3*sqrt(2)*sqrt(d)*log(-(sqrt
(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(
d) - cos(2*f*x + 2*e) - sin(2*f*x + 2*e) - 1)/sin(2*f*x + 2*e))*sin(2*f*x
+ 2*e) - 8*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*(cos(2*f*x + 2*
e) + 1))/(d*f*sin(2*f*x + 2*e))
```

## Sympy [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(cot(e + f*x)**3/sqrt(d*cot(e + f*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.18

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= \frac{3\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\cdots\right.}{12\,d^2f}$$

input
```
integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f
*x + e))^(3/2))/(d^2*f)
```

## Giac [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^3}{\sqrt{d\cot\left(fx+e\right)}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^3/sqrt(d*cot(f*x + e)), x)
```

## Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{2\,(d\cot(e+f\,x))^{3/2}}{3\,d^2\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

input
```
int(cot(e + f*x)^3/(d*cot(e + f*x))^(1/2),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
 - (2*(d*cot(e + f*x))^(3/2))/(3*d^2*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d
*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

## Reduce [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \sqrt{\cot(fx+e)}\,\cot(fx+e)^2\,dx\right)}{d}$$

input
```
int(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int(sqrt(cot(e + f*x))*cot(e + f*x)**2,x))/d
```

# 3.214 $\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1635
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1636
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . . 1636
Maple [B] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . . 1641
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1642
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1643
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1643
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1644
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1644
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1645

## Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}d^{3/2}f} + \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}}$$

output
```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f+1/2*arctan
h(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(3/
2)/f+2/5*d/f/(d*cot(f*x+e))^(5/2)-2/d/f/(d*cot(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.55

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{-5\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}+5\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}+2\left(-5+\tan^2(e+fx)\right)}{5df\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

output
```
(-5*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) + 5*ArcTanh[(-
Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) + 2*(-5 + Tan[e + f*x]^2))/
(5*d*f*Sqrt[d*Cot[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.30, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 2030, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^2\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$d^2\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int \frac{1}{(d\cot(e+fx))^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \ 3042$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int \frac{1}{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \ 3955$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \ 3042$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \ 3957$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \ 266$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \ 826$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \ 1476$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot\right.\right.}{df}}{d^2}\right.$$

$$\downarrow 1082$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot}{d^4\cot^4(}\right.}{df}}{d^2}\right.$$

$$\downarrow 217$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+}{d^2}\right.$$

$$\downarrow 1479$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(}{2\sqrt{2}\sqrt{d}}\right.\right.}{df}}{d^2}\right.$$

$$\downarrow 25$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e}{2\sqrt{2}\sqrt{d}}\right.\right.}{df}}{d^2}\right.$$

$\Big\downarrow$ 27

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e\cdots}{2\sqrt{d}}\cdots}{df}}{d^2}\cdots\right.$$

$\Big\downarrow$ 1103

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+\cdots}{2\sqrt{2}\sqrt{d}}\cdots}{df}}{d^2}\cdots\right.$$

input
```
Int[Tan[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

output
```
d^2*(2/(5*d*f*(d*Cot[e + f*x])^(5/2)) - (2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2
*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[
1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*
d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + S
qrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))
/(d*f))/d^2)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(140) = 280$.

Time = 2.56 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.88

| method | result |
|---|---|
| default | $-\frac{\left(\sqrt{2}\left(24-4\sec(fx+e)^2\right)+\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}\arctan\left(\frac{\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+\cos(fx+e)-1}{\cos(fx+e)-1}\right)(-10\cot(fx+e)-\right.}{}$ |

input
```
int(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20/f/(d*cot(f*x+e))^(1/2)/d*(2^(1/2)*(24-4*sec(f*x+e)^2)+(-sin(f*x+e)*c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-10*cot(f*x+e)
-10*csc(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*ln(-(cot(f
*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(
-5*cot(f*x+e)-5*csc(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*sin(f*x+e)*(-2*sin(f*x+e)*cos(
f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*cos(f*x+e)+2)/(cos(
f*x+e)-1))*(5*cot(f*x+e)+5*csc(f*x+e))+arctan((-sin(f*x+e)*(-2*sin(f*x+e)*
cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-sin(f*x
+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(10*cot(f*x+e)+10*csc(f*x+e)))*2^(1
/2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$

$$-\frac{10\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d}{\tan(fx+e)}}}{\sqrt{d}}+1\right)+10\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d}{\tan(fx+e)}}}{\sqrt{d}}-1\right)-5\sqrt{2}\sqrt{d}\log\left(\frac{\frac{\sqrt{2}\sqrt{\frac{d}{\tan(fx+e)}}\,\mathrm{ta}}{\sqrt{d}}}{\mathrm{ta}}\right.}{}$$

input
```
integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/20*(10*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt(d/tan(f*x + e))/sqrt(d) + 1)
 + 10*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt(d/tan(f*x + e))/sqrt(d) - 1) - 5
*sqrt(2)*sqrt(d)*log((sqrt(2)*sqrt(d/tan(f*x + e))*tan(f*x + e)/sqrt(d) +
tan(f*x + e) + 1)/tan(f*x + e)) + 5*sqrt(2)*sqrt(d)*log(-(sqrt(2)*sqrt(d/t
an(f*x + e))*tan(f*x + e)/sqrt(d) - tan(f*x + e) - 1)/tan(f*x + e)) - 8*(t
an(f*x + e)^3 - 5*tan(f*x + e))*sqrt(d/tan(f*x + e)))/(d^2*f)
```

## Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(tan(e + f*x)**2/(d*cot(e + f*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.18

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{d^3\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\ldots}{}\right.}{d^4}\ldots\right)}{20\,f}$$

input
```
integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-1/20*d^3*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan
(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(
2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^
2 - 5*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

## Giac [F]

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\tan\left(fx+e\right)^2}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/(d*cot(f*x + e))^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.53

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\frac{2\,d}{5}-\frac{2\,d}{\tan(e+f\,x)^2}}{f\,\left(\frac{d}{\tan(e+f\,x)}\right)^{5/2}} - \frac{{(-1)}^{1/4}\,\mathrm{atan}\left(\frac{{(-1)}^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{d^{3/2}\,f} + \frac{{(-1)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-1)}^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

input
```
int(tan(e + f*x)^2/(d*cot(e + f*x))^(3/2),x)
```

output
```
((2*d)/5 - (2*d)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1/4)*
atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f) + ((-1)^(1/
4)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

## Reduce [F]

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cot(fx+e)}\,\tan(fx+e)^2}{\cot(fx+e)^2}dx\right)}{d^2}$$

input

```
int(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x)
```

output

```
(sqrt(d)*int((sqrt(cot(e + f*x))*tan(e + f*x)**2)/cot(e + f*x)**2,x))/d**2
```

# 3.215 $\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1646
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1646
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . 1647
Maple [B] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . 1652
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1653
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1653
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1654
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1654
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1655
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1655


## Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}d^{3/2}f} + \frac{2}{3f(d\cot(e+fx))^{3/2}}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f+1/2*arcta
nh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(3
/2)/f+2/3/f/(d*cot(f*x+e))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.54

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{-2+3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))^{3/4}+3\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx}{3f(d\cot(e+fx))^{3/2}}$$

input
```
Integrate[Tan[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

output
```
-1/3*(-2 + 3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4) + 3*A
rcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4))/(f*(d*Cot[e + f*x
])^(3/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.34, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.789, Rules used = {3042, 25, 2030, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$d\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3955}$$

$$d\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}dx}{d^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$d\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int\frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx}{d^2}\right)$$

$$\Big\downarrow \; 3957$$

$$d\left(\frac{\int\frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \; 266$$

$$d\left(\frac{2\int\frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \; 755$$

$$d\left(\frac{2\left(\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \; 1476$$

$$d\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\ldots\right.$$

$$\Big\downarrow \; 1082$$

$$d\left(\frac{2\left(\frac{\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{\ldots}{3df\ldots}\right.$$

$$\Big\downarrow \; 217$$

$$d\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\Big\downarrow$ 1479

$$d\left(\frac{2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\right.}{\sqrt{2}\sqrt{d}}}{}\right.}{df}\right.$$

$\Big\downarrow$ 25

$$d\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}\right.}{df}\right.$$

$\Big\downarrow$ 27

$$d\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}\right.}{df}\right.$$

$\Big\downarrow$ 1103

$$d\left(\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{df}\right.$$

input
```
Int[Tan[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

output
```
d*(2/(3*d*f*(d*Cot[e + f*x])^(3/2)) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Co
t[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/
(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^
2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x]
 + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/(d*f))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(121) = 242$.

Time = 1.41 (sec) , antiderivative size = 501, normalized size of antiderivative = 3.23

| method | result |
|---|---|
| default | $-\frac{\left(\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)-2\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+\csc(fx+e)-\sin(fx+e)-2\cos(fx+e)+2}{\cos(fx+e)-1}\right)\sqrt{-\frac{\sin(fx+e)}{(1+\cos(}}\right.}{}$ |

input
```
int(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/f/(d*cot(f*x+e))^(1/2)/d*(ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*
sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-si
n(f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)*(-3*cot(f*x+e)-3*csc(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+
e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-6*cot(f*x+e)-6*csc(f*x+e))+(-
sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x+e)-2
*cot(f*x+e)+2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
+csc(f*x+e)-sin(f*x+e)-2*cos(f*x+e)+2)/(cos(f*x+e)-1))*(3*cot(f*x+e)+3*csc
(f*x+e))+(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*arctan((-sin(f*x+
e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*
x+e)-1))*(6*cot(f*x+e)+6*csc(f*x+e))-4*2^(1/2)*tan(f*x+e))*2^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.26

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{8\sqrt{\frac{d}{\tan(fx+e)}}\tan\left(fx+e\right)^2 + 6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d}{\tan(fx+e)}}}{\sqrt{d}}+1\right) + 6\sqrt{2}\sqrt{d}\,\mathrm{arct}}{}$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/12*(8*sqrt(d/tan(f*x + e))*tan(f*x + e)^2 + 6*sqrt(2)*sqrt(d)*arctan(sqr
t(2)*sqrt(d/tan(f*x + e))/sqrt(d) + 1) + 6*sqrt(2)*sqrt(d)*arctan(sqrt(2)*
sqrt(d/tan(f*x + e))/sqrt(d) - 1) + 3*sqrt(2)*sqrt(d)*log((sqrt(2)*sqrt(d/
tan(f*x + e))*tan(f*x + e)/sqrt(d) + tan(f*x + e) + 1)/tan(f*x + e)) - 3*s
qrt(2)*sqrt(d)*log(-(sqrt(2)*sqrt(d/tan(f*x + e))*tan(f*x + e)/sqrt(d) - t
an(f*x + e) - 1)/tan(f*x + e)))/(d^2*f)
```

## Sympy [F]

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\tan\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(tan(e + f*x)/(d*cot(e + f*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.23

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{d^2\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\right.}{d^2}\right.}{12\,f}$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/12*d^2*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2
*(d/tan(f*x + e))^(3/2)))/f
```

## Giac [F]

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)/(d*cot(f*x + e))^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2}{3f\left(\frac{d}{\tan(e+fx)}\right)^{3/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}$$

input
```
int(tan(e + f*x)/(d*cot(e + f*x))^(3/2),x)
```

output
```
2/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d/tan(e + f
*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d/ta
n(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f)
```

## Reduce [F]

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cot(fx+e)}\,\tan(fx+e)}{\cot(fx+e)^2}dx\right)}{d^2}$$

input
```
int(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(cot(e + f*x))*tan(e + f*x))/cot(e + f*x)**2,x))/d**2
```

# 3.216 $\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . 1656
Mathematica [A] (verified) . . . . . . . . . . 1656
Rubi [A] (warning: unable to verify) . . . . . . . . . . 1657
Maple [A] (verified) . . . . . . . . . . 1661
Fricas [B] (verification not implemented) . . . . . . . . . . 1662
Sympy [F] . . . . . . . . . . 1663
Maxima [A] (verification not implemented) . . . . . . . . . . 1663
Giac [F] . . . . . . . . . . 1664
Mupad [B] (verification not implemented) . . . . . . . . . . 1664
Reduce [F] . . . . . . . . . . 1665

## Optimal result

Integrand size = 12, antiderivative size = 157

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}d^{3/2}f} + \frac{2}{df\sqrt{d\cot(e+fx)}}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f-1/2*arcta
nh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(3
/2)/f+2/d/f/(d*cot(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.52

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2+\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx}\right.}{df\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[(d*Cot[e + f*x])^(-3/2),x]
```

output
```
(2 + ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) - ArcTanh[(-C
ot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4))/(d*f*Sqrt[d*Cot[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$$\Big\downarrow 826$$

$$\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$$\Big\downarrow 1476$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)\right.}{df}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$$\Big\downarrow 1082$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right.}{df}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$$\Big\downarrow 217$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$$\Big\downarrow 1479$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\cdots\right.}{}\right.\right.}{df}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$$\Big\downarrow 25$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\right.}}{\sqrt{}}\right.}{df}$$
$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$$\Big\downarrow 27$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\right.}{\sqrt{}}\right.}{df}$$
$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$$\Big\downarrow 1103$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/}\right.}{}\right.}{df}$$
$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

input
```
Int[(d*Cot[e + f*x])^(-3/2),x]
```

output
```
2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f
*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2
]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2
]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e +
 f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/(d*f)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| derivativedivides | $-\frac{2d\left(-\frac{1}{d^2\sqrt{d\cot(fx+e)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\cdots\right)\right)}{8d^2(d^2)^{\frac{1}{4}}}\right)}{f}$ |
| default | $-\frac{2d\left(-\frac{1}{d^2\sqrt{d\cot(fx+e)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\cdots\right)\right)}{8d^2(d^2)^{\frac{1}{4}}}\right)}{f}$ |

input
```
int(1/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f*d*(-1/d^2/(d*cot(f*x+e))^(1/2)-1/8/d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*cot
(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e
)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/
(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f
*x+e))^(1/2)+1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(124) = 248$.

Time = 0.10 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.40

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\frac{2\sqrt{2}(d\cos(2fx+2e)+d)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}+1\right)}{\sqrt{d}}+\frac{2\sqrt{2}(d\cos(2fx+2e)+d)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos\cdots}{\sin\cdots}}}{\cdots}\right)}{\sqrt{d}}\cdots}{\cdots}$$

input
```
integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*(d*cos(2*f*x + 2*e) + d)*arctan(sqrt(2)*sqrt((d*cos(2*f*x +
 2*e) + d)/sin(2*f*x + 2*e))/sqrt(d) + 1)/sqrt(d) + 2*sqrt(2)*(d*cos(2*f*x
 + 2*e) + d)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)
)/sqrt(d) - 1)/sqrt(d) - sqrt(2)*(d*cos(2*f*x + 2*e) + d)*log((sqrt(2)*sqr
t((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(d) + co
s(2*f*x + 2*e) + sin(2*f*x + 2*e) + 1)/sin(2*f*x + 2*e))/sqrt(d) + sqrt(2)
*(d*cos(2*f*x + 2*e) + d)*log(-(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(
2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(d) - cos(2*f*x + 2*e) - sin(2*f*x + 2*
e) - 1)/sin(2*f*x + 2*e))/sqrt(d) + 8*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*
f*x + 2*e))*sin(2*f*x + 2*e))/(d^2*f*cos(2*f*x + 2*e) + d^2*f)
```

## Sympy [F]

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\cot{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral((d*cot(e + f*x))**(-3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.19

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{d\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{}{\tan}}\right.}{}}{d^2}\right.}{4f}$$

input
```
integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/4*d*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x +
 e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqr
t(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqr
t(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sqrt
(d/tan(f*x + e))))/f
```

## Giac [F]

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(-3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2}{d\,f\,\sqrt{d\cot(e+f\,x)}} + \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

input
```
int(1/(d*cot(e + f*x))^(3/2),x)
```

output
```
2/(d*f*(d*cot(e + f*x))^(1/2)) + ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f
*x))^(1/2))/d^(1/2)))/(d^(3/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e
 + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

## Reduce [F]

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cot(fx+e)}}{\cot(fx+e)^2}dx\right)}{d^2}$$

input
```
int(1/(d*cot(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(cot(e + f*x))/cot(e + f*x)**2,x))/d**2
```

# 3.217 $\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result . . . 1666
Mathematica [A] (verified) . . . 1666
Rubi [A] (warning: unable to verify) . . . 1667
Maple [A] (verified) . . . 1671
Fricas [B] (verification not implemented) . . . 1671
Sympy [F] . . . 1672
Maxima [A] (verification not implemented) . . . 1672
Giac [F] . . . 1673
Mupad [B] (verification not implemented) . . . 1673
Reduce [F] . . . 1674

## Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx=\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}d^{3/2}f}$$

output
```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f-1/2*arctan
h(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(3/
2)/f
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx=\frac{\sqrt{\cot(e+fx)}\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot(e+\right.\right.}{2\sqrt{}}$$

input
```
Integrate[Cot[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

output
```
(Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[1
 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[
e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2
]*d*f*Sqrt[d*Cot[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2030, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx}{d}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{755}$$

$$-\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$\Big\downarrow$ 1476

$$-\frac{2\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{\cdots}}{2d}\cdots\right)}{f}$$

$\Big\downarrow$ 1082

$$-\frac{2\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$\Big\downarrow$ 217

$$-\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}$$

$\Big\downarrow$ 1479

$$-\frac{2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\cdots}{\cdots}\right)}{f}$$

$\Big\downarrow$ 25

$$-\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\right)}{f}$$

$\Big\downarrow$ 27

$$-\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\right)}{f}$$

$\Big\downarrow$ 1103

$$-\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}$$

input
```
Int[Cot[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcT
an[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[
d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) +
 Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqr
t[d]))/(2*d)))/f
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| derivativedivides | $-\frac{\left(d^{2}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{d \cot \left(f x +e \right)+\left(d^{2}\right)^{\frac{1}{4}} \sqrt{d \cot \left(f x +e \right)}\, \sqrt{2}+\sqrt{d^{2}}}{d \cot \left(f x +e \right)-\left(d^{2}\right)^{\frac{1}{4}} \sqrt{d \cot \left(f x +e \right)}\, \sqrt{2}+\sqrt{d^{2}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{d \cot \left(f x +e \right)}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2 \arctan \left(-\frac{\sqrt{2}\, \sqrt{d \cot}}{\left(d^{2}\right.}\right.\right.}{4 f \,d^{2}}$ |
| default | $-\frac{\left(d^{2}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{d \cot \left(f x +e \right)+\left(d^{2}\right)^{\frac{1}{4}} \sqrt{d \cot \left(f x +e \right)}\, \sqrt{2}+\sqrt{d^{2}}}{d \cot \left(f x +e \right)-\left(d^{2}\right)^{\frac{1}{4}} \sqrt{d \cot \left(f x +e \right)}\, \sqrt{2}+\sqrt{d^{2}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{d \cot \left(f x +e \right)}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2 \arctan \left(-\frac{\sqrt{2}\, \sqrt{d \cot}}{\left(d^{2}\right.}\right.\right.}{4 f \,d^{2}}$ |

input
```
int(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*(d^2)^(1/4)/d^2*2^(1/2)*(ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e)
)^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2
)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+
1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(106) = 212$.

Time = 0.08 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.98

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}+1\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}-1\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(\frac{\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\sin(2fx+2e)}{\sqrt{d}}+\cos(2fx-}{\sin(2fx+2e)}\right.}{\sqrt{d}}}{4df}$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2
*e))/sqrt(d) + 1)/sqrt(d) + 2*sqrt(2)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2
*e) + d)/sin(2*f*x + 2*e))/sqrt(d) - 1)/sqrt(d) + sqrt(2)*log((sqrt(2)*sqr
t((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(d) + co
s(2*f*x + 2*e) + sin(2*f*x + 2*e) + 1)/sin(2*f*x + 2*e))/sqrt(d) - sqrt(2)
*log(-(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x +
 2*e)/sqrt(d) - cos(2*f*x + 2*e) - sin(2*f*x + 2*e) - 1)/sin(2*f*x + 2*e))
/sqrt(d))/(d*f)
```

## Sympy [F]

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(cot(e + f*x)/(d*cot(e + f*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.20

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-}{4f}$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e
)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(
d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

## Giac [F]

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)/(d*cot(f*x + e))^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.42

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f} + \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}$$

input
```
int(cot(e + f*x)/(d*cot(e + f*x))^(3/2),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)
*f) + ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(
d^(3/2)*f)
```

## Reduce [F]

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cot(fx+e)}}{\cot(fx+e)}dx\right)}{d^2}$$

input
```
int(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(cot(e + f*x))/cot(e + f*x),x))/d**2
```

# 3.218 $\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1675
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . 1675
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . 1676
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1680
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . 1680
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1681
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . 1681
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1682
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . 1682
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1683


## Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}d^{3/2}f}$$

output
```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f+1/2*arctan
h(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(3/
2)/f
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\left(-\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right)\sqrt[4]{-\cot(e+}}{d^2 f\cot^{\frac{3}{4}}(e+fx)}$$

input
```
Integrate[Cot[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

output
```
((-ArcTan[(-Cot[e + f*x]^2)^(1/4)] + ArcTanh[(-Cot[e + f*x]^2)^(1/4)])*(-C
ot[e + f*x])^(1/4)*Sqrt[d*Cot[e + f*x]])/(d^2*f*Cot[e + f*x]^(3/4))
```

## Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.29, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2030, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}$$

$\downarrow$ 1476

$$-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right.\right.}{df}$$

$\downarrow$ 1082

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot\right.}{df}$$

$\downarrow$ 217

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}$$

$\downarrow$ 1479

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}{}\right.}{df}$$

$\downarrow$ 25

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2\right.}{}\right.}{df}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2\right.}{}\right.}{df}$$

$\downarrow$ 1103

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d\right.}{}\right.}{df}$$

input
```
Int[Cot[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcT
an[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[
2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d
+ Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/
2))/(d*f)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| derivativedivides | $-\frac{\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{d\cot(fx+e}}{\left(d^2\right)^{\frac{1}{4}}}\right.\right.}{4fd\left(d^2\right)^{\frac{1}{4}}}$ |
| default | $-\frac{\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{d\cot(fx+e}}{\left(d^2\right)^{\frac{1}{4}}}\right.\right.}{4fd\left(d^2\right)^{\frac{1}{4}}}$ |

input
```
int(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/d/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^
(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*
2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(106) = 212$.

Time = 0.08 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.99

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}+1\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}-1\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\frac{\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\sin(2fx+2e)}{\sqrt{d}}+\cos(2fx-}{\sin(2fx+2e)}\right.}{\sqrt{d}}}{4df}$$

input
```
integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2
*e))/sqrt(d) + 1)/sqrt(d) + 2*sqrt(2)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2
*e) + d)/sin(2*f*x + 2*e))/sqrt(d) - 1)/sqrt(d) - sqrt(2)*log((sqrt(2)*sqr
t((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(d) + co
s(2*f*x + 2*e) + sin(2*f*x + 2*e) + 1)/sin(2*f*x + 2*e))/sqrt(d) + sqrt(2)
*log(-(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x +
 2*e)/sqrt(d) - cos(2*f*x + 2*e) - sin(2*f*x + 2*e) - 1)/sin(2*f*x + 2*e))
/sqrt(d))/(d*f)
```

## Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot^2{(e+fx)}}{(d\cot{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(cot(e + f*x)**2/(d*cot(e + f*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.23

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}} + }{4\,df}$$

input
```
integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e
)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(
d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/(d*f)
```

## Giac [F]

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot\left(fx+e\right)^2}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^2/(d*cot(f*x + e))^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.43

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

input
```
int(cot(e + f*x)^2/(d*cot(e + f*x))^(3/2),x)
```

output
```
((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f
) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2
)*f)
```

## Reduce [F]

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \sqrt{\cot(fx+e)}dx\right)}{d^2}$$

input
```
int(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(cot(e + f*x)),x))/d**2
```

# 3.219 $\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . 1684
Mathematica [A] (verified) . . . . . . . . . . 1684
Rubi [A] (warning: unable to verify) . . . . . . . . . . 1685
Maple [A] (verified) . . . . . . . . . . 1690
Fricas [B] (verification not implemented) . . . . . . . . . . 1690
Sympy [F] . . . . . . . . . . 1691
Maxima [A] (verification not implemented) . . . . . . . . . . 1691
Giac [F] . . . . . . . . . . 1692
Mupad [B] (verification not implemented) . . . . . . . . . . 1692
Reduce [F] . . . . . . . . . . 1692

## Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}d^{3/2}f} - \frac{2\sqrt{d\cot(e+fx)}}{d^2f}$$

output

```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f+1/2*arcta
nh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(3
/2)/f-2*(d*cot(f*x+e))^(1/2)/d^2/f
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\cot^{\frac{3}{2}}(e+fx)\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}} + 2\sqrt{\cot(e+fx)} + \frac{\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e\right.}{2\sqrt{2}}\right.}{f(d\cot(e+fx))^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^3/(d*Cot[e + f*x])^(3/2),x]
```

output
```
-((Cot[e + f*x]^(3/2)*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] - Ar
cTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] + 2*Sqrt[Cot[e + f*x]] + Log[
1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[
2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2])))/(f*(d*Cot[e + f*x])^(3
/2)))
```

## Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.31, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\cot(e+fx))^{3/2}dx}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx}{d^3}$$

$$\Big\downarrow \text{3954}$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^3}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^3}$$

$\Big\downarrow$ 266

$$\frac{\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^3}$$

$\Big\downarrow$ 755

$$\frac{\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^3}$$

$\Big\downarrow$ 1476

$$\frac{\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+f}{2d}}{f}}{d^3}$$

$\Big\downarrow$ 1082

$$\frac{\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e-}{f}}{d^3}$$

$\Big\downarrow$ 217

$$\frac{\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^3}$$

$\Big\downarrow$ 1479

$$\frac{\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\mathrm{a}}{2d}}{f}}{d^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arctan}}{2d}\right.}{f}}{d^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arctan}}{2d}\right.}{f}}{d^3}$$

$\Big\downarrow$ 1103

$$\frac{\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}}{d^3}$$

input
```
Int[Cot[e + f*x]^3/(d*Cot[e + f*x])^(3/2),x]
```

output
```
((-2*d*Sqrt[d*Cot[e + f*x]])/f + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot
[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(
Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2
*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x]
+ d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/f)/d^3
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $-\frac{2\left(\sqrt{d\cot(fx+e)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arcta}}{8}}{f\,d^2}$ |
| default | $-\frac{2\left(\sqrt{d\cot(fx+e)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arcta}}{8}}{f\,d^2}$ |

input
```
int(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d^2*((d*cot(f*x+e))^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(
d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(
1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/
4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1
/2)+1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(124) = 248$.

Time = 0.08 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.92

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}+1\right)+2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}-\right.}{}$$

input
```
integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*
f*x + 2*e))/sqrt(d) + 1) + 2*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt((d*cos(2*
f*x + 2*e) + d)/sin(2*f*x + 2*e))/sqrt(d) - 1) + sqrt(2)*sqrt(d)*log((sqrt
(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(
d) + cos(2*f*x + 2*e) + sin(2*f*x + 2*e) + 1)/sin(2*f*x + 2*e)) - sqrt(2)*
sqrt(d)*log(-(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(
2*f*x + 2*e)/sqrt(d) - cos(2*f*x + 2*e) - sin(2*f*x + 2*e) - 1)/sin(2*f*x
+ 2*e)) - 8*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))/(d^2*f)
```

## Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot^3{(e+fx)}}{(d\cot{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(cot(e + f*x)**3/(d*cot(e + f*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.16

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx}}\right.}{2\sqrt{d}}\right.}{}$$

input
```
integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x +
e)))/(d^2*f)
```

## Giac [F]

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx=\int \frac{\cot\left(fx+e\right)^3}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^3/(d*cot(f*x + e))^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.49

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx=-\frac{2\sqrt{d\cot\left(e+f\,x\right)}}{d^2\,f}-\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}-\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}$$

input
```
int(cot(e + f*x)^3/(d*cot(e + f*x))^(3/2),x)
```

output
```
- (2*(d*cot(e + f*x))^(1/2))/(d^2*f) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot
(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)
*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f)
```

## Reduce [F]

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx=\frac{\sqrt{d}\left(-2\sqrt{\cot\left(fx+e\right)}-\left(\int \frac{\sqrt{\cot(fx+e)}}{\cot(fx+e)}dx\right)f\right)}{d^2 f}$$

input
```
int(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*( - 2*sqrt(cot(e + f*x)) - int(sqrt(cot(e + f*x))/cot(e + f*x),x)
*f))/(d**2*f)
```

# 3.220 $\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1694
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1694
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . 1695
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1699
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1700
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1700
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 1701
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1701
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1702
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1702


## Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}d^{3/2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3d^3f}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f-1/2*arcta
nh(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(3
/2)/f-2/3*(d*cot(f*x+e))^(3/2)/d^3/f
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.63

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\cot^{\frac{5}{4}}(e+fx)\left(-3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}+3\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\text{co}}\right.}{3f(d\cot(e+fx))^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^4/(d*Cot[e + f*x])^(3/2),x]
```

output
```
-1/3*(Cot[e + f*x]^(5/4)*(-3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x
])^(1/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) + 2*Co
t[e + f*x]^(7/4)))/(f*(d*Cot[e + f*x])^(3/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.26, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\cot(e+fx))^{5/2}dx}{d^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx}{d^4}$$

$$\Big\downarrow \text{3954}$$

$$\frac{d^2\left(-\int \sqrt{d\cot(e+fx)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\left(-\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\frac{d^3\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \ 266$$

$$\frac{\frac{2d^3\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \ 826$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \ 1476$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}\right.}{f}}{d^4}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-2d(}{d^4}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \ 1479$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx\right.}{\sqrt{2}\sqrt{d}}\right.\right.}{f}}{d^4}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\right.}{\sqrt{2}\sqrt{d}}\right.\right.}{f}}{d^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\right.}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^4}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e\right.}{2\sqrt{2}\sqrt{d}}\right.}{f}}{d^4}$$

input
```
Int[Cot[e + f*x]^4/(d*Cot[e + f*x])^(3/2),x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(3/2))/(3*f) + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[
d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f
*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Co
t[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] +
 d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/f)/d^4
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| derivativedivides | $-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8(d^2)^{\frac{1}{4}}}\right)}{f d^3}$ |
| default | $-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8(d^2)^{\frac{1}{4}}}\right)}{f d^3}$ |

input
```
int(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d^3*(1/3*(d*cot(f*x+e))^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(
f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)
+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(
d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*
x+e))^(1/2)+1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(124) = 248$.

Time = 0.09 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.26

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}+1\right)\sin(2fx+2e)+6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{}$$

input
```
integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/12*(6*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2
*f*x + 2*e))/sqrt(d) + 1)*sin(2*f*x + 2*e) + 6*sqrt(2)*sqrt(d)*arctan(sqrt
(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))/sqrt(d) - 1)*sin(2*f*x
 + 2*e) - 3*sqrt(2)*sqrt(d)*log((sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin
(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(d) + cos(2*f*x + 2*e) + sin(2*f*x + 2
*e) + 1)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e) + 3*sqrt(2)*sqrt(d)*log(-(sqrt
(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(
d) - cos(2*f*x + 2*e) - sin(2*f*x + 2*e) - 1)/sin(2*f*x + 2*e))*sin(2*f*x
+ 2*e) - 8*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*(cos(2*f*x + 2*
e) + 1))/(d^2*f*sin(2*f*x + 2*e))
```

## Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(cot(e + f*x)**4/(d*cot(e + f*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.18

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{3\,d^2\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\,\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\,\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\,\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\,\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\,\mathrm{lo}\cdots}{\cdots}\right.}{1\cdots}$$

input
```
integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f
*x + e))^(3/2))/(d^3*f)
```

## Giac [F]

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot\left(fx+e\right)^4}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^4/(d*cot(f*x + e))^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f} - \frac{2\,(d\cot(e+fx))^{3/2}}{3\,d^3\,f} - \frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}f}$$

input
```
int(cot(e + f*x)^4/(d*cot(e + f*x))^(3/2),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
 - (2*(d*cot(e + f*x))^(3/2))/(3*d^3*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d
*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

## Reduce [F]

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \sqrt{\cot(fx+e)}\,\cot(fx+e)^2\,dx\right)}{d^2}$$

input
```
int(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(cot(e + f*x))*cot(e + f*x)**2,x))/d**2
```

# 3.221 $\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1703
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1704
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . . . . 1704
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1710
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1710
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1711
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1711
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1712
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1712
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1713

## Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}+\sqrt{d}\cot(e+fx)}\right)}{\sqrt{2}d^{3/2}f} + \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f}$$

output

```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)/f-1/2*arctan
h(2^(1/2)*(d*cot(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*cot(f*x+e)))*2^(1/2)/d^(3/
2)/f+2*(d*cot(f*x+e))^(1/2)/d^2/f-2/5*(d*cot(f*x+e))^(5/2)/d^4/f
```

## Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.96

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\cot^{\frac{3}{2}}(e+fx)\left(-10\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)+10\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)-40\sqrt{\text{co}}\right.}{}$$

input

```
Integrate[Cot[e + f*x]^5/(d*Cot[e + f*x])^(3/2),x]
```

output

```
-1/20*(Cot[e + f*x]^(3/2)*(-10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x
]]] + 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] - 40*Sqrt[Cot[e +
f*x]] + 8*Cot[e + f*x]^(5/2) - 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]
] + Cot[e + f*x]] + 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e +
 f*x]]))/(f*(d*Cot[e + f*x])^(3/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.29, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {2030, 3042, 3954, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (d\cot(e+fx))^{7/2}dx}{d^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx}{d^5}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-d^2\int (d\cot(e+fx))^{3/2}dx-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-d^2\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-d^2\left(d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-d^2\left(d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \text{3957}$$

$$\frac{-d^2\left(\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \text{266}$$

$$\frac{-d^2\left(\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \text{755}$$

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \text{1476}$$

$$-d^2\left(\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d}}{2d}\right.}{f}\right.$$

$$d^5$$

$\downarrow$ 1082

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{}}{}\right.$$

$$d^5$$

$\downarrow$ 217

$$-d^2\left(\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e}{5}$$

$$d^5$$

$\downarrow$ 1479

$$-d^2\left(\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+f\right.}{\sqrt{2}\sqrt{d}}\right.}{f}\right.$$

$$d^5$$

$\downarrow$ 25

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\cdots\right)}{f}\cdots\right)}{d^5}\cdots$$

$\Big\downarrow$ 27

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\cdots\right)}{f}\cdots\right)}{d^5}\cdots$$

$\Big\downarrow$ 1103

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e\cdots\right)}{2\sqrt{2}\sqrt{d}}}{2d}\cdots\right)}{f}\cdots\right)}{d^5}\cdots$$

input
```
Int[Cot[e + f*x]^5/(d*Cot[e + f*x])^(3/2),x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(5/2))/(5*f) - d^2*((-2*d*Sqrt[d*Cot[e + f*x]])/f
+ (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
 ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2
*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[
d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2
]*Sqrt[d]))/(2*d)))/f))/d^5
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| derivativedivides | $-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{5}{2}}}{5}-d^2\sqrt{d\cot(fx+e)}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d}}{(}\right.\right.}{8}\right.}{f d^4}$ |
| default | $-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{5}{2}}}{5}-d^2\sqrt{d\cot(fx+e)}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d}}{(}\right.\right.}{8}\right.}{f d^4}$ |

input
```
int(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d^4*(1/5*(d*cot(f*x+e))^(5/2)-d^2*(d*cot(f*x+e))^(1/2)+1/8*d^2*(d^2)^
(1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(
d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(
1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1
/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(142) = 284.

Time = 0.13 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.18

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$

$$-\frac{\frac{10\sqrt{2}(d\cos(2fx+2e)-d)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}+1\right)}{\sqrt{d}}+\frac{10\sqrt{2}(d\cos(2fx+2e)-d)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}}{\sqrt{d}}-1\right)}{\sqrt{d}}+\frac{5\sqrt{2}(d\cos}{}}{}$$

input
```
integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/20*(10*sqrt(2)*(d*cos(2*f*x + 2*e) - d)*arctan(sqrt(2)*sqrt((d*cos(2*f*
x + 2*e) + d)/sin(2*f*x + 2*e))/sqrt(d) + 1)/sqrt(d) + 10*sqrt(2)*(d*cos(2
*f*x + 2*e) - d)*arctan(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x +
2*e))/sqrt(d) - 1)/sqrt(d) + 5*sqrt(2)*(d*cos(2*f*x + 2*e) - d)*log((sqrt(
2)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(d
) + cos(2*f*x + 2*e) + sin(2*f*x + 2*e) + 1)/sin(2*f*x + 2*e))/sqrt(d) - 5
*sqrt(2)*(d*cos(2*f*x + 2*e) - d)*log(-(sqrt(2)*sqrt((d*cos(2*f*x + 2*e) +
 d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e)/sqrt(d) - cos(2*f*x + 2*e) - sin(2*
f*x + 2*e) - 1)/sin(2*f*x + 2*e))/sqrt(d) - 16*sqrt((d*cos(2*f*x + 2*e) +
d)/sin(2*f*x + 2*e))*(3*cos(2*f*x + 2*e) - 2))/(d^2*f*cos(2*f*x + 2*e) - d
^2*f)
```

## Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot^5\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(cot(e + f*x)**5/(d*cot(e + f*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.11

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{10\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{5}{2}}\log\left(\sqrt{2}\right.}{}$$

input
```
integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-1/20*(10*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/t
an(f*x + e)))/sqrt(d)) + 10*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + 5*sqrt(2)*d^(5/2)*log(sqrt(2)*
sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 5*sqrt(2)*d^(5/2)*log
(-sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 40*d^2*sqrt
(d/tan(f*x + e)) + 8*(d/tan(f*x + e))^(5/2))/(d^4*f)
```

## Giac [F]

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx=\int \frac{\cot\left(fx+e\right)^5}{\left(d\cot\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^5/(d*cot(f*x + e))^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.52

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx=\frac{2\sqrt{d\cot\left(e+f\,x\right)}}{d^2\,f}-\frac{2\,(d\cot(e+f\,x))^{5/2}}{5\,d^4\,f}+\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}+\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}\,1\mathrm{i}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

input
```
int(cot(e + f*x)^5/(d*cot(e + f*x))^(3/2),x)
```

output
```
(2*(d*cot(e + f*x))^(1/2))/(d^2*f) - (2*(d*cot(e + f*x))^(5/2))/(5*d^4*f)
+ ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/
2)*f) + ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2)*1i)/d^(1/2)))/
(d^(3/2)*f)
```

## Reduce [F]

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(-2\sqrt{\cot(fx+e)}\,\cot(fx+e)^2 + 10\sqrt{\cot(fx+e)} + 5\left(\int \frac{\sqrt{\cot(fx+e)}}{\cot(fx+e)}dx\right) f\right)}{5d^2f}$$

input
```
int(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*( - 2*sqrt(cot(e + f*x))*cot(e + f*x)**2 + 10*sqrt(cot(e + f*x))
+ 5*int(sqrt(cot(e + f*x))/cot(e + f*x),x)*f))/(5*d**2*f)
```

# 3.222 $\int \cot^m(e+fx)\tan^n(e+fx)\,dx$

Optimal result . . . . . . . . . . 1714
Mathematica [A] (verified) . . . . . . . . . . 1714
Rubi [A] (verified) . . . . . . . . . . 1715
Maple [F] . . . . . . . . . . 1716
Fricas [F] . . . . . . . . . . 1717
Sympy [F] . . . . . . . . . . 1717
Maxima [F] . . . . . . . . . . 1717
Giac [F] . . . . . . . . . . 1718
Mupad [F(-1)] . . . . . . . . . . 1718
Reduce [F] . . . . . . . . . . 1718

## Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \frac{\cot^m(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

output
```
cot(f*x+e)^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x+e)^
2)*tan(f*x+e)^(1+n)/f/(1-m+n)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \frac{\cot^m(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

input
```
Integrate[Cot[e + f*x]^m*Tan[e + f*x]^n,x]
```

output
```
(Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e
+ f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3084, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(e+fx)^m\tan(e+fx)^n dx$$

$$\Big\downarrow \text{3084}$$

$$\tan^m(e+fx)\cot^m(e+fx)\int \tan^{n-m}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^m(e+fx)\cot^m(e+fx)\int \tan(e+fx)^{n-m}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^m(e+fx)\cot^m(e+fx)\int \frac{\tan^{n-m}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\cot^m(e+fx)\tan^{n+1}(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

input
```
Int[Cot[e + f*x]^m*Tan[e + f*x]^n,x]
```

output
```
(Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e
+ f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3084
```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m   Int[(b*Tan[e
+ f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
  !IntegerQ[n]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [F]

$$\int \cot (fx+e)^m \tan (fx+e)^n \, dx$$

input
```
int(cot(f*x+e)^m*tan(f*x+e)^n,x)
```

output
```
int(cot(f*x+e)^m*tan(f*x+e)^n,x)
```

## Fricas [F]

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx=\int \cot\left(fx+e\right)^m\tan\left(fx+e\right)^n\,dx$$

input
```
integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="fricas")
```

output
```
integral(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

## Sympy [F]

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx=\int \tan^n\left(e+fx\right)\cot^m\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**m*tan(f*x+e)**n,x)
```

output
```
Integral(tan(e + f*x)**n*cot(e + f*x)**m, x)
```

## Maxima [F]

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx=\int \cot\left(fx+e\right)^m\tan\left(fx+e\right)^n\,dx$$

input
```
integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

## Giac [F]

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \int \cot(fx+e)^m \tan(fx+e)^n\,dx$$

input
```
integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \int \cot(e+f\,x)^m \tan(e+f\,x)^n\,dx$$

input
```
int(cot(e + f*x)^m*tan(e + f*x)^n,x)
```

output
```
int(cot(e + f*x)^m*tan(e + f*x)^n, x)
```

## Reduce [F]

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \int \tan(fx+e)^n \cot(fx+e)^m\,dx$$

input
```
int(cot(f*x+e)^m*tan(f*x+e)^n,x)
```

output
```
int(tan(e + f*x)**n*cot(e + f*x)**m,x)
```

# 3.223 $\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1719
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1719
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1720
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1721
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1722
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1722
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1722
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1723
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1723
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1723

## Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \frac{\cot^m(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1-m+n)}$$

output
```
cot(f*x+e)^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x+e)^
2)*(b*tan(f*x+e))^(1+n)/b/f/(1-m+n)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \frac{\cot^{-1+m}(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)(b\tan(e+fx))^n}{f(1-m+n)}$$

input
```
Integrate[Cot[e + f*x]^m*(b*Tan[e + f*x])^n,x]
```

output
```
(Cot[e + f*x]^(-1 + m)*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2,
-Tan[e + f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - m + n))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3084, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(e+fx)^m(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3084}$$

$$\cot^m(e+fx)(b\tan(e+fx))^m\int (b\tan(e+fx))^{n-m}dx$$

$$\Big\downarrow \text{3042}$$

$$\cot^m(e+fx)(b\tan(e+fx))^m\int (b\tan(e+fx))^{n-m}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\cot^m(e+fx)(b\tan(e+fx))^m\int \frac{(b\tan(e+fx))^{n-m}}{\tan^2(e+fx)b^2+b^2}d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\cot^m(e+fx)(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),-\tan^2(e+fx)\right)}{bf(-m+n+1)}$$

input
```
Int[Cot[e + f*x]^m*(b*Tan[e + f*x])^n,x]
```

output
```
(Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e
+ f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - m + n))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3084
```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m   Int[(b*Tan[e
+ f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
  !IntegerQ[n]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [F]

$$\int \cot (fx+e)^m (b \tan (fx+e))^n \, dx$$

input
```
int(cot(f*x+e)^m*(b*tan(f*x+e))^n,x)
```

output
```
int(cot(f*x+e)^m*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\cot(fx+e)^m\,dx$$

input
```
integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

## Sympy [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(e+fx))^n\cot^m(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*cot(e + f*x)**m, x)
```

## Maxima [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\cot(fx+e)^m\,dx$$

input
```
integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

## Giac [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \cot(fx+e)^m\,dx$$

input
```
integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int \cot(e+f\,x)^m\,(b\tan(e+f\,x))^n\,dx$$

input
```
int(cot(e + f*x)^m*(b*tan(e + f*x))^n,x)
```

output
```
int(cot(e + f*x)^m*(b*tan(e + f*x))^n, x)
```

## Reduce [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = b^n\left(\int \tan(fx+e)^n \cot(fx+e)^m\,dx\right)$$

input
```
int(cot(f*x+e)^m*(b*tan(f*x+e))^n,x)
```

output
```
b**n*int(tan(e + f*x)**n*cot(e + f*x)**m,x)
```

# 3.224 $\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1724
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 1724
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1725
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1726
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1727
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1727
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1727
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1728
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1728
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1728

## Optimal result

Integrand size = 19, antiderivative size = 64

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \frac{(a\cot(e+fx))^m \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

output
```
(a*cot(f*x+e))^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x
+e)^2)*tan(f*x+e)^(1+n)/f/(1-m+n)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \frac{(a\cot(e+fx))^m \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

input
```
Integrate[(a*Cot[e + f*x])^m*Tan[e + f*x]^n,x]
```

output
```
((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Ta
n[e + f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3084, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^n(e+fx)(a\cot(e+fx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(e+fx)^n(a\cot(e+fx))^m dx$$

$$\Big\downarrow \ 3084$$

$$\tan^m(e+fx)(a\cot(e+fx))^m\int \tan^{n-m}(e+fx)dx$$

$$\Big\downarrow \ 3042$$

$$\tan^m(e+fx)(a\cot(e+fx))^m\int \tan(e+fx)^{n-m}dx$$

$$\Big\downarrow \ 3957$$

$$\frac{\tan^m(e+fx)(a\cot(e+fx))^m\int \frac{\tan^{n-m}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 278$$

$$\frac{\tan^{n+1}(e+fx)(a\cot(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

input
```
Int[(a*Cot[e + f*x])^m*Tan[e + f*x]^n,x]
```

output
```
((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Ta
n[e + f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

**Defintions of rubi rules used**

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3084
```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m   Int[(b*Tan[e
+ f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
  !IntegerQ[n]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [F]

$$\int (a\cot(fx+e))^m \tan(fx+e)^n \, dx$$

input
```
int((a*cot(f*x+e))^m*tan(f*x+e)^n,x)
```

output
```
int((a*cot(f*x+e))^m*tan(f*x+e)^n,x)
```

## Fricas [F]

$$\int (a\cot(e+fx))^m\tan^n(e+fx)\,dx=\int (a\cot(fx+e))^m\tan(fx+e)^n\,dx$$

input
```
integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="fricas")
```

output
```
integral((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

## Sympy [F]

$$\int (a\cot(e+fx))^m\tan^n(e+fx)\,dx=\int (a\cot(e+fx))^m\tan^n(e+fx)\,dx$$

input
```
integrate((a*cot(f*x+e))**m*tan(f*x+e)**n,x)
```

output
```
Integral((a*cot(e + f*x))**m*tan(e + f*x)**n, x)
```

## Maxima [F]

$$\int (a\cot(e+fx))^m\tan^n(e+fx)\,dx=\int (a\cot(fx+e))^m\tan(fx+e)^n\,dx$$

input
```
integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="maxima")
```

output
```
integrate((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

## Giac [F]

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \int (a\cot(fx+e))^m \tan(fx+e)^n\,dx$$

input
```
integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="giac")
```

output
```
integrate((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \int \tan(e+f\,x)^n\,(a\cot(e+f\,x))^m\,dx$$

input
```
int(tan(e + f*x)^n*(a*cot(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^n*(a*cot(e + f*x))^m, x)
```

## Reduce [F]

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = a^m\left(\int \tan(fx+e)^n \cot(fx+e)^m\,dx\right)$$

input
```
int((a*cot(f*x+e))^m*tan(f*x+e)^n,x)
```

output
```
a**m*int(tan(e + f*x)**n*cot(e + f*x)**m,x)
```

# 3.225 $\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1729
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . 1729
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1730
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1731
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1732
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1732
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1732
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1733
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1733
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1733

## Optimal result

Integrand size = 21, antiderivative size = 69

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{(a\cot(e+fx))^m \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)(b\tan(e+fx))^{1}}{bf(1-m+n)}$$

output
```
(a*cot(f*x+e))^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x
+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1-m+n)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{a(a\cot(e+fx))^{-1+m} \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)(b\tan(e+f}{f(1-m+n)}$$

input
```
Integrate[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
(a*(a*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m +
n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - m + n))
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3084, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3084}$$

$$(a\cot(e+fx))^m(b\tan(e+fx))^m\int (b\tan(e+fx))^{n-m}dx$$

$$\Big\downarrow \text{3042}$$

$$(a\cot(e+fx))^m(b\tan(e+fx))^m\int (b\tan(e+fx))^{n-m}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b(a\cot(e+fx))^m(b\tan(e+fx))^m\int \frac{(b\tan(e+fx))^{n-m}}{\tan^2(e+fx)b^2+b^2}d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(a\cot(e+fx))^m(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),-\tan^2(e+fx)\right)}{bf(-m+n+1)}$$

input
```
Int[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Ta
n[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - m + n))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3084
```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m   Int[(b*Tan[e
+ f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
  !IntegerQ[n]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [F]

$$\int (a\cot(fx+e))^m (b\tan(fx+e))^n\,dx$$

input
```
int((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
int((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\cot(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Sympy [F]

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\cot(e+fx))^m\,(b\tan(e+fx))^n\,dx$$

input
```
integrate((a*cot(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((a*cot(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

## Maxima [F]

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\cot(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Giac [F]

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\cot(fx+e))^m(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\cot(e+f\,x))^m\,(b\tan(e+f\,x))^n\,dx$$

input
```
int((a*cot(e + f*x))^m*(b*tan(e + f*x))^n,x)
```

output
```
int((a*cot(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

## Reduce [F]

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx=b^na^m\left(\int \tan(fx+e)^n\cot(fx+e)^m\,dx\right)$$

input
```
int((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
b**n*a**m*int(tan(e + f*x)**n*cot(e + f*x)**m,x)
```

# 3.226 $\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result . . . 1734
Mathematica [A] (verified) . . . 1734
Rubi [A] (verified) . . . 1735
Maple [A] (verified) . . . 1736
Fricas [A] (verification not implemented) . . . 1737
Sympy [F] . . . 1737
Maxima [A] (verification not implemented) . . . 1737
Giac [A] (verification not implemented) . . . 1738
Mupad [B] (verification not implemented) . . . 1738
Reduce [F] . . . 1739

## Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx=\frac{2(d\tan(e+fx))^{3/2}}{3df}+\frac{4(d\tan(e+fx))^{7/2}}{7d^3f}+\frac{2(d\tan(e+fx))^{11/2}}{11d^5f}$$

output
```
2/3*(d*tan(f*x+e))^(3/2)/d/f+4/7*(d*tan(f*x+e))^(7/2)/d^3/f+2/11*(d*tan(f*
x+e))^(11/2)/d^5/f
```

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx=\frac{2(45+28\cos(2(e+fx))+4\cos(4(e+fx)))\sec^4(e+fx)(d\tan(e+fx))^{3/2}}{231df}$$

input
```
Integrate[Sec[e + f*x]^6*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(45 + 28*Cos[2*(e + f*x)] + 4*Cos[4*(e + f*x)])*Sec[e + f*x]^4*(d*Tan[e
 + f*x])^(3/2))/(231*d*f)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^6\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \sqrt{d\tan(e+fx)}\left(\tan^2(e+fx)+1\right)^2 d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(e+fx))^{9/2}}{d^4}+\frac{2(d\tan(e+fx))^{5/2}}{d^2}+\sqrt{d\tan(e+fx)}\right) d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(e+fx))^{11/2}}{11d^5}+\frac{4(d\tan(e+fx))^{7/2}}{7d^3}+\frac{2(d\tan(e+fx))^{3/2}}{3d}}{f}$$

input
```
Int[Sec[e + f*x]^6*Sqrt[d*Tan[e + f*x]],x]
```

output
```
((2*(d*Tan[e + f*x])^(3/2))/(3*d) + (4*(d*Tan[e + f*x])^(7/2))/(7*d^3) + (
2*(d*Tan[e + f*x])^(11/2))/(11*d^5))/f
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{4d^2(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^4(d\tan(fx+e))^{\frac{3}{2}}}{3}}{f\,d^5}$ | 52 |
| default | $\frac{\frac{2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{4d^2(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^4(d\tan(fx+e))^{\frac{3}{2}}}{3}}{f\,d^5}$ | 52 |

input
```
int(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/d^5*(1/11*(d*tan(f*x+e))^(11/2)+2/7*d^2*(d*tan(f*x+e))^(7/2)+1/3*d^4*(
d*tan(f*x+e))^(3/2))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\left(32\cos\left(fx+e\right)^4+24\cos\left(fx+e\right)^2+21\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{231\,f\cos\left(fx+e\right)^5}$$

input
```
integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/231*(32*cos(f*x + e)^4 + 24*cos(f*x + e)^2 + 21)*sqrt(d*sin(f*x + e)/cos
(f*x + e))*sin(f*x + e)/(f*cos(f*x + e)^5)
```

## Sympy [F]

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(e+fx\right)}\sec^6\left(e+fx\right)\,dx$$

input
```
integrate(sec(f*x+e)**6*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**6, x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\left(21\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}+66\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^2+77\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}d^4\right)}{231\,d^5f}$$

input
```
integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
2/231*(21*(d*tan(f*x + e))^(11/2) + 66*(d*tan(f*x + e))^(7/2)*d^2 + 77*(d*
tan(f*x + e))^(3/2)*d^4)/(d^5*f)
```

## Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$=\frac{2\left(21\sqrt{d\tan(fx+e)}d^5\tan(fx+e)^5+66\sqrt{d\tan(fx+e)}d^5\tan(fx+e)^3+77\sqrt{d\tan(fx+e)}d^5\tan\right.}{231\,d^5f}$$

input
```
integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
2/231*(21*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e)^5 + 66*sqrt(d*tan(f*x + e)
)*d^5*tan(f*x + e)^3 + 77*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e))/(d^5*f)
```

## Mupad [B] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.99

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{231\,f}-\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{231\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)}$$
$$-\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,32i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^2}+\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,768i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^3}$$
$$-\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,160i}{11\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^4}+\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{11\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^5}$$

input
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^6,x)
```

output
```
((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*768i)/
(77*f*(exp(e*2i + f*x*2i) + 1)^3) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(e
xp(e*2i + f*x*2i) + 1))^(1/2)*64i)/(231*f*(exp(e*2i + f*x*2i) + 1)) - ((-(
d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*32i)/(77*f
*(exp(e*2i + f*x*2i) + 1)^2) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*
2i + f*x*2i) + 1))^(1/2)*64i)/(231*f) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i)
)/(exp(e*2i + f*x*2i) + 1))^(1/2)*160i)/(11*f*(exp(e*2i + f*x*2i) + 1)^4)
+ ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*64i)
/(11*f*(exp(e*2i + f*x*2i) + 1)^5)
```

## Reduce [F]

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx=\sqrt{d}\left(\int \sqrt{\tan\left(fx+e\right)}\,\sec\left(fx+e\right)^6dx\right)$$

input
```
int(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*sec(e + f*x)**6,x)
```

# 3.227 $\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result . . . . . 1740
Mathematica [A] (verified) . . . . . 1740
Rubi [A] (verified) . . . . . 1741
Maple [A] (verified) . . . . . 1742
Fricas [A] (verification not implemented) . . . . . 1743
Sympy [F] . . . . . 1743
Maxima [A] (verification not implemented) . . . . . 1743
Giac [A] (verification not implemented) . . . . . 1744
Mupad [B] (verification not implemented) . . . . . 1744
Reduce [F] . . . . . 1745

## Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(d\tan(e+fx))^{3/2}}{3df}+\frac{2(d\tan(e+fx))^{7/2}}{7d^3f}$$

output
```
2/3*(d*tan(f*x+e))^(3/2)/d/f+2/7*(d*tan(f*x+e))^(7/2)/d^3/f
```

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(4+3\sec^2(e+fx))\,(d\tan(e+fx))^{3/2}}{21df}$$

input
```
Integrate[Sec[e + f*x]^4*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(4 + 3*Sec[e + f*x]^2)*(d*Tan[e + f*x])^(3/2))/(21*d*f)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^4\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \sqrt{d\tan(e+fx)}\left(\tan^2(e+fx)+1\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(e+fx))^{5/2}}{d^2}+\sqrt{d\tan(e+fx)}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(e+fx))^{7/2}}{7d^3}+\frac{2(d\tan(e+fx))^{3/2}}{3d}}{f}$$

input
```
Int[Sec[e + f*x]^4*Sqrt[d*Tan[e + f*x]],x]
```

output
```
((2*(d*Tan[e + f*x])^(3/2))/(3*d) + (2*(d*Tan[e + f*x])^(7/2))/(7*d^3))/f
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^2(d\tan(fx+e))^{\frac{3}{2}}}{3}}{f\,d^3}$ | 37 |
| default | $\frac{\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^2(d\tan(fx+e))^{\frac{3}{2}}}{3}}{f\,d^3}$ | 37 |

input
```
int(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/d^3*(1/7*(d*tan(f*x+e))^(7/2)+1/3*d^2*(d*tan(f*x+e))^(3/2))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\,\left(4\,\cos\left(fx+e\right)^2+3\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\,\sin\left(fx+e\right)}{21\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/21*(4*cos(f*x + e)^2 + 3)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)
/(f*cos(f*x + e)^3)
```

## Sympy [F]

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(e+fx\right)}\sec^4\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**4*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\left(3\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}+7\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}d^2\right)}{21\,d^3f}$$

input
```
integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
2/21*(3*(d*tan(f*x + e))^(7/2) + 7*(d*tan(f*x + e))^(3/2)*d^2)/(d^3*f)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{2\left(3\sqrt{d\tan(fx+e)}d^3\tan(fx+e)^3+7\sqrt{d\tan(fx+e)}d^3\tan(fx+e)\right)}{21\,d^3f}$$

input
```
integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
2/21*(3*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)^3 + 7*sqrt(d*tan(f*x + e))*d
^3*tan(f*x + e))/(d^3*f)
```

## Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 4.84

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,8i}{21\,f}-\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,8i}{21\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)}$$
$$+\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,24i}{7\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^2}-\frac{\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,16i}{7\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^3}$$

input
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^4,x)
```

output
```
((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*24i)/(
7*f*(exp(e*2i + f*x*2i) + 1)^2) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp
(e*2i + f*x*2i) + 1))^(1/2)*8i)/(21*f*(exp(e*2i + f*x*2i) + 1)) - ((-(d*(e
xp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*8i)/(21*f) - (
(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*16i)/(7
*f*(exp(e*2i + f*x*2i) + 1)^3)
```

## Reduce [F]

$$\int \sec^4(e + fx)\sqrt{d\tan(e + fx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(fx + e\right)}\,\sec\left(fx + e\right)^4 dx\right)$$

input
```
int(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*sec(e + f*x)**4,x)
```

# 3.228 $\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1746
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1746
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1747
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1748
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1748
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1749
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 1749
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 1749
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 1750
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1750

## Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(d\tan(e+fx))^{3/2}}{3df}$$

output
```
2/3*(d*tan(f*x+e))^(3/2)/d/f
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(d\tan(e+fx))^{3/2}}{3df}$$

input
```
Integrate[Sec[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(e+fx)^2\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \; 3087$$

$$\frac{\int \sqrt{d\tan(e+fx)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 17$$

$$\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

input
```
Int[Sec[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2(d\tan(fx+e))^{\frac{3}{2}}}{3df}$ | 19 |
| default | $\frac{2(d\tan(fx+e))^{\frac{3}{2}}}{3df}$ | 19 |

input
```
int(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*tan(f*x+e))^(3/2)/d/f
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin(fx+e)}{3\,f\cos(fx+e)}$$

input
```
integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e))
```

## Sympy [F]

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan{(e+fx)}}\sec^2{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**2*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\,(d\tan{(fx+e)})^{\frac{3}{2}}}{3\,df}$$

input
```
integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(d*tan(f*x + e))^(3/2)/(d*f)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\,\sqrt{d\tan{(fx+e)}}\tan{(fx+e)}}{3\,f}$$

input
```
integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(d*tan(f*x + e))*tan(f*x + e)/f
```

## Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\,\sin\left(2\,e+2\,f\,x\right)\,\sqrt{\frac{d\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

input
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^2,x)
```

output
```
(2*sin(2*e + 2*f*x)*((d*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(
3*f*(cos(2*e + 2*f*x) + 1))
```

## Reduce [F]

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(fx+e\right)}\,\sec\left(fx+e\right)^2 dx\right)$$

input
```
int(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*sec(e + f*x)**2,x)
```

# 3.229 $\int \sqrt{d\tan(e+fx)}\,dx$


Optimal result . . . 1751
Mathematica [A] (verified) . . . 1752
Rubi [A] (warning: unable to verify) . . . 1752
Maple [A] (verified) . . . 1756
Fricas [A] (verification not implemented) . . . 1756
Sympy [F] . . . 1757
Maxima [A] (verification not implemented) . . . 1757
Giac [A] (verification not implemented) . . . 1758
Mupad [B] (verification not implemented) . . . 1758
Reduce [F] . . . 1759


## Optimal result

Integrand size = 12, antiderivative size = 137

$$\int \sqrt{d\tan(e+fx)}\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}+\sqrt{d}\tan(e+fx)}\right)}{\sqrt{2}f}$$

output
```
-1/2*d^(1/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*
d^(1/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d^(1/
2)*arctanh(2^(1/2)*(d*tan(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*tan(f*x+e)))*2^(1
/2)/f
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int \sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{\left(\arctan\left(\sqrt[4]{-\tan^2(e+fx)}\right) - \operatorname{arctanh}\left(\sqrt[4]{-\tan^2(e+fx)}\right)\right)\sqrt[4]{-\tan(e+fx)}\sqrt{d\tan(e+fx)}}{f\tan^{\frac{3}{4}}(e+fx)}$$

input
```
Integrate[Sqrt[d*Tan[e + f*x]],x]
```

output
```
((ArcTan[(-Tan[e + f*x]^2)^(1/4)] - ArcTanh[(-Tan[e + f*x]^2)^(1/4)])*(-Ta
n[e + f*x])^(1/4)*Sqrt[d*Tan[e + f*x]])/(f*Tan[e + f*x]^(3/4))
```

## Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\tan(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{d\tan(e+fx)}dx$$
$$\Big\downarrow \text{3957}$$
$$\frac{d\int \frac{\sqrt{d\tan(e+fx)}}{\tan^2(e+fx)d^2+d^2}d(d\tan(e+fx))}{f}$$
$$\Big\downarrow \text{266}$$

$$\frac{2d\int \frac{d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}}{f}$$

$\downarrow$ 826

$$\frac{2d\left(\frac{1}{2}\int \frac{d^2\tan^2(e+fx)+d}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}\right)}{f}$$

$\downarrow$ 1476

$$\frac{2d\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}\right.\right.}{f}$$

$\downarrow$ 1082

$$\frac{2d\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\tan(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan\right.}{f}$$

$\downarrow$ 217

$$\frac{2d\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}\right)}{f}$$

$\downarrow$ 1479

$$\frac{2d\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}{}\right.\right.}{f}$$

$\downarrow$ 25

$$\frac{2d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}{}\right.\right.}{f}$$

$\downarrow$ 27

$$\frac{2d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.}{f}$$

$\downarrow$ 1103

$$\frac{2d\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(e+fx)+d^2\tan^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d\right.}{}\right.}{f}$$

input
```
Int[Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*d*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[e + f*x]]/(Sqrt[2]*Sqrt[d])) + Arc
Tan[1 + Sqrt[2]*Sqrt[d]*Tan[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt
[2]*d^(3/2)*Tan[e + f*x] + d^2*Tan[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d
 + Sqrt[2]*d^(3/2)*Tan[e + f*x] + d^2*Tan[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))
/2))/f
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| derivativedivides | $\frac{d\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{d\tan(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}\right.\right.}{4f(d^2)^{\frac{1}{4}}}$ |
| default | $\frac{d\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{d\tan(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}\right.\right.}{4f(d^2)^{\frac{1}{4}}}$ |

input
```
int((d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*d/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(
1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2
^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-
2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.05

$$\int \sqrt{d\tan(e+fx)}\,dx = \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d}{d}\right)+2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}-d}{d}\right)-\sqrt{2}\sqrt{d}\log\left(d\tan\left(fx+e\right.\right.}{4f}$$

input
```
integrate((d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/d
) + 2*sqrt(2)*sqrt(d)*arctan((sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) - d)/d)
 - sqrt(2)*sqrt(d)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(
d) + d) + sqrt(2)*sqrt(d)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e)
)*sqrt(d) + d))/f
```

## Sympy [F]

$$\int \sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan{(e+fx)}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int \sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{\cdots}\right.}{\sqrt{d}}\right.}{4f}$$

input
```
integrate((d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x +
e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2
*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) + sqr
t(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x + e
) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d))/f
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.28

$$\int \sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} + \frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} - \frac{\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\,\mathrm{ta}}\right.}{f}}{4d}$$

input
```
integrate((d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*s
qrt(d*tan(f*x + e)))/sqrt(abs(d)))/f + 2*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f -
sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqr
t(abs(d)) + abs(d))/f + sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) - sqrt(2)*
sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f)/d
```

## Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.36

$$\int \sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{(-1)^{1/4}\sqrt{d}\left(\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right) - \mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)\right)}{f}$$

input
```
int((d*tan(e + f*x))^(1/2),x)
```

output
```
((-1)^(1/4)*d^(1/2)*(atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)) - a
tanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))))/f
```

## Reduce [F]

$$\int \sqrt{d\tan(e+fx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(fx+e\right)}dx\right)$$

input
```
int((d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x)),x)
```

# 3.230 $\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result . . . 1760
Mathematica [A] (verified) . . . 1761
Rubi [A] (verified) . . . 1761
Maple [B] (verified) . . . 1765
Fricas [B] (verification not implemented) . . . 1766
Sympy [F] . . . 1767
Maxima [A] (verification not implemented) . . . 1767
Giac [A] (verification not implemented) . . . 1768
Mupad [F(-1)] . . . 1768
Reduce [F] . . . 1769

## Optimal result

Integrand size = 21, antiderivative size = 169

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} - \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}(1+\tan(e+fx))}\right)}{4\sqrt{2}f} + \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df}$$

output
```
-1/8*d^(1/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/8*
d^(1/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/8*d^(1/
2)*arctanh(2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2)/(1+tan(f*x+e)))*2^(1/2)/f+
1/2*cos(f*x+e)^2*(d*tan(f*x+e))^(3/2)/d/f
```

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx =$$

$$-\frac{\left(\arcsin(\cos(e+fx)-\sin(e+fx))\csc(e+fx)+\csc(e+fx)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin\right.\right.}{8f}$$

input
```
Integrate[Cos[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

output
```
-1/8*((ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x] + Csc[e + f*x]*Log
[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]] - 2*Sqrt[Sin[2*(e +
 f*x)]])*Sqrt[Sin[2*(e + f*x)]]*Sqrt[d*Tan[e + f*x]])/f
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.33, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3087, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)^2}dx$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{\int \frac{\sqrt{d\tan(e+fx)}}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{\frac{1}{4}\int \frac{\sqrt{d\tan(e+fx)}}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{\int \frac{d^3\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}}{2d}+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}d\int \frac{d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \; 826$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\int \frac{\tan(e+fx)d+d}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}\right)+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}+\frac{1}{2}\int \frac{1}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}\right.\right.}{f}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d\tan(e+fx)-1}d\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d\tan(e+fx)-1}d\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e-\right.}{f}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}\right)+\frac{(d\tan(e+}{2d(\tan^2(e+}}{f}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{\sqrt{}}\right.}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{\sqrt{}}\right.}{f}$$

$$\downarrow 1103$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(d\tan(e+fx)-\sqrt{2}\sqrt{d}\sqrt{d\tan(e+fx)}+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(d\tan(\right.}{}\right.}{f}$$

input
```
Int[Cos[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

output
```
((d*((-(ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d
])) + ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d])
)/2 + (Log[d + d*Tan[e + f*x] - Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[e + f*x]]]/(2*S
qrt[2]*Sqrt[d]) - Log[d + d*Tan[e + f*x] + Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[e +
f*x]]]/(2*Sqrt[2]*Sqrt[d]))/2))/2 + (d*Tan[e + f*x])^(3/2)/(2*d*(1 + Tan[e
 + f*x]^2)))/f
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(129) = 258$.

Time = 1.19 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.38

| method | result |
|---|---|
| default | $\left(-\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)+2\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+\csc(fx+e)-\sin(fx+e)-2\cos(fx+e)+2}{\cos(fx+e)-1}\right)+\ln\left(\frac{2\sin(fx+\right.\right.$ |

input
```
int(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f*(-ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-2*sin(f*x+
e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*cos(f*x+e)+2
)/(cos(f*x+e)-1))+ln((2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e)
)^2)^(1/2)-cot(f*x+e)*cos(f*x+e)+sin(f*x+e)+2*cos(f*x+e)+2*cot(f*x+e)-csc(
f*x+e)-2)/(cos(f*x+e)-1))+2*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)-1))+2*arctan((sin(f*x+e)*
(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos(f*x+e
)-1))+(4+4*cos(f*x+e))*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2))*cos(f*x+e)*(d*tan(f*x+e))^(1/2)/(1+cos(f*x+e))/(-sin(f*x+e)*cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(129) = 258$.

Time = 0.15 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.60

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{16\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) - 2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{d\cos(fx+e)-d\sin(fx+e)}\right) - \sqrt{2}\sqrt{d}\arctan\left(\right.}{}$$

input
```
integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/32*(16*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 2*s
qrt(2)*sqrt(d)*arctan(-sqrt(2)*sqrt(d)*sqrt(d*sin(f*x + e)/cos(f*x + e))*c
os(f*x + e)/(d*cos(f*x + e) - d*sin(f*x + e))) - sqrt(2)*sqrt(d)*arctan(1/
2*(2*d*cos(f*x + e)^2 - 2*d*cos(f*x + e)*sin(f*x + e) + sqrt(2)*sqrt(d)*sq
rt(d*sin(f*x + e)/cos(f*x + e)) - 2*d)/(d*cos(f*x + e)^2 + d*cos(f*x + e)*
sin(f*x + e) - d)) - sqrt(2)*sqrt(d)*arctan(-1/2*(2*d*cos(f*x + e)^2 - 2*d
*cos(f*x + e)*sin(f*x + e) - sqrt(2)*sqrt(d)*sqrt(d*sin(f*x + e)/cos(f*x +
 e)) - 2*d)/(d*cos(f*x + e)^2 + d*cos(f*x + e)*sin(f*x + e) - d)) - sqrt(2
)*sqrt(d)*log(4*d*cos(f*x + e)*sin(f*x + e) + 2*sqrt(2)*(cos(f*x + e)^2 +
cos(f*x + e)*sin(f*x + e))*sqrt(d)*sqrt(d*sin(f*x + e)/cos(f*x + e)) + d)
+ sqrt(2)*sqrt(d)*log(4*d*cos(f*x + e)*sin(f*x + e) - 2*sqrt(2)*(cos(f*x +
 e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(d)*sqrt(d*sin(f*x + e)/cos(f*x + e
)) + d))/f
```

## Sympy [F]

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(e+fx)}\cos^2(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**2*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*cos(e + f*x)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.14

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\right.}{\sqrt{d}}}{16\,df}$$

input
```
integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*
x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) +
 sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x
 + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) + 8*(d*tan(f*x
+ e))^(3/2)*d^2/(d^2*tan(f*x + e)^2 + d^2))/(d*f)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.29

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{\frac{8\sqrt{d\tan(fx+e)}d^3\tan(fx+e)}{\left(d^2\tan(fx+e)^2+d^2\right)f} + \frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} + \frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} - }{16\,d}$$

input

```
integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output

```
1/16*(8*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)/((d^2*tan(f*x + e)^2 + d^2)*
f) + 2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*s
qrt(d*tan(f*x + e)))/sqrt(abs(d)))/f + 2*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f -
sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqr
t(abs(d)) + abs(d))/f + sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) - sqrt(2)*
sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f)/d
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \cos\left(e+f\,x\right)^2\,\sqrt{d\tan\left(e+f\,x\right)}\,dx$$

input

```
int(cos(e + f*x)^2*(d*tan(e + f*x))^(1/2),x)
```

output

```
int(cos(e + f*x)^2*(d*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx=\sqrt{d}\left(\int \sqrt{\tan(fx+e)}\,\cos(fx+e)^2\,dx\right)$$

input
```
int(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*cos(e + f*x)**2,x)
```

# 3.231 $\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result . . . 1770
Mathematica [C] (verified) . . . 1770
Rubi [A] (verified) . . . 1771
Maple [B] (verified) . . . 1774
Fricas [C] (verification not implemented) . . . 1774
Sympy [F] . . . 1775
Maxima [F] . . . 1775
Giac [F] . . . 1776
Mupad [F(-1)] . . . 1776
Reduce [F] . . . 1776

## Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{4\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{5f\sqrt{\sin(2e+2fx)}} + \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

output
```
4/5*cos(f*x+e)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f
/sin(2*f*x+2*e)^(1/2)+4/5*cos(f*x+e)*(d*tan(f*x+e))^(3/2)/d/f+2/5*sec(f*x+
e)*(d*tan(f*x+e))^(3/2)/d/f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.46 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\sqrt{d\tan(e+fx)}\left(-4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(e+fx)\right)\sec(e+fx)\tan(e+fx)+3\sqrt{\sec^2(\right.}{15f\sqrt{\sec^2(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*Sqrt[d*Tan[e + f*x]]*(-4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]
^2]*Sec[e + f*x]*Tan[e + f*x] + 3*Sqrt[Sec[e + f*x]^2]*(2*Sin[e + f*x] + S
ec[e + f*x]*Tan[e + f*x])))/(15*f*Sqrt[Sec[e + f*x]^2])
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3093, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^3\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{2}{5}\int \sec(e+fx)\sqrt{d\tan(e+fx)}dx+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \sec(e+fx)\sqrt{d\tan(e+fx)}dx+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3093}$$

$$\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-2\int \cos(e+fx)\sqrt{d\tan(e+fx)}dx\right)+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-2\int\frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)}dx\right)+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3095}$$

$$\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}}\right)+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}}\right)+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)\sqrt{d\tan(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}}\right)+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)\sqrt{d\tan(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}}\right)+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}\right)$$

input
```
Int[Sec[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*Sec[e + f*x]*(d*Tan[e + f*x])^(3/2))/(5*d*f) + (2*((-2*Cos[e + f*x]*Ell
ipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqrt[Sin[2*e + 2*f*x]])
 + (2*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2))/(d*f)))/5
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(94) = 188$.

Time = 1.34 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.76

| method | result |
|---|---|
| default | $\frac{\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}\,\sqrt{d\tan(fx+e)}\left(\sqrt{2\cot(fx+e)-2\csc(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \mathrm{EllipticE}\left(\sqrt{\csc(fx+e)-\cot\right.\right.}{}$ |

input
```
int(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/f*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(d*tan(f*x+e))^(1/
2)/(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*((2*cot(f*x+e)-2*csc(f*
x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticE((csc(f*x+e)-cot(f*x
+e)+1)^(1/2),1/2*2^(1/2))*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(2*cot(f*x+e)+2*
csc(f*x+e))+(2*cot(f*x+e)-2*csc(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(
1/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(csc(f*x+e)-co
t(f*x+e)+1)^(1/2)*(-cot(f*x+e)-csc(f*x+e))-2*cot(f*x+e)+csc(f*x+e)+sec(f*x
+e)^2*csc(f*x+e))*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.72

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx =$$

$$-\frac{2\left(i\sqrt{i\,d}\cos\left(fx+e\right)^2 E(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\mid -1) - i\sqrt{-i\,d}\cos\left(fx+e\right)^2 E(\arcsin\right.}{}$$

input
```
integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(I*sqrt(I*d)*cos(f*x + e)^2*elliptic_e(arcsin(cos(f*x + e) + I*sin(f*
x + e)), -1) - I*sqrt(-I*d)*cos(f*x + e)^2*elliptic_e(arcsin(cos(f*x + e)
- I*sin(f*x + e)), -1) - I*sqrt(I*d)*cos(f*x + e)^2*elliptic_f(arcsin(cos(
f*x + e) + I*sin(f*x + e)), -1) + I*sqrt(-I*d)*cos(f*x + e)^2*elliptic_f(a
rcsin(cos(f*x + e) - I*sin(f*x + e)), -1) - (2*cos(f*x + e)^2 + 1)*sqrt(d*
sin(f*x + e)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2)
```

## Sympy [F]

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx=\int \sqrt{d\tan\left(e+fx\right)}\sec^3\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**3*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**3, x)
```

## Maxima [F]

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx=\int \sqrt{d\tan\left(fx+e\right)}\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))*sec(f*x + e)^3, x)
```

## Giac [F]

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))*sec(f*x + e)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{\cos\left(e+f\,x\right)^3}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^3,x)
```

output
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^3, x)
```

## Reduce [F]

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(fx+e\right)}\,\sec\left(fx+e\right)^3 dx\right)$$

input
```
int(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*sec(e + f*x)**3,x)
```

# 3.232 $\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$


Optimal result . . . 1777
Mathematica [C] (verified) . . . 1777
Rubi [A] (verified) . . . 1778
Maple [B] (verified) . . . 1780
Fricas [C] (verification not implemented) . . . 1781
Sympy [F] . . . 1781
Maxima [F] . . . 1782
Giac [F] . . . 1782
Mupad [F(-1)] . . . 1782
Reduce [F] . . . 1783


## Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{2\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}} + \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}$$

output
```
2*cos(f*x+e)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/s
in(2*f*x+2*e)^(1/2)+2*cos(f*x+e)*(d*tan(f*x+e))^(3/2)/d/f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{2\left(-3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\right)\sin(e+fx)\sqrt{d\tan(e+fx)}}{3f}$$

input
```
Integrate[Sec[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(-2*(-3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e +
 f*x]^2])*Sin[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*f)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-2\int \cos(e+fx)\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-2\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)}dx$$

$$\Big\downarrow \text{3095}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)\sqrt{d\tan(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)\sqrt{d\tan(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

input
```
Int[Sec[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(-2*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqr
t[Sin[2*e + 2*f*x]]) + (2*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2))/(d*f)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(70) = 140$.

Time = 1.14 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.55

| method | result |
|---|---|
| default | $\frac{\csc(fx+e)\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{2\cot(fx+e)-2\csc(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\,(-\cos(fx+e)-1)\,\mathrm{EllipticF}\left(\sqrt{\right.\right.}{}$ |

input
```
int(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*csc(f*x+e)*((csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(2*cot(f*x+e)-2*csc(f*x+
e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*(-cos(f*x+e)-1)*EllipticF((csc(
f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))+2*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*
(2*cot(f*x+e)-2*csc(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*(1+cos(
f*x+e))*EllipticE((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))-2*cos(f*x+e
)+2)*(d*tan(f*x+e))^(1/2)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)/(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.75

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{-i\sqrt{i\,d}E(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\,|-1)+i\sqrt{-i\,d}E(\arcsin\left(\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)}{}$$

input
```
integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(I*d)*elliptic_e(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) + I*sq
rt(-I*d)*elliptic_e(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1) + I*sqrt(I*
d)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) - I*sqrt(-I*d)*el
liptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1) + 2*sqrt(d*sin(f*x + e
)/cos(f*x + e))*sin(f*x + e))/f
```

## Sympy [F]

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(e+fx\right)}\sec\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*sec(e + f*x), x)
```

## Maxima [F]

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))*sec(f*x + e), x)
```

## Giac [F]

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))*sec(f*x + e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x),x)
```

output
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x), x)
```

## Reduce [F]

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan(fx+e)}\,\sec(fx+e)\,dx\right)$$

input
```
int(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*sec(e + f*x),x)
```

# 3.233 $\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$


Optimal result . . . 1784
Mathematica [C] (verified) . . . 1784
Rubi [A] (verified) . . . 1785
Maple [B] (verified) . . . 1786
Fricas [F] . . . 1787
Sympy [F] . . . 1787
Maxima [F] . . . 1788
Giac [F] . . . 1788
Mupad [F(-1)] . . . 1788
Reduce [F] . . . 1789


## Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx=\frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

output
```
-cos(f*x+e)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/si
n(2*f*x+2*e)^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$=\frac{2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\sin(e+fx)\sqrt{d\tan(e+fx)}}{3f}$$

input
```
Integrate[Cos[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^2]*
Sin[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*f)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)}dx$$

$$\Big\downarrow \text{3095}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

input
```
Int[Cos[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqrt[S
in[2*e + 2*f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(45) = 90$.

Time = 0.85 (sec) , antiderivative size = 229, normalized size of antiderivative = 4.87

| method | result |
|---|---|
| default | $\frac{\left((1-\cos(fx+e))\cot(fx+e)-\frac{\sqrt{2\cot(fx+e)-2\csc(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \text{EllipticE}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\csc(fx-}}{2}\right.}{}$ |

input
```
int(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((1-cos(f*x+e))*cot(f*x+e)-1/2*(2*cot(f*x+e)-2*csc(f*x+e)+2)^(1/2)*(-c
sc(f*x+e)+cot(f*x+e))^(1/2)*EllipticE((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*
2^(1/2))*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(2*cot(f*x+e)+2*csc(f*x+e))-1/2*(
2*cot(f*x+e)-2*csc(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*Elliptic
F((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(csc(f*x+e)-cot(f*x+e)+1)^(
1/2)*(-cot(f*x+e)-csc(f*x+e)))*(d*tan(f*x+e))^(1/2)
```

## Fricas [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

## Sympy [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(e+fx\right)}\cos\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*cos(e + f*x), x)
```

## Maxima [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

## Giac [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \cos(e+f\,x)\,\sqrt{d\tan(e+f\,x)}\,dx$$

input
```
int(cos(e + f*x)*(d*tan(e + f*x))^(1/2),x)
```

output
```
int(cos(e + f*x)*(d*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx=\sqrt{d}\left(\int\sqrt{\tan\left(fx+e\right)}\,\cos\left(fx+e\right)dx\right)$$

input
```
int(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*cos(e + f*x),x)
```

# 3.234 $\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result . . . 1790
Mathematica [C] (verified) . . . 1790
Rubi [A] (verified) . . . 1791
Maple [B] (verified) . . . 1793
Fricas [F] . . . 1793
Sympy [F(-1)] . . . 1794
Maxima [F] . . . 1794
Giac [F(-2)] . . . 1794
Mupad [F(-1)] . . . 1795
Reduce [F] . . . 1795

## Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}} + \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

output
```
-1/2*cos(f*x+e)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/
f/sin(2*f*x+2*e)^(1/2)+1/3*cos(f*x+e)^3*(d*tan(f*x+e))^(3/2)/d/f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{\sqrt{d\tan(e+fx)}\left(\sqrt{\sec^2(e+fx)}(\sin(e+fx)+\sin(3(e+fx)))+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2\right.\right.}{12f\sqrt{\sec^2(e+fx)}}$$

input
```
Integrate[Cos[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Sqrt[d*Tan[e + f*x]]*(Sqrt[Sec[e + f*x]^2]*(Sin[e + f*x] + Sin[3*(e + f*x
)]) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sec[e + f*x]*Tan
[e + f*x]))/(12*f*Sqrt[Sec[e + f*x]^2])
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3092, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)^3}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{1}{2}\int \cos(e+fx)\sqrt{d\tan(e+fx)}dx+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)}dx+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3095}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{2\sqrt{\sin(2e+2fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{2\sqrt{\sin(2e+2fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}+\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}}$$

input
```
Int[Cos[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(2*f*Sqrt
[Sin[2*e + 2*f*x]]) + (Cos[e + f*x]^3*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(72) = 144$.

Time = 1.10 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.94

| method | result |
|---|---|
| default | $-\frac{\sqrt{d\tan(fx+e)}\left(\sqrt{2\cot(fx+e)-2\csc(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \mathrm{EllipticE}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\csc(}\right.}{}$ |

input
```
int(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/f*(d*tan(f*x+e))^(1/2)*((2*cot(f*x+e)-2*csc(f*x+e)+2)^(1/2)*(-csc(f*
x+e)+cot(f*x+e))^(1/2)*EllipticE((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/
2))*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(6*cot(f*x+e)+6*csc(f*x+e))+(2*cot(f*x
+e)-2*csc(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticF((csc(f*
x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(-3*
cot(f*x+e)-3*csc(f*x+e))+cot(f*x+e)*(4*cos(f*x+e)^3+2*cos(f*x+e)-6))
```

## Fricas [F]

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx=\int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^3, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(d*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx=\int \cos\left(e+f\,x\right)^3\,\sqrt{d\tan\left(e+f\,x\right)}\,dx$$

input
```
int(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2),x)
```

output
```
int(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx=\sqrt{d}\left(\int \sqrt{\tan\left(fx+e\right)}\,\cos\left(fx+e\right)^3dx\right)$$

input
```
int(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*cos(e + f*x)**3,x)
```

# 3.235 $\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal result . . . 1796
Mathematica [C] (verified) . . . 1796
Rubi [A] (verified) . . . 1797
Maple [B] (verified) . . . 1800
Fricas [F] . . . 1800
Sympy [F(-1)] . . . 1801
Maxima [F] . . . 1801
Giac [F] . . . 1801
Mupad [F(-1)] . . . 1802
Reduce [F] . . . 1802

## Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{7\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{20f\sqrt{\sin(2e+2fx)}} + \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df} + \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

output
```
-7/20*cos(f*x+e)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)
/f/sin(2*f*x+2*e)^(1/2)+7/30*cos(f*x+e)^3*(d*tan(f*x+e))^(3/2)/d/f+1/5*cos
(f*x+e)^5*(d*tan(f*x+e))^(3/2)/d/f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.77 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\left(20\sin(2(e+fx))+3\sin(4(e+fx))+28\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\text{t}\right.\right.}{120f}$$

input
```
Integrate[Cos[e + f*x]^5*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Cos[e + f*x]*Sqrt[d*Tan[e + f*x]]*(20*Sin[2*(e + f*x)] + 3*Sin[4*(e + f*x
)] + 28*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x
]^2]*Tan[e + f*x]))/(120*f)
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3092, 3042, 3092, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)^5}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{7}{10}\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}dx+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)^3}dx+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3092}$$

$$\frac{7}{10}\left(\frac{1}{2}\int \cos(e+fx)\sqrt{d\tan(e+fx)}dx+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}\right)+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{1}{2}\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)}dx+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}\right)+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3095}$$

$$\frac{7}{10}\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}\right)+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}\right)+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3052}$$

$$\frac{7}{10}\left(\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{2\sqrt{\sin(2e+2fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}\right)+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{2\sqrt{\sin(2e+2fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}\right)+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{7}{10}\left(\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}+\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}}\right)$$

input
```
Int[Cos[e + f*x]^5*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Cos[e + f*x]^5*(d*Tan[e + f*x])^(3/2))/(5*d*f) + (7*((Cos[e + f*x]*Ellipt
icE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(2*f*Sqrt[Sin[2*e + 2*f*x]])
+ (Cos[e + f*x]^3*(d*Tan[e + f*x])^(3/2))/(3*d*f)))/10
```

**Defintions of rubi rules used**

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(98) = 196$.

Time = 1.48 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.25

| method | result |
|---|---|
| default | $\frac{\left(-\frac{\sqrt{2\cot(fx+e)-2\csc(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\,\operatorname{EllipticF}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,(-21\cot(f}{120}\right.}{}$ |

input
```
int(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/120*(2*cot(f*x+e)-2*csc(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(
1/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(csc(f*x+e)-co
t(f*x+e)+1)^(1/2)*(-21*cot(f*x+e)-21*csc(f*x+e))-1/120*(2*cot(f*x+e)-2*csc
(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*(csc(f*x+e)-cot(f*x+e)+1)^
(1/2)*EllipticE((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(42*cot(f*x+e
)+42*csc(f*x+e))-1/120*cot(f*x+e)*(24*cos(f*x+e)^5+4*cos(f*x+e)^3+14*cos(f
*x+e)-42))*(d*tan(f*x+e))^(1/2)
```

## Fricas [F]

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\cos(fx+e)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(d*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\cos(fx+e)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

## Giac [F]

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\cos(fx+e)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \cos\left(e+f\,x\right)^5\,\sqrt{d\tan\left(e+f\,x\right)}\,dx$$

input
```
int(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2),x)
```

output
```
int(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(fx+e\right)}\,\cos\left(fx+e\right)^5dx\right)$$

input
```
int(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*cos(e + f*x)**5,x)
```

# 3.236 $\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . 1803
Mathematica [A] (verified) . . . 1803
Rubi [A] (verified) . . . 1804
Maple [A] (verified) . . . 1805
Fricas [A] (verification not implemented) . . . 1806
Sympy [F(-1)] . . . 1806
Maxima [A] (verification not implemented) . . . 1806
Giac [A] (verification not implemented) . . . 1807
Mupad [B] (verification not implemented) . . . 1807
Reduce [F] . . . 1808

## Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2(d\tan(a+bx))^{5/2}}{5bd}+\frac{4(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{13/2}}{13bd^5}$$

output
```
2/5*(d*tan(b*x+a))^(5/2)/b/d+4/9*(d*tan(b*x+a))^(9/2)/b/d^3+2/13*(d*tan(b*
x+a))^(13/2)/b/d^5
```

## Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2d\left(-32-8\sec^2(a+bx)-5\sec^4(a+bx)+45\sec^6(a+bx)\right)\sqrt{d\tan(a+bx)}}{585b}$$

input
```
Integrate[Sec[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*(-32 - 8*Sec[a + b*x]^2 - 5*Sec[a + b*x]^4 + 45*Sec[a + b*x]^6)*Sqrt[
d*Tan[a + b*x]])/(585*b)
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^6(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (d\tan(a+bx))^{3/2}\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(a+bx))^{11/2}}{d^4}+\frac{2(d\tan(a+bx))^{7/2}}{d^2}+(d\tan(a+bx))^{3/2}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(a+bx))^{13/2}}{13d^5}+\frac{4(d\tan(a+bx))^{9/2}}{9d^3}+\frac{2(d\tan(a+bx))^{5/2}}{5d}}{b}$$

input
```
Int[Sec[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

output
```
((2*(d*Tan[a + b*x])^(5/2))/(5*d) + (4*(d*Tan[a + b*x])^(9/2))/(9*d^3) + (
2*(d*Tan[a + b*x])^(13/2))/(13*d^5))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{2(d\tan(bx+a))^{\frac{13}{2}}}{13}+\frac{4d^2(d\tan(bx+a))^{\frac{9}{2}}}{9}+\frac{2d^4(d\tan(bx+a))^{\frac{5}{2}}}{5}}{d^5b}$ | 52 |
| default | $\frac{\frac{2(d\tan(bx+a))^{\frac{13}{2}}}{13}+\frac{4d^2(d\tan(bx+a))^{\frac{9}{2}}}{9}+\frac{2d^4(d\tan(bx+a))^{\frac{5}{2}}}{5}}{d^5b}$ | 52 |

input
```
int(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d^5/b*(1/13*(d*tan(b*x+a))^(13/2)+2/9*d^2*(d*tan(b*x+a))^(9/2)+1/5*d^4*(
d*tan(b*x+a))^(5/2))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\left(32\,d\cos\left(bx+a\right)^6+8\,d\cos\left(bx+a\right)^4+5\,d\cos\left(bx+a\right)^2-45\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{585\,b\cos\left(bx+a\right)^6}$$

input
```
integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/585*(32*d*cos(b*x + a)^6 + 8*d*cos(b*x + a)^4 + 5*d*cos(b*x + a)^2 - 45
*d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x + a)^6)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+a)**6*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(45\left(d\tan\left(bx+a\right)\right)^{\frac{13}{2}}+130\left(d\tan\left(bx+a\right)\right)^{\frac{9}{2}}d^2+117\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}d^4\right)}{585\,bd^5}$$

input
```
integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/585*(45*(d*tan(b*x + a))^(13/2) + 130*(d*tan(b*x + a))^(9/2)*d^2 + 117*(
d*tan(b*x + a))^(5/2)*d^4)/(b*d^5)
```

## Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(45\sqrt{d\tan(bx+a)}d^6\tan(bx+a)^6+130\sqrt{d\tan(bx+a)}d^6\tan(bx+a)^4+117\sqrt{d\tan(\dots$$

input
```
integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/585*(45*sqrt(d*tan(b*x + a))*d^6*tan(b*x + a)^6 + 130*sqrt(d*tan(b*x + a
))*d^6*tan(b*x + a)^4 + 117*sqrt(d*tan(b*x + a))*d^6*tan(b*x + a)^2)/(b*d^
5)
```

## Mupad [B] (verification not implemented)

Time = 7.15 (sec) , antiderivative size = 392, normalized size of antiderivative = 5.85

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{64\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{585\,b} - \frac{64\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{585\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)} - \frac{32\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{195\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^2} + \frac{1216\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{117\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^3} - \frac{3488\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{117\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^4} + \frac{384\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{13\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^5} - \frac{128\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{13\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^6}$$

input
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^6,x)
```

output
```
(1216*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)
)/(117*b*(exp(a*2i + b*x*2i) + 1)^3) - (64*d*(-(d*(exp(a*2i + b*x*2i)*1i -
 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(585*b*(exp(a*2i + b*x*2i) + 1)) -
(32*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/
(195*b*(exp(a*2i + b*x*2i) + 1)^2) - (64*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1
i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(585*b) - (3488*d*(-(d*(exp(a*2i + b*
x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(117*b*(exp(a*2i + b*x*2i
) + 1)^4) + (384*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i)
+ 1))^(1/2))/(13*b*(exp(a*2i + b*x*2i) + 1)^5) - (128*d*(-(d*(exp(a*2i + b
*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(13*b*(exp(a*2i + b*x*2i
) + 1)^6)
```

## Reduce [F]

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\sqrt{d}\,d\left(2\sqrt{\tan(bx+a)}\,\sec(bx+a)^6-\left(\int\frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)^6}{\tan(bx+a)}dx\right)b\right)}{13b}$$

input
```
int(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*d*(2*sqrt(tan(a + b*x))*sec(a + b*x)**6 - int((sqrt(tan(a + b*x))
*sec(a + b*x)**6)/tan(a + b*x),x)*b))/(13*b)
```

# 3.237 $\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . 1809
Mathematica [A] (verified) . . . 1809
Rubi [A] (verified) . . . 1810
Maple [A] (verified) . . . 1811
Fricas [A] (verification not implemented) . . . 1812
Sympy [F] . . . 1812
Maxima [A] (verification not implemented) . . . 1812
Giac [A] (verification not implemented) . . . 1813
Mupad [B] (verification not implemented) . . . 1813
Reduce [F] . . . 1814

## Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2(d\tan(a+bx))^{5/2}}{5bd}+\frac{2(d\tan(a+bx))^{9/2}}{9bd^3}$$

output
```
2/5*(d*tan(b*x+a))^(5/2)/b/d+2/9*(d*tan(b*x+a))^(9/2)/b/d^3
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2d(-4-\sec^2(a+bx)+5\sec^4(a+bx))\sqrt{d\tan(a+bx)}}{45b}$$

input
```
Integrate[Sec[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*(-4 - Sec[a + b*x]^2 + 5*Sec[a + b*x]^4)*Sqrt[d*Tan[a + b*x]])/(45*b)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^4(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (d\tan(a+bx))^{3/2}\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(a+bx))^{7/2}}{d^2}+(d\tan(a+bx))^{3/2}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(a+bx))^{9/2}}{9d^3}+\frac{2(d\tan(a+bx))^{5/2}}{5d}}{b}$$

input
```
Int[Sec[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

output
```
((2*(d*Tan[a + b*x])^(5/2))/(5*d) + (2*(d*Tan[a + b*x])^(9/2))/(9*d^3))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{2(d\tan(bx+a))^{\frac{9}{2}}}{9}+\frac{2d^2(d\tan(bx+a))^{\frac{5}{2}}}{5}}{b\,d^3}$ | 37 |
| default | $\frac{\frac{2(d\tan(bx+a))^{\frac{9}{2}}}{9}+\frac{2d^2(d\tan(bx+a))^{\frac{5}{2}}}{5}}{b\,d^3}$ | 37 |

input
```
int(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d^3*(1/9*(d*tan(b*x+a))^(9/2)+1/5*d^2*(d*tan(b*x+a))^(5/2))
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\left(4\,d\cos\left(bx+a\right)^4+d\cos\left(bx+a\right)^2-5\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45\,b\cos\left(bx+a\right)^4}$$

input
```
integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/45*(4*d*cos(b*x + a)^4 + d*cos(b*x + a)^2 - 5*d)*sqrt(d*sin(b*x + a)/co
s(b*x + a))/(b*cos(b*x + a)^4)
```

## Sympy [F]

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}\sec^4\left(a+bx\right)\,dx$$

input
```
integrate(sec(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(5\left(d\tan\left(bx+a\right)\right)^{\frac{9}{2}}+9\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}d^2\right)}{45\,bd^3}$$

input
```
integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/45*(5*(d*tan(b*x + a))^(9/2) + 9*(d*tan(b*x + a))^(5/2)*d^2)/(b*d^3)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(5\sqrt{d\tan(bx+a)}d^4\tan(bx+a)^4+9\sqrt{d\tan(bx+a)}d^4\tan(bx+a)^2\right)}{45\,bd^3}$$

input
```
integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/45*(5*sqrt(d*tan(b*x + a))*d^4*tan(b*x + a)^4 + 9*sqrt(d*tan(b*x + a))*d
^4*tan(b*x + a)^2)/(b*d^3)
```

## Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 276, normalized size of antiderivative = 6.13

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{8\,d\sqrt{-\frac{d\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{45\,b} - \frac{8\,d\sqrt{-\frac{d\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{45\,b\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1)} + \frac{56\,d\sqrt{-\frac{d\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{15\,b\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1)^2} - \frac{64\,d\sqrt{-\frac{d\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{9\,b\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1)^3} + \frac{32\,d\sqrt{-\frac{d\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{9\,b\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1)^4}$$

input
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^4,x)
```

output
```
(56*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/
(15*b*(exp(a*2i + b*x*2i) + 1)^2) - (8*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i)
)/(exp(a*2i + b*x*2i) + 1))^(1/2))/(45*b*(exp(a*2i + b*x*2i) + 1)) - (8*d*
(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(45*b)
 - (64*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2
))/(9*b*(exp(a*2i + b*x*2i) + 1)^3) + (32*d*(-(d*(exp(a*2i + b*x*2i)*1i -
1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(9*b*(exp(a*2i + b*x*2i) + 1)^4)
```

## Reduce [F]

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\sqrt{d}\,d\left(2\sqrt{\tan\left(bx+a\right)}\,\sec\left(bx+a\right)^4-\left(\int\frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)^4}{\tan(bx+a)}dx\right)b\right)}{9b}$$

input
```
int(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*d*(2*sqrt(tan(a + b*x))*sec(a + b*x)**4 - int((sqrt(tan(a + b*x))
*sec(a + b*x)**4)/tan(a + b*x),x)*b))/(9*b)
```

# 3.238 $\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$


Optimal result . . . 1815
Mathematica [A] (verified) . . . 1815
Rubi [A] (verified) . . . 1816
Maple [A] (verified) . . . 1817
Fricas [B] (verification not implemented) . . . 1817
Sympy [F] . . . 1818
Maxima [A] (verification not implemented) . . . 1818
Giac [A] (verification not implemented) . . . 1818
Mupad [B] (verification not implemented) . . . 1819
Reduce [F] . . . 1819


## Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

output
```
2/5*(d*tan(b*x+a))^(5/2)/b/d
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

input
```
Integrate[Sec[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*(d*Tan[a + b*x])^(5/2))/(5*b*d)
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(a+bx)^2(d\tan(a+bx))^{3/2}dx$$
$$\Big\downarrow \text{3087}$$
$$\frac{\int (d\tan(a+bx))^{3/2}d\tan(a+bx)}{b}$$
$$\Big\downarrow \text{17}$$
$$\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

input
```
Int[Sec[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*(d*Tan[a + b*x])^(5/2))/(5*b*d)
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2(d\tan(bx+a))^{\frac{5}{2}}}{5bd}$ | 19 |
| default | $\frac{2(d\tan(bx+a))^{\frac{5}{2}}}{5bd}$ | 19 |

input
```
int(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(d*tan(b*x+a))^(5/2)/b/d
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(18) = 36.

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\left(d\cos\left(bx+a\right)^2-d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{5\,b\cos\left(bx+a\right)^2}$$

input
```
integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(d*cos(b*x + a)^2 - d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x +
 a)^2)
```

## Sympy [F]

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(a+bx))^{\frac{3}{2}}\sec^2(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,(d\tan(bx+a))^{\frac{5}{2}}}{5\,bd}$$

input
```
integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/5*(d*tan(b*x + a))^(5/2)/(b*d)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,\sqrt{d\tan(bx+a)}d\tan(bx+a)^2}{5\,b}$$

input
```
integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/5*sqrt(d*tan(b*x + a))*d*tan(b*x + a)^2/b
```

## Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.55

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,d\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}\,\left(\cos\left(2\,a+2\,b\,x\right)-2\,\cos\left(4\,a+4\,b\,x\right)-\cos\left(6\,a+6\,b\,x\right)+2\right)}{5\,b\,\left(15\,\cos\left(2\,a+2\,b\,x\right)+6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)+10\right)}$$

input
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^2,x)
```

output
```
(2*d*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)*(cos(2*a + 2*b*x)
 - 2*cos(4*a + 4*b*x) - cos(6*a + 6*b*x) + 2))/(5*b*(15*cos(2*a + 2*b*x) +
 6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) + 10))
```

## Reduce [F]

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\sqrt{d}\,d\left(2\sqrt{\tan\left(bx+a\right)}\,\sec\left(bx+a\right)^2-\left(\int\frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)^2}{\tan(bx+a)}dx\right)b\right)}{5b}$$

input
```
int(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*d*(2*sqrt(tan(a + b*x))*sec(a + b*x)**2 - int((sqrt(tan(a + b*x))
*sec(a + b*x)**2)/tan(a + b*x),x)*b))/(5*b)
```

# 3.239 $\int (d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . 1820
Mathematica [A] (verified) . . . 1821
Rubi [A] (warning: unable to verify) . . . 1821
Maple [A] (verified) . . . 1825
Fricas [A] (verification not implemented) . . . 1826
Sympy [F] . . . 1827
Maxima [A] (verification not implemented) . . . 1827
Giac [F(-2)] . . . 1828
Mupad [B] (verification not implemented) . . . 1828
Reduce [F] . . . 1829

## Optimal result

Integrand size = 12, antiderivative size = 155

$$\int (d\tan(a+bx))^{3/2}\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} - \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{\sqrt{2}b} + \frac{2d\sqrt{d\tan(a+bx)}}{b}$$

output
```
1/2*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b-1/2*d
^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b-1/2*d^(3/2
)*arctanh(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))*2^(1/
2)/b+2*d*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\int (d\tan(a+bx))^{3/2}\,dx = \frac{\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan\right.}}{}\right.}{b\tan^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[(d*Tan[a + b*x])^(3/2),x]
```

output
```
((ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt
[Tan[a + b*x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x
]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*Sqr
t[2]) + 2*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(b*Tan[a + b*x]^(3/2
))
```

## Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.30, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b} - d^2\int \frac{1}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-d^2\int \frac{1}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \; 3957$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-\frac{d^3\int \frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)d^2+d^2)}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \; 266$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-\frac{2d^3\int \frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \; 755$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-\frac{2d^3\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{b}$$

$$\Big\downarrow \; 1476$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d}{2d}\right)}{b}$$

$$\Big\downarrow \; 1082$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{b}$$

$$\Big\downarrow \; 217$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-\frac{2d^3\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{b}$$

$$\Big\downarrow \; 1479$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d} + \frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right.}{\sqrt{2}\sqrt{d}}}{}\right.}{b}$$

$\Big\downarrow$ 25

$$\frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d} + \frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}} - \frac{\text{ar}}{}}{2d}\right.}{b}$$

$\Big\downarrow$ 27

$$\frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}}{2d} + \frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}} - \frac{\text{ar}}{}}{2d}\right.}{b}$$

$\Big\downarrow$ 1103

$$\frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d} + \frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}} - \frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{b}$$

input
```
Int[(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) +
ArcTan[1 + Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*
Log[d - Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d
]) + Log[d + Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]
*Sqrt[d]))/(2*d)))/b + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| derivativedivides | $\frac{2d\left(\sqrt{d\tan(bx+a)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan}{8}\right.}{b}$ |
| default | $\frac{2d\left(\sqrt{d\tan(bx+a)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan}{8}\right.}{b}$ |

input
```
int((d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*d*((d*tan(b*x+a))^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(b*x+a)+(d^2
)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(b*x+a)-(d^2)^(1/4
)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*
(d*tan(b*x+a))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)
+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

$$\int (d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{2\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d}{d}\right)+2\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}-d}{d}\right)+\sqrt{2}d^{\frac{3}{2}}\log\left(d\tan(bx+a)+\right.}{}$$

input
```
integrate((d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/
d) + 2*sqrt(2)*d^(3/2)*arctan((sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) - d)/d
) + sqrt(2)*d^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt
(d) + d) - sqrt(2)*d^(3/2)*log(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a
))*sqrt(d) + d) - 8*sqrt(d*tan(b*x + a))*d)/b
```

## Sympy [F]

$$\int (d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(a+bx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10

$$\int (d\tan(a+bx))^{3/2}\,dx =$$
$$-\frac{2\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+\sqrt{2}d^{\frac{5}{2}}\log\left(d\tan\right.}{}$$

input
```
integrate((d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d)) + 2*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + sqrt(2)*d^(5/2)*log(d*tan(b*x + a
) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - sqrt(2)*d^(5/2)*log(d*tan(
b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x +
a))*d^2)/(b*d)
```

## Giac [F(-2)]

Exception generated.

$$\int (d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[3,9]%%%}+%%%{4,[3,7]%%%}+%%%{6,[3,5]%%%}+%%%{4,[3,3
]%%%}+%%%
```

## Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int (d\tan(a+bx))^{3/2}\,dx = \frac{2\,d\,\sqrt{d\tan\left(a+b\,x\right)}}{b} + \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{b} + \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{b}$$

input
```
int((d*tan(a + b*x))^(3/2),x)
```

output
```
(2*d*(d*tan(a + b*x))^(1/2))/b + ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*t
an(a + b*x))^(1/2))/d^(1/2))*1i)/b + ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)
*(d*tan(a + b*x))^(1/2))/d^(1/2))*1i)/b
```

## Reduce [F]

$$\int (d\tan(a+bx))^{3/2}\,dx = \frac{\sqrt{d}\,d\left(2\sqrt{\tan(bx+a)}-\left(\int \frac{\sqrt{\tan(bx+a)}}{\tan(bx+a)}dx\right)b\right)}{b}$$

input
```
int((d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*d*(2*sqrt(tan(a + b*x)) - int(sqrt(tan(a + b*x))/tan(a + b*x),x)*
b))/b
```

# 3.240 $\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1830
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 1831
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1831
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1835
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . 1836
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1837
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . 1837
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . 1838
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1838
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1839

## Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}(1+\tan(a+bx))}\right)}{4\sqrt{2}b}-\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b}$$

output

```
-1/8*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b+1/8*
d^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b+1/8*d^(3/
2)*arctanh(2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2)/(1+tan(b*x+a)))*2^(1/2)/b-
1/2*d*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.66

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$
$$-\frac{d\csc(a+bx)\left(\sin(a+bx)+\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-\log\left(\cos(a+bx)+\sin\right.\right.}{8b}$$

input
```
Integrate[Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/8*(d*Csc[a + b*x]*(Sin[a + b*x] + ArcSin[Cos[a + b*x] - Sin[a + b*x]]*S
qrt[Sin[2*(a + b*x)]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a +
b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/b
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.35, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3087, 252, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(d\tan(a+bx))^{3/2}}{\sec(a+bx)^2}dx$$
$$\Big\downarrow \ 3087$$
$$\frac{\int \frac{(d\tan(a+bx))^{3/2}}{(\tan^2(a+bx)+1)^2}d\tan(a+bx)}{b}$$
$$\Big\downarrow \ 252$$

$$\frac{\frac{1}{4}d^2\int \frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)+1)}d\tan(a+bx)-\frac{d\sqrt{d\tan(a+bx)}}{2(\tan^2(a+bx)+1)}}{b}$$

$\downarrow$ 266

$$\frac{\frac{1}{2}d\int \frac{1}{\tan^2(a+bx)+1}d\sqrt{d\tan(a+bx)}-\frac{d\sqrt{d\tan(a+bx)}}{2(\tan^2(a+bx)+1)}}{b}$$

$\downarrow$ 755

$$\frac{\frac{1}{2}d\left(\frac{\int \frac{d^2(d-d\tan(a+bx))}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2(\tan(a+bx)d+d)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)-\frac{d\sqrt{d\tan(a+bx)}}{2(\tan^2(a+bx)+1)}}{b}$$

$\downarrow$ 27

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}+\frac{1}{2}d\int \frac{\tan(a+bx)d+d}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{d\sqrt{d\tan(a+bx)}}{2(\tan^2(a+bx)+1)}}{b}$$

$\downarrow$ 1476

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}+\frac{1}{2}d\left(\frac{1}{2}\int \frac{1}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{}{\tan(a+b}\right.\right.}{b}$$

$\downarrow$ 1082

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}+\frac{1}{2}d\left(\frac{\int \frac{1}{-d\tan(a+bx)-1}d\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d\tan(a+bx)-1}d\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx}}{\sqrt{d}}\right.}{\sqrt{2}\sqrt{d}}\right.\right.}{b}$$

$\downarrow$ 217

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}+\frac{1}{2}d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)\right)-\frac{d\sqrt{d\tan}}{2(\tan^2(a}}{b}$$

$\downarrow$ 1479

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\left(-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}d\left(\frac{\arctan}{}\right.\right.}{b}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\left(\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}d\left(\frac{\arctan\left(\sqrt{2}\right.}{\sqrt{}}\right.\right.}{b}$$

$\downarrow$ 27

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\left(\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}d\left(\frac{\arctan\left(\sqrt{2}\right.}{\sqrt{}}\right.\right.}{b}$$

$\downarrow$ 1103

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}d\left(\frac{\log\left(d\tan(a+bx)+\sqrt{2}\sqrt{d}\sqrt{d\tan(a+bx)}+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(d\tan\right.}{}\right.\right.}{b}$$

input
```
Int[Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
((d*((d*(-(ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*Sqr
t[d])) + ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[
d])))/2 + (d*(-1/2*Log[d + d*Tan[a + b*x] - Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[a +
 b*x]]]/(Sqrt[2]*Sqrt[d]) + Log[d + d*Tan[a + b*x] + Sqrt[2]*Sqrt[d]*Sqrt[
d*Tan[a + b*x]]]/(2*Sqrt[2]*Sqrt[d])))/2))/2 - (d*Sqrt[d*Tan[a + b*x]])/(2
*(1 + Tan[a + b*x]^2)))/b
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(127) = 254$.

Time = 2.03 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.42

| method | result |
|---|---|
| default | $-\frac{\left(\cos(bx+a)(4\cos(bx+a)+4)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}+2\arctan\left(\frac{\sin(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}-\cos(bx+a)+1}{-1+\cos(bx+a)}\right)+\ln\left(-\frac{\cot(b\ldots}{\ldots}\right.\right.}{\ldots}$ |

input
```
int(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/b*(cos(b*x+a)*(4*cos(b*x+a)+4)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)
+1)^2)^(1/2)+2*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))+ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(
b*x+a)+2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*co
s(b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))-ln((2*sin(b*x+a)*(-2*si
n(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cot(b*x+a)*cos(b*x+a)+sin(b*x+
a)+2*cos(b*x+a)-csc(b*x+a)+2*cot(b*x+a)-2)/(-1+cos(b*x+a)))+2*arctan((sin(
b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1
+cos(b*x+a))))*cos(b*x+a)*(d*tan(b*x+a))^(1/2)*d/(cos(b*x+a)+1)/(-sin(b*x+
a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(127) = 254$.

Time = 0.15 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.58

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{16\,d\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)^2-2\sqrt{2}d^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}(\cos(bx+a)-\sin(bx+a))}{2\sqrt{d}\sin(bx+a)}\right)+\sqrt{2}d^{\frac{3}{2}}\arctan\left(\frac{2\,d\cos\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/32*(16*d*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2 - 2*sqrt(2)*d
^(3/2)*arctan(-1/2*sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x + a))*(cos(b*x + a)
 - sin(b*x + a))/(sqrt(d)*sin(b*x + a))) + sqrt(2)*d^(3/2)*arctan(1/2*(2*d
*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) + sqrt(2)*sqrt(d)*sqrt(d*s
in(b*x + a)/cos(b*x + a)) - 2*d)/(d*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*
x + a) - d)) + sqrt(2)*d^(3/2)*arctan(-1/2*(2*d*cos(b*x + a)^2 - 2*d*cos(b
*x + a)*sin(b*x + a) - sqrt(2)*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) -
 2*d)/(d*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a) - d)) - sqrt(2)*d^(3
/2)*log(4*d*cos(b*x + a)*sin(b*x + a) + 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*
x + a)*sin(b*x + a))*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + d) + sqrt
(2)*d^(3/2)*log(4*d*cos(b*x + a)*sin(b*x + a) - 2*sqrt(2)*(cos(b*x + a)^2
+ cos(b*x + a)*sin(b*x + a))*sqrt(d)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + d
))/b
```

## Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.13

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+\sqrt{2}$$

input
```
integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
1/16*(2*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d)) + 2*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + sqrt(2)*d^(5/2)*log(d*tan(b*x + a
) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - sqrt(2)*d^(5/2)*log(d*tan(
b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x +
a))*d^4/(d^2*tan(b*x + a)^2 + d^2))/(b*d)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.34

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\frac{2\sqrt{2}d^2\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{2\sqrt{2}d^2\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{\sqrt{2}d^2\sqrt{|d|}\log}{}}{}$$

input
```
integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(2)*d^2*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
+ 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 2*sqrt(2)*d^2*sqrt(abs(d))*arc
tan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(
d)))/b + sqrt(2)*d^2*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(
b*x + a))*sqrt(abs(d)) + abs(d))/b - sqrt(2)*d^2*sqrt(abs(d))*log(d*tan(b*
x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 8*sqrt(d*
tan(b*x + a))*d^4/((d^2*tan(b*x + a)^2 + d^2)*b))/d
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \cos(a+b\,x)^2\,(d\tan(a+b\,x))^{3/2}\,dx$$

input
```
int(cos(a + b*x)^2*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)^2*(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx=\sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\cos(bx+a)^2\tan(bx+a)\,dx\right)d$$

input
```
int(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*cos(a + b*x)**2*tan(a + b*x),x)*d
```

# 3.241 $\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . 1840
Mathematica [C] (verified) . . . 1841
Rubi [A] (verified) . . . 1841
Maple [A] (verified) . . . 1844
Fricas [C] (verification not implemented) . . . 1845
Sympy [F] . . . 1845
Maxima [F] . . . 1846
Giac [F] . . . 1846
Mupad [F(-1)] . . . 1846
Reduce [F] . . . 1847

## Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{4d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{77b\sqrt{d\tan(a+bx)}} - \frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b} - \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b} + \frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}$$

output
```
-4/77*d^2*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^
(1/2)/b/(d*tan(b*x+a))^(1/2)-4/77*d*sec(b*x+a)*(d*tan(b*x+a))^(1/2)/b-2/77
*d*sec(b*x+a)^3*(d*tan(b*x+a))^(1/2)/b+2/11*d*sec(b*x+a)^5*(d*tan(b*x+a))^
(1/2)/b
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.73 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$
$$-\frac{d\sec^5(a+bx)\left(-23+6\cos(2(a+bx))+\cos(4(a+bx))+16\cos^6(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\right.\right.}{154b}$$

input
```
Integrate[Sec[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/154*(d*Sec[a + b*x]^5*(-23 + 6*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 16
*Cos[a + b*x]^6*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec
[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

## Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3091, 3042, 3093, 3042, 3093, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^5(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\int \frac{\sec^5(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\int\frac{\sec(a+bx)^5}{\sqrt{d\tan(a+bx)}}dx$$

$\big\downarrow$ 3093

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\int\frac{\sec^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{2\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7bd}\right)$$

$\big\downarrow$ 3042

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\int\frac{\sec(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx+\frac{2\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7bd}\right)$$

$\big\downarrow$ 3093

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\left(\frac{2}{3}\int\frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)+\frac{2\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7bd}\right)$$

$\big\downarrow$ 3042

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\left(\frac{2}{3}\int\frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)+\frac{2\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7bd}\right)$$

$\big\downarrow$ 3094

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)+\frac{2\sec^3(a+bx)\sqrt{d\tan(a}{7bd}\right.$$

$\big\downarrow$ 3042

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)+\frac{2\sec^3(a+bx)\sqrt{d\tan(a}{7bd}\right.$$

$\big\downarrow$ 3053

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b} - \frac{1}{11}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3\sqrt{d\tan(a+bx)}}+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)+\frac{2\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7bd}\right)$$

$\big\downarrow$ 3042

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b} - \frac{1}{11}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3\sqrt{d\tan(a+bx)}}+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)+\frac{2\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7bd}\right)$$

$\big\downarrow$ 3120

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b} - \frac{1}{11}d^2\left(\frac{2\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7bd}+\frac{6}{7}\left(\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}+\frac{2\sqrt{\sin(2a+2bx)}\sec(a+bx)\,\text{EllipticF}\ldots}{3b\sqrt{d\tan(a+bx)}}\right.\right.$$

input
```
Int[Sec[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*Sec[a + b*x]^5*Sqrt[d*Tan[a + b*x]])/(11*b) - (d^2*((2*Sec[a + b*x]^3
*Sqrt[d*Tan[a + b*x]])/(7*b*d) + (6*((2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a
 + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*Sqrt[d*Tan[a + b*x]]) + (2*Sec[a + b*
x]*Sqrt[d*Tan[a + b*x]])/(3*b*d)))/7))/11
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| default | $-\frac{2\sqrt{d\tan(bx+a)}\,d\left(2\sec(bx+a)+\sec(bx+a)^3-7\sec(bx+a)^5+\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \text{Ellipt}\right.}{77b}$ |

input
```
int(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77/b*(d*tan(b*x+a))^(1/2)*d*(2*sec(b*x+a)+sec(b*x+a)^3-7*sec(b*x+a)^5+(
-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*Ellipti
cF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(csc(b*x+a)-cot(b*x+a)+1)^
(1/2)*(2*cot(b*x+a)+2*csc(b*x+a)))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(2\sqrt{i\,d}d\cos(bx+a)^5F(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+2\sqrt{-i\,d}d\cos(bx+a\right.}{77}$$

input
```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/77*(2*sqrt(I*d)*d*cos(b*x + a)^5*elliptic_f(arcsin(cos(b*x + a) + I*sin(
b*x + a)), -1) + 2*sqrt(-I*d)*d*cos(b*x + a)^5*elliptic_f(arcsin(cos(b*x +
 a) - I*sin(b*x + a)), -1) - (2*d*cos(b*x + a)^4 + d*cos(b*x + a)^2 - 7*d)
*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a)^5)
```

## Sympy [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(a+bx))^{\frac{3}{2}}\sec^5(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**5*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**5, x)
```

## Maxima [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)^5\,dx$$

input
```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^5, x)
```

## Giac [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)^5\,dx$$

input
```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\cos(a+b\,x)^5}\,dx$$

input
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^5,x)
```

output
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^5, x)
```

## Reduce [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\sqrt{d}\,d\left(2\sqrt{\tan(bx+a)}\,\sec(bx+a)^5-\left(\int\frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)^5}{\tan(bx+a)}dx\right)b\right)}{11b}$$

input
```
int(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*d*(2*sqrt(tan(a + b*x))*sec(a + b*x)**5 - int((sqrt(tan(a + b*x))
*sec(a + b*x)**5)/tan(a + b*x),x)*b))/(11*b)
```

# 3.242 $\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1848
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1848
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1849
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1852
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . 1852
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1853
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1853
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1853
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1854
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1854

## Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{21b\sqrt{d\tan(a+bx)}} - \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}$$

output
```
-2/21*d^2*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^
(1/2)/b/(d*tan(b*x+a))^(1/2)-2/21*d*sec(b*x+a)*(d*tan(b*x+a))^(1/2)/b+2/7*
d*sec(b*x+a)^3*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.74

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d\sec^3(a+bx)\left(-5+\cos(2(a+bx))+4\cos^4(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec\right.}{21b}$$

input
```
Integrate[Sec[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/21*(d*Sec[a + b*x]^3*(-5 + Cos[2*(a + b*x)] + 4*Cos[a + b*x]^4*Hypergeo
metric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan
[a + b*x]])/b
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3091, 3042, 3093, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^3(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{1}{7}d^2\int \frac{\sec^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{1}{7}d^2\int \frac{\sec(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{1}{7}d^2\left(\frac{2}{3}\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{1}{7}d^2\left(\frac{2}{3}\int\frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx + \frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)$$

$\big\downarrow$ 3094

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{1}{7}d^2\left(\frac{2\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} + \frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)$$

$\big\downarrow$ 3042

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{1}{7}d^2\left(\frac{2\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} + \frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)$$

$\big\downarrow$ 3053

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{1}{7}d^2\left(\frac{2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3\sqrt{d\tan(a+bx)}} + \frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)$$

$\big\downarrow$ 3042

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{1}{7}d^2\left(\frac{2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3\sqrt{d\tan(a+bx)}} + \frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)$$

$\big\downarrow$ 3120

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{1}{7}d^2\left(\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd} + \frac{2\sqrt{\sin(2a+2bx)}\sec(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b\sqrt{d\tan(a+bx)}}\right)$$

input
```
Int[Sec[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*Sec[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(7*b) - (d^2*((2*EllipticF[a - P
i/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*Sqrt[d*Tan[a + b*x
]]) + (2*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b*d)))/7
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.18

| method | result |
|---|---|
| default | $-\frac{2\sqrt{d\tan(bx+a)}\,d\left(\sec(bx+a)-3\sec(bx+a)^3+\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot\ldots\right.}{21b}$ |

input
```
int(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21/b*(d*tan(b*x+a))^(1/2)*d*(sec(b*x+a)-3*sec(b*x+a)^3+(csc(b*x+a)-cot(
b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+
a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(cot(b*x+
a)+csc(b*x+a)))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(\sqrt{i\,d}d\cos(bx+a)^3\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+\sqrt{-i\,d}d\cos(bx+a)^3\ldots\right.}{21\,b\cos(bx+\ldots}$$

input
```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/21*(sqrt(I*d)*d*cos(b*x + a)^3*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*
x + a)), -1) + sqrt(-I*d)*d*cos(b*x + a)^3*elliptic_f(arcsin(cos(b*x + a)
- I*sin(b*x + a)), -1) - (d*cos(b*x + a)^2 - 3*d)*sqrt(d*sin(b*x + a)/cos(
b*x + a)))/(b*cos(b*x + a)^3)
```

## Sympy [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(a+bx))^{\frac{3}{2}}\sec^3(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**3, x)
```

## Maxima [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)^3\,dx$$

input
```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^3, x)
```

## Giac [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)^3\,dx$$

input
```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\cos{(a+b\,x)}^3}\,dx$$

input
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^3,x)
```

output
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^3, x)
```

## Reduce [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\sqrt{d}\,d\left(2\sqrt{\tan(bx+a)}\,\sec(bx+a)^3-\left(\int\frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)^3}{\tan(bx+a)}dx\right)b\right)}{7b}$$

input
```
int(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*d*(2*sqrt(tan(a + b*x))*sec(a + b*x)**3 - int((sqrt(tan(a + b*x))
*sec(a + b*x)**3)/tan(a + b*x),x)*b))/(7*b)
```

# 3.243 $\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . 1855
Mathematica [C] (verified) . . . 1855
Rubi [A] (verified) . . . 1856
Maple [A] (verified) . . . 1858
Fricas [C] (verification not implemented) . . . 1858
Sympy [F] . . . 1859
Maxima [F] . . . 1859
Giac [F] . . . 1859
Mupad [F(-1)] . . . 1860
Reduce [F] . . . 1860

## Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

output
```
-1/3*d^2*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(
1/2)/b/(d*tan(b*x+a))^(1/2)+2/3*d*sec(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\cos(a+bx)\left(\sec^2(a+bx)-\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)}{3b}$$

input
```
Integrate[Sec[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*Cos[a + b*x]*(Sec[a + b*x]^2 - Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[
a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3091, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{1}{3}d^2\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{1}{3}d^2\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3094}$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{d^2\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{d^2\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sec[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/3*(d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]]
)/(b*Sqrt[d*Tan[a + b*x]]) + (2*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.54

| method | result |
|---|---|
| default | $\frac{\sqrt{d\tan(bx+a)}\, d\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{EllipticF}\left(\sqrt{\csc(bx+a)}\right.\right.}{3b}$ |

input
```
int(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*(d*tan(b*x+a))^(1/2)*d*((csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+
a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x
+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)-csc(b*x+a))+2*sec(b*x+a)
)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\sqrt{i\,d}d\cos(bx+a)\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+\sqrt{-i\,d}d\cos(bx+a)\,F(\mathrm{arc}}{3\,b\cos(bx+a)}$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(I*d)*d*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x +
 a)), -1) + sqrt(-I*d)*d*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) - I*s
in(b*x + a)), -1) + 2*d*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a)
)
```

## Sympy [F]

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(a+bx))^{\frac{3}{2}}\sec(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x), x)
```

## Maxima [F]

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a), x)
```

## Giac [F]

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\cos(a+b\,x)}\,dx$$

input
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x),x)
```

output
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x), x)
```

## Reduce [F]

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\sqrt{d}\,d\left(2\sqrt{\tan(bx+a)}\,\sec(bx+a)-\left(\int \frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)}{\tan(bx+a)}dx\right)b\right)}{3b}$$

input
```
int(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*d*(2*sqrt(tan(a + b*x))*sec(a + b*x) - int((sqrt(tan(a + b*x))*se
c(a + b*x))/tan(a + b*x),x)*b))/(3*b)
```

# 3.244 $\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1861
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . 1861
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1862
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1864
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1864
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1865
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1865
Giac [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1865
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1866
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1866

## Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{d^2\operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

output
```
1/2*d^2*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1
/2)/b/(d*tan(b*x+a))^(1/2)-d*cos(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{d\cos(a+bx)\left(-1+\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a}}{b}$$

input
```
Integrate[Cos[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*Cos[a + b*x]*(-1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sq
rt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3090, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sec(a+bx)}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{1}{2}d^2\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}d^2\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3094}$$

$$\frac{d^2\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \textbf{3053}$$

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \textbf{3120}$$

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

input
```
Int[Cos[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*
b*Sqrt[d*Tan[a + b*x]]) - (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.58

| method | result |
|---|---|
| default | $\frac{\left(-\frac{\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1},\frac{\sqrt{2}}{2}\right)(-\cot(bx+}{2}}{b}$ |

input
```
int(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(
1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1
/2),1/2*2^(1/2))*(-cot(b*x+a)-csc(b*x+a))-cos(b*x+a))*(d*tan(b*x+a))^(1/2)
*d
```

## Fricas [F]

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx=\int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)\,dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)*tan(b*x + a), x)
```

## Sympy [F]

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(a+bx)})^{\frac{3}{2}}\cos{(a+bx)}\,dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*cos(a + b*x), x)
```

## Maxima [F]

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(bx+a)})^{\frac{3}{2}}\cos{(bx+a)}\;dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

## Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \cos(a+b\,x)\ (d\tan(a+b\,x))^{3/2}\,dx$$

input
```
int(cos(a + b*x)*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)*(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\cos(bx+a)\tan(bx+a)\,dx\right)d$$

input
```
int(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*cos(a + b*x)*tan(a + b*x),x)*d
```

# 3.245 $\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . 1867
Mathematica [C] (verified) . . . 1867
Rubi [A] (verified) . . . 1868
Maple [C] (warning: unable to verify) . . . 1871
Fricas [F] . . . 1872
Sympy [F(-1)] . . . 1872
Maxima [F] . . . 1872
Giac [F(-2)] . . . 1873
Mupad [F(-1)] . . . 1873
Reduce [F] . . . 1873

## Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d^2 \operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\tan(a+bx)}} + \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b} - \frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

output
```
1/12*d^2*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(
1/2)/b/(d*tan(b*x+a))^(1/2)+1/6*d*cos(b*x+a)*(d*tan(b*x+a))^(1/2)/b-1/3*d*
cos(b*x+a)^3*(d*tan(b*x+a))^(1/2)/b
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{\cos(a+bx)\left(\sqrt[4]{-1}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sqrt{\sec^2(a+bx)}+\cos(2(a+bx))\sqrt{\tan}\right.}{6b\tan^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/6*(Cos[a + b*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a +
 b*x]]], -1]*Sqrt[Sec[a + b*x]^2] + Cos[2*(a + b*x)]*Sqrt[Tan[a + b*x]])*(
d*Tan[a + b*x])^(3/2))/(b*Tan[a + b*x]^(3/2))
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3090, 3042, 3092, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sec(a+bx)^3}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{1}{6}d^2\int \frac{\cos(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}d^2\int \frac{1}{\sec(a+bx)\sqrt{d\tan(a+bx)}}dx-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$$\Big\downarrow \text{3092}$$

$$\frac{1}{6}d^2\left(\frac{1}{2}\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}d^2\left(\frac{1}{2}\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$$\Big\downarrow \text{3094}$$

$$\frac{1}{6}d^2\left(\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}d^2\left(\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$$\Big\downarrow \text{3053}$$

$$\frac{1}{6}d^2\left(\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}d^2\left(\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{6}d^2\left(\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}+\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{d\tan(a+bx)}}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

input
```
Int[Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/3*(d*Cos[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/b + (d^2*((EllipticF[a - Pi/4
 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Tan[a + b*x]])
 + (Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/(b*d)))/6
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.49 (sec) , antiderivative size = 1588, normalized size of antiderivative = 14.70

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1588 |

input
```
int(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/b*(d*tan(b*x+a))^(1/2)*d*(2^(1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x
+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-2*s
in(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b
*x+a)+2)/(-1+cos(b*x+a)))*(-6*cos(b*x+a)^2-6*cos(b*x+a)+3*cot(b*x+a)+3*csc
(b*x+a))+(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*c
os(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))*(12*cos
(b*x+a)^2+12*cos(b*x+a))+2^(1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^
2)^(1/2)*ln((2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2
)-cot(b*x+a)*cos(b*x+a)+sin(b*x+a)+2*cos(b*x+a)-csc(b*x+a)+2*cot(b*x+a)-2)
/(-1+cos(b*x+a)))*(6*cos(b*x+a)^2+6*cos(b*x+a)-3*cot(b*x+a)-3*csc(b*x+a))+
ln((2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cot(b*x
+a)*cos(b*x+a)+sin(b*x+a)+2*cos(b*x+a)-csc(b*x+a)+2*cot(b*x+a)-2)/(-1+cos(
b*x+a)))*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(-12*cos(b*x+a)^2
-12*cos(b*x+a))+2^(1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*
arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b
*x+a)+1)/(-1+cos(b*x+a)))*(-12*cos(b*x+a)^2-12*cos(b*x+a)+6*cot(b*x+a)+6*c
sc(b*x+a))+arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^
(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1
)^2)^(1/2)*(24*cos(b*x+a)^2+24*cos(b*x+a))+2^(1/2)*(-2*sin(b*x+a)*cos(b...
```

## Fricas [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)^3*tan(b*x + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \cos\left(a+b\,x\right)^3\,{\left(d\tan(a+b\,x)\right)}^{3/2}\,dx$$

input
```
int(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(bx+a\right)}\,\cos\left(bx+a\right)^3\tan\left(bx+a\right)dx\right)d$$

input
```
int(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*cos(a + b*x)**3*tan(a + b*x),x)*d
```

# 3.246 $\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal result . . . . . . . . . . 1874
Mathematica [C] (warning: unable to verify) . . . . . . . . . . 1875
Rubi [A] (verified) . . . . . . . . . . 1875
Maple [C] (warning: unable to verify) . . . . . . . . . . 1879
Fricas [F] . . . . . . . . . . 1880
Sympy [F(-1)] . . . . . . . . . . 1880
Maxima [F] . . . . . . . . . . 1880
Giac [F(-2)] . . . . . . . . . . 1881
Mupad [F(-1)] . . . . . . . . . . 1881
Reduce [F] . . . . . . . . . . 1881

## Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{24b\sqrt{d\tan(a+bx)}} + \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b} + \frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b} - \frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

output
```
1/24*d^2*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(
1/2)/b/(d*tan(b*x+a))^(1/2)+1/12*d*cos(b*x+a)*(d*tan(b*x+a))^(1/2)/b+1/30*
d*cos(b*x+a)^3*(d*tan(b*x+a))^(1/2)/b-1/5*d*cos(b*x+a)^5*(d*tan(b*x+a))^(1
/2)/b
```

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\cos(2(a+bx))\csc(a+bx)\left(10\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sqrt{\sec^2(a\cdots}{120b\sqrt{\tan(a+bx)}\,(-\cdots}$$

input
```
Integrate[Cos[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Cos[2*(a + b*x)]*Csc[a + b*x]*(10*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/
4)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Sec[a + b*x]^2] + (-3 + 10*Cos[2*(a + b*x
)] + 3*Cos[4*(a + b*x)])*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(120*
b*Sqrt[Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3090, 3042, 3092, 3042, 3092, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sec(a+bx)^5}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{1}{10}d^2\int \frac{\cos^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx - \frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{10}d^2\int \frac{1}{\sec(a+bx)^3\sqrt{d\tan(a+bx)}}dx-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$$\Big\downarrow \text{3092}$$

$$\frac{1}{10}d^2\left(\frac{5}{6}\int \frac{\cos(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{10}d^2\left(\frac{5}{6}\int \frac{1}{\sec(a+bx)\sqrt{d\tan(a+bx)}}dx+\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$$\Big\downarrow \text{3092}$$

$$\frac{1}{10}d^2\left(\frac{5}{6}\left(\frac{1}{2}\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)+\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{10}d^2\left(\frac{5}{6}\left(\frac{1}{2}\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)+\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$$\Big\downarrow \text{3094}$$

$$\frac{1}{10}d^2\left(\frac{5}{6}\left(\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)+\frac{\cos^3(a+bx)\sqrt{d\tan(a+b}}{3bd}\right.$$
$$-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{10}d^2\left(\frac{5}{6}\left(\frac{\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)+\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right.$$
$$\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$\downarrow$ 3053

$$\frac{1}{10}d^2\left(\frac{5}{6}\left(\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)+\frac{\cos^3(a+bx)\sqrt{d\tan(}{3bd}\right.$$
$$\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$\downarrow$ 3042

$$\frac{1}{10}d^2\left(\frac{5}{6}\left(\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)+\frac{\cos^3(a+bx)\sqrt{d\tan(}{3bd}\right.$$
$$\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$\downarrow$ 3120

$$\frac{1}{10}d^2\left(\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}+\frac{5}{6}\left(\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}+\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\operatorname{EllipticF}(a}{2b\sqrt{d\tan(a+bx)}}\right.\right.$$
$$\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

input
```
Int[Cos[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/5*(d*Cos[a + b*x]^5*Sqrt[d*Tan[a + b*x]])/b + (d^2*((Cos[a + b*x]^3*Sqr
t[d*Tan[a + b*x]])/(3*b*d) + (5*((EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x
]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Tan[a + b*x]]) + (Cos[a + b*x]*Sqrt[
d*Tan[a + b*x]])/(b*d)))/6))/10
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.79 (sec) , antiderivative size = 1526, normalized size of antiderivative = 11.22

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1526 |

input
```
int(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/480/b*(d*tan(b*x+a))^(1/2)*(15*2^(1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b
*x+a)+1)^2)^(1/2)*ln(2*cot(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)+2*csc(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2
*cot(b*x+a)+2)*(-2*cos(b*x+a)^2+cot(b*x+a)+csc(b*x+a)-2*cos(b*x+a))+60*ln(
2*cot(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+2*csc(b*x+a
)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cot(b*x+a)+2)*(-sin(
b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(cos(b*x+a)^2+cos(b*x+a))+15*2^(
1/2)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-2*cot(b*x+a)*(-
2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*csc(b*x+a)*(-2*sin(b*x+a
)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cot(b*x+a)+2)*(2*cos(b*x+a)^2-cot(b
*x+a)-csc(b*x+a)+2*cos(b*x+a))+60*ln(-2*cot(b*x+a)*(-2*sin(b*x+a)*cos(b*x+
a)/(cos(b*x+a)+1)^2)^(1/2)-2*csc(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x
+a)+1)^2)^(1/2)-2*cot(b*x+a)+2)*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^
(1/2)*(-cos(b*x+a)^2-cos(b*x+a))+30*2^(1/2)*arctan((sin(b*x+a)*(-2*sin(b*x
+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))*(-2*
sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(-2*cos(b*x+a)^2+cot(b*x+a)+
csc(b*x+a)-2*cos(b*x+a))+120*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/
2)*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-co
s(b*x+a)+1)/(-1+cos(b*x+a)))*(cos(b*x+a)^2+cos(b*x+a))+30*2^(1/2)*(-2*sin(
b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*(-2*sin(b*...
```

## Fricas [F]

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)^5\,dx$$

input
```
integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)^5*tan(b*x + a), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**5*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)^5\,dx$$

input
```
integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a)^5, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \cos(a+b\,x)^5\,(d\tan(a+b\,x))^{3/2}\,dx$$

input
```
int(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(bx+a)}\,\cos(bx+a)^5\tan(bx+a)\,dx\right)d$$

input
```
int(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(a + b*x))*cos(a + b*x)**5*tan(a + b*x),x)*d
```

# 3.247 $\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx$


Optimal result . . . 1882
Mathematica [A] (verified) . . . 1882
Rubi [A] (verified) . . . 1883
Maple [A] (verified) . . . 1884
Fricas [A] (verification not implemented) . . . 1885
Sympy [F(-1)] . . . 1885
Maxima [A] (verification not implemented) . . . 1885
Giac [A] (verification not implemented) . . . 1886
Mupad [B] (verification not implemented) . . . 1886
Reduce [F] . . . 1887


## Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(d\tan(e+fx))^{7/2}}{7df} + \frac{4(d\tan(e+fx))^{11/2}}{11d^3f} + \frac{2(d\tan(e+fx))^{15/2}}{15d^5f}$$

output
```
2/7*(d*tan(f*x+e))^(7/2)/d/f+4/11*(d*tan(f*x+e))^(11/2)/d^3/f+2/15*(d*tan(
f*x+e))^(15/2)/d^5/f
```

## Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(117+44\cos(2(e+fx))+4\cos(4(e+fx)))\sec^4(e+fx)(d\tan(e+fx))^{7/2}}{1155df}$$

input
```
Integrate[Sec[e + f*x]^6*(d*Tan[e + f*x])^(5/2),x]
```

output
```
(2*(117 + 44*Cos[2*(e + f*x)] + 4*Cos[4*(e + f*x)])*Sec[e + f*x]^4*(d*Tan[
e + f*x])^(7/2))/(1155*d*f)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(e+fx)^6(d\tan(e+fx))^{5/2}dx$$

$$\Big\downarrow \; 3087$$

$$\frac{\int (d\tan(e+fx))^{5/2}\left(\tan^2(e+fx)+1\right)^2 d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 244$$

$$\frac{\int \left(\frac{(d\tan(e+fx))^{13/2}}{d^4}+\frac{2(d\tan(e+fx))^{9/2}}{d^2}+(d\tan(e+fx))^{5/2}\right) d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{2(d\tan(e+fx))^{15/2}}{15d^5}+\frac{4(d\tan(e+fx))^{11/2}}{11d^3}+\frac{2(d\tan(e+fx))^{7/2}}{7d}}{f}$$

input
```
Int[Sec[e + f*x]^6*(d*Tan[e + f*x])^(5/2),x]
```

output
```
((2*(d*Tan[e + f*x])^(7/2))/(7*d) + (4*(d*Tan[e + f*x])^(11/2))/(11*d^3) +
 (2*(d*Tan[e + f*x])^(15/2))/(15*d^5))/f
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\frac{\frac{2(d\tan(fx+e))^{\frac{15}{2}}}{15}+\frac{4d^2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{2d^4(d\tan(fx+e))^{\frac{7}{2}}}{7}}{d^5f}$$

input
```
int(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x)
```

output
```
2/d^5/f*(1/15*(d*tan(f*x+e))^(15/2)+2/11*d^2*(d*tan(f*x+e))^(11/2)+1/7*d^4
*(d*tan(f*x+e))^(7/2))
```

## Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{2\left(32\,d^2\cos\left(fx+e\right)^6+24\,d^2\cos\left(fx+e\right)^4+21\,d^2\cos\left(fx+e\right)^2-77\,d^2\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{1155\,f\cos\left(fx+e\right)^7}$$

input
```
integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-2/1155*(32*d^2*cos(f*x + e)^6 + 24*d^2*cos(f*x + e)^4 + 21*d^2*cos(f*x +
e)^2 - 77*d^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x +
 e)^7)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**6*(d*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\left(77\left(d\tan\left(fx+e\right)\right)^{\frac{15}{2}}+210\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}d^2+165\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^4\right)}{1155\,d^5 f}$$

input
```
integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
2/1155*(77*(d*tan(f*x + e))^(15/2) + 210*(d*tan(f*x + e))^(11/2)*d^2 + 165
*(d*tan(f*x + e))^(7/2)*d^4)/(d^5*f)
```

## Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\left(77\,\sqrt{d\tan\left(fx+e\right)}d^7\tan\left(fx+e\right)^7+210\,\sqrt{d\tan\left(fx+e\right)}d^7\tan\left(fx+e\right)^5+165\,\sqrt{d\tan$$

$$1155\,d^5 f$$

input
```
integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
2/1155*(77*sqrt(d*tan(f*x + e))*d^7*tan(f*x + e)^7 + 210*sqrt(d*tan(f*x +
e))*d^7*tan(f*x + e)^5 + 165*sqrt(d*tan(f*x + e))*d^7*tan(f*x + e)^3)/(d^5
*f)
```

## Mupad [B] (verification not implemented)

Time = 11.23 (sec) , antiderivative size = 474, normalized size of antiderivative = 7.07

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
int((d*tan(e + f*x))^(5/2)/cos(e + f*x)^6,x)
```

output
```
(d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*64
i)/(1155*f) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i)
+ 1))^(1/2)*64i)/(1155*f*(exp(e*2i + f*x*2i) + 1)) + (d^2*(-(d*(exp(e*2i +
 f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*32i)/(385*f*(exp(e*2i +
 f*x*2i) + 1)^2) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x
*2i) + 1))^(1/2)*2432i)/(231*f*(exp(e*2i + f*x*2i) + 1)^3) + (d^2*(-(d*(ex
p(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*1504i)/(33*f*(e
xp(e*2i + f*x*2i) + 1)^4) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e
*2i + f*x*2i) + 1))^(1/2)*4288i)/(55*f*(exp(e*2i + f*x*2i) + 1)^5) + (d^2*
(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*896i)/(
15*f*(exp(e*2i + f*x*2i) + 1)^6) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))
/(exp(e*2i + f*x*2i) + 1))^(1/2)*256i)/(15*f*(exp(e*2i + f*x*2i) + 1)^7)
```

## Reduce [F]

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(fx+e)}\,\sec(fx+e)^6\tan(fx+e)^2\,dx\right)d^2$$

input
```
int(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*sec(e + f*x)**6*tan(e + f*x)**2,x)*d**2
```

# 3.248 $\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1888
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . 1888
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1889
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1890
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . 1891
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1891
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . 1891
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . 1892
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . 1892
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1893


## Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(d\tan(e+fx))^{7/2}}{7df} + \frac{2(d\tan(e+fx))^{11/2}}{11d^3f}$$

output
```
2/7*(d*tan(f*x+e))^(7/2)/d/f+2/11*(d*tan(f*x+e))^(11/2)/d^3/f
```

## Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(9+2\cos(2(e+fx)))\sec^2(e+fx)(d\tan(e+fx))^{7/2}}{77df}$$

input
```
Integrate[Sec[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

output
```
(2*(9 + 2*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(d*Tan[e + f*x])^(7/2))/(77*d*f
)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^4(d\tan(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (d\tan(e+fx))^{5/2}\left(\tan^2(e+fx)+1\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(e+fx))^{9/2}}{d^2}+(d\tan(e+fx))^{5/2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(e+fx))^{11/2}}{11d^3}+\frac{2(d\tan(e+fx))^{7/2}}{7d}}{f}$$

input
```
Int[Sec[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

output
```
((2*(d*Tan[e + f*x])^(7/2))/(7*d) + (2*(d*Tan[e + f*x])^(11/2))/(11*d^3))/
f
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 99.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{2d^2(d\tan(fx+e))^{\frac{7}{2}}}{7}}{f\,d^3}$ | 37 |
| default | $\frac{\frac{2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{2d^2(d\tan(fx+e))^{\frac{7}{2}}}{7}}{f\,d^3}$ | 37 |

input
```
int(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/d^3*(1/11*(d*tan(f*x+e))^(11/2)+1/7*d^2*(d*tan(f*x+e))^(7/2))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.53

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{2\left(4\,d^2\cos\left(fx+e\right)^4+3\,d^2\cos\left(fx+e\right)^2-7\,d^2\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\,\sin\left(fx+e\right)}{77\,f\cos\left(fx+e\right)^5}$$

input
```
integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-2/77*(4*d^2*cos(f*x + e)^4 + 3*d^2*cos(f*x + e)^2 - 7*d^2)*sqrt(d*sin(f*x
 + e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e)^5)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**4*(d*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\left(7\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}+11\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^2\right)}{77\,d^3f}$$

input
```
integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
2/77*(7*(d*tan(f*x + e))^(11/2) + 11*(d*tan(f*x + e))^(7/2)*d^2)/(d^3*f)
```

## Giac [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\left(7\sqrt{d\tan(fx+e)}d^5\tan(fx+e)^5+11\sqrt{d\tan(fx+e)}d^5\tan(fx+e)^3\right)}{77\,d^3f}$$

input
```
integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
2/77*(7*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e)^5 + 11*sqrt(d*tan(f*x + e))*
d^5*tan(f*x + e)^3)/(d^3*f)
```

## Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 352, normalized size of antiderivative = 7.82

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,8i}{77\,f} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,8i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,296i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^2} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,944i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^3} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,160i}{11\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^4} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{11\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^5}$$

input
```
int((d*tan(e + f*x))^(5/2)/cos(e + f*x)^4,x)
```

output
```
(d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*8i
)/(77*f) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1
))^(1/2)*8i)/(77*f*(exp(e*2i + f*x*2i) + 1)) - (d^2*(-(d*(exp(e*2i + f*x*2
i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*296i)/(77*f*(exp(e*2i + f*x*2
i) + 1)^2) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) +
 1))^(1/2)*944i)/(77*f*(exp(e*2i + f*x*2i) + 1)^3) - (d^2*(-(d*(exp(e*2i +
 f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*160i)/(11*f*(exp(e*2i +
 f*x*2i) + 1)^4) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x
*2i) + 1))^(1/2)*64i)/(11*f*(exp(e*2i + f*x*2i) + 1)^5)
```

## Reduce [F]

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(fx+e)}\,\sec(fx+e)^4\tan(fx+e)^2\,dx\right)d^2$$

input
```
int(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*sec(e + f*x)**4*tan(e + f*x)**2,x)*d**2
```

# 3.249 $\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal result . . . 1894
Mathematica [A] (verified) . . . 1894
Rubi [A] (verified) . . . 1895
Maple [A] (verified) . . . 1896
Fricas [B] (verification not implemented) . . . 1896
Sympy [F(-1)] . . . 1897
Maxima [A] (verification not implemented) . . . 1897
Giac [A] (verification not implemented) . . . 1897
Mupad [B] (verification not implemented) . . . 1898
Reduce [F] . . . 1898

## Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx=\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

output
```
2/7*(d*tan(f*x+e))^(7/2)/d/f
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx=\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

input
```
Integrate[Sec[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

output
```
(2*(d*Tan[e + f*x])^(7/2))/(7*d*f)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(e+fx)^2(d\tan(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{\int (d\tan(e+fx))^{5/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

input
```
Int[Sec[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

output
```
(2*(d*Tan[e + f*x])^(7/2))/(7*d*f)
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7df}$ | 19 |
| default | $\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7df}$ | 19 |

input
```
int(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*(d*tan(f*x+e))^(7/2)/d/f
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{2\left(d^2\cos\left(fx+e\right)^2-d^2\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{7\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-2/7*(d^2*cos(f*x + e)^2 - d^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x
+ e)/(f*cos(f*x + e)^3)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**2*(d*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\,(d\tan(fx+e))^{\frac{7}{2}}}{7\,df}$$

input
```
integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
2/7*(d*tan(f*x + e))^(7/2)/(d*f)
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\,\sqrt{d\tan(fx+e)}d^2\tan(fx+e)^3}{7\,f}$$

input
```
integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
2/7*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e)^3/f
```

## Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 230, normalized size of antiderivative = 10.45

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,2i}{7\,f} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,12i}{7\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,24i}{7\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^2} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,16i}{7\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^3}$$

input
```
int((d*tan(e + f*x))^(5/2)/cos(e + f*x)^2,x)
```

output
```
(d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*2i
)/(7*f) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1)
)^(1/2)*12i)/(7*f*(exp(e*2i + f*x*2i) + 1)) + (d^2*(-(d*(exp(e*2i + f*x*2i
)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*24i)/(7*f*(exp(e*2i + f*x*2i)
+ 1)^2) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1)
)^(1/2)*16i)/(7*f*(exp(e*2i + f*x*2i) + 1)^3)
```

## Reduce [F]

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(fx+e)}\,\sec(fx+e)^2\tan(fx+e)^2\,dx\right)d^2$$

input
```
int(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*sec(e + f*x)**2*tan(e + f*x)**2,x)*d**2
```

# 3.250 $\int (d\tan(e+fx))^{5/2}\,dx$

Optimal result . . . 1899
Mathematica [A] (verified) . . . 1900
Rubi [A] (warning: unable to verify) . . . 1900
Maple [A] (verified) . . . 1904
Fricas [A] (verification not implemented) . . . 1905
Sympy [F] . . . 1906
Maxima [A] (verification not implemented) . . . 1906
Giac [F(-2)] . . . 1907
Mupad [B] (verification not implemented) . . . 1907
Reduce [F] . . . 1908

## Optimal result

Integrand size = 12, antiderivative size = 156

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}+\sqrt{d}\tan(e+fx)}\right)}{\sqrt{2}f} + \frac{2d(d\tan(e+fx))^{3/2}}{3f}$$

output

```
1/2*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-1/2*d
^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+1/2*d^(5/2
)*arctanh(2^(1/2)*(d*tan(f*x+e))^(1/2)/(d^(1/2)+d^(1/2)*tan(f*x+e)))*2^(1/
2)/f+2/3*d*(d*tan(f*x+e))^(3/2)/f
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.65

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{d(d\tan(e+fx))^{3/2}\left(-3\arctan\left(\sqrt[4]{-\tan^2(e+fx)}\right)\sqrt[4]{-\tan(e+fx)}+3\text{arctanh}\left(\sqrt[4]{-\tan\right.\right.}{3f\tan^{\frac{7}{4}}(e+fx)}$$

input
```
Integrate[(d*Tan[e + f*x])^(5/2),x]
```

output
```
(d*(d*Tan[e + f*x])^(3/2)*(-3*ArcTan[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*
x])^(1/4) + 3*ArcTanh[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*x])^(1/4) + 2*T
an[e + f*x]^(7/4)))/(3*f*Tan[e + f*x]^(7/4))
```

## Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\tan(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-d^2\int \sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-d^2\int\sqrt{d\tan(e+fx)}dx$$

$\Big\downarrow$ 3957

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{d^3\int\frac{\sqrt{d\tan(e+fx)}}{\tan^2(e+fx)d^2+d^2}d(d\tan(e+fx))}{f}$$

$\Big\downarrow$ 266

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\int\frac{d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}}{f}$$

$\Big\downarrow$ 826

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\int\frac{d^2\tan^2(e+fx)+d}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}-\frac{1}{2}\int\frac{d-d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}\right)}{f}$$

$\Big\downarrow$ 1476

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fa\right.\right.}{f}$$

$\Big\downarrow$ 1082

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\tan^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\tan^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\tan(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\,\mathrm{ta}\right.}{f}$$

$\Big\downarrow$ 217

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}\right)}{f}$$

$\Big\downarrow$ 1479

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\right.}{}\right.}{f}$$

$\downarrow$ 25

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}}{}\right.}{f}$$

$\downarrow$ 27

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}}{}\right.}{f}$$

$\downarrow$ 1103

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(e+fx)+d^2\tan^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}\right.}{}\right.}{f}$$

input
```
Int[(d*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
ArcTan[1 + Sqrt[2]*Sqrt[d]*Tan[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - S
qrt[2]*d^(3/2)*Tan[e + f*x] + d^2*Tan[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Lo
g[d + Sqrt[2]*d^(3/2)*Tan[e + f*x] + d^2*Tan[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d
]))/2))/f + (2*d*(d*Tan[e + f*x])^(3/2))/(3*f)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| derivativedivides | $\frac{2d\left(\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f}$ |
| default | $\frac{2d\left(\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f}$ |

input
```
int((d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*d*(1/3*(d*tan(f*x+e))^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x
+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d
^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2
)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06

$$\int (d\tan(e+fx))^{5/2}\,dx =$$
$$-\frac{6\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d}{d}\right)+6\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}-d}{d}\right)-3\sqrt{2}d^{\frac{5}{2}}\log\left(d\tan(fx+e)\right.}{}$$

input
```
integrate((d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(6*sqrt(2)*d^(5/2)*arctan((sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)
/d) + 6*sqrt(2)*d^(5/2)*arctan((sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) - d)/
d) - 3*sqrt(2)*d^(5/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*s
qrt(d) + d) + 3*sqrt(2)*d^(5/2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d*tan(f*
x + e))*sqrt(d) + d) - 8*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e))/f
```

## Sympy [F]

$$\int (d\tan(e+fx))^{5/2}\,dx = \int (d\tan{(e+fx)})^{\frac{5}{2}}\;dx$$

input
```
integrate((d*tan(f*x+e))**(5/2),x)
```

output
```
Integral((d*tan(e + f*x))**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.13

$$\int (d\tan(e+fx))^{5/2}\,dx = -\frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e}\right.}{\sqrt{d}}\right.}{12\,df}$$

input
```
integrate((d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
-1/12*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e
) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(
f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) - 8*(d*tan(f
*x + e))^(3/2)*d^2)/(d*f)
```

## Giac [F(-2)]

Exception generated.

$$\int (d\tan(e+fx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[4,14]%%%}+%%%{6,[4,12]%%%}+%%%{15,[4,10]%%%}+%%%{20
,[4,8]%%%
```

## Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.47

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{2\,d\,(d\tan(e+f\,x))^{3/2}}{3\,f} - \frac{(-1)^{1/4}\,d^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{f} + \frac{(-1)^{1/4}\,d^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{f}$$

input
```
int((d*tan(e + f*x))^(5/2),x)
```

output
```
(2*d*(d*tan(e + f*x))^(3/2))/(3*f) - ((-1)^(1/4)*d^(5/2)*atan(((-1)^(1/4)*
(d*tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(1/4)*d^(5/2)*atanh(((-1)^(1/4
)*(d*tan(e + f*x))^(1/2))/d^(1/2)))/f
```

## Reduce [F]

$$\int (d\tan(e+fx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan\left(fx+e\right)}\,\tan\left(fx+e\right)^2 dx\right) d^2$$

input
```
int((d*tan(f*x+e))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*tan(e + f*x)**2,x)*d**2
```

# 3.251 $\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal result . . . 1909
Mathematica [A] (verified) . . . 1910
Rubi [A] (verified) . . . 1910
Maple [B] (verified) . . . 1914
Fricas [B] (verification not implemented) . . . 1915
Sympy [F(-1)] . . . 1916
Maxima [A] (verification not implemented) . . . 1916
Giac [A] (verification not implemented) . . . 1917
Mupad [F(-1)] . . . 1917
Reduce [F] . . . 1918

## Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{3d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}-\frac{3d^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}(1+\tan(e+fx))}\right)}{4\sqrt{2}f}-\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f}$$

output
```
-3/8*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+3/8*
d^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-3/8*d^(5/
2)*arctanh(2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2)/(1+tan(f*x+e)))*2^(1/2)/f-
1/2*d*cos(f*x+e)^2*(d*tan(f*x+e))^(3/2)/f
```

## Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.64

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx =$$
$$-\frac{d^2\left(3\arcsin(\cos(e+fx)-\sin(e+fx))\csc(e+fx)+3\csc(e+fx)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\right.\right.}{8f}$$

input
```
Integrate[Cos[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

output
```
-1/8*(d^2*(3*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x] + 3*Csc[e +
f*x]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]] + 2*Sqrt[Si
n[2*(e + f*x)]])*Sqrt[Sin[2*(e + f*x)]]*Sqrt[d*Tan[e + f*x]])/f
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.35, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3087, 252, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(d\tan(e+fx))^{5/2}}{\sec(e+fx)^2}dx$$
$$\Big\downarrow \text{ 3087}$$
$$\frac{\int \frac{(d\tan(e+fx))^{5/2}}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{ 252}$$

$$\frac{\frac{3}{4}d^2\int \frac{\sqrt{d\tan(e+fx)}}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{d(d\tan(e+fx))^{3/2}}{2(\tan^2(e+fx)+1)}}{f}$$

$\Big\downarrow$ 266

$$\frac{\frac{3}{2}d\int \frac{d^3\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}-\frac{d(d\tan(e+fx))^{3/2}}{2(\tan^2(e+fx)+1)}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{3}{2}d^3\int \frac{d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}-\frac{d(d\tan(e+fx))^{3/2}}{2(\tan^2(e+fx)+1)}}{f}$$

$\Big\downarrow$ 826

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\int \frac{\tan(e+fx)d+d}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}\right)-\frac{d(d\tan(e+fx))^{3/2}}{2(\tan^2(e+fx)+1)}}{f}$$

$\Big\downarrow$ 1476

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}+\frac{1}{2}\int \frac{1}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx}\right.\right.}{f}$$

$\Big\downarrow$ 1082

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d\tan(e+fx)-1}d\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d\tan(e+fx)-1}d\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e\right.}{f}$$

$\Big\downarrow$ 217

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}\right)-\frac{d(d\tan(e-}{2(\tan^2(e-}}{f}$$

$\Big\downarrow$ 1479

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\right.}{}\right.\right.}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{f}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(d\tan(e+fx)-\sqrt{2}\sqrt{d}\sqrt{d\tan(e+fx)}+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(d\tan\right.}{}\right.\right.}{f}$$

input
```
Int[Cos[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

output
```
((3*d^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sq
rt[d])) + ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt
[d]))/2 + (Log[d + d*Tan[e + f*x] - Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[e + f*x]]]/
(2*Sqrt[2]*Sqrt[d]) - Log[d + d*Tan[e + f*x] + Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[
e + f*x]]]/(2*Sqrt[2]*Sqrt[d]))/2))/2 - (d*(d*Tan[e + f*x])^(3/2))/(2*(1 +
 Tan[e + f*x]^2)))/f
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(127) = 254$.

Time = 2.79 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.44

| method | result |
|---|---|
| default | $-\frac{\left(-3\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)-2\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+\csc(fx+e)-\sin(fx+e)-2\cos(fx+e)+2}{\cos(fx+e)-1}\right)+3\ln\left(-\right.\right.}{}$ |

input
```
int(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/f*(-3*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*sin(f*x+e)*(-2*sin(f
*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*cos(f*x+e
)+2)/(cos(f*x+e)-1))+3*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e
)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-
2*cos(f*x+e)+2)/(cos(f*x+e)-1))-6*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)-1))-6*arctan((sin(f
*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(cos
(f*x+e)-1))+(4+4*cos(f*x+e))*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2))*cos(f*x+e)*(d*tan(f*x+e))^(1/2)*d^2/(1+cos(f*x+e))/(-sin(
f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(127) = 254$.

Time = 0.16 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.65

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx =$$

$$-\frac{16\,d^2\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+6\,\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{d\cos(fx+e)-d\sin(fx+e)}\right)+3\,\sqrt{2}d^{\frac{5}{2}}\operatorname{arctan}\cdots}{}$$

input
```
integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/32*(16*d^2*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
+ 6*sqrt(2)*d^(5/2)*arctan(-sqrt(2)*sqrt(d)*sqrt(d*sin(f*x + e)/cos(f*x +
e))*cos(f*x + e)/(d*cos(f*x + e) - d*sin(f*x + e))) + 3*sqrt(2)*d^(5/2)*ar
ctan(1/2*(2*d*cos(f*x + e)^2 - 2*d*cos(f*x + e)*sin(f*x + e) + sqrt(2)*sqr
t(d)*sqrt(d*sin(f*x + e)/cos(f*x + e)) - 2*d)/(d*cos(f*x + e)^2 + d*cos(f*
x + e)*sin(f*x + e) - d)) + 3*sqrt(2)*d^(5/2)*arctan(-1/2*(2*d*cos(f*x + e
)^2 - 2*d*cos(f*x + e)*sin(f*x + e) - sqrt(2)*sqrt(d)*sqrt(d*sin(f*x + e)/
cos(f*x + e)) - 2*d)/(d*cos(f*x + e)^2 + d*cos(f*x + e)*sin(f*x + e) - d))
 + 3*sqrt(2)*d^(5/2)*log(4*d*cos(f*x + e)*sin(f*x + e) + 2*sqrt(2)*(cos(f*
x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(d)*sqrt(d*sin(f*x + e)/cos(f*x
+ e)) + d) - 3*sqrt(2)*d^(5/2)*log(4*d*cos(f*x + e)*sin(f*x + e) - 2*sqrt(
2)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(d)*sqrt(d*sin(f*x + e
)/cos(f*x + e)) + d))/f
```

## Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**2*(d*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.16

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(fx+e)+\right.}{}\right.}{16\,df}$$

input
```
integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
1/16*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e)
 + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f
*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) - 8*(d*tan(f*
x + e))^(3/2)*d^4/(d^2*tan(f*x + e)^2 + d^2))/(d*f)
```

## Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.38

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx =$$

$$-\frac{1}{16}\left(\frac{8\sqrt{d\tan(fx+e)}d^2\tan(fx+e)}{\left(d^2\tan(fx+e)^2+d^2\right)f}-\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{df}-\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\right.$$

input
```
integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
-1/16*(8*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e)/((d^2*tan(f*x + e)^2 + d^2)
*f) - 6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*
sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) - 6*sqrt(2)*abs(d)^(3/2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))
/(d*f) + 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*
x + e))*sqrt(abs(d)) + abs(d))/(d*f) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*
x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f))*d^2
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \int \cos(e+f\,x)^2\,(d\tan(e+f\,x))^{5/2}\,dx$$

input
```
int(cos(e + f*x)^2*(d*tan(e + f*x))^(5/2),x)
```

output
```
int(cos(e + f*x)^2*(d*tan(e + f*x))^(5/2), x)
```

## Reduce [F]

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \sqrt{d}\left(\int \sqrt{\tan(fx+e)}\cos(fx+e)^2\tan(fx+e)^2\,dx\right)d^2$$

input
```
int(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*cos(e + f*x)**2*tan(e + f*x)**2,x)*d**2
```

# 3.252 $\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal result . . . 1919
Mathematica [A] (verified) . . . 1920
Rubi [A] (verified) . . . 1920
Maple [B] (verified) . . . 1925
Fricas [B] (verification not implemented) . . . 1925
Sympy [F(-1)] . . . 1926
Maxima [A] (verification not implemented) . . . 1926
Giac [A] (verification not implemented) . . . 1927
Mupad [F(-1)] . . . 1928
Reduce [F] . . . 1928

## Optimal result

Integrand size = 21, antiderivative size = 195

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{3d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f} + \frac{3d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f} - \frac{3d^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}(1+\tan(e+fx))}\right)}{32\sqrt{2}f} + \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f}$$

output
```
-3/64*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f+3/6
4*d^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*2^(1/2)/f-3/64*d^
(5/2)*arctanh(2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2)/(1+tan(f*x+e)))*2^(1/2)
/f+3/16*d*cos(f*x+e)^2*(d*tan(f*x+e))^(3/2)/f-1/4*d*cos(f*x+e)^4*(d*tan(f*
x+e))^(3/2)/f
```

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.64

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx =$$

$$-\frac{d^2\left(3\arcsin(\cos(e+fx)-\sin(e+fx))\csc(e+fx)\sqrt{\sin(2(e+fx))}+3\csc(e+fx)\log\left(\cos(e+fx)+\right.\right.}{6}$$

input
```
Integrate[Cos[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

output
```
-1/64*(d^2*(3*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x]*Sqrt[Sin[2*
(e + f*x)]] + 3*Csc[e + f*x]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*
(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] - 2*Sin[2*(e + f*x)] + 2*Sin[4*(e + f*
x)])*Sqrt[d*Tan[e + f*x]])/f
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.34, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3087, 252, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(e+fx))^{5/2}}{\sec(e+fx)^4}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \frac{(d\tan(e+fx))^{5/2}}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{3}{8}d^2\int \frac{\sqrt{d\tan(e+fx)}}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)-\frac{d(d\tan(e+fx))^{3/2}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$\Big\downarrow$ 253

$$\frac{\frac{3}{8}d^2\left(\frac{1}{4}\int \frac{\sqrt{d\tan(e+fx)}}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}\right)-\frac{d(d\tan(e+fx))^{3/2}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$\Big\downarrow$ 266

$$\frac{\frac{3}{8}d^2\left(\frac{\int \frac{d^3\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}}{2d}+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}\right)-\frac{d(d\tan(e+fx))^{3/2}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\int \frac{d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}\right)-\frac{d(d\tan(e+fx))^{3/2}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$\Big\downarrow$ 826

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\int \frac{\tan(e+fx)d+d}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}\right)+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}\right)-\frac{d(d\text{t}}{4(\text{tan}}}{f}$$

$\Big\downarrow$ 1476

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}+\frac{1}{2}\int \frac{1}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+}\right.\right.\right.}{f}$$

$\Big\downarrow$ 1082

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d\tan(e+fx)-1}d\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d\tan(e+fx)-1}d\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\text{t}}\right.\right.}{f}$$

$\Big\downarrow$ 217

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}\right)+\frac{(d\text{t}}{2d(\text{t}}\right.}{f}$$

$\Big\downarrow$ 1479

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\text{arc}}{}\right.\right.\right.}{f}$$

$\downarrow$ 25

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\text{arcta}}{}\right.\right.\right.}{f}$$

$\downarrow$ 27

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\text{arcta}}{}\right.\right.\right.}{f}$$

$\downarrow$ 1103

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(d\tan(e+fx)-\sqrt{2}\sqrt{d}\sqrt{d\tan(e+fx)}+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log}{}\right.\right.\right.}{f}$$

input
```
Int[Cos[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

output
```
(-1/4*(d*(d*Tan[e + f*x])^(3/2))/(1 + Tan[e + f*x]^2)^2 + (3*d^2*((d*((-(A
rcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d])) + Arc
Tan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]))/2 + (Lo
g[d + d*Tan[e + f*x] - Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[e + f*x]]]/(2*Sqrt[2]*Sq
rt[d]) - Log[d + d*Tan[e + f*x] + Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[e + f*x]]]/(2
*Sqrt[2]*Sqrt[d]))/2))/2 + (d*Tan[e + f*x])^(3/2)/(2*d*(1 + Tan[e + f*x]^2
))))/8)/f
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(151) = 302$.

Time = 61.21 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.57

| method | result |
|---|---|
| default | $-\frac{\left(-3\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)-2\sin(fx+e)\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(1+\cos(fx+e))^2}}+\csc(fx+e)-\sin(fx+e)-2\cos(fx+e)+2}{\cos(fx+e)-1}\right)+3\ln\left(-\cdots\right.\right.}{}$ |

input
```
int(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/128/f*(-3*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*sin(f*x+e)*(-2*sin(
f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*cos(f*x+
e)+2)/(cos(f*x+e)-1))+3*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+
e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)
-2*cos(f*x+e)+2)/(cos(f*x+e)-1))-6*arctan((sin(f*x+e)*(-2*sin(f*x+e)*cos(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)-1))-6*arctan((sin(
f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+cos(f*x+e)-1)/(co
s(f*x+e)-1))+(4*(4*cos(f*x+e)^2-3)*(1+cos(f*x+e))*sin(f*x+e)-8*cos(f*x+e)*
(cos(f*x+e)^2-1)*(1+cos(f*x+e)))*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^
2)^(1/2)+cos(f*x+e)*(-8*cos(f*x+e)-8)*2^(1/2)*sin(f*x+e)^2*(-sin(f*x+e)*co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)*(d*tan(f*x+e))^(1/2)*d^2/(-si
n(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)/(1+cos(f*x+e))*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(151) = 302$.

Time = 0.16 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.35

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx =$$

$$-\frac{6\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{d\cos(fx+e)-d\sin(fx+e)}\right)+3\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{2d\cos(fx+e)^2-2d\cos(fx+e)\sin(fx+e)+\sqrt{2}\sqrt{d}\sqrt{\frac{d\sin(\cdots}{\cos(\cdots}}\cdots}{2\left(d\cos(fx+e)^2+d\cos(fx+e)\sin(fx+e)-d\right)\cdots}\right.\cdots}{}$$

input
```
integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/256*(6*sqrt(2)*d^(5/2)*arctan(-sqrt(2)*sqrt(d)*sqrt(d*sin(f*x + e)/cos(
f*x + e))*cos(f*x + e)/(d*cos(f*x + e) - d*sin(f*x + e))) + 3*sqrt(2)*d^(5
/2)*arctan(1/2*(2*d*cos(f*x + e)^2 - 2*d*cos(f*x + e)*sin(f*x + e) + sqrt(
2)*sqrt(d)*sqrt(d*sin(f*x + e)/cos(f*x + e)) - 2*d)/(d*cos(f*x + e)^2 + d*
cos(f*x + e)*sin(f*x + e) - d)) + 3*sqrt(2)*d^(5/2)*arctan(-1/2*(2*d*cos(f
*x + e)^2 - 2*d*cos(f*x + e)*sin(f*x + e) - sqrt(2)*sqrt(d)*sqrt(d*sin(f*x
 + e)/cos(f*x + e)) - 2*d)/(d*cos(f*x + e)^2 + d*cos(f*x + e)*sin(f*x + e)
 - d)) + 3*sqrt(2)*d^(5/2)*log(4*d*cos(f*x + e)*sin(f*x + e) + 2*sqrt(2)*(
cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(d)*sqrt(d*sin(f*x + e)/co
s(f*x + e)) + d) - 3*sqrt(2)*d^(5/2)*log(4*d*cos(f*x + e)*sin(f*x + e) - 2
*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(d)*sqrt(d*sin(f
*x + e)/cos(f*x + e)) + d) + 16*(4*d^2*cos(f*x + e)^3 - 3*d^2*cos(f*x + e)
)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e))/f
```

## Sympy [F(-1)]

Timed out.

$$\int \cos^4(e + fx)(d\tan(e + fx))^{5/2}\, dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4*(d*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.15

$$\int \cos^4(e + fx)(d\tan(e + fx))^{5/2}\, dx = \frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(fx+e)+\right.}{}\right.}{1}$$

input
```
integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
1/128*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e
) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(
f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) + 8*(3*(d*ta
n(f*x + e))^(7/2)*d^4 - (d*tan(f*x + e))^(3/2)*d^6)/(d^4*tan(f*x + e)^4 +
2*d^4*tan(f*x + e)^2 + d^4))/(d*f)
```

## Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.32

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{1}{128}\,d^2\left(\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{df}+\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\right.}}{2\sqrt{|d|}}\right.}{df}\right.$$

input
```
integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
1/128*d^2*(6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
 + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) + 6*sqrt(2)*abs(d)^(3/2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs
(d)))/(d*f) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*t
an(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) + 3*sqrt(2)*abs(d)^(3/2)*log(d*t
an(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) +
8*(3*sqrt(d*tan(f*x + e))*d^4*tan(f*x + e)^3 - sqrt(d*tan(f*x + e))*d^4*ta
n(f*x + e))/((d^2*tan(f*x + e)^2 + d^2)^2*f))
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx=\int \cos\left(e+f\,x\right)^4\,\left(d\tan(e+f\,x)\right)^{5/2}\,dx$$

input
```
int(cos(e + f*x)^4*(d*tan(e + f*x))^(5/2),x)
```

output
```
int(cos(e + f*x)^4*(d*tan(e + f*x))^(5/2), x)
```

## Reduce [F]

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx=\sqrt{d}\,\left(\int \sqrt{\tan\left(fx+e\right)}\,\cos\left(fx+e\right)^4\tan\left(fx+e\right)^2dx\right)d^2$$

input
```
int(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x)
```

output
```
sqrt(d)*int(sqrt(tan(e + f*x))*cos(e + f*x)**4*tan(e + f*x)**2,x)*d**2
```

# 3.253 $\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal result . . . 1929
Mathematica [C] (verified) . . . 1930
Rubi [A] (verified) . . . 1930
Maple [A] (verified) . . . 1933
Fricas [C] (verification not implemented) . . . 1933
Sympy [F] . . . 1934
Maxima [F] . . . 1934
Giac [F] . . . 1935
Mupad [F(-1)] . . . 1935
Reduce [F] . . . 1935

## Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{4\operatorname{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{7f\sqrt{d\tan(e+fx)}} + \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df} + \frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

output
```
4/7*InverseJacobiAM(e-1/4*Pi+f*x,2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/
f/(d*tan(f*x+e))^(1/2)+4/7*sec(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f+2/7*sec(f*x
+e)^3*(d*tan(f*x+e))^(1/2)/d/f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.62 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\left((2+\cos(2(e+fx)))\sec^4(e+fx)+4\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\right)\sin(e+fx)}{7f\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^5/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*((2 + Cos[2*(e + f*x)])*Sec[e + f*x]^4 + 4*Hypergeometric2F1[1/4, 1/2,
5/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^2])*Sin[e + f*x])/(7*f*Sqrt[d*Tan[
e + f*x]])
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3093, 3042, 3093, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^5}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{6}{7}\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \frac{\sec(e+fx)^3}{\sqrt{d\tan(e+fx)}}dx+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$$\Big\downarrow \text{3093}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\right)+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\right)+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$$\Big\downarrow \text{3094}$$

$$\frac{6}{7}\left(\frac{2\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{3\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\right)+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{2\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{3\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\right)+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$$\Big\downarrow \text{3053}$$

$$\frac{6}{7}\left(\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\right)+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\right)+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{6}{7}\left(\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{3f\sqrt{d\tan(e+fx)}}\right)$$

input
```
Int[Sec[e + f*x]^5/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*Sec[e + f*x]^3*Sqrt[d*Tan[e + f*x]])/(7*d*f) + (6*((2*EllipticF[e - Pi/
4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(3*f*Sqrt[d*Tan[e + f*x]]
) + (2*Sec[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*d*f)))/7
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

| method | result |
|---|---|
| default | $\frac{\frac{2\tan(fx+e)\sec(fx+e)^3\left(2\cos(fx+e)^2+1\right)}{7}+\frac{2\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{2\cot(fx+e)-2\csc(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\,\operatorname{EllipticF}\left(\sqrt{\right.}{7}}{\sqrt{d\tan(fx+e)}\,f}$ |

input
```
int(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7/f/(d*tan(f*x+e))^(1/2)*(tan(f*x+e)*sec(f*x+e)^3*(2*cos(f*x+e)^2+1)+(cs
c(f*x+e)-cot(f*x+e)+1)^(1/2)*(2*cot(f*x+e)-2*csc(f*x+e)+2)^(1/2)*(-csc(f*x
+e)+cot(f*x+e))^(1/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2
))*(2+2*sec(f*x+e)))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx =$$
$$-\frac{2\left(2\sqrt{i\,d}\cos\left(fx+e\right)^3F(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\mid-1)+2\sqrt{-i\,d}\cos\left(fx+e\right)^3F(\arcsin\right.}{7\,df\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/7*(2*sqrt(I*d)*cos(f*x + e)^3*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*
x + e)), -1) + 2*sqrt(-I*d)*cos(f*x + e)^3*elliptic_f(arcsin(cos(f*x + e)
- I*sin(f*x + e)), -1) - (2*cos(f*x + e)^2 + 1)*sqrt(d*sin(f*x + e)/cos(f*
x + e)))/(d*f*cos(f*x + e)^3)
```

## Sympy [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx=\int \frac{\sec^5{(e+fx)}}{\sqrt{d\tan{(e+fx)}}}\,dx$$

input
```
integrate(sec(f*x+e)**5/(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)**5/sqrt(d*tan(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx=\int \frac{\sec{(fx+e)}^5}{\sqrt{d\tan{(fx+e)}}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^5/sqrt(d*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx=\int \frac{\sec\left(fx+e\right)^5}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^5/sqrt(d*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)^5\,\sqrt{d\tan\left(e+f\,x\right)}}\,dx$$

input
```
int(1/(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx=\frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sec(fx+e)^5}{\tan(fx+e)}dx\right)}{d}$$

input
```
int(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(e + f*x))*sec(e + f*x)**5)/tan(e + f*x),x))/d
```

# 3.254 $\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1936
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . 1936
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1937
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1939
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . 1939
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1940
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1940
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1940
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1941
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1941

## Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{3f\sqrt{d\tan(e+fx)}} + \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

output
```
2/3*InverseJacobiAM(e-1/4*Pi+f*x,2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/
f/(d*tan(f*x+e))^(1/2)+2/3*sec(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\left(\sec^2(e+fx)+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\right)\sin(e+fx)}{3f\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(Sec[e + f*x]^2 + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[e + f*x]^2]*S
qrt[Sec[e + f*x]^2])*Sin[e + f*x])/(3*f*Sqrt[d*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3093, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^3}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{2}{3}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3094}$$

$$\frac{2\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{3\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{3\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{3f\sqrt{d\tan(e+fx)}}$$

input
```
Int[Sec[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(3*f*
Sqrt[d*Tan[e + f*x]]) + (2*Sec[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| default | $\frac{\frac{2\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{2\cot(fx+e)-2\csc(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \mathrm{EllipticF}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1},\frac{\sqrt{2}}{2}\right)(1+\sec(fx+e))}{3}}{\sqrt{d\tan(fx+e)}\,f}$ |

input
```
int(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/f/(d*tan(f*x+e))^(1/2)*((csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(2*cot(f*x+e)-
2*csc(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticF((csc(f*x+e)
-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(1+sec(f*x+e))+sec(f*x+e)*tan(f*x+e))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2\left(\sqrt{i\,d}\cos\left(fx+e\right)F(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\,|\,-1)+\sqrt{-i\,d}\cos\left(fx+e\right)F(\arcsin\left(\cos\right.\right.}{3\,df\cos\left(fx+e\right)}$$

input
```
integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(I*d)*cos(f*x + e)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x +
e)), -1) + sqrt(-I*d)*cos(f*x + e)*elliptic_f(arcsin(cos(f*x + e) - I*sin(
f*x + e)), -1) - sqrt(d*sin(f*x + e)/cos(f*x + e)))/(d*f*cos(f*x + e))
```

## Sympy [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx=\int \frac{\sec^3{(e+fx)}}{\sqrt{d\tan{(e+fx)}}}\,dx$$

input
```
integrate(sec(f*x+e)**3/(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)**3/sqrt(d*tan(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx=\int \frac{\sec{(fx+e)}^3}{\sqrt{d\tan{(fx+e)}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx=\int \frac{\sec{(fx+e)}^3}{\sqrt{d\tan{(fx+e)}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^3\,\sqrt{d\tan\left(e+f\,x\right)}}\,dx$$

input
```
int(1/(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sec(fx+e)^3}{\tan(fx+e)}dx\right)}{d}$$

input
```
int(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(e + f*x))*sec(e + f*x)**3)/tan(e + f*x),x))/d
```

# 3.255 $\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal result . . . . . 1942
Mathematica [C] (verified) . . . . . 1942
Rubi [A] (verified) . . . . . 1943
Maple [B] (verified) . . . . . 1945
Fricas [C] (verification not implemented) . . . . . 1945
Sympy [F] . . . . . 1946
Maxima [F] . . . . . 1946
Giac [F] . . . . . 1946
Mupad [F(-1)] . . . . . 1947
Reduce [F] . . . . . 1947

## Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\operatorname{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{f\sqrt{d\tan(e+fx)}}$$

output
```
InverseJacobiAM(e-1/4*Pi+f*x,2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d
*tan(f*x+e))^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2\sqrt[4]{-1}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right),-1\right)\sec^3(e+fx)\sqrt{\tan(e+fx)}}{f\sqrt{d\tan(e+fx)}\left(1+\tan^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Sec[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(-2*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[e + f*x]]], -1]*Sec
[e + f*x]^3*Sqrt[Tan[e + f*x]])/(f*Sqrt[d*Tan[e + f*x]]*(1 + Tan[e + f*x]^
2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3094}$$

$$\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{\sqrt{d\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{\sqrt{d\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{f\sqrt{d\tan(e+fx)}}$$

input
```
Int[Sec[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(f*Sqrt
[d*Tan[e + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(43) = 86.

Time = 0.91 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.17

| method | result |
|---|---|
| default | $\frac{\text{EllipticF}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-\csc(fx+e)+\cot(fx+e)}\,\sqrt{2\cot(fx+e)-2\csc(fx+e)+2}\,\sqrt{\csc(fx+e)-\cot(fx+e)\cdots}}{f\sqrt{d\tan(fx+e)}}$ |

input
```
int(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-csc(f*x+e)+co
t(f*x+e))^(1/2)*(2*cot(f*x+e)-2*csc(f*x+e)+2)^(1/2)*(csc(f*x+e)-cot(f*x+e)
+1)^(1/2)/(d*tan(f*x+e))^(1/2)*(1+sec(f*x+e))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{\sqrt{i\,d}F(\arcsin(\cos(fx+e)+i\sin(fx+e))\mid -1)+\sqrt{-i\,d}F(\arcsin(\cos(fx+e)-i\sin(fx+e))\mid \cdots}{df}$$

input
```
integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(I*d)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) + sqrt(-
I*d)*elliptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1))/(d*f)
```

## Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\,dx$$

input
```
integrate(sec(f*x+e)/(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)/sqrt(d*tan(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{d\tan\left(e+f\,x\right)}}\,dx$$

input
```
int(1/(cos(e + f*x)*(d*tan(e + f*x))^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)*(d*tan(e + f*x))^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sec(fx+e)}{\tan(fx+e)}dx\right)}{d}$$

input
```
int(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(e + f*x))*sec(e + f*x))/tan(e + f*x),x))/d
```

# 3.256 $\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal result . . . 1948
Mathematica [C] (warning: unable to verify) . . . 1948
Rubi [A] (verified) . . . 1949
Maple [A] (verified) . . . 1951
Fricas [F] . . . 1952
Sympy [F] . . . 1952
Maxima [F] . . . 1952
Giac [F] . . . 1953
Mupad [F(-1)] . . . 1953
Reduce [F] . . . 1953

## Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\operatorname{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{2f\sqrt{d\tan(e+fx)}} + \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

output
```
1/2*InverseJacobiAM(e-1/4*Pi+f*x,2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/
f/(d*tan(f*x+e))^(1/2)+cos(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f
```

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.66

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\cos(2(e+fx))\sec(e+fx)\left(\sqrt[4]{-1}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right),-1\right)\sec^2(e+fx)-\sqrt{\sec^2}\right.}{f\sqrt{\sec^2(e+fx)}\sqrt{d\tan(e+fx)}\left(-1+\tan^2(e+fx)\right)}$$

input
```
Integrate[Cos[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Cos[2*(e + f*x)]*Sec[e + f*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*
Sqrt[Tan[e + f*x]]], -1]*Sec[e + f*x]^2 - Sqrt[Sec[e + f*x]^2]*Sqrt[Tan[e
+ f*x]])*Sqrt[Tan[e + f*x]])/(f*Sqrt[Sec[e + f*x]^2]*Sqrt[d*Tan[e + f*x]]*
(-1 + Tan[e + f*x]^2))
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3092, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{1}{2}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3094}$$

$$\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3053}$$

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{2f\sqrt{d\tan(e+fx)}}$$

input
```
Int[Cos[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sq
rt[d*Tan[e + f*x]]) + (Cos[e + f*x]*Sqrt[d*Tan[e + f*x]])/(d*f)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.46

| method | result |
|---|---|
| default | $\frac{\frac{\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{2\cot(fx+e)-2\csc(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \text{EllipticF}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1},\frac{\sqrt{2}}{2}\right)(1+\sec(fx+e))}{2}}{f\sqrt{d\tan(fx+e)}}$ |

input
```
int(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(2*cot(f*x+e)-2*csc(f*x+e)+2)^(1/
2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2
),1/2*2^(1/2))*(1+sec(f*x+e))+sin(f*x+e))/(d*tan(f*x+e))^(1/2)
```

## Fricas [F]

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos(fx+e)}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*cos(f*x + e)/(d*tan(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$

input
```
integrate(cos(f*x+e)/(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(cos(e + f*x)/sqrt(d*tan(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos(fx+e)}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos(fx+e)}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos(e+f\,x)}{\sqrt{d\tan(e+f\,x)}}\,dx$$

input
```
int(cos(e + f*x)/(d*tan(e + f*x))^(1/2),x)
```

output
```
int(cos(e + f*x)/(d*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(fx+e)}\,\cos(fx+e)}{\tan(fx+e)}dx\right)}{d}$$

input
```
int(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(e + f*x))*cos(e + f*x))/tan(e + f*x),x))/d
```

# 3.257 $\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal result . . . 1954
Mathematica [C] (verified) . . . 1955
Rubi [A] (verified) . . . 1955
Maple [C] (verified) . . . 1958
Fricas [F] . . . 1959
Sympy [F(-1)] . . . 1959
Maxima [F] . . . 1959
Giac [F] . . . 1960
Mupad [F(-1)] . . . 1960
Reduce [F] . . . 1960

## Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{5\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{12f\sqrt{d\tan(e+fx)}} + \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

output
```
5/12*InverseJacobiAM(e-1/4*Pi+f*x,2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)
/f/(d*tan(f*x+e))^(1/2)+5/6*cos(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f+1/3*cos(f*
x+e)^3*(d*tan(f*x+e))^(1/2)/d/f
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{11\sin(e+fx)+\sin(3(e+fx))-10\sqrt[4]{-1}\cos(e+fx)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right),-1\right)\sqrt{\phantom{x}}}{12f\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[Cos[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(11*Sin[e + f*x] + Sin[3*(e + f*x)] - 10*(-1)^(1/4)*Cos[e + f*x]*EllipticF
[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[e + f*x]]], -1]*Sqrt[Sec[e + f*x]^2]*Sqrt[T
an[e + f*x]])/(12*f*Sqrt[d*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3092, 3042, 3092, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^3\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{5}{6}\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{6}\int \frac{1}{\sec(e+fx)\sqrt{d\tan(e+fx)}}dx+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \ 3092$$

$$\frac{5}{6}\left(\frac{1}{2}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\right)+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{6}\left(\frac{1}{2}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\right)+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \ 3094$$

$$\frac{5}{6}\left(\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\right)+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{6}\left(\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\right)+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \ 3053$$

$$\frac{5}{6}\left(\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\right)+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{6}\left(\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\right)+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \ 3120$$

$$\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{5}{6}\left(\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\operatorname{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{2f\sqrt{d\tan(e+fx)}}\right)$$

input
```
Int[Cos[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Cos[e + f*x]^3*Sqrt[d*Tan[e + f*x]])/(3*d*f) + (5*((EllipticF[e - Pi/4 +
f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqrt[d*Tan[e + f*x]]) +
(Cos[e + f*x]*Sqrt[d*Tan[e + f*x]])/(d*f)))/6
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.77 (sec) , antiderivative size = 1507, normalized size of antiderivative = 13.83

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1507 |

input
```
int(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/f/(d*tan(f*x+e))^(1/2)*(2^(1/2)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-2*sin
(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*cos(f*x
+e)+2)/(cos(f*x+e)-1))*(sin(f*x+e)*(6*cos(f*x+e)+6)-3-3*sec(f*x+e))+sin(f*
x+e)*(-12*cos(f*x+e)-12)*(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*l
n(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)+csc(f*x+e)-sin(f*x+e)-2*cos(f*x+e)+2)/(cos(f*x
+e)-1))+2^(1/2)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*ln((2*si
n(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cot(f*x+e)*cos(
f*x+e)+sin(f*x+e)+2*cos(f*x+e)+2*cot(f*x+e)-csc(f*x+e)-2)/(cos(f*x+e)-1))*
(sin(f*x+e)*(-6*cos(f*x+e)-6)+3+3*sec(f*x+e))+sin(f*x+e)*(12*cos(f*x+e)+12
)*(-sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*ln((2*sin(f*x+e)*(-2*sin
(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cot(f*x+e)*cos(f*x+e)+sin(f*x+e
)+2*cos(f*x+e)+2*cot(f*x+e)-csc(f*x+e)-2)/(cos(f*x+e)-1))+2^(1/2)*(-2*sin(
f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*arctan((sin(f*x+e)*(-2*sin(f*x+e
)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)-1))*(sin(f*
x+e)*(12*cos(f*x+e)+12)-6-6*sec(f*x+e))+sin(f*x+e)*(-24*cos(f*x+e)-24)*(-s
in(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*arctan((sin(f*x+e)*(-2*sin(f*
x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)-1))+2^(1
/2)*(-2*sin(f*x+e)*cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*arctan((sin(f*x+e...
```

## Fricas [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos(fx+e)^3}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^3/(d*tan(f*x + e)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(d*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos(fx+e)^3}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx=\int \frac{\cos\left(fx+e\right)^3}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx=\int \frac{\cos\left(e+f\,x\right)^3}{\sqrt{d\tan\left(e+f\,x\right)}}\,dx$$

input
```
int(cos(e + f*x)^3/(d*tan(e + f*x))^(1/2),x)
```

output
```
int(cos(e + f*x)^3/(d*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx=\frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(fx+e)}\,\cos(fx+e)^3}{\tan(fx+e)}dx\right)}{d}$$

input
```
int(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(e + f*x))*cos(e + f*x)**3)/tan(e + f*x),x))/d
```

# 3.258 $\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1961
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1961
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1962
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1963
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1964
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1964
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1964
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 1965
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1965
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1966

## Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{d\tan(a+bx)}}+\frac{4(d\tan(a+bx))^{3/2}}{3bd^3}+\frac{2(d\tan(a+bx))^{7/2}}{7bd^5}$$

output
```
-2/b/d/(d*tan(b*x+a))^(1/2)+4/3*(d*tan(b*x+a))^(3/2)/b/d^3+2/7*(d*tan(b*x+
a))^(7/2)/b/d^5
```

## Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{-42+(22+6\sec^2(a+bx))\tan^2(a+bx)}{21bd\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Sec[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-42 + (22 + 6*Sec[a + b*x]^2)*Tan[a + b*x]^2)/(21*b*d*Sqrt[d*Tan[a + b*x]
])
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^6}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \frac{\left(\tan^2(a+bx)+1\right)^2}{(d\tan(a+bx))^{3/2}}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(a+bx))^{5/2}}{d^4}+\frac{2\sqrt{d\tan(a+bx)}}{d^2}+\frac{1}{(d\tan(a+bx))^{3/2}}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(a+bx))^{7/2}}{7d^5}+\frac{4(d\tan(a+bx))^{3/2}}{3d^3}-\frac{2}{d\sqrt{d\tan(a+bx)}}}{b}$$

input
```
Int[Sec[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2/(d*Sqrt[d*Tan[a + b*x]]) + (4*(d*Tan[a + b*x])^(3/2))/(3*d^3) + (2*(d*
Tan[a + b*x])^(7/2))/(7*d^5))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

| method | result | size |
|---|---|---|
| default | $-\frac{2\left(32-8\sec(bx+a)^{2}-3\sec(bx+a)^{4}\right)}{21b\sqrt{d\tan(bx+a)}\,d}$ | 41 |

input
```
int(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21/b/(d*tan(b*x+a))^(1/2)/d*(32-8*sec(b*x+a)^2-3*sec(b*x+a)^4)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(32\cos\left(bx+a\right)^4-8\cos\left(bx+a\right)^2-3\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{21\,bd^2\cos\left(bx+a\right)^3\sin\left(bx+a\right)}$$

input
```
integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/21*(32*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 3)*sqrt(d*sin(b*x + a)/cos(b
*x + a))/(b*d^2*cos(b*x + a)^3*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^6\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)**6/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sec(a + b*x)**6/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(\frac{21}{\sqrt{d\tan(bx+a)}}-\frac{3\,(d\tan(bx+a))^{\frac{7}{2}}+14\,(d\tan(bx+a))^{\frac{3}{2}}d^2}{d^4}\right)}{21\,bd}$$

input
```
integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/21*(21/sqrt(d*tan(b*x + a)) - (3*(d*tan(b*x + a))^(7/2) + 14*(d*tan(b*x
 + a))^(3/2)*d^2)/d^4)/(b*d)
```

## Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(\frac{21}{\sqrt{d\tan(bx+a)}b}-\frac{3\sqrt{d\tan(bx+a)}b^6d^{27}\tan(bx+a)^3+14\sqrt{d\tan(bx+a)}b^6d^{27}\tan(bx+a)}{b^7d^{28}}\right)}{21\,d}$$

input
```
integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2/21*(21/(sqrt(d*tan(b*x + a))*b) - (3*sqrt(d*tan(b*x + a))*b^6*d^27*tan(
b*x + a)^3 + 14*sqrt(d*tan(b*x + a))*b^6*d^27*tan(b*x + a))/(b^7*d^28))/d
```

## Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 268, normalized size of antiderivative = 4.12

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\left(\frac{20\mathrm{i}}{21\,b\,d^2}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,64\mathrm{i}}{21\,b\,d^2}\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1} + \frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,20\mathrm{i}}{21\,b\,d^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)} + \frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,24\mathrm{i}}{7\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2} - \frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3}$$

input
```
int(1/(cos(a + b*x)^6*(d*tan(a + b*x))^(3/2)),x)
```

output
```
((-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*20i)/(
21*b*d^2*(exp(a*2i + b*x*2i) + 1)) - ((20i/(21*b*d^2) + (exp(a*2i + b*x*2i
)*64i)/(21*b*d^2))*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i)
+ 1))^(1/2))/(exp(a*2i + b*x*2i) - 1) + ((-(d*(exp(a*2i + b*x*2i)*1i - 1i)
)/(exp(a*2i + b*x*2i) + 1))^(1/2)*24i)/(7*b*d^2*(exp(a*2i + b*x*2i) + 1)^2
) - ((-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*16
i)/(7*b*d^2*(exp(a*2i + b*x*2i) + 1)^3)
```

## Reduce [F]

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)^6}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sec(a + b*x)**6)/tan(a + b*x)**2,x))/d**2
```

# 3.259 $\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$


Optimal result . . . . . . . . . . 1967
Mathematica [A] (verified) . . . . . . . . . . 1967
Rubi [A] (verified) . . . . . . . . . . 1968
Maple [A] (verified) . . . . . . . . . . 1969
Fricas [A] (verification not implemented) . . . . . . . . . . 1970
Sympy [F] . . . . . . . . . . 1970
Maxima [A] (verification not implemented) . . . . . . . . . . 1970
Giac [A] (verification not implemented) . . . . . . . . . . 1971
Mupad [B] (verification not implemented) . . . . . . . . . . 1971
Reduce [F] . . . . . . . . . . 1971


## Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{d\tan(a+bx)}}+\frac{2(d\tan(a+bx))^{3/2}}{3bd^3}$$

output
```
-2/b/d/(d*tan(b*x+a))^(1/2)+2/3*(d*tan(b*x+a))^(3/2)/b/d^3
```

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2(-3+\tan^2(a+bx))}{3bd\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Sec[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*(-3 + Tan[a + b*x]^2))/(3*b*d*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^4}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \frac{\tan^2(a+bx)+1}{(d\tan(a+bx))^{3/2}}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{\sqrt{d\tan(a+bx)}}{d^2}+\frac{1}{(d\tan(a+bx))^{3/2}}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(a+bx))^{3/2}}{3d^3}-\frac{2}{d\sqrt{d\tan(a+bx)}}}{b}$$

input
```
Int[Sec[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2/(d*Sqrt[d*Tan[a + b*x]]) + (2*(d*Tan[a + b*x])^(3/2))/(3*d^3))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

| method | result | size |
|---|---|---|
| default | $-\frac{2\left(4-\sec(bx+a)^2\right)}{3b\sqrt{d\tan(bx+a)}\,d}$ | 31 |

input
```
int(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b/(d*tan(b*x+a))^(1/2)/d*(4-sec(b*x+a)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(4\cos\left(bx+a\right)^2-1\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{3\,bd^2\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

input
```
integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(4*cos(b*x + a)^2 - 1)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*d^2*cos(b
*x + a)*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^4\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sec(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(\frac{3}{\sqrt{d\tan(bx+a)}}-\frac{(d\tan(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,bd}$$

input
```
integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/3*(3/sqrt(d*tan(b*x + a)) - (d*tan(b*x + a))^(3/2)/d^2)/(b*d)
```

## Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{d\tan(bx+a)}\tan(bx+a)}{bd} - \frac{3}{\sqrt{d\tan(bx+a)}b}\right)}{3\,d}$$

input
```
integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/3*(sqrt(d*tan(b*x + a))*tan(b*x + a)/(b*d) - 3/(sqrt(d*tan(b*x + a))*b))
/d
```

## Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{4\left(\sin\left(2\,a+2\,b\,x\right)+\sin\left(4\,a+4\,b\,x\right)\right)\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{3\,b\,d^2\sin\left(2\,a+2\,b\,x\right)^2}$$

input
```
int(1/(cos(a + b*x)^4*(d*tan(a + b*x))^(3/2)),x)
```

output
```
-(4*(sin(2*a + 2*b*x) + sin(4*a + 4*b*x))*((d*sin(2*a + 2*b*x))/(cos(2*a +
 2*b*x) + 1))^(1/2))/(3*b*d^2*sin(2*a + 2*b*x)^2)
```

## Reduce [F]

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)^4}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sec(a + b*x)**4)/tan(a + b*x)**2,x))/d**2
```

# 3.260 $\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result . . . . . 1972
Mathematica [A] (verified) . . . . . 1972
Rubi [A] (verified) . . . . . 1973
Maple [A] (verified) . . . . . 1974
Fricas [B] (verification not implemented) . . . . . 1974
Sympy [F] . . . . . 1975
Maxima [A] (verification not implemented) . . . . . 1975
Giac [A] (verification not implemented) . . . . . 1975
Mupad [B] (verification not implemented) . . . . . 1976
Reduce [F] . . . . . 1976

## Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

output
```
-2/b/d/(d*tan(b*x+a))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Sec[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
-2/(b*d*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(a+bx)^2}{(d\tan(a+bx))^{3/2}}dx$$
$$\Big\downarrow \text{ 3087}$$
$$\frac{\int \frac{1}{(d\tan(a+bx))^{3/2}}d\tan(a+bx)}{b}$$
$$\Big\downarrow \text{ 17}$$
$$-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sec[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
-2/(b*d*Sqrt[d*Tan[a + b*x]])
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{2}{bd\sqrt{d\tan(bx+a)}}$ | 19 |
| default | $-\frac{2}{bd\sqrt{d\tan(bx+a)}}$ | 19 |

input
```
int(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b/d/(d*tan(b*x+a))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(18) = 36.

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{bd^2\sin(bx+a)}$$

input
```
integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)/(b*d^2*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sec(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{\sqrt{d\tan(bx+a)}bd}$$

input
```
integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(d*tan(b*x + a))*b*d)
```

## Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{\sqrt{d\tan(bx+a)}bd}$$

input
```
integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2/(sqrt(d*tan(b*x + a))*b*d)
```

## Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\sin(2a+2bx)\sqrt{\frac{d\sin(2a+2bx)}{\cos(2a+2bx)+1}}}{b\,d^2\sin(a+bx)^2}$$

input
```
int(1/(cos(a + b*x)^2*(d*tan(a + b*x))^(3/2)),x)
```

output
```
-(sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2))/(b
*d^2*sin(a + b*x)^2)
```

## Reduce [F]

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)^2}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sec(a + b*x)**2)/tan(a + b*x)**2,x))/d**2
```

# 3.261 $\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx$


Optimal result . . . 1977
Mathematica [A] (verified) . . . 1977
Rubi [A] (warning: unable to verify) . . . 1978
Maple [A] (verified) . . . 1982
Fricas [A] (verification not implemented) . . . 1983
Sympy [F] . . . 1984
Maxima [A] (verification not implemented) . . . 1984
Giac [F(-1)] . . . 1985
Mupad [B] (verification not implemented) . . . 1985
Reduce [F] . . . 1985


## Optimal result

Integrand size = 12, antiderivative size = 156

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}+\sqrt{d}\tan(a+bx)}\right)}{\sqrt{2}bd^{3/2}} - \frac{2}{bd\sqrt{d\tan(a+bx)}}$$

output
```
1/2*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(3/2)-1/2*a
rctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(3/2)+1/2*arctan
h(2^(1/2)*(d*tan(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*tan(b*x+a)))*2^(1/2)/b/d^(
3/2)-2/b/d/(d*tan(b*x+a))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \frac{-2-\arctan\left(\sqrt[4]{-\tan^2(a+bx)}\right)\sqrt[4]{-\tan^2(a+bx)}+\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(a+\right.}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[(d*Tan[a + b*x])^(-3/2),x]
```

output
```
(-2 - ArcTan[(-Tan[a + b*x]^2)^(1/4)]*(-Tan[a + b*x]^2)^(1/4) + ArcTanh[(-
Tan[a + b*x]^2)^(1/4)]*(-Tan[a + b*x]^2)^(1/4))/(b*d*Sqrt[d*Tan[a + b*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3955}$$

$$-\frac{\int \sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 3957}$$

$$-\frac{\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))}{bd}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{2\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{bd}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 826

$$-\frac{2\left(\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)}{bd}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 1476

$$-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}\right.\right.}{bd}$$

$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 1082

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan\right.}{bd}$$

$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 217

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)}{bd}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 1479

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{}\right.}{}\right.\right.}{bd}$$

$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 25

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{bd}$$

$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 27

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{bd}$$

$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 1103

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d\right.}{}\right.\right.}{bd}$$

$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[(d*Tan[a + b*x])^(-3/2),x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcT
an[1 + Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[
2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d
+ Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/
2))/(b*d) - 2/(b*d*Sqrt[d*Tan[a + b*x]])
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| derivativedivides | $\frac{2d\left(-\frac{1}{d^2\sqrt{d\tan(bx+a)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\,\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(bx+a)}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8d^2\left(d^2\right)^{\frac{1}{4}}}}{b}$ |
| default | $\frac{2d\left(-\frac{1}{d^2\sqrt{d\tan(bx+a)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\,\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\tan(bx+a)}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8d^2\left(d^2\right)^{\frac{1}{4}}}}{b}$ |

input
```
int(1/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*d*(-1/d^2/(d*tan(b*x+a))^(1/2)-1/8/d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(
b*x+a)-(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(b*x+a)
+(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(
d^2)^(1/4)*(d*tan(b*x+a))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(b*
x+a))^(1/2)+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}}{\sqrt{d}}+1\right)\tan(bx+a)+2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}}{\sqrt{d}}-1\right)\tan(bx+a)-\sqrt{}}{}$$

input
```
integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt(d*tan(b*x + a))/sqrt(d) + 1)*t
an(b*x + a) + 2*sqrt(2)*sqrt(d)*arctan(sqrt(2)*sqrt(d*tan(b*x + a))/sqrt(d
) - 1)*tan(b*x + a) - sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d*tan(b*x + a))/sqr
t(d) + tan(b*x + a) + 1)*tan(b*x + a) + sqrt(2)*sqrt(d)*log(-sqrt(2)*sqrt(
d*tan(b*x + a))/sqrt(d) + tan(b*x + a) + 1)*tan(b*x + a) + 8*sqrt(d*tan(b*
x + a)))/(b*d^2*tan(b*x + a))
```

## Sympy [F]

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(-3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.07

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)}{\sqrt{d}}}{4\,bd}$$

input
```
integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b*x + a
)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a) + sqrt
(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x + a)
 - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + 8/sqrt(d*tan(b*x +
a)))/(b*d)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.49

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \frac{(-1)^{1/4}\,\text{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)}{b\,d^{3/2}} - \frac{(-1)^{1/4}\,\text{atan}\left(\frac{(-1)^{1/4}\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)}{b\,d^{3/2}} - \frac{2}{b\,d\,\sqrt{d\tan\left(a+b\,x\right)}}$$

input
```
int(1/(d*tan(a + b*x))^(3/2),x)
```

output
```
((-1)^(1/4)*atanh(((-1)^(1/4)*(d*tan(a + b*x))^(1/2))/d^(1/2)))/(b*d^(3/2)
) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*tan(a + b*x))^(1/2))/d^(1/2)))/(b*d^(3
/2)) - 2/(b*d*(d*tan(a + b*x))^(1/2))
```

## Reduce [F]

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(1/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(tan(a + b*x))/tan(a + b*x)**2,x))/d**2
```

# 3.262 $\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$


Optimal result . . . 1987
Mathematica [A] (verified) . . . 1988
Rubi [A] (verified) . . . 1988
Maple [B] (verified) . . . 1993
Fricas [B] (verification not implemented) . . . 1994
Sympy [F] . . . 1994
Maxima [A] (verification not implemented) . . . 1995
Giac [A] (verification not implemented) . . . 1995
Mupad [F(-1)] . . . 1996
Reduce [F] . . . 1996


## Optimal result

Integrand size = 21, antiderivative size = 191

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} + \frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}(1+\tan(a+bx))}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}}$$

output
```
5/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(3/2)-5/8*a
rctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*2^(1/2)/b/d^(3/2)+5/8*arctan
h(2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2)/(1+tan(b*x+a)))*2^(1/2)/b/d^(3/2)-5
/2/b/d/(d*tan(b*x+a))^(1/2)+1/2*cos(b*x+a)^2/b/d/(d*tan(b*x+a))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.60

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\csc(a+bx)\left(-17\cos(a+bx)+\cos(3(a+bx))+5\arcsin(\cos(a+bx)-\sin(a-\right.}{}$$

input
```
Integrate[Cos[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Csc[a + b*x]*(-17*Cos[a + b*x] + Cos[3*(a + b*x)] + 5*ArcSin[Cos[a + b*x]
 - Sin[a + b*x]]*Sqrt[Sin[2*(a + b*x)]] + 5*Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]])*Sqrt[d*Tan[a + b*x]])/
(8*b*d^2)
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.29, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3087, 253, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(a+bx)^2(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \frac{1}{(d\tan(a+bx))^{3/2}(\tan^2(a+bx)+1)^2}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{5}{4}\int \frac{1}{(d\tan(a+bx))^{3/2}(\tan^2(a+bx)+1)}d\tan(a+bx)+\frac{1}{2d(\tan^2(a+bx)+1)\sqrt{d\tan(a+bx)}}}{b}$$

$$\Big\downarrow \; 264$$

$$\frac{\frac{5}{4}\left(-\frac{\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)+1}d\tan(a+bx)}{d^2}-\frac{2}{d\sqrt{d\tan(a+bx)}}\right)+\frac{1}{2d(\tan^2(a+bx)+1)\sqrt{d\tan(a+bx)}}}{b}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{5}{4}\left(-\frac{2\int \frac{d^3\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}}{d^3}-\frac{2}{d\sqrt{d\tan(a+bx)}}\right)+\frac{1}{2d(\tan^2(a+bx)+1)\sqrt{d\tan(a+bx)}}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{5}{4}\left(-\frac{2\int \frac{d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}}{d}-\frac{2}{d\sqrt{d\tan(a+bx)}}\right)+\frac{1}{2d(\tan^2(a+bx)+1)\sqrt{d\tan(a+bx)}}}{b}$$

$$\Big\downarrow \; 826$$

$$\frac{\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\int \frac{\tan(a+bx)d+d}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}\right)}{d}-\frac{2}{d\sqrt{d\tan(a+bx)}}\right)+\frac{1}{2d(\tan^2(a+bx)+1)\sqrt{d\tan(a+}}}{b}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}\right)-\frac{1}{2}\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d}}{d}}{b}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d\tan(a+bx)-1}d\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d\tan(a+bx)-1}d\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}\right)}{d}-\right.}{b}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}\right)}{d}-\frac{2}{d\sqrt{d\tan(a+bx)}}\right)+\frac{}{2d(}}{b}$$

$\downarrow$ 1479

$$\frac{\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a-}}{\sqrt{d}}\right.}{\sqrt{2}\sqrt{d}}\right.}{d}\right.}{b}$$

$\downarrow$ 25

$$\frac{\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+}}{\sqrt{d}}\right.}{\sqrt{2}\sqrt{d}}\right.}{d}\right.}{b}$$

$\downarrow$ 27

$$\frac{\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+}}{\sqrt{d}}\right.}{\sqrt{2}\sqrt{d}}\right.}{d}\right.}{b}$$

$\downarrow$ 1103

$$\frac{\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(d\tan(a+bx)-\sqrt{2}\sqrt{d}\sqrt{d\tan(a+bx)}+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(d\tan(a+bx)+\sqrt{2}\sqrt{d}\sqrt{d}\right.}{2\sqrt{2}\sqrt{d}}\right.}{d}\right.}{b}$$

input
```
Int[Cos[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(1/(2*d*Sqrt[d*Tan[a + b*x]]*(1 + Tan[a + b*x]^2)) + (5*((-2*((-(ArcTan[1
- (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 +
(Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d + d*
Tan[a + b*x] - Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[a + b*x]]]/(2*Sqrt[2]*Sqrt[d]) -
 Log[d + d*Tan[a + b*x] + Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[a + b*x]]]/(2*Sqrt[2]
*Sqrt[d]))/2))/d - 2/(d*Sqrt[d*Tan[a + b*x]])))/4)/b
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(147) = 294$.

Time = 6.29 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.64

| method | result |
|---|---|
| default | $-\frac{\left(5\ln\left(-\frac{\cot(bx+a)\cos(bx+a)-2\cot(bx+a)-2\sin(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}-2\cos(bx+a)+\csc(bx+a)-\sin(bx+a)+2}{-1+\cos(bx+a)}\right)\sin(bx+a)-5\right.}{}$ |

input
```
int(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/b*(5*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-2*sin(b*
x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)+csc(b*x+a)-sin(b*x+a)
+2)/(-1+cos(b*x+a)))*sin(b*x+a)-5*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+
2*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a
)+csc(b*x+a)-sin(b*x+a)+2)/(-1+cos(b*x+a)))*sin(b*x+a)+10*arctan((sin(b*x+
a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos
(b*x+a)))*sin(b*x+a)+10*arctan((sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(
b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*sin(b*x+a)+(-32*cos(b*x+
a)*(cos(b*x+a)+1)*sin(b*x+a)-4*(cos(b*x+a)^2-5)*(cos(b*x+a)+1))*(-2*sin(b*
x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+sin(b*x+a)*cos(b*x+a)*(32*cos(b*x+
a)+32)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2))/(-sin(b*x+
a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(cos(b*x+a)+1)/d/(d*tan(b*x+a))^(1/2
)*2^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(147) = 294.

Time = 0.17 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.42

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{10\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{\sqrt{d}(\cos(bx+a)-\sin(bx+a))}\right)\sin(bx+a)+5\sqrt{2}\sqrt{d}\arctan\left(\frac{2\cos(\ldots}{\ldots}\right.}{\ldots}$$

input
```
integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/32*(10*sqrt(2)*sqrt(d)*arctan(-sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x + a))
*cos(b*x + a)/(sqrt(d)*(cos(b*x + a) - sin(b*x + a))))*sin(b*x + a) + 5*sq
rt(2)*sqrt(d)*arctan(1/2*(2*cos(b*x + a)^2 - 2*cos(b*x + a)*sin(b*x + a) +
 sqrt(2)*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) - 2)/(cos(b*x + a)^2 +
cos(b*x + a)*sin(b*x + a) - 1))*sin(b*x + a) + 5*sqrt(2)*sqrt(d)*arctan(-1
/2*(2*cos(b*x + a)^2 - 2*cos(b*x + a)*sin(b*x + a) - sqrt(2)*sqrt(d*sin(b*
x + a)/cos(b*x + a))/sqrt(d) - 2)/(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x +
 a) - 1))*sin(b*x + a) + 5*sqrt(2)*sqrt(d)*log(4*cos(b*x + a)*sin(b*x + a)
 + 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x +
 a)/cos(b*x + a))/sqrt(d) + 1)*sin(b*x + a) - 5*sqrt(2)*sqrt(d)*log(4*cos(
b*x + a)*sin(b*x + a) - 2*sqrt(2)*(cos(b*x + a)^2 + cos(b*x + a)*sin(b*x +
 a))*sqrt(d*sin(b*x + a)/cos(b*x + a))/sqrt(d) + 1)*sin(b*x + a) + 16*(cos
(b*x + a)^3 - 5*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*d^2*si
n(b*x + a))
```

## Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(cos(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.07

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$

$$-\frac{\frac{10\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{10\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{5\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+\right.}{\sqrt{d}}}{16\,bd}$$

input
```
integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-1/16*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b*x +
 a)))/sqrt(d))/sqrt(d) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - 5*sqrt(2)*log(d*tan(b*x + a) +
 sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + 5*sqrt(2)*log(d*tan(b
*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + 8*(5*d^2*tan
(b*x + a)^2 + 4*d^2)/((d*tan(b*x + a))^(5/2) + sqrt(d*tan(b*x + a))*d^2))/
(b*d)
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.32

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$

$$-\frac{\frac{10\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}+\frac{10\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}-\frac{5\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{\right.}}{bd^2}}{16\,d}$$

input
```
integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-1/16*(10*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 10*sqrt(2)*abs(d)^(3/2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs
(d)))/(b*d^2) - 5*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d
*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) + 5*sqrt(2)*abs(d)^(3/2)*log
(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d
^2) + 8*(5*d^2*tan(b*x + a)^2 + 4*d^2)/((sqrt(d*tan(b*x + a))*d^2*tan(b*x
+ a)^2 + sqrt(d*tan(b*x + a))*d^2)*b))/d
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)^2/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)^2/(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\cos(bx+a)^2}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*cos(a + b*x)**2)/tan(a + b*x)**2,x))/d**2
```

# 3.263 $\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1997
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1998
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1998
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2001
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 2002
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2002
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2003
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2003
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2003
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2004

## Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{24\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^2\sqrt{\sin(2a+2bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}$$

output
```
-2*sec(b*x+a)^3/b/d/(d*tan(b*x+a))^(1/2)+24/5*cos(b*x+a)*EllipticE(cos(a+1
/4*Pi+b*x),2^(1/2))*(d*tan(b*x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)+24/5*c
os(b*x+a)*(d*tan(b*x+a))^(3/2)/b/d^3+12/5*sec(b*x+a)*(d*tan(b*x+a))^(3/2)/
b/d^3
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.81 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\csc(a+bx)\sqrt{d\tan(a+bx)}\left(-8\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\right.}{5bd^2\sqrt{\sec^2(a-}}$$

input
```
Integrate[Sec[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]]*(-8*Hypergeometric2F1[3/4, 3/2, 7/4,
-Tan[a + b*x]^2]*Tan[a + b*x]^2 + Sqrt[Sec[a + b*x]^2]*(-5 + 12*Sin[a + b*
x]^2 + Tan[a + b*x]^2)))/(5*b*d^2*Sqrt[Sec[a + b*x]^2])
```

## Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3088, 3042, 3093, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(a+bx)^5}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \; 3088$$

$$\frac{6\int \sec^3(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2} - \frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{6\int \sec(a+bx)^3\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3093

$$\frac{6\left(\frac{2}{5}\int \sec(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3042

$$\frac{6\left(\frac{2}{5}\int \sec(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3093

$$\frac{6\left(\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-2\int \cos(a+bx)\sqrt{d\tan(a+bx)}dx\right)+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3042

$$\frac{6\left(\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-2\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx\right)+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3095

$$\frac{6\left(\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\sin(a+bx)}}\right)+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3042

$$\frac{6\left(\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\sin(a+bx)}}\right)+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3052

$$\frac{6\left(\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}\right)+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3042

$$\frac{6\left(\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}\right)+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3119

$$\frac{6\left(\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}+\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\cos(a+bx)E\left(a+bx-\frac{\pi}{4}|2\right)\sqrt{d\tan(a+bx)}}{b\sqrt{\sin(2a+2bx)}}\right)\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sec[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Sec[a + b*x]^3)/(b*d*Sqrt[d*Tan[a + b*x]]) + (6*((2*Sec[a + b*x]*(d*Ta
n[a + b*x])^(3/2))/(5*b*d) + (2*((-2*Cos[a + b*x]*EllipticE[a - Pi/4 + b*x
, 2]*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Sin[2*a + 2*b*x]]) + (2*Cos[a + b*x]*(d
*Tan[a + b*x])^(3/2))/(b*d)))/5))/d^2
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.57

| method | result |
|---|---|
| default | $\frac{-\frac{24}{5}+\frac{24\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{EllipticE}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,(1+\mathrm{se}}{5}}{}$ |

input
```
int(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/b/(d*tan(b*x+a))^(1/2)*(-12+12*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-
csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2
*2^(1/2))*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(1+sec(b*x+a))+6*(csc(b*x+a)-cot
(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x
+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-1-sec(
b*x+a))+6*sec(b*x+a)+sec(b*x+a)^3)/d
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.61

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$
$$-\frac{2\left(6i\sqrt{i\,d}\cos\left(bx+a\right)^2 E(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)\sin\left(bx+a\right)-6i\sqrt{-i\,d}\cos\left(bx+a\right.\right.}{}$$

input
```
integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(6*I*sqrt(I*d)*cos(b*x + a)^2*elliptic_e(arcsin(cos(b*x + a) + I*sin(
b*x + a)), -1)*sin(b*x + a) - 6*I*sqrt(-I*d)*cos(b*x + a)^2*elliptic_e(arc
sin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - 6*I*sqrt(I*d)*cos(b
*x + a)^2*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x +
a) + 6*I*sqrt(-I*d)*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a) - I*sin(
b*x + a)), -1)*sin(b*x + a) + (12*cos(b*x + a)^4 - 6*cos(b*x + a)^2 - 1)*s
qrt(d*sin(b*x + a)/cos(b*x + a)))/(b*d^2*cos(b*x + a)^2*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^5\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)**5/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sec(a + b*x)**5/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

## Giac [F]

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^5\,\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2)),x)
```

output
```
int(1/(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)^5}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sec(a + b*x)**5)/tan(a + b*x)**2,x))/d**2
```

# 3.264 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$


Optimal result . . . 2005
Mathematica [C] (verified) . . . 2005
Rubi [A] (verified) . . . 2006
Maple [B] (verified) . . . 2009
Fricas [C] (verification not implemented) . . . 2009
Sympy [F] . . . 2010
Maxima [F] . . . 2010
Giac [F] . . . 2011
Mupad [F(-1)] . . . 2011
Reduce [F] . . . 2011


## Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}} + \frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}$$

output
```
-2*sec(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+4*cos(b*x+a)*EllipticE(cos(a+1/4*Pi
+b*x),2^(1/2))*(d*tan(b*x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)+4*cos(b*x+a
)*(d*tan(b*x+a))^(3/2)/b/d^3
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\csc(a+bx)\sqrt{d\tan(a+bx)}\left(3\cos(2(a+bx))\sqrt{\sec^2(a+bx)} + 4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a\right.\right.}{3bd^2\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Sec[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]]*(3*Cos[2*(a + b*x)]*Sqrt[Sec[a + b*x
]^2] + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Tan[a + b*x]^2)
)/(3*b*d^2*Sqrt[Sec[a + b*x]^2])
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3088, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^3}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3088}$$

$$\frac{2\int \sec(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \sec(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3093}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-2\int \cos(a+bx)\sqrt{d\tan(a+bx)}dx\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-2\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3095}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\sin(a+bx)}}\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\sin(a+bx)}}\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\cos(a+bx)E\left(a+bx-\frac{\pi}{4}|2\right)\sqrt{d\tan(a+bx)}}{b\sqrt{\sin(2a+2bx)}}\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sec[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Sec[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) + (2*((-2*Cos[a + b*x]*Ellipt
icE[a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Sin[2*a + 2*b*x]]) +
(2*Cos[a + b*x]*(d*Tan[a + b*x])^(3/2))/(b*d)))/d^2
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(97) = 194.

Time = 1.21 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.49

| method | result |
|---|---|
| default | $\frac{4\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1},\right.}{}$ |

input
```
int(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/(d*tan(b*x+a))^(1/2)/d*(2*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+
a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticE((csc(b*x
+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))-(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*c
sc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((
csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))+2*(-2*csc(b*x+a)+2*cot(b*x+a)+
2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1
)^(1/2),1/2*2^(1/2))*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*sec(b*x+a)-(csc(b*x+a
)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+co
t(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*sec
(b*x+a)-2+sec(b*x+a))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.65

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$
$$-\frac{2\left(i\sqrt{i\,d}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)-i\sqrt{-i\,d}E(\arcsin(\cos(bx+a)-i\,\mathrm{s}\right.}{}$$

input
```
integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(I*d)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(
b*x + a) - I*sqrt(-I*d)*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)),
-1)*sin(b*x + a) - I*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x
+ a)), -1)*sin(b*x + a) + I*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*
sin(b*x + a)), -1)*sin(b*x + a) + (2*cos(b*x + a)^2 - 1)*sqrt(d*sin(b*x +
a)/cos(b*x + a)))/(b*d^2*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=\int \frac{\sec^3\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sec(a + b*x)**3/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=\int \frac{\sec\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

## Giac [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^3\,\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2)),x)
```

output
```
int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)^3}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sec(a + b*x)**3)/tan(a + b*x)**2,x))/d**2
```

# 3.265 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result . . . . . 2012
Mathematica [C] (verified) . . . . . 2012
Rubi [A] (verified) . . . . . 2013
Maple [B] (verified) . . . . . 2015
Fricas [C] (verification not implemented) . . . . . 2016
Sympy [F] . . . . . 2016
Maxima [F] . . . . . 2017
Giac [F] . . . . . 2017
Mupad [F(-1)] . . . . . 2017
Reduce [F] . . . . . 2018

## Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{2\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

output
```
-2*cos(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+2*cos(b*x+a)*EllipticE(cos(a+1/4*Pi
+b*x),2^(1/2))*(d*tan(b*x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=-\frac{2\sin(a+bx)\left(3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{3b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Sec[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Sin[a + b*x]*(3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*
Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(3*b*(d*Tan[a + b*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3088, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3088}$$

$$-\frac{2\int \cos(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx}{d^2}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3095}$$

$$-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sec[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (2*Cos[a + b*x]*EllipticE[a
 - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(73) = 146$.

Time = 0.96 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.92

| method | result |
|---|---|
| default | $\frac{2\cot(bx+a)\csc(bx+a)\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(-1+2\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a}\right.}{}$ |

input
```
int(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*cot(b*x+a)*csc(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2
)*(-1+2*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/
2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2
),1/2*2^(1/2))-(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)
+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+
1)^(1/2),1/2*2^(1/2))+csc(b*x+a)^2*(1-cos(b*x+a))^2)*(1-cos(b*x+a))/(-sin(
b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(csc(b*x+a)^2*(1-cos(b*x+a))^2-1
)^2/d/(d*tan(b*x+a))^(1/2)*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.17

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)^2 + i\sqrt{i\,d}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a) - i\sqrt{-i\,d}E$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2 + I*sqrt(I*d)*ellipti
c_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) - I*sqrt(-I*d)
*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - I*sq
rt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a)
 + I*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(
b*x + a))/(b*d^2*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sec(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

## Giac [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)\,\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(a + b*x)*(d*tan(a + b*x))^(3/2)),x)
```

output
```
int(1/(cos(a + b*x)*(d*tan(a + b*x))^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sec(a + b*x))/tan(a + b*x)**2,x))/d**2
```

# 3.266 $\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$


Optimal result . . . 2019
Mathematica [C] (verified) . . . 2019
Rubi [A] (verified) . . . 2020
Maple [B] (verified) . . . 2022
Fricas [F] . . . 2022
Sympy [F] . . . 2023
Maxima [F] . . . 2023
Giac [F] . . . 2023
Mupad [F(-1)] . . . 2024
Reduce [F] . . . 2024


## Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{3\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

output
```
-2*cos(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+3*cos(b*x+a)*EllipticE(cos(a+1/4*Pi
+b*x),2^(1/2))*(d*tan(b*x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sin(a+bx)\left(1+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Cos[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Sin[a + b*x]*(1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sq
rt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(b*(d*Tan[a + b*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3089, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(a+bx)(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{3\int \cos(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx}{d^2}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3095}$$

$$-\frac{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{3\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Cos[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (3*Cos[a + b*x]*EllipticE[a
 - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(73) = 146.

Time = 1.05 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.38

| method | result |
|---|---|
| default | $\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\operatorname{EllipticE}\left(\sqrt{cs\right.\right.}{}$ |

input
```
int(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1/
2)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/d*((csc(b*x+a)-cot(b*x+
a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^
(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(6+6*sec(b*x+
a))+(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(
-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/
2*2^(1/2))*(-3-3*sec(b*x+a))+2*cos(b*x+a)-6)*2^(1/2)
```

## Fricas [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=\int \frac{\cos(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*cos(b*x + a)/(d^2*tan(b*x + a)^2), x)
```

## Sympy [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(cos(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

## Giac [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos(a+b\,x)}{(d\tan(a+b\,x))^{3/2}}\,dx$$

input
```
int(cos(a + b*x)/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)/(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\cos(bx+a)}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*cos(a + b*x))/tan(a + b*x)**2,x))/d**2
```

# 3.267 $\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2025
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2025
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2026
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2029
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2029
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2030
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2030
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2030
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2031
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2031

## Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sqrt{d\tan(a+bx)}}{2bd^2\sqrt{\sin(2a+2bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3}$$

output
```
-2*cos(b*x+a)^3/b/d/(d*tan(b*x+a))^(1/2)+7/2*cos(b*x+a)*EllipticE(cos(a+1/
4*Pi+b*x),2^(1/2))*(d*tan(b*x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)-7/3*cos
(b*x+a)^3*(d*tan(b*x+a))^(3/2)/b/d^3
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.58 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sin(a+bx)\left(-13+\cos(2(a+bx))-14\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a\right.\right.}{6b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Cos[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Sin[a + b*x]*(-13 + Cos[2*(a + b*x)] - 14*Hypergeometric2F1[3/4, 3/2, 7/4
, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(6*b*(d*Tan[a + b
*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3089, 3042, 3092, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(a+bx)^3(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{7\int \cos^3(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)^3}dx}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3092}$$

$$-\frac{7\left(\frac{1}{2}\int \cos(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7\left(\frac{1}{2}\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3095}$$

$$-\frac{7\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\sin(a+bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\sin(a+bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{7\left(\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7\left(\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{7\left(\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}+\frac{\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

input

```
Int[Cos[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output

```
(-2*Cos[a + b*x]^3)/(b*d*Sqrt[d*Tan[a + b*x]]) - (7*((Cos[a + b*x]*Ellipti
cE[a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(2*b*Sqrt[Sin[2*a + 2*b*x]]) +
 (Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b*d)))/d^2
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(101) = 202.

Time = 1.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.45

| method | result |
|---|---|
| default | $\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\sqrt{\csc(bx+a)-\cot(bx+a)+1}\ \mathrm{EllipticE}\left(\sqrt{\mathrm{cs}}\right.\right.}{}$ |

input
```
int(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/b*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1
/2)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/d*((-2*csc(b*x+a)+2*co
t(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*(csc(b*x+a)-cot(b*x+a)+1)
^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(42+42*sec(b
*x+a))+(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)
*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(csc(b*x+a)-cot(b*
x+a)+1)^(1/2)*(-21-21*sec(b*x+a))+4*cos(b*x+a)^3+14*cos(b*x+a)-42)*2^(1/2)
```

## Fricas [F]

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*cos(b*x + a)^3/(d^2*tan(b*x + a)^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

## Giac [F]

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)^3/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)^3/(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\cos(bx+a)^3}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*cos(a + b*x)**3)/tan(a + b*x)**2,x))/d**2
```

# 3.268 $\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2032
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2033
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2033
Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2036
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2037
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2037
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2038
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2038
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2038
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2039

## Optimal result

Integrand size = 21, antiderivative size = 142

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{20bd^2\sqrt{\sin(2a+2bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}$$

output
```
-2*cos(b*x+a)^5/b/d/(d*tan(b*x+a))^(1/2)+77/20*cos(b*x+a)*EllipticE(cos(a+
1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)-77/30
*cos(b*x+a)^3*(d*tan(b*x+a))^(3/2)/b/d^3-11/5*cos(b*x+a)^5*(d*tan(b*x+a))^
(3/2)/b/d^3
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.83 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sin(a+bx)\left(-277+34\cos(2(a+bx))+3\cos(4(a+bx))-308\,\text{Hypergeometr}\right.}{120b(d\tan(a+bx))^{3}}$$

input
```
Integrate[Cos[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Sin[a + b*x]*(-277 + 34*Cos[2*(a + b*x)] + 3*Cos[4*(a + b*x)] - 308*Hyper
geometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2*Tan[a +
b*x]^2))/(120*b*(d*Tan[a + b*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3089, 3042, 3092, 3042, 3092, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(a+bx)^5(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{11\int \cos^5(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11 \int \frac{\sqrt{d \tan(a+bx)}}{\sec(a+bx)^5}dx}{d^2}-\frac{2 \cos^5(a+bx)}{bd\sqrt{d \tan(a+bx)}}$$

$\downarrow$ 3092

$$-\frac{11\left(\frac{7}{10}\int \cos^3(a+bx)\sqrt{d \tan(a+bx)}dx+\frac{\cos^5(a+bx)(d \tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2 \cos^5(a+bx)}{bd\sqrt{d \tan(a+bx)}}$$

$\downarrow$ 3042

$$-\frac{11\left(\frac{7}{10}\int \frac{\sqrt{d \tan(a+bx)}}{\sec(a+bx)^3}dx+\frac{\cos^5(a+bx)(d \tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2 \cos^5(a+bx)}{bd\sqrt{d \tan(a+bx)}}$$

$\downarrow$ 3092

$$-\frac{11\left(\frac{7}{10}\left(\frac{1}{2}\int \cos(a+bx)\sqrt{d \tan(a+bx)}dx+\frac{\cos^3(a+bx)(d \tan(a+bx))^{3/2}}{3bd}\right)+\frac{\cos^5(a+bx)(d \tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2 \cos^5(a+bx)}{bd\sqrt{d \tan(a+bx)}}$$

$\downarrow$ 3042

$$-\frac{11\left(\frac{7}{10}\left(\frac{1}{2}\int \frac{\sqrt{d \tan(a+bx)}}{\sec(a+bx)}dx+\frac{\cos^3(a+bx)(d \tan(a+bx))^{3/2}}{3bd}\right)+\frac{\cos^5(a+bx)(d \tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2 \cos^5(a+bx)}{bd\sqrt{d \tan(a+bx)}}$$

$\downarrow$ 3095

$$-\frac{11\left(\frac{7}{10}\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d \tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\sin(a+bx)}}+\frac{\cos^3(a+bx)(d \tan(a+bx))^{3/2}}{3bd}\right)+\frac{\cos^5(a+bx)(d \tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2 \cos^5(a+bx)}{bd\sqrt{d \tan(a+bx)}}$$

$\downarrow$ 3042

$$-\frac{11\left(\frac{7}{10}\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d \tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\sin(a+bx)}}+\frac{\cos^3(a+bx)(d \tan(a+bx))^{3/2}}{3bd}\right)+\frac{\cos^5(a+bx)(d \tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2 \cos^5(a+bx)}{bd\sqrt{d \tan(a+bx)}}$$

$\downarrow$ 3052

$$-\frac{11\left(\frac{7}{10}\left(\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)+\frac{\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11\left(\frac{7}{10}\left(\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)+\frac{\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{11\left(\frac{\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd}+\frac{7}{10}\left(\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}+\frac{\cos(a+bx)E\left(a+bx-\frac{\pi}{4}|2\right)\sqrt{d\tan(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}\right)\right)}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Cos[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x]^5)/(b*d*Sqrt[d*Tan[a + b*x]]) - (11*((Cos[a + b*x]^5*(d*T
an[a + b*x])^(3/2))/(5*b*d) + (7*((Cos[a + b*x]*EllipticE[a - Pi/4 + b*x,
2]*Sqrt[d*Tan[a + b*x]])/(2*b*Sqrt[Sin[2*a + 2*b*x]]) + (Cos[a + b*x]^3*(d
*Tan[a + b*x])^(3/2))/(3*b*d)))/10))/d^2
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(127) = 254$.

Time = 1.79 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.00

| method | result |
|---|---|
| default | $\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(-462+462\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{EllipticE}\left(\sqrt{\csc(bx+a)-\cot(bx\cdots\right.\right.}{}$ |

input
```
int(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/240/b*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(
1/2)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(-462+462*(-2*csc(b*x
+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticE((csc(b*
x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(1+s
ec(b*x+a))+231*(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)
+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+
1)^(1/2),1/2*2^(1/2))*(-1-sec(b*x+a))+24*cos(b*x+a)^5+44*cos(b*x+a)^3+154*
cos(b*x+a))/d*2^(1/2)
```

## Fricas [F]

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=\int \frac{\cos\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*cos(b*x + a)^5/(d^2*tan(b*x + a)^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(cos(b*x+a)**5/(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

## Giac [F]

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^5}{\left(d\,\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)^5/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)^5/(d*tan(a + b*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\cos(bx+a)^5}{\tan(bx+a)^2}dx\right)}{d^2}$$

input
```
int(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*cos(a + b*x)**5)/tan(a + b*x)**2,x))/d**2
```

# 3.269 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal result . . . . . 2040
Mathematica [C] (warning: unable to verify) . . . . . 2040
Rubi [A] (verified) . . . . . 2041
Maple [A] (verified) . . . . . 2043
Fricas [C] (verification not implemented) . . . . . 2043
Sympy [F] . . . . . 2044
Maxima [F] . . . . . 2044
Giac [F] . . . . . 2045
Mupad [F(-1)] . . . . . 2045
Reduce [F] . . . . . 2045

## Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}} - \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\tan(a+bx)}}$$

output
```
-2/3*sec(b*x+a)/b/d/(d*tan(b*x+a))^(3/2)-1/3*InverseJacobiAM(a-1/4*Pi+b*x,
2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/d^2/(d*tan(b*x+a))^(1/2)
```

## Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.38

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2\cos(2(a+bx))\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(\sqrt{\sec^2(a+bx)}-\sqrt[4]{-1}\,\text{EllipticF}\left(i\right.\right.}{3bd^2\sqrt{d\tan(a+bx)}\left(-1+\tan^2(a\right.}$$

input
```
Integrate[Sec[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*Cos[2*(a + b*x)]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(Sqrt[Sec[a + b*x]^2
] - (-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Tan
[a + b*x]^(3/2)))/(3*b*d^2*Sqrt[d*Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3089, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3089}$$

$$-\frac{\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{3d^2}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{3d^2}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 3094}$$

$$-\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{ 3053}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3bd^2\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sec[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*Sec[a + b*x])/(3*b*d*(d*Tan[a + b*x])^(3/2)) - (EllipticF[a - Pi/4 + b
*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Tan[a + b*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| default | $-\frac{\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\ \mathrm{EllipticF}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\right.}{3b\sqrt{d\tan(bx+a)}\,d^2}$ |

input
```
int(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(d*tan(b*x+a))^(1/2)/d^2*((csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b
*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(
b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(1+sec(b*x+a))+2*csc(b*x+a))
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\left(\cos(bx+a)^2-1\right)\sqrt{i\,d}F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+(\cos(ba}{3\,(bd^3\,c}$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/3*((cos(b*x + a)^2 - 1)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin
(b*x + a)), -1) + (cos(b*x + a)^2 - 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*
x + a) - I*sin(b*x + a)), -1) + 2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*
x + a))/(b*d^3*cos(b*x + a)^2 - b*d^3)
```

## Sympy [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sec{(a+bx)}}{(d\tan{(a+bx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(sec(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

## Maxima [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sec{(bx+a)}}{(d\tan{(bx+a)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

## Giac [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sec(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{1}{\cos(a+b\,x)\,(d\tan(a+b\,x))^{5/2}}\,dx$$

input
```
int(1/(cos(a + b*x)*(d*tan(a + b*x))^(5/2)),x)
```

output
```
int(1/(cos(a + b*x)*(d*tan(a + b*x))^(5/2)), x)
```

## Reduce [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)}{\tan(bx+a)^3}dx\right)}{d^3}$$

input
```
int(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sec(a + b*x))/tan(a + b*x)**3,x))/d**3
```

# 3.270 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx$

Optimal result . . . 2046
Mathematica [C] (verified) . . . 2046
Rubi [A] (verified) . . . 2047
Maple [B] (verified) . . . 2049
Fricas [C] (verification not implemented) . . . 2050
Sympy [F] . . . 2051
Maxima [F] . . . 2051
Giac [F] . . . 2051
Mupad [F(-1)] . . . 2052
Reduce [F] . . . 2052

## Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}} - \frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}} - \frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

output
```
-2/5*sec(b*x+a)/b/d/(d*tan(b*x+a))^(5/2)-4/5*cos(b*x+a)/b/d^3/(d*tan(b*x+a
))^(1/2)+4/5*cos(b*x+a)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+a)
)^(1/2)/b/d^4/sin(2*b*x+2*a)^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.86 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = -\frac{2\left(4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sec^2(a+bx)+3(-2+\csc^2(a+bx)+\csc^4(a+bx))\sqrt{\phantom{x}}\right.}{15bd^4\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Sec[a + b*x]^3/(d*Tan[a + b*x])^(7/2),x]
```

output
```
(-2*(4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 +
3*(-2 + Csc[a + b*x]^2 + Csc[a + b*x]^4)*Sqrt[Sec[a + b*x]^2])*Sin[a + b*x
]*Sqrt[d*Tan[a + b*x]])/(15*b*d^4*Sqrt[Sec[a + b*x]^2])
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3088, 3042, 3088, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^3}{(d\tan(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3088}$$

$$\frac{2\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}dx}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}dx}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3088}$$

$$\frac{2\left(-\frac{2\int \cos(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(-\frac{2\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx}{d^2}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

$\Big\downarrow$ 3095

$$\frac{2\left(-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{2\left(-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

$\Big\downarrow$ 3052

$$\frac{2\left(-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{2\left(-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

$\Big\downarrow$ 3119

$$\frac{2\left(-\frac{2\cos(a+bx)E\left(a+bx-\frac{\pi}{4}|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

input
```
Int[Sec[a + b*x]^3/(d*Tan[a + b*x])^(7/2),x]
```

output
```
(-2*Sec[a + b*x])/(5*b*d*(d*Tan[a + b*x])^(5/2)) + (2*((-2*Cos[a + b*x])/(
b*d*Sqrt[d*Tan[a + b*x]]) - (2*Cos[a + b*x]*EllipticE[a - Pi/4 + b*x, 2]*S
qrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])))/(5*d^2)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(97) = 194$.

Time = 1.10 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.43

| method | result |
|---|---|
| default | $-\frac{\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\left(\sqrt{\csc(bx+a)-\cot(bx+a)+1}\,\sqrt{-2\csc(bx+a)+2\cot(bx+a)+2}\,\sqrt{-\csc(bx+a)+\cot(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{\right.\right.}{}$ |

input
```
int(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/b*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(-sin(b*x+a)*cos(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1/2)/d^3*((csc(b*x+a)-cot(b
*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/2)*(-csc(b*x+a)+cot(b*x+a
))^(1/2)*EllipticE((csc(b*x+a)-cot(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-2-2*sec(
b*x+a))+(csc(b*x+a)-cot(b*x+a)+1)^(1/2)*(-2*csc(b*x+a)+2*cot(b*x+a)+2)^(1/
2)*(-csc(b*x+a)+cot(b*x+a))^(1/2)*EllipticF((csc(b*x+a)-cot(b*x+a)+1)^(1/2
),1/2*2^(1/2))*(1+sec(b*x+a))+2+cot(b*x+a)*csc(b*x+a))*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.19

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx =$$

$$-\frac{2\left(\left(i\cos(bx+a)^2-i\right)\sqrt{i\,d}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\mid -1)\sin(bx+a)+\left(-i\cos(bx+a\right)\right.}{}$$

input
```
integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*((I*cos(b*x + a)^2 - I)*sqrt(I*d)*elliptic_e(arcsin(cos(b*x + a) + I*
sin(b*x + a)), -1)*sin(b*x + a) + (-I*cos(b*x + a)^2 + I)*sqrt(-I*d)*ellip
tic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (-I*cos(b*
x + a)^2 + I)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)),
-1)*sin(b*x + a) + (I*cos(b*x + a)^2 - I)*sqrt(-I*d)*elliptic_f(arcsin(cos
(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (2*cos(b*x + a)^4 - 3*cos(
b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((b*d^4*cos(b*x + a)^2 - b*
d^4)*sin(b*x + a))
```

## Sympy [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \int \frac{\sec^3{(a+bx)}}{(d\tan{(a+bx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)**3/(d*tan(b*x+a))**(7/2),x)
```

output
```
Integral(sec(a + b*x)**3/(d*tan(a + b*x))**(7/2), x)
```

## Maxima [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \int \frac{\sec{(bx+a)}^3}{(d\tan{(bx+a)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(7/2), x)
```

## Giac [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \int \frac{\sec{(bx+a)}^3}{(d\tan{(bx+a)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \int \frac{1}{\cos(a+b\,x)^3\,(d\tan(a+b\,x))^{7/2}}\,dx$$

input
```
int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(7/2)),x)
```

output
```
int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(7/2)), x)
```

## Reduce [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(bx+a)}\,\sec(bx+a)^3}{\tan(bx+a)^4}dx\right)}{d^4}$$

input
```
int(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x)
```

output
```
(sqrt(d)*int((sqrt(tan(a + b*x))*sec(a + b*x)**3)/tan(a + b*x)**4,x))/d**4
```

# 3.271 $\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2053
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2053
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2054
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2055
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2055
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2056
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2056
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2056
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2057
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2057

## Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{7}{6},-\frac{1}{2},-\frac{1}{6},\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

output
```
3/7*hypergeom([-7/6, -1/2],[-1/6],cos(f*x+e)^2)*sec(f*x+e)^(7/3)*sin(f*x+e
)/f/(sin(f*x+e)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\sqrt[3]{\sec(e+fx)}\Big(-3\sin(e+fx)+2\sqrt[6]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin(e+f}{7f}$$

input
```
Integrate[Sec[e + f*x]^(10/3)*Sin[e + f*x]^2,x]
```

output
```
(3*Sec[e + f*x]^(1/3)*(-3*Sin[e + f*x] + 2*(Cos[e + f*x]^2)^(1/6)*Hypergeo
metric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*Tan[e
 + f*x]))/(7*f)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sec^{\frac{10}{3}}(e+fx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(e+fx)^{10/3}}{\csc(e+fx)^2}dx$$
$$\Big\downarrow \text{3112}$$
$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin^2(e+fx)}{\cos^{\frac{10}{3}}(e+fx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin(e+fx)^2}{\cos(e+fx)^{10/3}}dx$$
$$\Big\downarrow \text{3056}$$
$$\frac{3\sin(e+fx)\sec^{\frac{7}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{7}{6},-\frac{1}{2},-\frac{1}{6},\cos^2(e+fx)\right)}{7f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(10/3)*Sin[e + f*x]^2,x]
```

output
```
(3*Hypergeometric2F1[-7/6, -1/2, -1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(7/3)*
Sin[e + f*x])/(7*f*Sqrt[Sin[e + f*x]^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

## Maple [F]

$$\int \sec (fx+e)^{\frac{10}{3}} \sin (fx+e)^2 \, dx$$

input
```
int(sec(f*x+e)^(10/3)*sin(f*x+e)^2,x)
```

output
```
int(sec(f*x+e)^(10/3)*sin(f*x+e)^2,x)
```

## Fricas [F]

$$\int \sec^{\frac{10}{3}}(e+fx) \sin^2(e+fx) \, dx = \int \sec (fx+e)^{\frac{10}{3}} \sin (fx+e)^2 \, dx$$

input
```
integrate(sec(f*x+e)^(10/3)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*sec(f*x + e)^(10/3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**(10/3)*sin(f*x+e)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{10}{3}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^(10/3)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(10/3)*sin(f*x + e)^2, x)
```

## Giac [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{10}{3}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^(10/3)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(10/3)*sin(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sin\left(e+f\,x\right)^2\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^{10/3} \,d x$$

input
```
int(sin(e + f*x)^2*(1/cos(e + f*x))^(10/3),x)
```

output
```
int(sin(e + f*x)^2*(1/cos(e + f*x))^(10/3), x)
```

## Reduce [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{10}{3}}\sin\left(fx+e\right)^{2}dx$$

input
```
int(sec(f*x+e)^(10/3)*sin(f*x+e)^2,x)
```

output
```
int(sec(e + f*x)**(1/3)*sec(e + f*x)**3*sin(e + f*x)**2,x)
```

# 3.272 $\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal result . . . 2058
Mathematica [A] (verified) . . . 2058
Rubi [A] (verified) . . . 2059
Maple [F] . . . 2060
Fricas [F] . . . 2060
Sympy [F(-1)] . . . 2061
Maxima [F] . . . 2061
Giac [F] . . . 2061
Mupad [F(-1)] . . . 2062
Reduce [F] . . . 2062

## Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\cos^2(e+fx)\right)\sec^{\frac{5}{3}}(e+fx)\sin(e+fx)}{5f\sqrt{\sin^2(e+fx)}}$$

output
```
3/5*hypergeom([-5/6, -1/2],[1/6],cos(f*x+e)^2)*sec(f*x+e)^(5/3)*sin(f*x+e)
/f/(sin(f*x+e)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = -\frac{3\left(-1+\cos^2(e+fx)\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\sin^2(e+fx)\right)\right)\sec^{\frac{5}{3}}(e+fx)\sin(e+fx)}{5f}$$

input
```
Integrate[Sec[e + f*x]^(8/3)*Sin[e + f*x]^2,x]
```

output
```
(-3*(-1 + (Cos[e + f*x]^2)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, Sin[e +
f*x]^2])*Sec[e + f*x]^(5/3)*Sin[e + f*x])/(5*f)
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sec^{\frac{8}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(e+fx)^{8/3}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{ 3112}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin^2(e+fx)}{\cos^{\frac{8}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin(e+fx)^2}{\cos(e+fx)^{8/3}}dx$$

$$\Big\downarrow \text{ 3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{5}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\cos^2(e+fx)\right)}{5f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(8/3)*Sin[e + f*x]^2,x]
```

output
```
(3*Hypergeometric2F1[-5/6, -1/2, 1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(5/3)*S
in[e + f*x])/(5*f*Sqrt[Sin[e + f*x]^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

## Maple [F]

$$\int \sec (fx+e)^{\frac{8}{3}} \sin (fx+e)^2 \, dx$$

input
```
int(sec(f*x+e)^(8/3)*sin(f*x+e)^2,x)
```

output
```
int(sec(f*x+e)^(8/3)*sin(f*x+e)^2,x)
```

## Fricas [F]

$$\int \sec^{\frac{8}{3}}(e+fx) \sin^2(e+fx) \, dx = \int \sec (fx+e)^{\frac{8}{3}} \sin (fx+e)^2 \, dx$$

input
```
integrate(sec(f*x+e)^(8/3)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*sec(f*x + e)^(8/3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**(8/3)*sin(f*x+e)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{8}{3}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^(8/3)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(8/3)*sin(f*x + e)^2, x)
```

## Giac [F]

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{8}{3}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^(8/3)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(8/3)*sin(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \sin\left(e+f\,x\right)^2\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^{8/3} \,d x$$

input
```
int(sin(e + f*x)^2*(1/cos(e + f*x))^(8/3),x)
```

output
```
int(sin(e + f*x)^2*(1/cos(e + f*x))^(8/3), x)
```

## Reduce [F]

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \sec\left(fx+e\right)^{\frac{8}{3}}\sin\left(fx+e\right)^{2}dx$$

input
```
int(sec(f*x+e)^(8/3)*sin(f*x+e)^2,x)
```

output
```
int(sec(e + f*x)**(2/3)*sec(e + f*x)**2*sin(e + f*x)**2,x)
```

# 3.273 $\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx$


Optimal result . . . 2063
Mathematica [A] (verified) . . . 2063
Rubi [A] (verified) . . . 2064
Maple [F] . . . 2065
Fricas [F] . . . 2065
Sympy [F(-1)] . . . 2066
Maxima [F] . . . 2066
Giac [F] . . . 2066
Mupad [F(-1)] . . . 2067
Reduce [F] . . . 2067


## Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},\cos^2(e+fx)\right)\sec^{\frac{4}{3}}(e+fx)\sin(e+fx)}{4f\sqrt{\sin^2(e+fx)}}$$

output
```
3/4*hypergeom([-2/3, -1/2],[1/3],cos(f*x+e)^2)*sec(f*x+e)^(4/3)*sin(f*x+e)
/f/(sin(f*x+e)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = -\frac{3\left(-1+\cos^2(e+fx)\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\sin^2(e+fx)\right)\right)\sec^{\frac{4}{3}}(e+fx)\sin(e+fx)}{4f}$$

input
```
Integrate[Sec[e + f*x]^(7/3)*Sin[e + f*x]^2,x]
```

output
```
(-3*(-1 + (Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e +
f*x]^2])*Sec[e + f*x]^(4/3)*Sin[e + f*x])/(4*f)
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sec^{\frac{7}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^{7/3}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{3112}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin^2(e+fx)}{\cos^{\frac{7}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin(e+fx)^2}{\cos(e+fx)^{7/3}}dx$$

$$\Big\downarrow \text{3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{4}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(7/3)*Sin[e + f*x]^2,x]
```

output
```
(3*Hypergeometric2F1[-2/3, -1/2, 1/3, Cos[e + f*x]^2]*Sec[e + f*x]^(4/3)*S
in[e + f*x])/(4*f*Sqrt[Sin[e + f*x]^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

## Maple [F]

$$\int \sec(fx+e)^{\frac{7}{3}}\sin(fx+e)^2\,dx$$

input
```
int(sec(f*x+e)^(7/3)*sin(f*x+e)^2,x)
```

output
```
int(sec(f*x+e)^(7/3)*sin(f*x+e)^2,x)
```

## Fricas [F]

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec(fx+e)^{\frac{7}{3}}\sin(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^(7/3)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*sec(f*x + e)^(7/3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**(7/3)*sin(f*x+e)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{7}{3}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^(7/3)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(7/3)*sin(f*x + e)^2, x)
```

## Giac [F]

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{7}{3}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^(7/3)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(7/3)*sin(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \sin\left(e+f\,x\right)^2\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^{7/3}\,dx$$

input
```
int(sin(e + f*x)^2*(1/cos(e + f*x))^(7/3),x)
```

output
```
int(sin(e + f*x)^2*(1/cos(e + f*x))^(7/3), x)
```

## Reduce [F]

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \sec\left(fx+e\right)^{\frac{7}{3}}\sin\left(fx+e\right)^2dx$$

input
```
int(sec(f*x+e)^(7/3)*sin(f*x+e)^2,x)
```

output
```
int(sec(e + f*x)**(1/3)*sec(e + f*x)**2*sin(e + f*x)**2,x)
```

# 3.274 $\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx$


Optimal result . . . 2068
Mathematica [A] (verified) . . . 2068
Rubi [A] (verified) . . . 2069
Maple [F] . . . 2070
Fricas [F] . . . 2070
Sympy [F(-1)] . . . 2071
Maxima [F] . . . 2071
Giac [F] . . . 2071
Mupad [F(-1)] . . . 2072
Reduce [F] . . . 2072


## Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\cos^2(e+fx)\right)\sec^{\frac{2}{3}}(e+fx)\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}$$

output
```
3/2*hypergeom([-1/2, -1/3],[2/3],cos(f*x+e)^2)*sec(f*x+e)^(2/3)*sin(f*x+e)
/f/(sin(f*x+e)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = -\frac{3\left(-1+\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\right)\sec^{\frac{2}{3}}(e+fx)\sin(e+fx)}{2f}$$

input
```
Integrate[Sec[e + f*x]^(5/3)*Sin[e + f*x]^2,x]
```

output
```
(-3*(-1 + (Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, Sin[e +
f*x]^2])*Sec[e + f*x]^(2/3)*Sin[e + f*x])/(2*f)
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sec^{\frac{5}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^{5/3}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{3112}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin^2(e+fx)}{\cos^{\frac{5}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin(e+fx)^2}{\cos(e+fx)^{5/3}}dx$$

$$\Big\downarrow \text{3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{2}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(5/3)*Sin[e + f*x]^2,x]
```

output
```
(3*Hypergeometric2F1[-1/2, -1/3, 2/3, Cos[e + f*x]^2]*Sec[e + f*x]^(2/3)*S
in[e + f*x])/(2*f*Sqrt[Sin[e + f*x]^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

## Maple [F]

$$\int \sec(fx+e)^{\frac{5}{3}} \sin(fx+e)^2 \, dx$$

input
```
int(sec(f*x+e)^(5/3)*sin(f*x+e)^2,x)
```

output
```
int(sec(f*x+e)^(5/3)*sin(f*x+e)^2,x)
```

## Fricas [F]

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec(fx+e)^{\frac{5}{3}} \sin(fx+e)^2 \, dx$$

input
```
integrate(sec(f*x+e)^(5/3)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*sec(f*x + e)^(5/3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**(5/3)*sin(f*x+e)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec{(fx+e)}^{\frac{5}{3}}\sin{(fx+e)}^2\,dx$$

input
```
integrate(sec(f*x+e)^(5/3)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(5/3)*sin(f*x + e)^2, x)
```

## Giac [F]

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec{(fx+e)}^{\frac{5}{3}}\sin{(fx+e)}^2\,dx$$

input
```
integrate(sec(f*x+e)^(5/3)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(5/3)*sin(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \sin\left(e+f\,x\right)^2\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^{5/3} \,d x$$

input
```
int(sin(e + f*x)^2*(1/cos(e + f*x))^(5/3),x)
```

output
```
int(sin(e + f*x)^2*(1/cos(e + f*x))^(5/3), x)
```

## Reduce [F]

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \sec\left(fx+e\right)^{\frac{5}{3}}\sin\left(fx+e\right)^{2}dx$$

input
```
int(sec(f*x+e)^(5/3)*sin(f*x+e)^2,x)
```

output
```
int(sec(e + f*x)**(2/3)*sec(e + f*x)*sin(e + f*x)**2,x)
```

# 3.275 $\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx$


Optimal result . . . . . 2073
Mathematica [A] (verified) . . . . . 2073
Rubi [A] (verified) . . . . . 2074
Maple [F] . . . . . 2075
Fricas [F] . . . . . 2075
Sympy [F(-1)] . . . . . 2076
Maxima [F] . . . . . 2076
Giac [F] . . . . . 2076
Mupad [F(-1)] . . . . . 2077
Reduce [F] . . . . . 2077


## Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\cos^2(e+fx)\right)\sqrt[3]{\sec(e+fx)}\sin(e+fx)}{f\sqrt{\sin^2(e+fx)}}$$

output
```
3*hypergeom([-1/2, -1/6],[5/6],cos(f*x+e)^2)*sec(f*x+e)^(1/3)*sin(f*x+e)/f
/(sin(f*x+e)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = -\frac{3\left(-1+\sqrt[6]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\right)\sqrt[3]{\sec(e+fx)}\sin(e+fx)}{f}$$

input
```
Integrate[Sec[e + f*x]^(4/3)*Sin[e + f*x]^2,x]
```

output
```
(-3*(-1 + (Cos[e + f*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, Sin[e +
f*x]^2])*Sec[e + f*x]^(1/3)*Sin[e + f*x])/f
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sec^{\frac{4}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(e+fx)^{4/3}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{ 3112}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin^2(e+fx)}{\cos^{\frac{4}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin(e+fx)^2}{\cos(e+fx)^{4/3}}dx$$

$$\Big\downarrow \text{ 3056}$$

$$\frac{3\sin(e+fx)\sqrt[3]{\sec(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(4/3)*Sin[e + f*x]^2,x]
```

output
```
(3*Hypergeometric2F1[-1/2, -1/6, 5/6, Cos[e + f*x]^2]*Sec[e + f*x]^(1/3)*S
in[e + f*x])/(f*Sqrt[Sin[e + f*x]^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

## Maple [F]

$$\int \sec(fx+e)^{\frac{4}{3}} \sin(fx+e)^2\, dx$$

input
```
int(sec(f*x+e)^(4/3)*sin(f*x+e)^2,x)
```

output
```
int(sec(f*x+e)^(4/3)*sin(f*x+e)^2,x)
```

## Fricas [F]

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\, dx = \int \sec(fx+e)^{\frac{4}{3}} \sin(fx+e)^2\, dx$$

input
```
integrate(sec(f*x+e)^(4/3)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*sec(f*x + e)^(4/3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**(4/3)*sin(f*x+e)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{4}{3}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^(4/3)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(4/3)*sin(f*x + e)^2, x)
```

## Giac [F]

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{4}{3}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^(4/3)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(4/3)*sin(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sin(e+f\,x)^2\left(\frac{1}{\cos(e+f\,x)}\right)^{4/3}\,dx$$

input
```
int(sin(e + f*x)^2*(1/cos(e + f*x))^(4/3),x)
```

output
```
int(sin(e + f*x)^2*(1/cos(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec(fx+e)^{\frac{4}{3}}\sin(fx+e)^2\,dx$$

input
```
int(sec(f*x+e)^(4/3)*sin(f*x+e)^2,x)
```

output
```
int(sec(e + f*x)**(1/3)*sec(e + f*x)*sin(e + f*x)**2,x)
```

# 3.276 $\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal result . . . 2078
Mathematica [A] (verified) . . . 2078
Rubi [A] (verified) . . . 2079
Maple [F] . . . 2080
Fricas [F] . . . 2080
Sympy [F(-1)] . . . 2081
Maxima [F] . . . 2081
Giac [F] . . . 2081
Mupad [F(-1)] . . . 2082
Reduce [F] . . . 2082

## Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{13}{6},-\frac{3}{2},-\frac{7}{6},\cos^2(e+fx)\right)\sec^{\frac{13}{3}}(e+fx)\sin(e+fx)}{13f\sqrt{\sin^2(e+fx)}}$$

output
```
3/13*hypergeom([-13/6, -3/2],[-7/6],cos(f*x+e)^2)*sec(f*x+e)^(13/3)*sin(f*
x+e)/f/(sin(f*x+e)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sqrt[3]{\sec(e+fx)}\left(27\sin(e+fx)-18\sqrt[6]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin(e+ f}{91f}$$

input
```
Integrate[Sec[e + f*x]^(16/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Sec[e + f*x]^(1/3)*(27*Sin[e + f*x] - 18*(Cos[e + f*x]^2)^(1/6)*Hyperge
ometric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*(-16
 + 7*Sec[e + f*x]^2)*Tan[e + f*x]))/(91*f)
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sec^{\frac{16}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^{16/3}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{3112}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin^4(e+fx)}{\cos^{\frac{16}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin(e+fx)^4}{\cos(e+fx)^{16/3}}dx$$

$$\Big\downarrow \text{3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{13}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{13}{6},-\frac{3}{2},-\frac{7}{6},\cos^2(e+fx)\right)}{13f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(16/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Hypergeometric2F1[-13/6, -3/2, -7/6, Cos[e + f*x]^2]*Sec[e + f*x]^(13/3
)*Sin[e + f*x])/(13*f*Sqrt[Sin[e + f*x]^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

## Maple [F]

$$\int \sec \left(fx+e\right)^{\frac{16}{3}} \sin \left(fx+e\right)^{4} dx$$

input
```
int(sec(f*x+e)^(16/3)*sin(f*x+e)^4,x)
```

output
```
int(sec(f*x+e)^(16/3)*sin(f*x+e)^4,x)
```

## Fricas [F]

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \sec \left(fx+e\right)^{\frac{16}{3}} \sin \left(fx+e\right)^{4}\,dx$$

input
```
integrate(sec(f*x+e)^(16/3)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sec(f*x + e)^(16/3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**(16/3)*sin(f*x+e)**4,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{16}{3}}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^(16/3)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(16/3)*sin(f*x + e)^4, x)
```

## Giac [F]

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{16}{3}}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^(16/3)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(16/3)*sin(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \sin\left(e+f\,x\right)^4\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^{16/3}\,dx$$

input
```
int(sin(e + f*x)^4*(1/cos(e + f*x))^(16/3),x)
```

output
```
int(sin(e + f*x)^4*(1/cos(e + f*x))^(16/3), x)
```

## Reduce [F]

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \sec\left(fx+e\right)^{\frac{16}{3}}\sin\left(fx+e\right)^4dx$$

input
```
int(sec(f*x+e)^(16/3)*sin(f*x+e)^4,x)
```

output
```
int(sec(e + f*x)**(1/3)*sec(e + f*x)**5*sin(e + f*x)**4,x)
```

# 3.277 $\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx$


Optimal result . . . 2083
Mathematica [A] (verified) . . . 2083
Rubi [A] (verified) . . . 2084
Maple [F] . . . 2085
Fricas [F] . . . 2085
Sympy [F(-1)] . . . 2086
Maxima [F] . . . 2086
Giac [F] . . . 2086
Mupad [F(-1)] . . . 2087
Reduce [F] . . . 2087


## Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{11}{6},-\frac{3}{2},-\frac{5}{6},\cos^2(e+fx)\right)\sec^{\frac{11}{3}}(e+fx)\sin(e+fx)}{11f\sqrt{\sin^2(e+fx)}}$$

output
```
3/11*hypergeom([-11/6, -3/2],[-5/6],cos(f*x+e)^2)*sec(f*x+e)^(11/3)*sin(f*
x+e)/f/(sin(f*x+e)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\left(\frac{9\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\sin^2(e+fx)\right)}{\sqrt[6]{\cos^2(e+fx)}}-(2+7\cos(2(e+fx)))\sec^4(e+fx)\right)\sin(e+fx)}{55f\sqrt[3]{\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^(14/3)*Sin[e + f*x]^4,x]
```

output
```
(3*((9*Hypergeometric2F1[1/2, 5/6, 3/2, Sin[e + f*x]^2])/(Cos[e + f*x]^2)^
(1/6) - (2 + 7*Cos[2*(e + f*x)])*Sec[e + f*x]^4)*Sin[e + f*x])/(55*f*Sec[e
 + f*x]^(1/3))
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sec^{\frac{14}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^{14/3}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{3112}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin^4(e+fx)}{\cos^{\frac{14}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin(e+fx)^4}{\cos(e+fx)^{14/3}}dx$$

$$\Big\downarrow \text{3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{11}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{11}{6},-\frac{3}{2},-\frac{5}{6},\cos^2(e+fx)\right)}{11f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(14/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Hypergeometric2F1[-11/6, -3/2, -5/6, Cos[e + f*x]^2]*Sec[e + f*x]^(11/3
)*Sin[e + f*x])/(11*f*Sqrt[Sin[e + f*x]^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

## Maple [F]

$$\int \sec (fx+e)^{\frac{14}{3}} \sin (fx+e)^4 \, dx$$

input
```
int(sec(f*x+e)^(14/3)*sin(f*x+e)^4,x)
```

output
```
int(sec(f*x+e)^(14/3)*sin(f*x+e)^4,x)
```

## Fricas [F]

$$\int \sec^{\frac{14}{3}}(e+fx) \sin^4(e+fx) \, dx = \int \sec (fx+e)^{\frac{14}{3}} \sin (fx+e)^4 \, dx$$

input
```
integrate(sec(f*x+e)^(14/3)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sec(f*x + e)^(14/3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**(14/3)*sin(f*x+e)**4,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{14}{3}}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^(14/3)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(14/3)*sin(f*x + e)^4, x)
```

## Giac [F]

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{14}{3}}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^(14/3)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(14/3)*sin(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sin\left(e+f\,x\right)^4\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^{14/3} \,d x$$

input
```
int(sin(e + f*x)^4*(1/cos(e + f*x))^(14/3),x)
```

output
```
int(sin(e + f*x)^4*(1/cos(e + f*x))^(14/3), x)
```

## Reduce [F]

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{14}{3}}\sin\left(fx+e\right)^4dx$$

input
```
int(sec(f*x+e)^(14/3)*sin(f*x+e)^4,x)
```

output
```
int(sec(e + f*x)**(2/3)*sec(e + f*x)**4*sin(e + f*x)**4,x)
```

# 3.278 $\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal result . . . 2088
Mathematica [A] (verified) . . . 2088
Rubi [A] (verified) . . . 2089
Maple [F] . . . 2090
Fricas [F] . . . 2090
Sympy [F(-1)] . . . 2091
Maxima [F] . . . 2091
Giac [F] . . . 2091
Mupad [F(-1)] . . . 2092
Reduce [F] . . . 2092

## Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},\cos^2(e+fx)\right)\sec^{\frac{10}{3}}(e+fx)\sin(e+fx)}{10f\sqrt{\sin^2(e+fx)}}$$

output
```
3/10*hypergeom([-5/3, -3/2],[-2/3],cos(f*x+e)^2)*sec(f*x+e)^(10/3)*sin(f*x
+e)/f/(sin(f*x+e)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\left(\frac{9\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\sin^2(e+fx)\right)}{\sqrt[3]{\cos^2(e+fx)}}+\sec^2(e+fx)\left(-13+4\sec^2(e+fx)\right)\right)\sin(e+fx)}{40f\sec^{\frac{2}{3}}(e+fx)}$$

input
```
Integrate[Sec[e + f*x]^(13/3)*Sin[e + f*x]^4,x]
```

output
```
(3*((9*Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^2])/(Cos[e + f*x]^2)^
(1/3) + Sec[e + f*x]^2*(-13 + 4*Sec[e + f*x]^2))*Sin[e + f*x])/(40*f*Sec[e
 + f*x]^(2/3))
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sec^{\frac{13}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^{13/3}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{3112}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin^4(e+fx)}{\cos^{\frac{13}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin(e+fx)^4}{\cos(e+fx)^{13/3}}dx$$

$$\Big\downarrow \text{3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{10}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},\cos^2(e+fx)\right)}{10f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(13/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Hypergeometric2F1[-5/3, -3/2, -2/3, Cos[e + f*x]^2]*Sec[e + f*x]^(10/3)
*Sin[e + f*x])/(10*f*Sqrt[Sin[e + f*x]^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

## Maple [F]

$$\int \sec\left(fx+e\right)^{\frac{13}{3}} \sin\left(fx+e\right)^{4} dx$$

input
```
int(sec(f*x+e)^(13/3)*sin(f*x+e)^4,x)
```

output
```
int(sec(f*x+e)^(13/3)*sin(f*x+e)^4,x)
```

## Fricas [F]

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{13}{3}} \sin\left(fx+e\right)^{4} \,dx$$

input
```
integrate(sec(f*x+e)^(13/3)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sec(f*x + e)^(13/3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**(13/3)*sin(f*x+e)**4,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{13}{3}}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^(13/3)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(13/3)*sin(f*x + e)^4, x)
```

## Giac [F]

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{13}{3}}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^(13/3)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(13/3)*sin(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \sin\left(e+f\,x\right)^4\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^{13/3}\,dx$$

input
```
int(sin(e + f*x)^4*(1/cos(e + f*x))^(13/3),x)
```

output
```
int(sin(e + f*x)^4*(1/cos(e + f*x))^(13/3), x)
```

## Reduce [F]

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \sec\left(fx+e\right)^{\frac{13}{3}}\sin\left(fx+e\right)^4dx$$

input
```
int(sec(f*x+e)^(13/3)*sin(f*x+e)^4,x)
```

output
```
int(sec(e + f*x)**(1/3)*sec(e + f*x)**4*sin(e + f*x)**4,x)
```

# 3.279 $\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal result . . . 2093
Mathematica [A] (verified) . . . 2093
Rubi [A] (verified) . . . 2094
Maple [F] . . . 2095
Fricas [F] . . . 2095
Sympy [F(-1)] . . . 2096
Maxima [F] . . . 2096
Giac [F] . . . 2096
Mupad [F(-1)] . . . 2097
Reduce [F] . . . 2097

## Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},\cos^2(e+fx)\right)\sec^{\frac{8}{3}}(e+fx)\sin(e+fx)}{8f\sqrt{\sin^2(e+fx)}}$$

output
```
3/8*hypergeom([-3/2, -4/3],[-1/3],cos(f*x+e)^2)*sec(f*x+e)^(8/3)*sin(f*x+e
)/f/(sin(f*x+e)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sec^{\frac{2}{3}}(e+fx)\left(-11\sin(e+fx)+9\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin(e+\right.}{16f}$$

input
```
Integrate[Sec[e + f*x]^(11/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Sec[e + f*x]^(2/3)*(-11*Sin[e + f*x] + 9*(Cos[e + f*x]^2)^(1/3)*Hyperge
ometric2F1[1/3, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + 2*Sec[e + f*x]*Ta
n[e + f*x]))/(16*f)
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sec^{\frac{11}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^{11/3}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{3112}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin^4(e+fx)}{\cos^{\frac{11}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin(e+fx)^4}{\cos(e+fx)^{11/3}}dx$$

$$\Big\downarrow \text{3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{8}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(11/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Hypergeometric2F1[-3/2, -4/3, -1/3, Cos[e + f*x]^2]*Sec[e + f*x]^(8/3)*
Sin[e + f*x])/(8*f*Sqrt[Sin[e + f*x]^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

## Maple [F]

$$\int \sec(fx+e)^{\frac{11}{3}} \sin(fx+e)^4 \, dx$$

input
```
int(sec(f*x+e)^(11/3)*sin(f*x+e)^4,x)
```

output
```
int(sec(f*x+e)^(11/3)*sin(f*x+e)^4,x)
```

## Fricas [F]

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{11}{3}} \sin(fx+e)^4 \, dx$$

input
```
integrate(sec(f*x+e)^(11/3)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sec(f*x + e)^(11/3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)**(11/3)*sin(f*x+e)**4,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \sec{(fx+e)}^{\frac{11}{3}}\sin{(fx+e)}^4\,dx$$

input
```
integrate(sec(f*x+e)^(11/3)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(11/3)*sin(f*x + e)^4, x)
```

## Giac [F]

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \sec{(fx+e)}^{\frac{11}{3}}\sin{(fx+e)}^4\,dx$$

input
```
integrate(sec(f*x+e)^(11/3)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(11/3)*sin(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4\left(\frac{1}{\cos(e+f\,x)}\right)^{11/3}\,dx$$

input
```
int(sin(e + f*x)^4*(1/cos(e + f*x))^(11/3),x)
```

output
```
int(sin(e + f*x)^4*(1/cos(e + f*x))^(11/3), x)
```

## Reduce [F]

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{11}{3}}\sin(fx+e)^4\,dx$$

input
```
int(sec(f*x+e)^(11/3)*sin(f*x+e)^4,x)
```

output
```
int(sec(e + f*x)**(2/3)*sec(e + f*x)**3*sin(e + f*x)**4,x)
```

# 3.280 $\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx$


Optimal result . . . 2098
Mathematica [A] (verified) . . . 2098
Rubi [A] (verified) . . . 2099
Maple [F] . . . 2100
Fricas [F] . . . 2100
Sympy [F(-1)] . . . 2101
Maxima [F] . . . 2101
Giac [F] . . . 2101
Mupad [F(-1)] . . . 2102
Reduce [F] . . . 2102


## Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{6},-\frac{1}{6},\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

output
```
3/7*hypergeom([-3/2, -7/6],[-1/6],cos(f*x+e)^2)*sec(f*x+e)^(7/3)*sin(f*x+e
)/f/(sin(f*x+e)^2)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sqrt[3]{\sec(e+fx)}\left(-10\sin(e+fx)+9\sqrt[6]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin(e+\right.}{7f}$$

input
```
Integrate[Sec[e + f*x]^(10/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Sec[e + f*x]^(1/3)*(-10*Sin[e + f*x] + 9*(Cos[e + f*x]^2)^(1/6)*Hyperge
ometric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*Tan[
e + f*x]))/(7*f)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sec^{\frac{10}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^{10/3}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{3112}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin^4(e+fx)}{\cos^{\frac{10}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin(e+fx)^4}{\cos(e+fx)^{10/3}}dx$$

$$\Big\downarrow \text{3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{7}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{6},-\frac{1}{6},\cos^2(e+fx)\right)}{7f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(10/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Hypergeometric2F1[-3/2, -7/6, -1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(7/3)*
Sin[e + f*x])/(7*f*Sqrt[Sin[e + f*x]^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

## Maple [F]

$$\int \sec(fx+e)^{\frac{10}{3}} \sin(fx+e)^4 \, dx$$

input
```
int(sec(f*x+e)^(10/3)*sin(f*x+e)^4,x)
```

output
```
int(sec(f*x+e)^(10/3)*sin(f*x+e)^4,x)
```

## Fricas [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{10}{3}} \sin(fx+e)^4 \, dx$$

input
```
integrate(sec(f*x+e)^(10/3)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sec(f*x + e)^(10/3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**(10/3)*sin(f*x+e)**4,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{10}{3}}\sin(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^(10/3)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(10/3)*sin(f*x + e)^4, x)
```

## Giac [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{10}{3}}\sin(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^(10/3)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(10/3)*sin(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4\left(\frac{1}{\cos(e+f\,x)}\right)^{10/3}\,dx$$

input
```
int(sin(e + f*x)^4*(1/cos(e + f*x))^(10/3),x)
```

output
```
int(sin(e + f*x)^4*(1/cos(e + f*x))^(10/3), x)
```

## Reduce [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{10}{3}}\sin(fx+e)^4\,dx$$

input
```
int(sec(f*x+e)^(10/3)*sin(f*x+e)^4,x)
```

output
```
int(sec(e + f*x)**(1/3)*sec(e + f*x)**3*sin(e + f*x)**4,x)
```

# 3.281 $\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx$

Optimal result . . . . . 2103
Mathematica [A] (verified) . . . . . 2103
Rubi [A] (verified) . . . . . 2104
Maple [F] . . . . . 2105
Fricas [F] . . . . . 2105
Sympy [F] . . . . . 2105
Maxima [F] . . . . . 2106
Giac [F] . . . . . 2106
Mupad [F(-1)] . . . . . 2106
Reduce [F] . . . . . 2107

## Optimal result

Integrand size = 21, antiderivative size = 57

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx=\frac{\cos^2(e+fx)^{13/6}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{13}{6},\frac{5}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^3(e+fx)}{3f}$$

output
```
1/3*(cos(f*x+e)^2)^(13/6)*hypergeom([3/2, 13/6],[5/2],sin(f*x+e)^2)*(d*sec
(f*x+e))^(4/3)*tan(f*x+e)^3/f
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx=\frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan(e+fx)}{4f\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^2,x]
```

output
```
(3*Hypergeometric2F1[-1/2, 2/3, 5/3, Sec[e + f*x]^2]*(d*Sec[e + f*x])^(4/3
)*Tan[e + f*x])/(4*f*Sqrt[-Tan[e + f*x]^2])
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(d\sec(e+fx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2(d\sec(e+fx))^{4/3}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{13/6}\tan^3(e+fx)(d\sec(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{13}{6},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

input
```
Int[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^2,x]
```

output
```
((Cos[e + f*x]^2)^(13/6)*Hypergeometric2F1[3/2, 13/6, 5/2, Sin[e + f*x]^2]
*(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^3)/(3*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (d\sec(fx+e))^{\frac{4}{3}} \tan(fx+e)^2 \, dx$$

input
```
int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x)
```

output
```
int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x)
```

## Fricas [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}} \tan(fx+e)^2 \, dx$$

input
```
integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*d*sec(f*x + e)*tan(f*x + e)^2, x)
```

## Sympy [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int (d\sec(e+fx))^{\frac{4}{3}} \tan^2(e+fx)\,dx$$

input
```
integrate((d*sec(f*x+e))**(4/3)*tan(f*x+e)**2,x)
```

output
```
Integral((d*sec(e + f*x))**(4/3)*tan(e + f*x)**2, x)
```

## Maxima [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^2, x)
```

## Giac [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(\frac{d}{\cos(e+f\,x)}\right)^{4/3}\,dx$$

input
```
int(tan(e + f*x)^2*(d/cos(e + f*x))^(4/3),x)
```

output
```
int(tan(e + f*x)^2*(d/cos(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = d^{\frac{4}{3}}\left(\int \sec(fx+e)^{\frac{4}{3}}\tan(fx+e)^2\,dx\right)$$

input
```
int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x)
```

output
```
d**(1/3)*int(sec(e + f*x)**(1/3)*sec(e + f*x)*tan(e + f*x)**2,x)*d
```

# 3.282 $\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2108
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2108
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2109
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2110
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2110
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2110
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2111
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2111
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2111
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2112

## Optimal result

Integrand size = 21, antiderivative size = 57

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx=\frac{\cos^2(e+fx)^{11/6}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{11}{6},\frac{5}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^3(e+fx)}{3f}$$

output
```
1/3*(cos(f*x+e)^2)^(11/6)*hypergeom([3/2, 11/6],[5/2],sin(f*x+e)^2)*(d*sec
(f*x+e))^(2/3)*tan(f*x+e)^3/f
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = -\frac{3\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},\sec^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sqrt{-\tan^2(e+fx)}}{2f}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^2,x]
```

output
```
(-3*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1/3, 4/3, Sec[e + f*x]^2]*(d*Sec[
e + f*x])^(2/3)*Sqrt[-Tan[e + f*x]^2])/(2*f)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(d\sec(e+fx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2(d\sec(e+fx))^{2/3}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{11/6}\tan^3(e+fx)(d\sec(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{11}{6},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^2,x]
```

output
```
((Cos[e + f*x]^2)^(11/6)*Hypergeometric2F1[3/2, 11/6, 5/2, Sin[e + f*x]^2]
*(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^3)/(3*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^{2}\,dx$$

input
```
int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x)
```

output
```
int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x)
```

## Fricas [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^{2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

## Sympy [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int (d\sec(e+fx))^{\frac{2}{3}}\tan^{2}(e+fx)\,dx$$

input
```
integrate((d*sec(f*x+e))**(2/3)*tan(f*x+e)**2,x)
```

output
```
Integral((d*sec(e + f*x))**(2/3)*tan(e + f*x)**2, x)
```

## Maxima [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

## Giac [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}\,dx$$

input
```
int(tan(e + f*x)^2*(d/cos(e + f*x))^(2/3),x)
```

output
```
int(tan(e + f*x)^2*(d/cos(e + f*x))^(2/3), x)
```

## Reduce [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = d^{\frac{2}{3}}\left(\int \sec(fx+e)^{\frac{2}{3}}\tan(fx+e)^2\,dx\right)$$

input
```
int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x)
```

output
```
d**(2/3)*int(sec(e + f*x)**(2/3)*tan(e + f*x)**2,x)
```

# 3.283 $\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx$

Optimal result . . . 2113
Mathematica [A] (verified) . . . 2113
Rubi [A] (verified) . . . 2114
Maple [F] . . . 2115
Fricas [F] . . . 2115
Sympy [F] . . . 2115
Maxima [F] . . . 2116
Giac [F] . . . 2116
Mupad [F(-1)] . . . 2116
Reduce [F] . . . 2117

## Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{5/3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{3},\frac{5}{2},\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3f}$$

output
```
1/3*(cos(f*x+e)^2)^(5/3)*hypergeom([3/2, 5/3],[5/2],sin(f*x+e)^2)*(d*sec(f
*x+e))^(1/3)*tan(f*x+e)^3/f
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = -\frac{3\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

output
```
(-3*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1/6, 7/6, Sec[e + f*x]^2]*(d*Sec[
e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])/f
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\sqrt[3]{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\sqrt[3]{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{5/3}\tan^3(e+fx)\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{3},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

output
```
((Cos[e + f*x]^2)^(5/3)*Hypergeometric2F1[3/2, 5/3, 5/2, Sin[e + f*x]^2]*(
d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^3)/(3*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (d\sec (fx+e))^{\frac{1}{3}} \tan (fx+e)^2 \, dx$$

input
```
int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

output
```
int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

## Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \int (d\sec (fx+e))^{\frac{1}{3}} \tan (fx+e)^2 \, dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

## Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt[3]{d\sec (e+fx)}\tan^2 (e+fx)\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/3)*tan(f*x+e)**2,x)
```

output
```
Integral((d*sec(e + f*x))**(1/3)*tan(e + f*x)**2, x)
```

## Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

## Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}dx$$

input
```
int(tan(e + f*x)^2*(d/cos(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^2*(d/cos(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = d^{\frac{1}{3}}\left(\int \sec(fx+e)^{\frac{1}{3}}\tan(fx+e)^2\,dx\right)$$

input
```
int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

output
```
d**(1/3)*int(sec(e + f*x)**(1/3)*tan(e + f*x)**2,x)
```

# 3.284 $\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal result . . . . . 2118
Mathematica [A] (verified) . . . . . 2118
Rubi [A] (verified) . . . . . 2119
Maple [F] . . . . . 2120
Fricas [F] . . . . . 2120
Sympy [F] . . . . . 2121
Maxima [F] . . . . . 2121
Giac [F] . . . . . 2121
Mupad [F(-1)] . . . . . 2122
Reduce [F] . . . . . 2122

## Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{4/3}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)\tan^3(e+fx)}{3f\sqrt[3]{d\sec(e+fx)}}$$

output
```
1/3*(cos(f*x+e)^2)^(4/3)*hypergeom([4/3, 3/2],[5/2],sin(f*x+e)^2)*tan(f*x+
e)^3/f/(d*sec(f*x+e))^(1/3)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^2/(d*Sec[e + f*x])^(1/3),x]
```

output
```
(-3*Hypergeometric2F1[-1/2, -1/6, 5/6, Sec[e + f*x]^2]*Tan[e + f*x])/(f*(d
*Sec[e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{4/3}\tan^3(e+fx)\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[Tan[e + f*x]^2/(d*Sec[e + f*x])^(1/3),x]
```

output
```
((Cos[e + f*x]^2)^(4/3)*Hypergeometric2F1[4/3, 3/2, 5/2, Sin[e + f*x]^2]*T
an[e + f*x]^3)/(3*f*(d*Sec[e + f*x])^(1/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \frac{\tan (fx+e)^2}{(d \sec (fx+e))^{\frac{1}{3}}} dx$$

input
```
int(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x)
```

output
```
int(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d \sec(e+fx)}} \, dx = \int \frac{\tan (fx+e)^2}{(d \sec (fx+e))^{\frac{1}{3}}} \, dx$$

input
```
integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2/(d*sec(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(d*sec(f*x+e))**(1/3),x)
```

output
```
Integral(tan(e + f*x)**2/(d*sec(e + f*x))**(1/3), x)
```

## Maxima [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int(tan(e + f*x)^2/(d/cos(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^2/(d/cos(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{1}{3}}}dx}{d^{\frac{1}{3}}}$$

input
```
int(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x)
```

output
```
int(tan(e + f*x)**2/sec(e + f*x)**(1/3),x)/d**(1/3)
```

# 3.285 $\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2123
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2123
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2124
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2125
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2125
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2125
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2126
Giac [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2126
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2126
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2127

## Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \frac{\cos^2(e+fx)^{7/6}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)\tan^3(e+fx)}{3f(d\sec(e+fx))^{2/3}}$$

output
```
1/3*(cos(f*x+e)^2)^(7/6)*hypergeom([7/6, 3/2],[5/2],sin(f*x+e)^2)*tan(f*x+
e)^3/f/(d*sec(f*x+e))^(2/3)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\sec^2(e+fx)\right)\tan(e+fx)}{2f(d\sec(e+fx))^{2/3}\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^2/(d*Sec[e + f*x])^(2/3),x]
```

output
```
(-3*Hypergeometric2F1[-1/2, -1/3, 2/3, Sec[e + f*x]^2]*Tan[e + f*x])/(2*f*
(d*Sec[e + f*x])^(2/3)*Sqrt[-Tan[e + f*x]^2])
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{(d\sec(e+fx))^{2/3}}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{7/6}\tan^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f(d\sec(e+fx))^{2/3}}$$

input
```
Int[Tan[e + f*x]^2/(d*Sec[e + f*x])^(2/3),x]
```

output
```
((Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 3/2, 5/2, Sin[e + f*x]^2]*T
an[e + f*x]^3)/(3*f*(d*Sec[e + f*x])^(2/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \frac{\tan (fx+e)^2}{(d\sec (fx+e))^{\frac{2}{3}}}dx$$

input
```
int(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x)
```

output
```
int(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x)
```

## Fricas [F]

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan (fx+e)^2}{(d\sec (fx+e))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2/(d*sec(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan^2 (e+fx)}{(d\sec (e+fx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(d*sec(f*x+e))**(2/3),x)
```

output
```
Integral(tan(e + f*x)**2/(d*sec(e + f*x))**(2/3), x)
```

## Maxima [F]

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan\left(fx+e\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(2/3), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Error: Bad Argument
Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

input
```
int(tan(e + f*x)^2/(d/cos(e + f*x))^(2/3),x)
```

output
```
int(tan(e + f*x)^2/(d/cos(e + f*x))^(2/3), x)
```

## Reduce [F]

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \frac{\int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{2}{3}}}dx}{d^{\frac{2}{3}}}$$

input
```
int(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x)
```

output
```
int(tan(e + f*x)**2/sec(e + f*x)**(2/3),x)/d**(2/3)
```

# 3.286 $\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx$


Optimal result . . . 2128
Mathematica [A] (verified) . . . 2128
Rubi [A] (verified) . . . 2129
Maple [F] . . . 2130
Fricas [F] . . . 2130
Sympy [F] . . . 2130
Maxima [F(-1)] . . . 2131
Giac [F] . . . 2131
Mupad [F(-1)] . . . 2131
Reduce [F] . . . 2132


## Optimal result

Integrand size = 21, antiderivative size = 57

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{19/6}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{19}{6},\frac{7}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^5(e+fx)}{5f}$$

output
```
1/5*(cos(f*x+e)^2)^(19/6)*hypergeom([5/2, 19/6],[7/2],sin(f*x+e)^2)*(d*sec
(f*x+e))^(4/3)*tan(f*x+e)^5/f
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \frac{3d\csc(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},\sec^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{4f}$$

input
```
Integrate[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^4,x]
```

output
```
(3*d*Csc[e + f*x]*Hypergeometric2F1[-3/2, 2/3, 5/3, Sec[e + f*x]^2]*(d*Sec
[e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])/(4*f)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(d\sec(e+fx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4(d\sec(e+fx))^{4/3}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{19/6}\tan^5(e+fx)(d\sec(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{19}{6},\frac{7}{2},\sin^2(e+fx)\right)}{5f}$$

input
```
Int[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^4,x]
```

output
```
((Cos[e + f*x]^2)^(19/6)*Hypergeometric2F1[5/2, 19/6, 7/2, Sin[e + f*x]^2]
*(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^5)/(5*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (d\sec(fx+e))^{\frac{4}{3}} \tan(fx+e)^4 \, dx$$

input
```
int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x)
```

output
```
int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x)
```

## Fricas [F]

$$\int (d\sec(e+fx))^{4/3} \tan^4(e+fx) \, dx = \int (d\sec(fx+e))^{\frac{4}{3}} \tan(fx+e)^4 \, dx$$

input
```
integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*d*sec(f*x + e)*tan(f*x + e)^4, x)
```

## Sympy [F]

$$\int (d\sec(e+fx))^{4/3} \tan^4(e+fx) \, dx = \int (d\sec(e+fx))^{\frac{4}{3}} \tan^4(e+fx) \, dx$$

input
```
integrate((d*sec(f*x+e))**(4/3)*tan(f*x+e)**4,x)
```

output
```
Integral((d*sec(e + f*x))**(4/3)*tan(e + f*x)**4, x)
```

## Maxima [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{d}{\cos(e+f\,x)}\right)^{4/3}\,dx$$

input
```
int(tan(e + f*x)^4*(d/cos(e + f*x))^(4/3),x)
```

output
```
int(tan(e + f*x)^4*(d/cos(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = d^{\frac{4}{3}}\left(\int \sec(fx+e)^{\frac{4}{3}}\tan(fx+e)^4\,dx\right)$$

input
```
int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x)
```

output
```
d**(1/3)*int(sec(e + f*x)**(1/3)*sec(e + f*x)*tan(e + f*x)**4,x)*d
```

# 3.287 $\int (d \sec(e+fx))^{2/3} \tan^4(e+fx)\, dx$


Optimal result . . . 2133
Mathematica [A] (verified) . . . 2133
Rubi [A] (verified) . . . 2134
Maple [F] . . . 2135
Fricas [F] . . . 2135
Sympy [F] . . . 2135
Maxima [F] . . . 2136
Giac [F] . . . 2136
Mupad [F(-1)] . . . 2136
Reduce [F] . . . 2137


## Optimal result

Integrand size = 21, antiderivative size = 57

$$\int (d \sec(e+fx))^{2/3} \tan^4(e+fx)\, dx = \frac{\cos^2(e+fx)^{17/6} \text{Hypergeometric2F1}\left(\frac{5}{2},\frac{17}{6},\frac{7}{2},\sin^2(e+fx)\right) (d \sec(e+fx))^{2/3} \tan^5(e+fx)}{5f}$$

output
```
1/5*(cos(f*x+e)^2)^(17/6)*hypergeom([5/2, 17/6],[7/2],sin(f*x+e)^2)*(d*sec
(f*x+e))^(2/3)*tan(f*x+e)^5/f
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (d \sec(e+fx))^{2/3} \tan^4(e+fx)\, dx = \frac{3 \cot(e+fx) \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},\sec^2(e+fx)\right) (d \sec(e+fx))^{2/3} \sqrt{-\tan^2(e+fx)}}{2f}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^4,x]
```

output
```
(3*Cot[e + f*x]*Hypergeometric2F1[-3/2, 1/3, 4/3, Sec[e + f*x]^2]*(d*Sec[e
 + f*x])^(2/3)*Sqrt[-Tan[e + f*x]^2])/(2*f)
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(d\sec(e+fx))^{2/3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^4(d\sec(e+fx))^{2/3}dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{\cos^2(e+fx)^{17/6}\tan^5(e+fx)(d\sec(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{17}{6},\frac{7}{2},\sin^2(e+fx)\right)}{5f}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^4,x]
```

output
```
((Cos[e + f*x]^2)^(17/6)*Hypergeometric2F1[5/2, 17/6, 7/2, Sin[e + f*x]^2]
*(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^5)/(5*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (d\sec (fx+e))^{\frac{2}{3}}\tan (fx+e)^{4}\,dx$$

input
```
int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x)
```

output
```
int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x)
```

## Fricas [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int (d\sec (fx+e))^{\frac{2}{3}}\tan (fx+e)^{4}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

## Sympy [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int (d\sec (e+fx))^{\frac{2}{3}}\tan^{4}(e+fx)\,dx$$

input
```
integrate((d*sec(f*x+e))**(2/3)*tan(f*x+e)**4,x)
```

output
```
Integral((d*sec(e + f*x))**(2/3)*tan(e + f*x)**4, x)
```

## Maxima [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

## Giac [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}dx$$

input
```
int(tan(e + f*x)^4*(d/cos(e + f*x))^(2/3),x)
```

output
```
int(tan(e + f*x)^4*(d/cos(e + f*x))^(2/3), x)
```

## Reduce [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = d^{\frac{2}{3}}\left(\int \sec(fx+e)^{\frac{2}{3}}\tan(fx+e)^4\,dx\right)$$

input
```
int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x)
```

output
```
d**(2/3)*int(sec(e + f*x)**(2/3)*tan(e + f*x)**4,x)
```

# 3.288 $\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2138
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2138
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2139
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2140
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2140
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2140
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2141
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2141
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2141
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2142

## Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{8/3}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{8}{3},\frac{7}{2},\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^5(e+fx)}{5f}$$

output
```
1/5*(cos(f*x+e)^2)^(8/3)*hypergeom([5/2, 8/3],[7/2],sin(f*x+e)^2)*(d*sec(f
*x+e))^(1/3)*tan(f*x+e)^5/f
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \frac{3\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

output
```
(3*Cot[e + f*x]*Hypergeometric2F1[-3/2, 1/6, 7/6, Sec[e + f*x]^2]*(d*Sec[e
 + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])/f
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\sqrt[3]{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4\sqrt[3]{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{8/3}\tan^5(e+fx)\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{8}{3},\frac{7}{2},\sin^2(e+fx)\right)}{5f}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

output
```
((Cos[e + f*x]^2)^(8/3)*Hypergeometric2F1[5/2, 8/3, 7/2, Sin[e + f*x]^2]*(
d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^5)/(5*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}} \tan(fx+e)^{4} dx$$

input
```
int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

output
```
int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

## Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{1}{3}} \tan(fx+e)^{4}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

## Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt[3]{d\sec(e+fx)}\tan^{4}(e+fx)\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/3)*tan(f*x+e)**4,x)
```

output
```
Integral((d*sec(e + f*x))**(1/3)*tan(e + f*x)**4, x)
```

## Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx=\int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

## Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx=\int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx=\int \tan(e+f\,x)^4\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}dx$$

input
```
int(tan(e + f*x)^4*(d/cos(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^4*(d/cos(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = d^{\frac{1}{3}}\left(\int \sec\left(fx+e\right)^{\frac{1}{3}}\tan\left(fx+e\right)^4dx\right)$$

input
```
int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

output
```
d**(1/3)*int(sec(e + f*x)**(1/3)*tan(e + f*x)**4,x)
```

# 3.289 $\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal result . . . . . 2143
Mathematica [A] (verified) . . . . . 2143
Rubi [A] (verified) . . . . . 2144
Maple [F] . . . . . 2145
Fricas [F] . . . . . 2145
Sympy [F] . . . . . 2146
Maxima [F] . . . . . 2146
Giac [F] . . . . . 2146
Mupad [F(-1)] . . . . . 2147
Reduce [F] . . . . . 2147

## Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{3},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)\tan^5(e+fx)}{5f\sqrt[3]{d\sec(e+fx)}}$$

output
```
1/5*(cos(f*x+e)^2)^(7/3)*hypergeom([7/3, 5/2],[7/2],sin(f*x+e)^2)*tan(f*x+
e)^5/f/(d*sec(f*x+e))^(1/3)
```

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)\tan^3(e+fx)}{f\sqrt[3]{d\sec(e+fx)}\left(-\tan^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^4/(d*Sec[e + f*x])^(1/3),x]
```

output
```
(-3*Hypergeometric2F1[-3/2, -1/6, 5/6, Sec[e + f*x]^2]*Tan[e + f*x]^3)/(f*
(d*Sec[e + f*x])^(1/3)*(-Tan[e + f*x]^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{7/3}\tan^5(e+fx)\text{Hypergeometric2F1}\left(\frac{7}{3},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)}{5f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[Tan[e + f*x]^4/(d*Sec[e + f*x])^(1/3),x]
```

output
```
((Cos[e + f*x]^2)^(7/3)*Hypergeometric2F1[7/3, 5/2, 7/2, Sin[e + f*x]^2]*T
an[e + f*x]^5)/(5*f*(d*Sec[e + f*x])^(1/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \frac{\tan (fx+e)^4}{(d\sec (fx+e))^{\frac{1}{3}}}dx$$

input
```
int(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x)
```

output
```
int(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan (fx+e)^4}{(d\sec (fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4/(d*sec(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(d*sec(f*x+e))**(1/3),x)
```

output
```
Integral(tan(e + f*x)**4/(d*sec(e + f*x))**(1/3), x)
```

## Maxima [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(fx+e)^4}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(fx+e)^4}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int(tan(e + f*x)^4/(d/cos(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^4/(d/cos(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\int \frac{\tan(fx+e)^4}{\sec(fx+e)^{\frac{1}{3}}}dx}{d^{\frac{1}{3}}}$$

input
```
int(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x)
```

output
```
int(tan(e + f*x)**4/sec(e + f*x)**(1/3),x)/d**(1/3)
```

# 3.290 $\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2148
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2148
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2149
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2150
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2150
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2150
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2151
Giac [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2151
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2151
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2152


## Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \frac{\cos^2(e+fx)^{13/6}\,\text{Hypergeometric2F1}\left(\frac{13}{6},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)\tan^5(e+fx)}{5f(d\sec(e+fx))^{2/3}}$$

output
```
1/5*(cos(f*x+e)^2)^(13/6)*hypergeom([13/6, 5/2],[7/2],sin(f*x+e)^2)*tan(f*
x+e)^5/f/(d*sec(f*x+e))^(2/3)
```

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},\sec^2(e+fx)\right)\tan^3(e+fx)}{2f(d\sec(e+fx))^{2/3}\left(-\tan^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^4/(d*Sec[e + f*x])^(2/3),x]
```

output
```
(-3*Hypergeometric2F1[-3/2, -1/3, 2/3, Sec[e + f*x]^2]*Tan[e + f*x]^3)/(2*
f*(d*Sec[e + f*x])^(2/3)*(-Tan[e + f*x]^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{(d\sec(e+fx))^{2/3}}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{13/6}\tan^5(e+fx)\,\text{Hypergeometric2F1}\left(\frac{13}{6},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)}{5f(d\sec(e+fx))^{2/3}}$$

input
```
Int[Tan[e + f*x]^4/(d*Sec[e + f*x])^(2/3),x]
```

output
```
((Cos[e + f*x]^2)^(13/6)*Hypergeometric2F1[13/6, 5/2, 7/2, Sin[e + f*x]^2]
*Tan[e + f*x]^5)/(5*f*(d*Sec[e + f*x])^(2/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \frac{\tan (fx+e)^4}{(d\sec (fx+e))^{\frac{2}{3}}}dx$$

input
```
int(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x)
```

output
```
int(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x)
```

## Fricas [F]

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan (fx+e)^4}{(d\sec (fx+e))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4/(d*sec(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan^4 (e+fx)}{(d\sec (e+fx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(d*sec(f*x+e))**(2/3),x)
```

output
```
Integral(tan(e + f*x)**4/(d*sec(e + f*x))**(2/3), x)
```

## Maxima [F]

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(fx+e)^4}{(d\sec(fx+e))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(2/3), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,0,0]%%%}+%%%{-1,[0,1,0,0,0]%%%} / %%%{1,[0,0,0,1,
1]%%%} Er
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

input
```
int(tan(e + f*x)^4/(d/cos(e + f*x))^(2/3),x)
```

output
```
int(tan(e + f*x)^4/(d/cos(e + f*x))^(2/3), x)
```

## Reduce [F]

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \frac{\int \frac{\tan(fx+e)^4}{\sec(fx+e)^{\frac{2}{3}}}dx}{d^{\frac{2}{3}}}$$

input
```
int(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x)
```

output
```
int(tan(e + f*x)**4/sec(e + f*x)**(2/3),x)/d**(2/3)
```

# 3.291 $\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal result . . . . . 2153
Mathematica [A] (verified) . . . . . 2154
Rubi [A] (warning: unable to verify) . . . . . 2154
Maple [A] (verified) . . . . . 2157
Fricas [B] (verification not implemented) . . . . . 2158
Sympy [F(-1)] . . . . . 2159
Maxima [F] . . . . . 2160
Giac [F] . . . . . 2160
Mupad [F(-1)] . . . . . 2160
Reduce [F] . . . . . 2161

## Optimal result

Integrand size = 25, antiderivative size = 178

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{\sqrt{b}d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{\sqrt{b}d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf}$$

output
```
-1/4*b^(1/2)*d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)
/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)+1/4*b^(1/2)*d^3*arctanh((b*si
n(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*si
n(f*x+e))^(1/2)+1/2*d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2)/b/f
```

## Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.74

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\left(-\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\text{arctanh}\left(\right.\right.}{4f\sqrt[4]{\sec^2(e+fx)}\sqrt{\tan(e+}}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]*(-ArcTan[Sqrt[Tan[e + f*x]]
/(Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/
4)] + 2*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^(3/2)))/(4*f*(Sec[e + f*x]^2)^
(1/4)*Sqrt[Tan[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.73, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3093, 3042, 3096, 3042, 3044, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{1}{4}d^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}d^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{3096}$$

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \sec(e+fx)\sqrt{b\sin(e+fx)}dx}{4\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{\cos(e+fx)}dx}{4\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{3044}$$

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \frac{b^2\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{4bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd^3\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{266}$$

$$\frac{bd^3\sqrt{b\tan(e+fx)}\int \frac{b^2\sin^2(e+fx)}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{827}$$

$$\frac{bd^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{216}$$

$$\frac{bd^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{219}$$

$$\frac{bd^3\sqrt{b\tan(e+fx)}\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(b*d^3*(-1/2*ArcTan[Sqrt[b]*Sin[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sin[e
+ f*x]]/(2*Sqrt[b]))*Sqrt[b*Tan[e + f*x]])/(2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[
b*Sin[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2*b*
f)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

## Maple [A] (verified)

Time = 5.69 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| default | $\frac{\sqrt{b\tan(fx+e)}\,d^2\sqrt{d\sec(fx+e)}\left(-\cos(fx+e)\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(fx+e)}{\cos(fx+e)-1}\right)-\cos(fx+e)\arctan\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(}{\cos(fx+e)-1}\right.\right.}{4f(1+\cos(fx+e))\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}}$ |

input
```
int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*(b*tan(f*x+e))^(1/2)*d^2*(d*sec(f*x+e))^(1/2)/(1+cos(f*x+e))/(sin(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)*(-cos(f*x+e)*arctanh((sin(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))-cos(f*x+e)*arctan((sin(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))+(sin(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)*(2*sin(f*x+e)+2*tan(f*x+e)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(144) = 288$.

Time = 0.29 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.43

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*(2*sqrt(-b*d)*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (
cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sq
rt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f
*x + e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) - s
qrt(-b*d)*d^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2
 - 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) -
8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*
x + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x
+ e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16
*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/
(f*cos(f*x + e)), -1/32*(2*sqrt(b*d)*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*co
s(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(
f*x + e) + 4)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x +
 e))/(b*d*cos(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*c
os(f*x + e) - sqrt(b*d)*d^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*
cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*s
in(f*x + e) - 8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x +
e))/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x +
e) + 8)) - 16*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)...
```

## Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan(e+f\,x)}\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}dx$$

input
```
int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(5/2), x)
```

## Reduce [F]

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \sqrt{d}\,\sqrt{b}\left(\int \sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)^2\,dx\right)d^2$$

input
```
int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*sqrt(b)*int(sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x)**2,
x)*d**2
```

# 3.292 $\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal result . . . . . 2162
Mathematica [C] (verified) . . . . . 2162
Rubi [A] (verified) . . . . . 2163
Maple [C] (verified) . . . . . 2165
Fricas [C] (verification not implemented) . . . . . 2165
Sympy [F] . . . . . 2166
Maxima [F] . . . . . 2166
Giac [F] . . . . . 2167
Mupad [F(-1)] . . . . . 2167
Reduce [F] . . . . . 2167

## Optimal result

Integrand size = 25, antiderivative size = 93

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}$$

output
```
d^2*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d
*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)+d^2*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*
x+e))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{d\sqrt{d\sec(e+fx)}\left(-3+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)\sin(e+fx)\sqrt{b\tan(e+fx)}}{3f}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
-1/3*(d*Sqrt[d*Sec[e + f*x]]*(-3 + Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e
 + f*x]^2]*(Sec[e + f*x]^2)^(1/4))*Sin[e + f*x]*Sqrt[b*Tan[e + f*x]])/f
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3093, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{1}{2}d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{1}{2}d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
-((d^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Se
c[e + f*x]]*Sqrt[Sin[e + f*x]])) + (d^2*(b*Tan[e + f*x])^(3/2))/(b*f*Sqrt[
d*Sec[e + f*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.89

| method | result |
|---|---|
| default | $-\frac{\csc(fx+e)\left(\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\operatorname{EllipticF}\left(\sqrt{1+i\cot(fx+\right.\right.}{}}$ |

input
```
int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*csc(f*x+e)*((1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(I*(csc(f*x+e)-cot(
f*x+e)))^(1/2)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*EllipticF((1+I*cot(f*x+
e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(cos(f*x+e)^2+cos(f*x+e))+(1-I*cot(f*x
+e)+I*csc(f*x+e))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((1+I*c
ot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1+I*cot(f*x+e)-I*csc(f*x+e))^(
1/2)*(-2*cos(f*x+e)^2-2*cos(f*x+e))+2^(1/2)*(cos(f*x+e)-1))*(b*tan(f*x+e))
^(1/2)*(d*sec(f*x+e))^(1/2)*d*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \frac{2\,d\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin(fx+e) - i\sqrt{-2i\,bd}d\text{weierstrassZeta}(4,0,\text{wei}}{}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x +
e) - I*sqrt(-2*I*b*d)*d*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, co
s(f*x + e) + I*sin(f*x + e))) + I*sqrt(2*I*b*d)*d*weierstrassZeta(4, 0, we
ierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/f
```

## Sympy [F]

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan(e+fx)}(d\sec(e+fx))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(3/2), x)
```

## Maxima [F]

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan(e+f\,x)}\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \sqrt{d}\,\sqrt{b}\left(\int \sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)\,dx\right)d$$

input
```
int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*sqrt(b)*int(sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x),x)*
d
```

# 3.293 $\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx$

Optimal result . . . 2168
Mathematica [A] (verified) . . . 2168
Rubi [A] (warning: unable to verify) . . . 2169
Maple [A] (verified) . . . 2172
Fricas [B] (verification not implemented) . . . 2172
Sympy [F] . . . 2173
Maxima [F] . . . 2173
Giac [F] . . . 2174
Mupad [F(-1)] . . . 2174
Reduce [F] . . . 2174

## Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = -\frac{\sqrt{b}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

output
```
-b^(1/2)*d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*
sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)+b^(1/2)*d*arctanh((b*sin(f*x+e))^(1
/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1
/2)
```

## Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.77

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \frac{\left(-\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right)\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f\sqrt[4]{\sec^2(e+fx)}\sqrt{\tan(e+fx)}}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

output
```
((-ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqrt[Tan[e
+ f*x]]/(Sec[e + f*x]^2)^(1/4)])*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]
)/(f*(Sec[e + f*x]^2)^(1/4)*Sqrt[Tan[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.67, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3096, 3042, 3044, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{d\sqrt{b\tan(e+fx)}\int \sec(e+fx)\sqrt{b\sin(e+fx)}dx}{\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{\cos(e+fx)}dx}{\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{d\sqrt{b\tan(e+fx)}\int \frac{b^2\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2bd\sqrt{b\tan(e+fx)}\int \frac{b^2\sin^2(e+fx)}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2bd\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}\right)}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2bd\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2bd\sqrt{b\tan(e+fx)}\left(\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*b*d*(-1/2*ArcTan[Sqrt[b]*Sin[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sin[e
+ f*x]]/(2*Sqrt[b]))*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*
Sin[e + f*x]])
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

## Maple [A] (verified)

Time = 5.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03

| method | result |
|---|---|
| default | $-\frac{\left(\arctan\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(fx+e)}{\cos(fx+e)-1}\right)+\operatorname{arctanh}\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(fx+e)}{\cos(fx+e)-1}\right)\right)\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,\cos(fx+e)}{f(1+\cos(fx+e))\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}}$ |

input
```
int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(arctan((sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1)
)+arctanh((sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1)))*
(b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/2)*cos(f*x+e)/(1+cos(f*x+e))/(sin(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(108) = 216.

Time = 0.21 (sec) , antiderivative size = 654, normalized size of antiderivative = 4.95

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(2*sqrt(-b*d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(f
*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(-b
*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x +
e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e))) - sqrt(-b*d)*log((b*d
*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 - (cos(f*x +
 e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*s
in(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b*d + 28*(b*d*cos(f*x
+ e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(
f*x + e)^2 - 2)*sin(f*x + e) + 8)))/f, -1/8*(2*sqrt(b*d)*arctan(1/4*(cos(f
*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f
*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*
sin(f*x + e))) - sqrt(b*d)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2
 - 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) -
8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x
 + e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x +
 e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/f]
```

## Sympy [F]

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan{(e+fx)}}\sqrt{d\sec{(e+fx)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x))*sqrt(d*sec(e + f*x)), x)
```

## Maxima [F]

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{d\sec{(fx+e)}}\sqrt{b\tan{(fx+e)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{d\sec(fx+e)}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan(e+fx)}\sqrt{\frac{d}{\cos(e+fx)}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \sqrt{d}\sqrt{b}\left(\int \sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}dx\right)$$

input
```
int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*sqrt(b)*int(sqrt(tan(e + f*x))*sqrt(sec(e + f*x)),x)
```

# 3.294 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal result . . . . . 2175
Mathematica [C] (verified) . . . . . 2175
Rubi [A] (verified) . . . . . 2176
Maple [C] (verified) . . . . . 2178
Fricas [C] (verification not implemented) . . . . . 2178
Sympy [F] . . . . . 2179
Maxima [F] . . . . . 2179
Giac [F] . . . . . 2179
Mupad [F(-1)] . . . . . 2180
Reduce [F] . . . . . 2180

## Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx=\frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*
sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx=\frac{2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}(b\tan(e+fx))^{3/2}}{3bf\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/4
)*(b*Tan[e + f*x])^(3/2))/(3*b*f*Sqrt[d*Sec[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e
+ f*x]]*Sqrt[Sin[e + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 253, normalized size of antiderivative = 4.60

| method | result |
|---|---|
| default | $-\frac{\csc(fx+e)\sqrt{b\tan(fx+e)}\left(\sqrt{1+i(-\csc(fx+e)+\cot(fx+e))}\,\sqrt{1-i(-\csc(fx+e)+\cot(fx+e))}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\,(\right.}{}$ |
| risch | $-\frac{i\sqrt{2}\sqrt{-\frac{ib\left(e^{2i(fx+e)}-1\right)}{e^{2i(fx+e)}+1}}}{f\sqrt{\frac{d\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}}+\frac{i\left(\frac{2i\left(-ibd\,e^{2i(fx+e)}+idb\right)}{bd\sqrt{e^{i(fx+e)}\left(-ibd\,e^{2i(fx+e)}+idb\right)}}-\frac{\sqrt{e^{i(fx+e)}+1}\,\sqrt{-2\,e^{i(fx+e)}+2}\,\sqrt{-e^{i(fx+e)}}\left(-2\,\mathrm{EllipticE}\left(\sqrt{e^{i(}}\right.\right.}{\sqrt{-ibd\,e^{3i(fx+e)}+idb\,e^{i}}}\right.}{f\sqrt{\frac{d\,e^{i(fx+e}}{e^{2i(fx+e)}}}}$ |

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*csc(f*x+e)*(b*tan(f*x+e))^(1/2)*((1+I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)
*(1-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*
(-cos(f*x+e)-1)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))
+2*(1+I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+cot(f*x+e)))^(1/
2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1+cos(f*x+e))*EllipticE((1+I*cot(f
*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))+2^(1/2)*(cos(f*x+e)-1))/(d*sec(f*x+
e))^(1/2)*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx$$
$$=\frac{i\sqrt{-2i\,bd}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e)))-i\sqrt{2i\,bd}\mathrm{weiers}}{df}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(-2*I*b*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x
+ e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*weierstrassZeta(4, 0, weierstras
sPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d*f)
```

## Sympy [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\tan{\left(e+fx \right)}}}{\sqrt{d\sec{\left(e+fx \right)}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x))/sqrt(d*sec(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/sqrt(d*sec(f*x + e)), x)
```

## Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/sqrt(d*sec(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)}dx\right)}{d}$$

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/sec(e + f*x),
x))/d
```

# 3.295 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal result . . . 2181
Mathematica [A] (verified) . . . 2181
Rubi [A] (verified) . . . 2182
Maple [A] (verified) . . . 2183
Fricas [A] (verification not implemented) . . . 2183
Sympy [A] (verification not implemented) . . . 2183
Maxima [F] . . . 2184
Giac [F] . . . 2184
Mupad [B] (verification not implemented) . . . 2184
Reduce [F] . . . 2185

## Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

output
```
2/3*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*(b*Tan[e + f*x])^(3/2))/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3085}$$

$$\frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*(b*Tan[e + f*x])^(3/2))/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

## Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

| method | result | size |
|---|---|---|
| default | $\frac{2\sin(fx+e)\sqrt{b\tan(fx+e)}}{3fd\sqrt{d\sec(fx+e)}}$ | 35 |

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/f*sin(f*x+e)*(b*tan(f*x+e))^(1/2)/d/(d*sec(f*x+e))^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{3\,d^2f}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e)/(d^2*f)
```

## Sympy [A] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \begin{cases} \frac{2\sqrt{b\tan(e+fx)}\tan(e+fx)}{3f(d\sec(e+fx))^{\frac{3}{2}}} & \text{for } f \neq 0 \\ \frac{x\sqrt{b\tan(e)}}{(d\sec(e))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(3/2),x)
```

output
```
Piecewise((2*sqrt(b*tan(e + f*x))*tan(e + f*x)/(3*f*(d*sec(e + f*x))**(3/2
)), Ne(f, 0)), (x*sqrt(b*tan(e))/(d*sec(e))**(3/2), True))
```

## Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx=\frac{\sin\left(2\,e+2\,f\,x\right)\,\sqrt{\frac{d}{\cos(e+f\,x)}}\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{3\,d^2\,f}$$

input
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(3/2),x)
```

output
```
(sin(2*e + 2*f*x)*(d/cos(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(cos(2*e +
2*f*x) + 1))^(1/2))/(3*d^2*f)
```

## Reduce [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^2}dx\right)}{d^2}$$

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/sec(e + f*x)*
*2,x))/d**2
```

# 3.296 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal result . . . 2186
Mathematica [C] (verified) . . . 2186
Rubi [A] (verified) . . . 2187
Maple [C] (verified) . . . 2189
Fricas [C] (verification not implemented) . . . 2189
Sympy [F] . . . 2190
Maxima [F] . . . 2190
Giac [F] . . . 2191
Mupad [F(-1)] . . . 2191
Reduce [F] . . . 2191

## Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx=\frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

output
```
-4/5*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^2
/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)+2/5*(b*tan(f*x+e))^(3/2)/b/f/(d*s
ec(f*x+e))^(5/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.81 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx=\frac{2\left(3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/4}\right)(b\tan(e}{15bf(d\sec(e+fx))^{5/2}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(2*(3 + 2*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^
2)^(5/4))*(b*Tan[e + f*x])^(3/2))/(15*b*f*(d*Sec[e + f*x])^(5/2))
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3092, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(4*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*
Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(5*b*f*(d*S
ec[e + f*x])^(5/2))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.79

| method | result |
|---|---|
| default | $-\frac{\csc(fx+e)\left((4+4\cos(fx+e))\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\ \mathrm{Elli}\right.}{}$ |

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/f*csc(f*x+e)*((4+4*cos(f*x+e))*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1
-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*Elli
pticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))+(-2*cos(f*x+e)-2)*(
1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I
*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(
1/2),1/2*2^(1/2))+(cos(f*x+e)^3+cos(f*x+e)-2)*2^(1/2))*(b*tan(f*x+e))^(1/2
)/d^2/(d*sec(f*x+e))^(1/2)*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2\sin\left(fx+e\right)+i\sqrt{-2i\,bd}\mathrm{weierstrassZeta}\right.}{}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
*sin(f*x + e) + I*sqrt(-2*I*b*d)*weierstrassZeta(4, 0, weierstrassPInverse
(4, 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*weierstrassZeta(4
, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d^3*f)
```

## Sympy [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan{\left(e+fx \right)}}}{\left(d\sec{\left(e+fx \right)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x))/(d*sec(e + f*x))**(5/2), x)
```

## Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(5/2), x)
```

## Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^3}dx\right)}{d^3}$$

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/sec(e + f*x)*
*3,x))/d**3
```

# 3.297 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2192
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2192
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2193
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2194
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2195
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2195
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2195
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2196
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 2196
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2196

## Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}} + \frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}}$$

output
```
2/7*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(7/2)+8/21*(b*tan(f*x+e))^(3/2
)/b/d^2/f/(d*sec(f*x+e))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{(19\sin(e+fx)+3\sin(3(e+fx)))\sqrt{b\tan(e+fx)}}{42d^3f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(7/2),x]
```

output
```
((19*Sin[e + f*x] + 3*Sin[3*(e + f*x)])*Sqrt[b*Tan[e + f*x]])/(42*d^3*f*Sq
rt[d*Sec[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3092, 3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}dx}{7d^2}+\frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}dx}{7d^2}+\frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3085}$$

$$\frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}}+\frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(2*(b*Tan[e + f*x])^(3/2))/(7*b*f*(d*Sec[e + f*x])^(7/2)) + (8*(b*Tan[e +
f*x])^(3/2))/(21*b*d^2*f*(d*Sec[e + f*x])^(3/2))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

## Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

| method | result | size |
|---|---|---|
| default | $\frac{2 \sin(fx+e)\left(3 \cos(fx+e)^{2}+4\right) \sqrt{b \tan(fx+e)}}{21 f\, d^{3} \sqrt{d \sec(fx+e)}}$ | 47 |

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/f*sin(f*x+e)*(3*cos(f*x+e)^2+4)*(b*tan(f*x+e))^(1/2)/d^3/(d*sec(f*x+e
))^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2\left(3\,\cos\left(fx+e\right)^3+4\,\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin\left(fx+e\right)}{21\,d^4f}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
2/21*(3*cos(f*x + e)^3 + 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(d/cos(f*x + e))*sin(f*x + e)/(d^4*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(7/2), x)
```

## Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx=\int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(7/2), x)
```

## Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx=\frac{\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(22\,\sin\left(2\,e+2\,f\,x\right)+3\,\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{84\,d^4\,f}$$

input
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(7/2),x)
```

output
```
((d/cos(e + f*x))^(1/2)*(22*sin(2*e + 2*f*x) + 3*sin(4*e + 4*f*x))*((b*sin
(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(84*d^4*f)
```

## Reduce [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx=\frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^4}dx\right)}{d^4}$$

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/sec(e + f*x)*
*4,x))/d**4
```

# 3.298 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx$


Optimal result . . . 2197
Mathematica [C] (verified) . . . 2197
Rubi [A] (verified) . . . 2198
Maple [C] (verified) . . . 2200
Fricas [C] (verification not implemented) . . . 2201
Sympy [F(-1)] . . . 2201
Maxima [F] . . . 2202
Giac [F] . . . 2202
Mupad [F(-1)] . . . 2202
Reduce [F] . . . 2203


## Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{8E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} + \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}} + \frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}$$

output
```
-8/15*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^
4/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)+2/9*(b*tan(f*x+e))^(3/2)/b/f/(d*
sec(f*x+e))^(9/2)+4/15*(b*tan(f*x+e))^(3/2)/b/d^2/f/(d*sec(f*x+e))^(5/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\left(17+5\cos(2(e+fx))+8\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sec^2(e}{45d^3f(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(9/2),x]
```

output
```
((17 + 5*Cos[2*(e + f*x)] + 8*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*
x]^2]*(Sec[e + f*x]^2)^(5/4))*Sin[e + f*x]*Sqrt[b*Tan[e + f*x]])/(45*d^3*f
*(d*Sec[e + f*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3092, 3042, 3092, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \ 3092$$

$$\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3092$$

$$\frac{2\left(\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\left(\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3096$$

$$\frac{2\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3121$$

$$\frac{2\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3119$$

$$\frac{2\left(\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(9/2),x]
```

output
```
(2*(b*Tan[e + f*x])^(3/2))/(9*b*f*(d*Sec[e + f*x])^(9/2)) + (2*((4*Ellipti
cE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*Sec[e + f*
x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(5*b*f*(d*Sec[e + f*x
])^(5/2))))/(3*d^2)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.10

| method | result |
|---|---|
| default | $-\frac{\csc(fx+e)\left((24\cos(fx+e)+24)\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\ \text{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\,\mathrm{cs}\right.\right.}{}$ |

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45/f*csc(f*x+e)*((24*cos(f*x+e)+24)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)
*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e
))^(1/2),1/2*2^(1/2))*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)+(-12*cos(f*x+e)-
12)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2
)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1+I*cot(f*x+
e)-I*csc(f*x+e))^(1/2)+(5*cos(f*x+e)^5+cos(f*x+e)^3+6*cos(f*x+e)-12)*2^(1/
2))*(b*tan(f*x+e))^(1/2)/d^4/(d*sec(f*x+e))^(1/2)*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{2\left(\left(5\,\cos\left(fx+e\right)^4+6\,\cos\left(fx+e\right)^2\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin\left(fx+e\right)+6i\,\sqrt{}\cdots}{\cdots}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
2/45*((5*cos(f*x + e)^4 + 6*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x +
e))*sqrt(d/cos(f*x + e))*sin(f*x + e) + 6*I*sqrt(-2*I*b*d)*weierstrassZeta
(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqr
t(2*I*b*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) -
I*sin(f*x + e))))/(d^5*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(9/2), x)
```

## Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(9/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{9/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(9/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(9/2), x)
```

## Reduce [F]

$$\int \frac{\sqrt{b \tan(e+fx)}}{(d \sec(e+fx))^{9/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^5}dx\right)}{d^5}$$

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/sec(e + f*x)*
*5,x))/d**5
```

# 3.299 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2204
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2205
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2205
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2208
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . 2209
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2209
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2210
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2210
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2210
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2211


## Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{b^2d^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{6f\sqrt{b\tan(e+fx)}} - \frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f}$$

output
```
-1/6*b^2*d^2*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*(d*sec(f*x+e))^
(1/2)*sin(f*x+e)^(1/2)/f/(b*tan(f*x+e))^(1/2)-1/6*b*d^2*(d*sec(f*x+e))^(1/
2)*(b*tan(f*x+e))^(1/2)/f+1/3*b*(d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2)/
f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.63

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx =$$
$$-\frac{b(d\sec(e+fx))^{5/2}\left(-2+\cos^2(e+fx)+\cos^4(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec}{6f}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
-1/6*(b*(d*Sec[e + f*x])^(5/2)*(-2 + Cos[e + f*x]^2 + Cos[e + f*x]^4*Hyper
geometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4))*Sqrt[
b*Tan[e + f*x]])/f
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3091, 3042, 3093, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\int\frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx$$

$\Big\downarrow$ 3093

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{1}{2}d^2\int\frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}\right)$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{1}{2}d^2\int\frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}\right)$$

$\Big\downarrow$ 3096

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}\right)$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}\right)$$

$\Big\downarrow$ 3121

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}\right)$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}+\frac{d^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}\right)$$

input
```
Int[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(b*(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]])/(3*f) - (b^2*((d^2*Ellipti
cF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(f*Sqrt
[b*Tan[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(b*f))
)/6
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.21

| method | result |
|---|---|
| default | $-\frac{\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,b\,d^{2}\left(\sqrt{2}\left(1-2\sec(fx+e)^{2}\right)+i\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{i(c}\right.}{12f}$ |

input
```
int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/f*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)*b*d^2*(2^(1/2)*(1-2*sec(
f*x+e)^2)+I*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+
e))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*cs
c(f*x+e))^(1/2),1/2*2^(1/2))*cot(f*x+e)*(1+cos(f*x+e)))*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx =$$

$$-\frac{\sqrt{-2i\,bd}bd^2\cos\left(fx+e\right)^2\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+\sqrt{2i\,bd}bd^2\cos\left(fx+\right.}{12\,f\,c}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(-2*I*b*d)*b*d^2*cos(f*x + e)^2*weierstrassPInverse(4, 0, cos(f
*x + e) + I*sin(f*x + e)) + sqrt(2*I*b*d)*b*d^2*cos(f*x + e)^2*weierstrass
PInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(b*d^2*cos(f*x + e)^2 -
2*b*d^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*
x + e)^2)
```

## Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(e+f\,x))^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2), x)
```

## Reduce [F]

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \frac{\sqrt{d}\,\sqrt{b}\,b\,d^2\Big(2\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)^2 - \Big(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\ }}{\mathrm{ta}}}{6f}$$

input
```
int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(b)*b*d**2*(2*sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f
*x)**2 - int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x)**2)/tan(e
 + f*x),x)*f))/(6*f)
```

# 3.300 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal result . . . 2212
Mathematica [A] (verified) . . . 2213
Rubi [A] (warning: unable to verify) . . . 2213
Maple [A] (verified) . . . 2217
Fricas [B] (verification not implemented) . . . 2217
Sympy [F(-1)] . . . 2218
Maxima [F] . . . 2219
Giac [F] . . . 2219
Mupad [F(-1)] . . . 2219
Reduce [F] . . . 2220

## Optimal result

Integrand size = 25, antiderivative size = 169

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{b^{3/2}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4f\sqrt{b\tan(e+fx)}} - \frac{b^{3/2}d\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4f\sqrt{b\tan(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f}$$

output
```
-1/4*b^(3/2)*d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(
b*sin(f*x+e))^(1/2)/f/(b*tan(f*x+e))^(1/2)-1/4*b^(3/2)*d*arctanh((b*sin(f*
x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/f/(b*tan(f*
x+e))^(1/2)+1/2*b*(d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2)/f
```

## Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{(d\sec(e+fx))^{3/2}\left(-\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+f}}\right.\right.}{4f\sec^2(e+fx)^{3/4}\tan^{\frac{3}{2}}(}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
((d*Sec[e + f*x])^(3/2)*(-ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)
] - ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + 2*(Sec[e + f*x]^2
)^(3/4)*Sqrt[Tan[e + f*x]])*(b*Tan[e + f*x])^(3/2))/(4*f*(Sec[e + f*x]^2)^
(3/4)*Tan[e + f*x]^(3/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3091, 3042, 3096, 3042, 3044, 27, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{1}{4}b^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{1}{4}b^2\int\frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^2d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}dx}{4\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^2d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\cos(e+fx)\sqrt{b\sin(e+fx)}}dx}{4\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{bd\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{b^2}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{4f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^3d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{4f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^3d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{2f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{756}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^3d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\int\frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}}{2b}\right)}{2f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^3d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{2f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^3d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{2f\sqrt{b\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
-1/2*(b^3*d*(ArcTan[Sqrt[b]*Sin[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Si
n[e + f*x]]/(2*b^(3/2)))*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(f*Sqr
t[b*Tan[e + f*x]]) + (b*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(2*f)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

## Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05

| method | result |
|---|---|
| default | $-\frac{d\sqrt{d\sec(fx+e)}\,b\sqrt{b\tan(fx+e)}\left(\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,(-2-2\sec(fx+e))+\cos(fx+e)\arctan\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(fx+e)}{\cos(fx+e)-1}\right)-\cos(\right.}{4f(1+\cos(fx+e))\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}}$ |

input
```
int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*d*(d*sec(f*x+e))^(1/2)*b*(b*tan(f*x+e))^(1/2)/(1+cos(f*x+e))/(sin(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)*((sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(-2-2*s
ec(f*x+e))+cos(f*x+e)*arctan((sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e
)/(cos(f*x+e)-1))-cos(f*x+e)*arctanh((sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*s
in(f*x+e)/(cos(f*x+e)-1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(135) = 270.

Time = 0.23 (sec) , antiderivative size = 769, normalized size of antiderivative = 4.55

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(2*sqrt(-b*d)*b*d*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (c
os(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqr
t(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*
x + e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) + sq
rt(-b*d)*b*d*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2
- 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8
*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x
 + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x +
 e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*
b*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e
)), -1/32*(2*sqrt(b*d)*b*d*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 +
 (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*
sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(
f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) -
sqrt(b*d)*b*d*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2
 + 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) -
8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x
 + e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x +
 e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*
b*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x ...
```

## Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(e+f\,x))^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{\sqrt{d}\,\sqrt{b}\,bd\left(2\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)-\left(\int\frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec}}{\tan(}\right.\right.}{4f}$$

input
```
int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(b)*b*d*(2*sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x)
 - int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x))/tan(e + f*x),x
)*f))/(4*f)
```

# 3.301 $\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx$

Optimal result . . . 2221
Mathematica [C] (verified) . . . 2221
Rubi [A] (verified) . . . 2222
Maple [C] (verified) . . . 2224
Fricas [C] (verification not implemented) . . . 2224
Sympy [F] . . . 2225
Maxima [F] . . . 2225
Giac [F] . . . 2225
Mupad [F(-1)] . . . 2226
Reduce [F] . . . 2226

## Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = -\frac{b^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}} + \frac{b\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f}$$

output
```
-b^2*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*(d*sec(f*x+e))^(1/2)*si
n(f*x+e)^(1/2)/f/(b*tan(f*x+e))^(1/2)+b*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e)
)^(1/2)/f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = -\frac{b\sqrt{d\sec(e+fx)}\left(-1+\frac{\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)}{\sqrt[4]{\sec^2(e+fx)}}\right)\sqrt{b\tan(e+fx)}}{f}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

output
```
-((b*Sqrt[d*Sec[e + f*x]]*(-1 + Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e +
f*x]^2]/(Sec[e + f*x]^2)^(1/4))*Sqrt[b*Tan[e + f*x]])/f)
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3091, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{1}{2}b^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{1}{2}b^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

output
```
-((b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e +
f*x]])/(f*Sqrt[b*Tan[e + f*x]])) + (b*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e +
f*x]])/f
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.62

| method | result |
|---|---|
| default | $\frac{\sqrt{2}\left(\sqrt{2}-i\,\mathrm{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\sqrt{1-i(-\csc(fx+e)+\cot(fx+e))}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\,\sqrt{1+i}}{2f}$ |

input
```
int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*2^(1/2)*(2^(1/2)-I*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2
*2^(1/2))*(1-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)
))^(1/2)*(1+I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*cot(f*x+e)*(1+cos(f*x+e)))*(
b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/2)*b
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx =$$

$$-\frac{\sqrt{-2i\,bd}b\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))+\sqrt{2i\,bd}b\text{weierstrassPInverse}(4,0,\cos}{2f}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
+ e)) + sqrt(2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
 + e)) - 2*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/f
```

## Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(e+fx))^{\frac{3}{2}}\sqrt{d\sec(e+fx)}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(3/2),x)
```

output
```
Integral((b*tan(e + f*x))**(3/2)*sqrt(d*sec(e + f*x)), x)
```

## Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int \sqrt{d\sec(fx+e)}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int \sqrt{d\sec(fx+e)}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(e+f\,x))^{3/2}\sqrt{\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \frac{\sqrt{d}\,\sqrt{b}\,b\left(2\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}-\left(\int\frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\tan(fx+e)}dx\right)f\right)}{2f}$$

input
```
int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(b)*b*(2*sqrt(tan(e + f*x))*sqrt(sec(e + f*x)) - int((sqrt(ta
n(e + f*x))*sqrt(sec(e + f*x)))/tan(e + f*x),x)*f))/(2*f)
```

# 3.302 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal result . . . 2227
Mathematica [C] (verified) . . . 2228
Rubi [A] (warning: unable to verify) . . . 2228
Maple [A] (verified) . . . 2231
Fricas [B] (verification not implemented) . . . 2232
Sympy [F] . . . 2233
Maxima [F] . . . 2233
Giac [F] . . . 2233
Mupad [F(-1)] . . . 2234
Reduce [F] . . . 2234

## Optimal result

Integrand size = 25, antiderivative size = 167

$$\begin{aligned}\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx &= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}} \\ &\quad + \frac{b^{3/2}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}} \\ &\quad + \frac{b^{3/2}d\,\mathrm{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}\end{aligned}$$

output
```
-2*d*csc(f*x+e)*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)+b^(3/2)*d*arct
an((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/
2)/(b*sin(f*x+e))^(3/2)+b^(3/2)*d*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2))*(b
*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)/(b*sin(f*x+e))^(3/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.40

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{4},\frac{9}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}(b\tan(e+fx))^{5/2}}{5bf\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[5/4, 5/4, 9/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/4
)*(b*Tan[e + f*x])^(5/2))/(5*b*f*Sqrt[d*Sec[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3096, 3042, 3044, 27, 262, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{d(b\tan(e+fx))^{3/2}\int \sec(e+fx)(b\sin(e+fx))^{3/2}dx}{(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d(b\tan(e+fx))^{3/2}\int \frac{(b\sin(e+fx))^{3/2}}{\cos(e+fx)}dx}{(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{d(b\tan(e+fx))^{3/2}\int \frac{b^2(b\sin(e+fx))^{3/2}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{bf(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd(b\tan(e+fx))^{3/2}\int \frac{(b\sin(e+fx))^{3/2}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{bd(b\tan(e+fx))^{3/2}\left(b^2\int \frac{1}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))-2\sqrt{b\sin(e+fx)}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{bd(b\tan(e+fx))^{3/2}\left(2b^2\int \frac{1}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}-2\sqrt{b\sin(e+fx)}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{756}$$

$$\frac{bd(b\tan(e+fx))^{3/2}\left(2b^2\left(\frac{\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}}{2b}\right)-2\sqrt{b\sin(e+fx)}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{bd(b\tan(e+fx))^{3/2}\left(2b^2\left(\frac{\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)-2\sqrt{b\sin(e+fx)}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{bd(b\tan(e+fx))^{3/2}\left(2b^2\left(\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)-2\sqrt{b\sin(e+fx)}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(b*d*(2*b^2*(ArcTan[Sqrt[b]*Sin[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Si
n[e + f*x]]/(2*b^(3/2))) - 2*Sqrt[b*Sin[e + f*x]])*(b*Tan[e + f*x])^(3/2))
/(f*(d*Sec[e + f*x])^(3/2)*(b*Sin[e + f*x])^(3/2))
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

## Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.25

| method | result |
|---|---|
| default | $\frac{\sec(fx+e)b\sqrt{b\tan(fx+e)}\left(\sin(fx+e)^2-\cos(fx+e)^2+2\cos(fx+e)-1\right)\left(\arctan\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\sin(fx+e)}{\cos(fx+e)-1}\right)\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}}{2f(\cos(fx+e)-1)^2\sqrt{d\sec(fx+e}}$ |

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*sec(f*x+e)*b*(b*tan(f*x+e))^(1/2)*(sin(f*x+e)^2-cos(f*x+e)^2+2*cos(f
*x+e)-1)*(arctan((sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e
)-1))*(sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)-arctanh((sin(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*(sin(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)*sin(f*x+e)+2*cos(f*x+e)-2)/(cos(f*x+e)-1)^2/(d*sec(f*x+e))^(1/
2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(139) = 278$.

Time = 0.49 (sec) , antiderivative size = 741, normalized size of antiderivative = 4.44

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(2*b*d*sqrt(-b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (c
os(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqr
t(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e))/(b*cos(f*x
+ e)^2 - (b*cos(f*x + e) + b)*sin(f*x + e) - b)) - b*d*sqrt(-b/d)*log((b*c
os(f*x + e)^4 - 72*b*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^
3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos
(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e)) + 28*(b*cos(f*x + e)^2 - 2*b)*s
in(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2
 - 2)*sin(f*x + e) + 8)) + 16*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/c
os(f*x + e))*cos(f*x + e))/(d*f), 1/8*(2*b*d*sqrt(b/d)*arctan(1/4*(cos(f*x
 + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x
 + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/d)*sq
rt(d/cos(f*x + e))/(b*cos(f*x + e)^2 + (b*cos(f*x + e) + b)*sin(f*x + e) -
 b)) + b*d*sqrt(b/d)*log((b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 - 8*(7*co
s(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x
+ e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e)) - 2
8*(b*cos(f*x + e)^2 - 2*b)*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*
x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*b*sqrt(b*sin(f*x
 + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(d*f)]
```

## Sympy [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(b\tan(e+fx))^{\frac{3}{2}}}{\sqrt{d\sec(e+fx)}}\,dx$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(1/2),x)
```

output
```
Integral((b*tan(e + f*x))**(3/2)/sqrt(d*sec(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/sqrt(d*sec(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/sqrt(d*sec(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(b\tan(e+f\,x))^{3/2}}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx=\frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\tan(fx+e)}{\sec(fx+e)}dx\right)b}{d}$$

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*tan(e + f*x))/
sec(e + f*x),x)*b)/d
```

# 3.303 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2235
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2235
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2236
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2238
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . 2238
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2239
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2239
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2240
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2240
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2240

## Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx=\frac{2b^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

output
```
2/3*b^2*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*(d*sec(f*x+e))^(1/2)
*sin(f*x+e)^(1/2)/d^2/f/(b*tan(f*x+e))^(1/2)-2/3*b*(b*tan(f*x+e))^(1/2)/f/
(d*sec(f*x+e))^(3/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx=\frac{2b\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)\sqrt{b\tan}}{3f(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*b*(-1 + Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^2]*(Sec[e + f*x]
^2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(3*f*(d*Sec[e + f*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3090, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{b^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e +
f*x]])/(3*d^2*f*Sqrt[b*Tan[e + f*x]]) - (2*b*Sqrt[b*Tan[e + f*x]])/(3*f*(d
*Sec[e + f*x])^(3/2))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.62

| method | result |
|---|---|
| default | $-\frac{b\sqrt{b\tan(fx+e)}\left(i\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\ \text{EllipticF}\left(\sqrt{1\cdot}\right.\right.}{3f\sqrt{d\sec(fx+e)}\,d}$ |

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f*b*(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2)/d*(I*(1+I*cot(f*x+e)-I*
csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot
(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))
*(-cot(f*x+e)-csc(f*x+e))+2^(1/2)*cos(f*x+e))*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{2\,b\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2-\sqrt{-2i\,bd}b\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)}{3\,d^2f}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(2*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x +
 e)^2 - sqrt(-2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*
x + e)) - sqrt(2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f
*x + e)))/(d^2*f)
```

## Sympy [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+fx))^{\frac{3}{2}}}{(d\sec(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(3/2),x)
```

output
```
Integral((b*tan(e + f*x))**(3/2)/(d*sec(e + f*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+fx))^{3/2}}{\left(\frac{d}{\cos(e+fx)}\right)^{3/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\tan(fx+e)}{\sec(fx+e)^2}dx\right)b}{d^2}$$

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*tan(e + f*x))/
sec(e + f*x)**2,x)*b)/d**2
```

# 3.304 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2241
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2241
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2242
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2243
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . 2243
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2243
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2244
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2244
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 2245
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2245

## Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx=\frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

output
```
2/5*(b*tan(f*x+e))^(5/2)/b/f/(d*sec(f*x+e))^(5/2)
```

## Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx=\frac{2b\sin^2(e+fx)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(2*b*Sin[e + f*x]^2*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*Sec[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3085$$

$$\frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(2*(b*Tan[e + f*x])^(5/2))/(5*b*f*(d*Sec[e + f*x])^(5/2))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

## Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

| method | result | size |
|---|---|---|
| default | $\frac{2 \sin(fx+e)^{2} \sqrt{b \tan(fx+e)}\, b}{5 f\, d^{2} \sqrt{d \sec(fx+e)}}$ | 38 |

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/f*sin(f*x+e)^2*(b*tan(f*x+e))^(1/2)*b/d^2/(d*sec(f*x+e))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(28) = 56.

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

$$\int \frac{(b \tan(e+fx))^{3/2}}{(d \sec(e+fx))^{5/2}}\, dx = -\frac{2 \left(b \cos\left(fx+e\right)^{3} - b \cos\left(fx+e\right)\right) \sqrt{\frac{b \sin(fx+e)}{\cos(fx+e)}} \sqrt{\frac{d}{\cos(fx+e)}}}{5\, d^{3} f}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-2/5*(b*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(d/cos(f*x + e))/(d^3*f)
```

## Sympy [A] (verification not implemented)

Time = 45.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{(b \tan(e+fx))^{3/2}}{(d \sec(e+fx))^{5/2}}\, dx = \begin{cases} \frac{2 (b \tan{(e+fx)})^{\frac{3}{2}} \tan{(e+fx)}}{5 f (d \sec{(e+fx)})^{\frac{5}{2}}} & \text{for}\ f \neq 0 \\ \frac{x (b \tan{(e)})^{\frac{3}{2}}}{(d \sec{(e)})^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(5/2),x)
```

output
```
Piecewise((2*(b*tan(e + f*x))**(3/2)*tan(e + f*x)/(5*f*(d*sec(e + f*x))**(
5/2)), Ne(f, 0)), (x*(b*tan(e))**(3/2)/(d*sec(e))**(5/2), True))
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(5/2), x)
```

## Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(5/2), x)
```

## Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{b\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(\cos\left(e+f\,x\right)-\cos\left(3\,e+3\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{10\,d^3\,f}$$

input
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(5/2),x)
```

output
```
(b*(d/cos(e + f*x))^(1/2)*(cos(e + f*x) - cos(3*e + 3*f*x))*((b*sin(2*e +
2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(10*d^3*f)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\,\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\tan(fx+e)}{\sec(fx+e)^3}dx\right)b}{d^3}$$

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*tan(e + f*x))/
sec(e + f*x)**3,x)*b)/d**3
```

# 3.305 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2246
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2246
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2247
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2250
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . 2250
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2251
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2251
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2251
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2252
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2252

## Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx=\frac{4b^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{21d^4f\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}+\frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}}$$

output
```
4/21*b^2*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*(d*sec(f*x+e))^(1/2
)*sin(f*x+e)^(1/2)/d^4/f/(b*tan(f*x+e))^(1/2)-2/7*b*(b*tan(f*x+e))^(1/2)/f
/(d*sec(f*x+e))^(7/2)+2/21*b*(b*tan(f*x+e))^(1/2)/d^2/f/(d*sec(f*x+e))^(3/
2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.81 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx=-\frac{b\left(1+3\cos(2(e+fx))-4\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)\sqrt{b\tan(e+fx}}{21d^2f(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(7/2),x]
```

output
```
-1/21*(b*(1 + 3*Cos[2*(e + f*x)] - 4*Hypergeometric2F1[1/4, 3/4, 5/4, -Tan
[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(d^2*f*(d*Sec[e
 + f*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3090, 3042, 3092, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{b^2\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}dx}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}dx}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3092}$$

$$\frac{b^2\left(\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{b^2\left(\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{b^2\left(\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{b^2\left(\frac{4\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(-2*b*Sqrt[b*Tan[e + f*x]])/(7*f*(d*Sec[e + f*x])^(7/2)) + (b^2*((4*Ellipt
icF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*d^2
*f*Sqrt[b*Tan[e + f*x]]) + (2*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec[e + f*x]
)^(3/2))))/(7*d^2)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| default | $-\frac{b\sqrt{b\tan(fx+e)}\left(\sqrt{2}\left(3\cos(fx+e)^3-\cos(fx+e)\right)+i\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{i(\csc(fx+e}}{21f\,d^3\sqrt{d\sec(fx+e)}}$ |

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21/f*b*(b*tan(f*x+e))^(1/2)/d^3/(d*sec(f*x+e))^(1/2)*(2^(1/2)*(3*cos(f*
x+e)^3-cos(f*x+e))+I*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I
*csc(f*x+e))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*
x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(-2*csc(f*x+e)-2*cot(f*x+e)))*2^(1/2
)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.89

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2\left(\sqrt{-2i\,bd}b\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))+\sqrt{2i\,bd}b\text{w}\right.}{}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
2/21*(sqrt(-2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
+ e)) + sqrt(2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
 + e)) - (3*b*cos(f*x + e)^4 - b*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f
*x + e))*sqrt(d/cos(f*x + e)))/(d^4*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(7/2), x)
```

## Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{3/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(7/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(7/2), x)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\tan(fx+e)}{\sec(fx+e)^4}dx\right)b}{d^4}$$

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*tan(e + f*x))/
sec(e + f*x)**4,x)*b)/d**4
```

# 3.306 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx$


Optimal result . . . 2253
Mathematica [A] (verified) . . . 2253
Rubi [A] (verified) . . . 2254
Maple [A] (verified) . . . 2256
Fricas [A] (verification not implemented) . . . 2256
Sympy [F(-1)] . . . 2257
Maxima [F] . . . 2257
Giac [F] . . . 2257
Mupad [B] (verification not implemented) . . . 2258
Reduce [F] . . . 2258


## Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}} + \frac{8b\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}$$

output
```
-2/9*b*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(9/2)+2/45*b*(b*tan(f*x+e))^(
1/2)/d^2/f/(d*sec(f*x+e))^(5/2)+8/45*b*(b*tan(f*x+e))^(1/2)/d^4/f/(d*sec(f
*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.53

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{b(13+5\cos(2(e+fx)))\sin^2(e+fx)\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(9/2),x]
```

output
```
(b*(13 + 5*Cos[2*(e + f*x)])*Sin[e + f*x]^2*Sqrt[b*Tan[e + f*x]])/(45*d^4*
f*Sqrt[d*Sec[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3090, 3042, 3092, 3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{b^2\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}dx}{9d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}dx}{9d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3092}$$

$$\frac{b^2\left(\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{5d^2}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}\right)}{9d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{5d^2}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}\right)}{9d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3085}$$

$$\frac{b^2\left(\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}\right)}{9d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(9/2),x]
```

output
```
(-2*b*Sqrt[b*Tan[e + f*x]])/(9*f*(d*Sec[e + f*x])^(9/2)) + (b^2*((2*Sqrt[b
*Tan[e + f*x]])/(5*b*f*(d*Sec[e + f*x])^(5/2)) + (8*Sqrt[b*Tan[e + f*x]])/
(5*b*d^2*f*Sqrt[d*Sec[e + f*x]])))/(9*d^2)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

## Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

| method | result | size |
|---|---|---|
| default | $\frac{2\left(5\cos(fx+e)^2+4\right)\sin(fx+e)^2\sqrt{b\tan(fx+e)}\,b}{45f\,d^4\sqrt{d\sec(fx+e)}}$ | 50 |

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/f*(5*cos(f*x+e)^2+4)*sin(f*x+e)^2*(b*tan(f*x+e))^(1/2)*b/d^4/(d*sec(f
*x+e))^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{2\left(5\,b\cos\left(fx+e\right)^5 - b\cos\left(fx+e\right)^3 - 4\,b\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{45\,d^5f}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
-2/45*(5*b*cos(f*x + e)^5 - b*cos(f*x + e)^3 - 4*b*cos(f*x + e))*sqrt(b*si
n(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(d^5*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(9/2), x)
```

## Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(9/2), x)
```

## Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{b\sqrt{\frac{d}{\cos(e+f\,x)}}\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}\,\left(21\,\cos\left(3\,e+3\,f\,x\right)-26\,\cos\left(e+f\,x\right)+5\,\cos\left(5\,e+5\,f\,x\right)\right)}{360\,d^5\,f}$$

input
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(9/2),x)
```

output
```
-(b*(d/cos(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(
1/2)*(21*cos(3*e + 3*f*x) - 26*cos(e + f*x) + 5*cos(5*e + 5*f*x)))/(360*d^
5*f)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\tan(fx+e)}{\sec(fx+e)^5}dx\right)b}{d^5}$$

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*tan(e + f*x))/
sec(e + f*x)**5,x)*b)/d**5
```

# 3.307 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2259
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2260
Rubi [A] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . 2260
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2264
Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . 2265
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2266
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2266
Giac [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2266
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2267
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2267

## Optimal result

Integrand size = 25, antiderivative size = 208

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \frac{3b^{5/2}d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3b^{5/2}d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f}$$

output
```
3/32*b^(5/2)*d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)
/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)-3/32*b^(5/2)*d^3*arctanh((b*s
in(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*s
in(f*x+e))^(1/2)-3/16*b*d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2)/f+1/
4*b*(d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2)/f
```

## Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.74

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \frac{d^4(b\tan(e+fx))^{5/2}\left(3\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\sec^2(e+fx)^{3/4} - 3\text{arcta}}{32f(d\,\text{se}}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2),x]
```

output
```
(d^4*(b*Tan[e + f*x])^(5/2)*(3*ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^
(1/4)]*(Sec[e + f*x]^2)^(3/4) - 3*ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]
^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4) + 2*Sec[e + f*x]^2*(-3 + 4*Sec[e + f*x]^
2)*Tan[e + f*x]^(3/2)))/(32*f*(d*Sec[e + f*x])^(3/2)*Tan[e + f*x]^(5/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.82, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3091, 3042, 3093, 3042, 3096, 3042, 3044, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{5/2}(d\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx))^{5/2}(d\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f} - \frac{3}{8}b^2\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\int(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{1}{4}d^2\int\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{1}{4}d^2\int\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \text{3096}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{d^3\sqrt{b\tan(e+fx)}\int\sec(e+fx)\sqrt{b\sin(e+fx)}dx}{4\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{d^3\sqrt{b\tan(e+fx)}\int\frac{\sqrt{b\sin(e+fx)}}{\cos(e+fx)}dx}{4\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \text{3044}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{d^3\sqrt{b\tan(e+fx)}\int\frac{b^2\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{4bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{bd^3\sqrt{b\tan(e+fx)}\int\frac{\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \ 266$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{bd^3\sqrt{b\tan(e+fx)}\int\frac{b^2\sin^2(e+fx)}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \ 827$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{bd^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{1}{2}\int\frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e\right.$$

$$\Big\downarrow \ 216$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{bd^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\,s}}{2bf}\right.$$

$$\Big\downarrow \ 219$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{bd^3\sqrt{b\tan(e+fx)}\left(\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

input
```
Int[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2),x]
```

output
```
(b*(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2))/(4*f) - (3*b^2*((b*d^3*(
-1/2*ArcTan[Sqrt[b]*Sin[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sin[e + f*x]]/
(2*Sqrt[b]))*Sqrt[b*Tan[e + f*x]])/(2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e
+ f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2*b*f)))/8
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

## Maple [A] (verified)

Time = 267.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| default | $-\frac{\sqrt{b\tan(fx+e)}\, b^2 d^2 \sqrt{d\sec(fx+e)} \left(-3\cos(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\, \sin(fx+e)}{\cos(fx+e)-1}\right)-3\cos(fx+e)\arctan\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e}}}{\cos(fx}\right.\right.}{32f(1+\cos(fx+e))\sqrt{\frac{\sin(}{(1+\cos}}}$ |

input
```
int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/f*(b*tan(f*x+e))^(1/2)*b^2*d^2*(d*sec(f*x+e))^(1/2)/(1+cos(f*x+e))/(
sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(-3*cos(f*x+e)*arctanh((sin(f*x+e)/(1+c
os(f*x+e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))-3*cos(f*x+e)*arctan((sin(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))+tan(f*x+e)*sec(f*x
+e)^2*(6*cos(f*x+e)^3+6*cos(f*x+e)^2-8*cos(f*x+e)-8)*(sin(f*x+e)/(1+cos(f*
x+e))^2)^(1/2))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(168) = 336$.

Time = 0.29 (sec) , antiderivative size = 852, normalized size of antiderivative = 4.10

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/256*(6*sqrt(-b*d)*b^2*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2
 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4
)*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*c
os(f*x + e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e)
^3 + 3*sqrt(-b*d)*b^2*d^2*cos(f*x + e)^3*log((b*d*cos(f*x + e)^4 - 72*b*d*
cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*s
in(f*x + e) - 8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(d/cos(f*x + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x +
 e))/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x +
 e) + 8)) - 16*(3*b^2*d^2*cos(f*x + e)^2 - 4*b^2*d^2)*sqrt(b*sin(f*x + e)/
cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3), 1/256
*(6*sqrt(b*d)*b^2*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos
(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(
b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x +
 e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e)^3 + 3*s
qrt(b*d)*b^2*d^2*cos(f*x + e)^3*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x +
 e)^2 + 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x +
e) - 8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/co
s(f*x + e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(
f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8...
```

## Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[4,14]%%%}+%%%{6,[4,12]%%%}+%%%{15,[4,10]%%%}+%%%{20
,[4,8]%%%
```

## Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \int (b\tan(e+f\,x))^{5/2}\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}dx$$

input
```
int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(5/2), x)
```

## Reduce [F]

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \sqrt{d}\,\sqrt{b}\left(\int \sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)^{2}\tan(fx+e)^{2}\,dx\right)b^{2}$$

input
```
int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x)
```

output
```
sqrt(d)*sqrt(b)*int(sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x)**2*
tan(e + f*x)**2,x)*b**2*d**2
```

# 3.308 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx$

Optimal result . . . 2268
Mathematica [C] (verified) . . . 2268
Rubi [A] (verified) . . . 2269
Maple [C] (verified) . . . 2272
Fricas [C] (verification not implemented) . . . 2272
Sympy [F(-1)] . . . 2273
Maxima [F] . . . 2273
Giac [F(-2)] . . . 2274
Mupad [F(-1)] . . . 2274
Reduce [F] . . . 2274

## Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \frac{b^2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f}$$

output
```
-1/2*b^2*d^2*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(
1/2)/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)-1/2*b*d^2*(b*tan(f*x+e))^(3/2
)/f/(d*sec(f*x+e))^(1/2)+1/3*b*(d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2)/f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.63 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \frac{bd^2\left(-3+2\sec^2(e+fx)+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt{\cdots}\right.}{6f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2),x]
```

output
```
(b*d^2*(-3 + 2*Sec[e + f*x]^2 + Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e +
f*x]^2]*(Sec[e + f*x]^2)^(1/4))*(b*Tan[e + f*x])^(3/2))/(6*f*Sqrt[d*Sec[e
+ f*x]])
```

## Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3091, 3042, 3093, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{5/2}(d\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx))^{5/2}(d\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{1}{2}d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{1}{2}d^2\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx\right)$$

$$\Big\downarrow \text{ 3096}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}\right)$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}\right)$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}\right)$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2),x]
```

output
```
(b*(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2))/(3*f) - (b^2*(-((d^2*Ell
ipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e + f*x]
]*Sqrt[Sin[e + f*x]])) + (d^2*(b*Tan[e + f*x])^(3/2))/(b*f*Sqrt[d*Sec[e +
f*x]])))/2
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.31

| method | result |
|---|---|
| default | $\frac{\sec(fx+e)^2 \csc(fx+e)\left(\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\,\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\ \mathrm{EllipticF}\left(\right.\right.}{}$ |

input
```
int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/f*sec(f*x+e)^2*csc(f*x+e)*((1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-
csc(f*x+e)+cot(f*x+e)))^(1/2)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*Elliptic
F((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(3*cos(f*x+e)^4+3*cos(f
*x+e)^3)+(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))
^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1+I*cot
(f*x+e)-I*csc(f*x+e))^(1/2)*(-6*cos(f*x+e)^4-6*cos(f*x+e)^3)+(3*cos(f*x+e)
^3-5*cos(f*x+e)^2+2)*2^(1/2))*(b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/2)*b^
2*d*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \frac{3i\sqrt{-2i\,bd}b^2d\cos{(fx+e)}^2\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,}{}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/12*(3*I*sqrt(-2*I*b*d)*b^2*d*cos(f*x + e)^2*weierstrassZeta(4, 0, weiers
trassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - 3*I*sqrt(2*I*b*d)*b^
2*d*cos(f*x + e)^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x
 + e) - I*sin(f*x + e))) - 2*(3*b^2*d*cos(f*x + e)^2 - 2*b^2*d)*sqrt(b*sin
(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)
^2)
```

## Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[4,14]%%%}+%%%{6,[4,12]%%%}+%%%{15,[4,10]%%%}+%%%{20
,[4,8]%%%
```

## Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \int (b\tan(e+f\,x))^{5/2}\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}dx$$

input
```
int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \sqrt{d}\,\sqrt{b}\left(\int\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)\tan(fx+e)^2\,dx\right)b^2d$$

input
```
int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x)
```

output
```
sqrt(d)*sqrt(b)*int(sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x)*tan
(e + f*x)**2,x)*b**2*d
```

# 3.309 $\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx$

Optimal result . . . . . 2276
Mathematica [A] (verified) . . . . . 2277
Rubi [A] (warning: unable to verify) . . . . . 2277
Maple [A] (verified) . . . . . 2280
Fricas [B] (verification not implemented) . . . . . 2281
Sympy [F(-1)] . . . . . 2282
Maxima [F] . . . . . 2283
Giac [F(-2)] . . . . . 2283
Mupad [F(-1)] . . . . . 2283
Reduce [F] . . . . . 2284

## Optimal result

Integrand size = 25, antiderivative size = 169

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \frac{3b^{5/2}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3b^{5/2}d\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f}$$

output
```
3/4*b^(5/2)*d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/
(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)-3/4*b^(5/2)*d*arctanh((b*sin(f*x
+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x
+e))^(1/2)+1/2*b*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2)/f
```

## Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \frac{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\left(3\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)-3\text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx}}\right.\right.}{4f\sqrt[4]{\sec^2(e+fx)}\tan^{\frac{5}{2}}(e+fx)}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2),x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)*(3*ArcTan[Sqrt[Tan[e + f*x]]/
(Sec[e + f*x]^2)^(1/4)] - 3*ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1
/4)] + 2*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^(3/2)))/(4*f*(Sec[e + f*x]^2)
^(1/4)*Tan[e + f*x]^(5/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3091, 3042, 3096, 3042, 3044, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{5/2}\sqrt{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx))^{5/2}\sqrt{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3}{4}b^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3}{4}b^2\int\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^2d\sqrt{b\tan(e+fx)}\int\sec(e+fx)\sqrt{b\sin(e+fx)}dx}{4\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^2d\sqrt{b\tan(e+fx)}\int\frac{\sqrt{b\sin(e+fx)}}{\cos(e+fx)}dx}{4\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3bd\sqrt{b\tan(e+fx)}\int\frac{b^2\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^3d\sqrt{b\tan(e+fx)}\int\frac{\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^3d\sqrt{b\tan(e+fx)}\int\frac{b^2\sin^2(e+fx)}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{827}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^3d\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{1}{2}\int\frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^3d\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^3d\sqrt{b\tan(e+fx)}\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-3*b^3*d*(-1/2*ArcTan[Sqrt[b]*Sin[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sin
[e + f*x]]/(2*Sqrt[b]))*Sqrt[b*Tan[e + f*x]])/(2*f*Sqrt[d*Sec[e + f*x]]*Sq
rt[b*Sin[e + f*x]]) + (b*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2*f
)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

## Maple [A] (verified)

Time = 5.64 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| default | $\frac{\sqrt{b\tan(fx+e)}\, b^{2} \sqrt{d\sec(fx+e)} \left(3\cos(fx+e)\, \operatorname{arctanh}\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^{2}}}\, \sin(fx+e)}{\cos(fx+e)-1}\right)+3\cos(fx+e)\, \arctan\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^{2}}}\, \sin}{\cos(fx+e)-1}\right.\right.}{4f(1+\cos(fx+e))\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^{2}}}}$ |

input
```
int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*(b*tan(f*x+e))^(1/2)*b^2*(d*sec(f*x+e))^(1/2)/(1+cos(f*x+e))/(sin(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)*(3*cos(f*x+e)*arctanh((sin(f*x+e)/(1+cos(f*x+
e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))+3*cos(f*x+e)*arctan((sin(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))+(sin(f*x+e)/(1+cos(f*x+e)
)^2)^(1/2)*(2*sin(f*x+e)+2*tan(f*x+e)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(135) = 270$.

Time = 0.23 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.66

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/32*(6*sqrt(-b*d)*b^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (c
os(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqr
t(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*
x + e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) + 3*
sqrt(-b*d)*b^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^
2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) -
 8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f
*x + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x
 + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 1
6*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))
/(f*cos(f*x + e)), 1/32*(6*sqrt(b*d)*b^2*arctan(1/4*(cos(f*x + e)^3 - 5*co
s(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(
f*x + e) + 4)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x +
 e))/(b*d*cos(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*c
os(f*x + e) + 3*sqrt(b*d)*b^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*
d*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))
*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e)
)*sqrt(d/cos(f*x + e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x
+ e))/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x
+ e) + 8)) + 16*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + ...
```

## Sympy [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \int \sqrt{d\sec{(fx+e)}}(b\tan{(fx+e)})^{\frac{5}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[4,14]%%%}+%%%{6,[4,12]%%%}+%%%{15,[4,10]%%%}+%%%{20
,[4,8]%%%
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \int {\left(b\,\mathrm{tan}\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \sqrt{d}\,\sqrt{b}\left(\int \sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\tan(fx+e)^2\,dx\right)b^2$$

input
```
int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x)
```

output
```
sqrt(d)*sqrt(b)*int(sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*tan(e + f*x)**2,
x)*b**2
```

# 3.310 $\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx$


Optimal result . . . . . 2285
Mathematica [C] (verified) . . . . . 2285
Rubi [A] (verified) . . . . . 2286
Maple [C] (verified) . . . . . 2288
Fricas [C] (verification not implemented) . . . . . 2288
Sympy [F(-1)] . . . . . 2289
Maxima [F] . . . . . 2289
Giac [F] . . . . . 2290
Mupad [F(-1)] . . . . . 2290
Reduce [F] . . . . . 2290


## Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = -\frac{3b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}$$

output
```
3*b^2*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/
(d*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)+b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+
e))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = -\frac{b\left(-1+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(5/2)/Sqrt[d*Sec[e + f*x]],x]
```

output
```
-((b*(-1 + Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x]
^2)^(1/4))*(b*Tan[e + f*x])^(3/2))/(f*Sqrt[d*Sec[e + f*x]]))
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3091, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3}{2}b^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3}{2}b^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x])^(5/2)/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(-3*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*S
ec[e + f*x]]*Sqrt[Sin[e + f*x]]) + (b*(b*Tan[e + f*x])^(3/2))/(f*Sqrt[d*Se
c[e + f*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.33

| method | result |
|---|---|
| default | $\frac{\sec(fx+e)\csc(fx+e)\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\ \text{EllipticE}\left(\sqrt{\right.}\right.}{}$ |

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*sec(f*x+e)*csc(f*x+e)*((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(
f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticE((
1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(6*cos(f*x+e)^2+6*cos(f*x+
e))+(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2
)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+
e))^(1/2),1/2*2^(1/2))*(-3*cos(f*x+e)^2-3*cos(f*x+e))+(2*cos(f*x+e)^2-3*co
s(f*x+e)+1)*2^(1/2))*(b*tan(f*x+e))^(1/2)*b^2/(d*sec(f*x+e))^(1/2)*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2\,b^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\,\sin(fx+e) - 3i\,\sqrt{-2i\,bd}b^2\text{weierstrassZeta}(4,0,\text{weie}}{}$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x
+ e) - 3*I*sqrt(-2*I*b*d)*b^2*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2*I*b*d)*b^2*weierstrassZet
a(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/sqrt(d*sec(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/sqrt(d*sec(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(b\tan(e+f\,x))^{5/2}}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx=\frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\tan(fx+e)^{2}}{\sec(fx+e)}dx\right)b^{2}}{d}$$

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*tan(e + f*x)**
2)/sec(e + f*x),x)*b**2)/d
```

# 3.311 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx$


Optimal result . . . . . . 2291
Mathematica [A] (verified) . . . . . . 2291
Rubi [A] (warning: unable to verify) . . . . . . 2292
Maple [A] (verified) . . . . . . 2295
Fricas [B] (verification not implemented) . . . . . . 2296
Sympy [F(-1)] . . . . . . 2297
Maxima [F] . . . . . . 2297
Giac [F(-2)] . . . . . . 2297
Mupad [F(-1)] . . . . . . 2298
Reduce [F] . . . . . . 2298


## Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

output
```
-b^(5/2)*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/d/f/(d*
sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)+b^(5/2)*arctanh((b*sin(f*x+e))^(1/2
)/b^(1/2))*(b*tan(f*x+e))^(1/2)/d/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1
/2)-2/3*b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.83

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{d\sec(e+fx)}\left(3\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right) - 3\text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right) + \sqrt[4]{\sec^2(e+fx)}\sin(2(e+\ldots}{3d^2f\sqrt[4]{\sec^2(e+fx)}\tan^{\frac{5}{2}}(e+fx)}$$

input
```
Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(3/2),x]
```

output
```
-1/3*(Sqrt[d*Sec[e + f*x]]*(3*ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(
1/4)] - 3*ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + (Sec[e + f*
x]^2)^(1/4)*Sin[2*(e + f*x)]*Sqrt[Tan[e + f*x]])*(b*Tan[e + f*x])^(5/2))/(
d^2*f*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^(5/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3090, 3042, 3096, 3042, 3044, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{b^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx}{d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx}{d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{b^2\sqrt{b\tan(e+fx)}\int \sec(e+fx)\sqrt{b\sin(e+fx)}dx}{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\tan(e+fx)}\int\frac{\sqrt{b\sin(e+fx)}}{\cos(e+fx)}dx}{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{b\sqrt{b\tan(e+fx)}\int\frac{b^2\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^3\sqrt{b\tan(e+fx)}\int\frac{\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b^3\sqrt{b\tan(e+fx)}\int\frac{b^2\sin^2(e+fx)}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2b^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{1}{2}\int\frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2b^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2b^3\sqrt{b\tan(e+fx)}\left(\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

input
```
Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*b^3*(-1/2*ArcTan[Sqrt[b]*Sin[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sin[e
+ f*x]]/(2*Sqrt[b]))*Sqrt[b*Tan[e + f*x]])/(d*f*Sqrt[d*Sec[e + f*x]]*Sqrt[
b*Sin[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(3*f*(d*Sec[e + f*x])^(3/2
))
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

## Maple [A] (verified)

Time = 5.66 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| default | $\frac{\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(fx+e)}{\cos(fx+e)-1}\right)\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,(-3\cot(fx+e)-3\csc(fx+e))}{3}+\frac{\arctan\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(fx+e)}{\cos(fx+e)-1}\right)\sqrt{\frac{\sin}{(1+\cos}}}{3}\right.}{f\sqrt{d\sec(fx+e)}\,d}$ |

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*arctanh((sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e
)-1))*(sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(-3*cot(f*x+e)-3*csc(f*x+e))+1/3
*arctan((sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))*(si
n(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(-3*cot(f*x+e)-3*csc(f*x+e))-2/3*sin(f*x+
e))*b^2*(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2)/d
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(138) = 276$.

Time = 0.58 (sec) , antiderivative size = 766, normalized size of antiderivative = 4.56

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/24*(16*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(
f*x + e)*sin(f*x + e) + 6*b^2*d*sqrt(-b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*
cos(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*co
s(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*
x + e))/(b*cos(f*x + e)^2 - (b*cos(f*x + e) + b)*sin(f*x + e) - b)) - 3*b^
2*d*sqrt(-b/d)*log((b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x
+ e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*
sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e)) + 28*(b*
cos(f*x + e)^2 - 2*b)*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e
)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/(d^2*f), -1/24*(16*b^2*sq
rt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) + 6*b^2*d*sqrt(b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (
cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sq
rt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e))/(b*cos(f*x
+ e)^2 + (b*cos(f*x + e) + b)*sin(f*x + e) - b)) - 3*b^2*d*sqrt(b/d)*log((
b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x +
e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/
cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e)) - 28*(b*cos(f*x + e)^2 - 2*b)
*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)
^2 - 2)*sin(f*x + e) + 8)))/(d^2*f)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(3/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[4,14]%%%}+%%%{6,[4,12]%%%}+%%%{15,[4,10]%%%}+%%%{20
,[4,8]%%%
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{5/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\tan(fx+e)^2}{\sec(fx+e)^2}dx\right)b^2}{d^2}$$

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*tan(e + f*x)**
2)/sec(e + f*x)**2,x)*b**2)/d**2
```

# 3.312 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2299
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2299
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2300
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2302
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2302
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2303
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2303
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2304
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2304
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2304


## Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{6b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

output
```
-6/5*b^2*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)
/d^2/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)-2/5*b*(b*tan(f*x+e))^(3/2)/f/
(d*sec(f*x+e))^(5/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.70 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{b\left(1+\cos(2(e+fx))-2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)(b\tan(e+fx))}{5d^2f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(5/2),x]
```

output
```
-1/5*(b*(1 + Cos[2*(e + f*x)] - 2*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e
+ f*x]^2]*(Sec[e + f*x]^2)^(1/4))*(b*Tan[e + f*x])^(3/2))/(d^2*f*Sqrt[d*Se
c[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3090, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{3b^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3b^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{3b^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3b^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{3b^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3b^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

input
```
Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(6*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqr
t[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(5*f*
(d*Sec[e + f*x])^(5/2))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.83

| method | result |
|---|---|
| default | $-\frac{\csc(fx+e)\left((6\cos(fx+e)+6)\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\ \mathrm{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+\right.\right.}{}$ |

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/f*csc(f*x+e)*((6*cos(f*x+e)+6)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(I
*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1
/2),1/2*2^(1/2))*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)+(-3*cos(f*x+e)-3)*(1-
I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*Ellipti
cF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1+I*cot(f*x+e)-I*csc(
f*x+e))^(1/2)+(-cos(f*x+e)^3+4*cos(f*x+e)-3)*2^(1/2))*(b*tan(f*x+e))^(1/2)
*b^2/(d*sec(f*x+e))^(1/2)/d^2*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{2\,b^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2\sin\left(fx+e\right)-3i\sqrt{-2i\,bd}b^2\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInv}}{}$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/5*(2*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x
 + e)^2*sin(f*x + e) - 3*I*sqrt(-2*I*b*d)*b^2*weierstrassZeta(4, 0, weiers
trassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2*I*b*d)*b^
2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
 + e))))/(d^3*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(5/2), x)
```

## Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx=\int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx=\int \frac{(b\tan(e+f\,x))^{5/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx=\frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\tan(fx+e)^{2}}{\sec(fx+e)^{3}}dx\right)b^{2}}{d^{3}}$$

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*tan(e + f*x)**
2)/sec(e + f*x)**3,x)*b**2)/d**3
```

# 3.313 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx$


Optimal result . . . 2305
Mathematica [A] (verified) . . . 2305
Rubi [A] (verified) . . . 2306
Maple [A] (verified) . . . 2307
Fricas [B] (verification not implemented) . . . 2307
Sympy [F(-1)] . . . 2308
Maxima [F] . . . 2308
Giac [F] . . . 2308
Mupad [B] (verification not implemented) . . . 2309
Reduce [F] . . . 2309


## Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx=\frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

output
```
2/7*(b*tan(f*x+e))^(7/2)/b/f/(d*sec(f*x+e))^(7/2)
```

## Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx=\frac{2b^2\sin^3(e+fx)\sqrt{b\tan(e+fx)}}{7d^3f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(2*b^2*Sin[e + f*x]^3*Sqrt[b*Tan[e + f*x]])/(7*d^3*f*Sqrt[d*Sec[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3085}$$

$$\frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

input
```
Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(2*(b*Tan[e + f*x])^(7/2))/(7*b*f*(d*Sec[e + f*x])^(7/2))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

## Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

| method | result | size |
|---|---|---|
| default | $\frac{2 \sin(fx+e)^{3} \sqrt{b \tan(fx+e)}\, b^{2}}{7 f\, d^{3} \sqrt{d \sec(fx+e)}}$ | 40 |

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/7/f*sin(f*x+e)^3*(b*tan(f*x+e))^(1/2)*b^2/d^3/(d*sec(f*x+e))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(28) = 56.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{2\left(b^2\cos\left(fx+e\right)^3-b^2\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin\left(fx+e\right)}{7\,d^4 f}$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
-2/7*(b^2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x +
 e))*sqrt(d/cos(f*x + e))*sin(f*x + e)/(d^4*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(7/2), x)
```

## Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(7/2), x)
```

## Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.12

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{b^2\sqrt{\frac{d}{\cos(e+f\,x)}}\,(2\,\sin(2\,e+2\,f\,x)-\sin(4\,e+4\,f\,x))\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{28\,d^4\,f}$$

input
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(7/2),x)
```

output
```
(b^2*(d/cos(e + f*x))^(1/2)*(2*sin(2*e + 2*f*x) - sin(4*e + 4*f*x))*((b*si
n(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(28*d^4*f)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\tan(fx+e)^2}{\sec(fx+e)^4}dx\right)b^2}{d^4}$$

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*tan(e + f*x)**
2)/sec(e + f*x)**4,x)*b**2)/d**4
```

# 3.314 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2310
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2310
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2311
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2314
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2314
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2315
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2315
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2316
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2316
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2316


## Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{4b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}} + \frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}$$

output
```
-4/15*b^2*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2
)/d^4/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)-2/9*b*(b*tan(f*x+e))^(3/2)/f
/(d*sec(f*x+e))^(9/2)+2/15*b*(b*tan(f*x+e))^(3/2)/d^2/f/(d*sec(f*x+e))^(5/
2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{b^3\left(1-5\cos(2(e+fx))+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sec^2(e\right.}{45d^4f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(9/2),x]
```

output
```
(b^3*(1 - 5*Cos[2*(e + f*x)] + 4*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e +
 f*x]^2]*(Sec[e + f*x]^2)^(5/4))*Sin[e + f*x]^2)/(45*d^4*f*Sqrt[d*Sec[e +
f*x]]*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3090, 3042, 3092, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{b^2\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3092}$$

$$\frac{b^2\left(\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{b^2\left(\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{b^2\left(\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{b^2\left(\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

input
```
Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(9/2),x]
```

output
```
(-2*b*(b*Tan[e + f*x])^(3/2))/(9*f*(d*Sec[e + f*x])^(9/2)) + (b^2*((4*Elli
pticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*Sec[e +
 f*x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(5*b*f*(d*Sec[e +
f*x])^(5/2))))/(3*d^2)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.15

| method | result |
|---|---|
| default | $-\frac{\csc(fx+e)\left((12\cos(fx+e)+12)\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\,\text{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc\right.\right.}{}$ |

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45/f*csc(f*x+e)*((12*cos(f*x+e)+12)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)
*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e
))^(1/2),1/2*2^(1/2))*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)+(-6*cos(f*x+e)-6
)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*
EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1+I*cot(f*x+e)
-I*csc(f*x+e))^(1/2)+(-5*cos(f*x+e)^5+8*cos(f*x+e)^3+3*cos(f*x+e)-6)*2^(1/
2))*(b*tan(f*x+e))^(1/2)*b^2/(d*sec(f*x+e))^(1/2)/d^4*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx =$$
$$-\frac{2\left(-3i\sqrt{-2i\,bd}b^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e)))+3i\sqrt{2i}\right.}{}$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
-2/45*(-3*I*sqrt(-2*I*b*d)*b^2*weierstrassZeta(4, 0, weierstrassPInverse(4
, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2*I*b*d)*b^2*weierstrassZe
ta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))) + (5*b^
2*cos(f*x + e)^4 - 3*b^2*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(d/cos(f*x + e))*sin(f*x + e))/(d^5*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(9/2), x)
```

## Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(9/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(e+fx))^{5/2}}{\left(\frac{d}{\cos(e+fx)}\right)^{9/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(9/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(9/2), x)
```

## Reduce [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\tan(fx+e)^2}{\sec(fx+e)^5}dx\right)b^2}{d^5}$$

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*tan(e + f*x)**
2)/sec(e + f*x)**5,x)*b**2)/d**5
```

# 3.315 $\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal result . . . 2317
Mathematica [A] (verified) . . . 2318
Rubi [A] (warning: unable to verify) . . . 2318
Maple [A] (verified) . . . 2322
Fricas [B] (verification not implemented) . . . 2322
Sympy [F(-1)] . . . 2323
Maxima [F] . . . 2324
Giac [F] . . . 2324
Mupad [F(-1)] . . . 2324
Reduce [F] . . . 2325

## Optimal result

Integrand size = 25, antiderivative size = 178

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{3d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{3d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf}$$

output
```
3/4*d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f
*x+e))^(1/2)/b^(1/2)/f/(b*tan(f*x+e))^(1/2)+3/4*d^3*arctanh((b*sin(f*x+e))
^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/b^(1/2)/f/(b*tan
(f*x+e))^(1/2)+1/2*d^2*(d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2)/b/f
```

## Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.75

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d^2(d\sec(e+fx))^{3/2}\left(3\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+3\text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right.}{4f\sec^2(e+fx)^{3/4}\sqrt{b\tan(e+}}$$

input
```
Integrate[(d*Sec[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(d^2*(d*Sec[e + f*x])^(3/2)*(3*ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^
(1/4)] + 3*ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + 2*(Sec[e +
 f*x]^2)^(3/4)*Sqrt[Tan[e + f*x]])*Sqrt[Tan[e + f*x]])/(4*f*(Sec[e + f*x]^
2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.73, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3093, 3042, 3096, 3042, 3044, 27, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3093}$$

$$\frac{3}{4}d^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{4}d^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$\Big\downarrow$ 3096

$$\frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}dx}{4\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$\Big\downarrow$ 3042

$$\frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\cos(e+fx)\sqrt{b\sin(e+fx)}}dx}{4\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$\Big\downarrow$ 3044

$$\frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{b^2}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{4bf\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$\Big\downarrow$ 27

$$\frac{3bd^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{4f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$\Big\downarrow$ 266

$$\frac{3bd^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{2f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$\Big\downarrow$ 756

$$\frac{3bd^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}}{2b}\right)}{2f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$\Big\downarrow$ 216

$$\frac{3bd^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{2f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{219}$$

$$\frac{3bd^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{2f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

input
```
Int[(d*Sec[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(3*b*d^3*(ArcTan[Sqrt[b]*Sin[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Sin[e
 + f*x]]/(2*b^(3/2)))*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(2*f*Sqrt
[b*Tan[e + f*x]]) + (d^2*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(2*b
*f)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

## Maple [A] (verified)

Time = 5.99 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.08

| method | result |
|---|---|
| default | $\frac{d^3\sqrt{d\sec(fx+e)}\left(3\sin(fx+e)\arctan\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\sin(fx+e)}{\cos(fx+e)-1}\right)-3\sin(fx+e)\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\sin(fx+e)}{\cos(fx+e)-1}\right)+\sqrt{\frac{\cdots}{(1\cdots}}\right.}{4f(1+\cos(fx+e))\sqrt{b\tan(fx+e)}\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}}$ |

input
```
int((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*d^3*(d*sec(f*x+e))^(1/2)/(1+cos(f*x+e))/(b*tan(f*x+e))^(1/2)/(sin(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)*(3*sin(f*x+e)*arctan((sin(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))-3*sin(f*x+e)*arctanh((sin(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))+(sin(f*x+e)/(1+cos(f*x+e)
)^2)^(1/2)*(2*tan(f*x+e)+2*sec(f*x+e)*tan(f*x+e)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(144) = 288.

Time = 0.34 (sec) , antiderivative size = 782, normalized size of antiderivative = 4.39

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*(6*b*d^3*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 -
 (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*
sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f
*x + e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d))*cos(f*x + e) - 3*b*d^3
*sqrt(-d/b)*cos(f*x + e)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 + 8*(
7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(
f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)
) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*c
os(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*d^3*sqrt(b*
sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b*f*cos(f*x + e)), 1/32*
(6*b*d^3*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*
x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*s
in(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2
 + (d*cos(f*x + e) + d)*sin(f*x + e) - d))*cos(f*x + e) + 3*b*d^3*sqrt(d/b
)*cos(f*x + e)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x
+ e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*
sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*c
os(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)
^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*d^3*sqrt(b*sin(f*x + e
)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b*f*cos(f*x + e))]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(7/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{\sqrt{b\tan(e+f\,x)}}\,dx$$

input
```
int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)^3}{\tan(fx+e)}dx\right)d^3}{b}$$

input
```
int((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x)**
3)/tan(e + f*x),x)*d**3)/b
```

# 3.316 $\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$


Optimal result . . . 2326
Mathematica [C] (verified) . . . 2326
Rubi [A] (verified) . . . 2327
Maple [C] (verified) . . . 2329
Fricas [C] (verification not implemented) . . . 2330
Sympy [F(-1)] . . . 2330
Maxima [F] . . . 2330
Giac [F] . . . 2331
Mupad [F(-1)] . . . 2331
Reduce [F] . . . 2331


## Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d^2\operatorname{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}} + \frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{bf}$$

output
```
d^2*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*(d*sec(f*x+e))^(1/2)*sin
(f*x+e)^(1/2)/f/(b*tan(f*x+e))^(1/2)+d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e
))^(1/2)/b/f
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.57 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d^2\sqrt{d\sec(e+fx)}\left(\cos(e+fx)\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\operatorname{sec}\right.}{f\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(d^2*Sqrt[d*Sec[e + f*x]]*(Cos[e + f*x]*Hypergeometric2F1[1/4, 3/4, 5/4, -
Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4)*Sin[e + f*x] + Tan[e + f*x]))/(f*Sq
rt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3093, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{1}{2}d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}$$

$$\Big\downarrow \text{3096}$$

$$\frac{d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}$$

$$\Big\downarrow \text{3121}$$

$$\frac{d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}$$

$\Big\downarrow$ 3042

$$\frac{d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}$$

$\Big\downarrow$ 3120

$$\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}+\frac{d^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(d^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*
x]])/(f*Sqrt[b*Tan[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f
*x]])/(b*f)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.59

| method | result |
|---|---|
| default | $\frac{\sqrt{d\sec(fx+e)}\, d^2 \left(i(1+\cos(fx+e))\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\ \text{Ellip}}{2f\sqrt{b\tan(fx+e)}}$ |

input
```
int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(d*sec(f*x+e))^(1/2)*d^2/(b*tan(f*x+e))^(1/2)*(I*(1+cos(f*x+e))*(1+I
*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(I*(cs
c(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),
1/2*2^(1/2))+2^(1/2)*tan(f*x+e))*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{-2i\,bd}d^2\text{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+\sqrt{2i\,bd}d^2\text{w}}{2}$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-2*I*b*d)*d^2*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
 + e)) + sqrt(2*I*b*d)*d^2*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(
f*x + e)) + 2*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/
(b*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{5}{2}}}{\sqrt{b\tan{(fx+e)}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx=\int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{b\tan(e+f\,x)}}\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx=\frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)^{2}}{\tan(fx+e)}dx\right)d^{2}}{b}$$

input
```
int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x)**
2)/tan(e + f*x),x)*d**2)/b
```

# 3.317 $\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$


Optimal result . . . . . . . . . . 2333
Mathematica [A] (verified) . . . . . . . . . . 2333
Rubi [A] (warning: unable to verify) . . . . . . . . . . 2334
Maple [A] (verified) . . . . . . . . . . 2337
Fricas [B] (verification not implemented) . . . . . . . . . . 2337
Sympy [F] . . . . . . . . . . 2338
Maxima [F] . . . . . . . . . . 2338
Giac [F] . . . . . . . . . . 2339
Mupad [F(-1)] . . . . . . . . . . 2339
Reduce [F] . . . . . . . . . . 2339


## Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{\sqrt{b}f\sqrt{b\tan(e+fx)}}$$

output
```
d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))
^(1/2)/b^(1/2)/f/(b*tan(f*x+e))^(1/2)+d*arctanh((b*sin(f*x+e))^(1/2)/b^(1/
2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/b^(1/2)/f/(b*tan(f*x+e))^(1/
2)
```

## Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\left(\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right) + \text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right)(d\sec(e+fx))^{3/2}\sqrt{}}{f\sec^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
((ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqrt[Tan[e +
 f*x]]/(Sec[e + f*x]^2)^(1/4)])*(d*Sec[e + f*x])^(3/2)*Sqrt[Tan[e + f*x]])
/(f*(Sec[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.67, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3096, 3042, 3044, 27, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}dx}{\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\cos(e+fx)\sqrt{b\sin(e+fx)}}dx}{\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{b^2}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \textbf{266}$$

$$\frac{2bd\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \textbf{756}$$

$$\frac{2bd\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}}{2b}\right)}{f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{2bd\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{2bd\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{f\sqrt{b\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*b*d*(ArcTan[Sqrt[b]*Sin[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Sin[e +
 f*x]]/(2*b^(3/2)))*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(f*Sqrt[b*T
an[e + f*x]])
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

## Maple [A] (verified)

Time = 5.71 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.06

| method | result | size |
|---|---|---|
| default | $-\frac{\sin(fx+e)\sqrt{d\sec(fx+e)}\, d\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(fx+e)}{\cos(fx+e)-1}\right)-\arctan\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(fx+e)}{\cos(fx+e)-1}\right)\right)}{f(1+\cos(fx+e))\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sqrt{b\tan(fx+e)}}$ | 139 |

input
```
int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*sin(f*x+e)*(d*sec(f*x+e))^(1/2)*d*(arctanh((sin(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))-arctan((sin(f*x+e)/(1+cos(f*x+e))^2)^
(1/2)*sin(f*x+e)/(cos(f*x+e)-1)))/(1+cos(f*x+e))/(sin(f*x+e)/(1+cos(f*x+e)
)^2)^(1/2)/(b*tan(f*x+e))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(107) = 214.

Time = 0.37 (sec) , antiderivative size = 653, normalized size of antiderivative = 4.98

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(2*d*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos
(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(
b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x +
e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d)) - d*sqrt(-d/b)*log((d*cos(f
*x + e)^4 - 72*d*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 -
8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x
 + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f
*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2
)*sin(f*x + e) + 8)))/f, 1/8*(2*d*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5
*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*c
os(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*
x + e))/(d*cos(f*x + e)^2 + (d*cos(f*x + e) + d)*sin(f*x + e) - d)) + d*sq
rt(d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3
+ (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*
sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x
+ e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*
(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/f]
```

## Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{3}{2}}}{\sqrt{b\tan{(e+fx)}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)/sqrt(b*tan(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{3}{2}}}{\sqrt{b\tan{(fx+e)}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{b\tan(e+f\,x)}}\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)}{\tan(fx+e)}dx\right)d}{b}$$

input
```
int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x))/
tan(e + f*x),x)*d)/b
```

# 3.318 $\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$


Optimal result . . . 2341
Mathematica [C] (verified) . . . 2341
Rubi [A] (verified) . . . 2342
Maple [C] (verified) . . . 2344
Fricas [C] (verification not implemented) . . . 2344
Sympy [F] . . . 2345
Maxima [F] . . . 2345
Giac [F] . . . 2345
Mupad [F(-1)] . . . 2346
Reduce [F] . . . 2346


## Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

output
```
2*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f
*x+e)^(1/2)/f/(b*tan(f*x+e))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.46 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx =\frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\sin(e+fx)}{f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*d*Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(3
/4)*Sin[e + f*x])/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3096}$$

$$\frac{\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]
])/(f*Sqrt[b*Tan[e + f*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.67

| method | result |
|---|---|
| default | $\frac{2i\,\mathrm{EllipticF}\left(\sqrt{-i(-\cot(fx+e)+\csc(fx+e)+i)}, \frac{\sqrt{2}}{2}\right)\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(-\cot(}}{f\sqrt{b\tan(fx+e)}\left(\csc(fx+e)^2(1-\cos(fx+e))^2+1\right)}$ |

input
```
int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/f*EllipticF((-I*(-cot(f*x+e)+csc(f*x+e)+I))^(1/2),1/2*2^(1/2))*(-I*(-c
sc(f*x+e)+cot(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(-co
t(f*x+e)+csc(f*x+e)+I))^(1/2)*(d*sec(f*x+e))^(1/2)*2^(1/2)/(b*tan(f*x+e))^
(1/2)/(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{-2i\,bd}\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e)) + \sqrt{2i\,bd}\text{weierstrassPInverse}(4,0,\cos(f}{bf}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) +
 sqrt(2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(
b*f)
```

## Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{d\sec{(e+fx)}}}{\sqrt{b\tan{(e+fx)}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*sec(e + f*x))/sqrt(b*tan(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}}}{\sqrt{b\tan{(fx+e)}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}}}{\sqrt{b\tan{(fx+e)}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\tan(fx+e)}dx\right)}{b}$$

input
```
int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/tan(e + f*x),
x))/b
```

# 3.319 $\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$


Optimal result . . . . . 2347
Mathematica [A] (verified) . . . . . 2347
Rubi [A] (verified) . . . . . 2348
Maple [A] (verified) . . . . . 2349
Fricas [A] (verification not implemented) . . . . . 2349
Sympy [A] (verification not implemented) . . . . . 2350
Maxima [F] . . . . . 2350
Giac [F] . . . . . 2350
Mupad [B] (verification not implemented) . . . . . 2351
Reduce [F] . . . . . 2351


## Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

output
```
2*(b*tan(f*x+e))^(1/2)/b/f/(d*sec(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[1/(Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(2*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3085}$$
$$\frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(2*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

## Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| default | $\frac{2\tan(fx+e)}{f\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}}$ | 32 |
| risch | $-\frac{i\sqrt{2}\,(\mathrm{e}^{2i(fx+e)}-1)}{\sqrt{\frac{d\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\,(\mathrm{e}^{2i(fx+e)}+1)\sqrt{-\frac{ib\left(\mathrm{e}^{2i(fx+e)}-1\right)}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$ | 90 |

input
```
int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*tan(f*x+e)/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)}{bdf}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(b*d
*f)
```

## Sympy [A] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \begin{cases} \frac{2\tan(e+fx)}{f\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}} & \text{for } f \neq 0 \\ \frac{x}{\sqrt{b\tan(e)}\sqrt{d\sec(e)}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Piecewise((2*tan(e + f*x)/(f*sqrt(b*tan(e + f*x))*sqrt(d*sec(e + f*x))), N
e(f, 0)), (x/(sqrt(b*tan(e))*sqrt(d*sec(e))), True))
```

## Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

## Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sin(e+fx)\sqrt{\frac{d}{\cos(e+fx)}}}{d\,f\,\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}$$

input
```
int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(1/2)),x)
```

output
```
(2*sin(e + f*x)*(d/cos(e + f*x))^(1/2))/(d*f*((b*sin(2*e + 2*f*x))/(cos(2*
e + 2*f*x) + 1))^(1/2))
```

## Reduce [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)\tan(fx+e)}dx\right)}{bd}$$

input
```
int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
*tan(e + f*x)),x))/(b*d)
```

# 3.320 $\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2352
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2352
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2353
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2355
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . 2355
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2356
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2356
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2357
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2357
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2357


## Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}} + \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

output
```
4/3*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*(d*sec(f*x+e))^(1/2)*sin
(f*x+e)^(1/2)/d^2/f/(b*tan(f*x+e))^(1/2)+2/3*(b*tan(f*x+e))^(1/2)/b/f/(d*s
ec(f*x+e))^(3/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\left(1+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+}{3bf(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(2*(1 + 2*Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^
2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3092, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3120

$$\frac{4\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

input
```
Int[1/((d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(4*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]
])/(3*d^2*f*Sqrt[b*Tan[e + f*x]]) + (2*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec
[e + f*x])^(3/2))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.06 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| default | $\frac{\left(i\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\ \text{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc\right.}{3f\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,d}$ |

input
```
int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/d*(I*(1+I*cot(f*x+e)-I*csc
(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*
x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(2
+2*sec(f*x+e))+2^(1/2)*sin(f*x+e))*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2+\sqrt{-2i\,bd}\text{weierstrassPI}\right.}{}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
2/3*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
 + sqrt(-2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))
 + sqrt(2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))
/(b*d^2*f)
```

## Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan{(e+fx)}}\,(d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(3/2)), x)
```

## Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{3}{2}}\sqrt{b\tan{(fx+e)}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

## Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan(e+fx)}\left(\frac{d}{\cos(e+fx)}\right)^{3/2}}\,dx$$

input
```
int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^2\tan(fx+e)}dx\right)}{b\,d^2}$$

input
```
int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
**2*tan(e + f*x)),x))/(b*d**2)
```

# 3.321 $\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal result . . . 2358
Mathematica [A] (verified) . . . 2358
Rubi [A] (verified) . . . 2359
Maple [A] (verified) . . . 2360
Fricas [A] (verification not implemented) . . . 2361
Sympy [A] (verification not implemented) . . . 2361
Maxima [F] . . . 2362
Giac [F] . . . 2362
Mupad [B] (verification not implemented) . . . 2362
Reduce [F] . . . 2363

## Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx=\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}+\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}$$

output
```
2/5*(b*tan(f*x+e))^(1/2)/b/f/(d*sec(f*x+e))^(5/2)+8/5*(b*tan(f*x+e))^(1/2)
/b/d^2/f/(d*sec(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx=\frac{(9+\cos(2(e+fx)))\sqrt{d\sec(e+fx)}\sin(e+fx)}{5d^3f\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
((9 + Cos[2*(e + f*x)])*Sqrt[d*Sec[e + f*x]]*Sin[e + f*x])/(5*d^3*f*Sqrt[b
*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3092, 3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{5d^2}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{5d^2}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3085}$$

$$\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}$$

input
```
Int[1/((d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(2*Sqrt[b*Tan[e + f*x]])/(5*b*f*(d*Sec[e + f*x])^(5/2)) + (8*Sqrt[b*Tan[e
+ f*x]])/(5*b*d^2*f*Sqrt[d*Sec[e + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

## Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

| method | result | size |
|---|---|---|
| default | $\frac{2\tan(fx+e)\left(\cos(fx+e)^2+4\right)}{5f\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,d^2}$ | 45 |

input
```
int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/f*tan(f*x+e)*(cos(f*x+e)^2+4)/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2
)/d^2
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\left(\cos\left(fx+e\right)^{3}+4\,\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{5\,bd^{3}f}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
2/5*(cos(f*x + e)^3 + 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sq
rt(d/cos(f*x + e))/(b*d^3*f)
```

## Sympy [A] (verification not implemented)

Time = 55.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \begin{cases} \frac{8\tan^{3}(e+fx)}{5f\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{\frac{5}{2}}} + \frac{2\tan(e+fx)}{f\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{\frac{5}{2}}} & \text{for } f \neq \\ \frac{x}{\sqrt{b\tan(e)}(d\sec(e))^{\frac{5}{2}}} & \text{otherw} \end{cases}$$

input
```
integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Piecewise((8*tan(e + f*x)**3/(5*f*sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(
5/2)) + 2*tan(e + f*x)/(f*sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(5/2)), N
e(f, 0)), (x/(sqrt(b*tan(e))*(d*sec(e))**(5/2)), True))
```

## Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

## Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

## Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{(17\sin(e+fx)+\sin(3e+3fx))\sqrt{\frac{d}{\cos(e+fx)}}}{10d^3f\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}$$

input
```
int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(5/2)),x)
```

output
```
((17*sin(e + f*x) + sin(3*e + 3*f*x))*(d/cos(e + f*x))^(1/2))/(10*d^3*f*((
b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

## Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^3\tan(fx+e)}dx\right)}{b\,d^3}$$

input
```
int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
**3*tan(e + f*x)),x))/(b*d**3)
```

# 3.322 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$


Optimal result . . . . . 2364
Mathematica [A] (verified) . . . . . 2365
Rubi [A] (warning: unable to verify) . . . . . 2365
Maple [A] (verified) . . . . . 2368
Fricas [B] (verification not implemented) . . . . . 2369
Sympy [F(-1)] . . . . . 2370
Maxima [F] . . . . . 2370
Giac [F] . . . . . 2370
Mupad [F(-1)] . . . . . 2371
Reduce [F] . . . . . 2371


## Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

output

```
-2*d^2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)-d^3*arctan((b*sin(f*x
+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/b^(3/2)/f/(d*sec(f*x+e))^(1/2)/(b
*sin(f*x+e))^(1/2)+d^3*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e)
)^(1/2)/b^(3/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.69

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{d(d\sec(e+fx))^{3/2}\left(-2\sin(e+fx)+\frac{\left(-\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\operatorname{arctanh}\left(\frac{}{\sqrt[4]{}}\right.\right.}{\sqrt[4]{\sec^2(e+}}\right.}{f(b\tan(e+fx))^{3/2}}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(d*(d*Sec[e + f*x])^(3/2)*(-2*Sin[e + f*x] + ((-ArcTan[Sqrt[Tan[e + f*x]]/
(Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4
)])*Cos[e + f*x]*Tan[e + f*x]^(3/2))/(Sec[e + f*x]^2)^(1/4)))/(f*(b*Tan[e
+ f*x])^(3/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.75, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3088, 3042, 3096, 3042, 3044, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3088$$

$$\frac{d^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx}{b^2} - \frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{d^2 \int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx}{b^2}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$\downarrow$ 3096

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \sec(e+fx)\sqrt{b\sin(e+fx)}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$\downarrow$ 3042

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{\cos(e+fx)}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$\downarrow$ 3044

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \frac{b^2\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{b^3f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$\downarrow$ 27

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$\downarrow$ 266

$$\frac{2d^3\sqrt{b\tan(e+fx)}\int \frac{b^2\sin^2(e+fx)}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$\downarrow$ 827

$$\frac{2d^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}\right)}{bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$\downarrow$ 216

$$\frac{2d^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$\downarrow$ 219

$$\frac{2d^3\sqrt{b\tan(e+fx)}\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*d^2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]) + (2*d^3*(-1/2*Ar
cTan[Sqrt[b]*Sin[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sin[e + f*x]]/(2*Sqrt
[b]))*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]
)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

## Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| default | $-\frac{\left(\sin(fx+e)\arctan\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(fx+e)}{\cos(fx+e)-1}\right)+\sin(fx+e)\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(fx+e)}{\cos(fx+e)-1}\right)+2\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\cos\right.}{f(1+\cos(fx+e))\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,b\sqrt{b\tan(fx+e)}}$ |

input
```
int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(sin(f*x+e)*arctan((sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)/(co
s(f*x+e)-1))+sin(f*x+e)*arctanh((sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*
x+e)/(cos(f*x+e)-1))+2*(sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+2*(s
in(f*x+e)/(1+cos(f*x+e))^2)^(1/2))*d^2*(d*sec(f*x+e))^(1/2)/(1+cos(f*x+e))
/(sin(f*x+e)/(1+cos(f*x+e))^2)^(1/2)/b/(b*tan(f*x+e))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(143) = 286.

Time = 0.30 (sec) , antiderivative size = 794, normalized size of antiderivative = 4.64

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(2*b*d^2*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 -
(cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*s
qrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*
x + e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d))*sin(f*x + e) - b*d^2*sq
rt(-d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3
 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b
*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*
x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 -
4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8))*sin(f*x + e) + 16*d^2*sqrt(b*sin
(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x
+ e)), -1/8*(2*b*d^2*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)
^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) +
 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*co
s(f*x + e)^2 + (d*cos(f*x + e) + d)*sin(f*x + e) - d))*sin(f*x + e) - b*d^
2*sqrt(d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e
)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqr
t(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(
f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2
+ 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8))*sin(f*x + e) + 16*d^2*sqrt(b*s
in(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin...
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{(b\tan\left(e+f\,x\right))^{3/2}}\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)^2}{\tan(fx+e)^2}dx\right)d^2}{b^2}$$

input
```
int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x)**
2)/tan(e + f*x)**2,x)*d**2)/b**2
```

# 3.323 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2372
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2372
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2373
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2375
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2376
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2376
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2377
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2377
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2377
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2378

## Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2*d^2/b/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)+2*d^2*EllipticE(cos(1
/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/f/(d*sec(f*x+e))^(1
/2)/sin(f*x+e)^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2d^2\left(3+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\tan^2(e+fx)\right)}{3bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*d^2*(3 + Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*
x]^2)^(1/4)*Tan[e + f*x]^2))/(3*b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*
x]])
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3088, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3088}$$

$$-\frac{d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{b^2}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{b^2}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3096}$$

$$-\frac{d^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*d^2)/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (2*d^2*Elliptic
E[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[d*Sec[e + f*x]]
*Sqrt[Sin[e + f*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.43

| method | result |
|---|---|
| default | $-\frac{\sqrt{2}\left(\sqrt{2}-\sqrt{1-i(-\csc(fx+e)+\cot(fx+e))}\,\sqrt{1+i(-\csc(fx+e)+\cot(fx+e))}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\left(-\operatorname{EllipticF}\left(\sqrt{1\cdots}\right.\right.\right.}{}$ |

input
```
int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*2^(1/2)*(2^(1/2)-(1-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1+I*(-csc(f*x+
e)+cot(f*x+e)))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(-EllipticF((1+I
*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)+2*EllipticE((1+I*c
ot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)-EllipticF((1+I*cot(f
*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))+2*EllipticE((1+I*cot(f*x+e)-I*csc(f
*x+e))^(1/2),1/2*2^(1/2))))*d*(d*sec(f*x+e))^(1/2)/b/(b*tan(f*x+e))^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\,d\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2 + i\sqrt{-2i\,bd}d\sin\left(fx+e\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInver}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-(2*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^
2 + I*sqrt(-2*I*b*d)*d*sin(f*x + e)*weierstrassZeta(4, 0, weierstrassPInve
rse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*d*sin(f*x + e)
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
+ e))))/(b^2*f*sin(f*x + e))
```

## Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec\left(e+fx\right))^{\frac{3}{2}}}{(b\tan\left(e+fx\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)/(b*tan(e + f*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{(b\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)}{\tan(fx+e)^2}dx\right)d}{b^2}$$

input
```
int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x))/
tan(e + f*x)**2,x)*d)/b**2
```

# 3.324 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2379
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2379
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2380
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2381
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2381
Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2381
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2382
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2382
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2382
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2383

## Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

output
```
-2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d \sec(e+fx)}}{(b \tan(e+fx))^{3/2}} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sqrt{d \sec(e+fx)}}{(b \tan(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{ 3085}$$
$$-\frac{2\sqrt{d \sec(e+fx)}}{bf\sqrt{b \tan(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

## Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

| method | result | size |
|---|---|---|
| default | $-\frac{2\sqrt{d\sec(fx+e)}}{bf\sqrt{b\tan(fx+e)}}$ | 29 |

input
```
int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)}{b^2 f\sin\left(fx+e\right)}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(b^
2*f*sin(f*x + e))
```

## Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \begin{cases} -\frac{2\sqrt{d\sec\left(e+fx\right)}\tan\left(e+fx\right)}{f\left(b\tan\left(e+fx\right)\right)^{\frac{3}{2}}} & \text{for}\ f\neq 0 \\ \frac{x\sqrt{d\sec\left(e\right)}}{\left(b\tan\left(e\right)\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Piecewise((-2*sqrt(d*sec(e + f*x))*tan(e + f*x)/(f*(b*tan(e + f*x))**(3/2)
), Ne(f, 0)), (x*sqrt(d*sec(e))/(b*tan(e))**(3/2), True))
```

## Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{d\sec(fx+e)}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{d\sec(fx+e)}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{d}{\cos(e+f\,x)}}}{b\,f\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

input
```
int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
-(2*(d/cos(e + f*x))^(1/2))/(b*f*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) +
 1))^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d}\,\sqrt{b}\,\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\tan(fx+e)\,b^2f}$$

input
```
int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
( - 2*sqrt(d)*sqrt(b)*sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/(tan(e + f*x)
*b**2*f)
```

# 3.325 $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$

Optimal result . . . . . 2384
Mathematica [C] (verified) . . . . . 2384
Rubi [A] (verified) . . . . . 2385
Maple [C] (verified) . . . . . 2387
Fricas [C] (verification not implemented) . . . . . 2388
Sympy [F] . . . . . 2388
Maxima [F] . . . . . 2389
Giac [F(-1)] . . . . . 2389
Mupad [F(-1)] . . . . . 2389
Reduce [F] . . . . . 2390

## Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx=-\frac{2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2/b/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)+4*EllipticE(cos(1/2*e+1/4
*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/f/(d*sec(f*x+e))^(1/2)/sin(
f*x+e)^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.52 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx=-\frac{2\left(3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/4}\sin^2(e+fx)\right)}{3bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
(-2*(3 + 2*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x]
^2)^(5/4)*Sin[e + f*x]^2))/(3*b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]
])
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3089, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3089}$$

$$-\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3096}$$

$$-\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-2/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (4*EllipticE[(e - Pi/
2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[
e + f*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.77

| method | result |
|---|---|
| default | $\frac{\left(4\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\,\text{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\,\text{cs}\right.\right.}{}$ |

input
```
int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b*(4*(1+I*cot(f*x+e)-I*csc(f
*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+
e)))^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))-2*(1
+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*
(-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1
/2),1/2*2^(1/2))+2^(1/2)+4*sec(f*x+e)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*
(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*El
lipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))-2*sec(f*x+e)*(1+I
*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-
csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2
),1/2*2^(1/2))-2*sec(f*x+e)*2^(1/2))*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2+i\sqrt{-2i\,bd}\sin\left(fx+e\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInvers}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
-2*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
+ I*sqrt(-2*I*b*d)*sin(f*x + e)*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*sin(f*x + e)*weier
strassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))
)/(b^2*d*f*sin(f*x + e))
```

## Sympy [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\tan(e+fx))^{\frac{3}{2}}\sqrt{d\sec(e+fx)}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Integral(1/((b*tan(e + f*x))**(3/2)*sqrt(d*sec(e + f*x))), x)
```

## Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(e+f\,x\right)\right)^{3/2}\,\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2)), x)
```

## Reduce [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)\tan(fx+e)^2}dx\right)}{b^2d}$$

input
```
int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
*tan(e + f*x)**2),x))/(b**2*d)
```

# 3.326 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$


Optimal result . . . 2391
Mathematica [A] (verified) . . . 2391
Rubi [A] (verified) . . . 2392
Maple [A] (verified) . . . 2393
Fricas [A] (verification not implemented) . . . 2394
Sympy [A] (verification not implemented) . . . 2394
Maxima [F] . . . 2395
Giac [F(-1)] . . . 2395
Mupad [B] (verification not implemented) . . . 2395
Reduce [F] . . . 2396


## Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{2}{3bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{8\sqrt{d\sec(e+fx)}}{3bd^2f\sqrt{b\tan(e+fx)}}$$

output
```
2/3/b/f/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2)-8/3*(d*sec(f*x+e))^(1/2)
/b/d^2/f/(b*tan(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{(-7+\cos(2(e+fx)))\sec^2(e+fx)}{3bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
((-7 + Cos[2*(e + f*x)])*Sec[e + f*x]^2)/(3*b*f*(d*Sec[e + f*x])^(3/2)*Sqr
t[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3089, 3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}dx}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}dx}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3085}$$

$$-\frac{8(b\tan(e+fx))^{3/2}}{3b^3f(d\sec(e+fx))^{3/2}}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}$$

input
```
Int[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-2/(b*f*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]) - (8*(b*Tan[e + f*x])
^(3/2))/(3*b^3*f*(d*Sec[e + f*x])^(3/2))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

## Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

| method | result | size |
|---|---|---|
| default | $\frac{\frac{2\cos(fx+e)}{3}-\frac{8\sec(fx+e)}{3}}{f\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,bd}$ | 48 |

input
```
int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/3*cos(f*x+e)-8/3*sec(f*x+e))/(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1
/2)/b/d
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(\cos\left(fx+e\right)^{3}-4\,\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{3\,b^{2}d^{2}f\sin\left(fx+e\right)}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
2/3*(cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sq
rt(d/cos(f*x + e))/(b^2*d^2*f*sin(f*x + e))
```

## Sympy [A] (verification not implemented)

Time = 25.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \begin{cases} -\frac{8\tan^{3}\left(e+fx\right)}{3f\left(b\tan\left(e+fx\right)\right)^{\frac{3}{2}}\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}-\frac{2\tan\left(e+fx\right)}{f\left(b\tan\left(e+fx\right)\right)^{\frac{3}{2}}\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}} & \text{fo} \\ \frac{x}{\left(b\tan\left(e\right)\right)^{\frac{3}{2}}\left(d\sec\left(e\right)\right)^{\frac{3}{2}}} & \text{o} \end{cases}$$

input
```
integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Piecewise((-8*tan(e + f*x)**3/(3*f*(b*tan(e + f*x))**(3/2)*(d*sec(e + f*x)
)**(3/2)) - 2*tan(e + f*x)/(f*(b*tan(e + f*x))**(3/2)*(d*sec(e + f*x))**(3
/2)), Ne(f, 0)), (x/((b*tan(e))**(3/2)*(d*sec(e))**(3/2)), True))
```

## Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{(\cos(2e+2fx)-7)\sqrt{\frac{d}{\cos(e+fx)}}}{3bd^2f\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}$$

input
```
int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(3/2)),x)
```

output
```
((cos(2*e + 2*f*x) - 7)*(d/cos(e + f*x))^(1/2))/(3*b*d^2*f*((b*sin(2*e + 2
*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

## Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^2\tan(fx+e)^2}dx\right)}{b^2d^2}$$

input
```
int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
**2*tan(e + f*x)**2),x))/(b**2*d**2)
```

# 3.327 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2397
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2397
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2398
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2401
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2401
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2402
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2402
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2403
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2403
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2403

## Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{24E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5b^2d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}}$$

output
```
-2/b/f/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2)+24/5*EllipticE(cos(1/2*e+
1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/d^2/f/(d*sec(f*x+e))^(1/
2)/sin(f*x+e)^(1/2)-12/5*(b*tan(f*x+e))^(3/2)/b^3/f/(d*sec(f*x+e))^(5/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.69 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.68

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{-11+\cos(2(e+fx))-8\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan\right.}{5bd^2f\sqrt{d\sec(e+fx)}\sqrt{b\tan(}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
(-11 + Cos[2*(e + f*x)] - 8*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]
^2]*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^2)/(5*b*d^2*f*Sqrt[d*Sec[e + f*x]]
*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3089, 3042, 3092, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{6\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{6\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3092}$$

$$-\frac{6\left(\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{6\left(\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3096}$$

$$-\frac{6\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{6\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{6\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{6\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{6\left(\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

input
```
Int[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-2/(b*f*(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) - (6*((4*EllipticE[(e
 - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*Sec[e + f*x]]*S
qrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(5*b*f*(d*Sec[e + f*x])^(5
/2))))/b^2
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.08

| method | result |
|---|---|
| default | $\frac{\sec(fx+e)\left((24\cos(fx+e)+24)\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\ \mathrm{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc(\right.\right.}{}$ |

input
```
int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/f*sec(f*x+e)*((24*cos(f*x+e)+24)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(
-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))
^(1/2),1/2*2^(1/2))*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)+(-12*cos(f*x+e)-12
)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*
EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1+I*cot(f*x+e)
-I*csc(f*x+e))^(1/2)+(cos(f*x+e)^3+6*cos(f*x+e)-12)*2^(1/2))/(d*sec(f*x+e)
)^(1/2)/(b*tan(f*x+e))^(1/2)/b/d^2*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\left(6i\sqrt{-2i\,bd}\sin(fx+e)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e)))\right.}{}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
-2/5*(6*I*sqrt(-2*I*b*d)*sin(f*x + e)*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqrt(2*I*b*d)*sin(f*x +
e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*
x + e))) - (cos(f*x + e)^4 - 6*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x
 + e))*sqrt(d/cos(f*x + e)))/(b^2*d^3*f*sin(f*x + e))
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

## Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(e+f\,x\right)\right)^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^{3}\tan(fx+e)^{2}}dx\right)}{b^{2}d^{3}}$$

input
```
int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
**3*tan(e + f*x)**2),x))/(b**2*d**3)
```

# 3.328 $\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx$

Optimal result . . . 2404
Mathematica [A] (verified) . . . 2405
Rubi [A] (warning: unable to verify) . . . 2405
Maple [A] (verified) . . . 2409
Fricas [B] (verification not implemented) . . . 2409
Sympy [F(-1)] . . . 2410
Maxima [F] . . . 2411
Giac [F(-1)] . . . 2411
Mupad [F(-1)] . . . 2411
Reduce [F] . . . 2412

## Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{b^{5/2}f\sqrt{b\tan(e+fx)}} + \frac{d^3\operatorname{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{b^{5/2}f\sqrt{b\tan(e+fx)}}$$

output
```
-2/3*d^2*(d*sec(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(3/2)+d^3*arctan((b*sin(f
*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/b^(5/2)/f/
(b*tan(f*x+e))^(1/2)+d^3*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+
e))^(1/2)*(b*sin(f*x+e))^(1/2)/b^(5/2)/f/(b*tan(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\cos(e+fx)(d\sec(e+fx))^{7/2}\sin(e+fx)}{3f(b\tan(e+fx))^{5/2}} + \frac{\left(\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right)\cos^2(e+fx)(d\sec(e+fx))^{7/2}\tan^{\frac{5}{2}}(e+fx)}{f\sec^2(e+fx)^{3/4}(b\tan(e+fx))^{5/2}}$$

input
```
Integrate[(d*Sec[e + f*x])^(7/2)/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*Cos[e + f*x]*(d*Sec[e + f*x])^(7/2)*Sin[e + f*x])/(3*f*(b*Tan[e + f*x]
)^(5/2)) + ((ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + ArcTanh[S
qrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)])*Cos[e + f*x]^2*(d*Sec[e + f*x])
^(7/2)*Tan[e + f*x]^(5/2))/(f*(Sec[e + f*x]^2)^(3/4)*(b*Tan[e + f*x])^(5/2
))
```

## Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3088, 3042, 3096, 3042, 3044, 27, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3088}$$

$$\frac{d^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx}{b^2} - \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx}{b^2}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}dx}{b^2\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\cos(e+fx)\sqrt{b\sin(e+fx)}}dx}{b^2\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{b^2}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{b^3f\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{bf\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{756}$$

$$\frac{2d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}}{2b}\right)}{bf\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{bf\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{bf\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

input
```
Int[(d*Sec[e + f*x])^(7/2)/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*d^2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2)) + (2*d^3*(A
rcTan[Sqrt[b]*Sin[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Sin[e + f*x]]/(2
*b^(3/2)))*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(b*f*Sqrt[b*Tan[e +
f*x]])
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

## Maple [A] (verified)

Time = 6.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

| method | result |
|---|---|
| default | $\frac{\csc(fx+e)\left((3\cos(fx+e)-3)\operatorname{arctanh}\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(fx+e)}{\cos(fx+e)-1}\right)+(-3\cos(fx+e)+3)\arctan\left(\frac{\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,\sin(fx+e)}{\cos(fx+e)-1}\right)-\right.}{3f\sqrt{b\tan(fx+e)}\sqrt{\frac{\sin(fx+e)}{(1+\cos(fx+e))^2}}\,b^2}$ |

input
```
int((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f*csc(f*x+e)*((3*cos(f*x+e)-3)*arctanh((sin(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)*sin(f*x+e)/(cos(f*x+e)-1))+(-3*cos(f*x+e)+3)*arctan((sin(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)*sin(f*x+e)/(cos(f*x+e)-1))-2*(sin(f*x+e)/(1+cos(f*x+e))
^2)^(1/2))*d^3*(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/(sin(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)/b^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(142) = 284.

Time = 0.33 (sec) , antiderivative size = 850, normalized size of antiderivative = 4.94

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(16*d^3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f
*x + e) - 6*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(-d/b)*arctan(1/4*(cos(f*x
+ e)^3 - 5*cos(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x
+ e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sq
rt(d/cos(f*x + e))/(d*cos(f*x + e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) -
 d)) + 3*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(-d/b)*log((d*cos(f*x + e)^4 -
 72*d*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x +
 e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt
(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 7
2*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x +
 e) + 8)))/(b^3*f*cos(f*x + e)^2 - b^3*f), 1/24*(16*d^3*sqrt(b*sin(f*x + e
)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e) + 6*(b*d^3*cos(f*x + e)^
2 - b*d^3)*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(
f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b
*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)
^2 + (d*cos(f*x + e) + d)*sin(f*x + e) - d)) + 3*(b*d^3*cos(f*x + e)^2 - b
*d^3)*sqrt(d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x
 + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))
*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*
cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x ...
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(7/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e))^(5/2), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{(b\tan(e+f\,x))^{5/2}}\,dx$$

input
```
int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(5/2),x)
```

output
```
int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)^3}{\tan(fx+e)^3}dx\right)d^3}{b^3}$$

input
```
int((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x)**
3)/tan(e + f*x)**3,x)*d**3)/b**3
```

# 3.329 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2413
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2413
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2414
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2416
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . 2417
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2417
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2418
Giac [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2418
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2418
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2419


## Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} + \frac{2d^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}$$

output
```
-2/3*d^2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(3/2)+2/3*d^2*InverseJaco
biAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/b
^2/f/(b*tan(f*x+e))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.82 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{2d^2\sqrt{d\sec(e+fx)}\left(-\cot^2(e+fx)+\frac{\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)}{\sqrt[4]{\sec^2(e+fx)}}\right)\sqrt{b\,\text{ta}}}{3b^3f}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(2*d^2*Sqrt[d*Sec[e + f*x]]*(-Cot[e + f*x]^2 + Hypergeometric2F1[1/4, 3/4,
 5/4, -Tan[e + f*x]^2]/(Sec[e + f*x]^2)^(1/4))*Sqrt[b*Tan[e + f*x]])/(3*b^
3*f)
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3088, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3088}$$

$$\frac{d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3096}$$

$$\frac{d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3121$$

$$\frac{d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3120$$

$$\frac{2d^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*d^2*Sqrt[d*Sec[e + f*x]])/(3*b*f*(b*Tan[e + f*x])^(3/2)) + (2*d^2*Elli
pticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b
^2*f*Sqrt[b*Tan[e + f*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| default | $\frac{d^2\sqrt{d\sec(fx+e)}\left(i(1+\cos(fx+e))\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\right.}{3f\,b^2\sqrt{b\tan(fx+e)}}$ |

input
```
int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f*d^2*(d*sec(f*x+e))^(1/2)/b^2/(b*tan(f*x+e))^(1/2)*(I*(1+cos(f*x+e))*
(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-
I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^
(1/2),1/2*2^(1/2))-2^(1/2)*cot(f*x+e))*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{2\,d^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2 + \left(d^2\cos(fx+e)^2 - d^2\right)\sqrt{-2i\,bd}\text{weier}$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x
+ e)^2 + (d^2*cos(f*x + e)^2 - d^2)*sqrt(-2*I*b*d)*weierstrassPInverse(4,
0, cos(f*x + e) + I*sin(f*x + e)) + (d^2*cos(f*x + e)^2 - d^2)*sqrt(2*I*b*
d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b^3*f*cos(f*
x + e)^2 - b^3*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(5/2), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{(b\tan(e+f\,x))^{5/2}}\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(5/2),x)
```

output
```
int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}\,\sec(fx+e)^2}{\tan(fx+e)^3}dx\right)d^2}{b^3}$$

input
```
int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x)**
2)/tan(e + f*x)**3,x)*d**2)/b**3
```

# 3.330 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx$


Optimal result . . . 2420
Mathematica [A] (verified) . . . 2420
Rubi [A] (verified) . . . 2421
Maple [A] (verified) . . . 2422
Fricas [B] (verification not implemented) . . . 2422
Sympy [A] (verification not implemented) . . . 2422
Maxima [F] . . . 2423
Giac [F(-1)] . . . 2423
Mupad [B] (verification not implemented) . . . 2424
Reduce [B] (verification not implemented) . . . 2424


## Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

output
```
-2/3*(d*sec(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(3/2)
```

## Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3085}$$

$$-\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

## Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

| method | result | size |
|---|---|---|
| default | $-\frac{2 \csc(fx+e) d \sqrt{d \sec(fx+e)}}{3 f \, b^{2} \sqrt{b \tan(fx+e)}}$ | 36 |

input
```
int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/f*csc(f*x+e)*d*(d*sec(f*x+e))^(1/2)/b^2/(b*tan(f*x+e))^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{(d \sec(e+fx))^{3/2}}{(b \tan(e+fx))^{5/2}} \, dx = \frac{2 \, d \sqrt{\frac{b \sin(fx+e)}{\cos(fx+e)}} \sqrt{\frac{d}{\cos(fx+e)}} \cos(fx+e)}{3 \, (b^{3} f \cos(fx+e)^{2} - b^{3} f)}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
2/3*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/
(b^3*f*cos(f*x + e)^2 - b^3*f)
```

## Sympy [A] (verification not implemented)

Time = 42.99 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \frac{(d \sec(e+fx))^{3/2}}{(b \tan(e+fx))^{5/2}} \, dx = \begin{cases} -\frac{2 (d \sec(e+fx))^{\frac{3}{2}} \tan(e+fx)}{3 f (b \tan(e+fx))^{\frac{5}{2}}} & \text{for}\ f \neq 0 \\ \frac{x (d \sec(e))^{\frac{3}{2}}}{(b \tan(e))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Piecewise((-2*(d*sec(e + f*x))**(3/2)*tan(e + f*x)/(3*f*(b*tan(e + f*x))**
(5/2)), Ne(f, 0)), (x*(d*sec(e))**(3/2)/(b*tan(e))**(5/2), True))
```

## Maxima [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(5/2), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\,d\,\sqrt{\frac{d}{\cos(e+f\,x)}}}{3\,b^2\,f\,\sin\left(e+f\,x\right)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

input
```
int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(5/2),x)
```

output
```
-(2*d*(d/cos(e + f*x))^(1/2))/(3*b^2*f*sin(e + f*x)*((b*sin(2*e + 2*f*x))/
(cos(2*e + 2*f*x) + 1))^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\sqrt{d}\,\sqrt{b}\,\sqrt{\tan\left(fx+e\right)}\,\sqrt{\sec\left(fx+e\right)}\,\sec\left(fx+e\right)d}{3\tan\left(fx+e\right)^2 b^3 f}$$

input
```
int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x)
```

output
```
( - 2*sqrt(d)*sqrt(b)*sqrt(tan(e + f*x))*sqrt(sec(e + f*x))*sec(e + f*x)*d
)/(3*tan(e + f*x)**2*b**3*f)
```

# 3.331 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2425
Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2425
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2426
Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2428
Fricas [C] (verification not implemented) . . . . . . . . . . . . . . . . . . 2429
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2429
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2429
Giac [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2430
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2430
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2430

## Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}$$

output
```
-2/3*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(3/2)-4/3*InverseJacobiAM(1/2
*e-1/4*Pi+1/2*f*x,2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/b^2/f/(b*
tan(f*x+e))^(1/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.70 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2d^2\left(\csc^2(e+fx)+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*d^2*(Csc[e + f*x]^2 + 2*Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]
^2]*(Sec[e + f*x]^2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(3*b^3*f*(d*Sec[e + f*x]
)^(3/2))
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3089, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$-\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3121}$$

$$-\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3120}$$

$$-\frac{4\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*Sqrt[d*Sec[e + f*x]])/(3*b*f*(b*Tan[e + f*x])^(3/2)) - (4*EllipticF[(e
 - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2*f*Sqr
t[b*Tan[e + f*x]])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| default | $-\frac{\sqrt{d\sec(fx+e)}\,\left(i(2\cos(fx+e)+2)\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\ \mathrm{El}}{3 f\, b^{2}\sqrt{b\tan(fx+e)}}$ |

input
```
int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f*(d*sec(f*x+e))^(1/2)/b^2/(b*tan(f*x+e))^(1/2)*(I*(2*cos(f*x+e)+2)*(
1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(I*
(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/
2),1/2*2^(1/2))+2^(1/2)*cot(f*x+e))*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2 - \sqrt{-2i\,bd}\left(\cos\left(fx+e\right)^2-1\right)\mathrm{weierstra}\right.}{}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
 - sqrt(-2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstrassPInverse(4, 0, cos(f*x +
 e) + I*sin(f*x + e)) - sqrt(2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstrassPInv
erse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b^3*f*cos(f*x + e)^2 - b^3*f)
```

## Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{d\sec\left(e+fx\right)}}{\left(b\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(d*sec(e + f*x))/(b*tan(e + f*x))**(5/2), x)
```

## Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(5/2), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{\left(b\tan\left(e+f\,x\right)\right)^{5/2}}\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(5/2),x)
```

output
```
int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(5/2), x)
```

## Reduce [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\tan(fx+e)^3}dx\right)}{b^3}$$

input
```
int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/tan(e + f*x)*
*3,x))/b**3
```

# 3.332 $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx$

Optimal result . . . . . 2432
Mathematica [A] (verified) . . . . . 2432
Rubi [A] (verified) . . . . . 2433
Maple [A] (verified) . . . . . 2434
Fricas [A] (verification not implemented) . . . . . 2435
Sympy [A] (verification not implemented) . . . . . 2435
Maxima [F] . . . . . 2436
Giac [F] . . . . . 2436
Mupad [B] (verification not implemented) . . . . . 2436
Reduce [F] . . . . . 2437

## Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}} - \frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}$$

output
```
-2/3/b/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2)-8/3*(b*tan(f*x+e))^(1/2
)/b^3/f/(d*sec(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2(3+\csc^2(e+fx))\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)),x]
```

output
```
(-2*(3 + Csc[e + f*x]^2)*Sqrt[b*Tan[e + f*x]])/(3*b^3*f*Sqrt[d*Sec[e + f*x
]])
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3089, 3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(e+fx))^{5/2}\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(e+fx))^{5/2}\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3085}$$

$$-\frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}-\frac{2}{3bf(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)),x]
```

output
```
-2/(3*b*f*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)) - (8*Sqrt[b*Tan[e +
f*x]])/(3*b^3*f*Sqrt[d*Sec[e + f*x]])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

## Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

| method | result | size |
|---|---|---|
| default | $\frac{2\cot(fx+e)-\frac{8\sec(fx+e)\csc(fx+e)}{3}}{f\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,b^{2}}$ | 52 |

input
```
int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/f/(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2)/b^2*(3*cot(f*x+e)-4*sec(f*
x+e)*csc(f*x+e))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(3\cos\left(fx+e\right)^{3}-4\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{3\left(b^{3}df\cos\left(fx+e\right)^{2}-b^{3}df\right)}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas
")
```

output
```
-2/3*(3*cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(d/cos(f*x + e))/(b^3*d*f*cos(f*x + e)^2 - b^3*d*f)
```

## Sympy [A] (verification not implemented)

Time = 55.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \begin{cases} -\frac{8\tan^{3}(e+fx)}{3f(b\tan(e+fx))^{\frac{5}{2}}\sqrt{d\sec(e+fx)}} - \frac{2\tan(e+fx)}{3f(b\tan(e+fx))^{\frac{5}{2}}\sqrt{d\sec(e+fx)}} & \text{for } f \\ \frac{x}{(b\tan(e))^{\frac{5}{2}}\sqrt{d\sec(e)}} & \text{othe} \end{cases}$$

input
```
integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Piecewise((-8*tan(e + f*x)**3/(3*f*(b*tan(e + f*x))**(5/2)*sqrt(d*sec(e +
f*x))) - 2*tan(e + f*x)/(3*f*(b*tan(e + f*x))**(5/2)*sqrt(d*sec(e + f*x)))
, Ne(f, 0)), (x/((b*tan(e))**(5/2)*sqrt(d*sec(e))), True))
```

## Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}\,(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2)), x)
```

## Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}\,(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2)), x)
```

## Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \frac{\left(\frac{13\sin(e+f\,x)}{3} - \sin(3\,e+3\,f\,x)\right)\sqrt{\frac{d}{\cos(e+f\,x)}}}{b^2\,d\,f\,\left(\cos(2\,e+2\,f\,x)-1\right)\sqrt{\frac{b\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

input
```
int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(1/2)),x)
```

output
```
(((13*sin(e + f*x))/3 - sin(3*e + 3*f*x))*(d/cos(e + f*x))^(1/2))/(b^2*d*f
*(cos(2*e + 2*f*x) - 1)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2
))
```

## Reduce [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)\tan(fx+e)^3}dx\right)}{b^3d}$$

input
```
int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
*tan(e + f*x)**3),x))/(b**3*d)
```

# 3.333 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx$


Optimal result . . . . . 2438
Mathematica [C] (verified) . . . . . 2439
Rubi [A] (verified) . . . . . 2439
Maple [C] (verified) . . . . . 2442
Fricas [C] (verification not implemented) . . . . . 2442
Sympy [F(-1)] . . . . . 2443
Maxima [F] . . . . . 2443
Giac [F(-1)] . . . . . 2444
Mupad [F(-1)] . . . . . 2444
Reduce [F] . . . . . 2444


## Optimal result

Integrand size = 25, antiderivative size = 132

$$\begin{aligned}\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = &-\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\\&-\frac{8\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2d^2f\sqrt{b\tan(e+fx)}}\\&-\frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}}\end{aligned}$$

output
```
-2/3/b/f/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2)-8/3*InverseJacobiAM(1/2
*e-1/4*Pi+1/2*f*x,2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/b^2/d^2/f
/(b*tan(f*x+e))^(1/2)-4/3*(b*tan(f*x+e))^(1/2)/b^3/f/(d*sec(f*x+e))^(3/2)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.90 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.72

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \frac{(-3+\cos(2(e+fx)))\csc(e+fx) - 8\,\text{Hypergeometric2F1}\,(\ldots}{3b^2df\,\sqrt{d\sec(e+fx)}\sqrt{\ldots}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2)),x]
```

output
```
((-3 + Cos[2*(e + f*x)])*Csc[e + f*x] - 8*Hypergeometric2F1[1/4, 3/4, 5/4,
 -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4)*Sin[e + f*x])/(3*b^2*d*f*Sqrt[d*S
ec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3089, 3042, 3092, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(e+fx))^{5/2}(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(e+fx))^{5/2}(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{2\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}dx}{b^2} - \frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}dx}{b^2} - \frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3092}$$

$$-\frac{2\left(\frac{2\int\frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(\frac{2\int\frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$-\frac{2\left(\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2\left(\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2\left(\frac{4\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

input
```
Int[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2)),x]
```

output
```
-2/(3*b*f*(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)) - (2*((4*Elliptic
F[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*d^2*f
*Sqrt[b*Tan[e + f*x]]) + (2*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec[e + f*x])^
(3/2))))/b^2
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.23

| method | result |
|---|---|
| default | $-\frac{\left(i\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\ \operatorname{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\right.}{3f\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,b^2d}$ |

input
```
int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2)/b^2/d*(I*(1+I*cot(f*x+e)-
I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+c
ot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2
))*(4+4*sec(f*x+e))+csc(f*x+e)*(-cos(f*x+e)^2+2)*2^(1/2))*2^(1/2)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{2\left(2\sqrt{-2i\,bd}\left(\cos\left(fx+e\right)^2-1\right)\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+2\sqrt{2i\,bd}(\cos\left(\right.\right.}{3\left(b\right.}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas
")
```

output
```
-2/3*(2*sqrt(-2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstrassPInverse(4, 0, cos(
f*x + e) + I*sin(f*x + e)) + 2*sqrt(2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstr
assPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)) + (cos(f*x + e)^4 - 2*cos
(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b^3*
d^2*f*cos(f*x + e)^2 - b^3*d^2*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2)), x)
```

## Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(e+f\,x\right)\right)^{5/2}\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^{2}\tan(fx+e)^{3}}dx\right)}{b^{3}d^{2}}$$

input
```
int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
**2*tan(e + f*x)**3),x))/(b**3*d**2)
```

# 3.334 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2445
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2445
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2446
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2448
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2448
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2449
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2449
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2449
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2450
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2450

## Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}} - \frac{16\sqrt{b\tan(e+fx)}}{15b^3f(d\sec(e+fx))^{5/2}} - \frac{64\sqrt{b\tan(e+fx)}}{15b^3d^2f\sqrt{d\sec(e+fx)}}$$

output
```
-2/3/b/f/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2)-16/15*(b*tan(f*x+e))^(1
/2)/b^3/f/(d*sec(f*x+e))^(5/2)-64/15*(b*tan(f*x+e))^(1/2)/b^3/d^2/f/(d*sec
(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \frac{(-151+108\cos(2(e+fx))+3\cos(4(e+fx)))\csc(e+fx)\sqrt{\ldots}}{60b^2d^3f\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2)),x]
```

output
```
((-151 + 108*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)])*Csc[e + f*x]*Sqrt[d*Se
c[e + f*x]])/(60*b^2*d^3*f*Sqrt[b*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3089, 3042, 3092, 3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(e+fx))^{5/2}(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(e+fx))^{5/2}(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{8\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3092}$$

$$-\frac{8\left(\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{5d^2}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8\left(\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{5d^2}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3085}$$

$$-\frac{8\left(\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

input
```
Int[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2)),x]
```

output
```
-2/(3*b*f*(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)) - (8*((2*Sqrt[b*T
an[e + f*x]])/(5*b*f*(d*Sec[e + f*x])^(5/2)) + (8*Sqrt[b*Tan[e + f*x]])/(5
*b*d^2*f*Sqrt[d*Sec[e + f*x]])))/(3*b^2)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

## Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

| method | result | size |
|---|---|---|
| default | $\frac{\left(\frac{2\cos(fx+e)^4}{5}+\frac{16\cos(fx+e)^2}{5}-\frac{64}{15}\right)\sec(fx+e)\csc(fx+e)}{f\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,b^2d^2}$ | 65 |

input
```
int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/5*cos(f*x+e)^4+16/5*cos(f*x+e)^2-64/15)*sec(f*x+e)*csc(f*x+e)/(b*ta
n(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2)/b^2/d^2
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.84

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(3\cos\left(fx+e\right)^5+24\cos\left(fx+e\right)^3-32\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{15\left(b^3d^3f\cos\left(fx+e\right)^2-b^3d^3f\right)}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas
")
```

output
```
-2/15*(3*cos(f*x + e)^5 + 24*cos(f*x + e)^3 - 32*cos(f*x + e))*sqrt(b*sin(
f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b^3*d^3*f*cos(f*x + e)^2 - b^
3*d^3*f)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2)), x)
```

## Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2)), x)
```

## Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(105\,\sin\left(3\,e+3\,f\,x\right)-410\,\sin\left(e+f\,x\right)+3\,\sin\left(5\,e+5\,f\,x\right)\right)}{60\,b^2\,d^3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

input
```
int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(5/2)),x)
```

output
```
-((d/cos(e + f*x))^(1/2)*(105*sin(3*e + 3*f*x) - 410*sin(e + f*x) + 3*sin(
5*e + 5*f*x)))/(60*b^2*d^3*f*(cos(2*e + 2*f*x) - 1)*((b*sin(2*e + 2*f*x))/
(cos(2*e + 2*f*x) + 1))^(1/2))
```

## Reduce [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{b}\,\left(\int \frac{\sqrt{\tan(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^3\tan(fx+e)^3}dx\right)}{b^3d^3}$$

input
```
int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(b)*int((sqrt(tan(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
**3*tan(e + f*x)**3),x))/(b**3*d**3)
```

# 3.335 $\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$


Optimal result . . . . . . . . . . 2451
Mathematica [A] (verified) . . . . . . . . . . 2451
Rubi [A] (verified) . . . . . . . . . . 2452
Maple [F] . . . . . . . . . . 2453
Fricas [F] . . . . . . . . . . 2453
Sympy [F(-1)] . . . . . . . . . . 2454
Maxima [F] . . . . . . . . . . 2454
Giac [F] . . . . . . . . . . 2454
Mupad [F(-1)] . . . . . . . . . . 2455
Reduce [F] . . . . . . . . . . 2455


## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{7}{4},\sin^2(e+fx)\right)(b\sec(e+fx}{3df}$$

output
```
2/3*(cos(f*x+e)^2)^(17/12)*hypergeom([3/4, 17/12],[7/4],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2)/d/f
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \frac{3d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{2}{3},\frac{5}{3},\sec^2(e+fx)\right)(b\sec(e+fx))^{4/3}\sqrt[4]{-\tan^2(e}{4f\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(3*d*Hypergeometric2F1[1/4, 2/3, 5/3, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(4/
3)*(-Tan[e + f*x]^2)^(1/4))/(4*f*Sqrt[d*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{2\cos^2(e+fx)^{17/12}(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{7}{4},\sin^2(e+fx)\right)}{3df}$$

input
```
Int[(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[3/4, 17/12, 7/4, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (b \sec (fx+e))^{\frac{4}{3}} \sqrt{d \tan (fx+e)}dx$$

input
```
int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

output
```
int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

## Fricas [F]

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b \sec (fx+e))^{\frac{4}{3}} \sqrt{d \tan (fx+e)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*sec(f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(4/3)*(d*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec{(fx+e)})^{\frac{4}{3}}\sqrt{d\tan{(fx+e)}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

## Giac [F]

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec{(fx+e)})^{\frac{4}{3}}\sqrt{d\tan{(fx+e)}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(e+f\,x\right)}\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{4/3}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(4/3),x)
```

output
```
int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \sqrt{d}\,b^{\frac{4}{3}}\left(\int \sqrt{\tan\left(fx+e\right)}\,\sec\left(fx+e\right)^{\frac{4}{3}}\,dx\right)$$

input
```
int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*b**(1/3)*int(sqrt(tan(e + f*x))*sec(e + f*x)**(1/3)*sec(e + f*x),x
)*b
```

# 3.336 $\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2456
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2456
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2457
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2458
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2458
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2459
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2459
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2459
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2460
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2460

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{7}{4},\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}}{3df}$$

output
```
2/3*(cos(f*x+e)^2)^(11/12)*hypergeom([3/4, 11/12],[7/4],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2)/d/f
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{3d\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{4},\frac{7}{6},\sec^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}\sqrt[4]{-\tan^2(e+fx)}}{f\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(3*d*Hypergeometric2F1[1/6, 1/4, 7/6, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(1/
3)*(-Tan[e + f*x]^2)^(1/4))/(f*Sqrt[d*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{2\cos^2(e+fx)^{11/12}\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{7}{4},\sin^2(e+fx)\right)}{3df}$$

input
```
Int[(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^(11/12)*Hypergeometric2F1[3/4, 11/12, 7/4, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (b\sec (fx+e))^{\frac{1}{3}} \sqrt{d\tan (fx+e)}dx$$

input
```
int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

output
```
int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

## Fricas [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec (fx+e))^{\frac{1}{3}} \sqrt{d\tan (fx+e)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

## Sympy [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/3)*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral((b*sec(e + f*x))**(1/3)*sqrt(d*tan(e + f*x)), x)
```

## Maxima [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

## Giac [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(e+f\,x\right)}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{1/3} \,dx$$

input
```
int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/3),x)
```

output
```
int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \sqrt{d}\,b^{\frac{1}{3}}\left(\int \sqrt{\tan\left(fx+e\right)}\,\sec\left(fx+e\right)^{\frac{1}{3}}dx\right)$$

input
```
int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*b**(1/3)*int(sqrt(tan(e + f*x))*sec(e + f*x)**(1/3),x)
```

# 3.337 $\int \frac{\sqrt{d \tan(e+fx)}}{\sqrt[3]{b \sec(e+fx)}} \, dx$

Optimal result . . . . . . . . . . 2461
Mathematica [A] (verified) . . . . . . . . . . 2461
Rubi [A] (verified) . . . . . . . . . . 2462
Maple [F] . . . . . . . . . . 2463
Fricas [F] . . . . . . . . . . 2463
Sympy [F] . . . . . . . . . . 2464
Maxima [F] . . . . . . . . . . 2464
Giac [F] . . . . . . . . . . 2464
Mupad [F(-1)] . . . . . . . . . . 2465
Reduce [F] . . . . . . . . . . 2465

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{d \tan(e+fx)}}{\sqrt[3]{b \sec(e+fx)}} \, dx = \frac{2 \cos^2(e+fx)^{7/12} \text{Hypergeometric2F1}\left(\frac{7}{12}, \frac{3}{4}, \frac{7}{4}, \sin^2(e+fx)\right) (d \tan(e+fx))^{3/2}}{3df \sqrt[3]{b \sec(e+fx)}}$$

output
```
2/3*(cos(f*x+e)^2)^(7/12)*hypergeom([7/12, 3/4],[7/4],sin(f*x+e)^2)*(d*tan
(f*x+e))^(3/2)/d/f/(b*sec(f*x+e))^(1/3)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d \tan(e+fx)}}{\sqrt[3]{b \sec(e+fx)}} \, dx = -\frac{3d \text{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{4}, \frac{5}{6}, \sec^2(e+fx)\right) \sqrt[4]{-\tan^2(e+fx)}}{f \sqrt[3]{b \sec(e+fx)} \sqrt{d \tan(e+fx)}}$$

input
```
Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(1/3),x]
```

output
```
(-3*d*Hypergeometric2F1[-1/6, 1/4, 5/6, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^
(1/4))/(f*(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}dx$$

$$\Big\downarrow \ 3097$$

$$\frac{2\cos^2(e+fx)^{7/12}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)}{3df\sqrt[3]{b\sec(e+fx)}}$$

input
```
Int[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(1/3),x]
```

output
```
(2*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[7/12, 3/4, 7/4, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(3*d*f*(b*Sec[e + f*x])^(1/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \frac{\sqrt{d \tan (fx+e)}}{(b \sec (fx+e))^{\frac{1}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x)
```

output
```
int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int \frac{\sqrt{d \tan(e+fx)}}{\sqrt[3]{b \sec(e+fx)}}\,dx = \int \frac{\sqrt{d \tan (fx+e)}}{(b \sec (fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b*sec(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan\left(e+fx\right)}}{\sqrt[3]{b\sec\left(e+fx\right)}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(1/2)/(b*sec(f*x+e))**(1/3),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))/(b*sec(e + f*x))**(1/3), x)
```

## Maxima [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(1/3),x)
```

output
```
int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(fx+e)}}{\sec(fx+e)^{\frac{1}{3}}}dx\right)}{b^{\frac{1}{3}}}$$

input
```
int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x)
```

output
```
(sqrt(d)*int(sqrt(tan(e + f*x))/sec(e + f*x)**(1/3),x))/b**(1/3)
```

# 3.338 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx$


Optimal result . . . . . 2466
Mathematica [A] (verified) . . . . . 2466
Rubi [A] (verified) . . . . . 2467
Maple [F] . . . . . 2468
Fricas [F] . . . . . 2468
Sympy [F] . . . . . 2469
Maxima [F] . . . . . 2469
Giac [F] . . . . . 2469
Mupad [F(-1)] . . . . . 2470
Reduce [F] . . . . . 2470


## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{2\sqrt[12]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df(b\sec(e+fx))^{4/3}}$$

output
```
2/3*(cos(f*x+e)^2)^(1/12)*hypergeom([1/12, 3/4],[7/4],sin(f*x+e)^2)*(d*tan
(f*x+e))^(3/2)/d/f/(b*sec(f*x+e))^(4/3)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = -\frac{3d\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{4},\frac{1}{3},\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{4f(b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(4/3),x]
```

output
```
(-3*d*Hypergeometric2F1[-2/3, 1/4, 1/3, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^
(1/4))/(4*f*(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]])
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{2\sqrt[12]{\cos^2(e+fx)}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)}{3df(b\sec(e+fx))^{4/3}}$$

input
```
Int[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(4/3),x]
```

output
```
(2*(Cos[e + f*x]^2)^(1/12)*Hypergeometric2F1[1/12, 3/4, 7/4, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(3*d*f*(b*Sec[e + f*x])^(4/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \frac{\sqrt{d\tan (fx+e)}}{(b\sec (fx+e))^{\frac{4}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x)
```

output
```
int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x)
```

## Fricas [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan (fx+e)}}{(b\sec (fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b^2*sec(f*x + e)^2),
 x)
```

## Sympy [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan{(e+fx)}}}{(b\sec{(e+fx)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(1/2)/(b*sec(f*x+e))**(4/3),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))/(b*sec(e + f*x))**(4/3), x)
```

## Maxima [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan{(fx+e)}}}{(b\sec{(fx+e)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(4/3), x)
```

## Giac [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan{(fx+e)}}}{(b\sec{(fx+e)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(4/3),x)
```

output
```
int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(fx+e)}}{\sec(fx+e)^{\frac{4}{3}}}dx\right)}{b^{\frac{4}{3}}}$$

input
```
int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x)
```

output
```
(sqrt(d)*int(sqrt(tan(e + f*x))/(sec(e + f*x)**(1/3)*sec(e + f*x)),x))/(b*
*(1/3)*b)
```

# 3.339 $\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$

Optimal result . . . 2471
Mathematica [A] (verified) . . . 2471
Rubi [A] (verified) . . . 2472
Maple [F] . . . 2473
Fricas [F] . . . 2473
Sympy [F(-1)] . . . 2474
Maxima [F] . . . 2474
Giac [F] . . . 2474
Mupad [F(-1)] . . . 2475
Reduce [F] . . . 2475

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{23/12}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{23}{12},\frac{9}{4},\sin^2(e+fx)\right)(b\sec(e+f}{5df}$$

output
```
2/5*(cos(f*x+e)^2)^(23/12)*hypergeom([5/4, 23/12],[9/4],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(4/3)*(d*tan(f*x+e))^(5/2)/d/f
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{3d\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{2}{3},\frac{5}{3},\sec^2(e+fx)\right)(b\sec(e+fx))^{4/3}\sqrt{d\tan(e}}{4f\sqrt[4]{-\tan^2(e+fx)}}$$

input
```
Integrate[(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(3*d*Hypergeometric2F1[-1/4, 2/3, 5/3, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(4
/3)*Sqrt[d*Tan[e + f*x]])/(4*f*(-Tan[e + f*x]^2)^(1/4))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{2\cos^2(e+fx)^{23/12}(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{23}{12},\frac{9}{4},\sin^2(e+fx)\right)}{5df}$$

input
```
Int[(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(Cos[e + f*x]^2)^(23/12)*Hypergeometric2F1[5/4, 23/12, 9/4, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (b\sec (fx+e))^{\frac{4}{3}} (d\tan (fx+e))^{\frac{3}{2}}\, dx$$

input
```
int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

output
```
int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

## Fricas [F]

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\, dx = \int (b\sec (fx+e))^{\frac{4}{3}} (d\tan (fx+e))^{\frac{3}{2}}\, dx$$

input
```
integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*d*sec(f*x + e)*tan(
f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(4/3)*(d*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx=\int (d\tan(e+f\,x))^{3/2}\left(\frac{b}{\cos(e+f\,x)}\right)^{4/3}\,dx$$

input
```
int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(4/3),x)
```

output
```
int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx=\frac{3\sqrt{d}\,b^{\frac{4}{3}}d\left(2\sqrt{\tan(fx+e)}\,\sec(fx+e)^{\frac{4}{3}}-\left(\int\frac{\sqrt{\tan(fx+e)}\,\sec(fx+e)^{\frac{4}{3}}}{\tan(fx+e)}dx\right)f\right)}{11f}$$

input
```
int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

output
```
(3*sqrt(d)*b**(1/3)*b*d*(2*sqrt(tan(e + f*x))*sec(e + f*x)**(1/3)*sec(e +
f*x) - int((sqrt(tan(e + f*x))*sec(e + f*x)**(1/3)*sec(e + f*x))/tan(e + f
*x),x)*f))/(11*f)
```

# 3.340 $\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2476
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2476
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2477
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2478
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2478
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2479
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2479
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2479
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2480
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2480

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{9}{4},\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx}{5df}$$

output
```
2/5*(cos(f*x+e)^2)^(17/12)*hypergeom([5/4, 17/12],[9/4],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(1/3)*(d*tan(f*x+e))^(5/2)/d/f
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \frac{3d\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}}{f\sqrt[4]{-\tan^2(e+fx)}}$$

input
```
Integrate[(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(3*d*Hypergeometric2F1[-1/4, 1/6, 7/6, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(1
/3)*Sqrt[d*Tan[e + f*x]])/(f*(-Tan[e + f*x]^2)^(1/4))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \ 3097$$

$$\frac{2\cos^2(e+fx)^{17/12}\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{9}{4},\sin^2(e+fx)\right)}{5df}$$

input
```
Int[(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[5/4, 17/12, 9/4, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (b\sec(fx+e))^{\frac{1}{3}} (d\tan(fx+e))^{\frac{3}{2}} dx$$

input
```
int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

output
```
int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

## Fricas [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{1}{3}} (d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e), x)
```

## Sympy [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int \sqrt[3]{b\sec\left(e+fx\right)}(d\tan\left(e+fx\right))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/3)*(d*tan(f*x+e))**(3/2),x)
```

output
```
Integral((b*sec(e + f*x))**(1/3)*(d*tan(e + f*x))**(3/2), x)
```

## Maxima [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx=\int (d\tan(e+f\,x))^{3/2}\left(\frac{b}{\cos(e+f\,x)}\right)^{1/3}dx$$

input
```
int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/3),x)
```

output
```
int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx=\frac{3\sqrt{d}\,b^{\frac{1}{3}}d\left(2\sqrt{\tan(fx+e)}\,\sec(fx+e)^{\frac{1}{3}}-\left(\int\frac{\sqrt{\tan(fx+e)}\,\sec(fx+e)^{\frac{1}{3}}}{\tan(fx+e)}dx\right)f\right)}{5f}$$

input
```
int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

output
```
(3*sqrt(d)*b**(1/3)*d*(2*sqrt(tan(e + f*x))*sec(e + f*x)**(1/3) - int((sqr
t(tan(e + f*x))*sec(e + f*x)**(1/3))/tan(e + f*x),x)*f))/(5*f)
```

# 3.341 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx$


Optimal result . . . 2481
Mathematica [A] (verified) . . . 2481
Rubi [A] (verified) . . . 2482
Maple [F] . . . 2483
Fricas [F] . . . 2483
Sympy [F] . . . 2483
Maxima [F] . . . 2484
Giac [F] . . . 2484
Mupad [F(-1)] . . . 2484
Reduce [F] . . . 2485


## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx=\frac{2\cos^2(e+fx)^{13/12}\,\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df\sqrt[3]{b\sec(e+fx)}}$$

output
```
2/5*(cos(f*x+e)^2)^(13/12)*hypergeom([13/12, 5/4],[9/4],sin(f*x+e)^2)*(d*t
an(f*x+e))^(5/2)/d/f/(b*sec(f*x+e))^(1/3)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx=\frac{3\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{f\sqrt[3]{b\sec(e+fx)}}$$

input
```
Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(1/3),x]
```

output
```
(3*Cot[e + f*x]^3*Hypergeometric2F1[-1/4, -1/6, 5/6, Sec[e + f*x]^2]*(d*Ta
n[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(3/4))/(f*(b*Sec[e + f*x])^(1/3))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{ 3097}$$
$$\frac{2\cos^2(e+fx)^{13/12}(d\tan(e+fx))^{5/2}\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)}{5df\sqrt[3]{b\sec(e+fx)}}$$

input
```
Int[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(1/3),x]
```

output
```
(2*(Cos[e + f*x]^2)^(13/12)*Hypergeometric2F1[13/12, 5/4, 9/4, Sin[e + f*x
]^2]*(d*Tan[e + f*x])^(5/2))/(5*d*f*(b*Sec[e + f*x])^(1/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x)
```

output
```
int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b*sec
(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan(e+fx))^{\frac{3}{2}}}{\sqrt[3]{b\sec(e+fx)}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(3/2)/(b*sec(f*x+e))**(1/3),x)
```

output
```
Integral((d*tan(e + f*x))**(3/2)/(b*sec(e + f*x))**(1/3), x)
```

## Maxima [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan(e+f\,x))^{3/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(1/3),x)
```

output
```
int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(fx+e)}\,\tan(fx+e)}{\sec(fx+e)^{\frac{1}{3}}}dx\right)d}{b^{\frac{1}{3}}}$$

input
```
int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x)
```

output
```
(sqrt(d)*int((sqrt(tan(e + f*x))*tan(e + f*x))/sec(e + f*x)**(1/3),x)*d)/b
**(1/3)
```

# 3.342 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2486
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2486
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2487
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2488
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2488
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2488
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2489
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2489
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2489
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2490

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{2\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df(b\sec(e+fx))^{4/3}}$$

output
```
2/5*(cos(f*x+e)^2)^(7/12)*hypergeom([7/12, 5/4],[9/4],sin(f*x+e)^2)*(d*tan
(f*x+e))^(5/2)/d/f/(b*sec(f*x+e))^(4/3)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{3\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{4},\frac{1}{3},\sec^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{4f(b\sec(e+fx))^{4/3}}$$

input
```
Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(4/3),x]
```

output
```
(3*Cot[e + f*x]^3*Hypergeometric2F1[-2/3, -1/4, 1/3, Sec[e + f*x]^2]*(d*Ta
n[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(3/4))/(4*f*(b*Sec[e + f*x])^(4/3))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{2\cos^2(e+fx)^{7/12}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)}{5df(b\sec(e+fx))^{4/3}}$$

input
```
Int[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(4/3),x]
```

output
```
(2*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[7/12, 5/4, 9/4, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(5/2))/(5*d*f*(b*Sec[e + f*x])^(4/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x)
```

output
```
int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x)
```

## Fricas [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b^2*s
ec(f*x + e)^2), x)
```

## Sympy [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(e+fx))^{\frac{3}{2}}}{(b\sec(e+fx))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(3/2)/(b*sec(f*x+e))**(4/3),x)
```

output
```
Integral((d*tan(e + f*x))**(3/2)/(b*sec(e + f*x))**(4/3), x)
```

## Maxima [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx=\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(4/3), x)
```

## Giac [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx=\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx=\int \frac{(d\tan(e+f\,x))^{3/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(4/3),x)
```

output
```
int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\tan(fx+e)}\,\tan(fx+e)}{\sec(fx+e)^{\frac{4}{3}}}dx\right)d}{b^{\frac{4}{3}}}$$

input
```
int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x)
```

output
```
(sqrt(d)*int((sqrt(tan(e + f*x))*tan(e + f*x))/(sec(e + f*x)**(1/3)*sec(e
+ f*x)),x)*d)/(b**(1/3)*b)
```

# 3.343 $\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2491
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2491
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2492
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2493
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2493
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2494
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2494
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2494
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2495
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2495

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{3\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{13}{6},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+f}{7df}$$

output
```
3/7*(cos(f*x+e)^2)^(17/12)*hypergeom([7/6, 17/12],[13/6],sin(f*x+e)^2)*(b*
sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(7/3)/d/f
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{4},\frac{5}{4},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}}{f\sqrt[6]{-\tan^2(e+fx)}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(2*d*Hypergeometric2F1[-1/6, 1/4, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]
]*(d*Tan[e + f*x])^(1/3))/(f*(-Tan[e + f*x]^2)^(1/6))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{3\cos^2(e+fx)^{17/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{13}{6},\sin^2(e+fx)\right)}{7df}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(3*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[7/6, 17/12, 13/6, Sin[e + f*
x]^2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(7/3))/(7*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}dx$$

input
```
int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

## Fricas [F]

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sec\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*d*tan(f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)*(d*tan(f*x+e))**(4/3),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sec(fx+e)}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

## Giac [F]

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sec(fx+e)}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx=\int (d\tan(e+f\,x))^{4/3}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(1/2),x)
```

output
```
int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx=\frac{2d^{\frac{4}{3}}\sqrt{b}\left(3\tan\left(fx+e\right)^{\frac{1}{3}}\sqrt{\sec\left(fx+e\right)}-\left(\int\frac{\sqrt{\sec(fx+e)}}{\tan(fx+e)^{\frac{2}{3}}}dx\right)f\right)}{5f}$$

input
```
int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

output
```
(2*d**(1/3)*sqrt(b)*d*(3*tan(e + f*x)**(1/3)*sqrt(sec(e + f*x)) - int((tan
(e + f*x)**(1/3)*sqrt(sec(e + f*x)))/tan(e + f*x),x)*f))/(5*f)
```

# 3.344 $\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$

Optimal result . . . 2496
Mathematica [A] (verified) . . . 2496
Rubi [A] (verified) . . . 2497
Maple [F] . . . 2498
Fricas [F] . . . 2498
Sympy [F] . . . 2499
Maxima [F] . . . 2499
Giac [F] . . . 2499
Mupad [F(-1)] . . . 2500
Reduce [F] . . . 2500

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{3\cos^2(e+fx)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{5}{3},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}}{4df}$$

output
```
3/4*(cos(f*x+e)^2)^(11/12)*hypergeom([2/3, 11/12],[5/3],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3)/d/f
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}\sqrt[3]{-\tan^2(e+fx)}}{f(d\tan(e+fx))^{2/3}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(2*d*Hypergeometric2F1[1/4, 1/3, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]
*(-Tan[e + f*x]^2)^(1/3))/(f*(d*Tan[e + f*x])^(2/3))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{3\cos^2(e+fx)^{11/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{5}{3},\sin^2(e+fx)\right)}{4df}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(3*(Cos[e + f*x]^2)^(11/12)*Hypergeometric2F1[2/3, 11/12, 5/3, Sin[e + f*x
]^2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(4*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \sqrt{b \sec (fx+e)}\, (d \tan (fx+e))^{\frac{1}{3}}\, dx$$

input
```
int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int \sqrt{b \sec (e+fx)} \sqrt[3]{d \tan (e+fx)}\, dx = \int \sqrt{b \sec (fx+e)} (d \tan (fx+e))^{\frac{1}{3}}\, dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

## Sympy [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sec{(e+fx)}}\sqrt[3]{d\tan{(e+fx)}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/2)*(d*tan(f*x+e))**(1/3),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))*(d*tan(e + f*x))**(1/3), x)
```

## Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}}(d\tan{(fx+e)})^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}}(d\tan{(fx+e)})^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int (d\tan(e+f\,x))^{1/3}\,\sqrt{\frac{b}{\cos(e+f\,x)}}\,dx$$

input
```
int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(1/2),x)
```

output
```
int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = d^{\frac{1}{3}}\sqrt{b}\left(\int \tan(fx+e)^{\frac{1}{3}}\sqrt{\sec(fx+e)}dx\right)$$

input
```
int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

output
```
d**(1/3)*sqrt(b)*int(tan(e + f*x)**(1/3)*sqrt(sec(e + f*x)),x)
```

# 3.345 $\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$


Optimal result . . . . . . . . . . 2501
Mathematica [A] (verified) . . . . . . . . . . 2501
Rubi [A] (verified) . . . . . . . . . . 2502
Maple [F] . . . . . . . . . . 2503
Fricas [F] . . . . . . . . . . 2503
Sympy [F] . . . . . . . . . . 2504
Maxima [F] . . . . . . . . . . 2504
Giac [F] . . . . . . . . . . 2504
Mupad [F(-1)] . . . . . . . . . . 2505
Reduce [F] . . . . . . . . . . 2505


## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{3\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{4}{3},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{2/3}}{2df}$$

output
```
3/2*(cos(f*x+e)^2)^(7/12)*hypergeom([1/3, 7/12],[4/3],sin(f*x+e)^2)*(b*sec
(f*x+e))^(1/2)*(d*tan(f*x+e))^(2/3)/d/f
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{2}{3},\frac{5}{4},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}\left(-\tan^2(e+fx)\right)^{2/3}}{f(d\tan(e+fx))^{4/3}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(2*d*Hypergeometric2F1[1/4, 2/3, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]
*(-Tan[e + f*x]^2)^(2/3))/(f*(d*Tan[e + f*x])^(4/3))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{3\cos^2(e+fx)^{7/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{4}{3},\sin^2(e+fx)\right)}{2df}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(3*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[1/3, 7/12, 4/3, Sin[e + f*x]^
2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(2*d*f)
```

**Defintions of rubi rules used**

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \frac{\sqrt{b \sec (fx+e)}}{(d \tan (fx+e))^{\frac{1}{3}}}dx$$

input
```
int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int \frac{\sqrt{b \sec(e+fx)}}{\sqrt[3]{d \tan(e+fx)}}\, dx = \int \frac{\sqrt{b \sec (fx+e)}}{(d \tan (fx+e))^{\frac{1}{3}}}\, dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)/(d*tan(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sec{\left(e+fx \right)}}}{\sqrt[3]{d\tan{\left(e+fx \right)}}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/2)/(d*tan(f*x+e))**(1/3),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))/(d*tan(e + f*x))**(1/3), x)
```

## Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\left(d\tan\left(e+f\,x\right)\right)^{1/3}}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3),x)
```

output
```
int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}}{\tan(fx+e)^{\frac{1}{3}}}dx\right)}{d^{\frac{1}{3}}}$$

input
```
int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

output
```
(sqrt(b)*int(sqrt(sec(e + f*x))/tan(e + f*x)**(1/3),x))/d**(1/3)
```

# 3.346 $\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2506
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2506
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2507
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2508
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2508
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2509
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2509
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2509
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2510
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2510


## Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = -\frac{3\sqrt[12]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{5}{6},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}}{df\sqrt[3]{d\tan(e+fx)}}$$

output
```
-3*(cos(f*x+e)^2)^(1/12)*hypergeom([-1/6, 1/12],[5/6],sin(f*x+e)^2)*(b*sec
(f*x+e))^(1/2)/d/f/(d*tan(f*x+e))^(1/3)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{6},\frac{5}{4},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}(-\tan^2(e+fx))^{7}}{f(d\tan(e+fx))^{7/3}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(2*d*Hypergeometric2F1[1/4, 7/6, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]
*(-Tan[e + f*x]^2)^(7/6))/(f*(d*Tan[e + f*x])^(7/3))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{ 3097}$$

$$-\frac{3\sqrt[12]{\cos^2(e+fx)}\sqrt{b\sec(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{5}{6},\sin^2(e+fx)\right)}{df\sqrt[3]{d\tan(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(-3*(Cos[e + f*x]^2)^(1/12)*Hypergeometric2F1[-1/6, 1/12, 5/6, Sin[e + f*x
]^2]*Sqrt[b*Sec[e + f*x]])/(d*f*(d*Tan[e + f*x])^(1/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \frac{\sqrt{b\sec(fx+e)}}{(d\tan(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

## Fricas [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sec(fx+e)}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)/(d^2*tan(f*x + e)^2),
 x)
```

## Sympy [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sec\left(e+fx\right)}}{\left(d\tan\left(e+fx\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/2)/(d*tan(f*x+e))**(4/3),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))/(d*tan(e + f*x))**(4/3), x)
```

## Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

## Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{(d\tan(e+f\,x))^{4/3}}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3),x)
```

output
```
int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}}{\tan(fx+e)^{\frac{4}{3}}}dx\right)}{d^{\frac{4}{3}}}$$

input
```
int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

output
```
(sqrt(b)*int(sqrt(sec(e + f*x))/(tan(e + f*x)**(1/3)*tan(e + f*x)),x))/(d*
*(1/3)*d)
```

# 3.347 $\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2511
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2511
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2512
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2513
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2513
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2514
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2514
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2514
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2515
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2515

## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{3\cos^2(e+fx)^{23/12}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{13}{6},\sin^2(e+fx)\right)(b\sec(e+}{7df}$$

output
```
3/7*(cos(f*x+e)^2)^(23/12)*hypergeom([7/6, 23/12],[13/6],sin(f*x+e)^2)*(b*
sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(7/3)/d/f
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{3}{4},\frac{7}{4},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(}{3f\sqrt[6]{-\tan^2(e+fx)}}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(2*d*Hypergeometric2F1[-1/6, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3
/2)*(d*Tan[e + f*x])^(1/3))/(3*f*(-Tan[e + f*x]^2)^(1/6))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{3\cos^2(e+fx)^{23/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{13}{6},\sin^2(e+fx)\right)}{7df}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(3*(Cos[e + f*x]^2)^(23/12)*Hypergeometric2F1[7/6, 23/12, 13/6, Sin[e + f*
x]^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(7/3))/(7*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (b\sec(fx+e))^{\frac{3}{2}} (d\tan(fx+e))^{\frac{4}{3}} dx$$

input
```
int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

## Fricas [F]

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3} dx = \int (b\sec(fx+e))^{\frac{3}{2}} (d\tan(fx+e))^{\frac{4}{3}} dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*b*d*sec(f*x + e)*tan(
f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(3/2)*(d*tan(f*x+e))**(4/3),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

## Giac [F]

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (d\tan(e+f\,x))^{4/3}\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

input
```
int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(3/2),x)
```

output
```
int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{2d^{\frac{4}{3}}\sqrt{b}\,b\left(3\tan(fx+e)^{\frac{1}{3}}\sqrt{\sec(fx+e)}\,\sec(fx+e)-\left(\int\frac{\sqrt{\sec(fx+e)}\,\sec(fx}{\tan(fx+e)^{\frac{2}{3}}}\right.\right.}{11f}$$

input
```
int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

output
```
(2*d**(1/3)*sqrt(b)*b*d*(3*tan(e + f*x)**(1/3)*sqrt(sec(e + f*x))*sec(e +
f*x) - int((tan(e + f*x)**(1/3)*sqrt(sec(e + f*x))*sec(e + f*x))/tan(e + f
*x),x)*f))/(11*f)
```

# 3.348 $\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$


Optimal result . . . . . 2516
Mathematica [A] (verified) . . . . . 2516
Rubi [A] (verified) . . . . . 2517
Maple [F] . . . . . 2518
Fricas [F] . . . . . 2518
Sympy [F] . . . . . 2519
Maxima [F] . . . . . 2519
Giac [F] . . . . . 2519
Mupad [F(-1)] . . . . . 2520
Reduce [F] . . . . . 2520


## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{3\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{17}{12},\frac{5}{3},\sin^2(e+fx)\right)(b\sec(e+fx)}{4df}$$

output
```
3/4*(cos(f*x+e)^2)^(17/12)*hypergeom([2/3, 17/12],[5/3],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3)/d/f
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\sqrt[3]{-\tan^2(e}}{3f(d\tan(e+fx))^{2/3}}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(2*d*Hypergeometric2F1[1/3, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/
2)*(-Tan[e + f*x]^2)^(1/3))/(3*f*(d*Tan[e + f*x])^(2/3))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{3\cos^2(e+fx)^{17/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{17}{12},\frac{5}{3},\sin^2(e+fx)\right)}{4df}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(3*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[2/3, 17/12, 5/3, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3))/(4*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (b\sec (fx+e))^{\frac{3}{2}} (d\tan (fx+e))^{\frac{1}{3}} dx$$

input
```
int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sec (fx+e))^{\frac{3}{2}} (d\tan (fx+e))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*b*sec(f*x + e), x)
```

## Sympy [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sec{(e+fx)})^{\frac{3}{2}}\sqrt[3]{d\tan{(e+fx)}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(3/2)*(d*tan(f*x+e))**(1/3),x)
```

output
```
Integral((b*sec(e + f*x))**(3/2)*(d*tan(e + f*x))**(1/3), x)
```

## Maxima [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sec{(fx+e)})^{\frac{3}{2}}(d\tan{(fx+e)})^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sec{(fx+e)})^{\frac{3}{2}}(d\tan{(fx+e)})^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (d\tan(e+f\,x))^{1/3}\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

input
```
int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(3/2),x)
```

output
```
int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = d^{\frac{1}{3}}\sqrt{b}\left(\int \tan(fx+e)^{\frac{1}{3}}\sqrt{\sec(fx+e)}\,\sec(fx+e)\,dx\right)b$$

input
```
int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

output
```
d**(1/3)*sqrt(b)*int(tan(e + f*x)**(1/3)*sqrt(sec(e + f*x))*sec(e + f*x),x
)*b
```

# 3.349 $\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2521
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2521
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2522
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2523
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2523
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2523
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2524
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2524
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2524
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2525


## Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{3\cos^2(e+fx)^{13/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{4}{3},\sin^2(e+fx)\right)(b\sec(e+fx))^{3/}}{2df}$$

output
```
3/2*(cos(f*x+e)^2)^(13/12)*hypergeom([1/3, 13/12],[4/3],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(3/2)*(d*tan(f*x+e))^(2/3)/d/f
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{4},\frac{7}{4},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\left(-\tan^2(e+fx\right.}{3f(d\tan(e+fx))^{4/3}}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(2*d*Hypergeometric2F1[2/3, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/
2)*(-Tan[e + f*x]^2)^(2/3))/(3*f*(d*Tan[e + f*x])^(4/3))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}dx$$
$$\Big\downarrow \text{3097}$$
$$\frac{3\cos^2(e+fx)^{13/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{4}{3},\sin^2(e+fx)\right)}{2df}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(3*(Cos[e + f*x]^2)^(13/12)*Hypergeometric2F1[1/3, 13/12, 4/3, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(2/3))/(2*d*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

## Fricas [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sec(f*x + e)/(d*tan
(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sec(e+fx))^{\frac{3}{2}}}{\sqrt[3]{d\tan(e+fx)}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(3/2)/(d*tan(f*x+e))**(1/3),x)
```

output
```
Integral((b*sec(e + f*x))**(3/2)/(d*tan(e + f*x))**(1/3), x)
```

## Maxima [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx=\int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

## Giac [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx=\int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx=\int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{(d\tan(e+f\,x))^{1/3}}\,dx$$

input
```
int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3),x)
```

output
```
int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3), x)
```

## Reduce [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sec(fx+e)}{\tan(fx+e)^{\frac{1}{3}}}dx\right)b}{d^{\frac{1}{3}}}$$

input
```
int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sec(e + f*x))/tan(e + f*x)**(1/3),x)*b)/d
**(1/3)
```

# 3.350 $\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2526
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2526
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2527
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2528
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2528
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2529
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2529
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2529
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2530
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2530

## Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = -\frac{3\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{5}{6},\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}}{df\sqrt[3]{d\tan(e+fx)}}$$

output
```
-3*(cos(f*x+e)^2)^(7/12)*hypergeom([-1/6, 7/12],[5/6],sin(f*x+e)^2)*(b*sec
(f*x+e))^(3/2)/d/f/(d*tan(f*x+e))^(1/3)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{6},\frac{7}{4},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\left(-\tan^2(e+fx\right.}{3f(d\tan(e+fx))^{7/3}}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(2*d*Hypergeometric2F1[3/4, 7/6, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/
2)*(-Tan[e + f*x]^2)^(7/6))/(3*f*(d*Tan[e + f*x])^(7/3))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{3097}$$

$$-\frac{3\cos^2(e+fx)^{7/12}(b\sec(e+fx))^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{5}{6},\sin^2(e+fx)\right)}{df\sqrt[3]{d\tan(e+fx)}}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(-3*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[-1/6, 7/12, 5/6, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(3/2))/(d*f*(d*Tan[e + f*x])^(1/3))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

## Fricas [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sec(f*x + e)/(d^2*t
an(f*x + e)^2), x)
```

## Sympy [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(e+fx))^{\frac{3}{2}}}{(d\tan(e+fx))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(3/2)/(d*tan(f*x+e))**(4/3),x)
```

output
```
Integral((b*sec(e + f*x))**(3/2)/(d*tan(e + f*x))**(4/3), x)
```

## Maxima [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

## Giac [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\left(d\tan\left(e+f\,x\right)\right)^{4/3}}\,dx$$

input
```
int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3),x)
```

output
```
int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3), x)
```

## Reduce [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sec(fx+e)}{\tan(fx+e)^{\frac{4}{3}}}dx\right)b}{d^{\frac{4}{3}}}$$

input
```
int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sec(e + f*x))/(tan(e + f*x)**(1/3)*tan(e
+ f*x)),x)*b)/(d**(1/3)*d)
```

# 3.351 $\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx$

Optimal result . . . 2531
Mathematica [A] (verified) . . . 2531
Rubi [A] (verified) . . . 2532
Maple [C] (warning: unable to verify) . . . 2533
Fricas [A] (verification not implemented) . . . 2534
Sympy [F] . . . 2534
Maxima [A] (verification not implemented) . . . 2535
Giac [B] (verification not implemented) . . . 2535
Mupad [B] (verification not implemented) . . . 2536
Reduce [B] (verification not implemented) . . . 2536

## Optimal result

Integrand size = 19, antiderivative size = 67

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \frac{(b\sec(e+fx))^m}{fm} - \frac{2(b\sec(e+fx))^{2+m}}{b^2f(2+m)} + \frac{(b\sec(e+fx))^{4+m}}{b^4f(4+m)}$$

output
```
(b*sec(f*x+e))^m/f/m-2*(b*sec(f*x+e))^(2+m)/b^2/f/(2+m)+(b*sec(f*x+e))^(4+
m)/b^4/f/(4+m)
```

## Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \frac{(b\sec(e+fx))^m (8+6m+m^2-2m(4+m)\sec^2(e+fx)+m(2+m)\sec^4(e+fx))}{fm(2+m)(4+m)}$$

input
```
Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^5,x]
```

output
```
((b*Sec[e + f*x])^m*(8 + 6*m + m^2 - 2*m*(4 + m)*Sec[e + f*x]^2 + m*(2 + m
)*Sec[e + f*x]^4))/(f*m*(2 + m)*(4 + m))
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^5(b\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{b\int (b\sec(e+fx))^{m-1}\left(1-\sec^2(e+fx)\right)^2 d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left((b\sec(e+fx))^{m-1}-\frac{2(b\sec(e+fx))^{m+1}}{b^2}+\frac{(b\sec(e+fx))^{m+3}}{b^4}\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{m+4}}{b^5(m+4)}-\frac{2(b\sec(e+fx))^{m+2}}{b^3(m+2)}+\frac{(b\sec(e+fx))^m}{bm}\right)}{f}$$

input
```
Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^5,x]
```

output
```
(b*((b*Sec[e + f*x])^m/(b*m) - (2*(b*Sec[e + f*x])^(2 + m))/(b^3*(2 + m))
+ (b*Sec[e + f*x])^(4 + m)/(b^5*(4 + m))))/f
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 7.92 (sec) , antiderivative size = 6067, normalized size of antiderivative = 90.55

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 6067 |

input
```
int((b*sec(f*x+e))^m*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \frac{\left((m^2+6\,m+8)\cos\left(fx+e\right)^4-2\,(m^2+4\,m)\cos\left(fx+e\right)^2+m^2+2\,m\right)\left(\frac{b}{\cos(fx+e)}\right)^m}{(fm^3+6\,fm^2+8\,fm)\cos\left(fx+e\right)^4}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
((m^2 + 6*m + 8)*cos(f*x + e)^4 - 2*(m^2 + 4*m)*cos(f*x + e)^2 + m^2 + 2*m
)*(b/cos(f*x + e))^m/((f*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^4)
```

## Sympy [F]

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \begin{cases} x(b\sec(e))^m\tan^5(e) & \text{for } f \\ \frac{\int \frac{\tan^5(e+fx)}{\sec^4(e+fx)}\,dx}{b^4} & \text{for } m \\ \frac{\int \frac{\tan^5(e+fx)}{\sec^2(e+fx)}\,dx}{b^2} & \text{for } m \\ \frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f} & \text{for } m \\ \frac{m^2(b\sec(e+fx))^m\tan^4(e+fx)}{fm^3+6fm^2+8fm}+\frac{2m(b\sec(e+fx))^m\tan^4(e+fx)}{fm^3+6fm^2+8fm}-\frac{4m(b\sec(e+fx))^m\tan^2(e+fx)}{fm^3+6fm^2+8fm}+\frac{8(b\sec(e+fx))^m}{fm^3+6fm^2+8fm} & \text{other} \end{cases}$$

input
```
integrate((b*sec(f*x+e))**m*tan(f*x+e)**5,x)
```

output
```
Piecewise((x*(b*sec(e))**m*tan(e)**5, Eq(f, 0)), (Integral(tan(e + f*x)**5
/sec(e + f*x)**4, x)/b**4, Eq(m, -4)), (Integral(tan(e + f*x)**5/sec(e + f
*x)**2, x)/b**2, Eq(m, -2)), (log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x
)**4/(4*f) - tan(e + f*x)**2/(2*f), Eq(m, 0)), (m**2*(b*sec(e + f*x))**m*t
an(e + f*x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) + 2*m*(b*sec(e + f*x))**m*tan(e
 + f*x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) - 4*m*(b*sec(e + f*x))**m*tan(e + f
*x)**2/(f*m**3 + 6*f*m**2 + 8*f*m) + 8*(b*sec(e + f*x))**m/(f*m**3 + 6*f*m
**2 + 8*f*m), True))
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \frac{\frac{b^m\cos(fx+e)^{-m}}{m} - \frac{2\,b^m\cos(fx+e)^{-m}}{(m+2)\cos(fx+e)^2} + \frac{b^m\cos(fx+e)^{-m}}{(m+4)\cos(fx+e)^4}}{f}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
(b^m*cos(f*x + e)^(-m)/m - 2*b^m*cos(f*x + e)^(-m)/((m + 2)*cos(f*x + e)^2
) + b^m*cos(f*x + e)^(-m)/((m + 4)*cos(f*x + e)^4))/f
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(67) = 134.

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx$$
$$= \frac{\frac{b\left(\frac{b}{\cos(fx+e)}\right)^m}{m} - \frac{\frac{2\,b^4m\left(\frac{b}{\cos(fx+e)}\right)^m}{\cos(fx+e)^2} + \frac{8\,b^4\left(\frac{b}{\cos(fx+e)}\right)^m}{\cos(fx+e)^2} - \frac{b^4m\left(\frac{b}{\cos(fx+e)}\right)^m}{\cos(fx+e)^4} - \frac{2\,b^4\left(\frac{b}{\cos(fx+e)}\right)^m}{\cos(fx+e)^4}}{b^3m^2+6\,b^3m+8\,b^3}}{bf}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
(b*(b/cos(f*x + e))^m/m - (2*b^4*m*(b/cos(f*x + e))^m/cos(f*x + e)^2 + 8*b
^4*(b/cos(f*x + e))^m/cos(f*x + e)^2 - b^4*m*(b/cos(f*x + e))^m/cos(f*x +
e)^4 - 2*b^4*(b/cos(f*x + e))^m/cos(f*x + e)^4)/(b^3*m^2 + 6*b^3*m + 8*b^3
))/(b*f)
```

## Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.97

$$\int (b\sec(e+fx))^m\tan^5(e+fx)\,dx$$
$$= \frac{(\cos(4e+4fx)-\sin(4e+4fx)\ 1\mathrm{i})\left(\frac{b}{\cos(e+fx)}\right)^m\left(\frac{2\cos(4e+4fx)\,(\cos(4e+4fx)+\sin(4e+4fx)\,1\mathrm{i})}{fm}+\frac{(\cos(4e+4f}{}\right.}{16\left(\frac{\cos(2e+2fx)}{2}+\frac{1}{2}\right)^2}$$

input
```
int(tan(e + f*x)^5*(b/cos(e + f*x))^m,x)
```

output
```
((cos(4*e + 4*f*x) - sin(4*e + 4*f*x)*1i)*(b/cos(e + f*x))^m*((2*cos(4*e +
 4*f*x)*(cos(4*e + 4*f*x) + sin(4*e + 4*f*x)*1i))/(f*m) + ((cos(4*e + 4*f*
x) + sin(4*e + 4*f*x)*1i)*(4*m + 6*m^2 + 48))/(f*m*(6*m + m^2 + 8)) - (2*c
os(2*e + 2*f*x)*(cos(4*e + 4*f*x) + sin(4*e + 4*f*x)*1i)*(8*m + 4*m^2 - 32
))/(f*m*(6*m + m^2 + 8))))/(16*(cos(2*e + 2*f*x)/2 + 1/2)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int (b\sec(e+fx))^m\tan^5(e+fx)\,dx$$
$$= \frac{b^m\sec(fx+e)^m\left(\tan(fx+e)^4m^2+2\tan(fx+e)^4m-4\tan(fx+e)^2m+8\right)}{fm(m^2+6m+8)}$$

input
```
int((b*sec(f*x+e))^m*tan(f*x+e)^5,x)
```

output
```
(b**m*sec(e + f*x)**m*(tan(e + f*x)**4*m**2 + 2*tan(e + f*x)**4*m - 4*tan(
e + f*x)**2*m + 8))/(f*m*(m**2 + 6*m + 8))
```

# 3.352 $\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx$

Optimal result . . . 2538
Mathematica [A] (verified) . . . 2538
Rubi [A] (verified) . . . 2539
Maple [C] (warning: unable to verify) . . . 2540
Fricas [A] (verification not implemented) . . . 2541
Sympy [F] . . . 2542
Maxima [A] (verification not implemented) . . . 2542
Giac [A] (verification not implemented) . . . 2543
Mupad [B] (verification not implemented) . . . 2543
Reduce [B] (verification not implemented) . . . 2544

## Optimal result

Integrand size = 19, antiderivative size = 43

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = -\frac{(b\sec(e+fx))^m}{fm} + \frac{(b\sec(e+fx))^{2+m}}{b^2f(2+m)}$$

output
```
-(b*sec(f*x+e))^m/f/m+(b*sec(f*x+e))^(2+m)/b^2/f/(2+m)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = -\frac{(b\sec(e+fx))^m\,(2+m-m\sec^2(e+fx))}{fm(2+m)}$$

input
```
Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
-(((b*Sec[e + f*x])^m*(2 + m - m*Sec[e + f*x]^2))/(f*m*(2 + m)))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3(b\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{b\int -(b\sec(e+fx))^{m-1}\left(1-\sec^2(e+fx)\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int (b\sec(e+fx))^{m-1}\left(1-\sec^2(e+fx)\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left((b\sec(e+fx))^{m-1}-\frac{(b\sec(e+fx))^{m+1}}{b^2}\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{m+2}}{b^3(m+2)}-\frac{(b\sec(e+fx))^m}{bm}\right)}{f}$$

input
```
Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
(b*(-((b*Sec[e + f*x])^m/(b*m)) + (b*Sec[e + f*x])^(2 + m)/(b^3*(2 + m))))
/f
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.15 (sec) , antiderivative size = 2423, normalized size of antiderivative = 56.35

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2423 |

input
```
int((b*sec(f*x+e))^m*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/(2+m)/f/(exp(2*I*(f*x+e))+1)^2/m*exp(I*(f*x+e))^m*(exp(2*I*(f*x+e))+1)^
(-m)*b^m*2^m*(m*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^
3*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*exp
(I*(f*x+e)))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2
*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(
2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))+1))*m)*ex
p(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x
+e))/(exp(2*I*(f*x+e))+1))^2*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2
*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)*m)
*exp(-1/2*I*Pi*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3*m)*exp(1/2*
I*Pi*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*b)*m)*exp(4*I*
f*x)*exp(4*I*e)+2*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1)
)^3*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*e
xp(I*(f*x+e)))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))
^2*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(ex
p(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))+1))*m)*
exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f
*x+e))/(exp(2*I*(f*x+e))+1))^2*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp
(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)*
m)*exp(-1/2*I*Pi*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3*m)*exp...
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = -\frac{\left((m+2)\cos\left(fx+e\right)^2-m\right)\left(\frac{b}{\cos(fx+e)}\right)^m}{\left(fm^2+2\,fm\right)\cos\left(fx+e\right)^2}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
-((m + 2)*cos(f*x + e)^2 - m)*(b/cos(f*x + e))^m/((f*m^2 + 2*f*m)*cos(f*x
+ e)^2)
```

## Sympy [F]

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = \begin{cases} x(b\sec(e))^m \tan^3(e) & \text{for } f=0 \\ \frac{\int \frac{\tan^3(e+fx)}{\sec^2(e+fx)}\,dx}{b^2} & \text{for } m=-2 \\ -\frac{\log(\tan^2(e+fx)+1)}{2f} + \frac{\tan^2(e+fx)}{2f} & \text{for } m=0 \\ \frac{m(b\sec(e+fx))^m \tan^2(e+fx)}{fm^2+2fm} - \frac{2(b\sec(e+fx))^m}{fm^2+2fm} & \text{otherwise} \end{cases}$$

input
```
integrate((b*sec(f*x+e))**m*tan(f*x+e)**3,x)
```

output
```
Piecewise((x*(b*sec(e))**m*tan(e)**3, Eq(f, 0)), (Integral(tan(e + f*x)**3
/sec(e + f*x)**2, x)/b**2, Eq(m, -2)), (-log(tan(e + f*x)**2 + 1)/(2*f) +
tan(e + f*x)**2/(2*f), Eq(m, 0)), (m*(b*sec(e + f*x))**m*tan(e + f*x)**2/(
f*m**2 + 2*f*m) - 2*(b*sec(e + f*x))**m/(f*m**2 + 2*f*m), True))
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = -\frac{\frac{b^m \cos(fx+e)^{-m}}{m} - \frac{b^m \cos(fx+e)^{-m}}{(m+2)\cos(fx+e)^2}}{f}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
-(b^m*cos(f*x + e)^(-m)/m - b^m*cos(f*x + e)^(-m)/((m + 2)*cos(f*x + e)^2)
)/f
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int (b\sec(e+fx))^m\tan^3(e+fx)\,dx=-\frac{\frac{b\left(\frac{b}{\cos(fx+e)}\right)^m}{m}-\frac{b^2\left(\frac{b}{\cos(fx+e)}\right)^m}{(bm+2\,b)\cos(fx+e)^2}}{bf}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
-(b*(b/cos(f*x + e))^m/m - b^2*(b/cos(f*x + e))^m/((b*m + 2*b)*cos(f*x + e
)^2))/(b*f)
```

## Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int (b\sec(e+fx))^m\tan^3(e+fx)\,dx=$$
$$-\frac{\left(\frac{b}{\cos(e+f\,x)}\right)^m\,(8\,\cos(2\,e+2\,f\,x)-m+2\,\cos(4\,e+4\,f\,x)+m\,\cos(4\,e+4\,f\,x)+6)}{f\,m\,(m+2)\,(4\,\cos(2\,e+2\,f\,x)+\cos(4\,e+4\,f\,x)+3)}$$

input
```
int(tan(e + f*x)^3*(b/cos(e + f*x))^m,x)
```

output
```
-((b/cos(e + f*x))^m*(8*cos(2*e + 2*f*x) - m + 2*cos(4*e + 4*f*x) + m*cos(
4*e + 4*f*x) + 6))/(f*m*(m + 2)*(4*cos(2*e + 2*f*x) + cos(4*e + 4*f*x) + 3
))
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = \frac{b^m \sec(fx+e)^m \left(\tan(fx+e)^2 m-2\right)}{fm(m+2)}$$

input
```
int((b*sec(f*x+e))^m*tan(f*x+e)^3,x)
```

output
```
(b**m*sec(e + f*x)**m*(tan(e + f*x)**2*m - 2))/(f*m*(m + 2))
```

# 3.353 $\int (b\sec(e+fx))^m \tan(e+fx)\,dx$

Optimal result . . . 2545
Mathematica [A] (verified) . . . 2545
Rubi [A] (verified) . . . 2546
Maple [A] (verified) . . . 2547
Fricas [A] (verification not implemented) . . . 2547
Sympy [B] (verification not implemented) . . . 2548
Maxima [A] (verification not implemented) . . . 2548
Giac [A] (verification not implemented) . . . 2549
Mupad [B] (verification not implemented) . . . 2549
Reduce [B] (verification not implemented) . . . 2549

## Optimal result

Integrand size = 17, antiderivative size = 17

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{(b\sec(e+fx))^m}{fm}$$

output
```
(b*sec(f*x+e))^m/f/m
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{(b\sec(e+fx))^m}{fm}$$

input
```
Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x],x]
```

output
```
(b*Sec[e + f*x])^m/(f*m)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3086, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)(b\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{b\int (b\sec(e+fx))^{m-1}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{17}$$

$$\frac{(b\sec(e+fx))^m}{fm}$$

input
```
Int[(b*Sec[e + f*x])^m*Tan[e + f*x],x]
```

output
```
(b*Sec[e + f*x])^m/(f*m)
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| derivativedivides | $\frac{(b\sec(fx+e))^m}{fm}$ |
| default | $\frac{(b\sec(fx+e))^m}{fm}$ |
| risch | $\left(e^{i(fx+e)}\right)^m \left(e^{2i(fx+e)}+1\right)^{-m} 2^m b^m e^{\frac{i\pi m\left(-\operatorname{csgn}\left(\frac{i e^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^3+\operatorname{csgn}\left(\frac{i e^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^2 \operatorname{csgn}\left(i e^{i(fx+e)}\right)+\operatorname{csgn}\left(\frac{i e^{i(fx}}{e^{2i(fx}}\right.\right.}{}}$ |

input
```
int((b*sec(f*x+e))^m*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
(b*sec(f*x+e))^m/f/m
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{\left(\frac{b}{\cos(fx+e)}\right)^m}{fm}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

output
```
(b/cos(f*x + e))^m/(f*m)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.47

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \begin{cases} x\tan(e) & \text{for } f=0 \wedge m=0 \\ x(b\sec(e))^m \tan(e) & \text{for } f=0 \\ \frac{\log(\tan^2(e+fx)+1)}{2f} & \text{for } m=0 \\ \frac{(b\sec(e+fx))^m}{fm} & \text{otherwise} \end{cases}$$

input
```
integrate((b*sec(f*x+e))**m*tan(f*x+e),x)
```

output
```
Piecewise((x*tan(e), Eq(f, 0) & Eq(m, 0)), (x*(b*sec(e))**m*tan(e), Eq(f,
0)), (log(tan(e + f*x)**2 + 1)/(2*f), Eq(m, 0)), ((b*sec(e + f*x))**m/(f*m
), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{b^m \cos(fx+e)^{-m}}{fm}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

output
```
b^m*cos(f*x + e)^(-m)/(f*m)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{\left(\frac{b}{\cos(fx+e)}\right)^m}{fm}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

output
```
(b/cos(f*x + e))^m/(f*m)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^m}{f\,m}$$

input
```
int(tan(e + f*x)*(b/cos(e + f*x))^m,x)
```

output
```
(b/cos(e + f*x))^m/(f*m)
```

## Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{b^m \sec(fx+e)^m}{fm}$$

input
```
int((b*sec(f*x+e))^m*tan(f*x+e),x)
```

output
```
(b**m*sec(e + f*x)**m)/(f*m)
```

# 3.354 $\int \cot(e+fx)(b\sec(e+fx))^m\,dx$

Optimal result . . . . . 2550
Mathematica [A] (verified) . . . . . 2550
Rubi [A] (verified) . . . . . 2551
Maple [F] . . . . . 2552
Fricas [F] . . . . . 2552
Sympy [F] . . . . . 2553
Maxima [F] . . . . . 2553
Giac [F] . . . . . 2553
Mupad [F(-1)] . . . . . 2554
Reduce [F] . . . . . 2554

## Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

output
```
-hypergeom([1, 1/2*m],[1+1/2*m],sec(f*x+e)^2)*(b*sec(f*x+e))^m/f/m
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

input
```
Integrate[Cot[e + f*x]*(b*Sec[e + f*x])^m,x]
```

output
```
-((Hypergeometric2F1[1, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m
)/(f*m))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3086, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^m}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{b\int -\frac{(b\sec(e+fx))^{m-1}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(b\sec(e+fx))^{m-1}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(b\sec(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},\sec^2(e+fx)\right)}{fm}$$

input
```
Int[Cot[e + f*x]*(b*Sec[e + f*x])^m,x]
```

output
```
-((Hypergeometric2F1[1, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m
)/(f*m))
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [F]

$$\int \cot \left(fx+e\right) \left(b\sec \left(fx+e\right)\right)^{m} dx$$

input
```
int(cot(f*x+e)*(b*sec(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)*(b*sec(f*x+e))^m,x)
```

## Fricas [F]

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx=\int \left(b\sec \left(fx+e\right)\right)^{m} \cot \left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*cot(f*x + e), x)
```

## Sympy [F]

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec{(e+fx)})^m \cot{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)*(b*sec(f*x+e))**m,x)
```

output
```
Integral((b*sec(e + f*x))**m*cot(e + f*x), x)
```

## Maxima [F]

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec{(fx+e)})^m \cot{(fx+e)}\,dx$$

input
```
integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e), x)
```

## Giac [F]

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec{(fx+e)})^m \cot{(fx+e)}\,dx$$

input
```
integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int \cot(e+f\,x)\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

input
```
int(cot(e + f*x)*(b/cos(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)*(b/cos(e + f*x))^m, x)
```

## Reduce [F]

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = b^m\left(\int \sec\left(fx+e\right)^m\cot\left(fx+e\right)dx\right)$$

input
```
int(cot(f*x+e)*(b*sec(f*x+e))^m,x)
```

output
```
b**m*int(sec(e + f*x)**m*cot(e + f*x),x)
```

# 3.355 $\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx$

Optimal result . . . 2555
Mathematica [A] (verified) . . . 2555
Rubi [A] (verified) . . . 2556
Maple [F] . . . 2557
Fricas [F] . . . 2557
Sympy [F] . . . 2558
Maxima [F] . . . 2558
Giac [F] . . . 2558
Mupad [F(-1)] . . . 2559
Reduce [F] . . . 2559

## Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

output
```
hypergeom([2, 1/2*m],[1+1/2*m],sec(f*x+e)^2)*(b*sec(f*x+e))^m/f/m
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

input
```
Integrate[Cot[e + f*x]^3*(b*Sec[e + f*x])^m,x]
```

output
```
(Hypergeometric2F1[2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)/
(f*m)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3086, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^m}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{b\int \frac{(b\sec(e+fx))^{m-1}}{(1-\sec^2(e+fx))^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(b\sec(e+fx))^m \text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{m+2}{2},\sec^2(e+fx)\right)}{fm}$$

input
```
Int[Cot[e + f*x]^3*(b*Sec[e + f*x])^m,x]
```

output
```
(Hypergeometric2F1[2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)/
(f*m)
```

**Defintions of rubi rules used**

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [F]

$$\int \cot(fx+e)^3 (b\sec(fx+e))^m \, dx$$

input
```
int(cot(f*x+e)^3*(b*sec(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^3*(b*sec(f*x+e))^m,x)
```

## Fricas [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m \, dx = \int (b\sec(fx+e))^m \cot(fx+e)^3 \, dx$$

input
```
integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

## Sympy [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(e+fx))^m \cot^3(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**3*(b*sec(f*x+e))**m,x)
```

output
```
Integral((b*sec(e + f*x))**m*cot(e + f*x)**3, x)
```

## Maxima [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m \cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

## Giac [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m \cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \int \cot(e+f\,x)^3\left(\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(cot(e + f*x)^3*(b/cos(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^3*(b/cos(e + f*x))^m, x)
```

## Reduce [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = b^m\left(\int \sec(fx+e)^m\cot(fx+e)^3\,dx\right)$$

input
```
int(cot(f*x+e)^3*(b*sec(f*x+e))^m,x)
```

output
```
b**m*int(sec(e + f*x)**m*cot(e + f*x)**3,x)
```

# 3.356 $\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx$


Optimal result . . . 2560
Mathematica [A] (verified) . . . 2560
Rubi [A] (verified) . . . 2561
Maple [F] . . . 2562
Fricas [F] . . . 2562
Sympy [F] . . . 2563
Maxima [F] . . . 2563
Giac [F] . . . 2563
Mupad [F(-1)] . . . 2564
Reduce [F] . . . 2564


## Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\text{Hypergeometric2F1}\left(3,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

output
```
-hypergeom([3, 1/2*m],[1+1/2*m],sec(f*x+e)^2)*(b*sec(f*x+e))^m/f/m
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\text{Hypergeometric2F1}\left(3,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

input
```
Integrate[Cot[e + f*x]^5*(b*Sec[e + f*x])^m,x]
```

output
```
-((Hypergeometric2F1[3, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m
)/(f*m))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3086, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(b\sec(e+fx))^m}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \; 3086$$

$$\frac{b\int -\frac{(b\sec(e+fx))^{m-1}}{(1-\sec^2(e+fx))^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{b\int \frac{(b\sec(e+fx))^{m-1}}{(1-\sec^2(e+fx))^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \; 278$$

$$-\frac{(b\sec(e+fx))^m \text{Hypergeometric2F1}\left(3,\frac{m}{2},\frac{m+2}{2},\sec^2(e+fx)\right)}{fm}$$

input
```
Int[Cot[e + f*x]^5*(b*Sec[e + f*x])^m,x]
```

output

```
-((Hypergeometric2F1[3, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m
)/(f*m))
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [F]

$$\int \cot \left(fx+e\right)^{5} \left(b\sec \left(fx+e\right)\right)^{m} dx$$

input
```
int(cot(f*x+e)^5*(b*sec(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^5*(b*sec(f*x+e))^m,x)
```

## Fricas [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = \int \left(b\sec \left(fx+e\right)\right)^{m} \cot \left(fx+e\right)^{5}\,dx$$

input
```
integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

## Sympy [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec{(e+fx)})^m\cot^5{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**5*(b*sec(f*x+e))**m,x)
```

output
```
Integral((b*sec(e + f*x))**m*cot(e + f*x)**5, x)
```

## Maxima [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)})^m\cot{(fx+e)}^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

## Giac [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)})^m\cot{(fx+e)}^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx=\int \cot(e+f\,x)^5\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

input
```
int(cot(e + f*x)^5*(b/cos(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^5*(b/cos(e + f*x))^m, x)
```

## Reduce [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx=b^m\left(\int \sec\left(fx+e\right)^m\cot\left(fx+e\right)^5dx\right)$$

input
```
int(cot(f*x+e)^5*(b*sec(f*x+e))^m,x)
```

output
```
b**m*int(sec(e + f*x)**m*cot(e + f*x)**5,x)
```

# 3.357 $\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2565
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . 2565
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2566
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2567
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2567
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2567
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2568
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2568
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2568
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2569


## Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{\frac{5+m}{2}}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7}{2},\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^5(e+fx)}{5f}$$

output
```
1/5*(cos(f*x+e)^2)^(5/2+1/2*m)*hypergeom([5/2, 5/2+1/2*m],[7/2],sin(f*x+e)
^2)*(b*sec(f*x+e))^m*tan(f*x+e)^5/f
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{fm}$$

input
```
Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
(Cot[e + f*x]*Hypergeometric2F1[-3/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*S
ec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/(f*m)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4(b\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\tan^5(e+fx)\cos^2(e+fx)^{\frac{m+5}{2}}(b\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+5}{2},\frac{7}{2},\sin^2(e+fx)\right)}{5f}$$

input
```
Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
((Cos[e + f*x]^2)^((5 + m)/2)*Hypergeometric2F1[5/2, (5 + m)/2, 7/2, Sin[e
 + f*x]^2]*(b*Sec[e + f*x])^m*Tan[e + f*x]^5)/(5*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (b\sec (fx+e))^m \tan (fx+e)^4 \, dx$$

input
```
int((b*sec(f*x+e))^m*tan(f*x+e)^4,x)
```

output
```
int((b*sec(f*x+e))^m*tan(f*x+e)^4,x)
```

## Fricas [F]

$$\int (b\sec(e+fx))^m \tan^4(e+fx) \, dx = \int (b\sec (fx+e))^m \tan (fx+e)^4 \, dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

## Sympy [F]

$$\int (b\sec(e+fx))^m \tan^4(e+fx) \, dx = \int (b\sec (e+fx))^m \tan^4 (e+fx) \, dx$$

input
```
integrate((b*sec(f*x+e))**m*tan(f*x+e)**4,x)
```

output
```
Integral((b*sec(e + f*x))**m*tan(e + f*x)**4, x)
```

## Maxima [F]

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \int (b\sec(fx+e))^m \tan(fx+e)^4\,dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

## Giac [F]

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \int (b\sec(fx+e))^m \tan(fx+e)^4\,dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4 \left(\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(tan(e + f*x)^4*(b/cos(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^4*(b/cos(e + f*x))^m, x)
```

## Reduce [F]

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = b^m \left(\int \sec(fx+e)^m \tan(fx+e)^4\,dx\right)$$

input
```
int((b*sec(f*x+e))^m*tan(f*x+e)^4,x)
```

output
```
b**m*int(sec(e + f*x)**m*tan(e + f*x)**4,x)
```

# 3.358 $\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2570
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2570
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2571
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2572
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2572
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2572
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2573
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2573
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2573
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2574

## Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{\frac{3+m}{2}}\operatorname{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5}{2},\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^3(e+fx)}{3f}$$

output
```
1/3*(cos(f*x+e)^2)^(3/2+1/2*m)*hypergeom([3/2, 3/2+1/2*m],[5/2],sin(f*x+e)
^2)*(b*sec(f*x+e))^m*tan(f*x+e)^3/f
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx = \frac{\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\tan(e+fx)}{fm\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
(Hypergeometric2F1[-1/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^
m*Tan[e + f*x])/(f*m*Sqrt[-Tan[e + f*x]^2])
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(e+fx)^2(b\sec(e+fx))^m dx$$

$$\Big\downarrow \; 3097$$

$$\frac{\tan^3(e+fx)\cos^2(e+fx)^{\frac{m+3}{2}}(b\sec(e+fx))^m \text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

input
```
Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
((Cos[e + f*x]^2)^((3 + m)/2)*Hypergeometric2F1[3/2, (3 + m)/2, 5/2, Sin[e
 + f*x]^2]*(b*Sec[e + f*x])^m*Tan[e + f*x]^3)/(3*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (b\sec (fx+e))^m \tan (fx+e)^2 \, dx$$

input
```
int((b*sec(f*x+e))^m*tan(f*x+e)^2,x)
```

output
```
int((b*sec(f*x+e))^m*tan(f*x+e)^2,x)
```

## Fricas [F]

$$\int (b\sec(e+fx))^m \tan^2(e+fx) \, dx = \int (b\sec (fx+e))^m \tan (fx+e)^2 \, dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

## Sympy [F]

$$\int (b\sec(e+fx))^m \tan^2(e+fx) \, dx = \int (b\sec (e+fx))^m \tan^2 (e+fx) \, dx$$

input
```
integrate((b*sec(f*x+e))**m*tan(f*x+e)**2,x)
```

output
```
Integral((b*sec(e + f*x))**m*tan(e + f*x)**2, x)
```

## Maxima [F]

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx=\int (b\sec(fx+e))^m\tan(fx+e)^2\,dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

## Giac [F]

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx=\int (b\sec(fx+e))^m\tan(fx+e)^2\,dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx=\int \tan(e+f\,x)^2\left(\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(tan(e + f*x)^2*(b/cos(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^2*(b/cos(e + f*x))^m, x)
```

## Reduce [F]

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx = b^m\left(\int \sec(fx+e)^m\tan(fx+e)^2\,dx\right)$$

input
```
int((b*sec(f*x+e))^m*tan(f*x+e)^2,x)
```

output
```
b**m*int(sec(e + f*x)**m*tan(e + f*x)**2,x)
```

# 3.359 $\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2575
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2575
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2576
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2577
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2577
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2578
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2578
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2578
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2579
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2579

## Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-1+m)}\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1}{2},\sin^2(e+fx)\right)\left(b\sec(e+fx\right.}{f}$$

output
```
-(cos(f*x+e)^2)^(-1/2+1/2*m)*cot(f*x+e)*hypergeom([-1/2, -1/2+1/2*m],[1/2]
,sin(f*x+e)^2)*(b*sec(f*x+e))^m/f
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{fm}$$

input
```
Integrate[Cot[e + f*x]^2*(b*Sec[e + f*x])^m,x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[3/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*
Sec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/(f*m))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^m}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{3097}$$

$$-\frac{\cot(e+fx)\cos^2(e+fx)^{\frac{m-1}{2}}(b\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-1}{2},\frac{1}{2},\sin^2(e+fx)\right)}{f}$$

input
```
Int[Cot[e + f*x]^2*(b*Sec[e + f*x])^m,x]
```

output
```
-(((Cos[e + f*x]^2)^((-1 + m)/2)*Cot[e + f*x]*Hypergeometric2F1[-1/2, (-1
+ m)/2, 1/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \cot(fx+e)^2 (b\sec(fx+e))^m\, dx$$

input
```
int(cot(f*x+e)^2*(b*sec(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^2*(b*sec(f*x+e))^m,x)
```

## Fricas [F]

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\, dx = \int (b\sec(fx+e))^m \cot(fx+e)^2\, dx$$

input
```
integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

## Sympy [F]

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec{(e+fx)})^m\cot^2{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**2*(b*sec(f*x+e))**m,x)
```

output
```
Integral((b*sec(e + f*x))**m*cot(e + f*x)**2, x)
```

## Maxima [F]

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)})^m\cot{(fx+e)}^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

## Giac [F]

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)})^m\cot{(fx+e)}^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx=\int \cot(e+f\,x)^2\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

input
```
int(cot(e + f*x)^2*(b/cos(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^2*(b/cos(e + f*x))^m, x)
```

## Reduce [F]

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx=b^m\left(\int \sec\left(fx+e\right)^m\cot\left(fx+e\right)^2dx\right)$$

input
```
int(cot(f*x+e)^2*(b*sec(f*x+e))^m,x)
```

output
```
b**m*int(sec(e + f*x)**m*cot(e + f*x)**2,x)
```

# 3.360 $\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx$

Optimal result . . . . . 2580
Mathematica [A] (verified) . . . . . 2580
Rubi [A] (verified) . . . . . 2581
Maple [F] . . . . . 2582
Fricas [F] . . . . . 2582
Sympy [F] . . . . . 2583
Maxima [F] . . . . . 2583
Giac [F] . . . . . 2583
Mupad [F(-1)] . . . . . 2584
Reduce [F] . . . . . 2584

## Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-3+m)}\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),-\frac{1}{2},\sin^2(e+fx)\right)(b\sec(e+\ldots}{3f}$$

output
```
-1/3*(cos(f*x+e)^2)^(-3/2+1/2*m)*cot(f*x+e)^3*hypergeom([-3/2, -3/2+1/2*m]
,[-1/2],sin(f*x+e)^2)*(b*sec(f*x+e))^m/f
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = \frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{fm}$$

input
```
Integrate[Cot[e + f*x]^4*(b*Sec[e + f*x])^m,x]
```

output
```
(Cot[e + f*x]*Hypergeometric2F1[5/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Se
c[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/(f*m)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(b\sec(e+fx))^m}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{ 3097}$$

$$-\frac{\cot^3(e+fx)\cos^2(e+fx)^{\frac{m-3}{2}}(b\sec(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-3}{2},-\frac{1}{2},\sin^2(e+fx)\right)}{3f}$$

input
```
Int[Cot[e + f*x]^4*(b*Sec[e + f*x])^m,x]
```

output
```
-1/3*((Cos[e + f*x]^2)^((-3 + m)/2)*Cot[e + f*x]^3*Hypergeometric2F1[-3/2,
 (-3 + m)/2, -1/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \cot (fx+e)^4 (b\sec (fx+e))^m\,dx$$

input
```
int(cot(f*x+e)^4*(b*sec(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^4*(b*sec(f*x+e))^m,x)
```

## Fricas [F]

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec (fx+e))^m \cot (fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

## Sympy [F]

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(e+fx))^m \cot^4(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**4*(b*sec(f*x+e))**m,x)
```

output
```
Integral((b*sec(e + f*x))**m*cot(e + f*x)**4, x)
```

## Maxima [F]

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m \cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

## Giac [F]

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m \cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx=\int \cot(e+f\,x)^4\left(\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(cot(e + f*x)^4*(b/cos(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^4*(b/cos(e + f*x))^m, x)
```

## Reduce [F]

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx=b^m\left(\int \sec(fx+e)^m\cot(fx+e)^4\,dx\right)$$

input
```
int(cot(f*x+e)^4*(b*sec(f*x+e))^m,x)
```

output
```
b**m*int(sec(e + f*x)**m*cot(e + f*x)**4,x)
```

# 3.361 $\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx$

Optimal result . . . 2585
Mathematica [A] (verified) . . . 2585
Rubi [A] (verified) . . . 2586
Maple [F] . . . 2587
Fricas [F] . . . 2587
Sympy [F] . . . 2588
Maxima [F] . . . 2588
Giac [F] . . . 2588
Mupad [F(-1)] . . . 2589
Reduce [F] . . . 2589

## Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-5+m)}\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-5+m),-\frac{3}{2},\sin^2(e+fx)\right)(b\sec(e+}{5f}$$

output
```
-1/5*(cos(f*x+e)^2)^(-5/2+1/2*m)*cot(f*x+e)^5*hypergeom([-5/2, -5/2+1/2*m]
,[-3/2],sin(f*x+e)^2)*(b*sec(f*x+e))^m/f
```

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{fm}$$

input
```
Integrate[Cot[e + f*x]^6*(b*Sec[e + f*x])^m,x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[7/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*
Sec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/(f*m))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^m}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{3097}$$

$$-\frac{\cot^5(e+fx)\cos^2(e+fx)^{\frac{m-5}{2}}(b\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m-5}{2},-\frac{3}{2},\sin^2(e+fx)\right)}{5f}$$

input
```
Int[Cot[e + f*x]^6*(b*Sec[e + f*x])^m,x]
```

output
```
-1/5*((Cos[e + f*x]^2)^((-5 + m)/2)*Cot[e + f*x]^5*Hypergeometric2F1[-5/2,
 (-5 + m)/2, -3/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \cot(fx+e)^{6} (b\sec(fx+e))^{m} \, dx$$

input
```
int(cot(f*x+e)^6*(b*sec(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^6*(b*sec(f*x+e))^m,x)
```

## Fricas [F]

$$\int \cot^6(e+fx)(b\sec(e+fx))^m \, dx = \int (b\sec(fx+e))^{m} \cot(fx+e)^{6} \, dx$$

input
```
integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

## Sympy [F]

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(e+fx))^m \cot^6(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**6*(b*sec(f*x+e))**m,x)
```

output
```
Integral((b*sec(e + f*x))**m*cot(e + f*x)**6, x)
```

## Maxima [F]

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m \cot(fx+e)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

## Giac [F]

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m \cot(fx+e)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = \int \cot(e+f\,x)^6\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

input
```
int(cot(e + f*x)^6*(b/cos(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^6*(b/cos(e + f*x))^m, x)
```

## Reduce [F]

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = b^m\left(\int \sec\left(fx+e\right)^m\cot\left(fx+e\right)^6dx\right)$$

input
```
int(cot(f*x+e)^6*(b*sec(f*x+e))^m,x)
```

output
```
b**m*int(sec(e + f*x)**m*cot(e + f*x)**6,x)
```

# 3.362 $\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx$


Optimal result . . . . . . . . . . 2590
Mathematica [A] (verified) . . . . . . . . . . 2590
Rubi [A] (verified) . . . . . . . . . . 2591
Maple [F] . . . . . . . . . . 2592
Fricas [F] . . . . . . . . . . 2592
Sympy [F] . . . . . . . . . . 2593
Maxima [F] . . . . . . . . . . 2593
Giac [F] . . . . . . . . . . 2593
Mupad [F(-1)] . . . . . . . . . . 2594
Reduce [F] . . . . . . . . . . 2594


## Optimal result

Integrand size = 21, antiderivative size = 82

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx$$
$$= \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1+m+n),\frac{3+n}{2},\sin^2(e+fx)\right)(a\sec(e+fx))^m(b\tan}{bf(1+n)}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*m+1/2*n)*hypergeom([1/2+1/2*n, 1/2+1/2*m+1/2*n],[3
/2+1/2*n],sin(f*x+e)^2)*(a*sec(f*x+e))^m*(b*tan(f*x+e))^(1+n)/b/f/(1+n)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx$$
$$= \frac{b\,\text{Hypergeometric2F1}\left(\frac{m}{2},\frac{1-n}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(a\sec(e+fx))^m(b\tan(e+fx))^{-1+n}\left(-\tan^2(e+fx)\right.}{fm}$$

input
```
Integrate[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
(b*Hypergeometric2F1[m/2, (1 - n)/2, (2 + m)/2, Sec[e + f*x]^2]*(a*Sec[e +
 f*x])^m*(b*Tan[e + f*x])^(-1 + n)*(-Tan[e + f*x]^2)^((1 - n)/2))/(f*m)
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{(a\sec(e+fx))^m(b\tan(e+fx))^{n+1}\cos^2(e+fx)^{\frac{1}{2}(m+n+1)}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(m+n+1),\frac{n+3}{2},\sin^2(e-\right.}{bf(n+1)}$$

input
```
Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((Cos[e + f*x]^2)^((1 + m + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + m + n)
/2, (3 + n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(1 + n))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (a\sec(fx+e))^m (b\tan(fx+e))^n \,dx$$

input
```
int((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
int((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n \,dx = \int (a\sec(fx+e))^m (b\tan(fx+e))^n \,dx$$

input
```
integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Sympy [F]

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\sec{(e+fx)})^m\,(b\tan{(e+fx)})^n\,dx$$

input
```
integrate((a*sec(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((a*sec(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

## Maxima [F]

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\sec{(fx+e)})^m\,(b\tan{(fx+e)})^n\,dx$$

input
```
integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Giac [F]

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\sec{(fx+e)})^m\,(b\tan{(fx+e)})^n\,dx$$

input
```
integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx = \int (b\tan(e+f\,x))^n\left(\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

input
```
int((b*tan(e + f*x))^n*(a/cos(e + f*x))^m,x)
```

output
```
int((b*tan(e + f*x))^n*(a/cos(e + f*x))^m, x)
```

## Reduce [F]

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx = b^n a^m\left(\int \tan\left(fx+e\right)^n\sec\left(fx+e\right)^m dx\right)$$

input
```
int((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
b**n*a**m*int(tan(e + f*x)**n*sec(e + f*x)**m,x)
```

# 3.363 $\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2595
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2595
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2596
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2597
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 2598
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2598
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 2598
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2599
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2599
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2600

## Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx=\frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}+\frac{2(d\tan(a+bx))^{3+n}}{bd^3(3+n)}+\frac{(d\tan(a+bx))^{5+n}}{bd^5(5+n)}$$

output
```
(d*tan(b*x+a))^(1+n)/b/d/(1+n)+2*(d*tan(b*x+a))^(3+n)/b/d^3/(3+n)+(d*tan(b
*x+a))^(5+n)/b/d^5/(5+n)
```

## Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx$$
$$=\frac{d(d\tan(a+bx))^{-1+n}\left((8+6n+n^2+2(3+n)\cos(2(a+bx))+\cos(4(a+bx)))\sec^4(a+bx)\tan^2(a+b\right.}{b(1+n)(3+n)(5+n)}$$

input
```
Integrate[Sec[a + b*x]^6*(d*Tan[a + b*x])^n,x]
```

output
```
(d*(d*Tan[a + b*x])^(-1 + n)*((8 + 6*n + n^2 + 2*(3 + n)*Cos[2*(a + b*x)]
+ Cos[4*(a + b*x)])*Sec[a + b*x]^4*Tan[a + b*x]^2 + 8*(-Tan[a + b*x]^2)^((
1 - n)/2)))/(b*(1 + n)*(3 + n)*(5 + n))
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^6(d\tan(a+bx))^n dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (d\tan(a+bx))^n\left(\tan^2(a+bx)+1\right)^2 d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left((d\tan(a+bx))^n+\frac{2(d\tan(a+bx))^{n+2}}{d^2}+\frac{(d\tan(a+bx))^{n+4}}{d^4}\right) d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(d\tan(a+bx))^{n+5}}{d^5(n+5)}+\frac{2(d\tan(a+bx))^{n+3}}{d^3(n+3)}+\frac{(d\tan(a+bx))^{n+1}}{d(n+1)}}{b}$$

input
```
Int[Sec[a + b*x]^6*(d*Tan[a + b*x])^n,x]
```

output
```
((d*Tan[a + b*x])^(1 + n)/(d*(1 + n)) + (2*(d*Tan[a + b*x])^(3 + n))/(d^3*
(3 + n)) + (d*Tan[a + b*x])^(5 + n)/(d^5*(5 + n)))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\frac{\tan\left(bx+a\right) \mathrm{e}^{n\ln\left(d\tan\left(bx+a\right)\right)}}{b\left(1+n\right)}+\frac{\tan\left(bx+a\right)^{5} \mathrm{e}^{n\ln\left(d\tan\left(bx+a\right)\right)}}{b\left(5+n\right)}+\frac{2\tan\left(bx+a\right)^{3} \mathrm{e}^{n\ln\left(d\tan\left(bx+a\right)\right)}}{b\left(3+n\right)}$$

input
```
int(sec(b*x+a)^6*(d*tan(b*x+a))^n,x)
```

output
```
1/b/(1+n)*tan(b*x+a)*exp(n*ln(d*tan(b*x+a)))+1/b/(5+n)*tan(b*x+a)^5*exp(n*
ln(d*tan(b*x+a)))+2/b/(3+n)*tan(b*x+a)^3*exp(n*ln(d*tan(b*x+a)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \frac{\left(8\cos\left(bx+a\right)^4+4\left(n+1\right)\cos\left(bx+a\right)^2+n^2+4\,n+3\right)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^n\sin\left(bx+a\right)}{\left(bn^3+9\,bn^2+23\,bn+15\,b\right)\cos\left(bx+a\right)^5}$$

input
```
integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
(8*cos(b*x + a)^4 + 4*(n + 1)*cos(b*x + a)^2 + n^2 + 4*n + 3)*(d*sin(b*x +
 a)/cos(b*x + a))^n*sin(b*x + a)/((b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*cos(b*
x + a)^5)
```

## Sympy [F]

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \int \left(d\tan\left(a+bx\right)\right)^n\sec^6\left(a+bx\right)\,dx$$

input
```
integrate(sec(b*x+a)**6*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*sec(a + b*x)**6, x)
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \frac{\frac{d^n\tan(bx+a)^n\tan(bx+a)^5}{n+5}+\frac{2\,d^n\tan(bx+a)^n\tan(bx+a)^3}{n+3}+\frac{(d\tan(bx+a))^{n+1}}{d(n+1)}}{b}$$

input
```
integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
(d^n*tan(b*x + a)^n*tan(b*x + a)^5/(n + 5) + 2*d^n*tan(b*x + a)^n*tan(b*x
+ a)^3/(n + 3) + (d*tan(b*x + a))^(n + 1)/(d*(n + 1)))/b
```

## Giac [A] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \frac{\frac{(d\tan(bx+a))^n d^5\tan(bx+a)^5}{d^4n+5\,d^4} + \frac{2\,(d\tan(bx+a))^n d^3\tan(bx+a)^3}{d^2n+3\,d^2} + \frac{(d\tan(bx+a))^{n+1}}{n+1}}{bd}$$

input
```
integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
((d*tan(b*x + a))^n*d^5*tan(b*x + a)^5/(d^4*n + 5*d^4) + 2*(d*tan(b*x + a)
)^n*d^3*tan(b*x + a)^3/(d^2*n + 3*d^2) + (d*tan(b*x + a))^(n + 1)/(n + 1))
/(b*d)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \int \frac{{\left(d\,\tan\left(a+b\,x\right)\right)}^n}{{\cos\left(a+b\,x\right)}^6}\,dx$$

input
```
int((d*tan(a + b*x))^n/cos(a + b*x)^6,x)
```

output
```
int((d*tan(a + b*x))^n/cos(a + b*x)^6, x)
```

## Reduce [F]

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = d^n\left(\int \tan(bx+a)^n \sec(bx+a)^6\,dx\right)$$

input
```
int(sec(b*x+a)^6*(d*tan(b*x+a))^n,x)
```

output
```
d**n*int(tan(a + b*x)**n*sec(a + b*x)**6,x)
```

# 3.364 $\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx$


Optimal result . . . 2601
Mathematica [A] (verified) . . . 2601
Rubi [A] (verified) . . . 2602
Maple [A] (verified) . . . 2603
Fricas [A] (verification not implemented) . . . 2604
Sympy [F] . . . 2604
Maxima [A] (verification not implemented) . . . 2604
Giac [A] (verification not implemented) . . . 2605
Mupad [B] (verification not implemented) . . . 2605
Reduce [F] . . . 2606


## Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx=\frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}+\frac{(d\tan(a+bx))^{3+n}}{bd^3(3+n)}$$

output
```
(d*tan(b*x+a))^(1+n)/b/d/(1+n)+(d*tan(b*x+a))^(3+n)/b/d^3/(3+n)
```

## Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{d(d\tan(a+bx))^{-1+n}\left((2+n+\cos(2(a+bx)))\sec^2(a+bx)\tan^2(a+bx)+2(-\tan^2(a+bx))^{\frac{1-n}{2}}\right)}{b(1+n)(3+n)}$$

input
```
Integrate[Sec[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

output
```
(d*(d*Tan[a + b*x])^(-1 + n)*((2 + n + Cos[2*(a + b*x)])*Sec[a + b*x]^2*Ta
n[a + b*x]^2 + 2*(-Tan[a + b*x]^2)^((1 - n)/2)))/(b*(1 + n)*(3 + n))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^4(d\tan(a+bx))^n dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (d\tan(a+bx))^n\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left((d\tan(a+bx))^n+\frac{(d\tan(a+bx))^{n+2}}{d^2}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(d\tan(a+bx))^{n+3}}{d^3(n+3)}+\frac{(d\tan(a+bx))^{n+1}}{d(n+1)}}{b}$$

input
```
Int[Sec[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

output
```
((d*Tan[a + b*x])^(1 + n)/(d*(1 + n)) + (d*Tan[a + b*x])^(3 + n)/(d^3*(3 +
 n)))/b
```

### Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 56.52 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\tan(bx+a)\,e^{n\ln(d\tan(bx+a))}}{b(1+n)}+\frac{\tan(bx+a)^{3}e^{n\ln(d\tan(bx+a))}}{b(3+n)}$ | 58 |
| default | $\frac{\tan(bx+a)\,e^{n\ln(d\tan(bx+a))}}{b(1+n)}+\frac{\tan(bx+a)^{3}e^{n\ln(d\tan(bx+a))}}{b(3+n)}$ | 58 |
| risch | Expression too large to display | 5283 |

input
```
int(sec(b*x+a)^4*(d*tan(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
1/b/(1+n)*tan(b*x+a)*exp(n*ln(d*tan(b*x+a)))+1/b/(3+n)*tan(b*x+a)^3*exp(n*
ln(d*tan(b*x+a)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\left(2\cos\left(bx+a\right)^2+n+1\right)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^n\sin\left(bx+a\right)}{\left(bn^2+4\,bn+3\,b\right)\cos\left(bx+a\right)^3}$$

input
```
integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
(2*cos(b*x + a)^2 + n + 1)*(d*sin(b*x + a)/cos(b*x + a))^n*sin(b*x + a)/((
b*n^2 + 4*b*n + 3*b)*cos(b*x + a)^3)
```

## Sympy [F]

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n\sec^4(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**4*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*sec(a + b*x)**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\frac{d^n\tan(bx+a)^n\tan(bx+a)^3}{n+3}+\frac{(d\tan(bx+a))^{n+1}}{d(n+1)}}{b}$$

input
```
integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
(d^n*tan(b*x + a)^n*tan(b*x + a)^3/(n + 3) + (d*tan(b*x + a))^(n + 1)/(d*(
n + 1)))/b
```

## Giac [A] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\frac{(d\tan(bx+a))^n d^3\tan(bx+a)^3}{d^2n+3\,d^2} + \frac{(d\tan(bx+a))^{n+1}}{n+1}}{bd}$$

input
```
integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
((d*tan(b*x + a))^n*d^3*tan(b*x + a)^3/(d^2*n + 3*d^2) + (d*tan(b*x + a))^
(n + 1)/(n + 1))/(b*d)
```

## Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx =$$

$$-\frac{2\left(-\frac{d\,\sin(2\,a+2\,b\,x)}{2\sin(a+b\,x)^2-2}\right)^n (9\,\sin(2\,a+2\,b\,x)+6\,\sin(4\,a+4\,b\,x)+\sin(6\,a+6\,b\,x)+4\,n\,\sin(2\,a+2\,b\,x)+}{b\,(n^2+4\,n+3)\,\left(30\sin(a+b\,x)^2+12\sin(2\,a+2\,b\,x)^2+2\sin(3\,a+3\,b\,x)^2-32\right)}$$

input
```
int((d*tan(a + b*x))^n/cos(a + b*x)^4,x)
```

output
```
-(2*(-(d*sin(2*a + 2*b*x))/(2*sin(a + b*x)^2 - 2))^n*(9*sin(2*a + 2*b*x) +
 6*sin(4*a + 4*b*x) + sin(6*a + 6*b*x) + 4*n*sin(2*a + 2*b*x) + 2*n*sin(4*
a + 4*b*x)))/(b*(4*n + n^2 + 3)*(12*sin(2*a + 2*b*x)^2 + 2*sin(3*a + 3*b*x
)^2 + 30*sin(a + b*x)^2 - 32))
```

## Reduce [F]

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = d^n\left(\int \tan(bx+a)^n \sec(bx+a)^4\,dx\right)$$

input
```
int(sec(b*x+a)^4*(d*tan(b*x+a))^n,x)
```

output
```
d**n*int(tan(a + b*x)**n*sec(a + b*x)**4,x)
```

# 3.365 $\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx$


Optimal result . . . 2607
Mathematica [A] (verified) . . . 2607
Rubi [A] (verified) . . . 2608
Maple [A] (verified) . . . 2609
Fricas [A] (verification not implemented) . . . 2609
Sympy [F] . . . 2610
Maxima [A] (verification not implemented) . . . 2610
Giac [A] (verification not implemented) . . . 2610
Mupad [B] (verification not implemented) . . . 2611
Reduce [F] . . . 2611


## Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx=\frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx=\frac{\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

input
```
Integrate[Sec[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

output
```
(Tan[a + b*x]*(d*Tan[a + b*x])^n)/(b*(1 + n))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^2(d\tan(a+bx))^n dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (d\tan(a+bx))^n d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

input
```
Int[Sec[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

output
```
(d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n))
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{(d\tan(bx+a))^{1+n}}{bd(1+n)}$ | 25 |
| default | $\frac{(d\tan(bx+a))^{1+n}}{bd(1+n)}$ | 25 |
| risch | Expression too large to display | 1752 |

input
```
int(sec(b*x+a)^2*(d*tan(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^n\sin(bx+a)}{(bn+b)\cos(bx+a)}$$

input
```
integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
(d*sin(b*x + a)/cos(b*x + a))^n*sin(b*x + a)/((b*n + b)*cos(b*x + a))
```

## Sympy [F]

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(a+bx)})^n \sec^2{(a+bx)}\,dx$$

input
```
integrate(sec(b*x+a)**2*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*sec(a + b*x)**2, x)
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{(d\tan{(bx+a)})^{n+1}}{bd(n+1)}$$

input
```
integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
(d*tan(b*x + a))^(n + 1)/(b*d*(n + 1))
```

## Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{(d\tan{(bx+a)})^{n+1}}{bd(n+1)}$$

input
```
integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
(d*tan(b*x + a))^(n + 1)/(b*d*(n + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\sin\left(2\,a+2\,b\,x\right)\,{\left(\frac{d\,\sin\left(2\,a+2\,b\,x\right)}{2\,{\cos\left(a+b\,x\right)}^2}\right)}^n}{2\,b\,{\cos\left(a+b\,x\right)}^2\,\left(n+1\right)}$$

input
```
int((d*tan(a + b*x))^n/cos(a + b*x)^2,x)
```

output
```
(sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(2*cos(a + b*x)^2))^n)/(2*b*cos(a
+ b*x)^2*(n + 1))
```

## Reduce [F]

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = d^n\left(\int \tan\left(bx+a\right)^n\sec\left(bx+a\right)^2dx\right)$$

input
```
int(sec(b*x+a)^2*(d*tan(b*x+a))^n,x)
```

output
```
d**n*int(tan(a + b*x)**n*sec(a + b*x)**2,x)
```

# 3.366 $\int (d\tan(a+bx))^n\,dx$

Optimal result . . . . . 2612
Mathematica [A] (verified) . . . . . 2612
Rubi [A] (verified) . . . . . 2613
Maple [F] . . . . . 2614
Fricas [F] . . . . . 2614
Sympy [F] . . . . . 2615
Maxima [F] . . . . . 2615
Giac [F] . . . . . 2615
Mupad [F(-1)] . . . . . 2616
Reduce [F] . . . . . 2616

## Optimal result

Integrand size = 10, antiderivative size = 50

$$\int (d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b
/d/(1+n)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int (d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

input
```
Integrate[(d*Tan[a + b*x])^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*Tan[a + b*x]*
(d*Tan[a + b*x])^n)/(b*(1 + n))
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (d\tan(a+bx))^n dx$$

$$\Big\downarrow \; 3957$$

$$\frac{d\int \frac{(d\tan(a+bx))^n}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \; 278$$

$$\frac{(d\tan(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[(d*Tan[a + b*x])^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*
x])^(1 + n))/(b*d*(1 + n))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

## Maple [F]

$$\int (d\tan (bx+a))^n\,dx$$

input
```
int((d*tan(b*x+a))^n,x)
```

output
```
int((d*tan(b*x+a))^n,x)
```

## Fricas [F]

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan (bx+a))^n\,dx$$

input
```
integrate((d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n, x)
```

## Sympy [F]

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan{(a+bx)})^n\,dx$$

input
```
integrate((d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n, x)
```

## Maxima [F]

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan{(bx+a)})^n\,dx$$

input
```
integrate((d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n, x)
```

## Giac [F]

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan{(bx+a)})^n\,dx$$

input
```
integrate((d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan(a+b\,x))^n\,dx$$

input
```
int((d*tan(a + b*x))^n,x)
```

output
```
int((d*tan(a + b*x))^n, x)
```

## Reduce [F]

$$\int (d\tan(a+bx))^n\,dx = d^n\left(\int \tan(bx+a)^n\,dx\right)$$

input
```
int((d*tan(b*x+a))^n,x)
```

output
```
d**n*int(tan(a + b*x)**n,x)
```

# 3.367 $\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2617
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2617
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2618
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2619
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2619
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2620
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2620
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2620
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2621
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2621

## Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
hypergeom([2, 1/2+1/2*n],[3/2+1/2*n],-tan(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b
/d/(1+n)
```

## Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

input
```
Integrate[Cos[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

output
```
(Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*Tan[a + b*x]*
(d*Tan[a + b*x])^n)/(b*(1 + n))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3087, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^n}{\sec(a+bx)^2}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \frac{(d\tan(a+bx))^n}{(\tan^2(a+bx)+1)^2}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{278}$$

$$\frac{(d\tan(a+bx))^{n+1}\text{Hypergeometric2F1}\left(2,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Cos[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

output
```
(Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*
x])^(1 + n))/(b*d*(1 + n))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [F]

$$\int \cos \left(bx+a\right)^{2} \left(d\tan \left(bx+a\right)\right)^{n} dx$$

input
```
int(cos(b*x+a)^2*(d*tan(b*x+a))^n,x)
```

output
```
int(cos(b*x+a)^2*(d*tan(b*x+a))^n,x)
```

## Fricas [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int \left(d\tan \left(bx+a\right)\right)^{n} \cos \left(bx+a\right)^{2}\,dx$$

input
```
integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

## Sympy [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n \cos^2(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)**2*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*cos(a + b*x)**2, x)
```

## Maxima [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \cos(bx+a)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

## Giac [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \cos(bx+a)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int \cos\left(a+b\,x\right)^2\,\left(d\tan(a+b\,x)\right)^n\,dx$$

input
```
int(cos(a + b*x)^2*(d*tan(a + b*x))^n,x)
```

output
```
int(cos(a + b*x)^2*(d*tan(a + b*x))^n, x)
```

## Reduce [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = d^n\left(\int \tan\left(bx+a\right)^n\cos\left(bx+a\right)^2dx\right)$$

input
```
int(cos(b*x+a)^2*(d*tan(b*x+a))^n,x)
```

output
```
d**n*int(tan(a + b*x)**n*cos(a + b*x)**2,x)
```

# 3.368 $\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2622
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . 2622
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2623
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2624
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2624
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2625
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2625
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2625
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2626
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2626

## Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(3,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
hypergeom([3, 1/2+1/2*n],[3/2+1/2*n],-tan(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b
/d/(1+n)
```

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(3,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

input
```
Integrate[Cos[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

output
```
(Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*Tan[a + b*x]*
(d*Tan[a + b*x])^n)/(b*(1 + n))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3087, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^n}{\sec(a+bx)^4}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \frac{(d\tan(a+bx))^n}{(\tan^2(a+bx)+1)^3}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{278}$$

$$\frac{(d\tan(a+bx))^{n+1}\text{Hypergeometric2F1}\left(3,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Cos[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

output
```
(Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*
x])^(1 + n))/(b*d*(1 + n))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

## Maple [F]

$$\int \cos\left(bx+a\right)^{4} \left(d\tan\left(bx+a\right)\right)^{n} dx$$

input
```
int(cos(b*x+a)^4*(d*tan(b*x+a))^n,x)
```

output
```
int(cos(b*x+a)^4*(d*tan(b*x+a))^n,x)
```

## Fricas [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \int \left(d\tan\left(bx+a\right)\right)^{n} \cos\left(bx+a\right)^{4}\,dx$$

input
```
integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

## Sympy [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n \cos^4(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)**4*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*cos(a + b*x)**4, x)
```

## Maxima [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \cos(bx+a)^4\,dx$$

input
```
integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

## Giac [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \cos(bx+a)^4\,dx$$

input
```
integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \int \cos\left(a+b\,x\right)^4\,\left(d\tan(a+b\,x)\right)^n\,dx$$

input
```
int(cos(a + b*x)^4*(d*tan(a + b*x))^n,x)
```

output
```
int(cos(a + b*x)^4*(d*tan(a + b*x))^n, x)
```

## Reduce [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = d^n\left(\int \tan\left(bx+a\right)^n\cos\left(bx+a\right)^4dx\right)$$

input
```
int(cos(b*x+a)^4*(d*tan(b*x+a))^n,x)
```

output
```
d**n*int(tan(a + b*x)**n*cos(a + b*x)**4,x)
```

# 3.369 $\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result . . . . . 2627
Mathematica [A] (verified) . . . . . 2627
Rubi [A] (verified) . . . . . 2628
Maple [F] . . . . . 2629
Fricas [F] . . . . . 2629
Sympy [F] . . . . . 2630
Maxima [F] . . . . . 2630
Giac [F] . . . . . 2630
Mupad [F(-1)] . . . . . 2631
Reduce [F] . . . . . 2631

## Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{6+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{6+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec^5(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
(cos(b*x+a)^2)^(3+1/2*n)*hypergeom([3+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(b*
x+a)^2)*sec(b*x+a)^5*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \frac{d\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-n}{2},\frac{7}{2},\sec^2(a+bx)\right)\sec^5(a+bx)(d\tan(a+bx))^{-1+n}\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}}{5b}$$

input
```
Integrate[Sec[a + b*x]^5*(d*Tan[a + b*x])^n,x]
```

output
```
(d*Hypergeometric2F1[5/2, (1 - n)/2, 7/2, Sec[a + b*x]^2]*Sec[a + b*x]^5*(
d*Tan[a + b*x])^(-1 + n)*(-Tan[a + b*x]^2)^((1 - n)/2))/(5*b)
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(a+bx)^5(d\tan(a+bx))^n dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{\sec^5(a+bx)\cos^2(a+bx)^{\frac{n+6}{2}}(d\tan(a+bx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+6}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Sec[a + b*x]^5*(d*Tan[a + b*x])^n,x]
```

output
```
((Cos[a + b*x]^2)^((6 + n)/2)*Hypergeometric2F1[(1 + n)/2, (6 + n)/2, (3 +
 n)/2, Sin[a + b*x]^2]*Sec[a + b*x]^5*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 +
n))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \sec (bx+a)^5 (d\tan (bx+a))^n \, dx$$

input
```
int(sec(b*x+a)^5*(d*tan(b*x+a))^n,x)
```

output
```
int(sec(b*x+a)^5*(d*tan(b*x+a))^n,x)
```

## Fricas [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n \, dx = \int (d\tan (bx+a))^n \sec (bx+a)^5 \, dx$$

input
```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

## Sympy [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n \sec^5(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**5*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*sec(a + b*x)**5, x)
```

## Maxima [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \sec(bx+a)^5\,dx$$

input
```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

## Giac [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \sec(bx+a)^5\,dx$$

input
```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx=\int \frac{(d\tan(a+b\,x))^n}{\cos\left(a+b\,x\right)^5}\,dx$$

input
```
int((d*tan(a + b*x))^n/cos(a + b*x)^5,x)
```

output
```
int((d*tan(a + b*x))^n/cos(a + b*x)^5, x)
```

## Reduce [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx=d^n\left(\int \tan\left(bx+a\right)^n\sec\left(bx+a\right)^5dx\right)$$

input
```
int(sec(b*x+a)^5*(d*tan(b*x+a))^n,x)
```

output
```
d**n*int(tan(a + b*x)**n*sec(a + b*x)**5,x)
```

# 3.370 $\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx$


Optimal result . . . 2632
Mathematica [A] (verified) . . . 2632
Rubi [A] (verified) . . . 2633
Maple [F] . . . 2634
Fricas [F] . . . 2634
Sympy [F] . . . 2635
Maxima [F] . . . 2635
Giac [F] . . . 2635
Mupad [F(-1)] . . . 2636
Reduce [F] . . . 2636


## Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{4+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{4+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec^3(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
(cos(b*x+a)^2)^(2+1/2*n)*hypergeom([2+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(b*
x+a)^2)*sec(b*x+a)^3*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{d\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-n}{2},\frac{5}{2},\sec^2(a+bx)\right)\sec^3(a+bx)(d\tan(a+bx))^{-1+n}\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}}{3b}$$

input
```
Integrate[Sec[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

output
```
(d*Hypergeometric2F1[3/2, (1 - n)/2, 5/2, Sec[a + b*x]^2]*Sec[a + b*x]^3*(
d*Tan[a + b*x])^(-1 + n)*(-Tan[a + b*x]^2)^((1 - n)/2))/(3*b)
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^3(d\tan(a+bx))^n dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\sec^3(a+bx)\cos^2(a+bx)^{\frac{n+4}{2}}(d\tan(a+bx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+4}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Sec[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

output
```
((Cos[a + b*x]^2)^((4 + n)/2)*Hypergeometric2F1[(1 + n)/2, (4 + n)/2, (3 +
 n)/2, Sin[a + b*x]^2]*Sec[a + b*x]^3*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 +
n))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \sec (bx+a)^3 (d\tan (bx+a))^n \, dx$$

input
```
int(sec(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

output
```
int(sec(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

## Fricas [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n \, dx = \int (d\tan (bx+a))^n \sec (bx+a)^3 \, dx$$

input
```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

## Sympy [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n \sec^3(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**3*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*sec(a + b*x)**3, x)
```

## Maxima [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \sec(bx+a)^3\,dx$$

input
```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

## Giac [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \sec(bx+a)^3\,dx$$

input
```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \int \frac{(d\tan(a+b\,x))^n}{\cos\left(a+b\,x\right)^3}\,dx$$

input
```
int((d*tan(a + b*x))^n/cos(a + b*x)^3,x)
```

output
```
int((d*tan(a + b*x))^n/cos(a + b*x)^3, x)
```

## Reduce [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = d^n\left(\int \tan\left(bx+a\right)^n \sec\left(bx+a\right)^3 dx\right)$$

input
```
int(sec(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

output
```
d**n*int(tan(a + b*x)**n*sec(a + b*x)**3,x)
```

# 3.371 $\int \sec(a+bx)(d\tan(a+bx))^n\,dx$


Optimal result . . . . . . . . . . 2637
Mathematica [A] (verified) . . . . . . . . . . 2637
Rubi [A] (verified) . . . . . . . . . . 2638
Maple [F] . . . . . . . . . . 2639
Fricas [F] . . . . . . . . . . 2639
Sympy [F] . . . . . . . . . . 2640
Maxima [F] . . . . . . . . . . 2640
Giac [F] . . . . . . . . . . 2640
Mupad [F(-1)] . . . . . . . . . . 2641
Reduce [F] . . . . . . . . . . 2641


## Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{2+n}{2}}\operatorname{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
(cos(b*x+a)^2)^(1+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(b*
x+a)^2)*sec(b*x+a)*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \frac{\csc(a+bx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3}{2},\sec^2(a+bx)\right)(d\tan(a+bx))^n\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}}{b}$$

input
```
Integrate[Sec[a + b*x]*(d*Tan[a + b*x])^n,x]
```

output
```
(Csc[a + b*x]*Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Sec[a + b*x]^2]*(d*Ta
n[a + b*x])^n*(-Tan[a + b*x]^2)^((1 - n)/2))/b
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)(d\tan(a+bx))^n dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\sec(a+bx)\cos^2(a+bx)^{\frac{n+2}{2}}(d\tan(a+bx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Sec[a + b*x]*(d*Tan[a + b*x])^n,x]
```

output
```
((Cos[a + b*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (3 +
 n)/2, Sin[a + b*x]^2]*Sec[a + b*x]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n)
)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \sec (bx + a) (d \tan (bx + a))^n \, dx$$

input
```
int(sec(b*x+a)*(d*tan(b*x+a))^n,x)
```

output
```
int(sec(b*x+a)*(d*tan(b*x+a))^n,x)
```

## Fricas [F]

$$\int \sec(a + bx)(d \tan(a + bx))^n \, dx = \int (d \tan (bx + a))^n \sec (bx + a) \, dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*sec(b*x + a), x)
```

## Sympy [F]

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(a+bx)})^n \sec{(a+bx)}\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*sec(a + b*x), x)
```

## Maxima [F]

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(bx+a)})^n \sec{(bx+a)}\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*sec(b*x + a), x)
```

## Giac [F]

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(bx+a)})^n \sec{(bx+a)}\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*sec(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int \frac{(d\tan(a+b\,x))^n}{\cos(a+b\,x)}\,dx$$

input
```
int((d*tan(a + b*x))^n/cos(a + b*x),x)
```

output
```
int((d*tan(a + b*x))^n/cos(a + b*x), x)
```

## Reduce [F]

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = d^n\left(\int \tan(bx+a)^n\sec(bx+a)\,dx\right)$$

input
```
int(sec(b*x+a)*(d*tan(b*x+a))^n,x)
```

output
```
d**n*int(tan(a + b*x)**n*sec(a + b*x),x)
```

# 3.372 $\int \cos(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2642
Mathematica [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . 2642
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2643
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2644
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2644
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2645
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2645
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2645
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2646
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2646

## Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos(a+bx)\cos^2(a+bx)^{n/2}\,\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
cos(b*x+a)*(cos(b*x+a)^2)^(1/2*n)*hypergeom([1/2*n, 1/2+1/2*n],[3/2+1/2*n]
,sin(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.99 (sec) , antiderivative size = 452, normalized size of antiderivative = 6.28

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx =$$

$$-\frac{2\left(\text{AppellF1}\left(\frac{1+n}{2},n\right.\right.}{b(1+n)\left(-\text{AppellF1}\left(\frac{1+n}{2},n,1,\frac{3+n}{2},\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+\frac{\left(-\left(\left(\text{AppellF1}\left(\frac{3+n}{2},n,2,\frac{5+n}{2},\text{t}\right.\right.\right.\right.}{}\right.}$$

input
```
Integrate[Cos[a + b*x]*(d*Tan[a + b*x])^n,x]
```

output
```
(-2*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*
x)/2]^2] - 2*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan
[(a + b*x)/2]^2])*Cos[(a + b*x)/2]*Cos[a + b*x]*Sin[(a + b*x)/2]*(d*Tan[a
+ b*x])^n)/(b*(1 + n)*(-AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)
/2]^2, -Tan[(a + b*x)/2]^2] + ((-((AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Ta
n[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[(3 + n)/2, n, 3, (5 +
n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 +
 n, 1, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*n*AppellF1[
(3 + n)/2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*
(-1 + Cos[a + b*x])) + (3 + n)*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x]))*Sec[(a + b*x)/2]^2)
/(3 + n)))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^n}{\sec(a+bx)}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos(a+bx)\cos^2(a+bx)^{n/2}(d\tan(a+bx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Cos[a + b*x]*(d*Tan[a + b*x])^n,x]
```

output
```
(Cos[a + b*x]*(Cos[a + b*x]^2)^(n/2)*Hypergeometric2F1[n/2, (1 + n)/2, (3
+ n)/2, Sin[a + b*x]^2]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n))
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \cos (bx + a) (d \tan (bx + a))^n \, dx$$

input
```
int(cos(b*x+a)*(d*tan(b*x+a))^n,x)
```

output
```
int(cos(b*x+a)*(d*tan(b*x+a))^n,x)
```

## Fricas [F]

$$\int \cos(a + bx)(d \tan(a + bx))^n \, dx = \int (d \tan (bx + a))^n \cos (bx + a) \, dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*cos(b*x + a), x)
```

## Sympy [F]

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n \cos(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*cos(a + b*x), x)
```

## Maxima [F]

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \cos(bx+a)\,dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a), x)
```

## Giac [F]

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \cos(bx+a)\,dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int \cos\left(a+b\,x\right)\,\left(d\tan(a+b\,x)\right)^n\,dx$$

input
```
int(cos(a + b*x)*(d*tan(a + b*x))^n,x)
```

output
```
int(cos(a + b*x)*(d*tan(a + b*x))^n, x)
```

## Reduce [F]

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = d^n\left(\int \tan\left(bx+a\right)^n \cos\left(bx+a\right)dx\right)$$

input
```
int(cos(b*x+a)*(d*tan(b*x+a))^n,x)
```

output
```
d**n*int(tan(a + b*x)**n*cos(a + b*x),x)
```

# 3.373 $\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2647
Mathematica [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . 2647
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2648
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2649
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2650
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2650
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2650
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2651
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2651
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2651

## Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-2+n),\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(d\tan(a+bx}{bd(1+n)}$$

output
```
cos(b*x+a)^3*(cos(b*x+a)^2)^(-1+1/2*n)*hypergeom([-1+1/2*n, 1/2+1/2*n],[3/
2+1/2*n],sin(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 4.79 (sec) , antiderivative size = 1313, normalized size of antiderivative = 16.83

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

output
```
(4*(3 + n)*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[
(a + b*x)/2]^2] - 6*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2] + 12*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a +
 b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 8*AppellF1[(1 + n)/2, n, 4, (3 + n)/2,
Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^3*Cos[a + b*x]^
3*Sin[(a + b*x)/2]*(d*Tan[a + b*x])^n)/(b*(1 + n)*((3 + n)*AppellF1[(1 + n
)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a
+ b*x]) - 2*(AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan
[(a + b*x)/2]^2] - 12*AppellF1[(3 + n)/2, n, 3, (5 + n)/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2] + 36*AppellF1[(3 + n)/2, n, 4, (5 + n)/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 32*AppellF1[(3 + n)/2, n, 5, (5 + n)/
2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 + n,
 1, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 6*n*AppellF1[(3
+ n)/2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12
*n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2] + 8*n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] + 18*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a
+ b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 6*n*AppellF1[(1 + n
)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*
x)/2]^2 + 8*(3 + n)*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[(a + b*x)/...
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^n}{\sec(a+bx)^3}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{n-2}{2}}(d\tan(a+bx))^{n+1}\operatorname{Hypergeometric2F1}\left(\frac{n-2}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Cos[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

output
```
(Cos[a + b*x]^3*(Cos[a + b*x]^2)^((-2 + n)/2)*Hypergeometric2F1[(-2 + n)/2
, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1
+ n))
```

**Defintions of rubi rules used**

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \cos(bx+a)^3 (d\tan(bx+a))^n \, dx$$

input
```
int(cos(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

output
```
int(cos(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

## Fricas [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*(d*tan(b*x+a))**n,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

## Giac [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \int \cos(a+b\,x)^3\,(d\tan(a+b\,x))^n\,dx$$

input
```
int(cos(a + b*x)^3*(d*tan(a + b*x))^n,x)
```

output
```
int(cos(a + b*x)^3*(d*tan(a + b*x))^n, x)
```

## Reduce [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = d^n\left(\int \tan(bx+a)^n \cos(bx+a)^3\,dx\right)$$

input
```
int(cos(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

output
```
d**n*int(tan(a + b*x)**n*cos(a + b*x)**3,x)
```

# 3.374 $\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx$

Optimal result . . . 2652
Mathematica [A] (verified) . . . 2652
Rubi [A] (verified) . . . 2653
Maple [F] . . . 2654
Fricas [F] . . . 2654
Sympy [F] . . . 2655
Maxima [F] . . . 2655
Giac [F] . . . 2655
Mupad [F(-1)] . . . 2656
Reduce [F] . . . 2656

## Optimal result

Integrand size = 19, antiderivative size = 40

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = -\frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{2+m}{2},\csc^2(e+fx)\right)}{fm}$$

output
```
-(b*csc(f*x+e))^m*hypergeom([2, 1/2*m],[1+1/2*m],csc(f*x+e)^2)/f/m
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = -\frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(2,2-\frac{m}{2},3-\frac{m}{2},\sin^2(e+fx)\right)\sin^4(e+fx)}{f(-4+m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
-(((b*Csc[e + f*x])^m*Hypergeometric2F1[2, 2 - m/2, 3 - m/2, Sin[e + f*x]^
2]*Sin[e + f*x]^4)/(f*(-4 + m)))
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 25, 3086, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m}{\tan\left(e+fx-\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b\sec\left(\frac{1}{2}(2e-\pi)+fx\right)\right)^m}{\tan\left(\frac{1}{2}(2e-\pi)+fx\right)^3}dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{b\int \frac{(b\csc(e+fx))^{m-1}}{(1-\csc^2(e+fx))^2}d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{m+2}{2},\csc^2(e+fx)\right)}{fm}$$

input
```
Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
-(((b*Csc[e + f*x])^m*Hypergeometric2F1[2, m/2, (2 + m)/2, Csc[e + f*x]^2]
)/(f*m))
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [F]

$$\int (b\csc (fx+e))^{m} \tan (fx+e)^{3} dx$$

input
```
int((b*csc(f*x+e))^m*tan(f*x+e)^3,x)
```

output
```
int((b*csc(f*x+e))^m*tan(f*x+e)^3,x)
```

## Fricas [F]

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = \int (b\csc (fx+e))^{m} \tan (fx+e)^{3}\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

## Sympy [F]

$$\int (b\csc(e+fx))^m\tan^3(e+fx)\,dx=\int (b\csc(e+fx))^m\tan^3(e+fx)\,dx$$

input
```
integrate((b*csc(f*x+e))**m*tan(f*x+e)**3,x)
```

output
```
Integral((b*csc(e + f*x))**m*tan(e + f*x)**3, x)
```

## Maxima [F]

$$\int (b\csc(e+fx))^m\tan^3(e+fx)\,dx=\int (b\csc(fx+e))^m\tan(fx+e)^3\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

## Giac [F]

$$\int (b\csc(e+fx))^m\tan^3(e+fx)\,dx=\int (b\csc(fx+e))^m\tan(fx+e)^3\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

input
```
int(tan(e + f*x)^3*(b/sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^3*(b/sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = b^m\left(\int \csc\left(fx+e\right)^m \tan\left(fx+e\right)^3 dx\right)$$

input
```
int((b*csc(f*x+e))^m*tan(f*x+e)^3,x)
```

output
```
b**m*int(csc(e + f*x)**m*tan(e + f*x)**3,x)
```

# 3.375 $\int (b\csc(e+fx))^m \tan(e+fx)\,dx$

Optimal result . . . 2657
Mathematica [A] (verified) . . . 2657
Rubi [A] (verified) . . . 2658
Maple [F] . . . 2659
Fricas [F] . . . 2660
Sympy [F] . . . 2660
Maxima [F] . . . 2660
Giac [F] . . . 2661
Mupad [F(-1)] . . . 2661
Reduce [F] . . . 2661

## Optimal result

Integrand size = 17, antiderivative size = 39

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},\csc^2(e+fx)\right)}{fm}$$

output
```
(b*csc(f*x+e))^m*hypergeom([1, 1/2*m],[1+1/2*m],csc(f*x+e)^2)/f/m
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = -\frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(1,1-\frac{m}{2},2-\frac{m}{2},\sin^2(e+fx)\right)\sin^2(e+fx)}{f(-2+m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x],x]
```

output
```
-(((b*Csc[e + f*x])^m*Hypergeometric2F1[1, 1 - m/2, 2 - m/2, Sin[e + f*x]^
2]*Sin[e + f*x]^2)/(f*(-2 + m)))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 25, 3086, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m}{\tan\left(e+fx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b\sec\left(\frac{1}{2}(2e-\pi)+fx\right)\right)^m}{\tan\left(\frac{1}{2}(2e-\pi)+fx\right)}dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{b\int -\frac{(b\csc(e+fx))^{m-1}}{1-\csc^2(e+fx)}d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{(b\csc(e+fx))^{m-1}}{1-\csc^2(e+fx)}d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(b\csc(e+fx))^m\,\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},\csc^2(e+fx)\right)}{fm}$$

input
```
Int[(b*Csc[e + f*x])^m*Tan[e + f*x],x]
```

output
```
((b*Csc[e + f*x])^m*Hypergeometric2F1[1, m/2, (2 + m)/2, Csc[e + f*x]^2])/
(f*m)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [F]

$$\int (b\csc(fx+e))^m \tan(fx+e)\,dx$$

input
```
int((b*csc(f*x+e))^m*tan(f*x+e),x)
```

output
```
int((b*csc(f*x+e))^m*tan(f*x+e),x)
```

## Fricas [F]

$$\int (b\csc(e+fx))^m\tan(e+fx)\,dx=\int (b\csc(fx+e))^m\tan(fx+e)\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^m*tan(f*x + e), x)
```

## Sympy [F]

$$\int (b\csc(e+fx))^m\tan(e+fx)\,dx=\int (b\csc(e+fx))^m\tan(e+fx)\,dx$$

input
```
integrate((b*csc(f*x+e))**m*tan(f*x+e),x)
```

output
```
Integral((b*csc(e + f*x))**m*tan(e + f*x), x)
```

## Maxima [F]

$$\int (b\csc(e+fx))^m\tan(e+fx)\,dx=\int (b\csc(fx+e))^m\tan(fx+e)\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e), x)
```

## Giac [F]

$$\int (b\csc(e+fx))^m\tan(e+fx)\,dx=\int (b\csc(fx+e))^m\tan(fx+e)\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m\tan(e+fx)\,dx=\int \tan(e+f\,x)\left(\frac{b}{\sin(e+f\,x)}\right)^m\,dx$$

input
```
int(tan(e + f*x)*(b/sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)*(b/sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int (b\csc(e+fx))^m\tan(e+fx)\,dx=b^m\left(\int \csc(fx+e)^m\tan(fx+e)\,dx\right)$$

input
```
int((b*csc(f*x+e))^m*tan(f*x+e),x)
```

output
```
b**m*int(csc(e + f*x)**m*tan(e + f*x),x)
```

# 3.376 $\int \cot(e+fx)(b\csc(e+fx))^m\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2662
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2662
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2663
Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2664
Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2665
Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2665
Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2666
Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 2666
Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2666
Reduce [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 2667

## Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m}{fm}$$

output
```
-(b*csc(f*x+e))^m/f/m
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m}{fm}$$

input
```
Integrate[Cot[e + f*x]*(b*Csc[e + f*x])^m,x]
```

output
```
-((b*Csc[e + f*x])^m/(f*m))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 25, 3086, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(e+fx-\frac{\pi}{2}\right)\left(-\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(b\sec\left(\frac{1}{2}(2e-\pi)+fx\right)\right)^m\tan\left(\frac{1}{2}(2e-\pi)+fx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{b\int (b\csc(e+fx))^{m-1}d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{17}$$

$$-\frac{(b\csc(e+fx))^m}{fm}$$

input
```
Int[Cot[e + f*x]*(b*Csc[e + f*x])^m,x]
```

output
```
-((b*Csc[e + f*x])^m/(f*m))
```

### Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| derivativedivides | $-\frac{(b\csc(fx+e))^m}{fm}$ |
| default | $-\frac{(b\csc(fx+e))^m}{fm}$ |
| risch | $-\frac{2^m b^m \left(e^{i(fx+e)}\right)^m \left(e^{2i(fx+e)}-1\right)^{-m} e^{\frac{i\pi m\left(-\operatorname{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}-1}\right)^3+\operatorname{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}-1}\right)^2\operatorname{csgn}\left(ie^{i(fx+e)}\right)+\operatorname{csgn}\left(\frac{ie^{i}}{e^{2i(}}\right.\right.}{}}}{}$ |

input
```
int(cot(f*x+e)*(b*csc(f*x+e))^m,x,method=_RETURNVERBOSE)
```

output
```
-(b*csc(f*x+e))^m/f/m
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\left(\frac{b}{\sin(fx+e)}\right)^m}{fm}$$

input
```
integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

output
```
-(b/sin(f*x + e))^m/(f*m)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(14) = 28.

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.00

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = \begin{cases} x\cot(e) & \text{for } f=0 \wedge m=0 \\ x(b\csc(e))^m\cot(e) & \text{for } f=0 \\ -\frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\log(\tan(e+fx))}{f} & \text{for } m=0 \\ -\frac{(b\csc(e+fx))^m}{fm} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(f*x+e)*(b*csc(f*x+e))**m,x)
```

output
```
Piecewise((x*cot(e), Eq(f, 0) & Eq(m, 0)), (x*(b*csc(e))**m*cot(e), Eq(f,
0)), (-log(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f, Eq(m, 0)), (-
(b*csc(e + f*x))**m/(f*m), True))
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{b^m \sin{(fx+e)}^{-m}}{fm}$$

input
```
integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

output
```
-b^m*sin(f*x + e)^(-m)/(f*m)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\left(\frac{b}{\sin(fx+e)}\right)^m}{fm}$$

input
```
integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="giac")
```

output
```
-(b/sin(f*x + e))^m/(f*m)
```

## Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = \begin{cases} -\frac{\ln\left(\frac{b}{\sin(e+f\,x)}\right)}{f} & \text{if} \ \ m=0 \\ -\frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m}{f\,m} & \text{if} \ \ m\neq 0 \end{cases}$$

input
```
int(cot(e + f*x)*(b/sin(e + f*x))^m,x)
```

output
```
piecewise(m == 0, -log(b/sin(e + f*x))/f, m ~= 0, -(b/sin(e + f*x))^m/(f*m
))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{b^m\csc\left(fx+e\right)^m}{fm}$$

input
```
int(cot(f*x+e)*(b*csc(f*x+e))^m,x)
```

output
```
( - b**m*csc(e + f*x)**m)/(f*m)
```

# 3.377 $\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$

Optimal result . . . 2668
Mathematica [A] (verified) . . . 2668
Rubi [A] (verified) . . . 2669
Maple [C] (warning: unable to verify) . . . 2670
Fricas [A] (verification not implemented) . . . 2671
Sympy [F] . . . 2672
Maxima [A] (verification not implemented) . . . 2672
Giac [A] (verification not implemented) . . . 2673
Mupad [B] (verification not implemented) . . . 2673
Reduce [B] (verification not implemented) . . . 2674

## Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx=\frac{(b\csc(e+fx))^m}{fm}-\frac{(b\csc(e+fx))^{2+m}}{b^2f(2+m)}$$

output
```
(b*csc(f*x+e))^m/f/m-(b*csc(f*x+e))^(2+m)/b^2/f/(2+m)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx=\frac{(b\csc(e+fx))^m\left(2+m-m\csc^2(e+fx)\right)}{fm(2+m)}$$

input
```
Integrate[Cot[e + f*x]^3*(b*Csc[e + f*x])^m,x]
```

output
```
((b*Csc[e + f*x])^m*(2 + m - m*Csc[e + f*x]^2))/(f*m*(2 + m))
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 25, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(e+fx-\frac{\pi}{2}\right)^3\left(-\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(b\sec\left(\frac{1}{2}(2e-\pi)+fx\right)\right)^m\tan\left(\frac{1}{2}(2e-\pi)+fx\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{b\int -(b\csc(e+fx))^{m-1}\left(1-\csc^2(e+fx)\right)d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int (b\csc(e+fx))^{m-1}\left(1-\csc^2(e+fx)\right)d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left((b\csc(e+fx))^{m-1}-\frac{(b\csc(e+fx))^{m+1}}{b^2}\right)d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{(b\csc(e+fx))^{m+2}}{b^3(m+2)}-\frac{(b\csc(e+fx))^m}{bm}\right)}{f}$$

input
```
Int[Cot[e + f*x]^3*(b*Csc[e + f*x])^m,x]
```

output
```
-((b*(-((b*Csc[e + f*x])^m/(b*m)) + (b*Csc[e + f*x])^(2 + m)/(b^3*(2 + m))
))/f)
```

**Defintions of rubi rules used**

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.00 (sec) , antiderivative size = 3514, normalized size of antiderivative = 81.72

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 3514 |

input
```
int(cot(f*x+e)^3*(b*csc(f*x+e))^m,x,method=_RETURNVERBOSE)
```

output
```
1/(2+m)/f/(exp(2*I*(f*x+e))-1)^2/m*b^m*exp(I*(f*x+e))^m*(exp(2*I*(f*x+e))-
1)^(-m)*2^m*(m*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3
*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I*exp(
I*(f*x+e)))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*
csgn(I/(exp(2*I*(f*x+e))-1))*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2
*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))-1))*m)*exp
(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*b/(exp(2*I*(f
*x+e))-1)*exp(I*(f*x+e)))^2*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*
I*(f*x+e))-1))*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(I*b)*m)*
exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*m)*exp(1/2*I
*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn(I*b)*m)*exp(1/2*I
*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(b/(exp(2*I*(f*x+e))
-1)*exp(I*(f*x+e)))^2*m)*exp(1/2*I*Pi*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f
*x+e)))^3*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*c
sgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*m)*exp(-1/2*I*Pi*csgn(b/(exp(2*
I*(f*x+e))-1)*exp(I*(f*x+e)))^2*m)*exp(1/2*I*Pi*m)*exp(4*I*f*x)*exp(4*I*e)
+2*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3*m)*exp(1/2*
I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I*exp(I*(f*x+e)))*
m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I/(exp(
2*I*(f*x+e))-1))*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e...
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\left((m+2)\cos\left(fx+e\right)^2-2\right)\left(\frac{b}{\sin(fx+e)}\right)^m}{fm^2-(fm^2+2\,fm)\cos\left(fx+e\right)^2+2\,fm}$$

input
```
integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

output
```
-((m + 2)*cos(f*x + e)^2 - 2)*(b/sin(f*x + e))^m/(f*m^2 - (f*m^2 + 2*f*m)*
cos(f*x + e)^2 + 2*f*m)
```

## Sympy [F]

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx = \begin{cases} x(b\csc(e))^m\cot^3(e) & \text{for } f=0 \\ \frac{\int \frac{\cot^3(e+fx)}{\csc^2(e+fx)}\,dx}{b^2} & \text{for } m=-2 \\ \frac{\log(\tan^2(e+fx)+1)}{2f} - \frac{\log(\tan(e+fx))}{f} - \frac{1}{2f\tan^2(e+fx)} & \text{for } m=0 \\ -\frac{m(b\csc(e+fx))^m\cot^2(e+fx)}{fm^2+2fm} + \frac{2(b\csc(e+fx))^m}{fm^2+2fm} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(f*x+e)**3*(b*csc(f*x+e))**m,x)
```

output
```
Piecewise((x*(b*csc(e))**m*cot(e)**3, Eq(f, 0)), (Integral(cot(e + f*x)**3
/csc(e + f*x)**2, x)/b**2, Eq(m, -2)), (log(tan(e + f*x)**2 + 1)/(2*f) - l
og(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2), Eq(m, 0)), (-m*(b*csc(e + f*
x))**m*cot(e + f*x)**2/(f*m**2 + 2*f*m) + 2*(b*csc(e + f*x))**m/(f*m**2 +
2*f*m), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx = \frac{\frac{b^m \sin(fx+e)^{-m}}{m} - \frac{b^m \sin(fx+e)^{-m}}{(m+2)\sin(fx+e)^2}}{f}$$

input
```
integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

output
```
(b^m*sin(f*x + e)^(-m)/m - b^m*sin(f*x + e)^(-m)/((m + 2)*sin(f*x + e)^2))
/f
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx = \frac{\frac{b\left(\frac{b}{\sin(fx+e)}\right)^m}{m} - \frac{b^2\left(\frac{b}{\sin(fx+e)}\right)^m}{(bm+2b)\sin(fx+e)^2}}{bf}$$

input
```
integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="giac")
```

output
```
(b*(b/sin(f*x + e))^m/m - b^2*(b/sin(f*x + e))^m/((b*m + 2*b)*sin(f*x + e)
^2))/(b*f)
```

## Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$$
$$= \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m \left(m+4\sin\left(2\,e+2\,f\,x\right)^2+m\left(2\sin\left(2\,e+2\,f\,x\right)^2-1\right)-16\sin\left(e+f\,x\right)^2\right)}{f\,m\,\left(2\sin\left(2\,e+2\,f\,x\right)^2-8\sin\left(e+f\,x\right)^2\right)\,(m+2)}$$

input
```
int(cot(e + f*x)^3*(b/sin(e + f*x))^m,x)
```

output
```
((b/sin(e + f*x))^m*(m + 4*sin(2*e + 2*f*x)^2 + m*(2*sin(2*e + 2*f*x)^2 -
1) - 16*sin(e + f*x)^2))/(f*m*(2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2)*(m
 + 2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx = \frac{b^m\csc(fx+e)^m\left(-\cot(fx+e)^2m+2\right)}{fm(m+2)}$$

input
```
int(cot(f*x+e)^3*(b*csc(f*x+e))^m,x)
```

output
```
(b**m*csc(e + f*x)**m*( - cot(e + f*x)**2*m + 2))/(f*m*(m + 2))
```

# 3.378 $\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx$

Optimal result . . . 2675
Mathematica [A] (verified) . . . 2675
Rubi [A] (verified) . . . 2676
Maple [C] (warning: unable to verify) . . . 2677
Fricas [A] (verification not implemented) . . . 2678
Sympy [F(-1)] . . . 2678
Maxima [A] (verification not implemented) . . . 2679
Giac [B] (verification not implemented) . . . 2679
Mupad [B] (verification not implemented) . . . 2680
Reduce [B] (verification not implemented) . . . 2680

## Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m}{fm}+\frac{2(b\csc(e+fx))^{2+m}}{b^2f(2+m)}-\frac{(b\csc(e+fx))^{4+m}}{b^4f(4+m)}$$

output
```
-(b*csc(f*x+e))^m/f/m+2*(b*csc(f*x+e))^(2+m)/b^2/f/(2+m)-(b*csc(f*x+e))^(4
+m)/b^4/f/(4+m)
```

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m\left(8+6m+m^2-2m(4+m)\csc^2(e+fx)+m(2+m)\csc^4(e+fx)\right)}{fm(2+m)(4+m)}$$

input
```
Integrate[Cot[e + f*x]^5*(b*Csc[e + f*x])^m,x]
```

output
```
-(((b*Csc[e + f*x])^m*(8 + 6*m + m^2 - 2*m*(4 + m)*Csc[e + f*x]^2 + m*(2 +
 m)*Csc[e + f*x]^4))/(f*m*(2 + m)*(4 + m)))
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 25, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(e+fx-\frac{\pi}{2}\right)^5\left(-\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int\left(b\sec\left(\frac{1}{2}(2e-\pi)+fx\right)\right)^m\tan\left(\frac{1}{2}(2e-\pi)+fx\right)^5dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{b\int(b\csc(e+fx))^{m-1}\left(1-\csc^2(e+fx)\right)^2d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int\left((b\csc(e+fx))^{m-1}-\frac{2(b\csc(e+fx))^{m+1}}{b^2}+\frac{(b\csc(e+fx))^{m+3}}{b^4}\right)d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{(b\csc(e+fx))^{m+4}}{b^5(m+4)}-\frac{2(b\csc(e+fx))^{m+2}}{b^3(m+2)}+\frac{(b\csc(e+fx))^m}{bm}\right)}{f}$$

input
```
Int[Cot[e + f*x]^5*(b*Csc[e + f*x])^m,x]
```

output
```
-((b*((b*Csc[e + f*x])^m/(b*m) - (2*(b*Csc[e + f*x])^(2 + m))/(b^3*(2 + m)
) + (b*Csc[e + f*x])^(4 + m)/(b^5*(4 + m))))/f)
```

**Defintions of rubi rules used**

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.95 (sec) , antiderivative size = 8846, normalized size of antiderivative = 128.20

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 8846 |

input
```
int(cot(f*x+e)^5*(b*csc(f*x+e))^m,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.67

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx =$$
$$-\frac{\left(\left(m^{2}+6\,m+8\right)\cos\left(fx+e\right)^{4}-4\,\left(m+4\right)\cos\left(fx+e\right)^{2}+8\right)\left(\frac{b}{\sin\left(fx+e\right)}\right)^{m}}{\left(fm^{3}+6\,fm^{2}+8\,fm\right)\cos\left(fx+e\right)^{4}+fm^{3}+6\,fm^{2}-2\,\left(fm^{3}+6\,fm^{2}+8\,fm\right)\cos\left(fx+e\right)^{2}+8\,fm}$$

input
```
integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

output
```
-((m^2 + 6*m + 8)*cos(f*x + e)^4 - 4*(m + 4)*cos(f*x + e)^2 + 8)*(b/sin(f*
x + e))^m/((f*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^4 + f*m^3 + 6*f*m^2 - 2*
(f*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^2 + 8*f*m)
```

## Sympy [F(-1)]

Timed out.

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**5*(b*csc(f*x+e))**m,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\frac{b^m\sin(fx+e)^{-m}}{m} - \frac{2\,b^m\sin(fx+e)^{-m}}{(m+2)\sin(fx+e)^2} + \frac{b^m\sin(fx+e)^{-m}}{(m+4)\sin(fx+e)^4}}{f}$$

input
```
integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

output
```
-(b^m*sin(f*x + e)^(-m)/m - 2*b^m*sin(f*x + e)^(-m)/((m + 2)*sin(f*x + e)^
2) + b^m*sin(f*x + e)^(-m)/((m + 4)*sin(f*x + e)^4))/f
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(69) = 138.

Time = 0.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.20

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx$$
$$= -\frac{\frac{b\left(\frac{b}{\sin(fx+e)}\right)^m}{m} - \frac{\frac{2\,b^4m\left(\frac{b}{\sin(fx+e)}\right)^m}{\sin(fx+e)^2} + \frac{8\,b^4\left(\frac{b}{\sin(fx+e)}\right)^m}{\sin(fx+e)^2} - \frac{b^4m\left(\frac{b}{\sin(fx+e)}\right)^m}{\sin(fx+e)^4} - \frac{2\,b^4\left(\frac{b}{\sin(fx+e)}\right)^m}{\sin(fx+e)^4}}{b^3m^2+6\,b^3m+8\,b^3}}{bf}$$

input
```
integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="giac")
```

output
```
-(b*(b/sin(f*x + e))^m/m - (2*b^4*m*(b/sin(f*x + e))^m/sin(f*x + e)^2 + 8*
b^4*(b/sin(f*x + e))^m/sin(f*x + e)^2 - b^4*m*(b/sin(f*x + e))^m/sin(f*x +
 e)^4 - 2*b^4*(b/sin(f*x + e))^m/sin(f*x + e)^4)/(b^3*m^2 + 6*b^3*m + 8*b^
3))/(b*f)
```

## Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.22

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\left(\frac{b}{\sin(e+fx)}\right)^m \left(2\sin\left(2e+2fx\right)^2+\sin\left(4e+4fx\right)\,1\mathrm{i}-1\right)\,\left(\frac{2\left(2\sin(2e+2fx)^2-1\right)\left(-2\sin(2e+2fx)^2+\sin(4e+4\right.}{fm}\right.}{}$$

input
```
int(cot(e + f*x)^5*(b/sin(e + f*x))^m,x)
```

output
```
-((b/sin(e + f*x))^m*(sin(4*e + 4*f*x)*1i + 2*sin(2*e + 2*f*x)^2 - 1)*((2*
(2*sin(2*e + 2*f*x)^2 - 1)*(sin(4*e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1
))/(f*m) - ((sin(4*e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1)*(4*m + 6*m^2
+ 48))/(f*m*(6*m + m^2 + 8)) + (2*(2*sin(e + f*x)^2 - 1)*(sin(4*e + 4*f*x)
*1i - 2*sin(2*e + 2*f*x)^2 + 1)*(8*m + 4*m^2 - 32))/(f*m*(6*m + m^2 + 8)))
)/(16*sin(e + f*x)^4)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = \frac{b^m\csc\left(fx+e\right)^m\left(-\cot\left(fx+e\right)^4m^2-2\cot\left(fx+e\right)^4m+4\cot\left(fx+e\right)^2m-8\right)}{fm\left(m^2+6m+8\right)}$$

input
```
int(cot(f*x+e)^5*(b*csc(f*x+e))^m,x)
```

output
```
(b**m*csc(e + f*x)**m*( - cot(e + f*x)**4*m**2 - 2*cot(e + f*x)**4*m + 4*c
ot(e + f*x)**2*m - 8))/(f*m*(m**2 + 6*m + 8))
```

# 3.379 $\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx$

Optimal result . . . . . . . . . . 2681
Mathematica [B] (warning: unable to verify) . . . . . . . . . . 2682
Rubi [A] (verified) . . . . . . . . . . 2682
Maple [F] . . . . . . . . . . 2683
Fricas [F] . . . . . . . . . . 2684
Sympy [F] . . . . . . . . . . 2684
Maxima [F] . . . . . . . . . . 2684
Giac [F] . . . . . . . . . . 2685
Mupad [F(-1)] . . . . . . . . . . 2685
Reduce [F] . . . . . . . . . . 2685

## Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),-\frac{1}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(-3+m)}\tan^3(e+$$
$$3f$$

output
```
1/3*(b*csc(f*x+e))^m*hypergeom([-3/2, -3/2+1/2*m],[-1/2],cos(f*x+e)^2)*(si
n(f*x+e)^2)^(-3/2+1/2*m)*tan(f*x+e)^3/f
```

## Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 240 vs. $2(63) = 126$.

Time = 8.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.81

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = -\frac{\cos(e+fx)(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3}{2},\cos^2(e+fx)\right)\sin(e+fx)\sin^2(e+fx)^{\frac{1}{2}(\cdots}}{f} + \frac{4(b\csc(e+fx))^m\left(\frac{m\operatorname{Hypergeometric2F1}\left(1-m,\frac{1}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2},-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)^{-m}\tan\left(\frac{1}{2}(e+fx)\right)}{-1+m} - \frac{1}{2}\tan(e\cdots\right.}{f} + \frac{(b\csc(e+fx))^m\operatorname{Hypergeometric2F1}\left(-1-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}\tan(\cdots}{f(1-m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
-((Cos[e + f*x]*(b*Csc[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m)/2, 3/2,
Cos[e + f*x]^2]*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 + m)/2))/f) + (4*(b*Csc
[e + f*x])^m*((m*Hypergeometric2F1[1 - m, 1/2 - m/2, 3/2 - m/2, -Tan[(e +
f*x)/2]^2]*Tan[(e + f*x)/2])/((-1 + m)*(Sec[(e + f*x)/2]^2)^m) - Tan[e + f
*x]/2))/f + ((b*Csc[e + f*x])^m*Hypergeometric2F1[-1 - m/2, 1/2 - m/2, 3/2
 - m/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*(1 - m)*(Sec[e + f*x]^2)^(m/2))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b \sec \left(e+f x-\frac{\pi}{2}\right)\right)^m}{\tan \left(e+f x-\frac{\pi}{2}\right)^4} d x$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{\tan^3(e+fx)\sin^2(e+fx)^{\frac{m-3}{2}}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-3}{2},-\frac{1}{2},\cos^2(e+fx)\right)}{3f}$$

input
```
Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
((b*Csc[e + f*x])^m*Hypergeometric2F1[-3/2, (-3 + m)/2, -1/2, Cos[e + f*x]
^2]*(Sin[e + f*x]^2)^((-3 + m)/2)*Tan[e + f*x]^3)/(3*f)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \left(b \csc \left(f x+e\right)\right)^{m} \tan \left(f x+e\right)^{4} d x$$

input
```
int((b*csc(f*x+e))^m*tan(f*x+e)^4,x)
```

output
```
int((b*csc(f*x+e))^m*tan(f*x+e)^4,x)
```

## Fricas [F]

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^4\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

## Sympy [F]

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \int (b\csc(e+fx))^m \tan^4(e+fx)\,dx$$

input
```
integrate((b*csc(f*x+e))**m*tan(f*x+e)**4,x)
```

output
```
Integral((b*csc(e + f*x))**m*tan(e + f*x)**4, x)
```

## Maxima [F]

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^4\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

## Giac [F]

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^4\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4 \left(\frac{b}{\sin(e+f\,x)}\right)^m\,dx$$

input
```
int(tan(e + f*x)^4*(b/sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^4*(b/sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = b^m \left(\int \csc(fx+e)^m \tan(fx+e)^4\,dx\right)$$

input
```
int((b*csc(f*x+e))^m*tan(f*x+e)^4,x)
```

output
```
b**m*int(csc(e + f*x)**m*tan(e + f*x)**4,x)
```

# 3.380 $\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx$


Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2686
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . 2686
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2687
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2688
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2688
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2689
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2689
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2689
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2690
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2690


## Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(-1+m)}\tan(e+fx}{f}$$

output
```
(b*csc(f*x+e))^m*hypergeom([-1/2, -1/2+1/2*m],[1/2],cos(f*x+e)^2)*(sin(f*x
+e)^2)^(-1/2+1/2*m)*tan(f*x+e)/f
```

## Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.36

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(1-\frac{m}{2},\frac{3}{2}-\frac{m}{2},\frac{5}{2}-\frac{m}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}\tan^3(e+}{f(3-m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
((b*Csc[e + f*x])^m*Hypergeometric2F1[1 - m/2, 3/2 - m/2, 5/2 - m/2, -Tan[
e + f*x]^2]*Tan[e + f*x]^3)/(f*(3 - m)*(Sec[e + f*x]^2)^(m/2))
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m}{\tan\left(e+fx-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \; 3097$$

$$\frac{\tan(e+fx)\sin^2(e+fx)^{\frac{m-1}{2}}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-1}{2},\frac{1}{2},\cos^2(e+fx)\right)}{f}$$

input
```
Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
((b*Csc[e + f*x])^m*Hypergeometric2F1[-1/2, (-1 + m)/2, 1/2, Cos[e + f*x]^
2]*(Sin[e + f*x]^2)^((-1 + m)/2)*Tan[e + f*x])/f
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int (b\csc (fx+e))^{m} \tan (fx+e)^{2} dx$$

input
```
int((b*csc(f*x+e))^m*tan(f*x+e)^2,x)
```

output
```
int((b*csc(f*x+e))^m*tan(f*x+e)^2,x)
```

## Fricas [F]

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \int (b\csc (fx+e))^{m} \tan (fx+e)^{2}\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

## Sympy [F]

$$\int (b\csc(e+fx))^m\tan^2(e+fx)\,dx=\int (b\csc(e+fx))^m\tan^2(e+fx)\,dx$$

input
```
integrate((b*csc(f*x+e))**m*tan(f*x+e)**2,x)
```

output
```
Integral((b*csc(e + f*x))**m*tan(e + f*x)**2, x)
```

## Maxima [F]

$$\int (b\csc(e+fx))^m\tan^2(e+fx)\,dx=\int (b\csc(fx+e))^m\tan(fx+e)^2\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

## Giac [F]

$$\int (b\csc(e+fx))^m\tan^2(e+fx)\,dx=\int (b\csc(fx+e))^m\tan(fx+e)^2\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2 \left(\frac{b}{\sin(e+f\,x)}\right)^m dx$$

input
```
int(tan(e + f*x)^2*(b/sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^2*(b/sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = b^m \left(\int \csc(fx+e)^m \tan(fx+e)^2\,dx\right)$$

input
```
int((b*csc(f*x+e))^m*tan(f*x+e)^2,x)
```

output
```
b**m*int(csc(e + f*x)**m*tan(e + f*x)**2,x)
```

# 3.381 $\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx$

Optimal result . . . 2691
Mathematica [B] (warning: unable to verify) . . . 2691
Rubi [A] (verified) . . . 2692
Maple [F] . . . 2693
Fricas [F] . . . 2693
Sympy [F] . . . 2694
Maxima [F] . . . 2694
Giac [F] . . . 2694
Mupad [F(-1)] . . . 2695
Reduce [F] . . . 2695

## Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot^3(e+fx)(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{3+m}{2}}}{3f}$$

output
```
-1/3*cot(f*x+e)^3*(b*csc(f*x+e))^m*hypergeom([3/2, 3/2+1/2*m],[5/2],cos(f*
x+e)^2)*(sin(f*x+e)^2)^(3/2+1/2*m)/f
```

## Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(63) = 126.

Time = 1.02 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.95

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m\left(-4(1+m)\operatorname{Hypergeometric2F1}\left(1-m,\frac{1}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2},-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(-1\right.}{}$$

input
```
Integrate[Cot[e + f*x]^2*(b*Csc[e + f*x])^m,x]
```

output
```
-1/2*((b*Csc[e + f*x])^m*(-4*(1 + m)*Hypergeometric2F1[1 - m, 1/2 - m/2, 3
/2 - m/2, -Tan[(e + f*x)/2]^2] + (-1 + m)*Cot[(e + f*x)/2]^2*Hypergeometri
c2F1[-1/2 - m/2, -m, 1/2 - m/2, -Tan[(e + f*x)/2]^2] + (1 + m)*Hypergeomet
ric2F1[1/2 - m/2, -m, 3/2 - m/2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2])/(
f*(-1 + m^2)*(Sec[(e + f*x)/2]^2)^m)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(e+fx-\frac{\pi}{2}\right)^2\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3097}$$

$$-\frac{\cot^3(e+fx)\sin^2(e+fx)^{\frac{m+3}{2}}(b\csc(e+fx))^m\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+3}{2},\frac{5}{2},\cos^2(e+fx)\right)}{3f}$$

input
```
Int[Cot[e + f*x]^2*(b*Csc[e + f*x])^m,x]
```

output
```
-1/3*(Cot[e + f*x]^3*(b*Csc[e + f*x])^m*Hypergeometric2F1[3/2, (3 + m)/2,
5/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((3 + m)/2))/f
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \cot(fx+e)^2 (b\csc(fx+e))^m \, dx$$

input
```
int(cot(f*x+e)^2*(b*csc(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^2*(b*csc(f*x+e))^m,x)
```

## Fricas [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m \, dx = \int (b\csc(fx+e))^m \cot(fx+e)^2 \, dx$$

input
```
integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

## Sympy [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(e+fx))^m \cot^2(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**2*(b*csc(f*x+e))**m,x)
```

output
```
Integral((b*csc(e + f*x))**m*cot(e + f*x)**2, x)
```

## Maxima [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m \cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

## Giac [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m \cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx=\int \cot(e+f\,x)^2\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

input
```
int(cot(e + f*x)^2*(b/sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^2*(b/sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx=b^m\left(\int \csc\left(fx+e\right)^m\cot\left(fx+e\right)^2dx\right)$$

input
```
int(cot(f*x+e)^2*(b*csc(f*x+e))^m,x)
```

output
```
b**m*int(csc(e + f*x)**m*cot(e + f*x)**2,x)
```

# 3.382 $\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2696
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2696
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2697
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2698
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2698
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2699
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2699
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2699
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2700
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2700

## Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot^5(e+fx)(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{5+m}{2}}}{5f}$$

output
```
-1/5*cot(f*x+e)^5*(b*csc(f*x+e))^m*hypergeom([5/2, 5/2+1/2*m],[7/2],cos(f*
x+e)^2)*(sin(f*x+e)^2)^(5/2+1/2*m)/f
```

## Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot(e+fx)(b\csc(e+fx))^m \left(\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3}{2},\cos^2(e+fx)\right) - 2\operatorname{Hypergeometric2F1}\left(\right.\right.}{f}$$

input
```
Integrate[Cot[e + f*x]^4*(b*Csc[e + f*x])^m,x]
```

output
```
-((Cot[e + f*x]*(b*Csc[e + f*x])^m*(Hypergeometric2F1[1/2, (1 + m)/2, 3/2,
 Cos[e + f*x]^2] - 2*Hypergeometric2F1[1/2, (3 + m)/2, 3/2, Cos[e + f*x]^2
] + Hypergeometric2F1[1/2, (5 + m)/2, 3/2, Cos[e + f*x]^2])*(Sin[e + f*x]^
2)^((1 + m)/2))/f)
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan\left(e+fx-\frac{\pi}{2}\right)^4\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \ 3097$$

$$-\frac{\cot^5(e+fx)\sin^2(e+fx)^{\frac{m+5}{2}}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+5}{2},\frac{7}{2},\cos^2(e+fx)\right)}{5f}$$

input
```
Int[Cot[e + f*x]^4*(b*Csc[e + f*x])^m,x]
```

output
```
-1/5*(Cot[e + f*x]^5*(b*Csc[e + f*x])^m*Hypergeometric2F1[5/2, (5 + m)/2,
7/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((5 + m)/2))/f
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

## Maple [F]

$$\int \cot(fx+e)^4 (b\csc(fx+e))^m\,dx$$

input
```
int(cot(f*x+e)^4*(b*csc(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^4*(b*csc(f*x+e))^m,x)
```

## Fricas [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m \cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

## Sympy [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx=\int (b\csc(e+fx))^m\cot^4(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**4*(b*csc(f*x+e))**m,x)
```

output
```
Integral((b*csc(e + f*x))**m*cot(e + f*x)**4, x)
```

## Maxima [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx=\int (b\csc(fx+e))^m\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

## Giac [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx=\int (b\csc(fx+e))^m\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = \int \cot(e+f\,x)^4\left(\frac{b}{\sin(e+f\,x)}\right)^m dx$$

input
```
int(cot(e + f*x)^4*(b/sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^4*(b/sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = b^m\left(\int \csc(fx+e)^m \cot(fx+e)^4\,dx\right)$$

input
```
int(cot(f*x+e)^4*(b*csc(f*x+e))^m,x)
```

output
```
b**m*int(csc(e + f*x)**m*cot(e + f*x)**4,x)
```

# 3.383 $\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2701
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2701
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2702
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2704
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2704
Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2704
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2705
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2705
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2705
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2706

## Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{5/4}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(5-2m),\frac{1}{4}(9-2m),\sin^2(e+f\right.}{df(5-2m)}$$

output
```
2*(cos(f*x+e)^2)^(5/4)*(b*csc(f*x+e))^m*hypergeom([5/4, 5/4-1/2*m],[9/4-1/
2*m],sin(f*x+e)^2)*(d*tan(f*x+e))^(5/2)/d/f/(5-2*m)
```

## Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx = -\frac{2(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4}(5-2m),1-\frac{m}{2},\frac{1}{4}(9-2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}}{df(-5+2m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(5 - 2*m)/4, 1 - m/2, (9 - 2*m)/4
, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(5/2))/(d*f*(-5 + 2*m)*(Sec[e + f*x]^2
)^(m/2))
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3098, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^{3/2}(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\tan(e+fx))^{3/2}(b\csc(e+fx))^m dx$$

$$\Big\downarrow \text{3098}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{b}\right)^{-m} (d\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{b}\right)^{-m} (d\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}}(b\csc(e+fx))^{m+3}\int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{\frac{3}{2}-m}}{\cos^{\frac{3}{2}}(e+fx)}dx}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}}(b\csc(e+fx))^{m+3}\int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{\frac{3}{2}-m}}{\cos(e+fx)^{3/2}}dx}{bd}$$

$$\Big\downarrow \text{3057}$$

$$\frac{2\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(5-2m),\frac{1}{4}(9-2m),\sin^2(e+fx)\right)}{df(5-2m)}$$

input
```
Int[(b*Csc[e + f*x])^m*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(Cos[e + f*x]^2)^(5/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[5/4, (5 - 2
*m)/4, (9 - 2*m)/4, Sin[e + f*x]^2]*(d*Tan[e + f*x])^(5/2))/(d*f*(5 - 2*m)
)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3098
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (b \csc (fx+e))^m (d \tan (fx+e))^{\frac{3}{2}}\, dx$$

input
```
int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x)
```

output
```
int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x)
```

## Fricas [F]

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\, dx = \int (d \tan (fx+e))^{\frac{3}{2}} (b \csc (fx+e))^m\, dx$$

input
```
integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m*d*tan(f*x + e), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\, dx = \text{Timed out}$$

input
```
integrate((b*csc(f*x+e))**m*(d*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx=\int (d\tan(fx+e))^{\frac{3}{2}}\,(b\csc(fx+e))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(f*x + e))^(3/2)*(b*csc(f*x + e))^m, x)
```

## Giac [F]

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx=\int (d\tan(fx+e))^{\frac{3}{2}}\,(b\csc(fx+e))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(f*x + e))^(3/2)*(b*csc(f*x + e))^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx=\int (d\tan(e+f\,x))^{3/2}\left(\frac{b}{\sin(e+f\,x)}\right)^m\,dx$$

input
```
int((d*tan(e + f*x))^(3/2)*(b/sin(e + f*x))^m,x)
```

output
```
int((d*tan(e + f*x))^(3/2)*(b/sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx = \sqrt{d}\,b^m\left(\int \sqrt{\tan(fx+e)}\,\csc(fx+e)^m\tan(fx+e)\,dx\right)d$$

input
```
int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x)
```

output
```
sqrt(d)*b**m*int(sqrt(tan(e + f*x))*csc(e + f*x)**m*tan(e + f*x),x)*d
```

# 3.384 $\int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2707
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2707
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2708
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2710
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2710
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2710
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2711
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2711
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2711
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2712

## Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{3/4}(b\csc(e+fx))^m \,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(3-2m),\frac{1}{4}(7-2m),\sin^2(e+fx)\right)(d\tan(e}{df(3-2m)}$$

output
```
2*(cos(f*x+e)^2)^(3/4)*(b*csc(f*x+e))^m*hypergeom([3/4, 3/4-1/2*m],[7/4-1/
2*m],sin(f*x+e)^2)*(d*tan(f*x+e))^(3/2)/d/f/(3-2*m)
```

## Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx = -\frac{2(b\csc(e+fx))^m \,\text{Hypergeometric2F1}\left(\frac{1}{4}(3-2m),1-\frac{m}{2},\frac{1}{4}(7-2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{-}}{df(-3+2m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(3 - 2*m)/4, 1 - m/2, (7 - 2*m)/4
, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(-3 + 2*m)*(Sec[e + f*x]^2
)^(m/2))
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3098, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\tan(e+fx)}(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{d\tan(e+fx)}(b\csc(e+fx))^m dx$$

$$\Big\downarrow \text{3098}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{b}\right)^{-m} \sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{b}\right)^{-m} \sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}}(b\csc(e+fx))^{m+2}\int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}-m}}{\sqrt{\cos(e+fx)}}dx}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}}(b\csc(e+fx))^{m+2}\int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}-m}}{\sqrt{\cos(e+fx)}}dx}{bd}$$

$$\Big\downarrow \text{3057}$$

$$\frac{2\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(3-2m),\frac{1}{4}(7-2m),\sin^2(e+fx)\right.}{df(3-2m)}$$

input
```
Int[(b*Csc[e + f*x])^m*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^(3/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[3/4, (3 - 2
*m)/4, (7 - 2*m)/4, Sin[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(3 - 2*m)
)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3098
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (b\csc(fx+e))^m \sqrt{d\tan(fx+e)}dx$$

input
```
int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x)
```

output
```
int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x)
```

## Fricas [F]

$$\int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}(b\csc(fx+e))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

## Sympy [F]

$$\int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx = \int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx$$

input
```
integrate((b*csc(f*x+e))**m*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral((b*csc(e + f*x))**m*sqrt(d*tan(e + f*x)), x)
```

## Maxima [F]

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx=\int \sqrt{d\tan\left(fx+e\right)}(b\csc\left(fx+e\right))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

## Giac [F]

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx=\int \sqrt{d\tan\left(fx+e\right)}(b\csc\left(fx+e\right))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx=\int \sqrt{d\tan\left(e+f\,x\right)}\,{\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^m\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)*(b/sin(e + f*x))^m,x)
```

output
```
int((d*tan(e + f*x))^(1/2)*(b/sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx = \sqrt{d}\,b^m\left(\int \sqrt{\tan\left(fx+e\right)}\,\csc\left(fx+e\right)^m dx\right)$$

input
```
int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*b**m*int(sqrt(tan(e + f*x))*csc(e + f*x)**m,x)
```

# 3.385 $\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2713
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2713
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2714
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2716
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2716
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2716
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2717
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2717
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2717
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2718

## Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\sqrt[4]{\cos^2(e+fx)}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(1-2m),\frac{1}{4}(5-2m),\sin^2(e+fx)\right)\sqrt{d\tan(e}{df(1-2m)}$$

output
```
2*(cos(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^m*hypergeom([1/4, 1/4-1/2*m],[5/4-1/
2*m],sin(f*x+e)^2)*(d*tan(f*x+e))^(1/2)/d/f/(1-2*m)
```

## Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4}(1-2m),1-\frac{m}{2},\frac{1}{4}(5-2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{-}}{df(-1+2m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(1 - 2*m)/4, 1 - m/2, (5 - 2*m)/4
, -Tan[e + f*x]^2]*Sqrt[d*Tan[e + f*x]])/(d*f*(-1 + 2*m)*(Sec[e + f*x]^2)^
(m/2))
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3098, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3098}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}}(b\csc(e+fx))^{m+1}\int\sqrt{\cos(e+fx)}\left(\frac{\sin(e+fx)}{b}\right)^{-m-\frac{1}{2}}dx}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}}(b\csc(e+fx))^{m+1}\int\sqrt{\cos(e+fx)}\left(\frac{\sin(e+fx)}{b}\right)^{-m-\frac{1}{2}}dx}{bd}$$

$$\Big\downarrow \text{3057}$$

$$\frac{2\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(1-2m),\frac{1}{4}(5-2m),\sin^2(e+fx)\right)}{df(1-2m)}$$

input
```
Int[(b*Csc[e + f*x])^m/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^(1/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[1/4, (1 - 2
*m)/4, (5 - 2*m)/4, Sin[e + f*x]^2]*Sqrt[d*Tan[e + f*x]])/(d*f*(1 - 2*m))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3098
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}dx$$

input
```
int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x)
```

output
```
int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x)
```

## Fricas [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m/(d*tan(f*x + e)), x)
```

## Sympy [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx$$

input
```
integrate((b*csc(f*x+e))**m/(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral((b*csc(e + f*x))**m/sqrt(d*tan(e + f*x)), x)
```

## Maxima [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m/sqrt(d*tan(f*x + e)), x)
```

## Giac [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m/sqrt(d*tan(f*x + e)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m}{\sqrt{d\tan(e+f\,x)}}\,dx$$

input
```
int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(1/2),x)
```

output
```
int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(1/2), x)
```

## Reduce [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\sqrt{d}\,b^m\left(\int \frac{\sqrt{\tan(fx+e)}\,\csc(fx+e)^m}{\tan(fx+e)}dx\right)}{d}$$

input
```
int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*b**m*int((sqrt(tan(e + f*x))*csc(e + f*x)**m)/tan(e + f*x),x))/d
```

# 3.386 $\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2719
Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2719
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2720
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2722
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2722
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2722
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2723
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2723
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2723
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2724

## Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = -\frac{2(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(-1-2m),\frac{1}{4}(3-2m),\sin^2(e+fx)\right)}{df(1+2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}}$$

output
```
-2*(b*csc(f*x+e))^m*hypergeom([-1/4, -1/4-1/2*m],[3/4-1/2*m],sin(f*x+e)^2)
/d/f/(1+2*m)/(cos(f*x+e)^2)^(1/4)/(d*tan(f*x+e))^(1/2)
```

## Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = -\frac{2(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4}(-1-2m),1-\frac{m}{2},\frac{1}{4}(3-2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m}}{df(1+2m)\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[(b*Csc[e + f*x])^m/(d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(-1 - 2*m)/4, 1 - m/2, (3 - 2*m)/
4, -Tan[e + f*x]^2])/(d*f*(1 + 2*m)*(Sec[e + f*x]^2)^(m/2)*Sqrt[d*Tan[e +
f*x]])
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3098, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3098}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{(d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{(d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3082}$$

$$\frac{\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}} (b\csc(e+fx))^m \int \cos^{\frac{3}{2}}(e+fx)\left(\frac{\sin(e+fx)}{b}\right)^{-m-\frac{3}{2}}dx}{bd\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}} (b\csc(e+fx))^m \int \cos(e+fx)^{3/2}\left(\frac{\sin(e+fx)}{b}\right)^{-m-\frac{3}{2}}dx}{bd\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3057}$$

$$-\frac{2(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(-2m-1),\frac{1}{4}(3-2m),\sin^2(e+fx)\right)}{df(2m+1)\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}}$$

input
```
Int[(b*Csc[e + f*x])^m/(d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[-1/4, (-1 - 2*m)/4, (3 - 2*m)/4,
Sin[e + f*x]^2])/(d*f*(1 + 2*m)*(Cos[e + f*x]^2)^(1/4)*Sqrt[d*Tan[e + f*x]
])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3098
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

## Maple [F]

$$\int \frac{(b\csc (fx+e))^{m}}{(d\tan (fx+e))^{\frac{3}{2}}}dx$$

input
```
int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x)
```

output
```
int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x)
```

## Fricas [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{(b\csc (fx+e))^{m}}{(d\tan (fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m/(d^2*tan(f*x + e)^2), x)
```

## Sympy [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{(b\csc (e+fx))^{m}}{(d\tan (e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*csc(f*x+e))**m/(d*tan(f*x+e))**(3/2),x)
```

output
```
Integral((b*csc(e + f*x))**m/(d*tan(e + f*x))**(3/2), x)
```

## Maxima [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{(b\csc(fx+e))^m}{(d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m/(d*tan(f*x + e))^(3/2), x)
```

## Giac [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{(b\csc(fx+e))^m}{(d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m/(d*tan(f*x + e))^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m}{(d\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(3/2),x)
```

output
```
int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(3/2), x)
```

## Reduce [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\,b^m\left(\int \frac{\sqrt{\tan(fx+e)}\,\csc(fx+e)^m}{\tan(fx+e)^2}dx\right)}{d^2}$$

input
```
int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*b**m*int((sqrt(tan(e + f*x))*csc(e + f*x)**m)/tan(e + f*x)**2,x))
/d**2
```

# 3.387 $\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx$

Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2725
Mathematica [C] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . . . 2725
Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2726
Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2728
Fricas [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2728
Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2728
Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2729
Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2729
Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2729
Reduce [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2730

## Optimal result

Integrand size = 21, antiderivative size = 89

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{1+n}{2}}(a\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),\sin^2(e+fx)\right)(}{bf(1-m+n)}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*n)*(a*csc(f*x+e))^m*hypergeom([1/2+1/2*n, 1/2-1/2*
m+1/2*n],[3/2-1/2*m+1/2*n],sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1-m+n)
```

## Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.92 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.22

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx =$$

$$-\frac{a}{f(-1+m-n)\left((-3+m-n)\operatorname{AppellF1}\left(\frac{1}{2}(1-m+n),n,1-m,\frac{1}{2}(3-m+n),\tan^2\left(\frac{1}{2}(e+fx)\right),-\right.\right.}$$

input
```
Integrate[(a*Csc[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
-((a*(-3 + m - n)*AppellF1[(1 - m + n)/2, n, 1 - m, (3 - m + n)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(a*Csc[e + f*x])^(-1 + m)*(b*Tan[e + f*x
])^n)/(f*(-1 + m - n)*((-3 + m - n)*AppellF1[(1 - m + n)/2, n, 1 - m, (3 -
 m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((-1 + m)*AppellF1
[(3 - m + n)/2, n, 2 - m, (5 - m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + n*AppellF1[(3 - m + n)/2, 1 + n, 1 - m, (5 - m + n)/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3098, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3098}$$

$$\left(\frac{\sin(e+fx)}{a}\right)^m(a\csc(e+fx))^m\int \left(\frac{\sin(e+fx)}{a}\right)^{-m}(b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\sin(e+fx)}{a}\right)^m(a\csc(e+fx))^m\int \left(\frac{\sin(e+fx)}{a}\right)^{-m}(b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\cos^{n+1}(e+fx)(a\csc(e+fx))^{m+1}(b\tan(e+fx))^{n+1}\left(\frac{\sin(e+fx)}{a}\right)^{m-n}\int \cos^{-n}(e+fx)\left(\frac{\sin(e+fx)}{a}\right)^{n-m}dx}{ab}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{n+1}(e+fx)(a\csc(e+fx))^{m+1}(b\tan(e+fx))^{n+1}\left(\frac{\sin(e+fx)}{a}\right)^{m-n}\int\cos(e+fx)^{-n}\left(\frac{\sin(e+fx)}{a}\right)^{n-m}dx}{ab}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\cos^2(e+fx)^{\frac{n+1}{2}}(a\csc(e+fx))^m(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),\right.}{bf(-m+n+1)}$$

input
```
Int[(a*Csc[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n)/2)*(a*Csc[e + f*x])^m*Hypergeometric2F1[(1 + n)
/2, (1 - m + n)/2, (3 - m + n)/2, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(1 - m + n))
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3098
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

## Maple [F]

$$\int (a \csc (fx+e))^m (b \tan (fx+e))^n \, dx$$

input
```
int((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
int((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

## Fricas [F]

$$\int (a \csc(e+fx))^m (b \tan(e+fx))^n \, dx = \int (a \csc (fx+e))^m (b \tan (fx+e))^n \, dx$$

input
```
integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Sympy [F]

$$\int (a \csc(e+fx))^m (b \tan(e+fx))^n \, dx = \int (a \csc (e+fx))^m (b \tan (e+fx))^n \, dx$$

input
```
integrate((a*csc(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((a*csc(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

## Maxima [F]

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\csc(fx+e))^m(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Giac [F]

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\csc(fx+e))^m(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx = \int (b\tan(e+f\,x))^n\left(\frac{a}{\sin(e+f\,x)}\right)^m\,dx$$

input
```
int((b*tan(e + f*x))^n*(a/sin(e + f*x))^m,x)
```

output
```
int((b*tan(e + f*x))^n*(a/sin(e + f*x))^m, x)
```

## Reduce [F]

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx=b^na^m\left(\int \tan\left(fx+e\right)^n\csc\left(fx+e\right)^m dx\right)$$

input
```
int((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
b**n*a**m*int(tan(e + f*x)**n*csc(e + f*x)**m,x)
```

# CHAPTER 4

# Appendix

4.1 Listing of Grading functions . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2731
4.2 Links to plain text integration problems used in this report for each CAS . 2749

## 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

## Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
         return True
      else:
         return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

## SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

## 4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file